Minimal Flavour Violation in the Leptonic sector with Two Higgs Doublets

M. N. REBELO

CFTP / IST , UTL   LISBOA

GGI   neutrino Workshop 2012

WHAT IS ν ?

Work in collaboration with

F. J. BOTELLA , G. C. BRANCO , MIGUEL NEBOT
(IFIC/CSIC/Valencia)   (Lisbon)   (IFIC/CSIC/Valencia)
(IFIC/CSIC/Valencia)

Neutral currents have played an important role in the construction and experimental test of unified gauge theories

EPS Prize in 2009 to Gargamelle, CERN

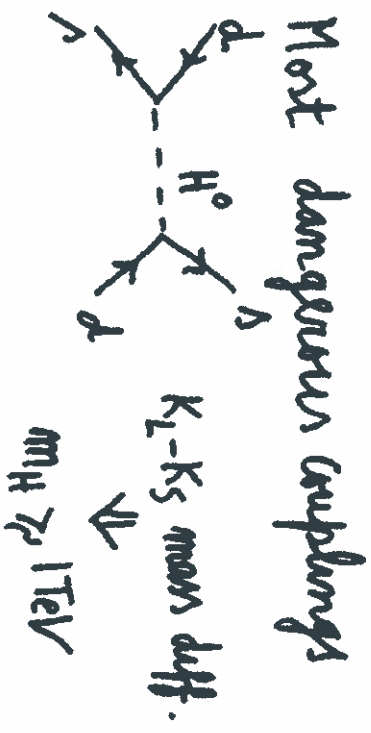In the Standard Model Flavour Changing Neutral Currents (FCNC) are forbidden at tree level
 - in the gauge sector, ie no Z FCNC
 - in the scalar sector, ie no HFCNC

Models with two or more Higgs doublets potentially large HFCNC

strict limits on FCNC processes!

In the SM , FCNC are generated only at loop level

⟹ very suppressed

$K^0 - \bar{K}^0$ mixing

$D^0 - \bar{D}^0$ mixing

$B_d^0 - \bar{B}_d^0$ mixing

$B_s^0 - \bar{B}_s^0$ mixing

rare Kaon decays

rare B-meson decays

CP violation

Most dangerous couplings

$K_L - K_S$ mass diff.

$m_H \gtrsim 1 TeV$

CP violation ε_K

$m_H \gtrsim 30 TeV$

processes that play a crucial role in
testing the SM and putting limits
in Models for Physics Beyond the SM

-2-

Proposed solutions, case of Multi-Higgs models

NFC

or

                                        Weinberg, Glashow (1977)
                                            Paschos (1977)

Existence of suppression factors in HFCNC

            Antaramian, Hall, Rasin (1992)
            Hall, Weinberg (1993)
            Joshipura, Rindani (1991)

first models of this type with no ad-hoc
assumptions suppression by small elements of
VCKM : BGL models

            Branco, Grimus, Lavoura (1996)

More recently, we have generalized BGL models to
larger class of models of "Minimal Flavour Violation"
type

About Minimal Flavour Violation

Buras, Gambino, Gorbahn, Jager, Silvestrini (2001)
D'Ambrosio, Giudice, Isidori, Strumia (2002)

Leptonic sector

Cirigliano, Grinstein, Isidori, Wise (2005)

$$G_F = U(3)^5$$ Largest symmetry of the gauge sector

flavour violation completely determined by Yukawa couplings

Our framework

- multi-Higgs models
- no Natural Flavour Conservation
- obey above condition (one of the defining
  ingredients of MFV framework)

"Higgs - mediated FCNC's: Natural Flavour Conservation vs.
Minimal Flavour Violation"

Buras, Carlucci, Gori, Isidori, arXiv:1005.5310 (JHEP)
Barbieri, Lodone, Straub, Jones-Perez ; Cenveso, Gerard ; .....

Question : Under what conditions the neutral Higgs couplings are only functions of V_{CKM} ?

