

# Neutrino diffraction : finite-size correction to Fermi's golden rule

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- **Is neutrino interference (diffraction) observable?**

(1) Neutrino is a quantum mechanical wave and interacts with matter extremely weakly.

(2) Neutrino should show a phenomenon of interference or diffraction, in a double slit-like experiment.

(3) However, it is very hard to control the neutrino , from the above (1). Hence a new method must be considered.

(4) Our answer and proposal ”

**Neutrino diffraction is observable . Use a pion (and other particle) decay process !”**

# Contents of this talk

- References

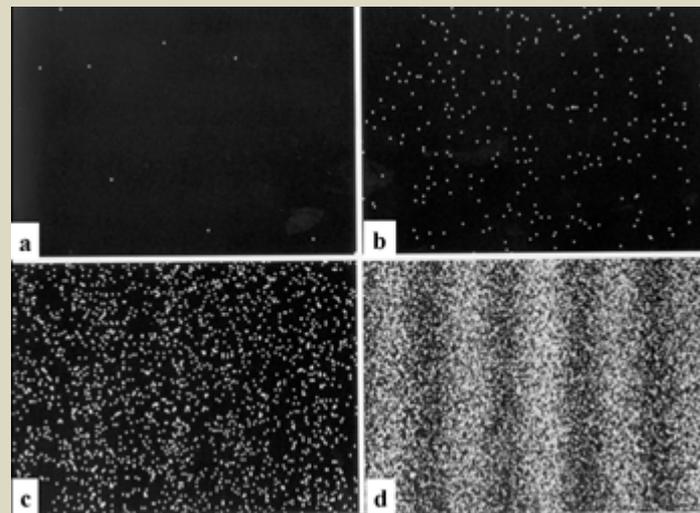
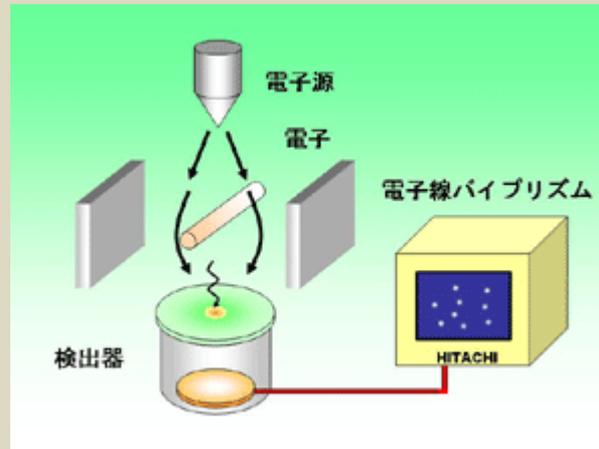
K.Ishikawa and Y. Tobita arXiv: 1206.2593, 1109.3105, 1109.4968 ,others [hep-ph], prog.theo.phy.(2009)

K.I and Yabuki, progress of theo.physics(2002),

K.I and T.Shimomura,progress of theo.physics(2005)

1. Introduction
2. Fermi's golden rule : finite-size correction
3. Probability to detect a neutrino in pion decay
4. Light-cone singularity and neutrino diffraction
5. Implications

# I. Single electron interference (Tonomura)



Number of events  
 $N_a < N_b < N_c < N_d$

interference pattern  
becomes clear and visible  
in d

We study single neutrino interference

# Unknown facts on neutrinos

- Absolute neutrino mass

Tritium decay electron spectrum

$\nu$  - less double Beta decay

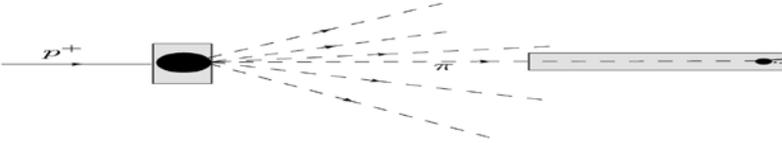
Cosmology gives bound.

Unclear now !

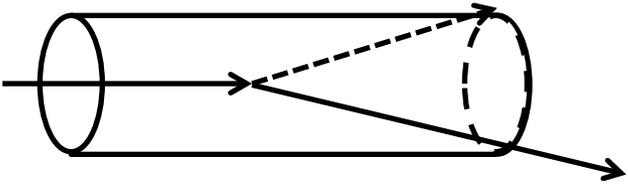
# Scattering ,interference, and diffraction of neutrino ( our works)

1. Neutrino is a quantum mechanical wave , so its distribution reveals single quantum interference.
2. Neutrino is identified from physical reactions with a nucleon in nucleus.
3. 1 and 2 are combined to the probability to detect the neutrino at a finite distance. This supplies ordinary results at the infinite distance and a new physical quantity from the finite-size correction.
4. Diffraction is a main part of the finite-size correction and is observed easily with a large number of events.

# Real pion decays

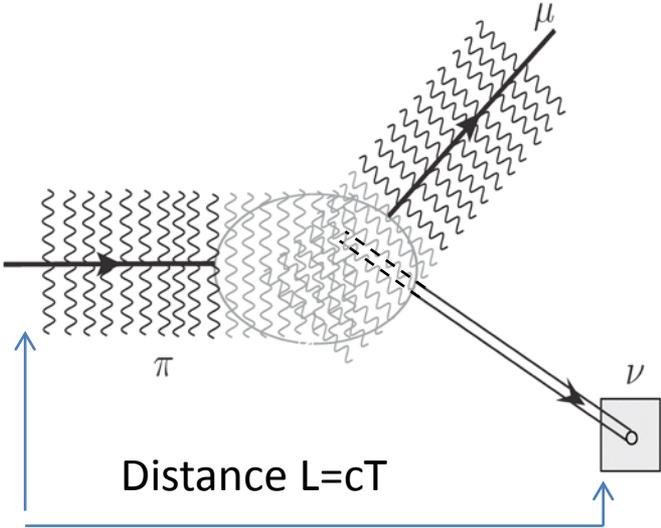


experiment



which?

Particle decay  
(traditional ?)



