Mueller Navelet jets at LHC: The first complete NLL BFKL study

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in collaboration with
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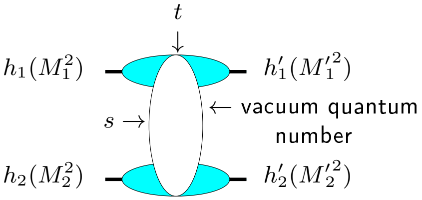
D. Colferai; F. Schwennsen, L. Szymanowski, S. Wallon
JHEP 1012:026 (2010) 1-72 [arXiv:1002.1365 [hep-ph]]

- One of the important longstanding theoretical questions raised by QCD is its behaviour in the perturbative Regge limit $s \gg -t$
- Based on theoretical grounds, one should identify and test suitable observables in order to test this peculiar dynamics



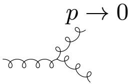
hard scales: $M_1^2,\, M_2^2 \gg \Lambda^2_{QCD}$ or $M'^2_1,\, M'^2_2 \gg \Lambda^2_{QCD}$ or $t \gg \Lambda^2_{QCD}$
where the $t-$channel exchanged state is the so-called hard Pomeron

How to test QCD in the perturbative Regge limit?

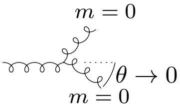
What kind of observables?

- perturbation theory should be applicable:
  selecting external or internal probes with transverse sizes $\ll 1/\Lambda_{QCD}$ or by choosing large t in order to provide the hard scale.

- governed by the *"soft"* perturbative dynamics of QCD



and *not* by its *collinear* dynamics



$\Rightarrow$ select semi-hard processes with $s \gg p_{T\,i}^2 \gg \Lambda_{QCD}^2$ where $p_{T\,i}^2$ are typical transverse scale, all of the same order.

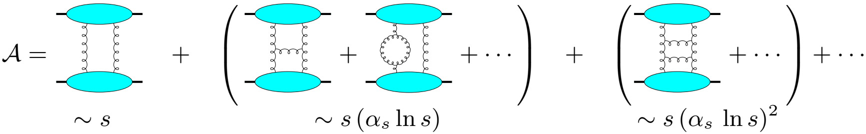
Some examples of processes

- inclusive: DIS (HERA), diffractive DIS, total $\gamma^*\gamma^*$ cross-section (LEP, ILC)
- semi-inclusive: forward jet and π^0 production in DIS, Mueller-Navelet double jets, diffractive double jets, high p_T central jet, in hadron-hadron colliders (Tevatron, LHC)
- exclusive: exclusive meson production in DIS, double diffractive meson production at e^+e^- colliders (ILC), ultraperipheral events at LHC ($\mathbb{P}$omeron, $\mathbb{O}$dderon)

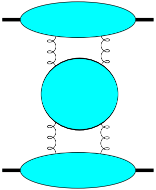
The specific case of QCD at large s

QCD in the perturbative Regge limit

- Small values of α_S (perturbation theory applies due to hard scales) can be compensated by large $\ln s$ enhancements.
  $\Rightarrow$ resummation of $\sum_n (\alpha_S \ln s)^n$ series (Balitski, Fadin, Kuraev, Lipatov)
  $\rightarrow$ introduction of a new arbitrary scale s_0 : $\ln s \rightarrow \ln \frac{s}{s_0}$



- this can be put in the following form :



$\leftarrow$ Impact factor

$\leftarrow$ Green's function

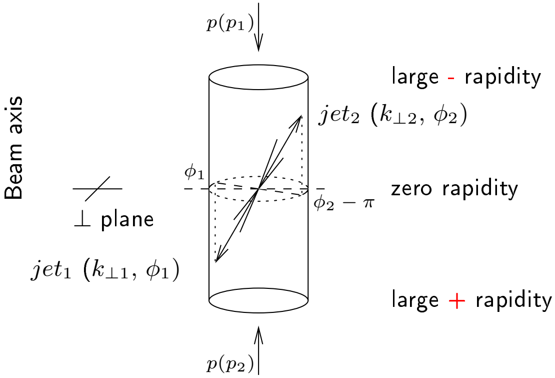
$\leftarrow$ Impact factor

- Higher order corrections to BFKL kernel are known at NLL order (Lipatov Fadin; Camici, Ciafaloni), now for arbitrary impact parameter
  $\alpha_S \sum_n (\alpha_S \ln s)^n$ resummation

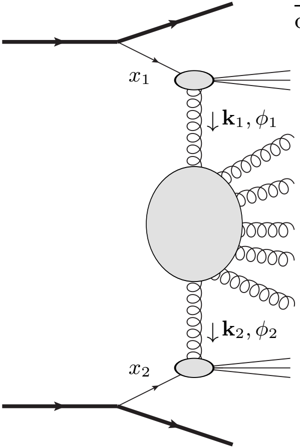
- impact factors are known in some cases at NLL
  - $\gamma^* \to \gamma^*$ at $t = 0$ (Bartels, Colferai, Gieseke, Kyrieleis, Qiao; Balitski, Chirilli)
  - forward jet production (Bartels, Colferai, Vacca)
  - inclusive production of a pair of hadrons separated by a large interval of rapidity (Ivanov, Papa)
  - $\gamma^*_L \to \rho_L$ in the forward limit (Ivanov, Kotsky, Papa)

Mueller-Navelet jets

- Consider two jets (hadrons flying within a narrow cone) separated by a large rapidity, i.e. each of them almost fly in the direction of the hadron "close" to it, and with very similar transverse momenta
- in a pure LO collinear treatment, these two jets should be emitted back to back at leading order: $\Delta\phi - \pi = 0$ ($\Delta\phi = \phi_1 - \phi_2$ = relative azimuthal angle) and $k_{\perp 1}$=$k_{\perp 2}$. There is no phase space for (untagged) emission between them



k_T-factorized differential cross-section



$$\frac{\mathrm{d}\sigma}{\mathrm{d}|\mathbf{k}_{J1}|\,\mathrm{d}|\mathbf{k}_{J2}|\,\mathrm{d}y_{J1}\,\mathrm{d}y_{J2}} = \int \mathrm{d}\phi_{J1}\,\mathrm{d}\phi_{J2} \int \mathrm{d}^2\mathbf{k}_1\,\mathrm{d}^2\mathbf{k}_2$$

$$\times\, \Phi(\mathbf{k}_{J1}, x_{J1}, -\mathbf{k}_1)$$

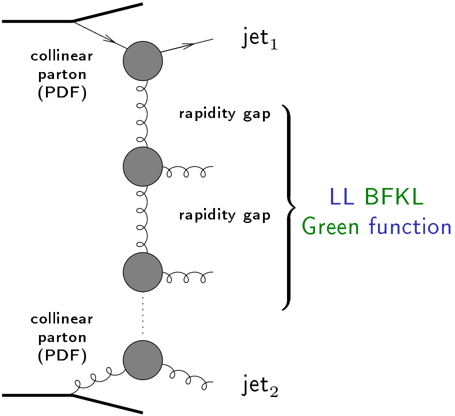
$$\times\, G(\mathbf{k}_1, \mathbf{k}_2, \hat{s})$$

$$\times\, \Phi(\mathbf{k}_{J2}, x_{J2}, \mathbf{k}_2)$$

with $\Phi(\mathbf{k}_{J2}, x_{J2}, \mathbf{k}_2) = \int \mathrm{d}x_2\, f(x_2) V(\mathbf{k}_2, x_2)$ $\quad f \equiv$ PDF $\quad x_J = \frac{|\mathbf{k}_J|}{\sqrt{s}} e^{y_J}$