The case of two Higgs doublets

Yukawa interactions

$$\mathcal{L}_Y = -\bar{Q}_L^0 \Gamma_1 \phi \, d_R^0 - \bar{Q}_L^0 \Gamma_2 \phi_2 \, d_R^0 - \bar{Q}_L^0 \Delta_1 \tilde{\phi} \, u_R^0 - \bar{Q}_L^0 \Delta_2 \tilde{\phi}_2 \, u_R^0 + h.c.$$

$$\tilde{\phi}_i = -i \tau_2 \phi_i^*$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}} \left(v_1 \Gamma_1 + v_2 \, e^{i\alpha} \, \Gamma_2 \right) \quad ; \quad M_u = \frac{1}{\sqrt{2}} \left(v_1 \Delta_1 + v_2 \, e^{-i\alpha} \, \Delta_2 \right)$$

Diagonalized by

$$U_{dL}^\dagger \, M_d \, U_{dR} = D_d \equiv diag \, (m_d, m_s, m_b)$$

$$U_{uL}^\dagger \, M_u \, U_{uR} = D_u \equiv diag \, (m_u, m_c, m_t)$$

Expansion around the new's

$$\Phi_j = e^{i\vec{\sigma}\cdot\vec{\eta}_j} \begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}}(v_j + \zeta_j + i\eta_j) \end{pmatrix} \qquad j = 1,2$$

We perform the following transformation

$$\begin{pmatrix} H^\circ \\ R \end{pmatrix} = O \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix} \quad ; \quad \begin{pmatrix} G^\circ \\ I \end{pmatrix} = O \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \quad ; \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = O \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

$$O = \frac{1}{v} \begin{pmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{pmatrix} \quad ; \quad v = \sqrt{v_1^2 + v_2^2} = (\sqrt{2}\, G_F)^{-1/2} \approx 246 \text{ GeV}$$

O mingles out

H° with couplings to quarks proportional to mass matrices
G° the neutral pseudo-goldstone boson
G^+ charged pseudo-goldstone boson

Physical neutral Higgs fields are combination of H_i°, R and I

Yukawa couplings in terms of quark mass eigenstates

$$\mathcal{L}_Y = \frac{\sqrt{2}\,H^+}{v}\,\bar{u}\,(V N_d \gamma_R - N_u^\dagger V \gamma_L)\,d + h.c. - \frac{H^0}{v}(\bar{u}\,D_u u + \bar{d}\,D_d d) -$$

$$-\frac{R}{v}\Big[\bar{u}(N_u\gamma_R + N_u^\dagger \gamma_L)u + \bar{d}(N_d\gamma_R + N_d^\dagger \gamma_L)d\Big] +$$

$$+\,i\,\frac{I}{v}\Big[\bar{u}(N_u\gamma_R - N_u^\dagger \gamma_L)u - \bar{d}(N_d\gamma_R - N_d^\dagger \gamma_L)d\Big]$$

$$\gamma_L = (1-\gamma_5)/2 \quad ; \quad \gamma_R = (1+\gamma_5)/2 \quad , \quad V \equiv V_{CKM}$$

Flavour changing neutral currents controlled by N_d, N_u

$$N_d = \frac{1}{\sqrt{2}}\,U_{dL}^\dagger\,(v_2 \Gamma_1 - v_1\,e^{i\alpha}\,\Gamma_2)\,U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}}\,U_{uL}^\dagger\,(v_2 \Delta_1 - v_1\,e^{-i\alpha}\,\Delta_2)\,U_{uR}$$

For generic two Higgs doublet models, N_u, N_d non-diagonal, arbitrary

For definiteness rewrite N_d :

$$N_d = \frac{\sqrt{2}}{v_1}\,D_d - \frac{v_2}{\sqrt{2}}\Big(\frac{v_2}{v_1} + \frac{v_1}{v_2}\Big)\,U_{dL}^\dagger\,e^{i\alpha}\,\Gamma_2\,U_{dR}$$

conserves flavour ↑ ⟶ leads to FCNC

$$N_d = \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}}\left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) U_{dL}^\dagger \, e^{i\alpha} \, \Gamma_2 \, U_{dR}$$