Decay product of wave is  
measured in wave packet

# 2. Fermi's Golden rule

2-1. Transition rate at a large T is:

$$f_{\alpha,\beta} = g \int_0^T dt e^{i\omega t} T_{\alpha,\beta} \approx g 2\pi \delta(\omega) T_{\alpha,\beta}, \quad \omega = E_\beta - E_\alpha \quad (1)$$

$$P = \int d\omega |f_{\alpha,\beta}|^2 = g^2 2\pi T \int d\omega \delta(\omega) |T_{\alpha,\beta}|^2 \quad (2)$$

$$P/T = g^2 2\pi \int d\omega \delta(\omega) |T_{\alpha,\beta}|^2 \quad (3)$$

Dirac(1927),  
Fermi Golden rule (1949),

2-1.T is large but finite. Compute a finite-T correction to above Eq.(3).

$$\int_0^T dt e^{i\omega t} = \frac{1}{i\omega} (e^{i\omega T} - 1) = e^{i\omega T/2} \frac{2 \sin \omega T}{\omega}$$

$$P/T = g^2 \int d\omega \left( \frac{2 \sin \omega T}{\omega} \right)^2 |f_{\alpha,\beta}|^2$$

$$= g^2 T \int dx \left( \frac{2 \sin x}{x} \right)^2 \left[ g(0) + \frac{x}{T} g'(0) + \dots \right]$$

$$= g^2 2\pi T g(0) \left[ 1 + \frac{1}{T} \times \infty + \dots \right],$$

Correction diverges

# 1/T correction diverged.

## For a correct calculation :

- To compute transition probability at a finite T, **boundary condition** should be taken into account.
- For a rigorous treatment of scatterings ,out-going (in-coming) state is expressed by a **wave packet** that is localized in space and time. **(LSZ,55)**
- Compute the finite-size (time) correction with the transition amplitude defined by wave packets. Then a unique value is obtained.

# What is the finite-size effect ?

1. Quantum mechanical transition in a finite time interval  $T$ .

$$i + \omega \rightarrow j \quad (1)$$

$$P_{ij} = |T_{ij}|^2 \frac{\sin^2(E_i + \hbar\omega - E_j)T/2}{(E_i + \hbar\omega - E_j)^2} \quad (2)$$

2. Violates the energy conservation  $E_i - E_j + \hbar\omega \neq 0$
3. States at ultra-violet energy region,  $J$ , gives a universal correction and is important in relativistic invariant system.

Compute a finite-size correction of the probability to detect the neutrino !!

# Transition in a finite time-interval

- $S[T]$ ;  $S$  matrix for a finite time interval  $T$ .

$$H = H_0 + H_1 \quad (1)$$

$$U(t) = e^{-iHt}, U_0 = e^{-iH_0t} \quad (2)$$

$$\Omega_{\pm}(T) = \lim_{t \rightarrow -\pm T} U(t)^\dagger U^{(0)}(t) \quad (3)$$

$$S - \text{matrix} : S[T] = \Omega_-^\dagger(T) \Omega_+(T) \quad (1)$$

*energy non - conservation;*

$$[S[T], H_0] = i \left( \left( \frac{\partial}{\partial T} \Omega_-(T) \right)^\dagger \Omega_+(T) - i \Omega_-^\dagger \frac{\partial}{\partial T} \Omega_+(T) \right) \quad (2)$$

$$\text{transition amplitude; } \langle \beta | S[T] | \alpha \rangle = \delta(E_\alpha - E_\beta) f_{\alpha,\beta} + \delta f \quad (1)$$

$$\text{probability; } \quad (2)$$

$$P = P_{\text{normal}} + P_{\text{non-conserving}}, P_{\text{non-conserving}} = \sum |\delta f|^2 \geq 0$$

Finite-size correction

# Wave packets dynamics

K.I and T.shimomura(prog.theor.phys:2005)

- Complete set

$$\langle \vec{x} | \vec{P}, \vec{X} \rangle = (\pi\sigma)^{-3/2} e^{i\vec{P}(\vec{x}-\vec{X}) - \frac{1}{2\sigma}(\vec{x}-\vec{X})^2} \quad (1)$$

$$\langle \vec{p} | \vec{P}, \vec{X} \rangle = N e^{-i\vec{p}\vec{X} - \frac{\sigma}{2}(\vec{p}-\vec{P})^2} \quad (2)$$

$$\int \frac{d\vec{P}d\vec{X}}{(2\pi)^3} |\vec{X}, \vec{P}, \sigma\rangle \langle \vec{X}, \vec{P}, \sigma| = 1$$

$$\langle t, \vec{x} | \vec{P}, \vec{X}, T_0 \rangle = N e^{-\frac{1}{2\sigma}(\vec{x}-\vec{X}-\vec{v}(t-T_0))^2 - iE(\vec{P})(t-T_0) + i\vec{P}(\vec{x}-\vec{X})}$$

$$\vec{v} = \frac{\partial}{\partial p_i} E(\vec{p})$$

Wave packet moves with this velocity .  $\vec{x} = \vec{v}t + \vec{X}$

And a slow phase  $\frac{m^2}{E} t$

# 3. Neutrino diffraction (ishikawa and tobita)

decay amplitude

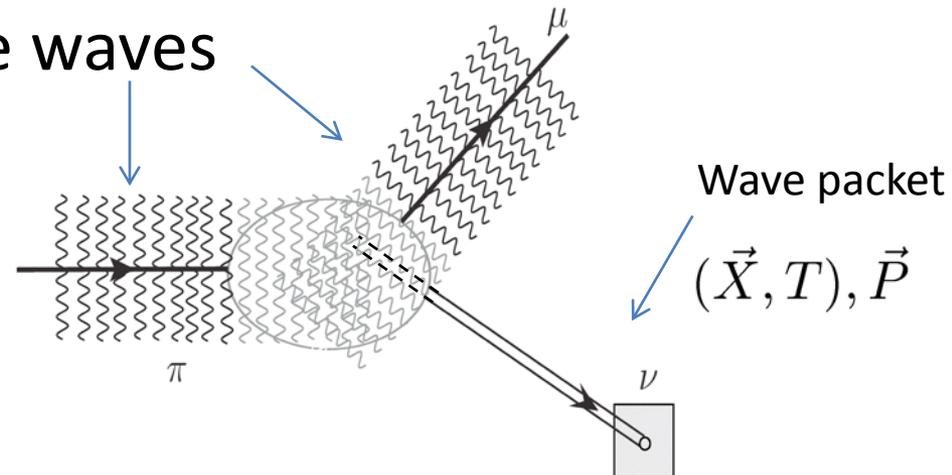
Neutrino's momentum and position are measured