Mueller Navelet jets at LL BFKL

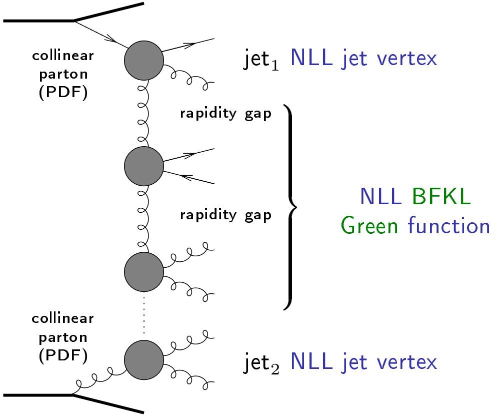
- in LL BFKL ($\sim \sum(\alpha_s \ln s)^n$), emission between these jets $\longrightarrow$ strong decorrelation between the relative azimuthal angle jets, incompatible with $p\bar{p}$ Tevatron collider data
- a collinear treatment at next-to-leading order (NLO) can describe the data
- important issue: non-conservation of energy-momentum along the BFKL ladder. A LL BFKL-based Monte Carlo combined with e-m conservation improves dramatically the situation (Orr and Stirling)



Multi-Regge kinematics (LL BFKL)

Mueller Navelet jets at NLL BFKL

- up to now, the subseries $\alpha_s \sum (\alpha_s \ln s)^n$ NLL was included only in the exchanged $\mathbb{P}$omeron state, and not inside the jet vertices Sabio Vera, Schwennsen Marquet, Royon
- the common belief was that these corrections should not be important



Quasi Multi-Regge kinematics (here for NLL BFKL)

Numerical implementation

Because of the structure of the NLL jet vertex, numerical implementation is quite delicate (requires special grouping of the terms, etc.)

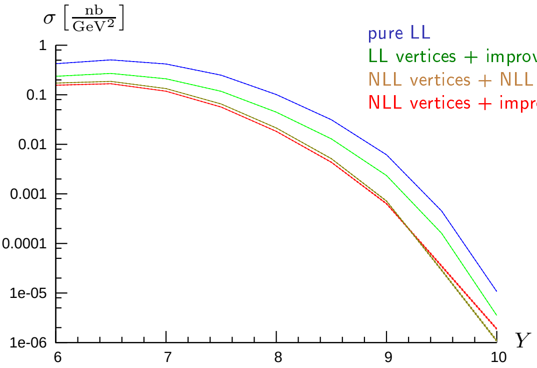
- First study done with a Mathematica code
  D. Colferai; F. Schwennsen, L. Szymanowski, S. Wallon
  JHEP 1012:026 (2010) 1-72
  rather slow $\Rightarrow$ access to a small number of configurations
- New Fortran code
  - much faster
  - Check of the Mathematica based results
  - Allows for k_J integration over a finite range and study of the $\Delta\phi$ distribution
  - Stability studies (PDFs, etc.) made easier
  - A comparison with the recent small R study of D. Yu. Ivanov et al. has been performed

In practice

Following results are with:

- $\sqrt{s} = 7$ TeV
- jet cone-algorithm with $R = 0.5$
- MSTW 2008 PDFs
- $\mu_R = \mu_F = \mu$ (imposed by the PDFs)
- μ and s_0 set equal to $\sqrt{k_{J1} k_{J2}}$
- two-loop running coupling $\alpha_s(\mu^2)$ with $\alpha_s(M_Z^2) = 0.1176$

Cross-section



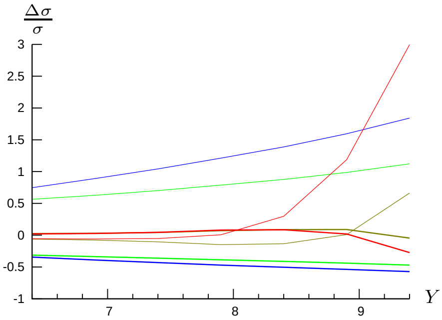
Differential cross section in dependence on Y for $|\mathbf{k}_{J1}| = |\mathbf{k}_{J2}| = 35\,\mathrm{GeV}$.

The effect of NLL vertex correction is very sizeable, comparable with NLL Green's function effects

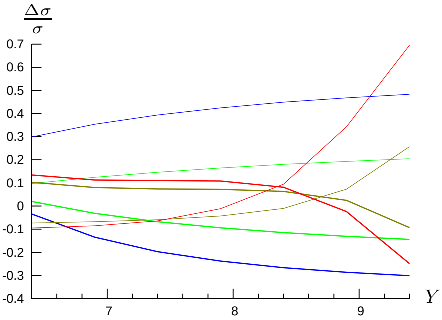
Energy-momentum conservation not satisfied by BFKL-like approaches $\Rightarrow$ validity restricted to $Y_{J,i} \ll \cosh^{-1}\frac{x_i\,E}{k_{J,i}}$, thus $Y = Y_1 + Y_2 \ll 8.4$ for $x \sim 1/3$

Cross-section: stability with respect to $\mu_R = \mu_F$ and s_0 changes

pure LL
LL vertices + improved collinear NLL Green's function
NLL vertices + NLL Green's function
NLL vertices + improved collinear NLL Green's function



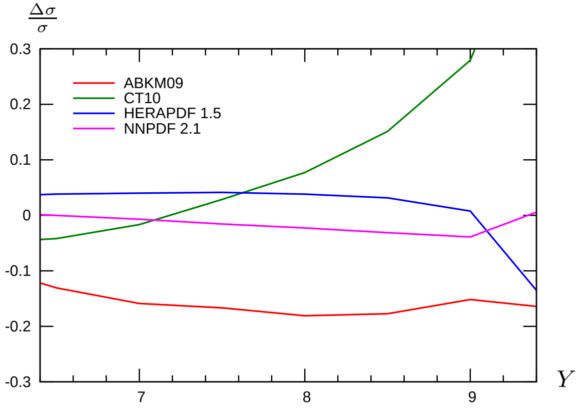
Relative effect of changing $\mu_R = \mu_F$ by factors 2 (thick) and $1/2$ (thin)



Relative effect of changing $\sqrt{s_0}$ by factors 2 (thick) and $1/2$ (thin)

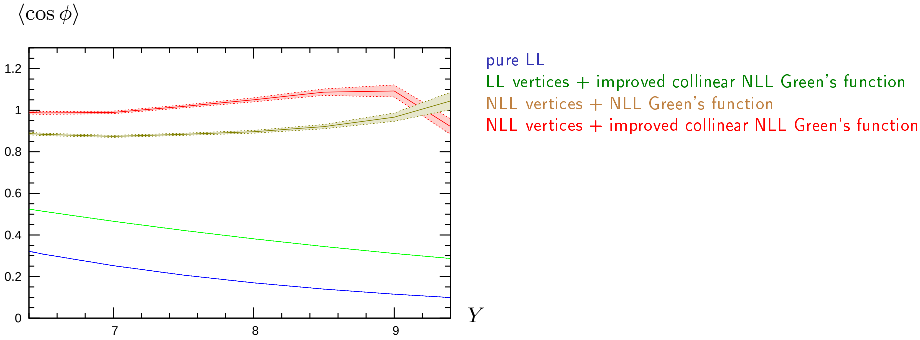
Cross-section: PDF errors

Relative variation of the cross section when using other PDF sets than MSTW 2008 (full NLL approach)



(very similar values for the LL computation)

