Physics of neutrino oscillations & flavor conversion

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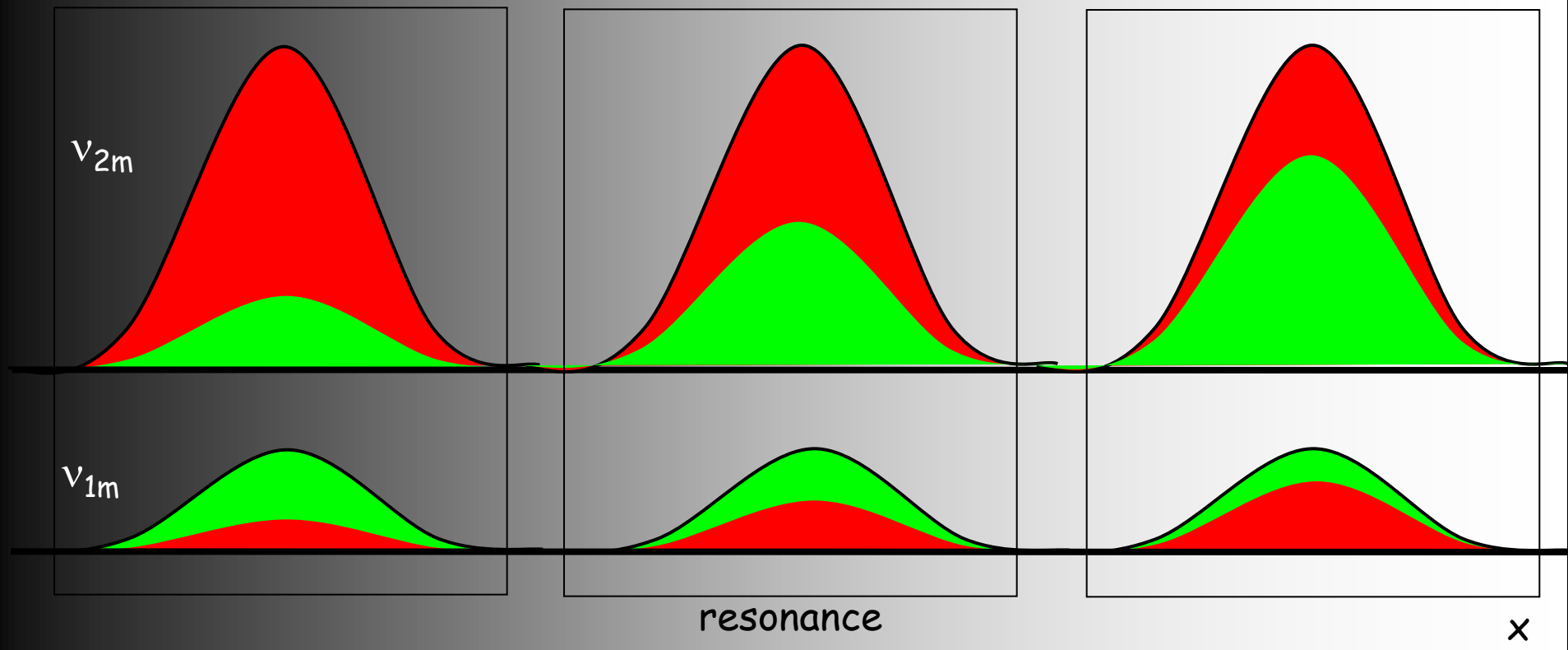
Invisibles network INT Training lectures
June 25 – 29, 2012

...in addition

Adiabatic conversion



if density changes slowly

- the amplitudes of the wave packets do not change
- flavors of the eigenstates follow the density change

Adiabatic conversion probability

Sun, Supernova

From high to low densities

Initial state: $\nu(0) = \nu_e = \cos\theta_m^0\, \nu_{1m}(0) + \sin\theta_m^0\, \nu_{2m}(0)$

Mixing angle in matter in initial state

Adiabatic evolution to the surface of the Sun (zero density):

$\nu_{1m}(0) \rightarrow \nu_1$
$\nu_{2m}(0) \rightarrow \nu_2$

Final state: $\nu(f) = \cos\theta_m^0\, \nu_1 + \sin\theta_m^0\, \nu_2\, e^{-i\phi}$

