Using BFKL resummation to fit DIS data

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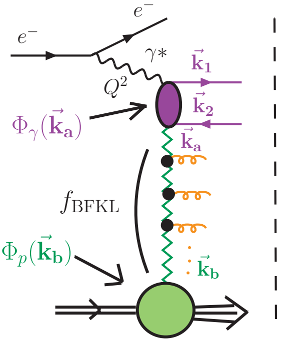
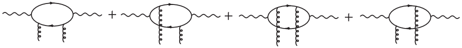
Outline

1. Kinematics and physical setup
2. Accounting for higher order corrections
   - Choice of energy scale
   - Running of the coupling and differential operator
   - Collinear improved resummation
3. Numerical analysis
4. Conclusions

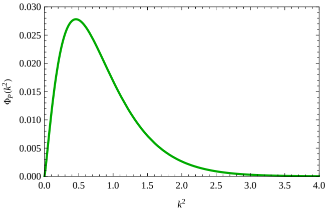
Physical setup



- Photon Impact Factor, $\Phi_\gamma(\bar{k}_a)$:



- Proton Impact Factor, $\Phi_p(\bar{k}_b)$:



Cross section:

$$F_2(x, Q^2) = \frac{F_c}{(2\pi)^4} \int \frac{d^2\mathbf{k}_a}{\mathbf{k}_a^2} \int \frac{d^2\mathbf{k}_b^2}{\mathbf{k}_b} \Phi_\gamma(\mathbf{k}_a) f(x, \mathbf{k}_a, \mathbf{k}_b)\, \Phi_p(\mathbf{k}_b)$$

Solutions to the BFKL equation

Leading Logarithmic Accuracy:

- Eigenstates: $\phi_{n,\nu}(\mathbf{q}) = \frac{1}{\pi\sqrt{2}}\left(\mathbf{q}^2\right)^{i\nu-1/2} \mathrm{e}^{in\theta}$
- Eigenvalues: $\mathcal{K}^{\mathrm{LL}} = \bar{\alpha}_s \chi_0(|n|, \nu)$

Next to Leading Logarithmic Accuracy:

- Eigenvalues: [Lipatov & Kotikov (2000), hep-ph/0004008]

$$\mathcal{K}^{NLL} = \bar{\alpha}_s \chi_0 + \bar{\alpha}_s^2 \left\{ \chi_1 + \frac{\beta_0}{8N_c}\chi_0 \left[i\,\mathcal{D}(\nu,\nu') + \log(\mu^2) + i\,\frac{\chi_0'}{\chi_0} \right] \right\} \delta(\nu-\nu')\,\delta_{n,n'}$$

- Gluon Green's function:

$$f(s,\mathbf{k},\mathbf{q}) = \frac{1}{2\pi^2}\sum_n \int_{-\infty}^{\infty} \mathrm{d}\nu \int_{\delta-i\infty}^{\delta+i\infty} \frac{\mathrm{d}\omega}{2\pi i} \frac{\mathrm{e}^{in(\theta_{\mathrm{q}}-\theta_{\mathrm{k}})}}{\omega - \mathcal{K}^{NLL}(\bar{\alpha}_s, 1/2+i\nu)} \frac{1}{\mathbf{q}^2}\left(\frac{\mathbf{q}^2}{\mathbf{k}^2}\right)^{1/2+i\nu}\left(\frac{s}{s_0}\right)^{\omega}$$

Choice of energy scale

- DIS-like choice:

$$s_0 = Q^2 \Rightarrow \left(\frac{s}{s_0}\right)^{\omega} = \left(\frac{1}{x}\right)^{\omega}$$

