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Transverse momentum dependent distributions and Q^2 evolution

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From a phenomenological point of view ...

The exploration of the 3-dimensional structure of the nucleon, both in momentum and in configuration space, is one of the major issues in high energy hadron physics.

Information on the 3-dimensional structure of the nucleon is embedded in the Transverse Momentum Dependent distribution and fragmentation functions (TMDs).

As explained
by E. Aschenauer
G. Schnell and M. Radici
in their talks

Huge amount of experimental data on spin asymmetries in several different processes show that TMD distribution and fragmentation functions exist and are non zero.

From a phenomenological point of view ...

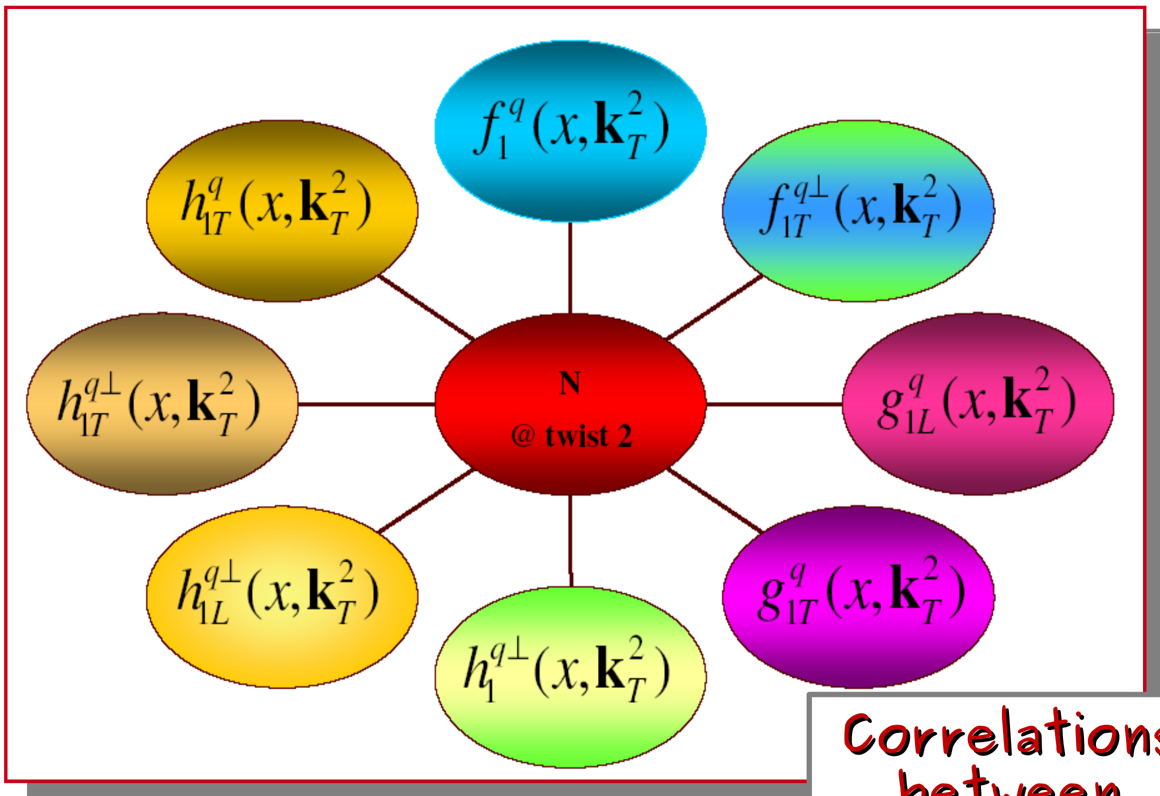
In a very simple phenomenological approach, cross sections and spin asymmetries are generated, in a factorized scheme, as convolutions of distribution and (or) fragmentation TMDs with elementary scattering cross sections.

This simple approach can successfully describe a wide range of experimental data.

As explained
by E. Aschenauer
G. Schnell and M. Radici
in their talks

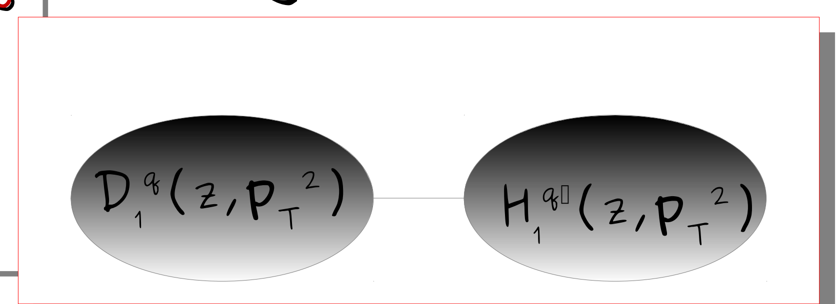
TMD distribution and fragmentation functions

Distribution



Correlations between spin and transverse momentum

Fragmentation

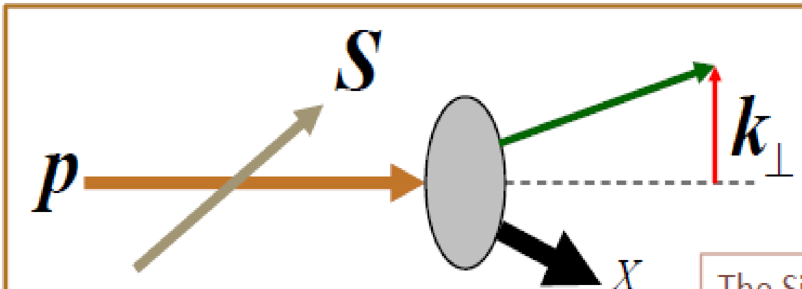


Sivers Distribution Function

$$f_{q/p,S}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

The Sivers function is related to the probability of finding an unpolarized quark inside a transversely polarized proton



The Sivers function inbeds the correlation between the proton spin and the quark transverse momentum

The Sivers function, is particularly interesting, as it provides information on the partonic orbital angular momentum

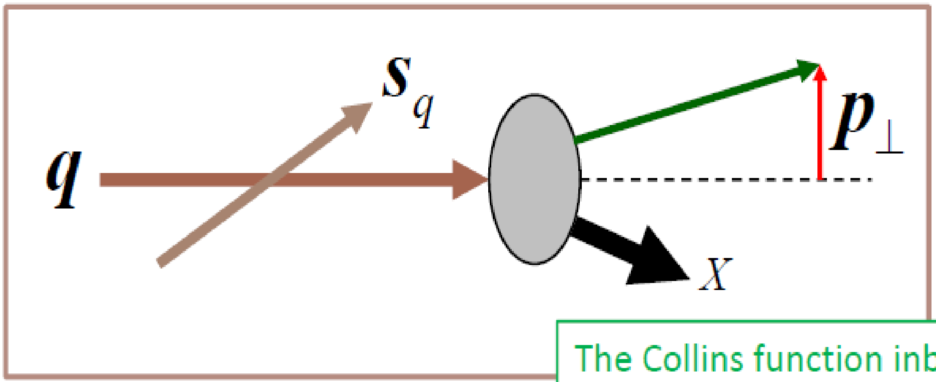
Collins Fragmentation Function

$$D_{h/q,s_q}(z,\boldsymbol{p}_\perp) = D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

$$= D_{h/q}(z,p_\perp) + \frac{p_\perp}{z\,M_h} H_1^{\perp q}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$

The Collins function is related to the probability that a transversely polarized struck quark will fragment into a spinless hadron

The Collins function is chirally odd



The Collins function inbeds the correlation between the fragmenting quark spin and the transverse momentum of the produced hadron

Transversity

- The transversity distribution function contains basic information on the spin structure of the nucleons.

- Being related to the expectation value of a chiral odd operator, it appears in physical processes which require a quark helicity flip; therefore it cannot be measured in usual DIS.

- Drell-Yan → planned experiments in polarized pp at PAX.

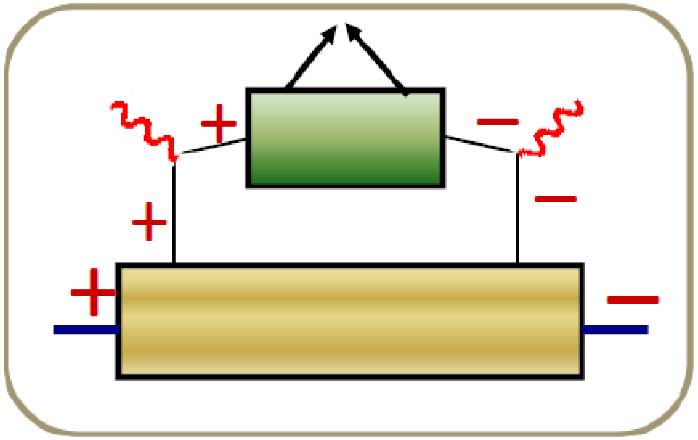
- At present, the only chance of gathering information on transversity is **SIDIS**, where it appears associated to the Collins fragmentation function.

- **DOUBLE PUZZLE**: we cannot determine transversity if we do not know the Collins fragmentation function.

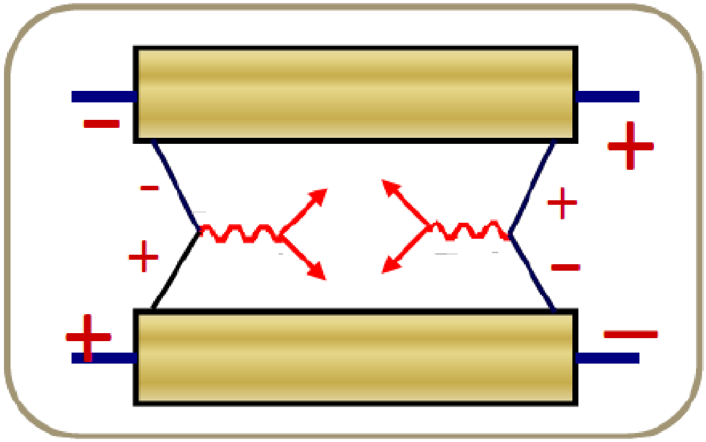
Transversity

- There is no gluon transversity distribution function
- Transversity cannot be studied in deep inelastic scattering because it is chirally odd
- Transversity can only appear in a cross-section convoluted to another chirally odd function

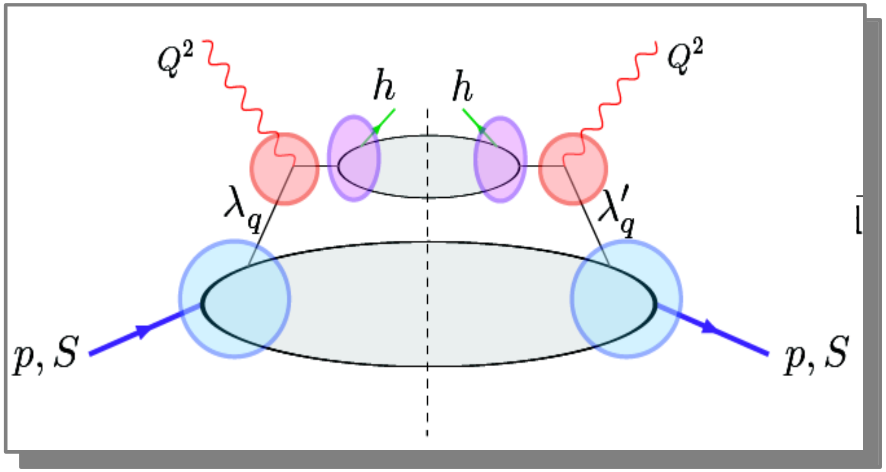
SIDIS



Drell -Yan



Factorization in SIDIS



TMD factorization holds at large Q^2 and $P_T \simeq k_\perp \simeq \Lambda_{QCD}$