$$f = \int d^4x \langle 0 | J_{V-A}^\mu | \pi \rangle \bar{u}(p_l) \gamma_\mu (1 - \gamma_5) \nu(p_\nu) e^{ip_l x + ip_\nu (x - X_\nu) - \frac{1}{2\sigma_\nu} (\vec{x} - \vec{X}_\nu - \vec{v}_\nu (t - T_\nu))^2}$$

$$= N \frac{2 \sin \omega T}{\omega} e^{-\frac{\sigma}{2} (\delta \vec{p})^2} \bar{u}(p_l) \gamma (1 - \gamma_5) \nu(p_\nu),$$

$$\omega = \delta E - \vec{v} \delta \vec{p}$$

Plane waves



**Integration over x and t =  
 interaction region is in a moving  
 frame**

# Wave packet effects

- 1 . Approximate conservation of the energy in the moving frame. (Pseudo-Doppler shift)
- 2 . Large T correction emerges for light particles, because there are many states of

$$\omega = \delta E - \vec{v}\delta\vec{p} = 0, |\vec{v}| \approx c$$

$$\delta E \neq 0, \delta\vec{p} \neq 0$$

$$f = N \frac{2 \sin \omega T}{\omega} e^{-\frac{\sigma}{2}(\delta\vec{p})^2} \bar{u}(p_l) \gamma(1 - \gamma_5) \nu(p_\nu),$$

$$\omega = \delta E - \vec{v}\delta\vec{p}$$

# 4. Rigorous calculation :Express the probability with a correlation function

decay amplitude

$$f = \int d^4x d\vec{k}_\nu \langle 0 | J_{V-A}^\mu | \pi \rangle \bar{u}(p_l) \gamma_\mu (1 - \gamma_5) \nu(k_\nu) e^{ip_l x + ik_\nu(x - X_\nu) - \frac{\sigma_\nu}{2}(k_\nu - p_\nu)^2}$$

Probability is expressed with a correlation function

$$\int \frac{d\vec{p}_l}{(2\pi)^3} \sum_{s_1, s_2} |f|^2 = \frac{N_2}{E_\nu} \int d^4x_1 d^4x_2 \Delta_{\pi, l}(\delta x) e^{i\phi(\delta x)}$$

Correlation function

$$\Delta_{\pi, l}(\delta x) = \frac{1}{(2\pi)^3} \int \frac{d^3 p_l}{E_l} (2p_\pi p_\nu p_\pi p_l - m_\pi^2 p_l p_\nu) e^{-i(p_\pi - p_l)\delta x}$$

$= m_l^2 p_l p_\nu$  (Only If energy-momentum is conserved)

1. Correlation function has a **light-cone singularity** which is generated by a superposition of relativistic waves.

2. The light-cone singularity is real and long-range and gives the finite-size correction. Since the energy conservation is violated, this has anomalous properties.

3. Wave packet ensures asymptotic boundary condition (LSZ) and leads new effects .<sup>14</sup>

# Correlation function

- Integration variable is change to  $q = p_l - p_\pi$

$$\begin{aligned} \Delta_{\pi,\mu}(\delta x = x_1 - x_2) &= \frac{1}{4\pi^4} \int d^4q \operatorname{Im} \left[ \frac{1}{q^2 + 2p_\pi \cdot q + \tilde{m}^2 - i\epsilon} \right] \theta(p_\pi^0 + q^0) \\ &\times \left\{ (2p_\pi \cdot p_\nu) p_\pi \cdot (p_\pi + q) - m_\pi^2 (q + p_\pi) \cdot p_\nu \right\} e^{-iq \cdot \delta x} \quad \tilde{m}^2 = m_\pi^2 - m_l^2 \end{aligned}$$

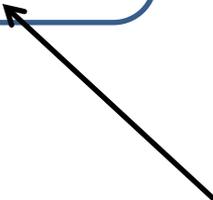
- Integral region is separated in two parts

$$\Delta_{\pi,\mu}(\delta x) = I_1 + I_2$$

$$q^0 > 0 \quad 0 > q^0 > -p_\pi^0$$



Finite size correction  
(energy non-conserving)



Normal term

# I\_1: Extract the light-cone singularity

About  $I_1$

$$I_1 = \left[ p_\pi \cdot p_\nu - ip_\nu \cdot \left( \frac{\partial}{\partial \delta x} \right) \right] \tilde{I}_1$$

$$\tilde{I}_1 = \int d^4 q \frac{\theta(q^0)}{4\pi^4} \text{Im} \left[ \frac{1}{q^2 + 2p_\pi \cdot q + \tilde{m}^2 - i\epsilon} \right] e^{iq \cdot \delta x}$$

$$= \int d^4 q \frac{\theta(q^0)}{4\pi^4} \text{Im} \left[ \frac{1}{q^2 + \tilde{m}^2 - i\epsilon} \right] e^{iq \cdot \delta x}$$

$$+ \sum_{n=1} \frac{1}{n!} \left( 2p_\pi \cdot \left( -i \frac{\partial}{\partial \delta x} \right) \frac{\partial}{\partial \tilde{m}^2} \right)^n \int d^4 q \frac{\theta(q^0)}{4\pi^4} \text{Im} \left[ \frac{1}{q^2 + \tilde{m}^2 - i\epsilon} \right] e^{iq \cdot \delta x}$$

To extract light-cone singularity expanding in  $2p_\pi \cdot q$   
**convergence condition:**  $2p_\pi \cdot p_\nu \leq m_\pi^2 - m_\mu^2$



Green's function

$$\begin{aligned} \int d^4 q \frac{\theta(q^0)}{4\pi^4} \text{Im} \left[ \frac{1}{q^2 + \tilde{m}^2 - i\epsilon} \right] e^{iq \cdot \delta x} &= \frac{2}{(2\pi)^3} \int d^4 q \theta(q^0) \delta(q^2 + \tilde{m}^2) e^{iq \cdot \delta x} \\ &= 2i \left[ \frac{\epsilon(\delta t)}{4\pi} \delta(\lambda) + f_{\text{short}} \right] \end{aligned}$$