Probability to find ν_e averaged over oscillations

$$P = |\langle \nu_e | \nu(f) \rangle|^2 = (\cos\theta \cos\theta_m^0)^2 + (\sin\theta \sin\theta_m^0)^2$$

$$= 0.5[\, 1 + \cos 2\theta_m^0 \cos 2\theta \,]$$

$$P = \sin^2\theta + \cos 2\theta \cos^2\theta_m^0$$

Two aspects of mixing

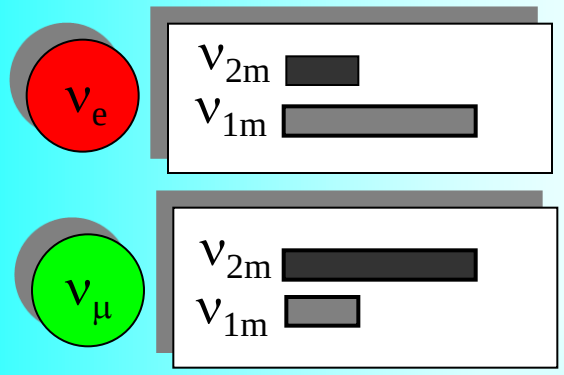
$$\nu_e = \cos\theta_m\, \nu_{1m} + \sin\theta_m \nu_{2m}$$
$$\nu_\mu = -\sin\theta_m \nu_{1m} + \cos\theta_m \nu_{2m}$$

inversely

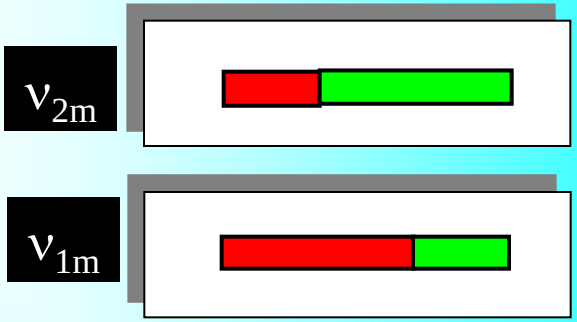
$$\nu_{2m} = \sin\theta_m\, \nu_e + \cos\theta_m\, \nu_\mu$$
$$\nu_{1m} = \cos\theta_m \nu_e - \sin\theta\, \nu_\mu\, \theta_m$$

coherent mixtures
of mass eigenstates

flavor composition of
the mass eigenstates



Wave packets



flavors of eigenstates

Vacuum $\theta_m \rightarrow \theta$

$$\nu_{1m} \rightarrow \nu_1$$
$$\nu_{2m} \rightarrow \nu_2$$

Level crossing

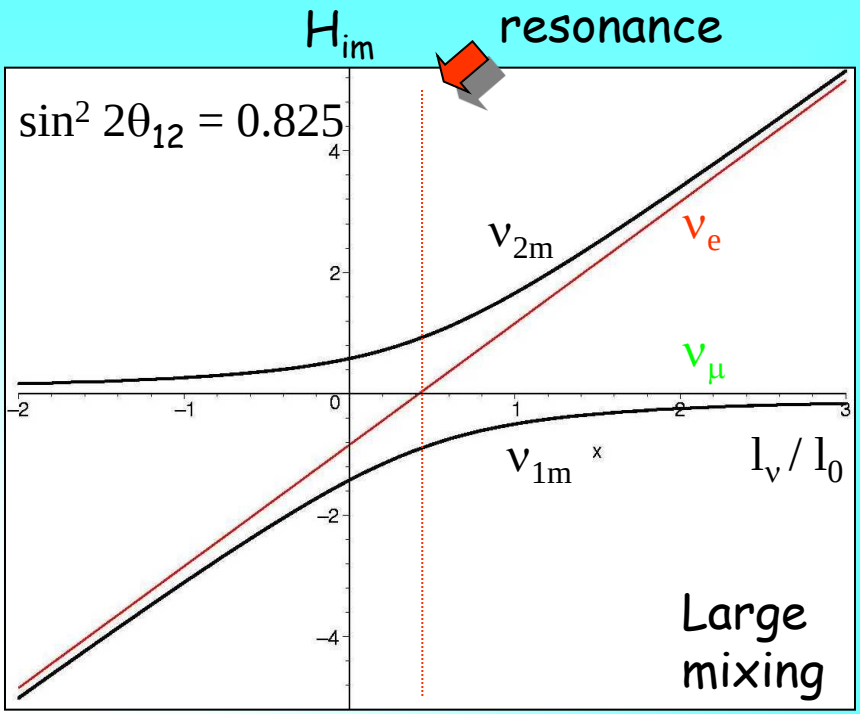
V. Rubakov, private comm.
N. Cabibbo, Savonlinna 1985
H. Bethe, PRL 57 (1986) 1271