We want N_d entirely controlled by V_{CKM} elements
(together with ratio of v_1 and v_2 and quark masses)

$$V_{CKM} = U_{uL}^\dagger \, U_{dL}$$

Obstacles : (i) Dependence on U_{dL} rather than V_{CKM}

(ii) Need to get rid of U_{dR}

Solution to first difficulty :

Flavour symmetry constraining $U_{uL} = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$V_{CKM} = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x & x & x \\ x & x & x \\ U_{d31} & U_{d32} & U_{d33} \end{pmatrix} = \begin{pmatrix} x & x & x \\ x & x & x \\ U_{d31} & U_{d32} & U_{d33} \end{pmatrix}$$

$(V_{CKM})_{3f} = (U_{dL})_{3f}$.

together with $\Gamma_2 \, U_{dR} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix} \Rightarrow$ only third row of U_{dL} appears in N_d

FCNC $\propto U_{dL}^\dagger \, e^{i\alpha} \, \Gamma_2 \, U_{dR}$

to get rid of $U_{dL}^\dagger \, e^{i\alpha} \, \Gamma_2 \, U_{dR}$, choose $\quad \Gamma_2 \propto P M_d$, P projection

$$U_{dL}^\dagger \, \Gamma_2 \, U_{dR} \propto U_{dL}^\dagger \, P M_d \, U_{dR} \propto U_{dL}^\dagger \, P \, U_{dL} \, D_d$$

for

$$P = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & 0 \\ x & 0 & 0 \end{pmatrix}$$

$$(U_{dL}^\dagger \, \Gamma_2)_{ij} = (U_{dL}^\dagger)_{i3} (\Gamma_2)_{3j} = (V_{CKM}^\dagger)_{i3} (\Gamma_2)_{3j}$$

$$(N_d)_{ij} = \frac{\sqrt{2}}{v_1}(D_d)_{ij} - \left(\frac{\sqrt{2}}{v_1} + \frac{v_2}{\sqrt{2}}\right)(V_{CKM}^\dagger)_{i3}(V_{CKM})_{3j}(D_d)_{ij}$$

Symmetry   BGL

$$Q_{L3} \longrightarrow e^{i\alpha}\, Q_{L3}^\circ \;;\; u_{R3} \longrightarrow e^{i\alpha}\, u_{R3}^\circ \;;\; \beta_2 \to e^{i\alpha}\beta_2 \qquad \alpha \neq 0, \pi$$

$$\Gamma_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix} \;;\; \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix} \;;\; \Delta_1 = \begin{bmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{bmatrix} \;;\; \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}$$

Both Higgs doublets have mm-zero yukawa couplings in up and down sectors

$$N_u = -\frac{v_1}{\sqrt{2}} \, \mathrm{diag}(0,0,m_t) + \frac{v_2}{\sqrt{2}} \, \mathrm{diag}(m_u, m_c, 0)$$

Six different models

$$(N_d)_{ij} = \frac{\sqrt{2}}{v_1}(D_d)_{ij} - \overline{\left(\frac{\sqrt{2}}{v_1} + \frac{v_1}{v_2}\right)(V_{CKM}^\dagger)_{i3}(V_{CKM})_{3j}}^{\,MFV}(D_d)_{ij}$$

$$N_d = -\frac{v_1}{v_2}\,diag(0,0,m_b) + \frac{\sqrt{2}}{v_1}\,diag(m_u,m_c,0)$$

FCNC only in the down sector
suppressed by the 3rd row of V_{CKM}

Strong and Natural suppression of the most constrained processes

R.I

$V_{td}V_{tb}^*$

$V_{cs}V_{cb}$

$V_{cs}V_{cb}$

$\Delta A = 2$ processes

$|V_{td}V_{tb}^*| \sim \lambda^5$ (λ^{10} suppression)