Light-cone singularity

$$f_{\text{short}} = -\frac{i\tilde{m}^2}{8\pi\sqrt{-\lambda}} \theta(-\lambda) \left\{ N_1 \left( \tilde{m}\sqrt{-\lambda} \right) - i\epsilon(\delta t) J_1 \left( \tilde{m}\sqrt{\lambda} \right) \right\} - \theta(\lambda) \frac{i\tilde{m}}{4\pi^2\sqrt{\lambda}} K_1 \left( \tilde{m}\sqrt{\lambda} \right), \quad \lambda = \delta t^2 - \delta \vec{x}^2$$

# Integration over space-time coordinates

$$\begin{aligned}
 \int \frac{d^3 p_l}{(2\pi)^3} \sum_{spin} |T|^2 &= \frac{N_2}{E_\nu} \int d^4 x_1 d^4 x_2 e^{-\frac{1}{2\sigma_\nu} \sum_i (\vec{x}_i - \vec{X}_\nu - \vec{v}_\nu (t_i - T_\nu))^2} \\
 &\quad \times \Delta_{\pi,l}(x_1 - x_2) e^{i p_\nu \cdot (x_1 - x_2)} \\
 &= \frac{N_3}{E_\nu} \int d^4 x_1 d^4 x_2 e^{-\frac{1}{2\sigma_\nu} \sum_i (\vec{x}_i - \vec{X}_\nu - \vec{v}_\nu (t_i - T_\nu))^2} \left[ i \frac{\epsilon(\delta t)}{4\pi} \delta(\lambda) + f'_{short} + I_2 \right] e^{i p_\nu \cdot \delta x} \\
 N_3 &= 8g^2 \{ p_\pi \cdot p_\nu (m_\pi^2 - 2p_\pi \cdot p_\nu) \} (\pi^2 / \sigma_\nu)^{\frac{3}{2}}
 \end{aligned}$$

**I\_1: Long-range term**

$$\begin{aligned}
 &\int d^4 x_1 d^4 x_2 e^{-\frac{1}{2\sigma_\nu} \sum_i (\vec{x}_i - \vec{X}_\nu - \vec{v}_\nu (t_i - T_\nu))^2} e^{i p_\nu \cdot \delta x} \times i \frac{\epsilon(\delta t)}{4\pi} \delta(\lambda) \\
 &= (\sigma_\nu \pi)^{\frac{3}{2}} \frac{\sigma_\nu}{2} i \int_0^T dt_1 dt_2 \frac{\epsilon(\delta t)}{|\delta t|} e^{i \frac{m_\nu^2}{2E_\nu} \delta t} \leftarrow \tilde{g}(\omega_\nu T)
 \end{aligned}$$

$L = cT$  is length of decay volume

**I\_2 short range term**

# Probability at a finite distance

- Probability is composed of the normal term and a T-dependent diffraction term, which gives a finite-size correction.

$$P = P^{normal} + P^{diffraction}$$

$$P^{normal}/T = \text{constant}$$

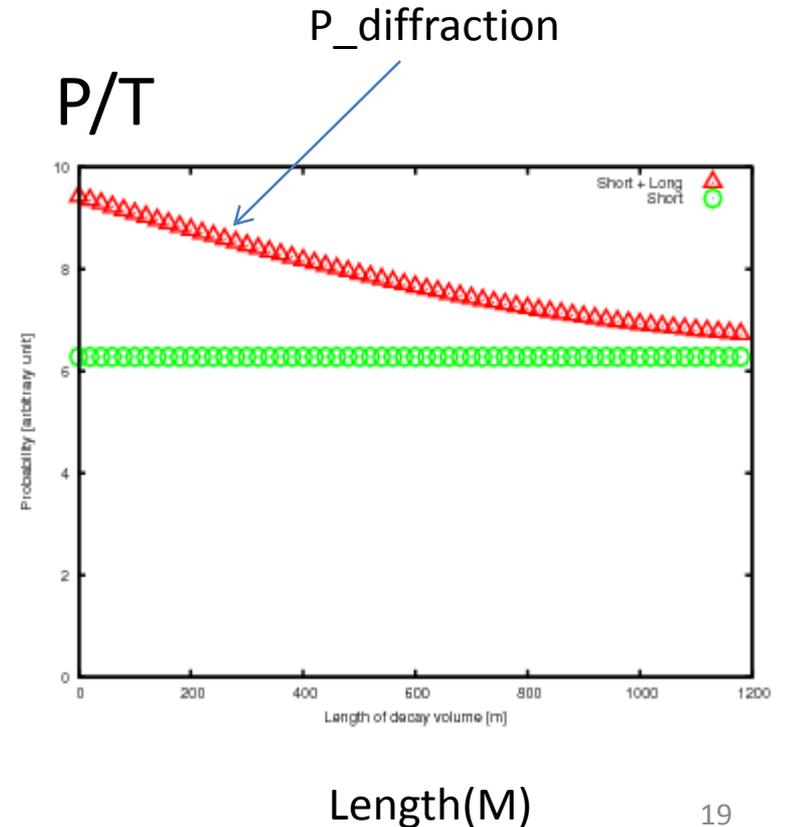
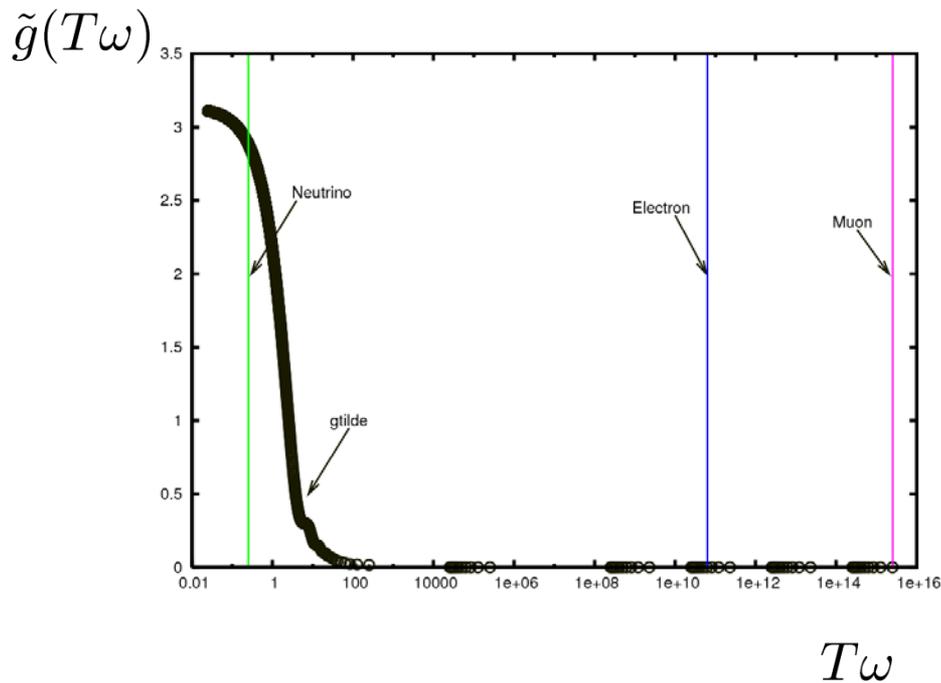
$$P^{diffraction} = C(\sigma)\tilde{g}(T\omega_\nu); \text{ universal function ,}$$

