NLO in BKP: Odderon and Integrability

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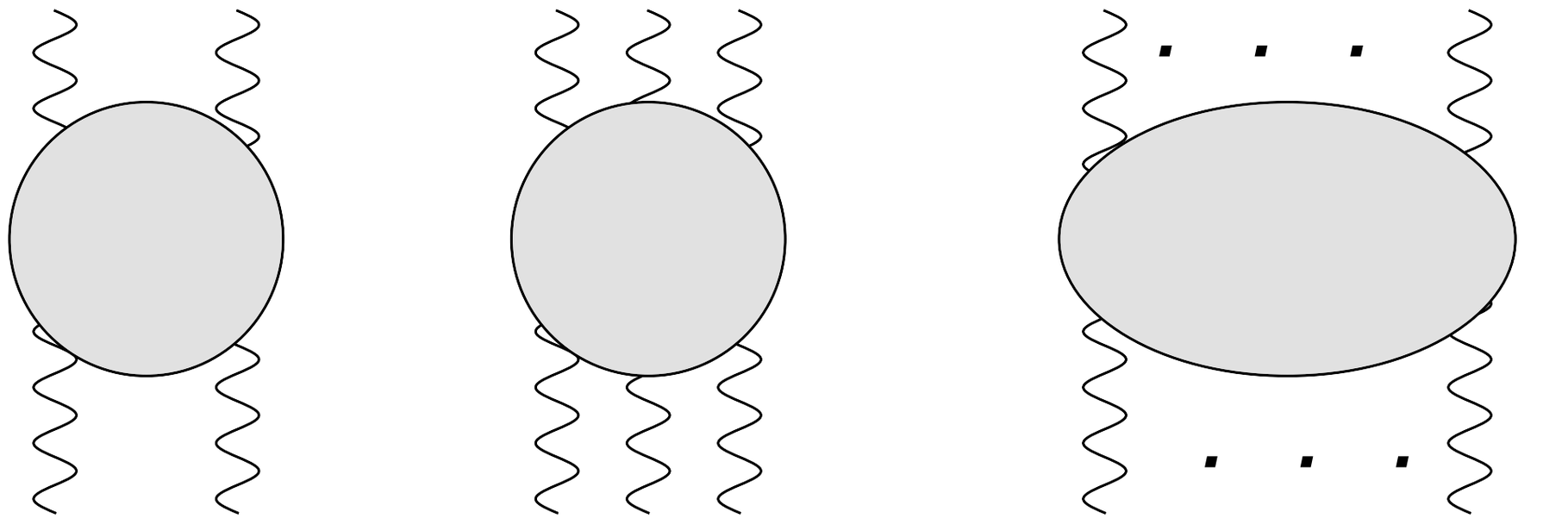
Based upon common work with:
V.S Fadin, L.N.Lipatov, G.-P.Vacca arXiv:1205.2530 [hep-th]

- Introduction: motivation
- BFKL revisited
- Ward identities for reggeized gluons
- BKP kernel

Introduction

Motivations for going beyond BFKL=2 gluon state:

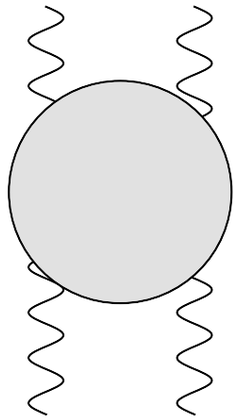


BFKL Pomeron *Odderon* *n gluon bound state*

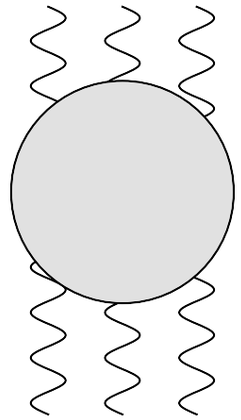
a) Odderon in pQCD
b) Integrability, AdS/CFT correspondence

Odderon:

SU(3) (QCD) has 2 Casimir operators:



BFKL Pomeron



Odderon

$$\omega_{BFKL} = \frac{4N_c \ln 2\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \qquad \omega_{Odderon} = 0 + ?$$

SU(n) has n Casimir operators:

$$\omega_2 = \frac{4N_c ln 2\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \qquad \omega_3 = 0 \qquad \omega_4 = ?$$