Two scales: $P_T \ll Q^2$

(Collins, Soper, Ji, Ma, Yuan, Qiu, Vogelsang, Collins, Metz)

$$\frac{d\sigma^{\ell(S_\ell)+p(S)\to\ell'+h+X}}{dx_B\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S} = \rho^{\ell,S_\ell}_{\lambda_\ell,\lambda'_\ell} \otimes \rho^{q/p,S}_{\lambda_q,\lambda'_q} \hat{f}_{q/p,S}(x,\boldsymbol{k}_\perp) \otimes \hat{M}_{\lambda_\ell,\lambda_q;\lambda_\ell,\lambda_q} \hat{M}^*_{\lambda'_\ell,\lambda'_q;\lambda'_\ell,\lambda'_q} \otimes \hat{D}^h_{\lambda_q,\lambda'_q}(z,\boldsymbol{p}_\perp)$$

TMD-PDF — **hard scattering** — **TMD-FF**

Unpolarized TMDs

TMD parametrizations

- In the Torino-Cagliari standard approach, TMDs are parametrized in a form in which the x and $k_\perp$ dependences are factorized and only the collinear part evolves in Q

Unpolarized TMD PDF

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

Collinear PDF

(DGLAP evolution)

Normalized Gaussian

(no evolution)

Unpolarized TMD FF

$$D_{h/q}(z, p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle},$$

Collinear FF (DGLAP evolution)

Normalized Gaussian (no evolution)

TMD parametrizations

Two key parameters in the parametrization of the unpolarized TMD distribution and fragmentation TMDs.

$$\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2, \qquad \langle p_\perp^2 \rangle = 0.20 \text{ GeV}^2.$$

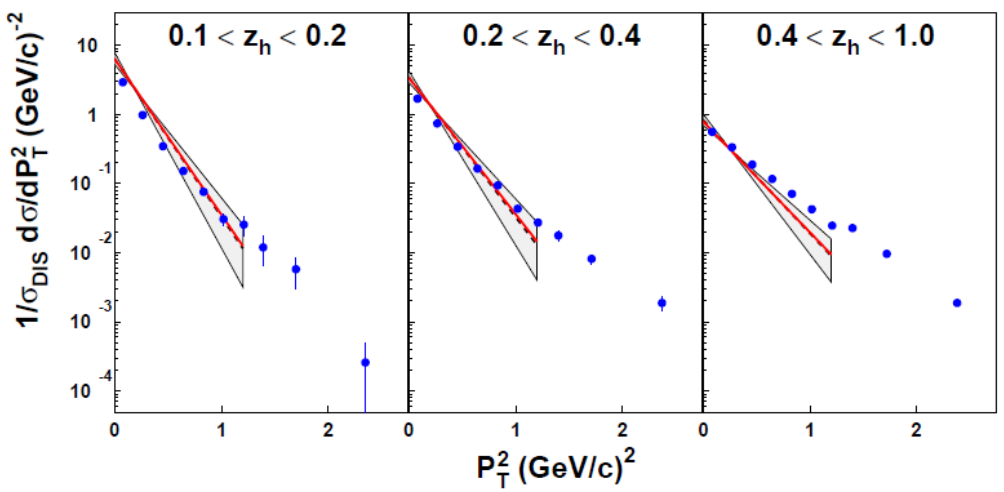
These parameters where extracted in 2005 mostly from EMC data. It is now about time to perform a new extraction of unpolarized TMDs with the outstanding high precision multiplicity data from the COMPASS and HERMES Collaborations, who have recently published their multiplicity measurements.

C. Adolph et al., arXiv:1305.7317 [hep-ex]

A. Airapetian et al., Phys. Rev. D87 (2013) 074029

Extracting the unpolarized TMD Gaussian widths from SIDIS data

Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin, Phys.Rev. D71 (2005) 074006

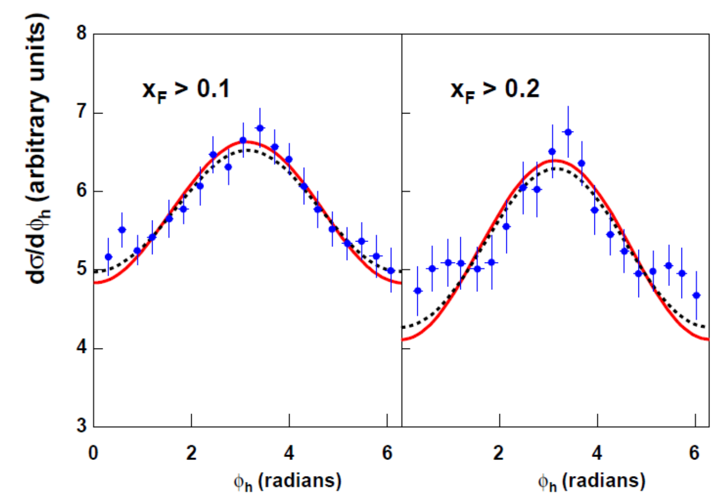


$$f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle},$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle},$$

$$\langle k_\perp^2 \rangle = 0.25 \ (\mathrm{GeV}/c)^2$$

$$\langle p_\perp^2 \rangle = 0.20 \ (\mathrm{GeV}/c)^2$$



Extracting the unpolarized TMD Gaussian widths from SIDIS data

- Data: Hermes (p and d targets, π^+, π^-, K^+, K^- production)

  1341 data points in (x, z, P_T, Q^2 bins)

  C. Adolph et al., arXiv:1305.7317 [hep-ex]

  Compass (d target, h^+, h^- production)

  18627 data points in (x, z, P_T, Q^2 bins)

  A. Airapetian et al., Phys. Rev. D87 (2013) 074029

- Parameterizations:

$$\widehat{f}_{q/p}(x,k_\perp;Q) = f_{q/p}(x;Q)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle},$$

CTEQ6L : (DGLAP evolution)

1 free parameter (no evolution)

DSS (DGLAP evolution)

1 free parameter (no evolution)

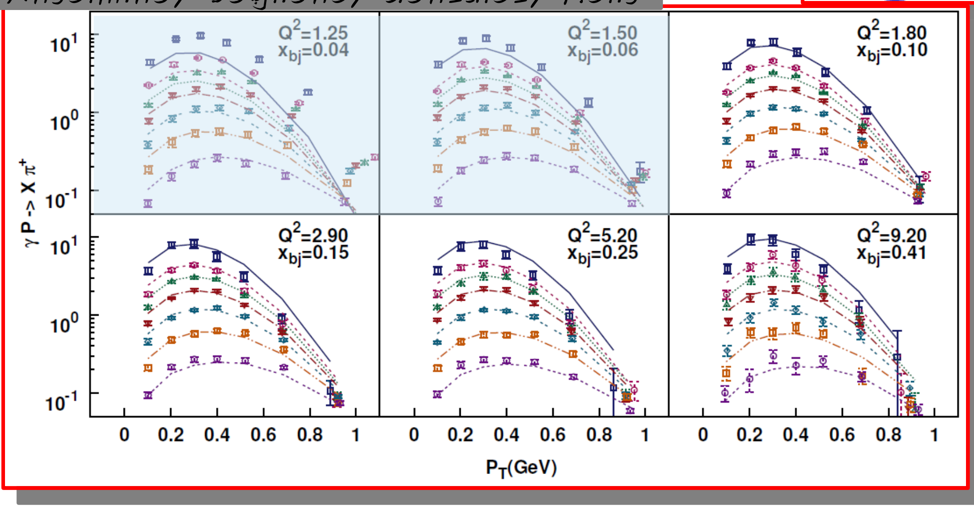
Phenomelogical study of SIDIS multiplicities