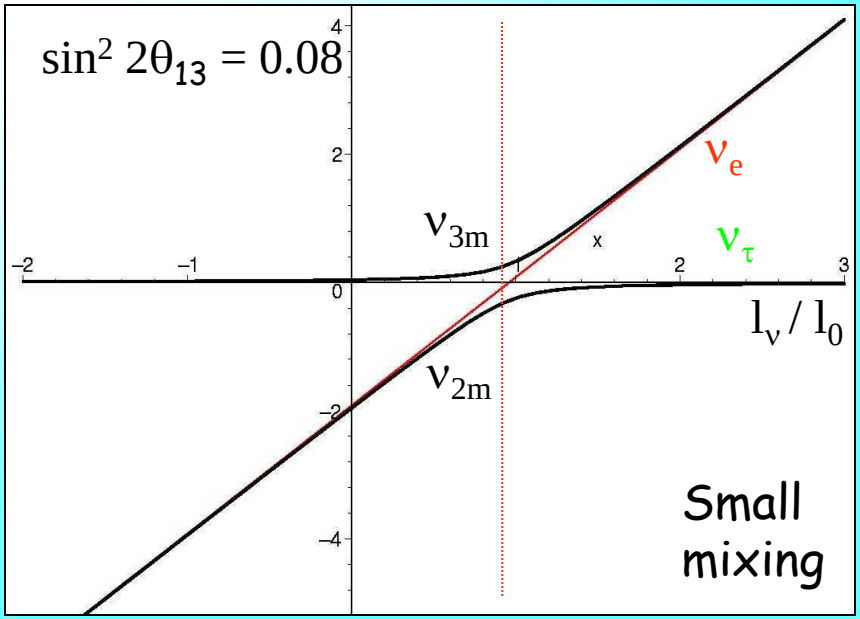
Dependence of the neutrino eigenvalues on the matter potential (density)

$$\frac{l_\nu}{l_0} = \frac{2E\,V}{\Delta m^2}$$

$$\frac{l_\nu}{l_0} = \cos 2\theta$$

Crossing point - resonance
- the level split is minimal
- the oscillation length is maximal





References

Mass hierarchy, 2-3 mixing and CP-phase with Huge Atmospheric Neutrino Detectors. E.Kh. Akhmedov, Soebur Razzaque, A.Yu. Smirnov. e-Print: arXiv:1205.7071 [hep-ph]

1-3 leptonic mixing and the neutrino oscillograms of the Earth. Evgeny K. Akhmedov, Michele Maltoni, Alexei Yu. Smirnov , JHEP 0705 (2007) 077 e-Print: hep-ph/0612285

Neutrino oscillograms of the Earth: Effects of 1-2 mixing and CP-violation. Evgeny Kh. Akhmedov, Michele Maltoni, Alexei Yu. Smirnov, JHEP 0806 (2008) 072 e-Print: arXiv:0804.1466 [hep-ph]

and references therein

Vaccum vs. matter

Can be treated on the same footing

vacuum:

matter with constant density,
(unless near topological defects)