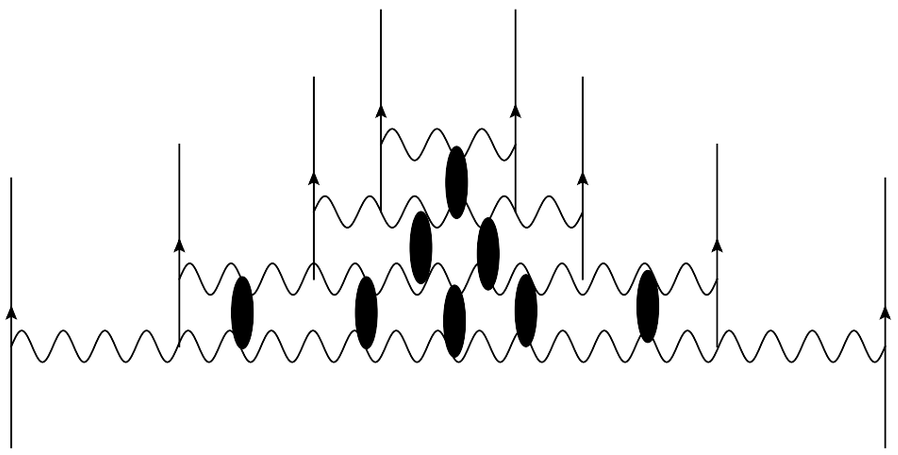
There is something to be understood

N=4 SYM may be soluble,
Integrability (anomalous dimensions)

AdS/CFT correspondence:
bridge from small to large coupling
Scattering amplitudes in multi-Regge kinematics

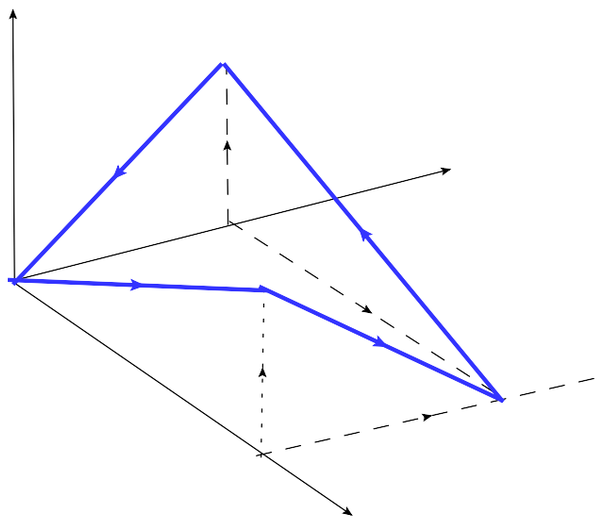
$\lambda \to 0$

leading log calculations,
integrability of BKP Hamiltonian:
needs NLO



$\lambda \to \infty$

Minimal surfaces of polygons



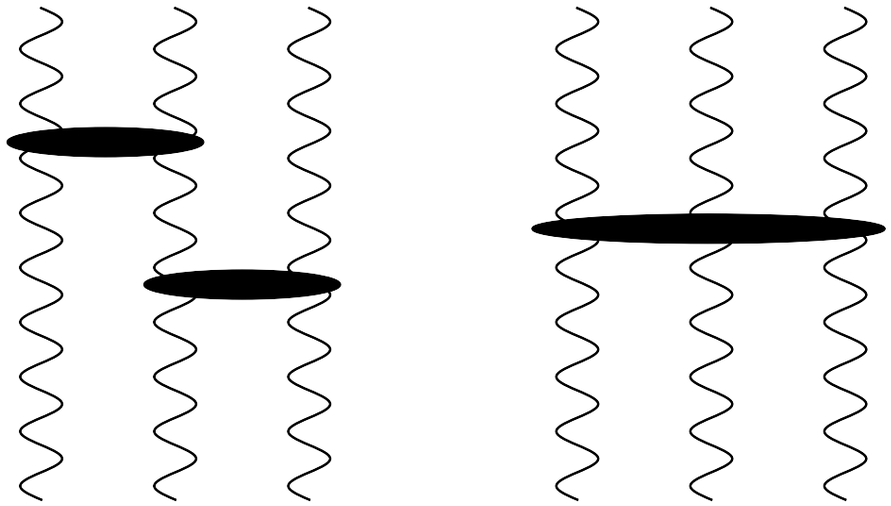
Y-equations in multiregge kinematics

JB,Kormilitzin,Kotanski,
Schomerus,Sprenger

Connect the integrable structures

NLO for BKP states needs

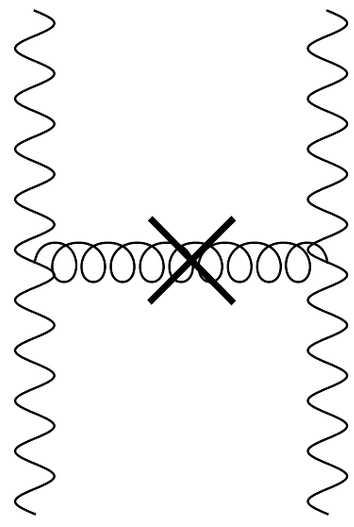
- 2 to 2 BFKL kernel in color octet state (large N_c)
- 3 to 3 kernel