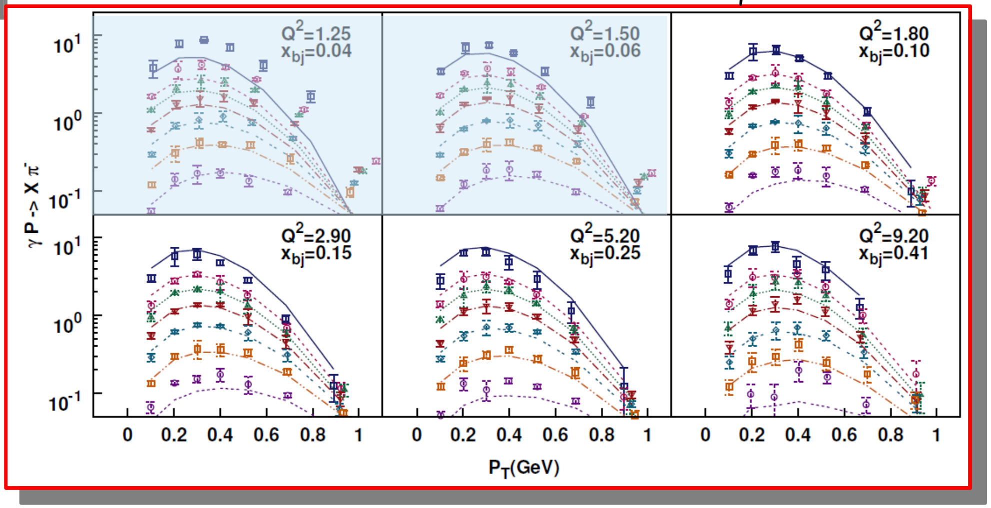
Preliminary

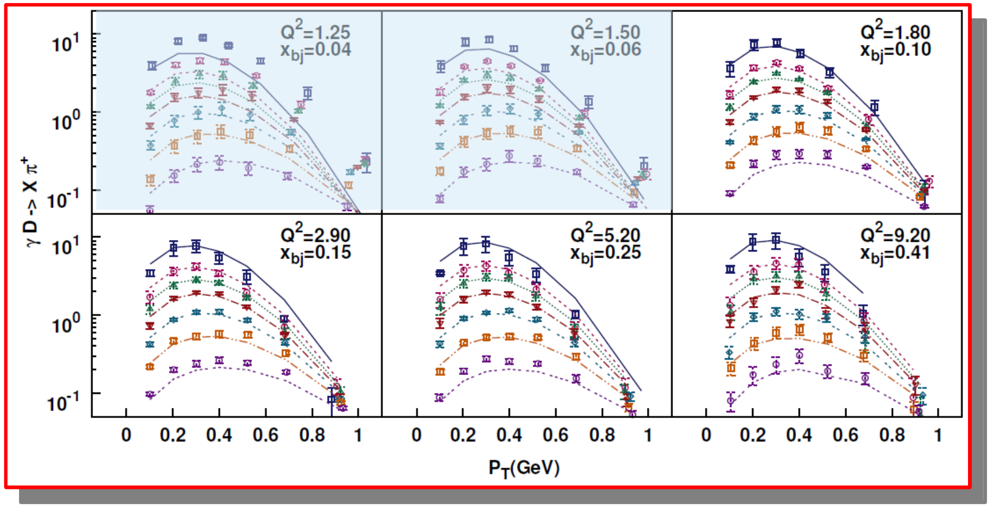
Anselmino, Boglione, Gonzalez, Melis

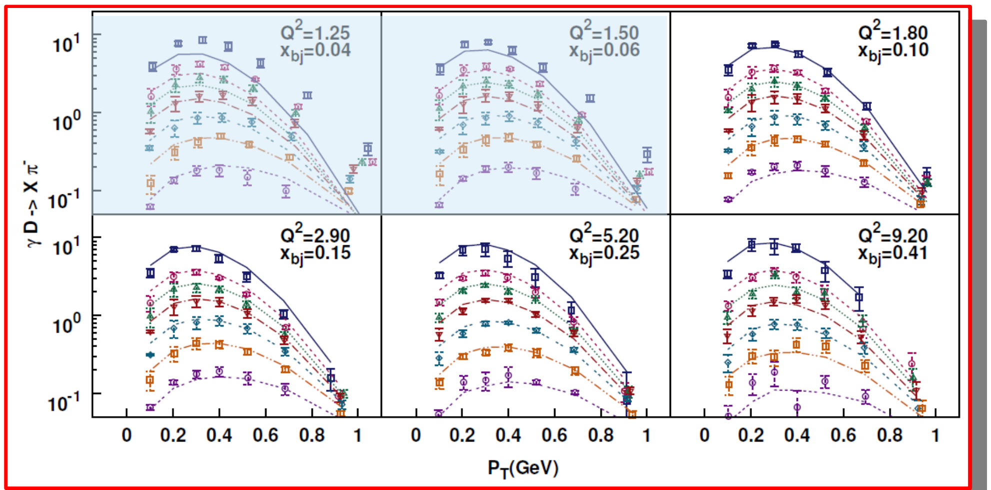
hermes

Our cuts: $Q^2 > 1.6$ GeV^2, $z < 0.7$, $P_T/Q < 1.0$









Extracting the unpolarized TMD Gaussian widths from SIDIS data

Preliminary

Anselmino, Boglione, Gonzalez, Melis

- HERMES fit

cuts: $Q^2 > 1.6$, $z < 0.7$, $P_T/Q < 1$

n. of fitted data points: 780

$\chi^2_{point} = 4.92$

$\langle k_\perp^2 \rangle = (0.39 \pm 0.15)\ GeV^2$

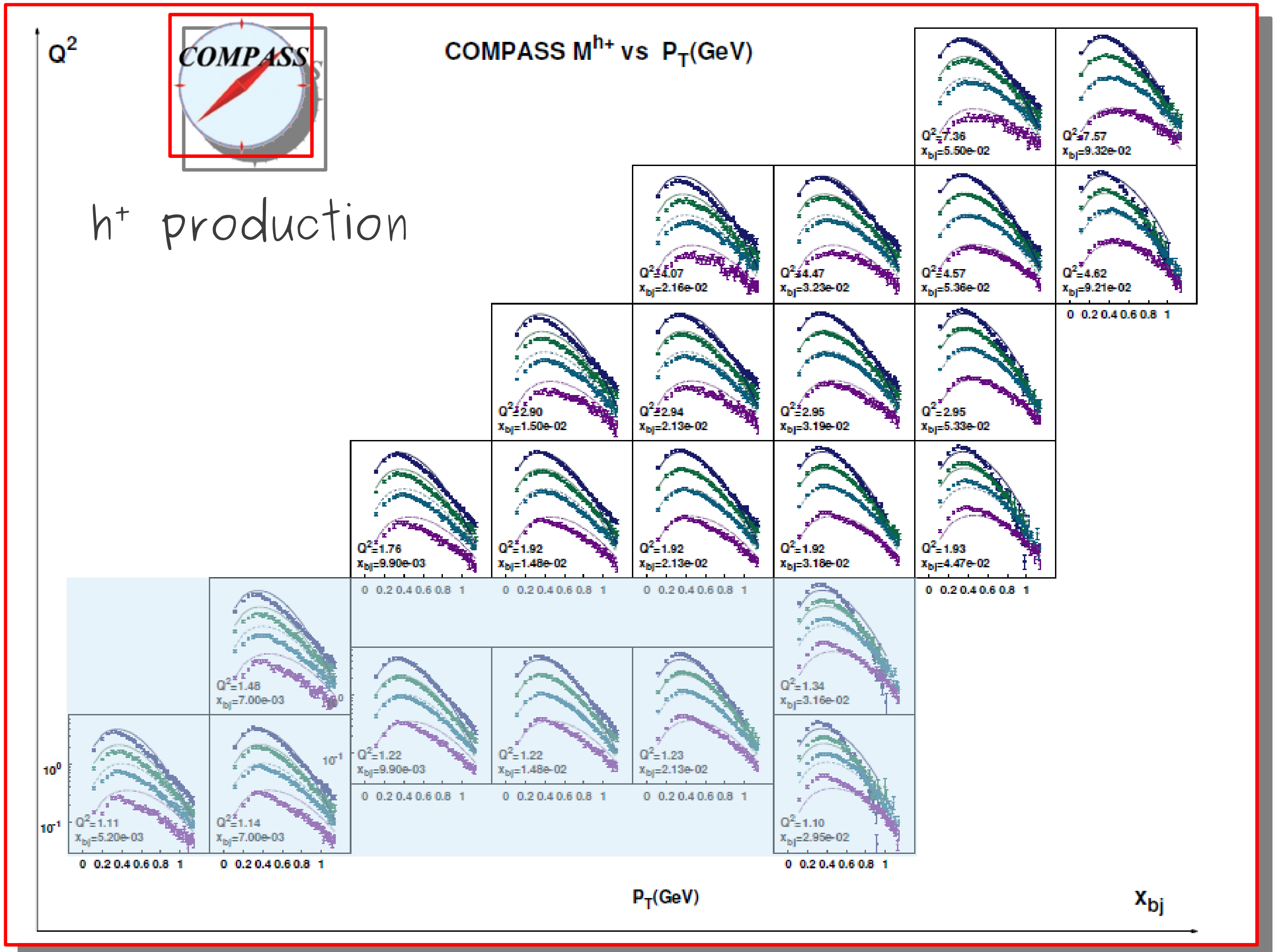
$\langle p_\perp^2 \rangle = (0.15 \pm 0.02)\ GeV^2$

Phenomelogical study of SIDIS multiplicities

Preliminary

Anselmino, Boglione, Gonzalez, Melis

Our cuts: $Q^2 > 1.6$ GeV^2 $\quad z < 0.7 \quad P_T/Q < 1.0$



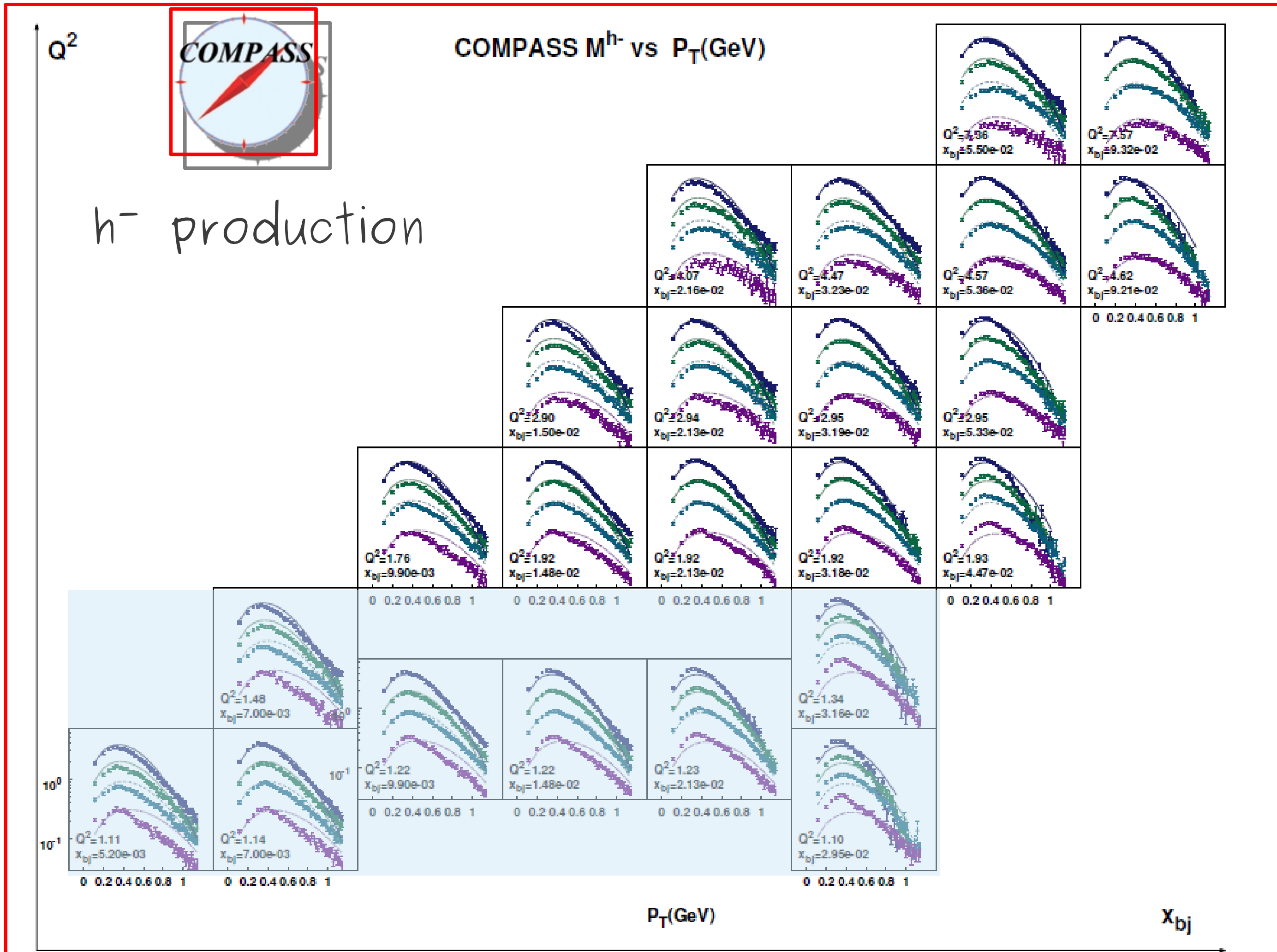
Phenomelogical study of SIDIS multiplicities