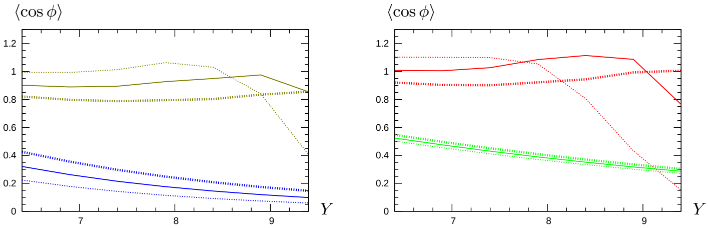
Azimuthal correlation



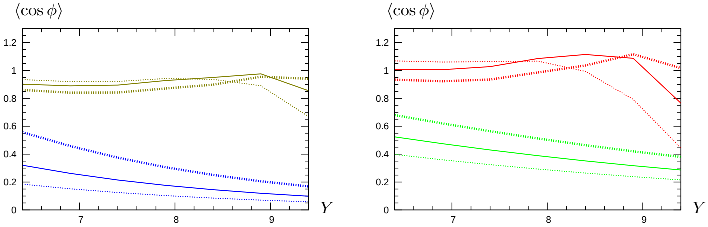
error bands: errors due to the Monte Carlo integration

LL $\rightarrow$ NLL vertices change results dramatically
Both NLL and improved NLL results are almost flat in Y

Azimuthal correlation: dependency with respect to $\mu_R = \mu_F$ and s_0 changes



Effect of changing $\mu_R = \mu_F$ by factors 2 (thick) and $1/2$ (thin)

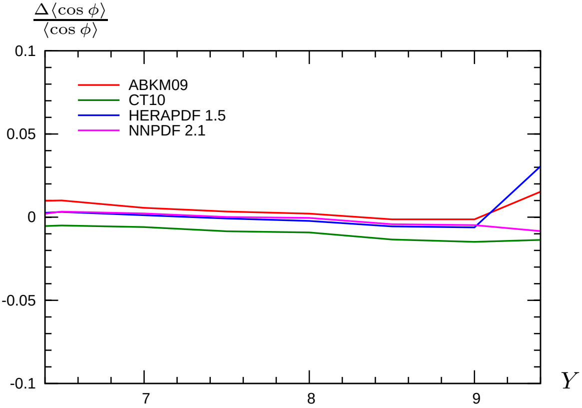


Effect of changing $\sqrt{s_0}$ by factors 2 (thick) and $1/2$ (thin)

- $\langle \cos\phi \rangle$ is still rather $\mu_R = \mu_F$ and s_0 dependent
- collinear resummation can lead to $\langle \cos\phi \rangle > 1(!)$ for small $\mu_R = \mu_F$

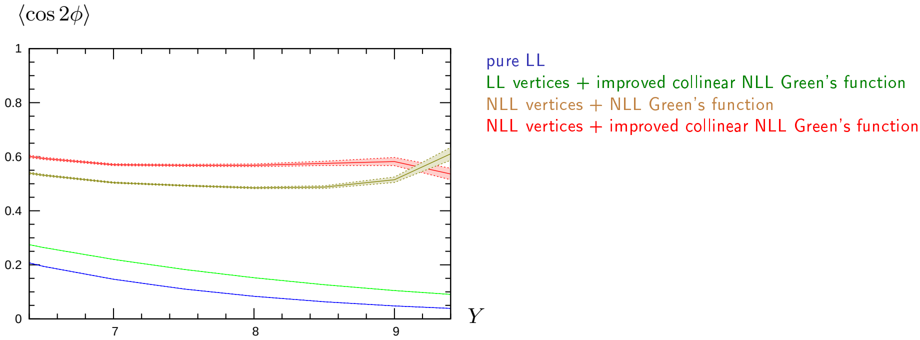
Azimuthal correlation: PDF errors

Relative variation of $\langle \cos\phi \rangle$ when using other PDF sets than MSTW 2008 (full NLL approach)



$\langle \cos\phi \rangle$ is much less sensitive to the PDFs than the cross section (at LL $\langle \cos\phi \rangle$ does not depend on the PDFs at all)

Azimuthal correlation: $\langle \cos 2\phi \rangle$

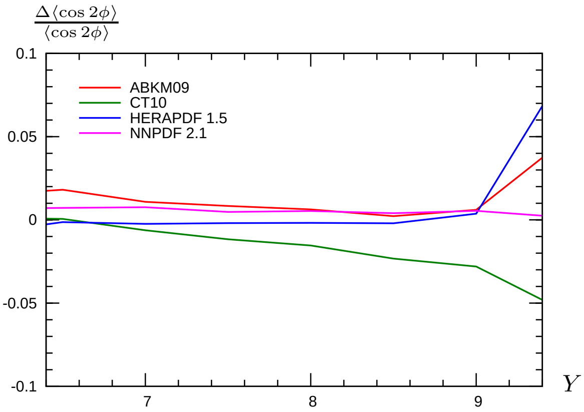


bands: errors due to the Monte Carlo integration

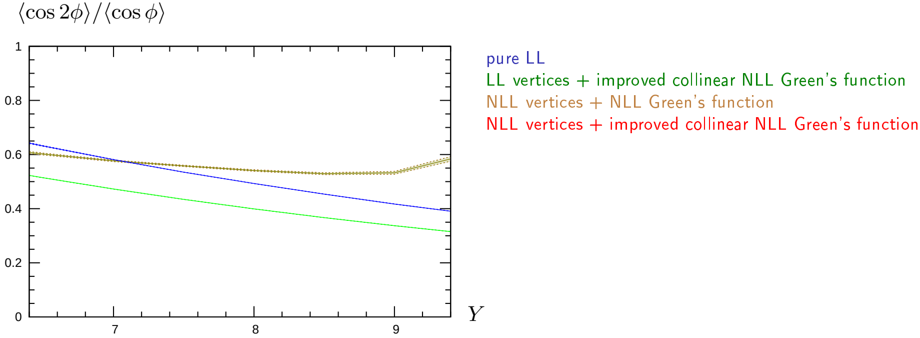
Both NLL and improved NLL results are almost flat in Y

Azimuthal correlation: PDF errors

Relative variation of $\langle \cos 2\phi \rangle$ when using other PDF sets than MSTW 2008



Ratio of azimuthal correlations $\langle \cos 2\phi \rangle / \langle \cos \phi \rangle$



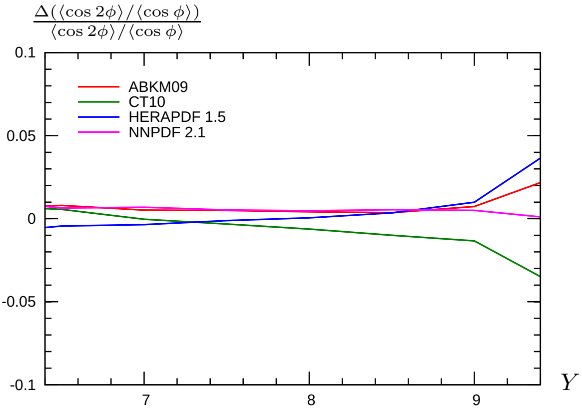
bands: errors due to the Monte Carlo integration