2 to 2

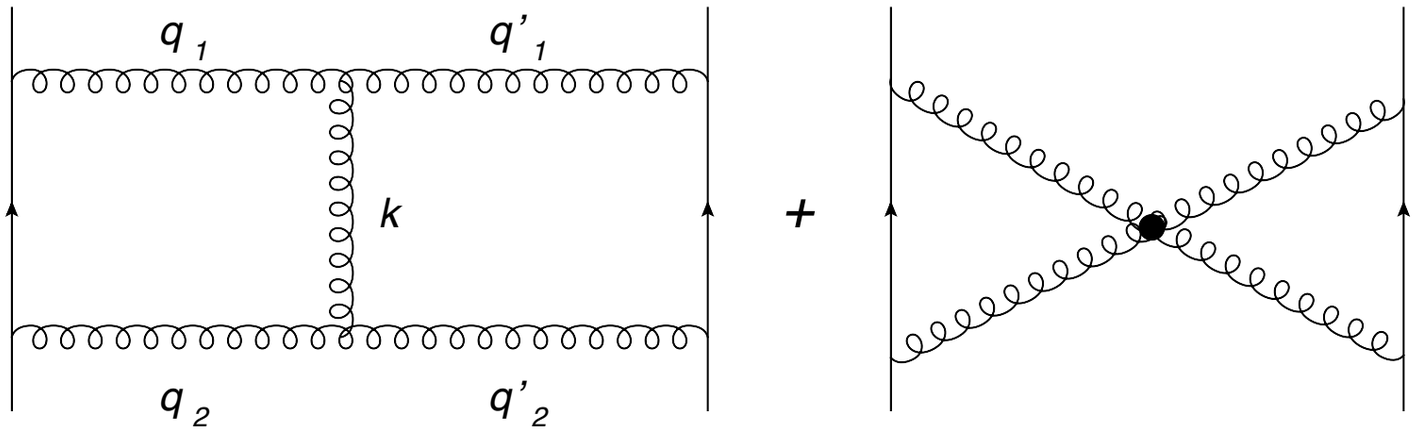
3 to 3

New kernel in QCD evolution equations: cannot use unitarity, needs longitudinal integration and Bose symmetry.



BFKL, revisited

BFKL derivation without unitarity:



+ crossed graphs

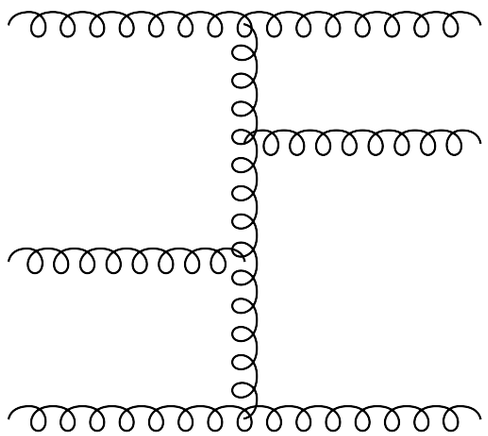
$$\frac{q^2}{k^+k^-} + \frac{-k^2q^2 + q_1^2{q_2'}^2 + {q_1'}^2q_2^2}{(k^+k^- + k^2)k^+k^-}$$