Preliminary

Anselmino, Boglione, Gonzalez, Melis

Our cuts: $Q^2 > 1.6\ GeV^2$

$z < 0.7$ $\quad P_T/Q < 1.0$



Extracting the unpolarized TMD Gaussian widths from SIDIS data

Preliminary

Anselmino, Boglione, Gonzalez, Melis

- COMPASS fit

cuts: $Q^2 > 1.6$, $z < 0.7$, $P_T/Q < 1$

n. of fitted data points: 10640

$\chi^2_{point} = 9.75$

$\langle k_\perp^2 \rangle = (0.49 \pm 0.20)\ GeV^2$

$\langle p_\perp^2 \rangle = (0.22 \pm 0.03)\ GeV^2$

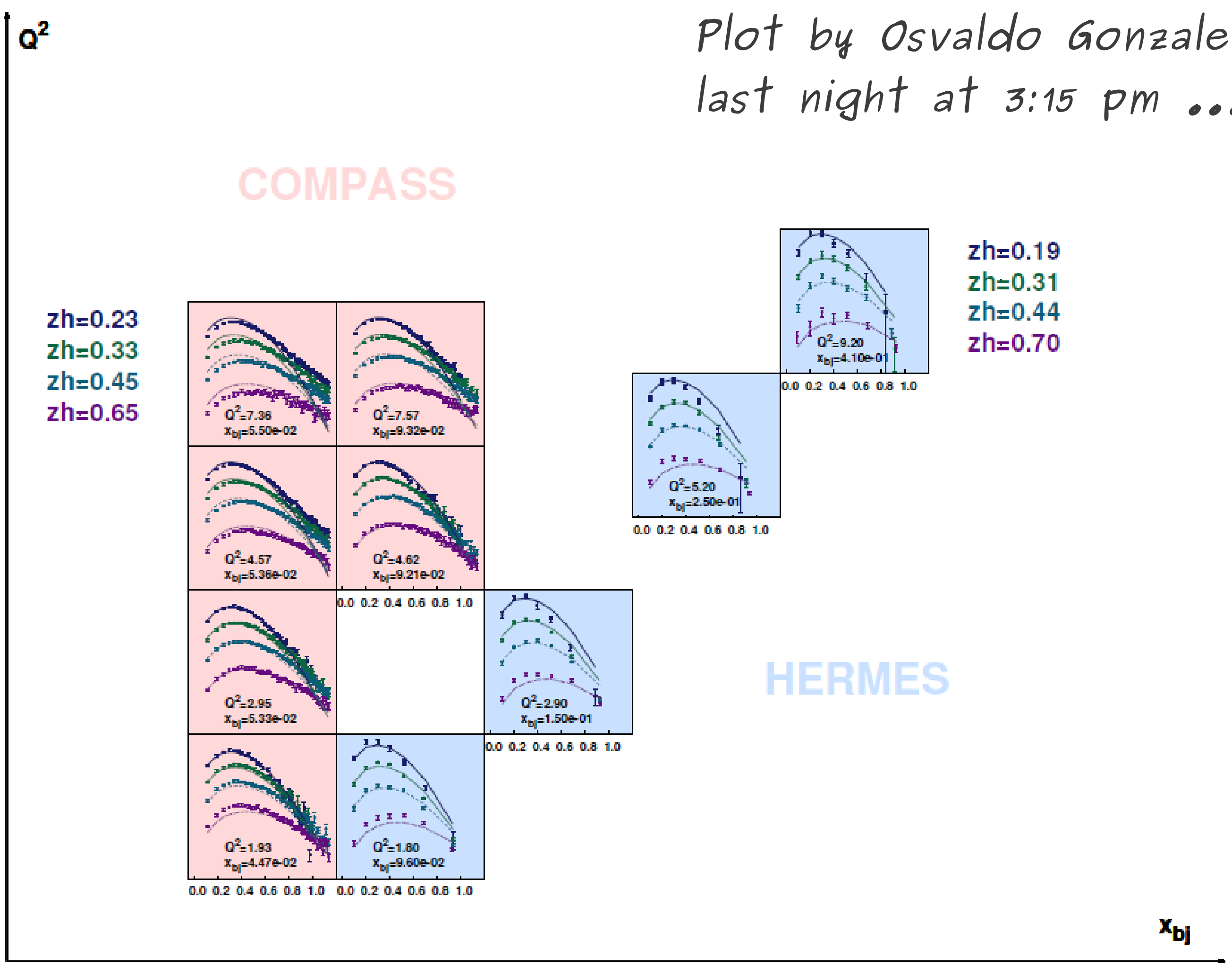
Extracting the unpolarized TMD Gaussian widths from SIDIS data

- HERMES + COMPASS fit

Very preliminary

Anselmino, Boglione, Gonzalez, Melis

Plot by Osvaldo Gonzalez last night at 3:15 pm ...



Extracting the unpolarized TMD Gaussian widths from SIDIS data

Very preliminary
Anselmino, Boglione, Gonzalez, Melis

- HERMES + COMPASS fit

cuts: $Q^2 > 1.68$, $z < 0.7$, $P_T/Q < 1$, $x > 0.08$

n. of fitted data points: 2873 (almost equally divided between HERMES and COMPASS)

$\chi^2_{point} = 7.47$

$\langle k_\perp^2 \rangle = (0.55 \pm 0.21)\ \mathrm{GeV}^2$

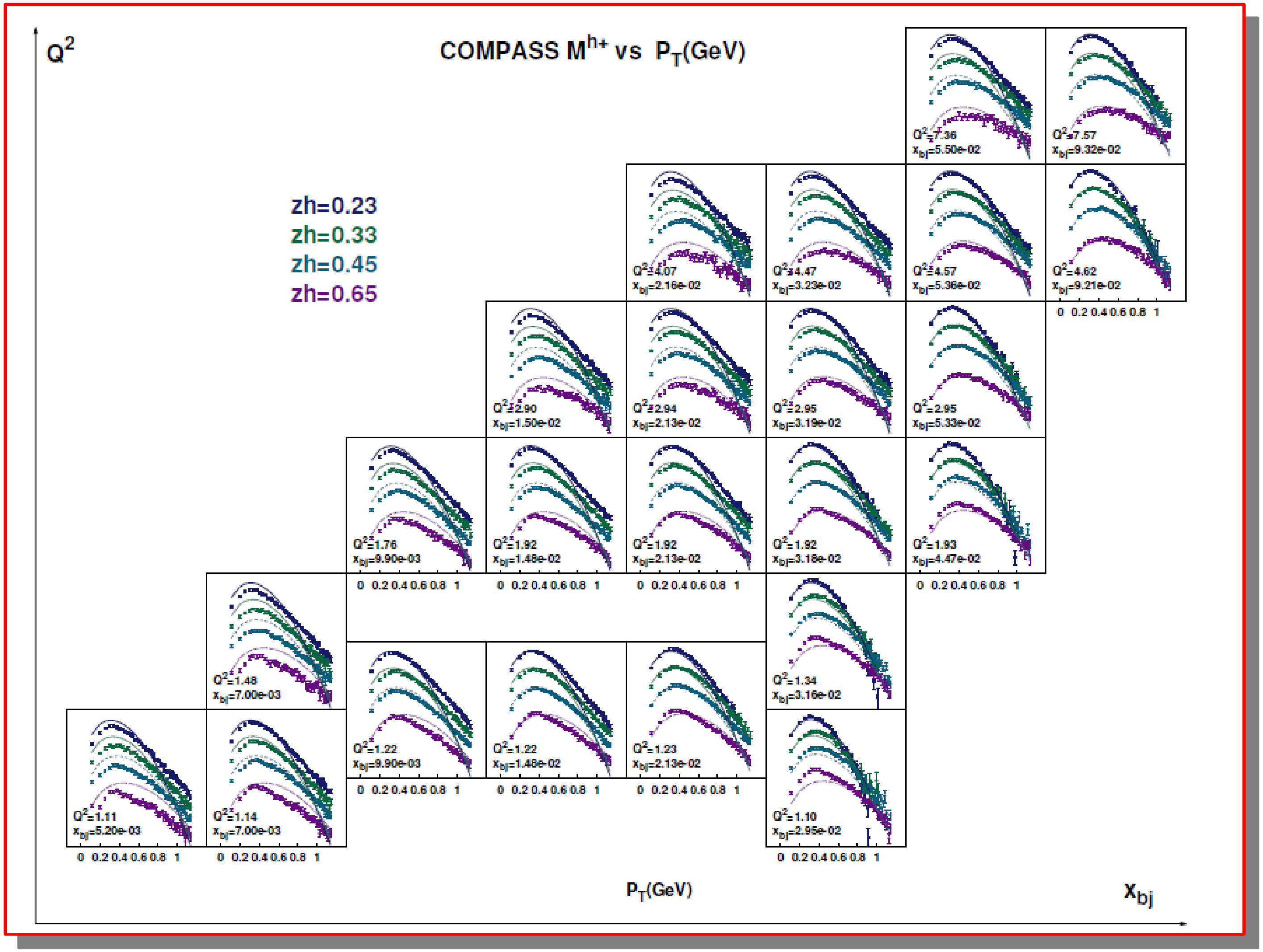
$\langle p_\perp^2 \rangle = (0.16 \pm 0.03)\ \mathrm{GeV}^2$

Extracting the unpolarized TMD Gaussian widths from SIDIS data

- HERMES + COMPASS fit

Very preliminary

Anselmino, Boglione, Gonzalez, Melis



Extracting the unpolarized TMD Gaussian widths from SIDIS data

Any sign of Q^2 evolution ?

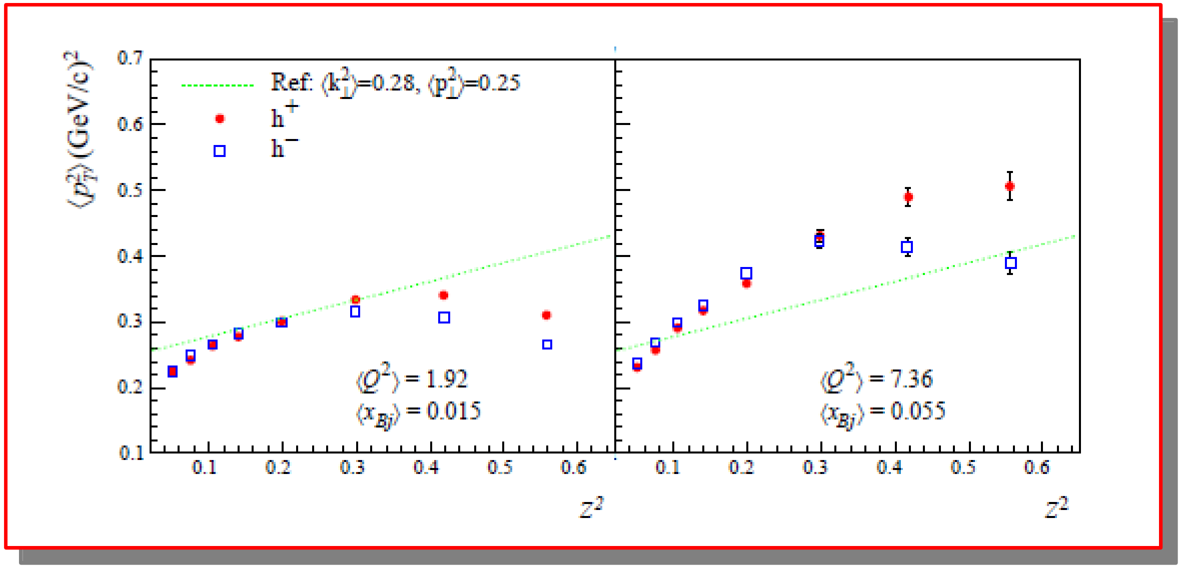
...

More work ...

More parameters ...

Extracting the unpolarized TMD Gaussian widths from SIDIS data

$$\langle p_T^2 \rangle_q = \langle p_\perp^2 \rangle_q + z^2 \langle k_\perp^2 \rangle_q$$



Kinematical integration cuts …

Evolution …

Sivers function
Transversity
Collins function

where "Nature doesn't seem to collaborate ..."