$\sim 10^{-4}$

may contribute significantly to $B_d - \bar{B}_d$ mixing

contribution to $B_s - \bar{B}_s$ mixing

How to find a general expansion of N_d^0, N_u^0 which conforms to the MFV requirements?

$$N_d^0 = U_{dL} N_d U_{dR}^\dagger = \frac{1}{\sqrt{2}}\left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2\right)$$

$$N_u^0 = U_{uL} N_u U_{uR}^\dagger = \frac{1}{\sqrt{2}}\left(v_2 \Delta_1 - v_1 e^{i\alpha} \Delta_2\right)$$

Necessary condition N_d^0, N_u^0 to be of MFV type:
Should be functions of M_d, M_u no other flavour dependence

Furthermore, N_d^0, N_u^0 should transform under WB appropriate form

$$Q_L^0 \rightarrow W_L Q_L^0 \quad , \quad d_R^0 \rightarrow W_R^d d_R^0 \quad ; \quad u_R^0 \rightarrow W_R^u u_R^0$$

$$M_d \rightarrow W_L^\dagger M_d W_R^d \quad ; \quad M_u \rightarrow W_L^\dagger M_u W_R^u$$

$$U_{dL} \rightarrow W_L^\dagger U_{dL} \quad ; \quad U_{uL} \rightarrow W_L^\dagger U_{uL} \quad ; \quad U_{dR} \rightarrow W_R^{d\dagger} U_{dR} \quad ; \quad U_{uR} \rightarrow W_R^{u\dagger} U_{uR}$$

$$N_{d,u} \equiv (M_{d,u})(M_{d,u}^\dagger) \quad , \quad N_{d,u} \rightarrow W_L^\dagger N_{d,u} W_L$$

$$N_d^0, N_u^0 \text{ transform as } N_d, N_u$$

It is convenient to write H_d, H_u in terms of projection operators

$$H_d = \sum_i m_{d_i}^2 P_i^{d_L} \ ; \quad P_i^{d_L} = U_{d_L} P_i U_{d_L}^\dagger \ ; \quad (P_i)_{jk} = \delta_{ij} \delta_{ik} \qquad u \leftrightarrow d$$

Botella, Nebot, Vives 2004

MFV expansion for N_d^0 and N_u^0

$$N_d^0 = \lambda_1 M_d + \lambda_{2i} U_{d_L} P_i U_{d_L}^\dagger M_d + \lambda_{3i} U_{u_L} P_i U_{u_L}^\dagger M_d + \cdots$$

$$N_u^0 = Z_1 M_u + Z_{2i} U_{u_L} P_i U_{u_L}^\dagger M_u + Z_{3i} U_{d_L} P_i U_{d_L}^\dagger M_u + \cdots$$

In green terms that do not lead to FCNC

In red terms that lead to FCNC

In the quark eigenstate basis

$$N_d = \lambda_1 D_d + \lambda_2 P_i D_d + \lambda_{3i} (V_{CKM})^\dagger P_i V_{CKM} D_d + \cdots$$

$$N_u = Z_1 D_u + Z_{2i} P_i D_u + Z_{3i} V_{CKM} P_i (V_{CKM})^\dagger D_u + \cdots$$

At this stage λ and Z coefficients appear as free parameters, MFV

Need for additional symmetries in order to constrain these coeff.