↑

needs crossed graph
to cancel the divergence

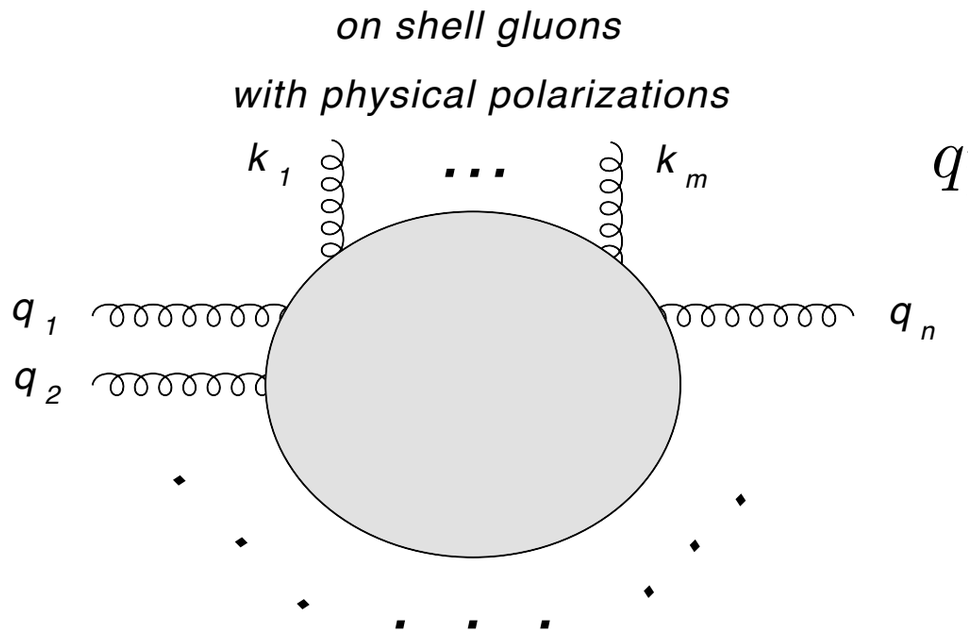
How to avoid summing over all permutations?

Idea: Ward identities for reggeized gluons,
makes life much simpler



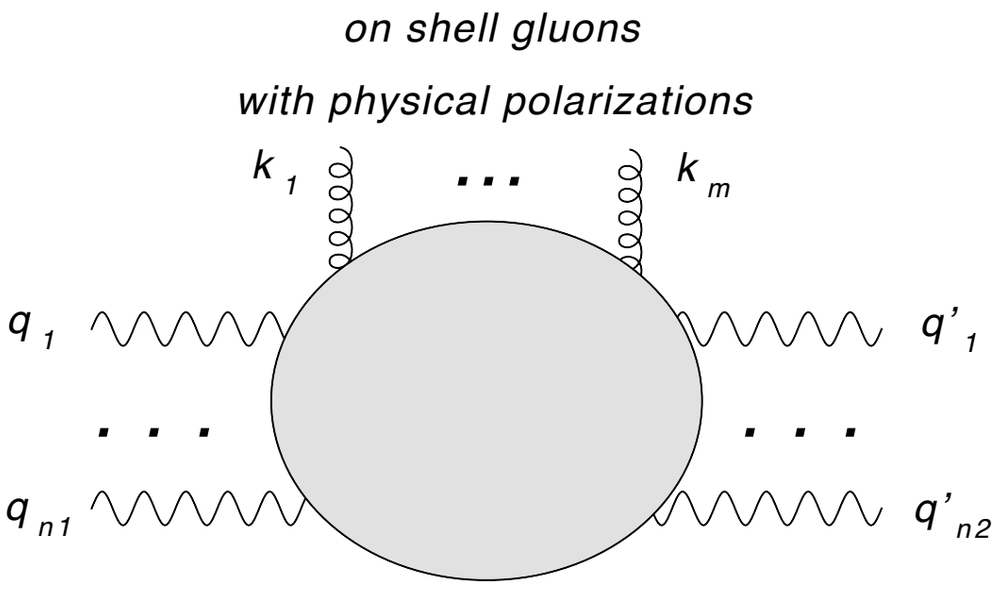
Ward identities for reggeized gluons

(Elementary) gluons:



$$q_1^{\mu_1} \dots q_n^{\mu_n} M_{\mu_1 \dots \mu_n}(q_1, \dots, q_n; k_1, \dots, k_m) = 0$$

Reggeized gluons:
Lipatov's effective gauge invariant action

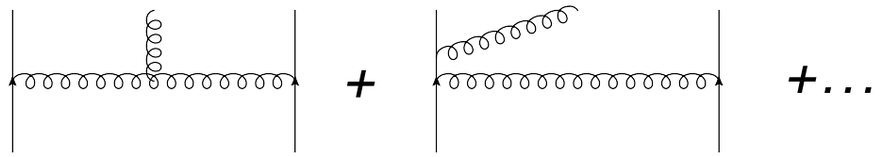


$$(n^-)^{\mu_1} \dots (n^-)^{\mu_{n_1}} M_{\mu_1 \dots \mu_{n_1} \nu_1 \dots \nu_{n_2}} (n^+)^{\nu_1} \dots (n^+)^{\nu_{n_2}} + \dots$$