- Product of the internal sacales: $s_0 = \sqrt{\mathbf{q}^2\mathbf{k}^2}$

$$f(x,\mathbf{k},\mathbf{q}) \propto \int \frac{d\gamma}{2\pi i} \int \frac{d\omega}{2\pi i} \frac{1}{\omega - \mathcal{K}(\bar{\alpha}_s,\gamma)} \frac{1}{\mathbf{q}^2} \left(\frac{\mathbf{q}^2}{\mathbf{k}^2}\right)^{\gamma} \left(\frac{Q^2/x}{\sqrt{\mathbf{q}^2\mathbf{k}^2}}\right)^{\omega}$$

$$= \int \frac{d\gamma}{2\pi i} \int \frac{d\omega}{2\pi i} \frac{1}{\omega - \mathcal{K}(\bar{\alpha}_s,\gamma)} \frac{1}{\mathbf{q}^2} \left(\mathbf{q}^2\right)^{\gamma+\omega/2} \left(\mathbf{k}^2\right)^{-(\gamma-\omega/2)} \left(\frac{Q^2}{x}\right)^{\omega}$$

Shift in ω for the impact factors:

$$\left(\mathbf{q}^2\right)^{\gamma+\omega/2} \Rightarrow \Phi_\gamma(\gamma) \rightarrow \Phi_\gamma(\gamma - \omega/2)$$

$$\left(\mathbf{k}^2\right)^{\gamma-\omega/2} \Rightarrow \Phi_P(\gamma) \rightarrow \Phi_P(\gamma + \omega/2)$$

NLL: Running coupling effects

- BFKL equation and gluon Green's function:

$$\langle \hat{f}_\omega \rangle = \frac{1}{\omega - \langle \hat{\mathcal{K}} \rangle} \quad \Rightarrow \quad F_2 \propto \int \frac{d\omega}{2\pi i} d\nu \, \phi_{\gamma*}(\nu) \left[\frac{1}{\omega} \sum_{j=0}^{\infty} \frac{\mathcal{K}^j}{\omega^j} \right] \phi_p(\nu) e^{\omega Y}$$

- Running inside the integral: $\bar{\alpha}_s(\mathbf{k}^2) = \bar{\alpha}_s(\mu^2) - \bar{\alpha}_s^2(\mu^2) \frac{\beta_0}{4N_c} \log\left(\frac{\mathbf{k}^2}{\mu^2}\right)$

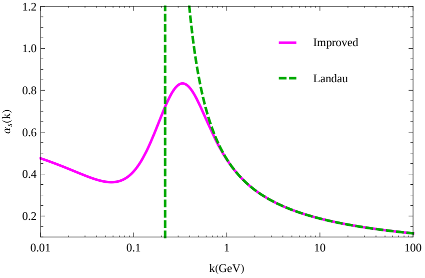
$$\langle \hat{\mathcal{K}} \rangle \equiv \langle n, \nu | \hat{\mathcal{K}} | \nu', n' \rangle \propto \int d^2\mathbf{k} \, \bar{\alpha}_s(\mathbf{k}^2) \times e^{i(\nu-\nu')\log(\mathbf{k}^2)} \, G(\mathbf{k}, n, n', \nu, \nu')$$

- Def. Differential operator: $\hat{\mathcal{D}}(\nu, \nu')$
  1. Symmetric choice: $\hat{\mathcal{D}}_1 = \frac{1}{2}(\partial_\nu - \partial_{\nu'}) \quad \Longrightarrow \quad \bar{\alpha}_s(Q * Q_0)$
  2. Acting on the photon: $\hat{\mathcal{D}}_2 = \partial_\nu \quad \Longrightarrow \quad \bar{\alpha}_s(Q^2)$
  3. Acting on the proton: $\hat{\mathcal{D}}_2 = -\partial'_\nu \quad \Longrightarrow \quad \bar{\alpha}_s(Q_0^2)$

A model for the running

Running coupling analytical in the infrared and compatible with power corrections to jet observables:

$$\bar{\alpha}_s\left(\mathbf{k}^2\right) = \frac{4N_c}{\beta_0}\left(\frac{1}{\ln\frac{\mathbf{k}^2}{\Lambda^2_{QCD}}} + 125\frac{\left(\Lambda^2_{QCD}+4\mathbf{k}^2\right)}{\left(\Lambda^2_{QCD}-\mathbf{k}^2\right)\left(4+\frac{\mathbf{k}^2}{\Lambda^2_{QCD}}\right)^4}\right)$$