BGL example again

corresponds to the following truncation of our MFV expansion

$$N_d^0 = \frac{\sqrt{2}}{v_1} M_d - \left(\frac{\sqrt{2}}{v_1} + \frac{v_1}{v_2}\right) U_{uL} P_3 U_{uL}^\dagger M_d$$

$$N_u^0 = \frac{\sqrt{2}}{v_1} M_u - \left(\frac{\sqrt{2}}{v_1} + \frac{v_1}{v_2}\right) U_{uL} P_3 U_{uL}^\dagger M_u$$

together with

$$N_d^0 = \frac{\sqrt{2}}{v_1} M_d - \frac{\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{v_1} + \frac{v_1}{\sqrt{2}}\right) e^{i\alpha} \Gamma_2$$

$$N_u^0 = \frac{\sqrt{2}}{v_1} M_u - \frac{\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{v_1} + \frac{v_1}{\sqrt{2}}\right) e^{-i\alpha} \Delta_2$$

implies BGL model fully defined in covariant way under WB transform.

$$\frac{\sqrt{2}}{\sqrt{2}} e^{i\alpha} \Gamma_2 = U_{uL} P_3 U_{uL}^\dagger M_d \; ; \quad \frac{\sqrt{2}}{\sqrt{2}} e^{-i\alpha} \Delta_2 = U_{uL} P_3 U_{uL}^\dagger M_u$$

we have

$$U_{uL} P_3 U_{uL}^\dagger \Gamma_2 = \Gamma_2 \; ; \quad U_{uL} P_3 U_{uL}^\dagger \Gamma_1 = 0 \; ; \quad U_{uL} P_3 U_{uL}^\dagger \Delta_2 = \Delta_2$$

$$U_{uL} P_3 U_{uL}^\dagger \Delta_1 = 0$$

BGL is the only implementation
of models where Higgs FCNC are
a function of V_{CKM} only (together
with v_1, v_2) which are based on an
Abelian symmetry obeying the sufficient
conditions of having M_u block diagonal
together with the existence of a matrix P
such that

$$P\Gamma_2 = \Gamma_2 \quad ; \quad P\Gamma_1 = 0$$

Ferreira, Silva arXiv : 1012287

13'

The leptonic sector

Required for completeness
- study of experimental implications
- study of stability under RGE

Models with two Higgs doublets with FCNC
- controlled by VCKM in the quark sector
- controlled by VPMNS in the leptonic sector

Case of Dirac neutrinos, straightforward

Minimal Flavour Violation with Majorana neutrinos

Low energy effective theory and stability

$$\mathcal{L}_{Majorana} = \frac{1}{2} \nu_L^{0T} C^{-1} m_\nu \nu_L^0 + h.c.$$

generated from effective dimension five operator

$$O = \sum_{i,d=1}^{2} \sum_{\alpha,\beta=e,\mu,\tau} \sum_{a,b,c,d=1}^{2} \kappa_{\alpha\beta}^{(ij)} \left(L_{L\alpha a}^T C^{-1} L_{L\beta c} \right) \left(\varepsilon^{ab} \phi_{i b} \right) \left(\varepsilon^{cd} \phi_{jd} \right)$$

$$\mathcal{L}_{Y_\ell} = - \overline{L_L^0} \, \pi_1 \, \phi_1 \, \ell_R^0 - \overline{L_L^0} \, \pi_2 \, \phi_2 \, \ell_R^0 + h.c.$$

$$\pi_1, \pi_2, \quad \kappa'', \kappa'^2, \kappa^{21}, \kappa^{22} \qquad \left(\kappa^{(ij)} \right)$$

$$L_{Li}^0 \longrightarrow exp(i\alpha) L_{Li}^0 \quad , \quad \phi_2 \longrightarrow exp(i\alpha) \phi_2$$

$$\alpha = \pi/2 \quad , \quad Z_4 \text{ symmetry}$$

Imposing this Z_4 symmetry implies: $\qquad (j = 3)$

$k^{(12)} = k^{(21)}$

$k^{(11)} = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad k^{(22)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\alpha = \pi/2$
ensures
$k_{33}^{(22)} \neq 0$

$\frac{1}{2} m_\nu = \frac{1}{2} v_1^2\, k^{(11)} + \frac{1}{2} v_2^2\, e^{2i\theta}\, k^{(22)}$

$\Pi_1 = \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix}, \qquad \Pi_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix}$