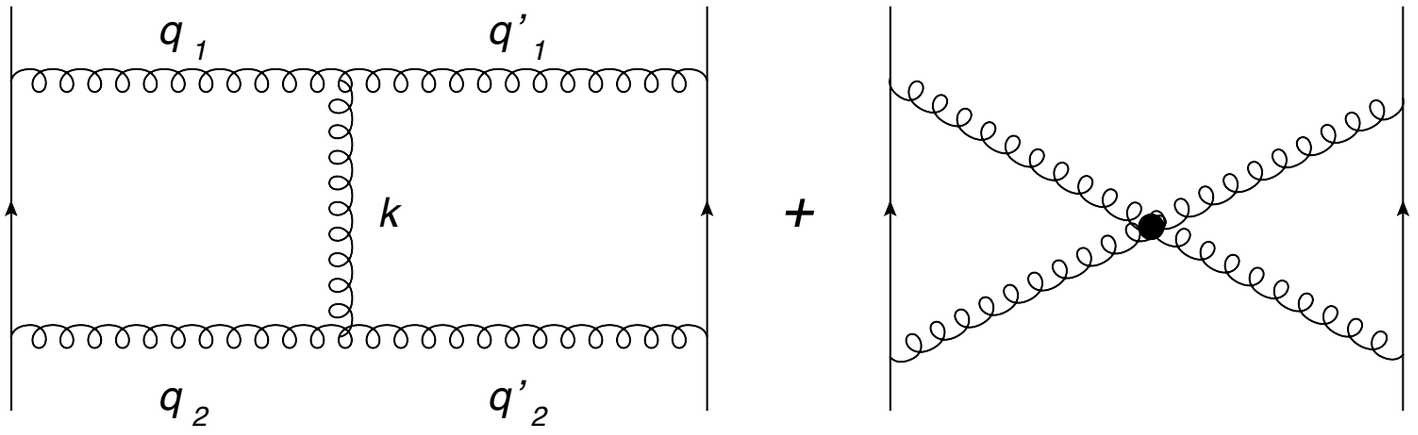
$$q^\mu = \frac{(n^-)^\mu}{2} q^+ + q_\perp^\mu$$

$$(n^-)^\mu \to \frac{-2}{q^+} q_\perp^\mu$$

$$\frac{(q_{1\perp})^{\mu_1}}{q_1^+} \dots \frac{(q_{n_1\perp})^{\mu_{n_1}}}{q_{n_1}^+} M_{\mu_1 \dots \mu_{n_1} \nu_1 \dots \nu_{n_2}} \frac{(q'_{1\perp})^{\nu_1}}{q'_1{}^-} \dots \frac{(q'_{n_2\perp})^{\nu_{n_2}}}{q'_{n_2}{}^-} + \dots$$



Apply to BFKL:
Use of Ward identities eliminates all induced terms
each diagram is convergent and has the required zero-properties.



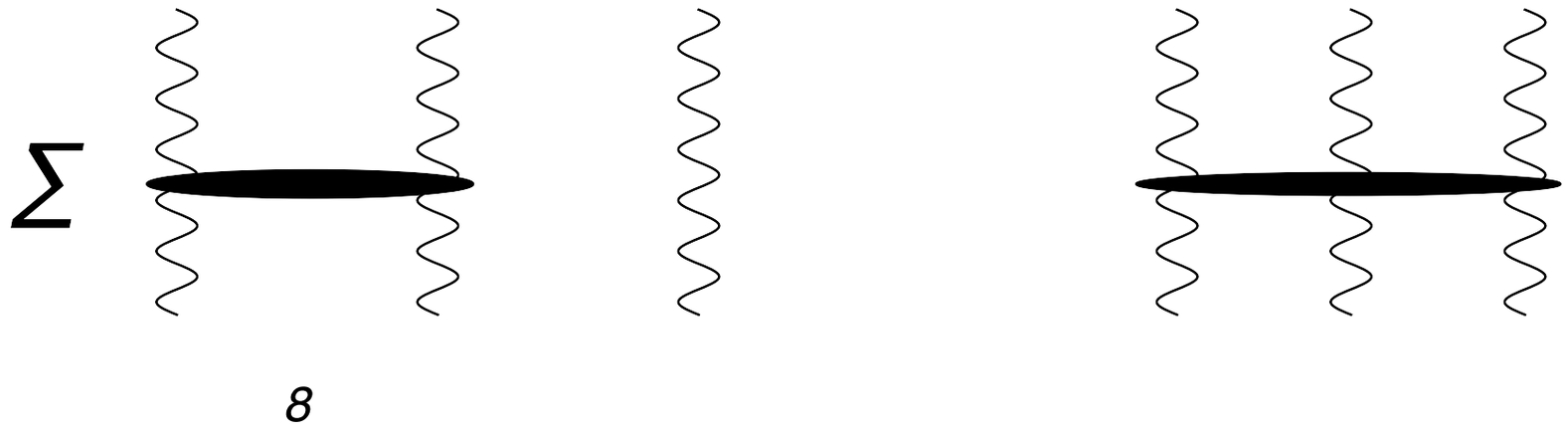
$$\frac{\cancel{q^2}}{\cancel{k^+k^-}} + \frac{-k^2q^2 + q_1^2{q_2'}^2 + {q_1'}^2q_2^2}{(k^+k^- + k^2)k^+k^-}$$

