Loop Computations with Lipatov's Effective Action

José Daniel Madrigal Martínez†

Instituto de Física Teórica UAM/CSIC, Madrid

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† *Based on work in collaboration with G. Chachamis, M. Hentschinski & A. Sabio Vera: arXiv:hep-ph/1202.0649v1* Nucl.Phys. **B861** (2012) 133 *and work to appear soon*

Lipatov's High-Energy Effective Action

Effective Field Theory (EFT) and High-Energy Limit (HEL)

- Effective field theory: powerful tool for multi-scale problems
- Semihard processes in Regge limit: $s \gg -t \gg \mu^2$
- Unitarity directly restored in EFT
- Takes the reggeized gluon as the relevant degree of freedom: captures simplicity of HEL
- Very powerful to compute reggeon vertices for NLO and NNLO BFKL (*tree-level*) [Kniehl, Basin & Saleev'06; Braun, Lipatov, Salykin & Vyazovsky'11...]

★ Lipatov's EFT can be derived (at LO) by integrating out heavy modes

- [Kirschner, Lipatov & Szymanowski'94,'95]

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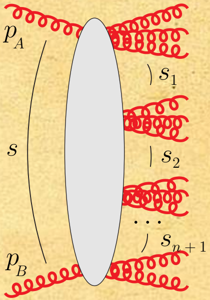
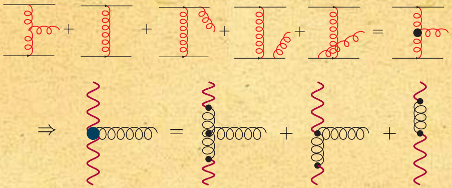
Generalized
Quasi-Multi-Regge Kinematics (QMRK)
[Fadin&Lipatov'89]



Clusters strongly ordered in rapidity:
$y_0 \gg y_1 \gg \cdots \gg y_{n+1}$,
$y_k = \frac{1}{2} \ln \frac{k^+}{k^-}$

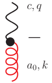
- Strong rapidity ordering simplifies polarization tensor of t-channel reggeons: $g_{\mu\nu} \to \frac{1}{2}(n^+)_\mu (n^-)^\nu + \mathcal{O}(1/s)$
- Reggeized gluons couple to quarks and gluons through effective vertices local in rapidity: ***Effective vertex*** $=$ ***Light-Cone Projection*** $+$ ***Induced Contributions***



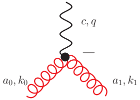
- Reggeon propagators are essentially transverse: $q_i^2 = -\boldsymbol{q}_i^2$

$p_a + p_b \to p_1 + p_2; \quad n^{+,-} = 2p_{a,b}/\sqrt{s}$,
$k = k^+ \frac{n^-}{2} + k^- \frac{n^+}{2} + \boldsymbol{k}$

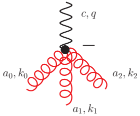
Feynman Rules for Lipatov's Effective Action



$$= \Delta^{\nu_0 -}_{a_0 c} = -i\boldsymbol{q}^2 \delta^{a_0 c} (n^-)^{\nu_0},$$



$$= g\boldsymbol{q}^2 f^{a_0 a_1 c} \frac{1}{k_0^-} (n^-)^{\nu_0} (n^-)^{\nu_1},$$



$$= \Delta^{\nu_0 \nu_1 \nu_2 -}_{a_0 a_1 a_2 c} = ig^2 \boldsymbol{q}^2 \left(\frac{f^{a_2 a_1 a} f^{a_0 a c}}{k_2^- k_0^-} + \frac{f^{a_2 a_0 a} f^{a_1 a c}}{k_2^- k_1^-} \right) (n^-)^{\nu_0} (n^-)^{\nu_1} (n^-)^{\nu_2},$$

$$= \frac{i}{2\boldsymbol{q}^2}.$$

[Antonov, Cherednikov, Kuraev & Lipatov'05]