Mass diagonal
Mixes flavor states

Flavor states
oscillate

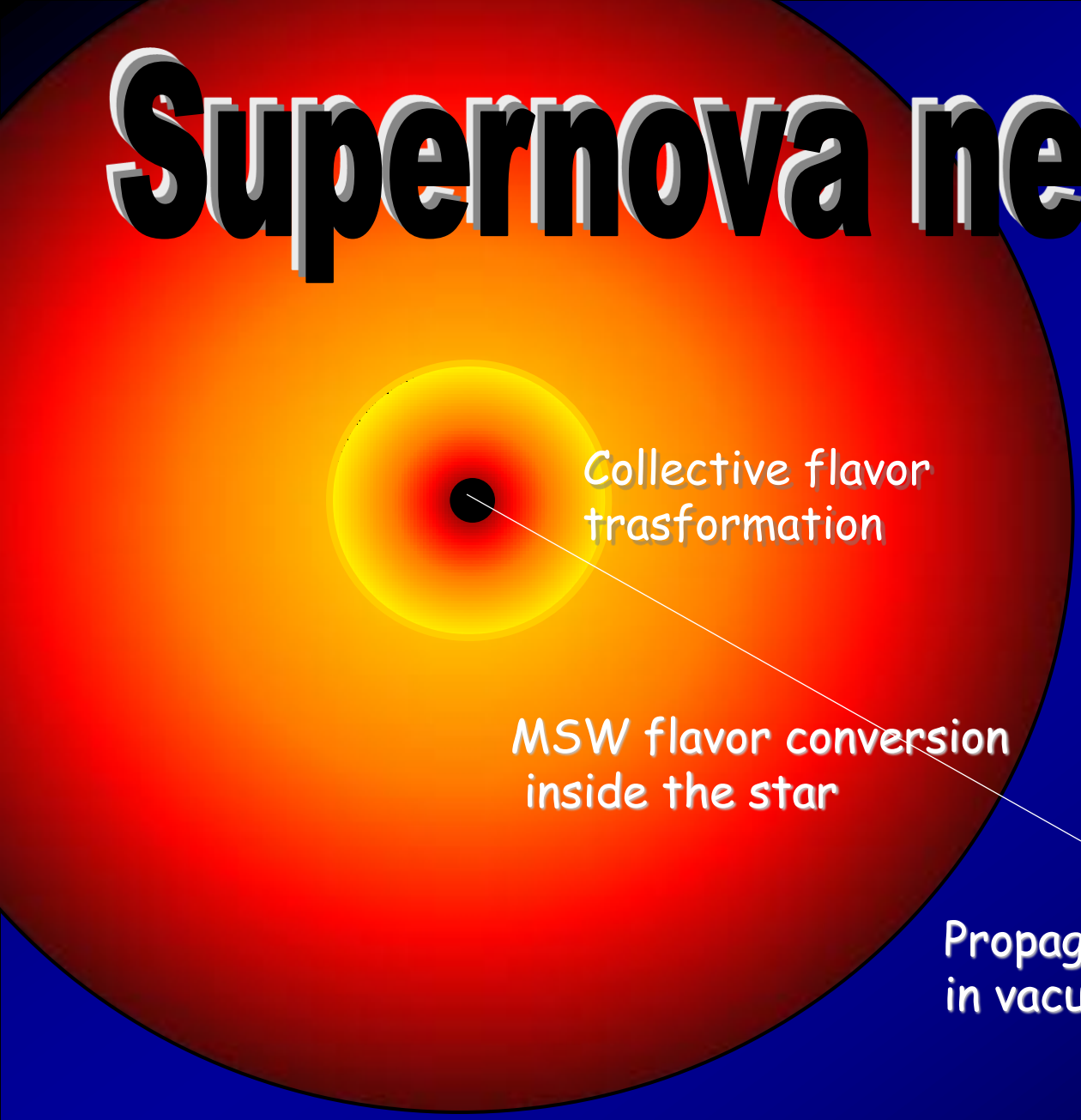
matter:

Flavor diagonal
Mixes mass eigenstates

Mass states oscillate
in matter

Flavor states and mass states change roles when
matter and vacuum exchange

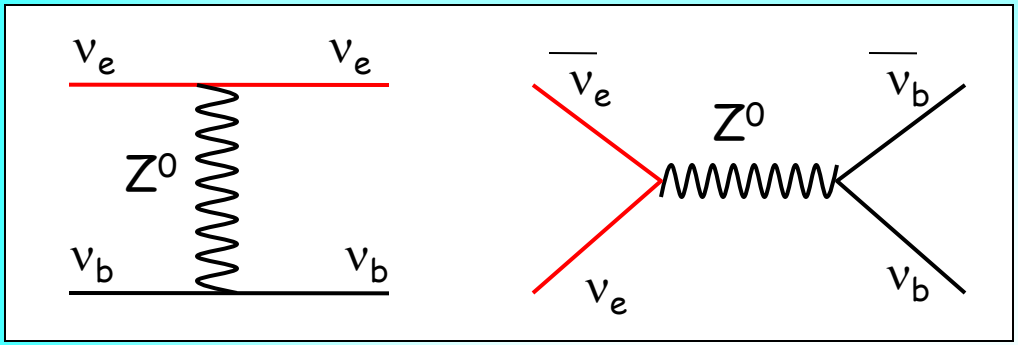
Supernova neutrinos



How flavor diagonal interactions can lead to flavor off-diagonal elements of the Hamiltonian?