Higgs FCNC in the charged sector

Stability: $\qquad k^{(12)} = k^{(21)} = 0$

$k^{(11)}\, \mathcal{P}_3^\nu \quad \mathcal{P}_3^\nu = 0 \qquad\qquad k^{(22)}\, \mathcal{P}_3^\nu \quad \mathcal{P}_3^\nu = k^{(22)}$

$\mathcal{P}_3^\nu\, \Pi_1 = 0 \qquad\qquad\qquad \mathcal{P}_3^\nu\, \Pi_2 = \Pi_2$

stable under renormalization

—16—

W. Grimus, L. Lavoura   2005

Seesaw framework

$$\mathcal{L}_Y + mass = -\bar{L}_L^0 \pi_1 \phi_1 \ell_R^0 - \bar{L}_L^0 \pi_2 \phi \ell_R^0 -$$

$$-\bar{L}_L^0 \Sigma_1 \phi_1 \tilde{\nu}_R^0 - \bar{L}_L^0 \Sigma_2 \tilde{\phi}_2 \nu_R^0 +$$

$$+\frac{1}{2} \nu_R^{0T} C^{-1} M_R \nu_R^0 + h.c.$$

$$m_\ell = \frac{1}{\sqrt{2}}(\nu_1 \pi_1 + \nu_2 e^{i\theta} \pi_2) \quad, \quad m_D = \frac{1}{\sqrt{2}}(\nu_1 \Sigma_1 + \nu_2 e^{-i\theta} \Sigma_2)$$

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} W_\mu^+ \bar{\ell}_L^0 \gamma^\mu \nu_L^0 + h.c$$

$$\mathcal{L}_Y (neutral, lepton) = -\bar{\ell}_L^0 \frac{1}{\sqrt{2}}[m_\ell H^0 + N_\ell^0 R + i N_\ell^0 I]\ell_R^0 -$$

$$-\bar{\nu}_L^0 \frac{1}{\sqrt{2}}[m_D H^0 + N_\nu^0 R + i N_\nu^0 I]\nu_R^0 + h.c.$$

$$N_\ell^0 = \frac{\nu_2}{\sqrt{2}}\pi_1 - \frac{\nu_1}{\sqrt{2}} e^{i\theta} \pi_2$$

$$N_\nu^0 = \frac{\nu_2}{\sqrt{2}}\Sigma_1 - \frac{\nu_1}{\sqrt{2}} e^{-i\theta} \Sigma_2$$

$$\mathcal{L}_{mass} = -\bar{\ell}_L^0 m_\ell \ell_R^0 + \frac{1}{2}\left(\nu_L^{0T}, (\nu_R^0)^{cT}\right) C^{-1} \mathcal{M}^* \begin{pmatrix} \nu_L^0 \\ (\nu_R^0)^c \end{pmatrix} + hc$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \qquad (\psi_L)^c = C\bar{\psi}_0^T (\psi_L)^*$$

BGL type example, Z_4 symmetry

$$L_{L3}^0 \to \exp(i\alpha) L_{L3}^0, \quad \nu_{R3}^0 \to \exp(i2\alpha)\nu_{R3}^0, \quad \phi_2 \to \exp(i\alpha)\phi_2$$

$$\alpha = \frac{\pi}{2}$$

$$\pi_1 = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ x & x & 0 \end{pmatrix}, \quad \pi_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & x & x \end{pmatrix},$$

$$\Sigma_1 = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{pmatrix}, \quad M_R = \begin{pmatrix} x & x & 0 \\ x & x & 0 \\ 0 & 0 & x \end{pmatrix}$$

New feature $m_{\nu i}$ from $m_{eff} = -m_D \frac{1}{M_R} m_D^T \qquad M_{33} \neq 0$

Three light neutrinos ν_i, plus heavy neutrinos N_i ↯

light–light, light–heavy, heavy–heavy couplings

H^0, R, I couplings

$U^\dagger m_{eff} U^* = d$, $\qquad m_D \dfrac{1}{D} m_D^T = -U\,d\,U^T$ (WB m_D diag)