$$= \frac{8}{k^+k^-}\frac{q_1q_2^*{q_1'}^*q_2' + cc.}{k^+k^- + k^2}$$

+ crossed graphs

Conclusion to be drawn: after the use of Ward identities
'physical polarization' $\sim (k_\perp)^\mu$ sometimes more convenient than
"unphysical polarization' $\sim (n^\pm)^\mu$

Apply to NLO BKP (3 to 3) kernel:



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The 2 to 2 kernel: color octet BFKL

$$\hat{\mathcal{K}}^{(8)} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_{\cdot r}^{(8)}$$

$$K_r^{(8)}(\vec{q}_1,\vec{q}_1',\vec{q}) = \frac{\bar{g}_\mu^2 \mu^{-2\epsilon}}{2\pi^{1+\epsilon}\Gamma(1-\epsilon)} \left\{ \left(\frac{\vec{q}_1^{\,2}\vec{q}_2'^{\,2} + \vec{q}_1'^{\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2} \right) \right.$$

$$\times \left(1 + \bar{g}_\mu^2 \left[\frac{\beta_0}{N_c\epsilon} + \left(\frac{\vec{k}^{\,2}}{\mu^2} \right)^{\epsilon} \left(-\frac{\beta_0}{N_c\epsilon} + \frac{67}{9} - 2\zeta(2) - \frac{10}{9}\frac{a_f}{N_c} - \frac{4}{9}\frac{a_S}{N_c} \right.\right.\right.$$

$$\left.\left.\left. + \epsilon \left(-\frac{404}{27} + 14\zeta(3) + \frac{\beta_0}{N_c}\zeta(2) + \frac{56}{27}\frac{a_f}{N_c} + \frac{26}{27}\frac{a_S}{N_c} \right) \right) \right] \right)$$

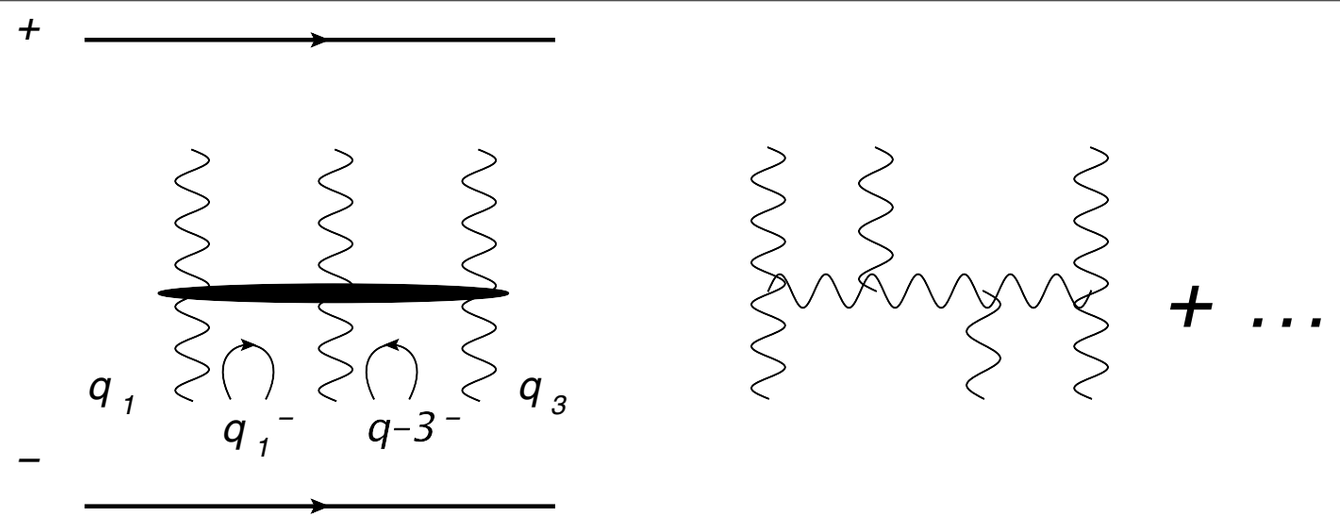
$$+\bar{g}_\mu^2 \left[\vec{q}^{\,2} \left(\frac{\beta_0}{N_c} \ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_1'^{\,2}}{\vec{q}^{\,2}\vec{k}^{\,2}} \right) + \frac{1}{2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}} \right) \ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}^{\,2}} \right) + \frac{1}{2}\ln\left(\frac{\vec{q}_1'^{\,2}}{\vec{q}^{\,2}} \right) \ln\left(\frac{\vec{q}_2'^{2}}{\vec{q}^{\,2}} \right) \right.\right.$$