[B. Webber (1998) hep-ph/9805484]

Collinear improved resummation

In double-Mellin space:

$$f \propto \int \frac{d\omega\, d\gamma}{\omega - \chi_0(\gamma)} \left(\frac{s}{k_a k_b}\right)^{\omega} \left(\frac{\mathbf{k}_a^2}{\mathbf{k}_b^2}\right)^{\gamma - 1/2} = \int \frac{d\omega\, d\gamma}{\omega - \chi_0(\gamma - \omega/2)} \left(\frac{s}{k_a^2}\right)^{\omega} \left(\frac{\mathbf{k}_a^2}{\mathbf{k}_b^2}\right)^{\gamma - 1/2}$$

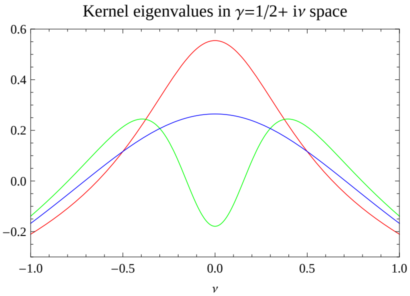
- This leads to big double logs (k-space) or poles (γ-space) when $\gamma \to 0, 1$ (collinear limit)
- Suggestion:

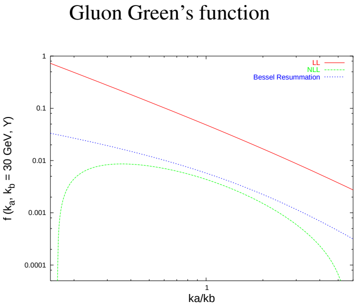
$$\chi_0(\gamma) \to \chi_0(\gamma + \omega/2)$$

  [Salam (1998),hep-ph/9806482], [Sabio Vera (2005), hep-ph/0505128]

- This ω-shift resums the double logs to all-orders (cancels the γ and $1-\gamma$ poles of the NLL kernel)
- It is consistent with the NLL BFKL solution

Collinear improved resummation



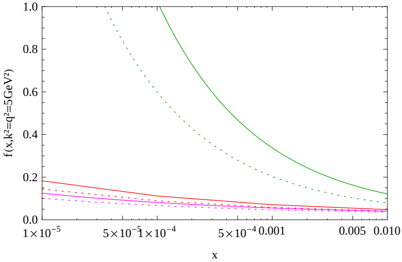


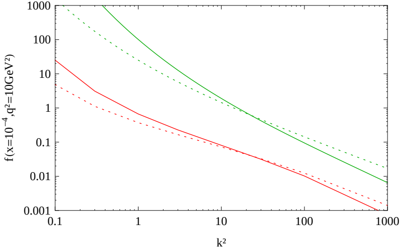
Numerical analysis

Dependence on the energy scale s_0

LL, NLL, collinear improved

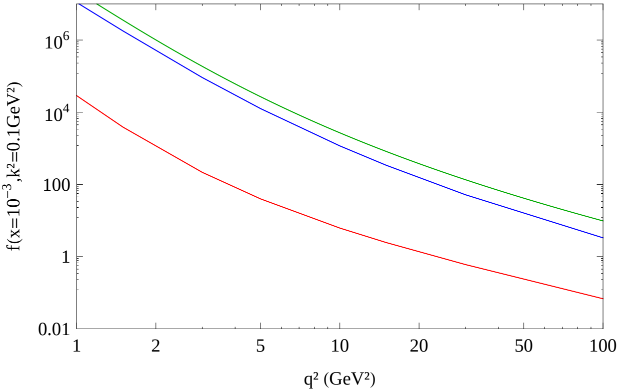
Solid lines $\Rightarrow s_0 = Q^2$, dotted lines $\Rightarrow s_0 = kq$