$$\omega_\nu = E_\nu - \vec{n}_\nu \cdot \vec{p}_\nu = \frac{m_\nu^2}{2E_\nu}$$

# Diffraction term + normal term (muon neutrino)

$$P = P_{normal} + P_{diffraction}$$

$$P_{diffraction} = C(\sigma)\tilde{g}(T\omega), \omega = \frac{m_\nu^2}{2E_\nu}$$



# For three flavors

$$P = P^{(0)}(long) + P^{diffraction}(short), \bar{m}_i^2 \gg \delta m^2$$

$$P^{(0)}(long) = \textit{flavour oscillation}(\delta m^2) \quad \text{standard}$$

Kayser, -, -, Akhmedov, et al, Smirnov, --

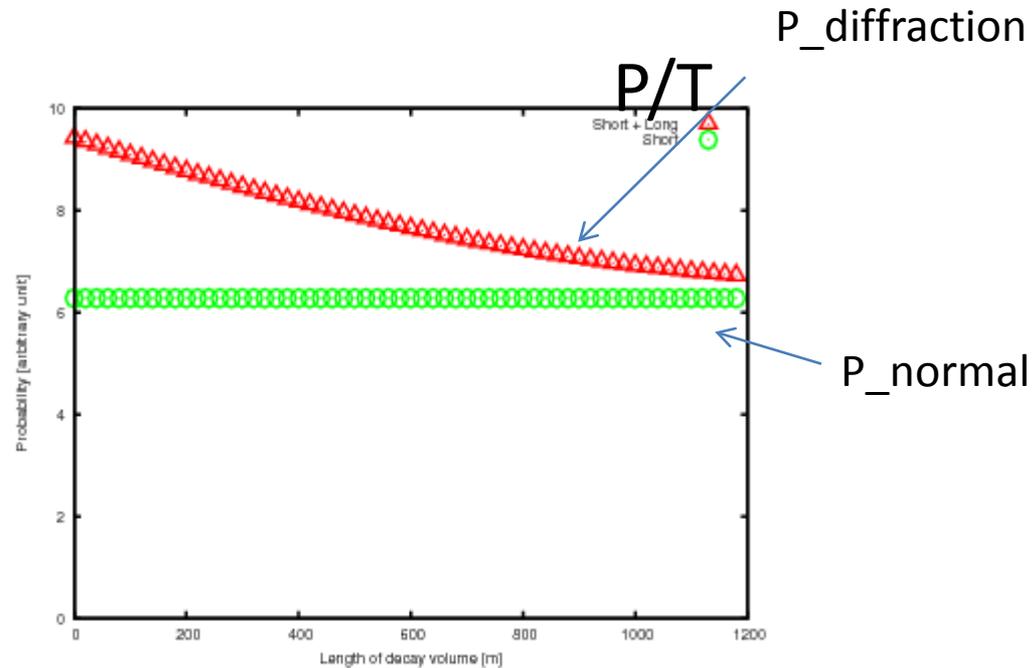
$$P^{diffraction}(short) = \textit{neutrino diffraction}(\bar{m}^2)$$

New term

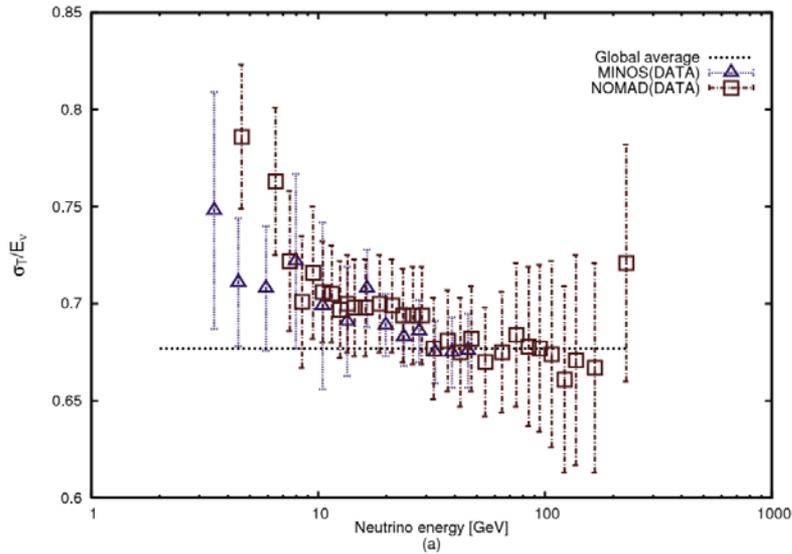
# 5. Diffraction term in a muon neutrino