Origin of collective effects

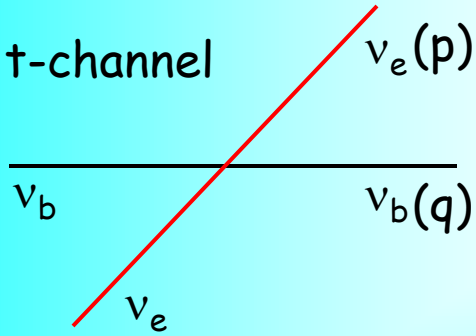
$\nu\nu$-scattering



Refraction in neutrino gases

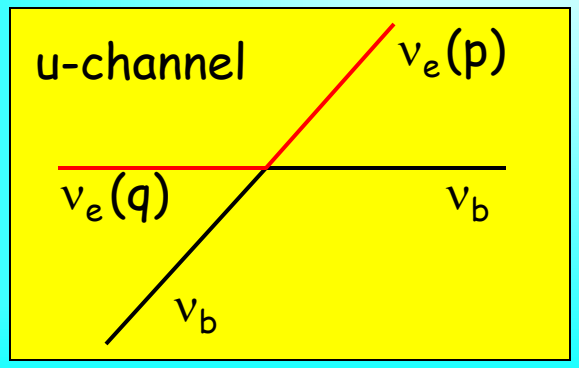
$$A = \sqrt{2}\, G_F (1 - v_e\, v_b)$$

velocities



ν_e

elastic forward scattering



J. Pantaleone

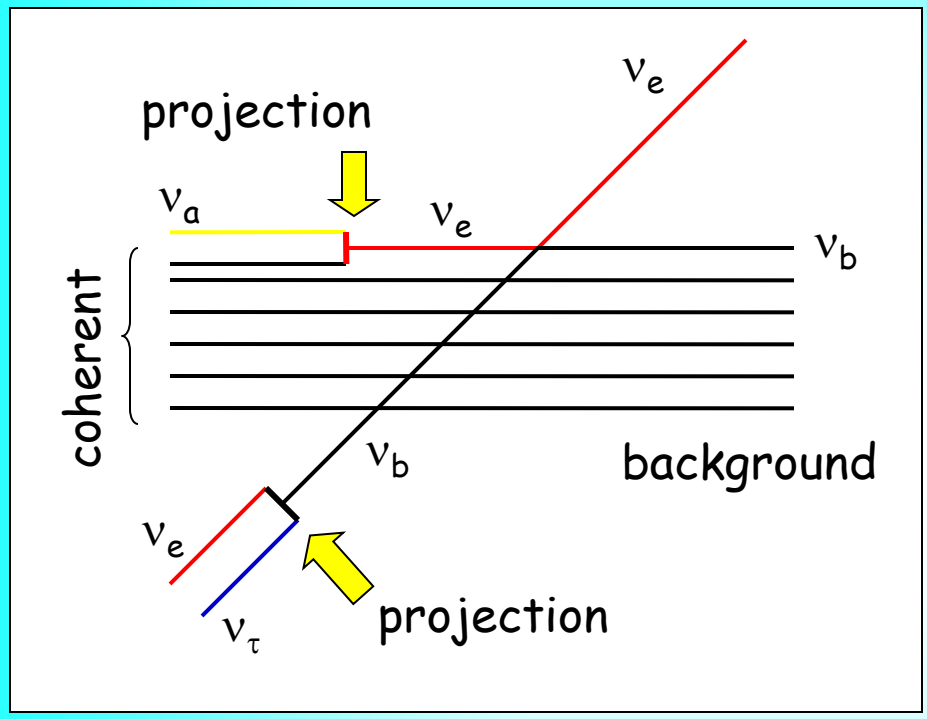
can lead to the coherent effect

Momentum exchange → flavor exchange → flavor mixing

Collective flavor transformations

Flavor exchange

J. Pantaleone
S. Samuel
V.A. Kostelecky



$\nu\nu$ – scattering in u-channel due to Z^0 - exchange

1. Momentum exchange → flavor exchange

2. Coherence if the background is in mixed state:

$$|\nu_{ib}\rangle = \Phi_{ie}\,|\nu_e\rangle + \Phi_{i\tau}\,|\nu_\tau\rangle$$

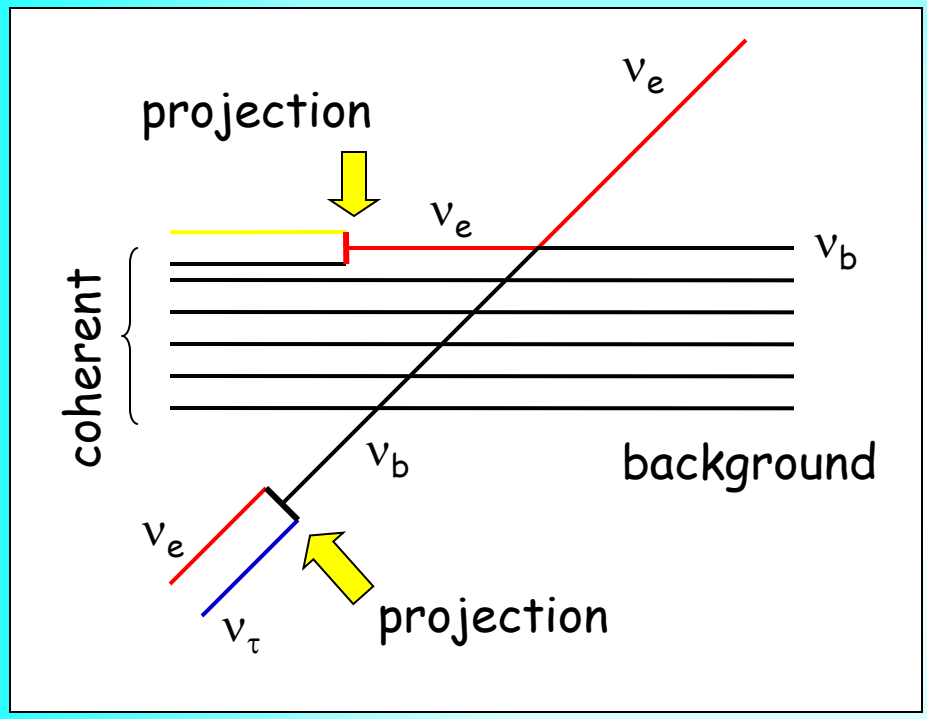
Coherent flavor changing transition

Probe neutrino = background neutrino

Potential depends on transition probability

Flavor exchange

J. Pantaleone
S. Samuel
V.A. Kostelecky



Flavor exchange between the beam (probe) and background neutrinos

If the background is in the mixed state:

$$|\nu_{ib}\rangle = \Phi_{ie}\,|\nu_e\rangle + \Phi_{i\tau}\,|\nu_\tau\rangle$$

$$B_{e\tau} \sim \Sigma_i\, \Phi_{ie}^{*}\Phi_{i\tau}$$

sum over particles of bg. w.f. give projections

Contribution to the Hamiltonian in the flavor basis

$$H_{\nu\nu} = \sqrt{2}\, G_F\, \Sigma_i\, (1 - v_e\, v_{ib}) \begin{pmatrix} |\Phi_{ie}|^2 & \Phi_{ie}^{*}\Phi_{i\tau} \\ \Phi_{ie}\Phi_{i\tau}^{*} & |\Phi_{i\tau}|^2 \end{pmatrix}$$

Evolution equation

Ensemble of neutrino polarization vectors $\mathbf{P}_\omega$

$$d_t \mathbf{P}_\omega = (-\omega \mathbf{B} + \lambda \mathbf{L} + \mu \mathbf{P}) \times \mathbf{P}_\omega$$

Negative frequencies for antineutrinos

Vacuum mixing term

$\mathbf{B} = (\sin 2\theta,\ 0,\ \cos 2\theta)$

$\omega = \Delta m^2 / 2E$

Usual matter potential

$\mathbf{L} = (0,\ 0,\ 1)$

$\lambda = V = \sqrt{2}\, G_F\, n_e$

Collective vector

$$\mathbf{P} = \int_{-\inf}^{+\inf} d\omega\ \mathbf{P}_\omega$$

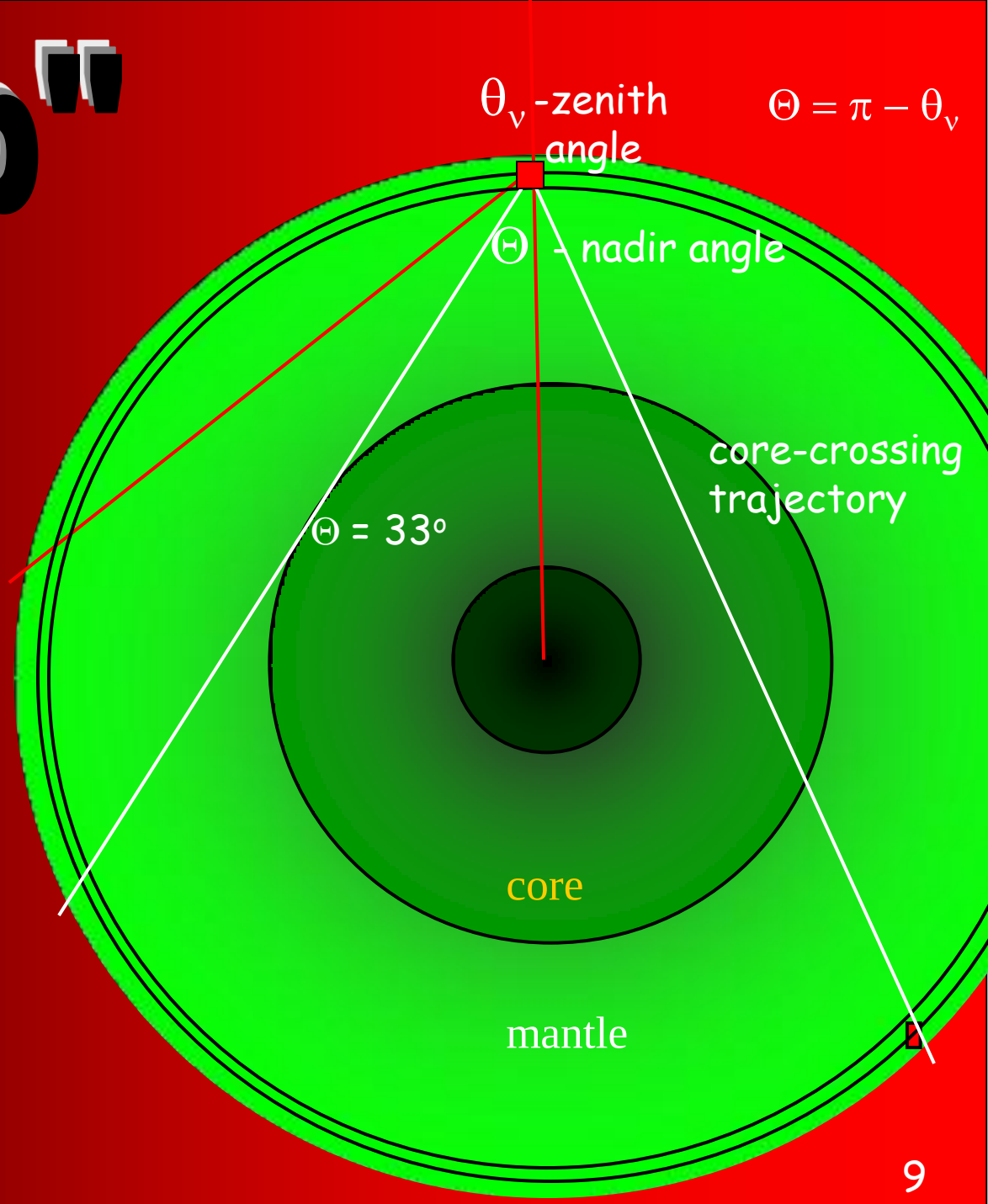
$\mu = \sqrt{2}\, G_F\, n_\nu\, (1 - \cos \theta_{\nu\nu})$