NLL collinear improved changed nothing compared to pure NLL

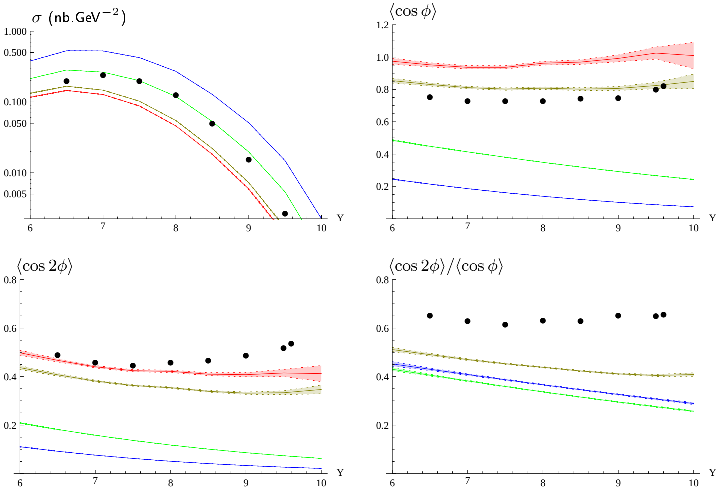
Based on comparisons for $\sqrt{s} = 14$ TeV (JHEP 1012:026 (2010) 1-72), it may be a good observable to distinguish between NLL BFKL and NLO DGLAP scenarii

Azimuthal correlation: PDF errors

Relative variation of $\frac{\langle \cos 2\phi \rangle}{\langle \cos \phi \rangle}$ when using other PDF sets than MSTW 2008



Comparison with NLO DGLAP for $\sqrt{s} = 14$ TeV

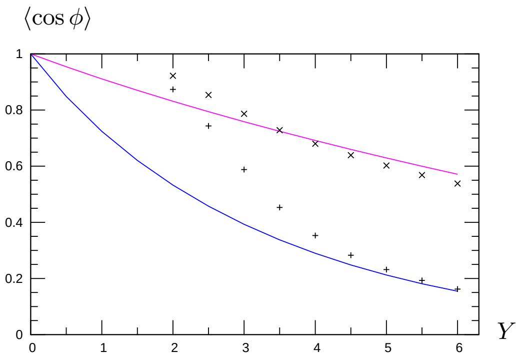


dots: based on the NLO DGLAP parton generator Dijet (thanks to M. Fontannaz)

We plan to do the same comparison for $\sqrt{s} = 7$ TeV

Comparison in the simplified NLL Green's function + LL jet vertices scenario

- The integration $\int_{k_{J\,min}}^{\infty} dk_J$ can be performed analytically
- A comparison with the numerical integration based on code provides a good test of stability, valid for large Y



blue: LL

magenta: NLL Green's function + LL jet vertices scenario Sabio Vera, Schwennsen

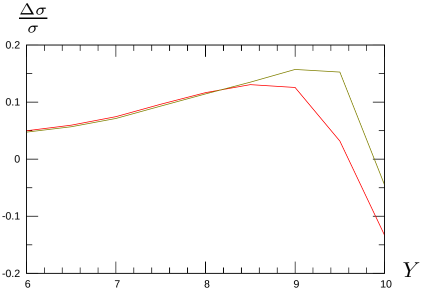
×: numerical dk_J integration $k_{J1} > 20$ GeV and $k_{J2} > 50$ GeV

Results: asym. config. ($|\mathbf{k}_{J1}| = 30\,\text{GeV}$, $|\mathbf{k}_{J2}| = 35\,\text{GeV}$)

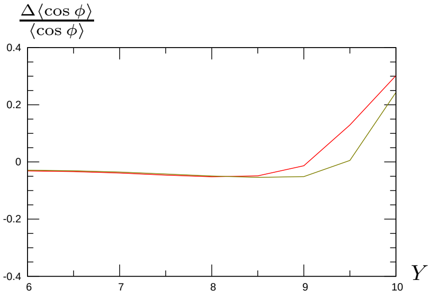
Recently a computation of the jet vertex at NLO in the small cone approximation ($R \ll 1$) was made.

F. Caporale, D. Yu. Ivanov, B. Murdaca, A. Papa, A. Perri

arXiv:1112.3752v2 [hep-ph]



Cross section



$\langle \cos\phi \rangle$

The comparison between the exact and approximate treatments shows good agreement even for a cone parameter $R \sim 0.5$

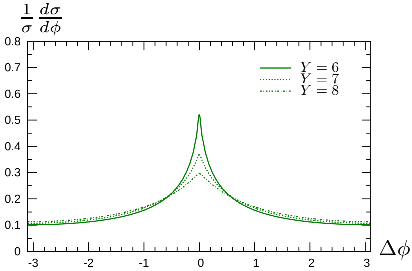
Note: $Y \ll 8$ for BFKL validity (e-m conservation issues)

Results: $\Delta\phi$ distribution

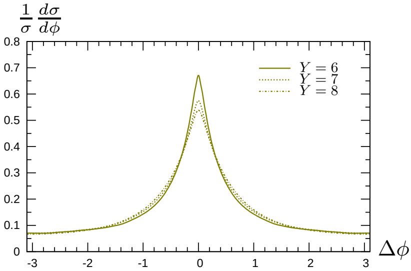
Computing $\langle\cos(n\phi)\rangle$ up to large values of n gives access to the angular distribution

$$\frac{1}{\sigma}\frac{d\sigma}{d\phi} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos\left(n\phi\right)\left\langle\cos\left(n\phi\right)\right\rangle\right\}$$

This is a quantity accessible at experiments like ATLAS and CMS



NLL Green's function + LL vertices



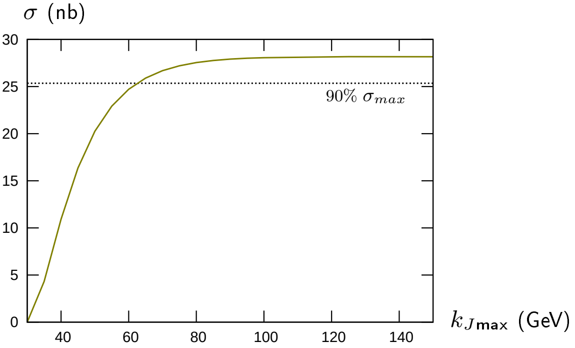
NLL Green's function + NLL vertices

Full NLL treatment predicts :

- Less decorrelation for same Y
- Slower decorrelation with increasing Y

Experimental data is integrated over some range, $k_{J\text{min}} \leq k_J$

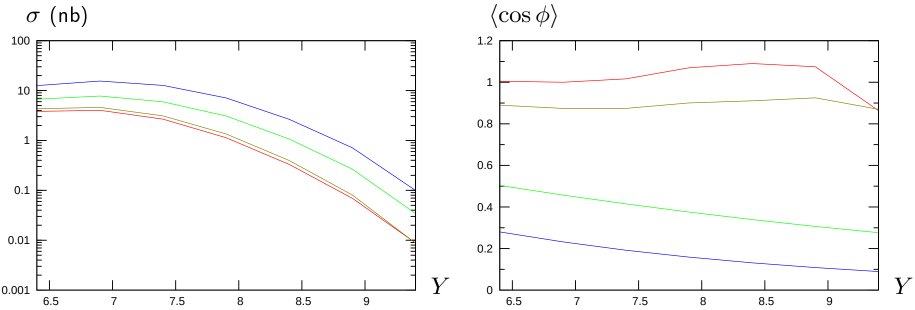
Growth of the cross section with increasing $k_{J\text{max}}$:



$\Rightarrow$ need to integrate up to $k_{J\text{max}} \sim 60$ GeV

- But the BFKL validity domain is limited: $Y_{J,i} \ll \cosh^{-1} \frac{x_i\, E}{k_{J,i}}$
  $\rightarrow$ A lower k_J means a larger validity domain : a k_J as small as possible is preferable
- With only a lower cut on k_J, one has to integrate over regions where the BFKL approach is not valid anymore : $k_J = 60$ GeV $\rightarrow Y_{J,i} \ll 7.3$
- For this reason it would be nice to have a measurement with also an upper cut on transverse momentum, $k_{J\text{min}} \leq k_J \leq k_{J\text{max}}$
- A measure with a $k_{J\text{min}}$ of 30 GeV seems to be possible
  Going down to 25 GeV would probably require a dedicated trigger

Results for 30 GeV $\leq k_J \leq$ 35 GeV



$k_{J\text{max}} = 35$ GeV $\Rightarrow$ computation should be valid for $Y_{J,i} \ll 8.4$

A rough estimation leads to $\sim 400\,000$ events for a relative rapidity $Y = 6.5$ and $\sim 100\,000$ events for $Y = 8$ with a luminosity of 100 pb^{-1}

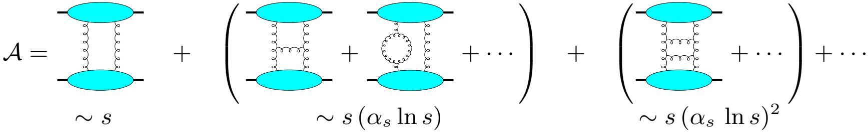
A k_J window of only 5 GeV doesn't seem feasible experimentally because of the resolution on transverse momentum of the jets

Conclusion

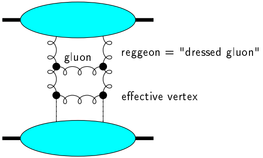
- The first complete NLL analysis of Mueller-Navelet jets has been performed
- The effect of NLL corrections to vertices is dramatic, similar to the NLL Green function corrections
- For the cross-section:
  makes prediction more stable with respect to variation of scales μ and s_0
- Surprisingly small decorrelation effect
  $\langle \cos \phi \rangle$ very flat in rapidity Y
  still rather dependent on the choice of scales
- Energy-momentum conservation could modify the picture, in particular for large values of Y
- We believe a measurement for low k_J would be interesting to study BFKL dynamics

QCD in the perturbative Regge limit

- Small values of α_S (perturbation theory applies due to hard scales) can be compensated by large $\ln s$ enhancements. $\Rightarrow$ resummation of $\sum_n (\alpha_S \, \ln s)^n$ series (Balitski, Fadin, Kuraev, Lipatov)



- this results in the effective BFKL ladder



$$\Longrightarrow \sigma_{tot}^{h_1\, h_2 \to anything} = \frac{1}{s} Im\mathcal{A} \sim s^{\alpha_{\mathbb{P}}(0)-1}$$

with $\alpha_{\mathbb{P}}(0) - 1 = C\, \alpha_s \quad (C > 0)$ Leading Log Pomeron
Balitsky, Fadin, Kuraev, Lipatov

Angular coefficients

$$\mathcal{C}_m \equiv \int \mathrm{d}\phi_{J1}\, \mathrm{d}\phi_{J2} \cos\left(m(\phi_{J,1} - \phi_{J,2} - \pi)\right)$$

$$\times \int \mathrm{d}^2\mathbf{k}_1\, \mathrm{d}^2\mathbf{k}_2\, \Phi(\mathbf{k}_{J1}, x_{J,1}, -\mathbf{k}_1)\, G(\mathbf{k}_1, \mathbf{k}_2, \hat{s})\, \Phi(\mathbf{k}_{J2}, x_{J,2}, \mathbf{k}_2).$$

- $m = 0 \implies$ cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}|\mathbf{k}_{J1}|\, \mathrm{d}|\mathbf{k}_{J2}|\, \mathrm{d}y_{J1}\, \mathrm{d}y_{J2}} = \mathcal{C}_0$$

- $m > 0 \implies$ azimutal decorrelation

$$\langle \cos(m\phi) \rangle \equiv \langle \cos\left(m(\phi_{J,1} - \phi_{J,2} - \pi)\right) \rangle = \frac{\mathcal{C}_m}{\mathcal{C}_0}$$

Rely on LL BFKL eigenfunctions

- LL BFKL eigenfunctions: $E_{n,\nu}(\mathbf{k}_1) = \frac{1}{\pi\sqrt{2}} \left(\mathbf{k}_1^2\right)^{i\nu - \frac{1}{2}} e^{in\phi_1}$
- decompose Φ on this basis
- use the known LL eigenvalue of the BFKL equation on this basis:

$$\omega(n,\nu) = \bar{\alpha}_s \chi_0 \left(|n|, \tfrac{1}{2} + i\nu\right)$$

with $\chi_0(n,\gamma) = 2\Psi(1) - \Psi\left(\gamma + \frac{n}{2}\right) - \Psi\left(1 - \gamma + \frac{n}{2}\right)$
$(\Psi(x) = \Gamma'(x)/\Gamma(x),\ \bar{\alpha}_s = N_c \alpha_s/\pi)$

- $\Longrightarrow$ master formula:

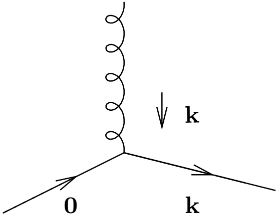
$$\mathcal{C}_m = (4 - 3\,\delta_{m,0}) \int \mathrm{d}\nu\, C_{m,\nu}(|\mathbf{k}_{J1}|, x_{J,1})\, C^*_{m,\nu}(|\mathbf{k}_{J2}|, x_{J,2}) \left(\frac{\hat{s}}{s_0}\right)^{\omega(m,\nu)}$$

$$\text{with} \quad C_{m,\nu}(|\mathbf{k}_J|, x_J) = \int \mathrm{d}\phi_J\, \mathrm{d}^2\mathbf{k}\, \mathrm{d}x\, f(x) V(\mathbf{k}, x) E_{m,\nu}(\mathbf{k}) \cos(m\phi_J)$$

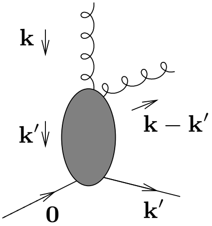
- at NLL, same master formula: just change $\omega(m,\nu)$ and V (although $E_{n,\nu}$ are not anymore eigenfunctions)

$\mathbf{k}, \mathbf{k}' =$ Euclidian two dimensional vectors

LL jet vertex:



NLL jet vertex:



NLL correction to the jet vertex: quark part (Bartels, Colferai, Vacca)