$$P = P_{normal} + P_{diffraction}$$

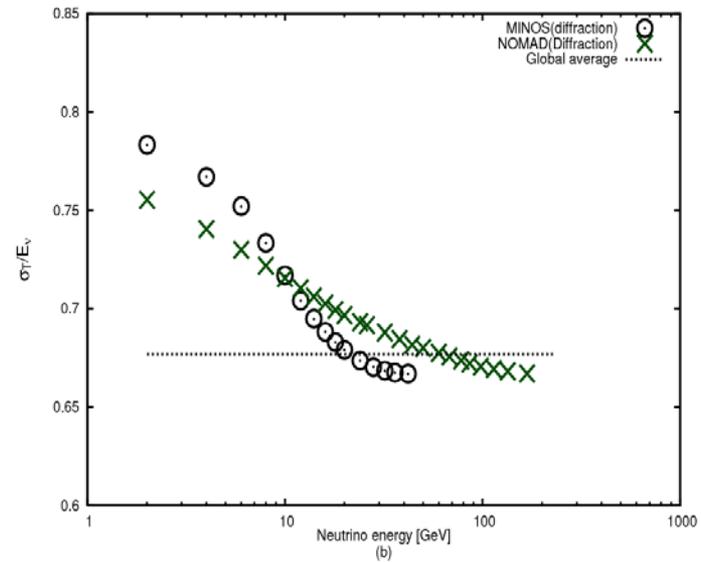
$$P_{diffraction} = C\tilde{g}(T\omega), \omega = \frac{m_\nu^2}{2E_\nu}$$



# $\nu_{\mu}$ -nucleon total cross section



Experiments.:  
NOMAD & MINOS



theory :  
normal+diffraction

# Electron mode: helicity suppression

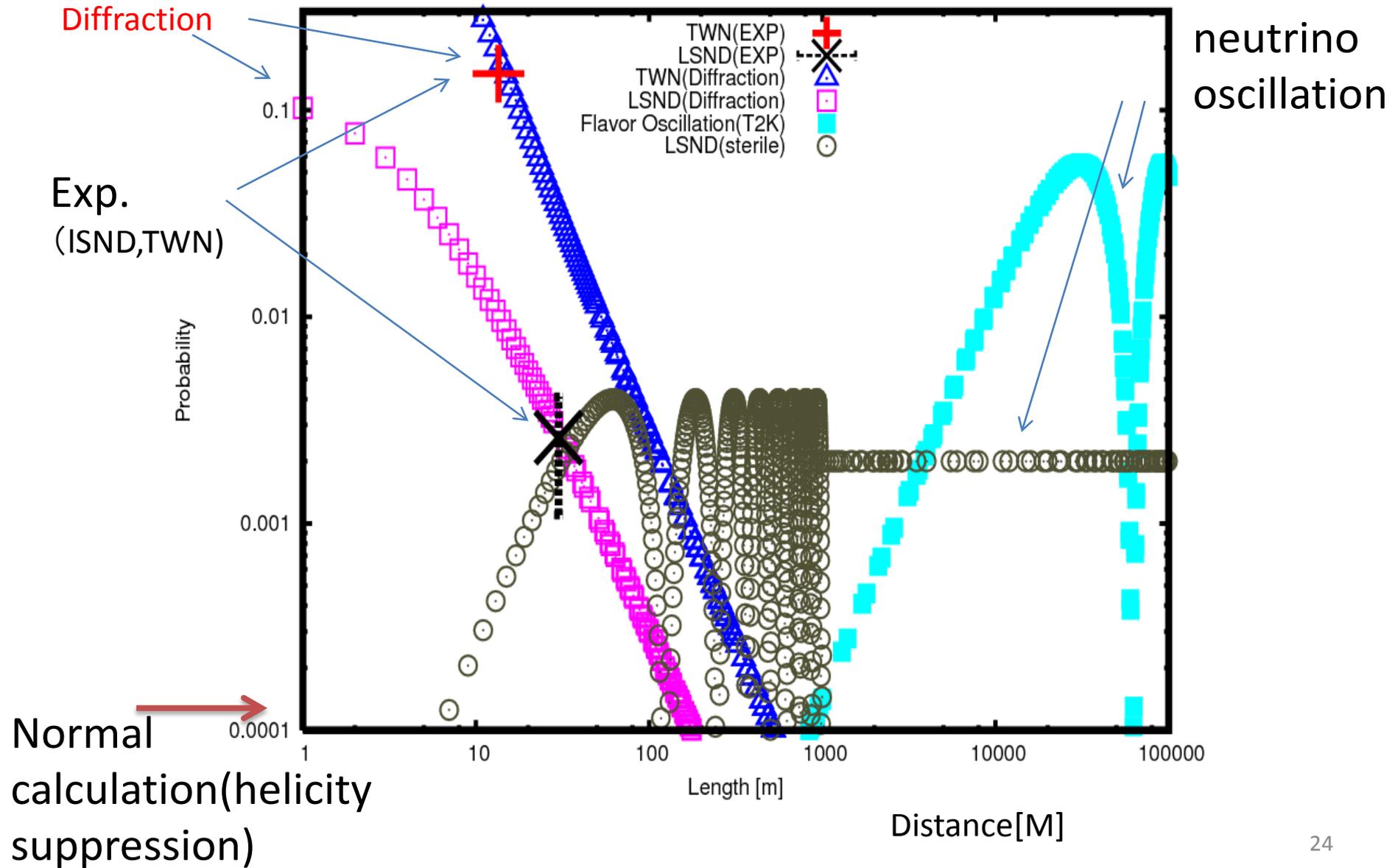
The normal term in the electron mode is suppressed by the angular momentum and energy-momentum conservation.