$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind}};$$

$$S_{\text{ind}} = \int d^4x \, \text{Tr} \left[\left(W_+[v(x)] - \mathscr{A}_+(x) \right) \partial_\perp^2 \mathscr{A}_-(x) \right] + \int d^4x \, \text{Tr} \left[\left(W_-[v(x)] - \mathscr{A}_-(x) \right) \partial_\perp^2 \mathscr{A}_+(x) \right];$$

$$W_\pm[v] = v_\pm \frac{1}{D_\pm} \partial_\pm = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + \cdots$$

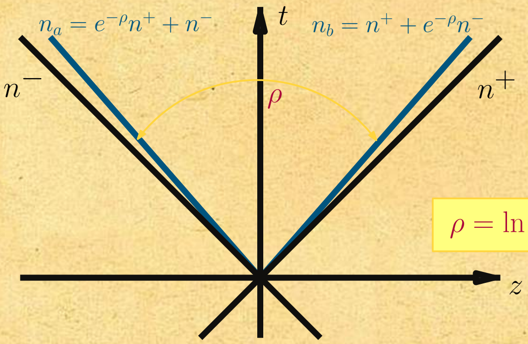
$\mathscr{A}_\pm$: reggeons, v_μ: gluons

Kinematical Constraints

$$\partial_\pm \mathscr{A}_\mp(x) = 0, \quad \sum_{i=0}^{r} k_i^\pm = 0$$

- Reggeon fields invariant under *local* gauge transformations

The Light-Cone Regularization



$\rho = \ln s$

Tilting the light-cone vectors appearing in the induced vertices

[Collins & Soper'81,'82]
[Korchemsky & Radyushkin'87]
[Hentschinski & Sabio Vera'11]

- Regularization needed to make sense of non-local operators $\frac{1}{\partial_\pm}$
- Rest of divergences managed with dimensional regularization
- $\rho \to \infty$ in the high-energy limit
- Pole prescription: principal value [Hentschinski'11]

The Gluon Regge Trajectory

Amplitudes in Multi-Regge Kinematics: Reggeization

$\mathcal{M}^{\text{LLA}}_{2\to 2+n} = \mathcal{M}^{\text{tree}}_{2\to 2+n} \prod_{i=1}^{n+1} s_i^{\omega(t_i)}$ [Lipatov'76] $\omega(t)$ =Regge Trajectory

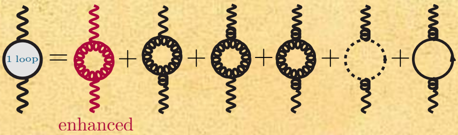
- Regge trajectory describes virtual contributions in the BFKL equation [Fadin, Kuraev & Lipatov'75,'77]; [Balitsky & Lipatov'78]

$$\omega \tilde{f}_\omega(\boldsymbol{q}_1, \boldsymbol{q}_2) = \delta^2(\boldsymbol{q}_1 - \boldsymbol{q}_2) + \int d^2\boldsymbol{\kappa}\, \mathcal{K}(\boldsymbol{q}_1, \boldsymbol{\kappa}) \tilde{f}_\omega(\boldsymbol{\kappa}, \boldsymbol{q}_2)$$

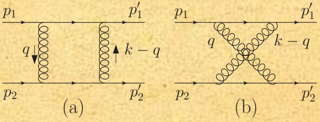
One-Loop Trajectory

Effective Action Diagrams

Enhanced means $\propto \rho = \ln s$

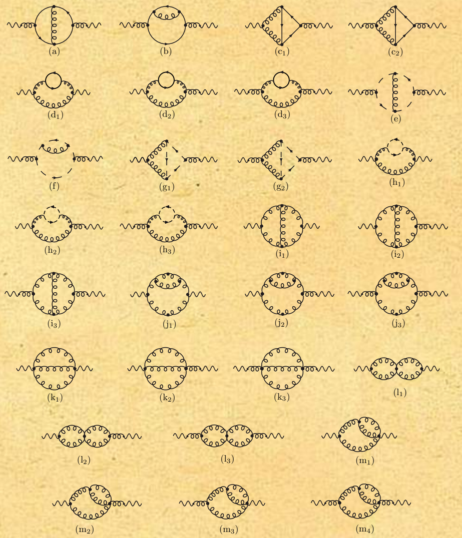


Usual QCD Diagrams



(+ non-enhanced contributions)