$$
\begin{aligned}
& V_{\mathrm{q}}^{(1)}(\mathbf{k},x) \\
= \; & \left[\left(\frac{3}{2}\ln\frac{\mathbf{k}^2}{\Lambda^2}-\frac{15}{4}\right)\frac{C_F}{\pi}+\left(\frac{85}{36}+\frac{\pi^2}{4}\right)\frac{C_A}{\pi}-\frac{5}{18}\frac{N_f}{\pi}-b_0\ln\frac{\mathbf{k}^2}{\mu^2}\right]V_{\mathrm{q}}^{(0)}(\mathbf{k},x) \\
& +\int \mathrm{d}z\left(\frac{C_F}{\pi}\frac{1-z}{2}+\frac{C_A}{\pi}\frac{z}{2}\right)V_{\mathrm{q}}^{(0)}(\mathbf{k},xz) \\
& +\frac{C_A}{\pi}\int\frac{\mathrm{d}^2\mathbf{k}'}{\pi}\int\mathrm{d}z\left[\frac{1+(1-z)^2}{2z}\right. \\
& \qquad\times\left((1-z)\frac{(\mathbf{k}-\mathbf{k}')\cdot\big((1-z)\mathbf{k}-\mathbf{k}'\big)}{(\mathbf{k}-\mathbf{k}')^2\big((1-z)\mathbf{k}-\mathbf{k}'\big)^2}h_{\mathrm{q}}^{(0)}(\mathbf{k}')\mathcal{S}_J^{(3)}(\mathbf{k}',\mathbf{k}-\mathbf{k}',xz;x)\right. \\
& \qquad\left.-\frac{1}{\mathbf{k}'^2}\Theta(\Lambda^2-\mathbf{k}'^2)V_{\mathrm{q}}^{(0)}(\mathbf{k},xz)\right) \\
& \qquad\left.-\frac{1}{z(\mathbf{k}-\mathbf{k}')^2}\Theta\big(|\mathbf{k}-\mathbf{k}'|-z(|\mathbf{k}-\mathbf{k}'|+|\mathbf{k}'|)\big)V_{\mathrm{q}}^{(0)}(\mathbf{k}',x)\right] \\
& +\frac{C_F}{2\pi}\int\mathrm{d}z\,\frac{1+z^2}{1-z}\int\frac{\mathrm{d}^2\mathbf{l}}{\pi\mathbf{l}^2} \\
& \qquad\times\left[\frac{\mathcal{N}C_F}{\mathbf{l}^2+(\mathbf{l}-\mathbf{k})^2}\Big(\mathcal{S}_J^{(3)}(z\mathbf{k}+(1-z)\mathbf{l},(1-z)(\mathbf{k}-\mathbf{l}),x(1-z);x)\right. \\
& \qquad\qquad+\mathcal{S}_J^{(3)}(\mathbf{k}-(1-z)\mathbf{l},(1-z)\mathbf{l},x(1-z);x)\Big) \\
& \qquad\left.-\Theta\left(\frac{\Lambda^2}{(1-z)^2}-\mathbf{l}^2\right)\left(V_{\mathrm{q}}^{(0)}(\mathbf{k},x)+V_{\mathrm{q}}^{(0)}(\mathbf{k},xz)\right)\right] \\
& -\frac{2C_F}{\pi}\int\mathrm{d}z\left(\frac{1}{1-z}\right)\int\frac{\mathrm{d}^2\mathbf{l}}{\pi\mathbf{l}^2}\left[\frac{\mathcal{N}C_F}{\mathbf{l}^2+(\mathbf{l}-\mathbf{k})^2}S_J^{(2)}(\mathbf{k},x)\right. \\
& \qquad\left.-\Theta\left(\frac{\Lambda^2}{(1-z)^2}-\mathbf{l}^2\right)V_{\mathrm{q}}^{(0)}(\mathbf{k},x)\right]
\end{aligned}
$$

NLL correction to the jet vertex: gluon part (Bartels, Colferai, Vacca)

$$
\begin{aligned}
& V_{\mathrm{g}}^{(1)}(\mathbf{k},x) \\
= & \left[\left(\frac{11}{6}\frac{C_A}{\pi}-\frac{1}{3}\frac{N_f}{\pi}\right)\ln\frac{\mathbf{k}^2}{\Lambda^2}+\left(\frac{\pi^2}{4}-\frac{67}{36}\right)\frac{C_A}{\pi}+\frac{13}{36}\frac{N_f}{\pi}-b_0\ln\frac{\mathbf{k}^2}{\mu^2}\right]V_{\mathrm{g}}^{(0)}(\mathbf{k},x) \\
& +\int \mathrm{d}z\,\frac{N_f}{\pi}\frac{C_F}{C_A}z(1-z)V_{\mathrm{g}}^{(0)}(\mathbf{k},xz) \\
& +\frac{N_f}{\pi}\int\frac{\mathrm{d}^2\mathbf{k}'}{\pi}\int_0^1\mathrm{d}z\,P_{\mathrm{qg}}(z)\left[\frac{h_{\mathrm{q}}^{(0)}(\mathbf{k}')}{(\mathbf{k}-\mathbf{k}')^2+\mathbf{k}'^2}\mathcal{S}_J^{(3)}(\mathbf{k}',\mathbf{k}-\mathbf{k}',xz;x)\right. \\
& \qquad\left.-\frac{1}{\mathbf{k}'^2}\Theta(\Lambda^2-\mathbf{k}'^2)V_{\mathrm{q}}^{(0)}(\mathbf{k},xz)\right] \\
& +\frac{N_f}{2\pi}\int\frac{\mathrm{d}^2\mathbf{k}'}{\pi}\int_0^1\mathrm{d}z\,P_{\mathrm{qg}}(z)\frac{\mathcal{N}C_A}{\left((1-z)\mathbf{k}-\mathbf{k}'\right)^2}\left[z(1-z)\frac{(\mathbf{k}-\mathbf{k}')\cdot\mathbf{k}'}{(\mathbf{k}-\mathbf{k}')^2\mathbf{k}'^2}\mathcal{S}_J^{(3)}(\mathbf{k}',\mathbf{k}-\mathbf{k}',xz;x)\right. \\
& \qquad\left.-\frac{1}{\mathbf{k}^2}\Theta\left(\Lambda^2-\left((1-z)\mathbf{k}-\mathbf{k}'\right)^2\right)\mathcal{S}_J^{(2)}(\mathbf{k},x)\right] \\
& +\frac{C_A}{\pi}\int_0^1\frac{\mathrm{d}z}{1-z}\left[(1-z)P(1-z)\right]\int\frac{\mathrm{d}^2\mathbf{l}}{\pi\mathbf{l}^2} \\
& \quad\times\left\{\frac{\mathcal{N}C_A}{\mathbf{l}^2+(\mathbf{l}-\mathbf{k})^2}\left[\mathcal{S}_J^{(3)}(z\mathbf{k}+(1-z)\mathbf{l},(1-z)(\mathbf{k}-\mathbf{l}),x(1-z);x)\right.\right. \\
& \qquad\left.+\mathcal{S}_J^{(3)}(\mathbf{k}-(1-z)\mathbf{l},(1-z)\mathbf{l},x(1-z);x)\right] \\
& \qquad\left.-\Theta\left(\frac{\Lambda^2}{(1-z)^2}-\mathbf{l}^2\right)\left[V_{\mathrm{g}}^{(0)}(\mathbf{k},x)+V_{\mathrm{g}}^{(0)}(\mathbf{k},xz)\right]\right\} \\
& -\frac{2C_A}{\pi}\int_0^1\frac{\mathrm{d}z}{1-z}\int\frac{\mathrm{d}^2\mathbf{l}}{\pi\mathbf{l}^2}\left[\frac{\mathcal{N}C_A}{\mathbf{l}^2+(\mathbf{l}-\mathbf{k})^2}\mathcal{S}_J^{(2)}(\mathbf{k},x)-\Theta\left(\frac{\Lambda^2}{(1-z)^2}-\mathbf{l}^2\right)V_{\mathrm{g}}^{(0)}(\mathbf{k},x)\right] \\
& +\frac{C_A}{\pi}\int\frac{\mathrm{d}^2\mathbf{k}'}{\pi}\int_0^1\mathrm{d}z\left[P(z)\left((1-z)\frac{(\mathbf{k}-\mathbf{k}')\cdot\left((1-z)\mathbf{k}-\mathbf{k}'\right)}{(\mathbf{k}-\mathbf{k}')^2\left((1-z)\mathbf{k}-\mathbf{k}'\right)^2}h_{\mathrm{g}}^{(0)}(\mathbf{k}')\right.\right. \\
& \qquad\left.\times\mathcal{S}_J^{(3)}(\mathbf{k}',\mathbf{k}-\mathbf{k}',xz;x)-\frac{1}{\mathbf{k}'^2}\Theta(\Lambda^2-\mathbf{k}'^2)V_{\mathrm{g}}^{(0)}(\mathbf{k},xz)\right) \\
& \qquad\left.-\frac{1}{z(\mathbf{k}-\mathbf{k}')^2}\Theta\left(|\mathbf{k}-\mathbf{k}'|-z(|\mathbf{k}-\mathbf{k}'|+|\mathbf{k}'|)\right)V_{\mathrm{g}}^{(0)}(\mathbf{k}',x)\right]
\end{aligned}
$$