(Steinberger, Rudermann-Finkelstein, Sasaki-Oneda-Ozaki, 49)

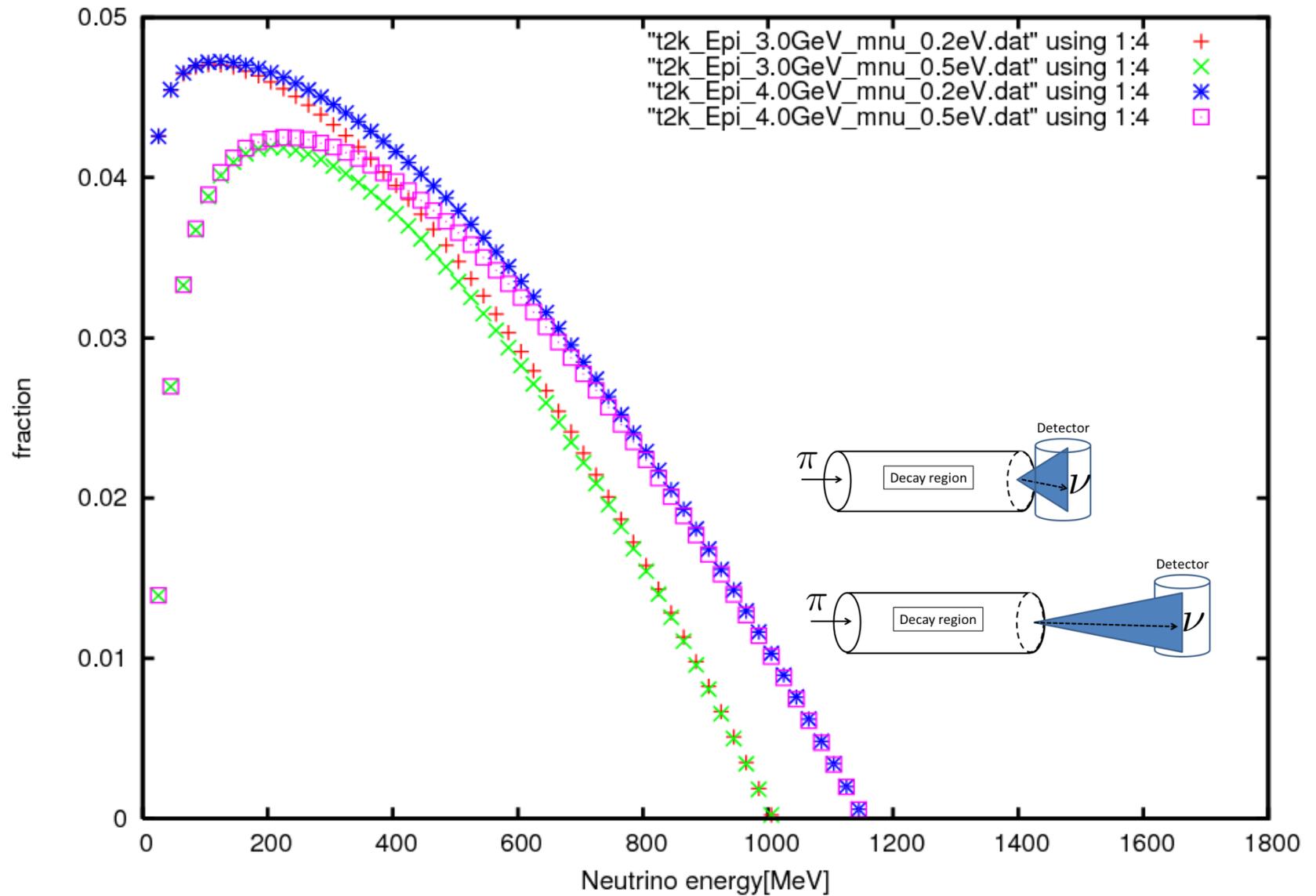
Since the finite-size correction does not conserve the energy, it violates the helicity suppression .

- When the neutrino is detected, the electron mode is enhanced.

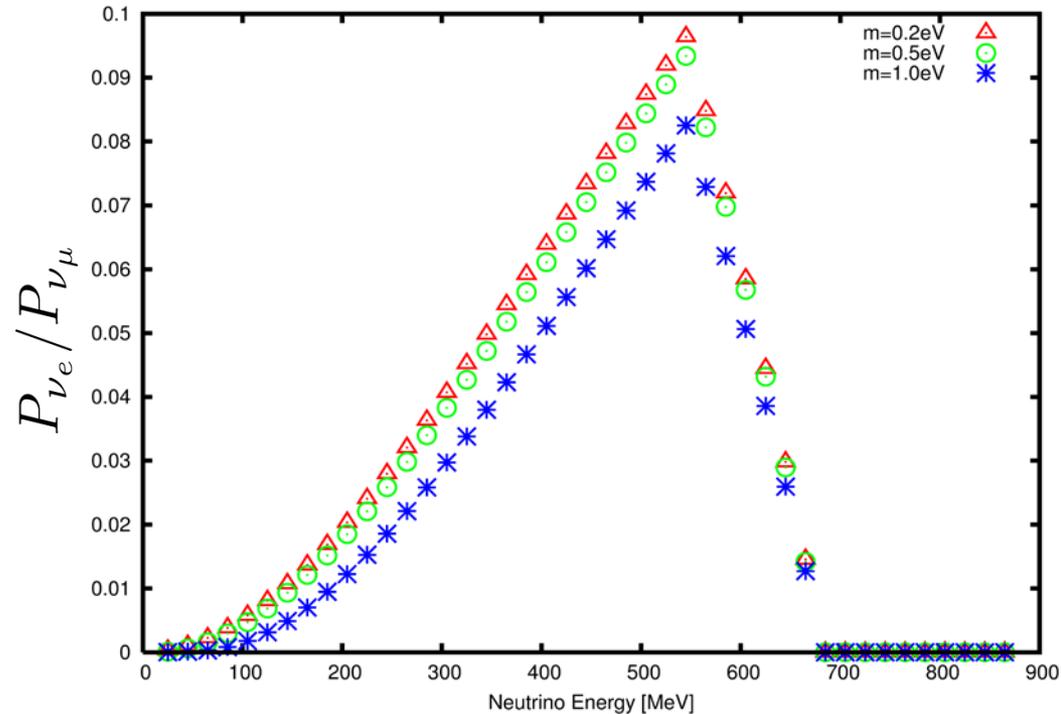
# Diffraction Events(enhances electron neutrino)