G. Schnell

TMD parametrizations

- In the Torino-Cagliari standard approach TMDs are parametrized in a form in which the x and $k_\perp$ dependences are factorized and only the collinear part evolves in Q

Sivers function

$$\begin{aligned}\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) &= 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q)\\ &= 2\mathcal{N}_q(x)f_{q/p}(x;Q)\sqrt{2e}\frac{k_\perp}{M_1}\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}\end{aligned}$$

Collinear PDF (DGLAP)

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2+\langle k_\perp^2\rangle}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{m_p}f_{1T}^\perp(x,k_\perp)$$

TMD parametrizations

Transversity

$$\Delta_T q(x,k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)[f_{q/p}(x)+\Delta q(x)]\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T},$$

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha(1-x)^\beta\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha\beta^\beta},$$

Collins function

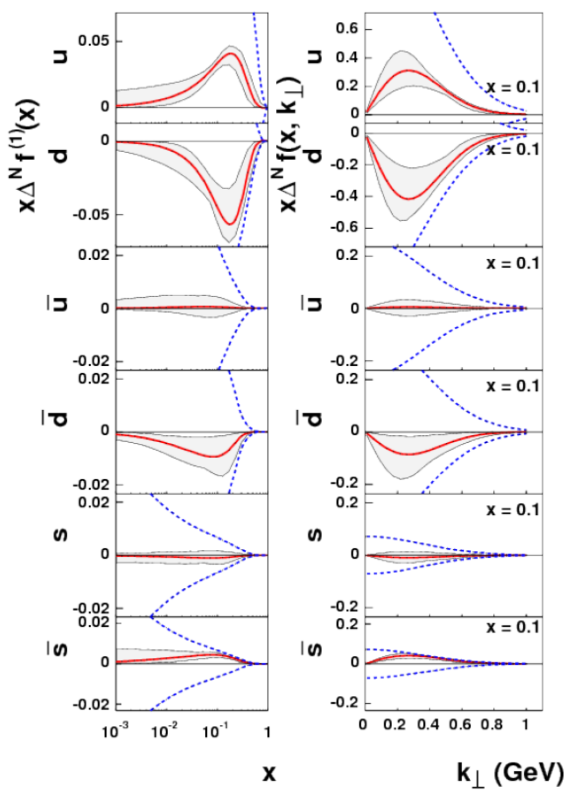
$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)D_{h/q}(z)h(p_\perp)\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle},$$

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma(1-z)^\delta\frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma\delta^\delta},$$

$$h(p_\perp) = \sqrt{2e}\frac{p_\perp}{M}e^{-p_\perp^2/M^2},$$

Extracting the Sivers function from SIDIS data

Anselmino et al, Eur. Phys. J. A39 (2009) 89



✔Valence quark

- $\Delta^N f_{u/p^\uparrow} > 0$ ⇨ $f_{1T}^{\perp u} < 0$
- $\Delta^N f_{d/p^\uparrow} < 0$ ⇨ $f_{1T}^{\perp d} > 0$

✔Sea quarks

- $\Delta^N f_{\bar{s}/p^\uparrow} > 0$ ⇨ $f_{1T}^{\perp \bar{s}} < 0$

→ $\Delta^N f_q^{(1)}(x) \equiv \int d^2 \boldsymbol{k}_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)$

SIDIS data from HERMES and COMPASS (deuterium target) available at that time could be successfully described in this simple scheme

Simultaneous extraction of transversity and Collins function

Old data:

BELLE : 2008 new analysis Phys. Rev. D 78, 032011 (2008)

HERMES: 2005 data, A. Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002.
2007 release, M. Diefenthaler, (2007), arXiv:0706.2242 [hep-ex].

COMPASS-d: 2007 data, E.S. Ageev et al., Nucl. Phys. B765 (2007) 31.
2008 release, M. Alekseev et al., (2008), arXiv:0802.2160 [hep-ex].

New data:

BELLE: 2012 erratum, R. Seidl et al., Phys. Rev. D 86, 039905(E) (2012)

HERMES: 2010 data, Phys. Lett. B 693, (2010)

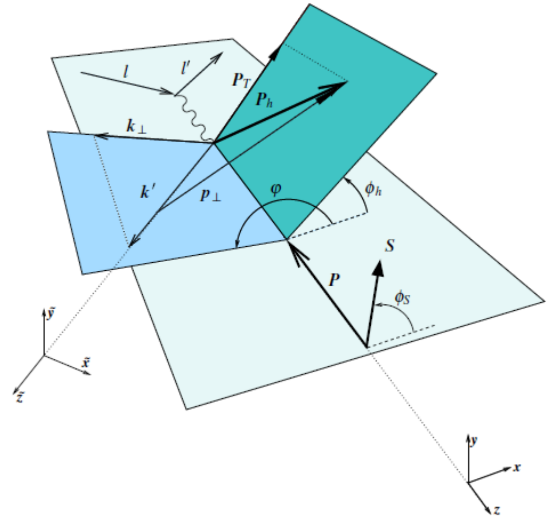
COMPASS-p: 2 runs: 2007 – Phys. Lett. B692 (2012) 240 – no hadron separation)
2010 – Charged hadron production (Transversity 2011)
– Hadron-separated data A. Martin, arXiv:1303.2076

BaBar ?

Simultaneous extraction of transversity and Collins function

Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin, Phys.Rev. D87 (2013) 094019

$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\,\frac{\int d\phi\, d\phi_S\, [d\sigma^{\uparrow} - d\sigma^{\downarrow}]\, \sin(\phi+\phi_S)}{\int d\phi\, d\phi_S\, [d\sigma^{\uparrow} + d\sigma^{\downarrow}]}$$

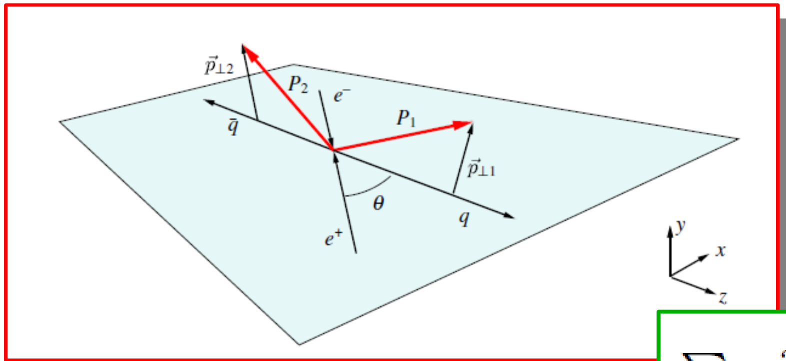


$$d\sigma^{\uparrow} - d\sigma^{\downarrow} = \sum_q h_{1q}(x, k_\perp) \otimes d\Delta\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes \Delta^N D_{h/q^{\uparrow}}(z, \boldsymbol{p}_\perp)$$

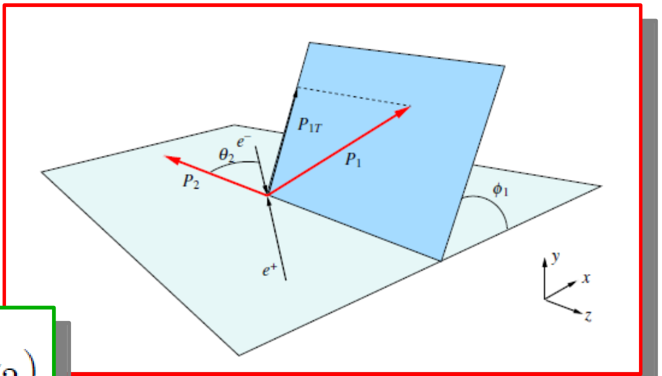
Transversity

Collins

BELLE A_{12} – thrust axis method



BELLE A_0 – hadron plane method

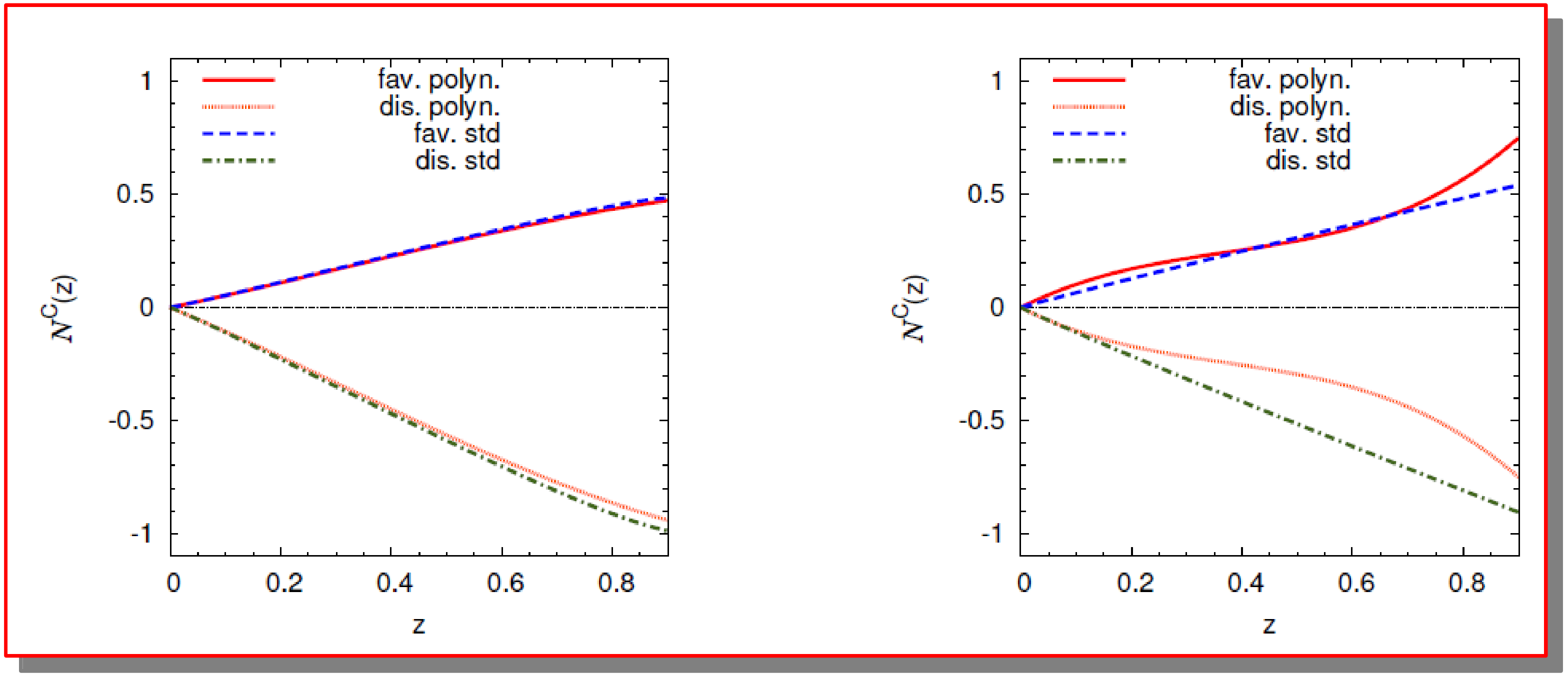


$$\frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^{\uparrow}}(z_1)\, \Delta^N D_{h_2/\bar{q}^{\uparrow}}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

Simultaneous extraction of transversity and Collins function

Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin, Phys.Rev. D87 (2013) 094019