$$\left. + \frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_1'^{\,2}} \right) \right) - \frac{\vec{q}_1^{\,2}\vec{q}_2'^{\,2} + \vec{q}_2^{\,2}\vec{q}_1'^{\,2}}{\vec{k}^{\,2}} \ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_1'^{\,2}} \right) + \frac{\vec{q}_1^{\,2}\vec{q}_2'^{\,2} - \vec{q}_2^{\,2}\vec{q}_1'^{\,2}}{\vec{k}^{\,2}} \ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_1'^{\,2}} \right)$$

$$\times \left(\frac{\beta_0}{N_c} - \frac{1}{2}\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_1'^{\,2}}{\vec{k}^{\,4}} \right) \right) + [\vec{q}^{\,2}(\vec{k}^{\,2} - \vec{q}_1^{\,2} - \vec{q}_1'^{\,2}) + 2\vec{q}_1^{\,2}\vec{q}_1'^{\,2} - \vec{q}_1^{\,2}\vec{q}_2'^{\,2} - \vec{q}_2^{\,2}\vec{q}_1'^{\,2}$$

$$\left. \left. + \frac{\vec{q}_1^{\,2}\vec{q}_2'^{\,2} - \vec{q}_2^{\,2}\vec{q}_1'^{\,2}}{\vec{k}^{\,2}} (\vec{q}_1^{\,2} - \vec{q}_1'^{\,2})] I(\vec{q}_1^{\,2}, \vec{q}_1'^{\,2}, \vec{k}^{\,2}) \right] \right\} + (\vec{q}_1 \leftrightarrow \vec{q}_2, \;\; \vec{q}_1' \leftrightarrow \vec{q}_2') \;\; ,$$

The 3 to 3 part:



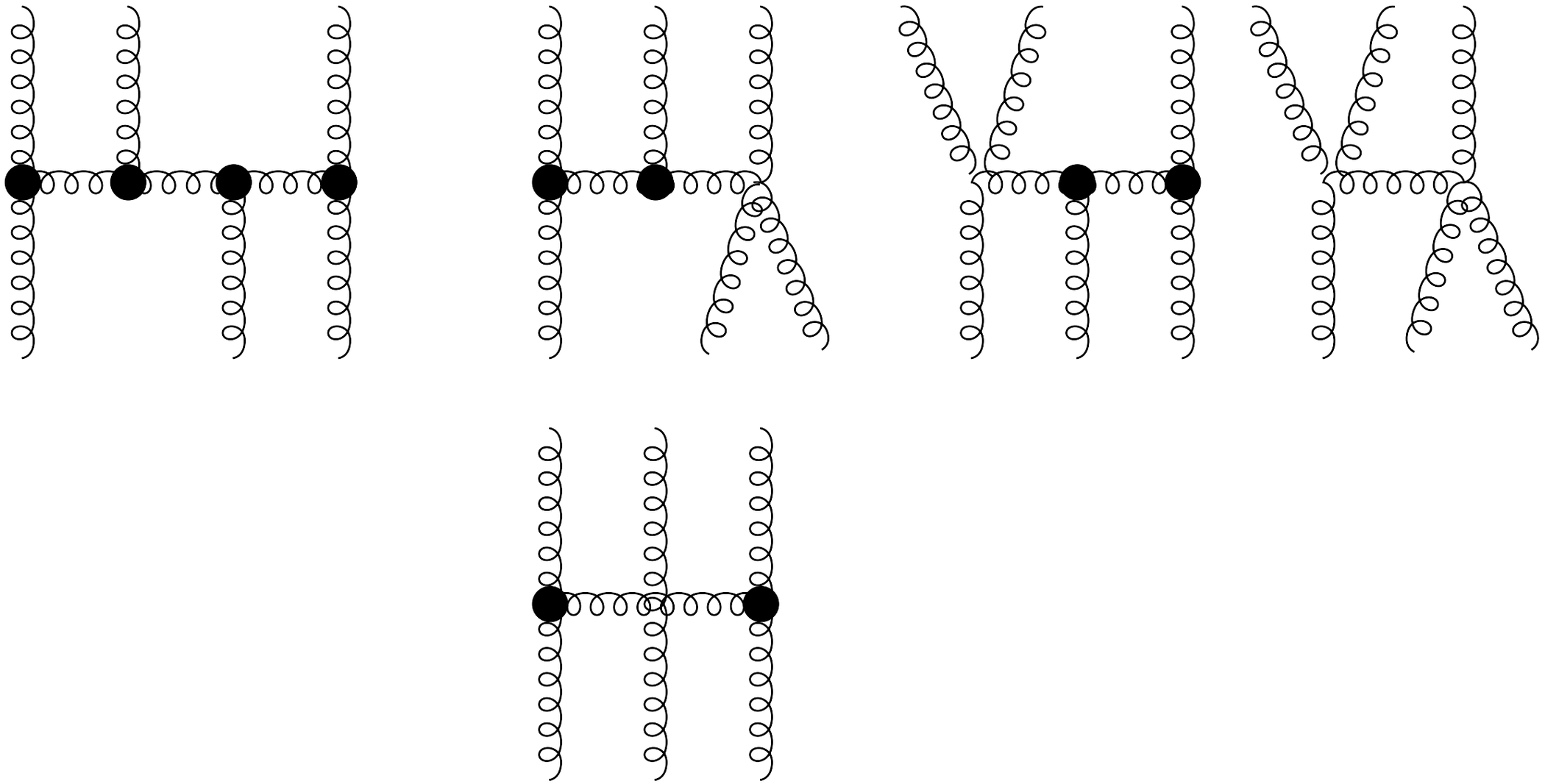
Tasks:
all s-channel lines are off-shell: longitudinal integrations
sum over 6 permutations

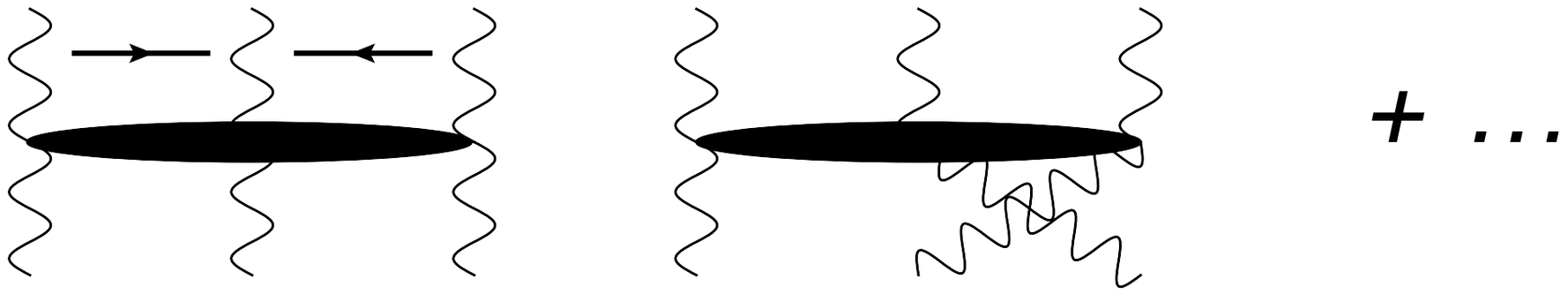
Use of Ward identities for all reggeons:

$$n^+ \to \frac{2q_\perp}{q^-}, \;\; n^- \to \frac{2q_\perp}{q^+}$$

Need to handle the singularities at $q^+ = 0, \;\; q^- = 0$
(similar to induced terms)

Diagrams:





Results:

- induced terms (almost) disappeared, back to QCD diagrams:
  with this polarization, reggeized gluon $\approx$ physical gluon
- individual diagrams are convergent
- diagrams have zero properties
- contribution only from 'opposite momenta'
- subtraction of LO contributions

Final result:
at present a sum of finite terms, not yet ready

Conclusions

NLO BKP is interesting,
phenomenology + theory

Done:

- 2 to 2 kernel in NLO
- new techniques based upon Ward identities
- 3 to 3 kernel (up to final form)

To be done:

- calculate Odderon intercept
- prove conformal invariance, integrability