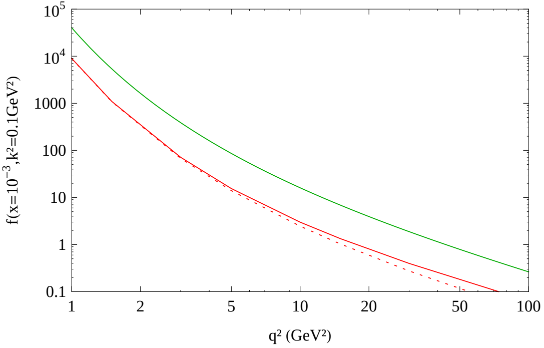
Plots for the gluon Green's function: action of the differential operator

- LO
- symmetric collinear improved
- asymmetric collinear improved (acting on the photon)
- IR finite running



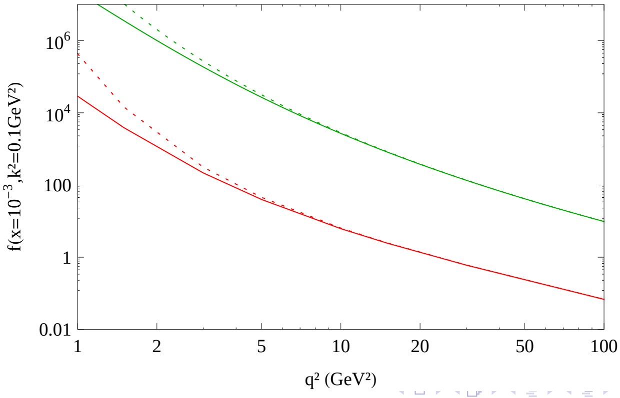
Plots for the gluon Green's function: differential operator

- Diff. operator acting on the photon
- LO, collinear improved
- Solid line ⇒ without exponentiating the non scale invariant part
- dotted line ⇒ exponentiating everything



Plots for the gluon Green's function: dependence on the model for the running

- Symmetric configuration.
- LO, collinear improved.
- Solid line ⇒ IR finite running , dotted line ⇒ perturbative $\bar{\alpha}_s$



A fit (work in progress)

Photon impact factor:

[Bialas, Navelet, Peschanski /0101179]

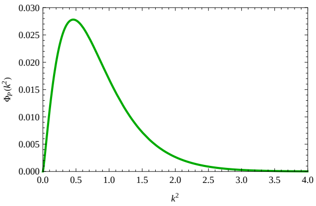
Proton impact factor:

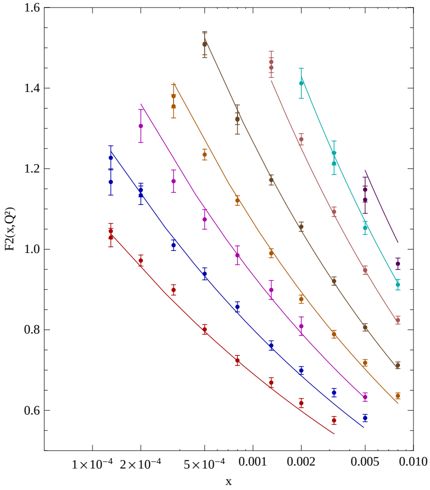
$$\phi_p(\mathbf{k}^2) = N\left(\frac{\mathbf{k}^2}{Q_0^2}\right)^{\delta} e^{-\mathbf{k}^2/Q_0^2}$$

Best fit: $\delta = 1.246$,

$Q_0^2 = 0.368\ \mathrm{GeV}^2$

$N = 0.0735$





Comments and conclusions

- Theoretical uncertainties:
  - choice of differential operator, exponentiation or not? what comes into the running?
  - choice of energy scale: symmetric one? DIS-like?
  - model for the running coupling
- The collinear improved resummation is needed to fit the data
- Other possible implementations:
  - Include quark masses

    [White, Peschanski, Thorne, hep-ph/0606169v1]
  - Include saturation effects to improve behavior at small Q^2