Standard param. of the Collins function

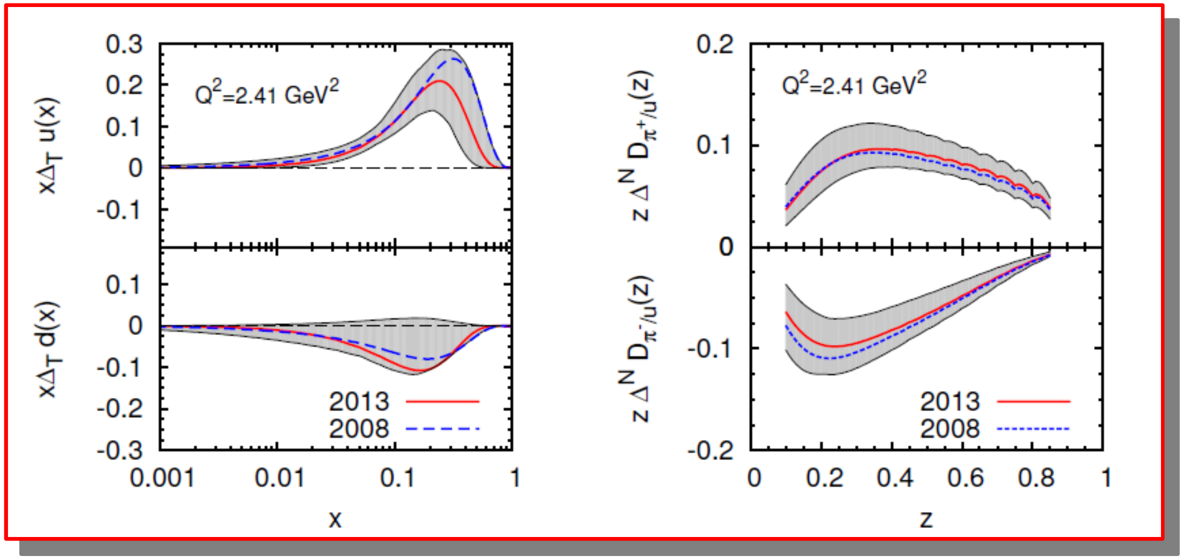


Polynom. param. of the Collins function

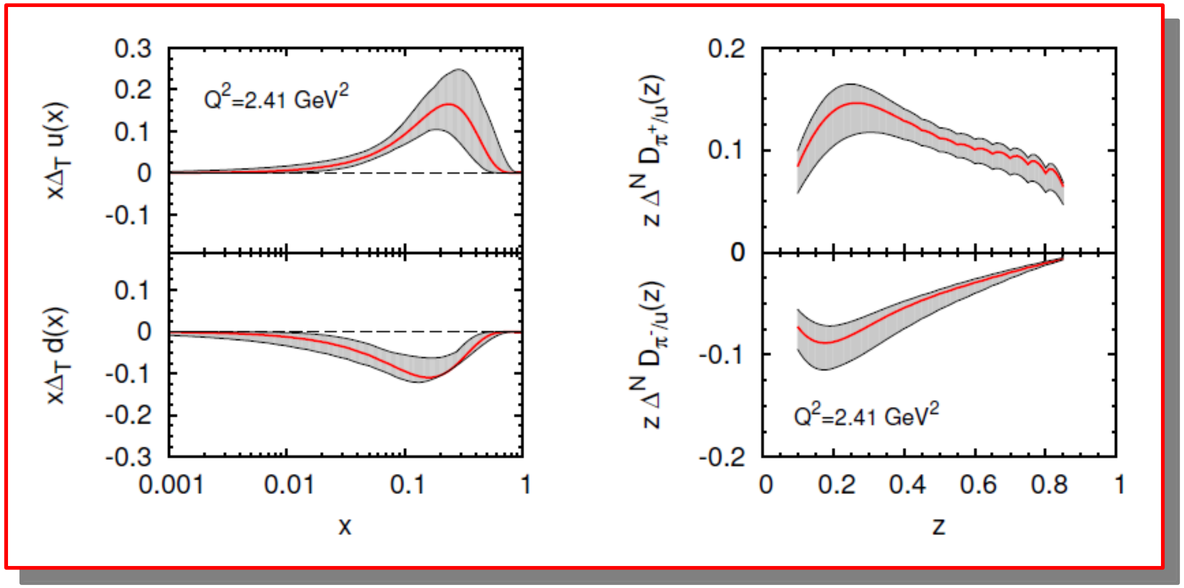
Simultaneous extraction of transversity and Collins function

Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin, Phys.Rev. D87 (2013) 094019

Standard. param. of the Collins function



Polynom. param. of the Collins function



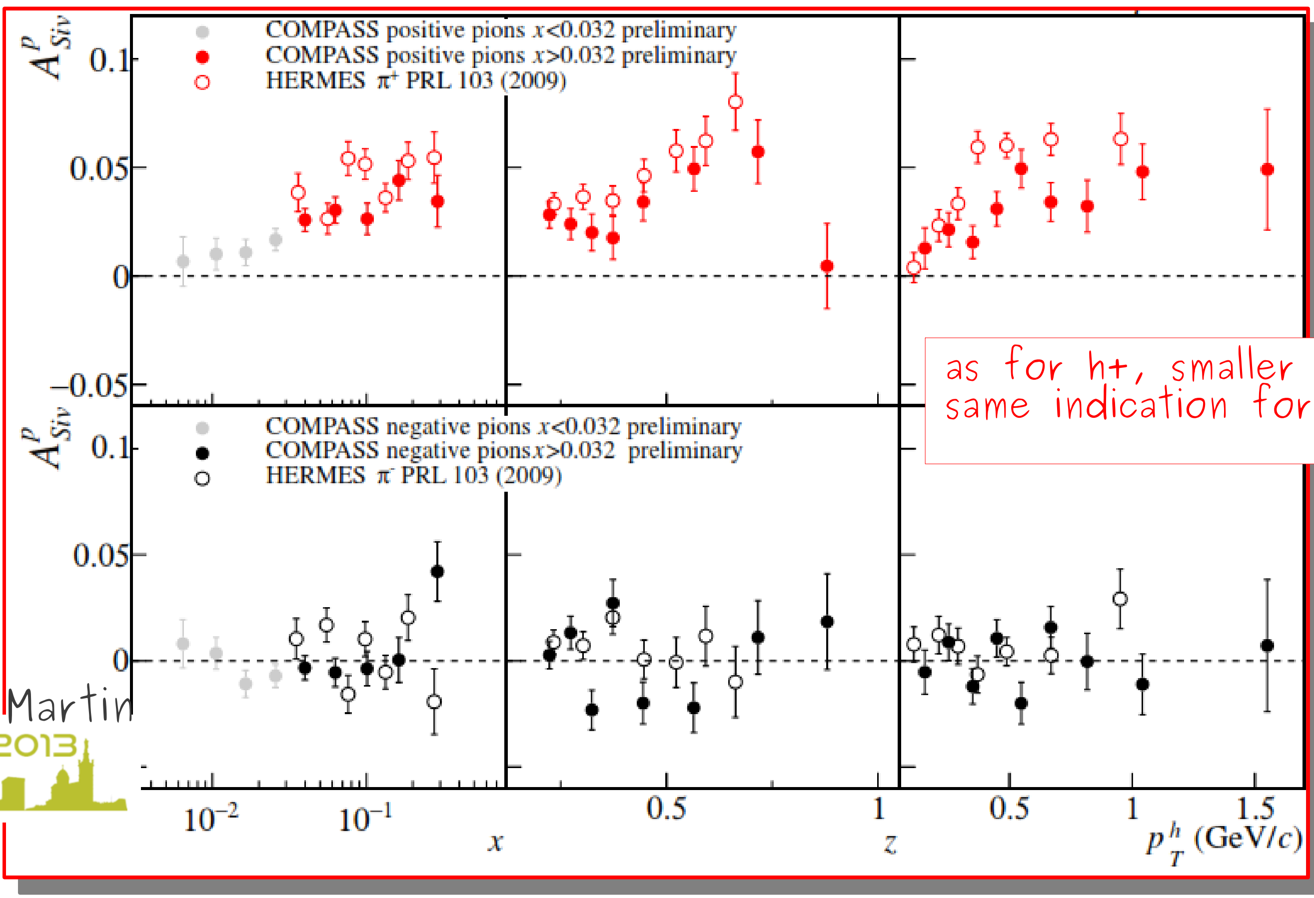
What about TMD evolution phenomenology ?

Do most recent SIDIS data suggest TMD evolution ?

Sivers asymmetry on proton (x > 0.032)

Charged pions (and kaons), 2010 data

Comparison with HERMES results



as for h+, smaller values measured by COMPASS; same indication for K+

π^+

COMPASS ● $\langle Q^2 \rangle$ = 3.2 GeV^2

hermes ○ $\langle Q^2 \rangle$ = 2.4 GeV^2

π^-

COMPASS ● $\langle Q^2 \rangle$ = 3.2 GeV^2

hermes ○ $\langle Q^2 \rangle$ = 2.4 GeV^2

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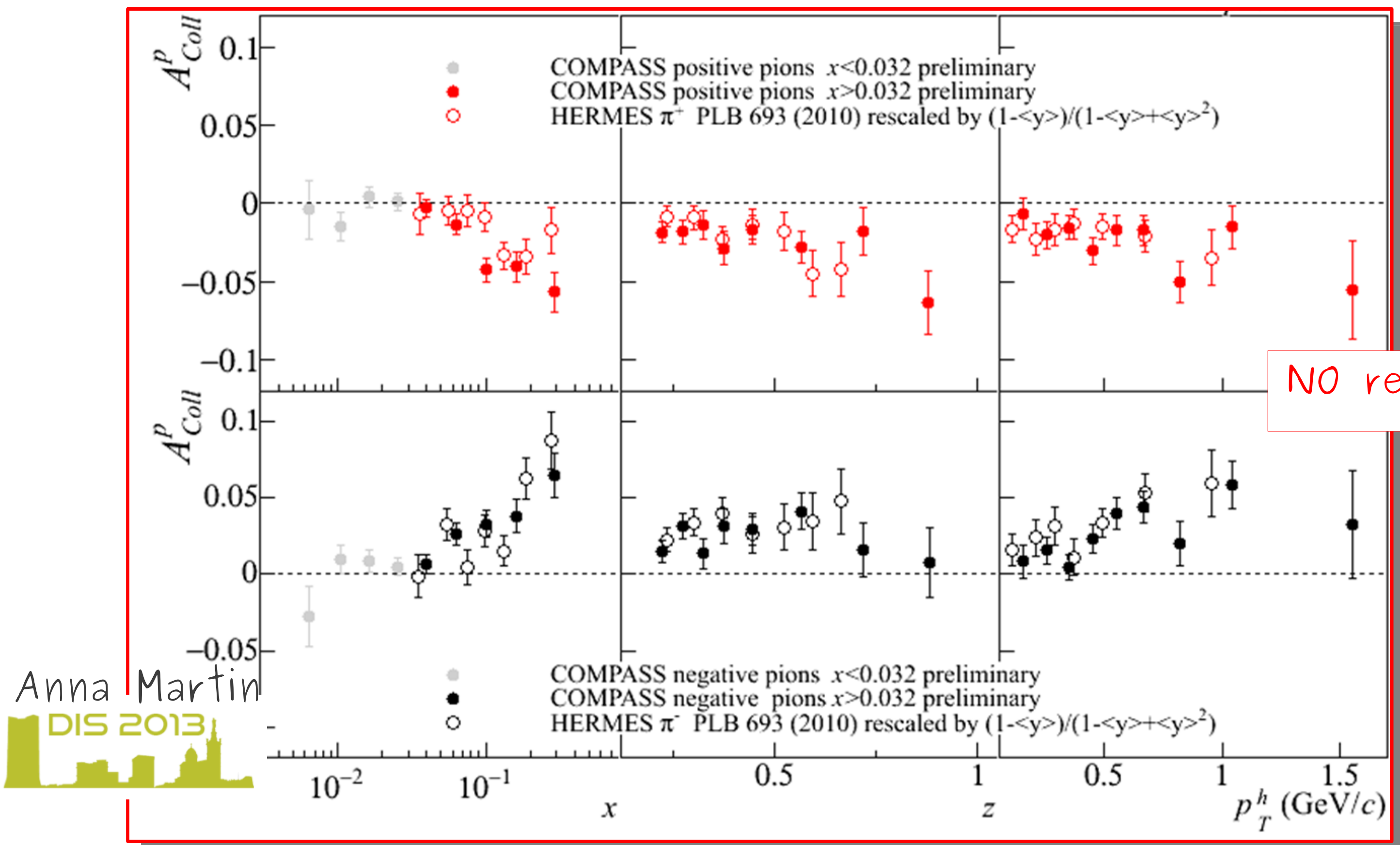

π+

COMPASS ⟨Q²⟩ = 3.2 GeV²

hermes ⟨Q²⟩ = 2.4 GeV²

NO relevant hints of TMD evolution

π-

COMPASS ⟨Q²⟩ = 3.2 GeV²

hermes ⟨Q²⟩ = 2.4 GeV²

TMD evolution literature

- Collins-Soper-Sterman resummation – Nucl. Phys. B250 (1985).
- Idilbi, Ji, Ma, Yuan – Phys.Lett. B 597, 299 (2004) – Phys. Rev. D70 (2004) 074021,
  Ji, Ma, Yuan – Phys. Rev. D71 (2005) 034005.