$m_D = i\,U\sqrt{d}\,O\sqrt{D}$ Casas and Ibarra, 2001

$(N_e)^\dagger_{ij} = \dfrac{\sqrt{2}}{v_L}(D\ell)^\dagger_{ij} - \left(\dfrac{\sqrt{2}}{v_L} + \dfrac{v_L}{v_R^2}\right)(U_\nu)_{i3}(U_\nu^\dagger)_{3i}(D\ell)^\dagger_{ij}$

light – light neutral couplings : diag, d

light – heavy neutral couplings : sensitive to O; d, D

heavy – heavy neutral couplings : diff, sensitive to O; d, D

H^+ couplings

$\dfrac{\sqrt{2}\,H^+}{v}\left(\bar{v}_L^\circ N_e \ell_R - \bar{v}_R^\circ N_v \ell_L^\circ\right) + h.c.$

Scalar Potential

Z_4 forbids $\quad \phi_1^\dagger \phi_2^\dagger$, $\quad \phi_1^\dagger \phi_2 \phi_i^\dagger \phi_i$, $\quad \phi_1^\dagger \phi_2 \phi_1^\dagger \phi_2$

ungauged accidental continuous symmetry
not a symmetry of full Lagrangean

after spontaneous gauge symmetry breaking →

$\longrightarrow$ pseudo Goldstone boson

solution : soft symmetry breaking $m_{12} \phi_1^\dagger \phi_2 + h.c.$

Conclusions

Multi-Higgs models are very interesting candidates for NP

There are new mechanisms beyond NFC to obtain strong suppression of FCNC as required by experiment

LHC results may bring surprises for the Higgs sector

Models with three Higgs doublets

...

Yukawa interactions

$$L_Y = -\bar{Q}_L^0 \Gamma_1 \phi_1 d_R^0 - \bar{Q}_L^0 \Gamma_2 \phi_2 d_R^0 - \bar{Q}_L^0 \Gamma_3 \phi_3 d_R^0 -$$
$$- \bar{Q}_L^0 \Delta_1 \tilde{\phi}_1 u_R^0 - \bar{Q}_L^0 \Delta_2 \tilde{\phi}_2 u_R^0 - \bar{Q}_L^0 \Delta_3 \tilde{\phi}_3 u_R^0 + h.c.$$

$$\tilde{\phi}_i = -i \tau_2 \phi_i^*$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}} \left(v_1 e^{i\alpha_1} \Gamma_1 + v_2 e^{i\alpha_2} \Gamma_2 + v_3 e^{i\alpha_3} \Gamma_3 \right)$$

$$M_u = \frac{1}{\sqrt{2}} \left(v_1 e^{-i\alpha_1} \Delta_1 + v_2 e^{-i\alpha_2} \Delta_2 + v_3 e^{-i\alpha_3} \Delta_3 \right)$$

after spontaneous symmetry breakdown

$$\phi_i = e^{i\alpha_i} \begin{pmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \rho_i + i\eta_i) \end{pmatrix}$$

-4-

We perform the following transformation

$$\begin{pmatrix} H^0 \\ R \\ R' \end{pmatrix} = O \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} \; , \quad \begin{pmatrix} G^0 \\ I \\ I' \end{pmatrix} = O \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$$

$$O = \begin{pmatrix} \dfrac{v_1}{v} & \dfrac{v_2}{v} & \dfrac{v_3}{v} \\[2mm] \dfrac{v_2}{v'} & -\dfrac{v_1}{v'} & 0 \\[2mm] \dfrac{v_1}{v''} & \dfrac{v_2}{v''} & \dfrac{-(v_1^2+v_2^2)/v_3}{v''} \end{pmatrix}$$

$$v = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$v' = \sqrt{v_1^2 + v_2^2}$$