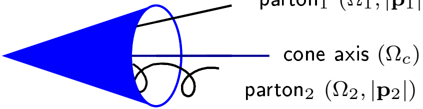
Jet algorithms

- a jet algorithm should be IR safe, both for soft and collinear singularities
- the most common jet algorithm are:
  - k_t algorithms (IR safe but time consuming for multiple jets configurations)
  - cone algorithm (not IR safe in general; can be made IR safe at NLO: Ellis, Kunszt, Soper)

Cone jet algorithm at NLO (Ellis, Kunszt, Soper)

- Should partons $(|\mathbf{p}_1|, \phi_1, y_1)$ and $(\mathbf{p}_2|, \phi_2, y_2)$ be combined in a single jet?
  $|\mathbf{p}_i|$ =transverse energy deposit in the calorimeter cell i of parameter $\Omega = (y_i, \phi_i)$ in $y - \phi$ plane
- define transverse energy of the jet: $p_J = |\mathbf{p}_1| + |\mathbf{p}_2|$
- jet axis:

$$\Omega_c \begin{cases} y_J = \dfrac{|\mathbf{p}_1|\, y_1 + |\mathbf{p}_2|\, y_2}{p_J} \\[2ex] \phi_J = \dfrac{|\mathbf{p}_1|\, \phi_1 + |\mathbf{p}_2|\, \phi_2}{p_J} \end{cases}$$



$\Omega = (y_i, \phi_i)$ in $y - \phi$ plane

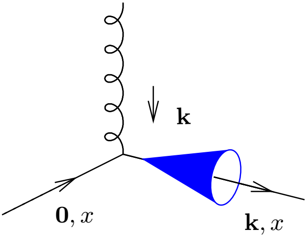
If distances $|\Omega_i - \Omega_c|^2 \equiv (y_i - y_c)^2 + (\phi_i - \phi_c)^2 < R^2$ $(i = 1$ and $i = 2)$

$\Longrightarrow$ partons 1 and 2 are in the same cone Ω_c

combined condition: $|\Omega_1 - \Omega_2| < \dfrac{|\mathbf{p}_1| + |\mathbf{p}_2|}{max(|\mathbf{p}_1|, |\mathbf{p}_2|)} R$

LL jet vertex and cone algorithm

$\mathbf{k}, \mathbf{k}'$ = Euclidian two dimensional vectors

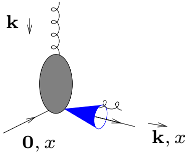


$$\mathcal{S}_J^{(2)}(k_\perp; x) = \delta\left(1 - \frac{x_J}{x}\right) |\mathbf{k}|\, \delta^{(2)}(\mathbf{k} - \mathbf{k}_J)$$

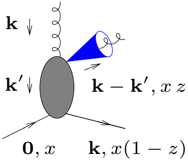
NLL jet vertex and cone algorithm

$\mathbf{k}, \mathbf{k}'$ = Euclidian two dimensional vectors

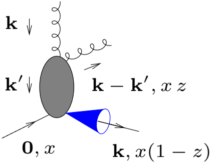
$$\mathcal{S}_J^{(3,\mathrm{cone})}(\mathbf{k}', \mathbf{k}-\mathbf{k}', xz; x) =$$



$$\mathcal{S}_J^{(2)}(\mathbf{k}, x)\, \Theta\left(\left[\frac{|\mathbf{k}-\mathbf{k}'|+|\mathbf{k}'|}{\max(|\mathbf{k}-\mathbf{k}'|,|\mathbf{k}'|)} R_{\mathrm{cone}}\right]^2 - \left[\Delta y^2 + \Delta\phi^2\right]\right)$$



$$+\ \mathcal{S}_J^{(2)}(\mathbf{k}-\mathbf{k}', xz)\, \Theta\left(\left[\Delta y^2 + \Delta\phi^2\right] - \left[\frac{|\mathbf{k}-\mathbf{k}'|+|\mathbf{k}'|}{\max(|\mathbf{k}-\mathbf{k}'|,|\mathbf{k}'|)} R_{\mathrm{cone}}\right]^2\right)$$



$$+\ \mathcal{S}_J^{(2)}(\mathbf{k}', x(1-z))\, \Theta\left(\left[\Delta y^2 + \Delta\phi^2\right] - \left[\frac{|\mathbf{k}-\mathbf{k}'|+|\mathbf{k}'|}{\max(|\mathbf{k}-\mathbf{k}'|,|\mathbf{k}'|)} R_{\mathrm{cone}}\right]^2\right),$$

Using a IR safe jet algorithm, Mueller-Navelet jets at NLL are finite

- UV sector:
  - the NLL impact factor contains UV divergencies $1/\epsilon$
  - they are absorbed by the renormalization of the coupling: $\alpha_S \longrightarrow \alpha_S(\mu_R)$
- IR sector:
  - PDF have IR collinear singularities: pole $1/\epsilon$ at LO
  - these collinear singularities can be compensated by collinear singularities of the two jets vertices and the real part of the BFKL kernel
  - the remaining collinear singularities compensate exactly among themselves
  - soft singularities of the real and virtual BFKL kernel, and of the jets vertices compensates among themselves

This was shown for both quark and gluon initiated vertices (Bartels, Colferai, Vacca)

NLL Green's function: rely on LL BFKL eigenfunctions

- NLL BFKL kernel is not conformal invariant
- LL $E_{n,\nu}$ are not anymore eigenfunction
- this can be overcome by considering the eigenvalue as an operator with a part containing $\frac{\partial}{\partial\nu}$
- it acts on the impact factor

$$\omega(n,\nu) = \bar{\alpha}_s \chi_0\left(|n|, \frac{1}{2}+i\nu\right) + \bar{\alpha}_s^2\left[\chi_1\left(|n|, \frac{1}{2}+i\nu\right) - \frac{\pi b_0}{2N_c}\chi_0\left(|n|, \frac{1}{2}+i\nu\right)\underbrace{\left\{-2\ln\mu_R^2 - i\frac{\partial}{\partial\nu}\ln\frac{C_{n,\nu}(|\mathbf{k}_{J1}|, x_{J,1})}{C_{n,\nu}(|\mathbf{k}_{J2}|, x_{J,2})}\right\}}_{2\ln\frac{|\mathbf{k}_{J1}|\cdot|\mathbf{k}_{J2}|}{\mu_R^2}}\right],$$

LL substraction and s_0

- one sums up $\sum(\alpha_s \ln \hat{s}/s_0)^n + \alpha_s \sum(\alpha_s \ln \hat{s}/s_0)^n \quad (\hat{s} = x_1\, x_2\, s)$
- at LL s_0 is arbitrary