- John Collins, "Foundations of perturbative QCD" (2011),
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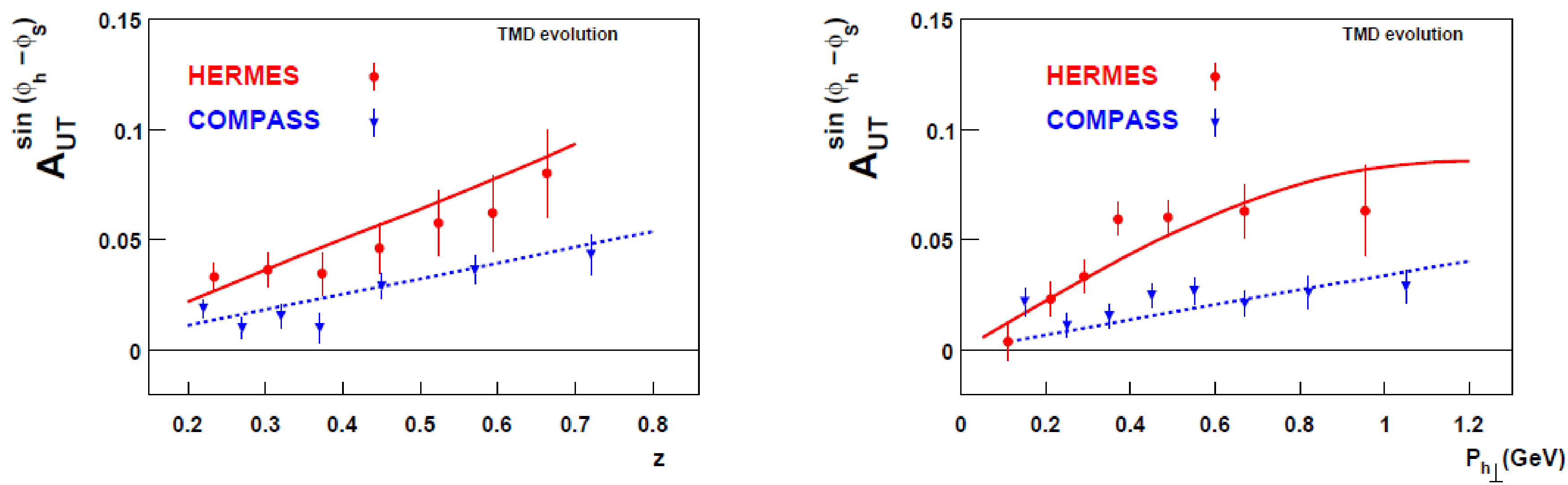
- Aybat, Rogers, Phys. Rev. D83, 114042(2011).
- Aybat, Collins, Qiu, Rogers, Phys. Rev. D85, 034043 (2011).
- Echevarria, Idilbi, Schafer, Scimemi, arXiv:1208.1281
  Echevarria, Idilbi, Scimemi, arXiv:1208.1281

- Aybat, Prokudin, Rogers, Phys. Rev. Lett. 108, 242003
- Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028

- Godbole, Misra, Mukherjee, Raswoot, arXiv:1304:2584
- Sun, Yuan, arXiv:1304.5037
- Boer, arXiv:1304.5387

Phenomenological results

Aybat, Prokudin, Rogers, Phys. Rev. Lett. 108, (2011) 242003



hermes

$\langle Q^2 \rangle = 2.4$ GeV^2

Q^2 in the range [1.3 – 6.2]

- No x dependence taken into account
- Sivers A_{UT} calculated at two fixed different values of Q^2 : 2.4 and 3.8 GeV^2
- Evolution effects are then compared.

COMPASS

$\langle Q^2 \rangle = 3.8$ GeV^2

Q^2 in the range [1.3 – 20.5] GeV^2

TMD evolution phenomenology

Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028

- *First step* : choose a TMD evolution scheme
- Let $F_\perp$ be either an unpolarized distribution or fragmentation function, or the first derivative of the Sivers distribution function, in the **impact parameter space.**
- In general terms, its TMD evolution equation can be written as

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

Aybat, Collins, Qiu, Rogers

TMD evolution phenomenology

Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028

- Let $\tilde{F}$ be either an unpolarized distribution or fragmentation function, or the first derivative of the Sivers distribution function, in the **impact parameter space.**
- In general terms, its TMD evolution equation can be written as

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$\widetilde{F}(x, \boldsymbol{b}_T; Q_0)$: Input function

$g_K(b_T)$: Unknown, but universal and scale Independent, input function

$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{ \ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(\mu') + \int_{Q_0}^{Q} \frac{d\mu}{\mu} \gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right) \right\}$$

$$\gamma_K(\mu) = \alpha_s(\mu) \frac{2\, C_F}{\pi}$$

$$\gamma_F(\mu; \frac{Q^2}{\mu^2}) = \alpha_s(\mu) \frac{C_F}{\pi} \left(\frac{3}{2} - \ln \frac{Q^2}{\mu^2} \right)$$

Parameterization of unknown functions

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$g_K(b_T) = \frac{1}{2}\, g_2\, b_T^2 \qquad \text{with} \qquad g_2 = 0.68$$

$$\widetilde{f}_{q/p}(x,b_T;Q_0) = f_{q/p}(x,Q_0)\exp\{-\alpha^2 b_T^2\}$$

$$f_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\alpha^2 = \langle k_\perp^2\rangle/4$$

g_2 alert !

- g_2 controls the b_T Gaussian width and its spreading as b_T varies.

$$g_K(b_T) = \frac{1}{2}\, g_2\, b_T^2 \qquad \text{with} \qquad g_2 = 0.68$$

$$b_{\max} = 0.5 \text{ GeV}^{-1}$$

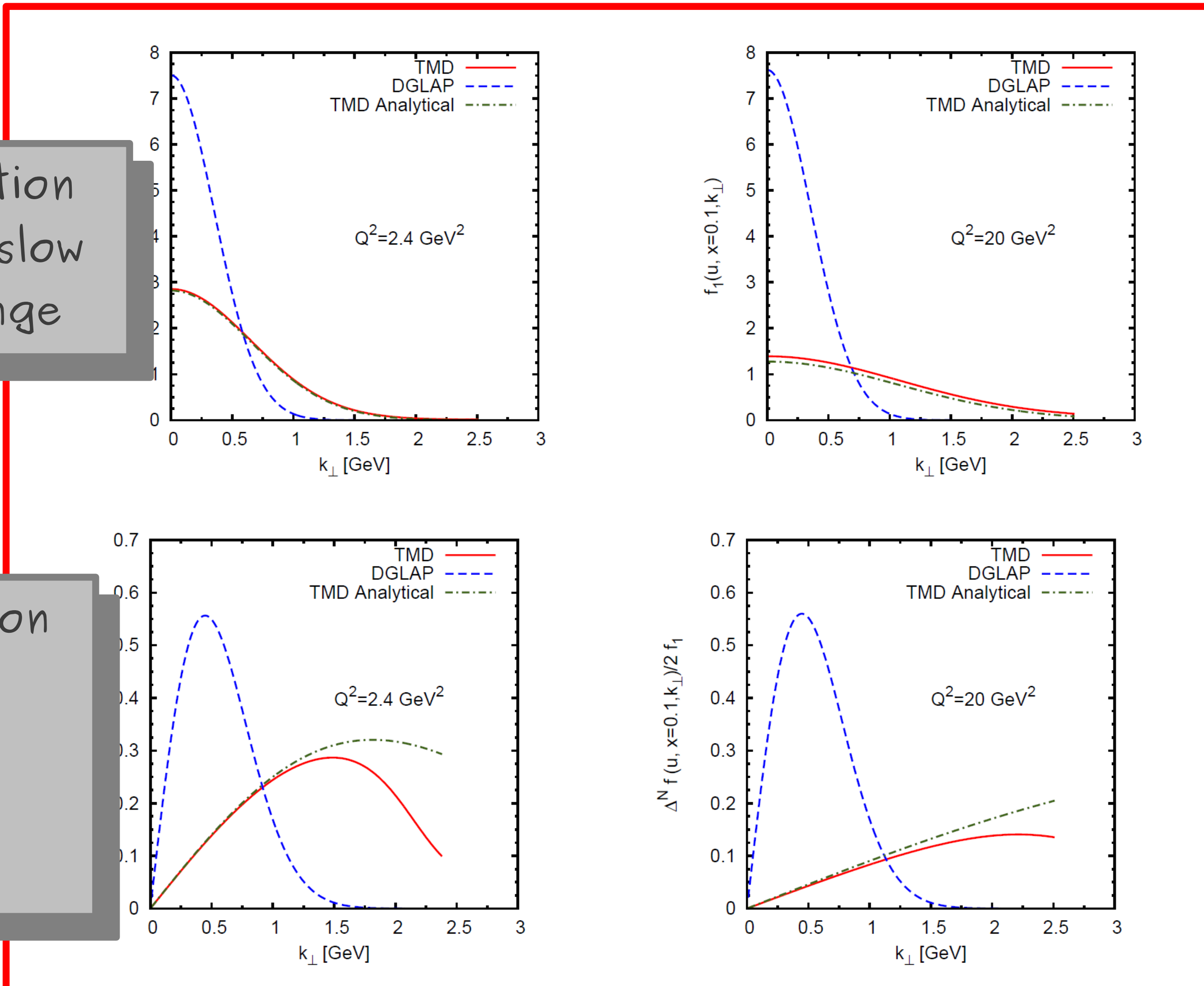
- We do not extract the value of g_2 from our fit
- We use a fixed value, previously determined in a fit of D-Y data. Landry, Brock, Nadolsky, Yuan, Phys. Rev. D67(2003) 073016

  We could have extracted it, and probably got a smaller value, but it is important to remember that **SIDIS data are very little sensitive to the precise value of g_2.**
- D-Y data, instead, are extremely sensitive to it: this requires a new, careful, global analysis on all SIDIS and D-Y, re-starting from unpolarized cross sections.