$$v'' = \sqrt{v_1^2 + v_2^2 + \left(v_1^2+v_2^2\right)^2/v_3^2}$$

O singles out

H^0 with couplings to quarks proportional to mass matrices

G the neutral pseudo-Goldstone boson

-5-

$$\mathcal{L}_\gamma \ (neutral) = -\frac{H^0}{v} (\bar{d}_L D_d d_R + \bar{u}_L D_u u_R) -$$

$$-\bar{d}_L \frac{1}{\sqrt{2}} N_d (R+iI)d_R - \bar{u}_L \frac{1}{\sqrt{2}} N_u (R-iI)u_R -$$

$$-\bar{d}_L \frac{1}{\sqrt{2}} N_d' (R'+iI')d_R - \bar{u}_L \frac{1}{\sqrt{2}} N_u' (R'-iI')u_R + h.c.$$

With

$$N_d = \frac{1}{\sqrt{2}} U_{dL}^\dagger (v_2 e^{i\alpha_1}\Gamma_1 - v_1 e^{i\alpha_2}\Gamma_2) U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}} U_{uL}^\dagger (v_2 e^{-i\alpha_1}\Delta_1 - v_1 e^{-i\alpha_2}\Delta_2) U_{uR}$$

$$N_d' = \frac{1}{\sqrt{2}} U_{dL}^\dagger (v_1 e^{i\alpha_1}\Gamma_1 + v_2 e^{i\alpha_2}\Gamma_2 + x e^{i\alpha_3}\Gamma_3) U_{dR}$$

$$N_u' = \frac{1}{\sqrt{2}} U_{uL}^\dagger (v_1 e^{-i\alpha_1}\Delta_1 + v_2 e^{-i\alpha_2}\Delta_2 + x e^{-i\alpha_3}\Delta_3) U_{uR}$$

$$x = -(v_1^2 + v_2^2)/v_3$$

Imposing the following discrete symmetry on the Lagrangian

$$Q_{L1}^0 \to \omega Q_{L1}^0 \quad , \quad Q_{L2}^0 \to \omega^2 Q_{L2}^0 \quad , \quad Q_{L3}^0 \to \omega^4 Q_{L3}^0$$

$$\ell_1^0 \to \omega \ell_1^0 \quad , \quad \ell_2^0 \to \omega^2 \ell_2^0 \quad , \quad \ell_3^0 \to \omega^4 \ell_3^0$$

$$u_{R1}^0 \to \omega^2 u_{R1}^0 \quad , \quad u_{R2}^0 \to \omega^4 u_{R2}^0 \quad , \quad u_{R3}^0 \to \omega^8 u_{R3}^0$$

$$d_{R1}^0 \to d_{R1}^0$$

$$\text{with} \quad \omega = \exp i\pi/4$$

restricts the yukawa coupling matrices. Following structure

$$\Gamma_1 = \begin{bmatrix} \times & \times & \times \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Gamma_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} \times & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Delta_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{bmatrix}$$

all three Higgs doublets have non-zero yukawa couplings
both in the up and down sectors

In this case there are Higgs mediated FCNC only down sector

$$(N_d)_{ij} = \frac{\sqrt{2}}{v_1}(D_d)_{ij} - \left(\frac{\sqrt{2}}{v_1} + \frac{v_1}{v_2}\right)(V_{CKM})_{i2}(V_{CKM}^\dagger)_{2j}(D)_{jj}$$
$$- \frac{\sqrt{2}}{v_1}(V_{CKM}^\dagger)_{i3}(V_{CKM})_{3j}(D)_{jj}$$

$$x = -(v_1^2 + v_2^2)/v_3$$

$$(N_d')_{ij} = (D_d)_{ij} - \frac{v_3 - x}{v_3}(V_{CKM}^\dagger)_{i3}(V_{CKM})_{3j}(D_d)_{jj}$$

N_d includes FCNC terms where the suppression factor in $\Delta S = 2$ transitions is only $(V_{cd}^* V_{cs})^2$, which then requires quite heavy neutral Higgs