- natural choice: $s_0 = \sqrt{s_{0,1}\, s_{0,2}}$ $\;s_{0,i}$ for each of the scattering objects
  - possible choice: $s_{0,i} = (|\mathbf{k}_J| + |\mathbf{k}_J - \mathbf{k}|)^2$ (Bartels, Colferai, Vacca)
    - but depend on $\mathbf{k}$, which is integrated over
    - $\hat{s}$ is not an external scale ($x_{1,2}$ are integrated over)
  - we prefer

$$\left.\begin{array}{l} s_{0,1} = (|\mathbf{k}_{J1}| + |\mathbf{k}_{J1} - \mathbf{k}_1|)^2 \;\to\; s'_{0,1} = \dfrac{x_1^2}{x_{J,1}^2}\mathbf{k}_{J1}^2 \\[2em] s_{0,2} = (|\mathbf{k}_{J2}| + |\mathbf{k}_{J2} - \mathbf{k}_2|)^2 \;\to\; s'_{0,2} = \dfrac{x_2^2}{x_{J,2}^2}\mathbf{k}_{J2}^2 \end{array}\right\} \quad \frac{\hat{s}}{s_0} \;\to\; \frac{\hat{s}}{s'_0} = \frac{x_{J,1}\, x_{J_2}\, s}{|\mathbf{k}_{J1}|\,|\mathbf{k}_{J2}|} = e^{y_{J,1}-y_{J,2}} \equiv e^Y$$

- $s_0 \to s'_0$ affects
  - the BFKL NLL Green function
  - the impact factors:

$$\Phi_{\rm NLL}(\mathbf{k}_i; s'_{0,i}) = \Phi_{\rm NLL}(\mathbf{k}_i; s_{0,i}) + \int \mathrm{d}^2\mathbf{k}'\, \Phi_{\rm LL}(\mathbf{k}'_i)\, \mathcal{K}_{\rm LL}(\mathbf{k}'_i, \mathbf{k}_i)\, \frac{1}{2} \ln \frac{s'_{0,i}}{s_{0,i}} \qquad (1)$$

- numerical stability (non azimuthal averaging of LL substraction) improved with the choice $s_{0,i} = (\mathbf{k}_i - 2\mathbf{k}_{Ji})^2$

  (then replaced by $s'_{0,i}$ after numerical integration)
- (1) can be used to test $s_0 \to \lambda\, s_0$ dependence

Collinear improved Green's function at NLL

- one may improve the NLL BFKL kernel for $n = 0$ by imposing its compatibility with DGLAP in the collinear limit
  Salam; Ciafaloni, Colferai
- usual (anti)collinear poles in $\gamma = 1/2 + i\nu$ (resp. $1-\gamma$) are shifted by $\omega/2$
- one practical implementation:
  - the new kernel $\bar{\alpha}_s \chi^{(1)}(\gamma, \omega)$ with shifted poles replaces

    $\bar{\alpha}_s \chi_0(\gamma, 0) + \bar{\alpha}_s^2 \chi_1(\gamma, 0)$
  - $\omega(0, \nu)$ is obtained by solving the implicit equation

    $$\omega(0, \nu) = \bar{\alpha}_s \chi^{(1)}(\gamma, \omega(0, \nu))$$

    for $\omega(n, \nu)$ numerically.
- there is no need for any jet vertex improvement because of the absence of γ and $1-\gamma$ poles (numerical proof using Cauchy theorem "backward")

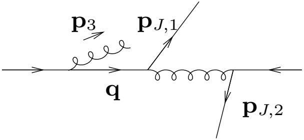
In practice

- MSTW 2008 PDFs (available as Mathematica packages)
- $\mu_R = \mu_F$ (this is imposed by the MSTW 2008 PDFs)
- two-loop running coupling $\alpha_s(\mu_R^2)$
- We use a ν grid (with a dense sampling around 0)
- all numerical calculations are done in Mathematica
- we use Cuba integration routines (in practice Vegas): precision 10^{-2} for 500.000 max points per integration
- mapping $|\mathbf{k}| = |\mathbf{k}_J| \tan(\xi\pi/2)$ for $\mathbf{k}$ integrations $\Rightarrow [0, \infty[\to [0, 1]$
- although formally the results should be finite, it requires a special grouping of the integrand in order to get stable results

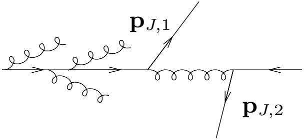
$\Longrightarrow$ 14 minimal stable basic blocks to be evaluated numerically

Motivation for asymmetric configurations

- Initial state radiation (unseen) produces divergencies if one touches the collinear singularity $\mathbf{q}^2 \to 0$

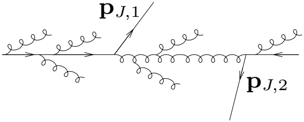


- they are compensated by virtual corrections
- this compensation is in practice difficult to implement when for some reason this additional emission is in a "corner" of the phase space (dip in the differential cross-section)
- this is the case when $\mathbf{p}_1 + \mathbf{p}_2 \to 0$
- this calls for a resummation of large remaing logs $\Rightarrow$ Sudakov resummation



Motivation for asymmetric configurations

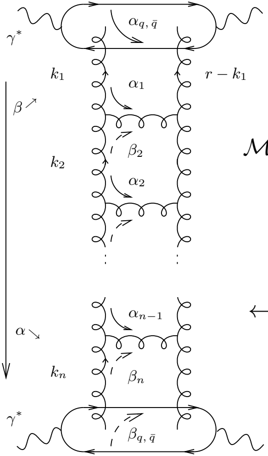
- since these resummation have never been investigated in this context, one should better avoid that region
- note that for BFKL, due to additional emission between the two jets, one may expect a less severe problem (at least a smearing in the dip region $|\mathbf{p}_1| \sim |\mathbf{p}_2|$)



- this may however not mean that the region $|\mathbf{p}_1| \sim |\mathbf{p}_2|$ is perfectly trustable even in a BFKL type of treatment
- we now investigate a region where NLL DGLAP is under control

Opening the boxes: Impact representation $\gamma^* \gamma^* \to \gamma^* \gamma^*$ as an example

- Sudakov decomposition: $k_i = \alpha_i \, p_1 + \beta_i \, p_2 + k_{\perp i}$ $\quad (p_1^2 = p_2^2 = 0, \; 2p_1 \cdot p_2 = s)$
- write $\quad d^4 k_i = \frac{s}{2} \, d\alpha_i \, d\beta_i \, d^2 k_{\perp i}$ $\quad$ ($\underline{k}$ = Eucl. $\leftrightarrow$ $k_\perp$ = Mink.)
- $t-$channel gluons have non-sense polarizations at large s: $\epsilon_{NS}^{up/down} = \frac{2}{s} \, p_{2/1}$



$\Rightarrow$ set $\alpha_1 = 0$ and $\int d\beta_1 \;\Rightarrow\; \Phi^{\gamma^* \to \gamma^*}(\underline{k}_1, \underline{r} - \underline{k}_1)$ impact factor

$$\mathcal{M} = \frac{is}{(2\pi)^2} \int \frac{d^2 \underline{k}}{\underline{k}^2} \, \Phi^{up}(\underline{k}, \, \underline{r} - \underline{k}) \int \frac{d^2 \underline{k}'}{\underline{k}'^2} \, \Phi^{down}(-\underline{k}', \, -\underline{r} + \underline{k}')$$
$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0} \right)^{\omega} G_\omega(\underline{k}, \underline{k}', \underline{r})$$

$\longleftarrow$ multi-Regge kinematics

$\Rightarrow$ set $\beta_n = 0$ and $\int d\alpha_n \;\Rightarrow\; \Phi^{\gamma^* \to \gamma^*}(-\underline{k}_n, -\underline{r} + \underline{k}_n)$