Phenomenological results

DGLAP evolution is extremely slow in this Q^2 range

TMD evolution Very rapidly widens and dilutes the functions



Sivers function from HERMES and COMPASS SIDIS data

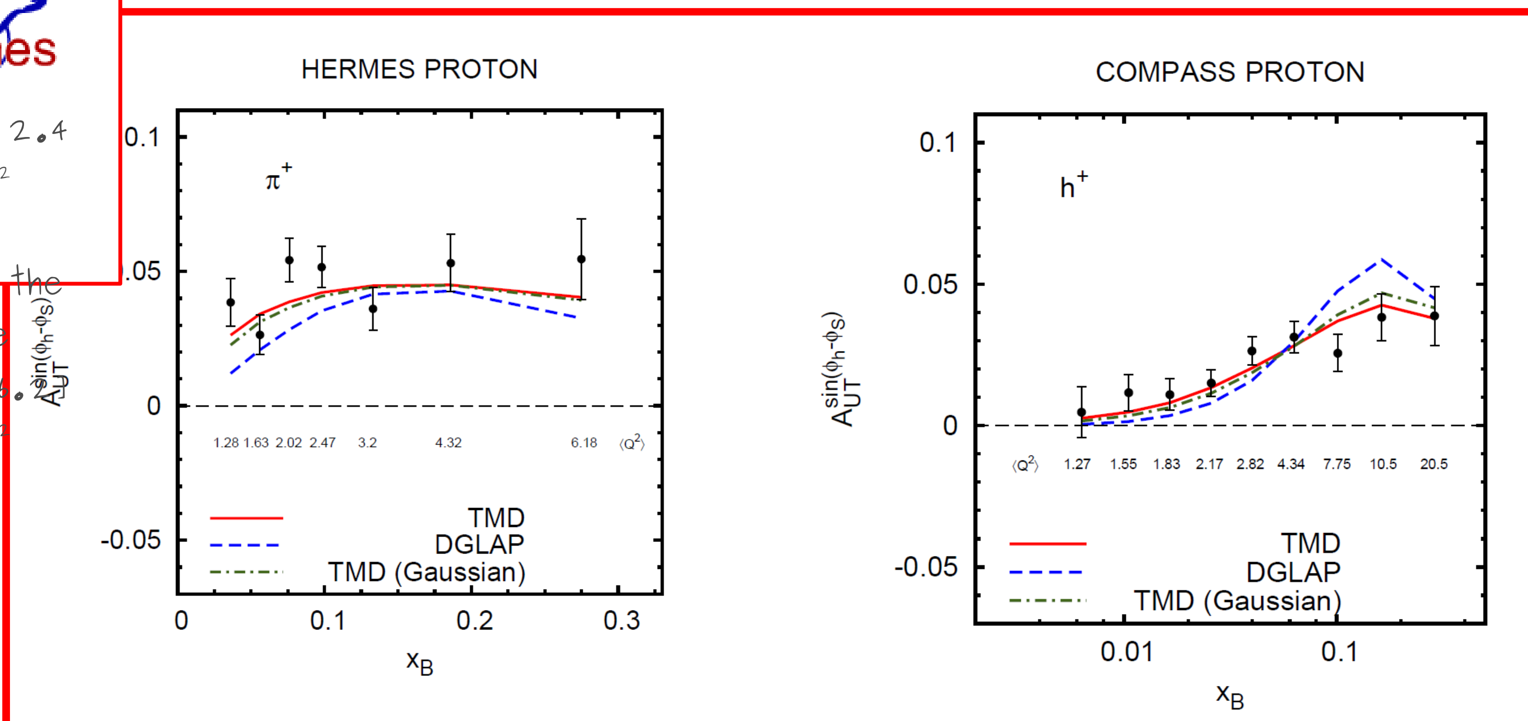
- 2 different fits:
- TMD-fit (computing TMD evolution equations numerically)
- DGLAP evolution equation for the collinear part of the TMD)

hermes

$\langle Q^2 \rangle$ = 2.4 GeV^2

Q^2 in the range [1.3 – 6.2] GeV^2

Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028



COMPASS

$\langle Q^2 \rangle$ = 3.8 GeV^2

Q^2 in the range [1.3 – 20.5] GeV^2

A. Airapetian et al., Phys. Rev. Lett. 103, (2009) 152002

C. Adolph et al., Phys. Lett. B717 (2012) 383

TMD evolution phenomenology of the Collins fragmentation function

- The Collins fragmentation function is chiral odd, so it always appears coupled with another chiral odd function. In SIDIS it is observed in the Collins effect, coupled to transversity.

- TMD evolution equations for the Collins functions where formulated by Zhong-Bo Kang in Phys.Rev. D83 (2011) 036006 while for transversity a recent study has been performed by Bacchetta and Prokudin in arXiv:1303.2129

TMD evolution phenomenology of the Collins fragmentation function

- While for phenomenologically testing the TMD evolution of the Sivers function we have difficulties in finding data which span a sufficiently large range of Q^2 values, for the Collins fragmentation function we have data at large Q as well, from e^+e^- scattering experiments (BELLE and BaBar, Q^2=100 GeV^2), where a convolution of two Collins functions appears.

- First experimental data from COMPASS and HERMES seem to hint to a very slow evolution, however a combined phenomenological study of TMD evolution in SIDIS and e^+e^- scattering would be highly desirable and certainly very interesting.

Drell-Yan studies

C. Riedl
M. Radici

SIDIS vs Drell-Yan

- In these first attempts to perform TMD evolution phenomenology, implications of different evolution schemes have only been tested on a restricted range of Q^2 values
- To perform a serious phenomenological study of TMD evolution we would need SIDIS data in a much larger range of Q^2 values – Electron – Ion Collider
- Meanwhile, we can look at Drell-Yan unpolarized data, which cover a much larger range of Q^2 values
- However, if we apply the parameters extracted from fitting SIDIS spin asymmetries data to the computation of Drell-Yan cross sections, we find a very strong suppression due to the fast $k_\perp$ broadening effects induced by the TMD evolution within the Aybat, Collins, Qiu, Rogers scheme with a large g_2

SIDIS vs Drell-Yan

- Numerator of the asymmetry in analytical approximation for a DY process

$$N_{DY} \propto \Delta^N f(x_1, Q_0) f(x_2, Q_0) \sqrt{2e} \frac{P_T}{M_1} \frac{\langle k_\perp^2 \rangle_{Siv}^2}{\langle k_{\perp 1}^2 \rangle \langle P_T^2 \rangle_{Siv}^2} e^{-P_T^2/\langle P_T^2 \rangle_{Siv}}$$

$$\langle P_T^2 \rangle_{Siv}^{DY} = \omega_{Siv}^2 + \omega_2^2$$

$$w_S^2(Q, Q_0) = \langle k_\perp^2 \rangle_S + 2 g_2 \ln \frac{Q}{Q_0}$$

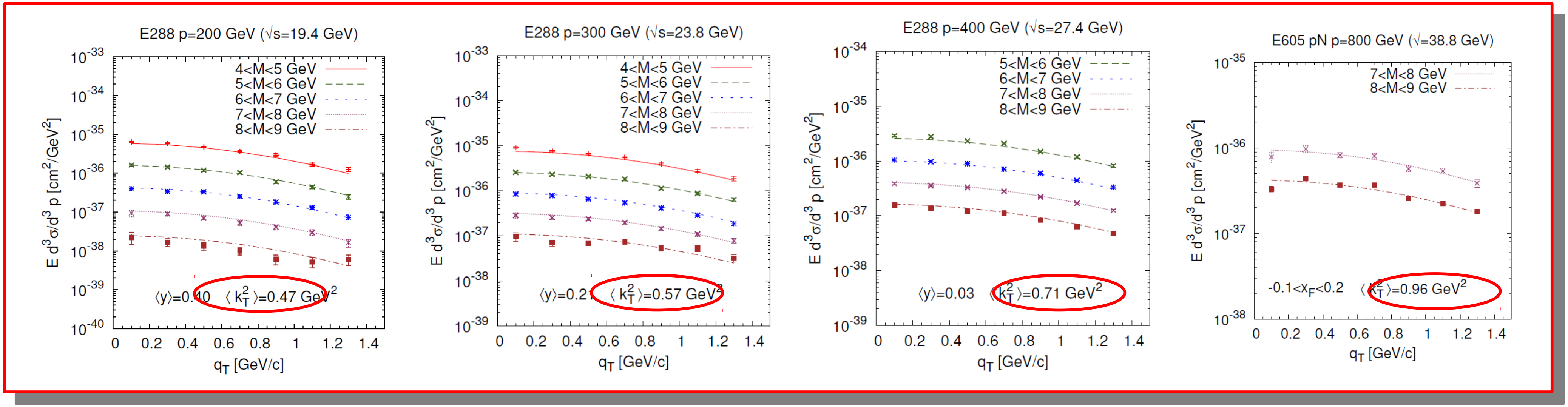
$$w^2(Q, Q_0) = \langle k_\perp^2 \rangle + 2\, g_2 \ln \frac{Q}{Q_0}$$

In Drell-Yan cross section the average value $\langle P_T^2 \rangle$ appears squared, strongly suppressing the asymmetry as it grows larger

- g_2 is more crucial for DY processes than for the present SIDIS data because of the larger range spanned by Q

Drell-Yan phenomenology

Stefano Melis preliminary studies



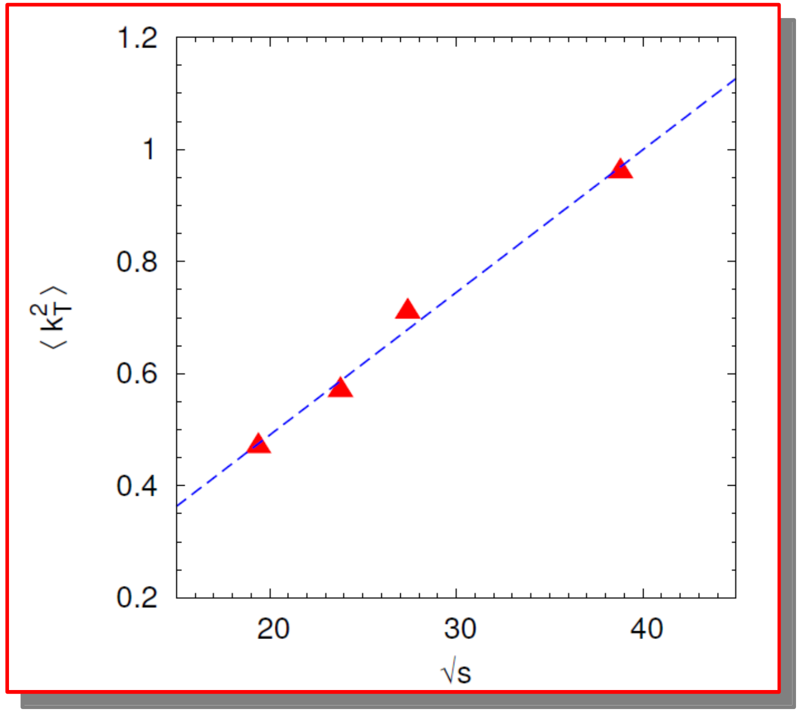
The fit on E288 and E605 Drell-Yan data is performed by assuming a Gaussian $k_\perp$ dependence with a DGLAP evolution of the factorized PDFs

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

The Gaussian width is fitted independently for each different energy data set. Notice that $\langle k_\perp^2\rangle$ grows as energy grows.

Schweitzer, Teckentrup, Metz, Phys.Rev. D81 (2010) 094019
D'Alesio, Murgia, Phys. Rev. D70 (2004) 074009

Stefano Melis preliminary studies



The dependence of $\langle k_\perp^2 \rangle$ on the energy is roughly linear

Loads of work for phenomenologists !

Summary and Outlook

Summary and Outlook

- As far as TMD evolution is concerned we have recently come a long way.
- We now have evolution schemes and some first attempts to the phenomenological study of the unpolarized distribution and fragmentation TMDs, of the TMD transversity and of the Sivers functions.
- These are very preliminary studies, which need to be refined and re-thought in a more consistent and appropriate way, especially as far as the parametrization of unknown phenomenological quantities is concerned.

Summary and Outlook

- With HERMES and COMPASS new experimental data on SIDIS multiplicities, we have to re-think and re-perform a solid, global analysis of Drell-Yan as well as SIDIS unpolarized cross sections, to determine the basic parameters for the phenomenological quantities needed for the implementation of the TMD evolution schemes.

- Afterwards, we can proceed on a firm footing to perform the same analysis for the Sivers, transversity and Collins TMD functions, keeping in mind the importance of finding phenomenological frameworks suitable for all processes.

- Looking forward to the next meeting !!!

You are warmly invited to:

Indiana-Illinois Workshop on Fragmentation Functions

www.indiana.edu/~ffwrkshp

- **December 12-14 2013** at Indiana University, Bloomington, IN
- Organizing Committee: D. Boer, E. Boglione, F. Giordano. M. Grosse-Perdekamp, M. Stratmann and A. Vossen