# Diffraction prediction of the electron neutrino diffraction(T2K、on axis)



# $\nu_e$ appearance at near detector

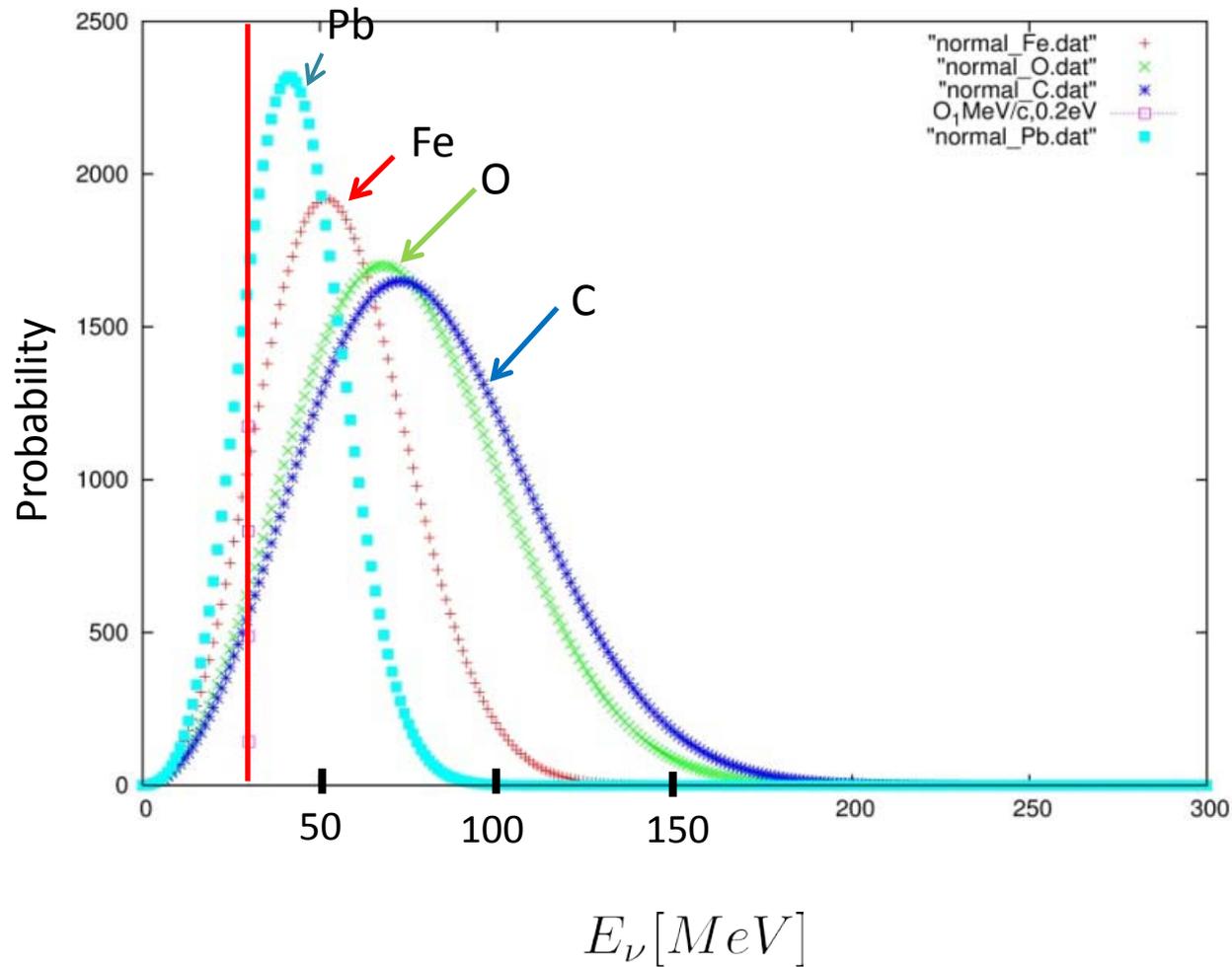


$$|\vec{p}_\pi| = 2[\text{GeV}], L = 110[\text{m}]$$

$L_{\text{decay-detector}} = 170[\text{m}]$ , axis = 2.5 degree, Detector 3[m]x3[m]

**$\sim 10\%$  excess (maximum)**

# Neutrino from decay of pion at rest



赤は通常の保存則での計算値( $\sim 30$ MeV)です

# Anomalous properties of the diffraction

- 1 neutrino diffraction is easily observed once the statistics becomes large. **Single quantum interference**
- 2 energy conservation is violated : **finite-size effect**
- 3 lepton number appears to be non-conserved.  
P(L) decreases with L , so unitarity appears to be violated. But they are not.: **finite-size effect and retarded effect.**
- 4 pion life time varies due to the measurement, **quantum Zeno effect** . However the majority of the pion are unchanged because the neutrino interacts with matter so weakly.

# Comparisons with previous experiments

- Diffraction effect has been observed but that has not been recognized. So, unusual events have been regarded as anomalous events. They are explained with the neutrino diffraction.
- High energy neutrino nucleon scattering cross section decreases with the energy slowly. This is understood by the diffraction effect of the neutrino process.
- High precision experiment may provide **the neutrino absolute mass.**

# The neutrino diffraction is a finite-size correction

- Why does the neutrino diffraction emerge?

1. Transition in a finite time interval  $T$ ,

$$i + \omega \rightarrow j \quad (1)$$

$$P_{ij} = |T_{ij}|^2 \frac{\sin^2(E_i + \hbar\omega - E_j)T/2}{(E_i + \hbar - E_j)^2} \quad (2)$$

, violates the energy conservation .

2. States at ultra-violet energy region

$$E_j \rightarrow \infty$$

give a universal correction.

3. Relativistic invariance  $\gg$  large momentum states

# Other channels on the neutrino processes

- 1. muon decay

$$\mu \rightarrow e + \nu_{\mu} + \bar{\nu}_e$$

- 2. neutron decay

$$n \rightarrow p + e + \bar{\nu}_e$$

- 3. nucleus decays

$$A \rightarrow A' + e + \bar{\nu}_e$$

$$A + e \rightarrow A' + \nu$$

- 4. neutrino scattering

$$\nu + A \rightarrow \nu + A$$

(In progress)

# New phenomena caused by finite-size effects

1. Emission of light particles .

Energy non-conserving transition lead background noises that has universal properties.

“theory of universal noises “

2. Interference and diffraction.

interference of a new scale that is very different from wave length “physics of a new scale ”

3. Energy shift : “pseudo-Doppler shift ”

# 6. conclusion

- Finite-size correction to the probability to detect the neutrino is large.
- With enough number of neutrino events, the neutrino diffraction, which is the main part of the finite-size correction, is easy to observe and may provide the absolute neutrino mass.