2-Loop Effective Action Diagrams



The **Regge trajectory** is an extremely important quantity:

- BFKL equation controls asymptotic rising of cross-sections at very high energies
- Includes as a piece the **cusp anomalous dimension**

$$\omega(-t) = \frac{1}{2}\int_{-t}^{\mu_{\mathrm{IR}}^2} \frac{d\boldsymbol{k}^2}{\boldsymbol{k}^2}\Gamma_{\mathrm{cusp}}(\alpha_s(\boldsymbol{k}^2)) + \Gamma_R(\alpha_s(-t)) + \text{poles in } (1/\epsilon_{\mathrm{IR}})$$

It is known

- at NLO in QCD

  [Fadin, Fiore & Kotsky'96]

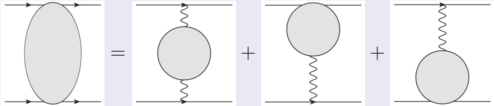
- to all orders in $\mathcal{N} = 4$ SYM

  [Kotikov & Lipatov'00; Beisert, Eden & Staudacher'07; Bartels, Lipatov & S.Vera'09]

THE RECIPE to Compute the 2-Loop Gluon Trajectory $\omega^{(2)}$

1. Determine the high energy limit of the 2-loop parton-parton scattering amplitude by dropping terms not ρ-enhanced (remember, $\rho = \ln s$)
2. Subtract non-local contributions to reggeized gluon self-energy to avoid double-counting
3. Divide by the tree-level HEL result
4. Remove all terms corresponding to combinations of 1-loop trajectory and 1-loop impact factors (reggeon-parton scattering vertices)

$$i\mathcal{M}^{(1)}_{q_a q_b \to q_1 q_2} =$$



Cancellation of ρ-divergences in full amplitude [HIGH-ENERGY FACTORIZATION]

5. Remove a term $\frac{1}{2}\ln^2(s/s_0)[\omega^{(1)}(t)]^2$ (logs arise from $s^\omega = 1 + \omega \ln s + \frac{1}{2!}\omega^2 \ln^2 s + \cdots, \quad \omega(t) = \omega^{(1)}(t) + \omega^{(2)}(t) + \cdots$)

This is an example of a general procedure

The Subtraction Procedure

- In Lipatov's action, interactions between partons and reggeons assumed to occur at $\Delta y < \eta \ll \ln s$ (locality in rapidity)
- QMRK clusters connected by reggeon propagators (non-local in rapidity)

However, when considering loops...

☐ The constraint has to be enforced in the momentum integrals, e.g. with a cutoff in rapidity [Bartels, Hentschinski & Lipatov'07]

☐ Alternatively, subtract non-local contributions, mediated by a reggeized gluon. In our case...

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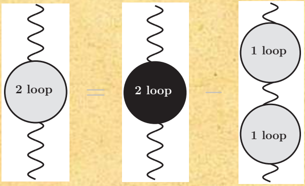

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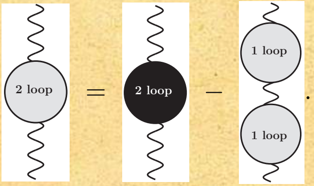
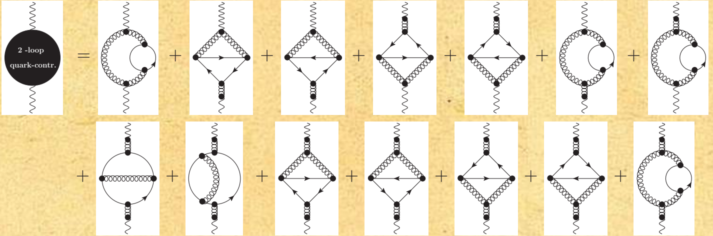


2-Loop Gluon Trajectory: Quark Part

Contributions to Unsubtracted 2-Loop Gluon Self-Energy



(only first diagram ρ-enhanced)

Subtractions

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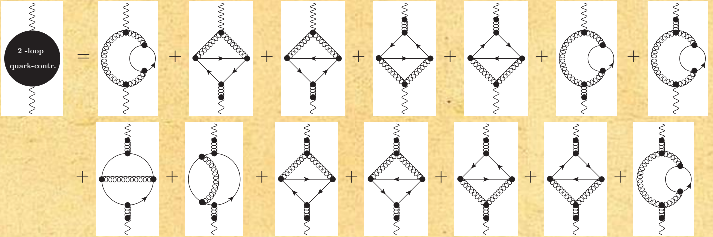

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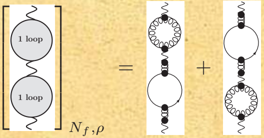

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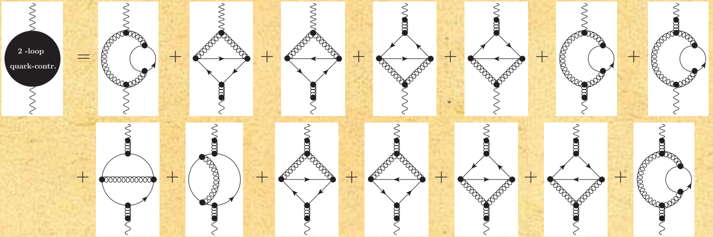

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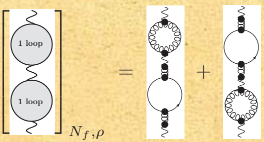

Result of the Computation

Exact Agreement with Previous Two-Loop Computation

[Fadin, Fiore & Kotsky'96; Fadin, Fiore & Quartarolo'96; Blümlein, Ravindran & van Neerven'98; Del Duca & Glover'01]

$$\omega_{n_f}^{(2)}\left(\epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right) = \bar{g}^4 \left(\frac{\boldsymbol{q}^2}{\mu^2}\right)^{2\epsilon} \frac{4n_f}{\epsilon N_c} \frac{\Gamma^2(2+\epsilon)}{\Gamma(4+2\epsilon)} \times \left[\frac{2\Gamma^2(1+\epsilon)}{\epsilon\,\Gamma(1+2\epsilon)} - \frac{3\Gamma(1-2\epsilon)\Gamma(1+\epsilon)\Gamma(1+2\epsilon)}{\epsilon\,\Gamma^2(1-\epsilon)\Gamma(1+3\epsilon)}\right];$$

$$\bar{g}^2 = \frac{g^2 N_c \Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}, \quad d = 4+2\epsilon$$

The procedure has been checked with the quark impact factor

[Hentschinski & Sabio Vera'11]

and the gluon impact factor

[Chachamis, Madrigal, Hentschinski & Sabio Vera'11, to appear soon]

And now the computation of the rest of the trajectory is almost done... [Chachamis, Hentschinski, JDM & Sabio Vera, work in progress]

- More difficult contributions, require a more powerful strategy
  1. Reduction to master integrals using integration by parts codes

     e.g. [Smirnov & Smirnov'08]
  2. Obtention of Mellin-Barnes representations and computation of residues relevant in Regge limit [Smirnov'99]
- General powerful procedure, which can be automatized

Conclusions

☐ Lipatov's effective action is a very powerful tool for computations in the high-energy limit
☐ The proposed regularization-subtraction procedure gives a systematic way to employ this action for loop computations
☐ Quark piece for 2-loop trajectory: exact agreement. Agreement also found for 1-loop jet vertex

Yet to be done...

★ Check further the procedure (e.g. computation of gluon jet vertex)
★ Automatization of the computation