The term describes collective effects

Neutrino propagation In the Earth

"Set-up"

Oscillations in multilayer medium



Applications:

flavor-to-flavor transitions

- accelerator
- atmospheric
- cosmic neutrinos

Oscillations in multilayer medium

Adiabaticity breaking at the borders of layers

Graphical representation

Equation of motion
(= spin in magnetic field)

$$\frac{dP}{dt} = (B \times P)$$

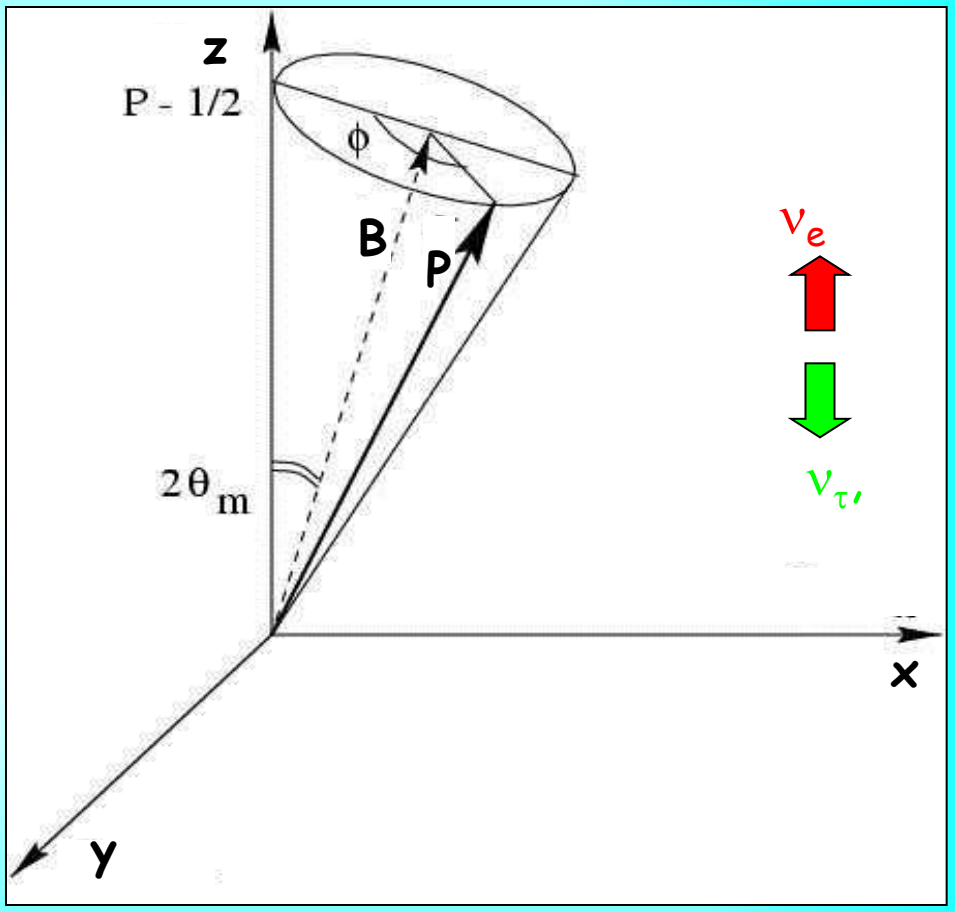
where ``magnetic field" vector:

$$B = \frac{2\pi}{l_m}(\sin 2\theta_m, 0, \cos 2\theta_m)$$

$$P = (\mathrm{Re}\, \nu_e^+ \nu_\tau,\ \mathrm{Im}\, \nu_e^+ \nu_\tau,\ \nu_e^+ \nu_e - 1/2)$$

Phase of oscillations

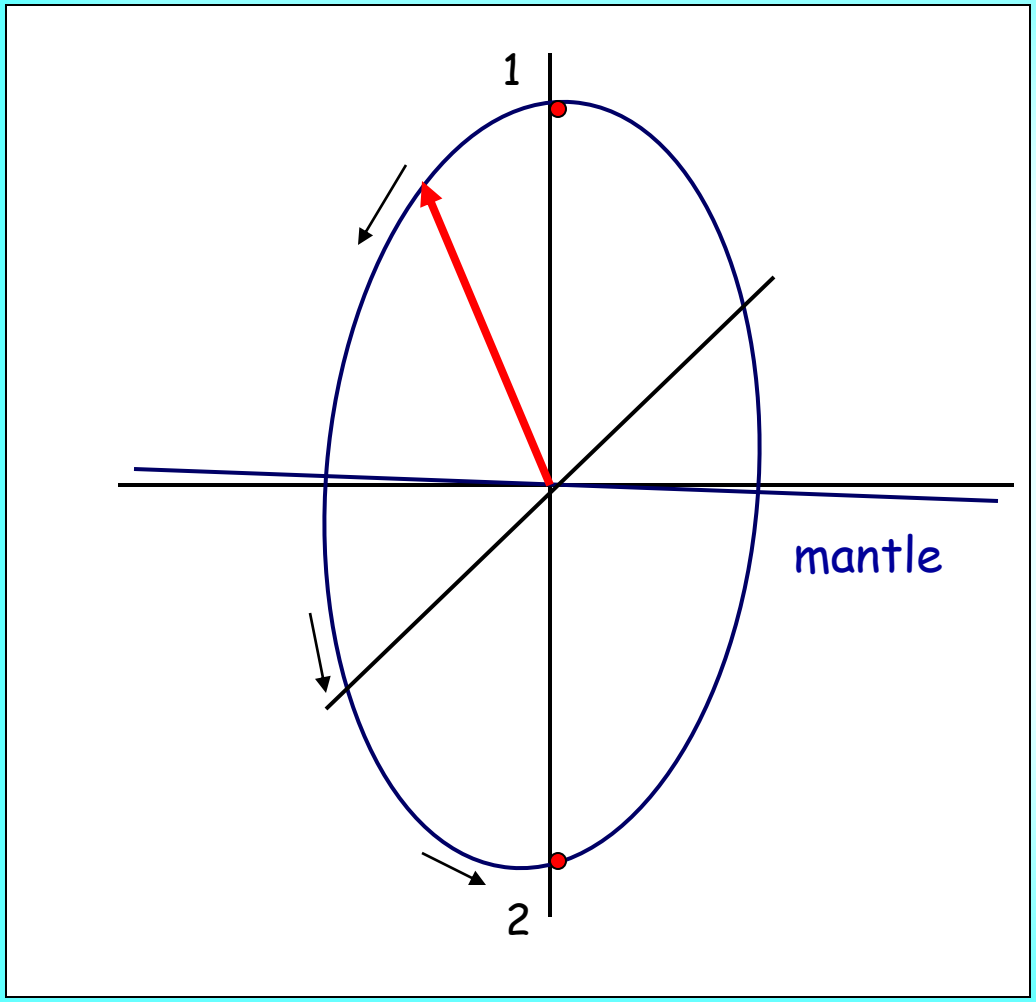
$$\phi = 2\pi t / l_m$$

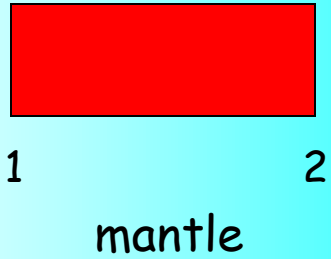


Probability to find ν_e

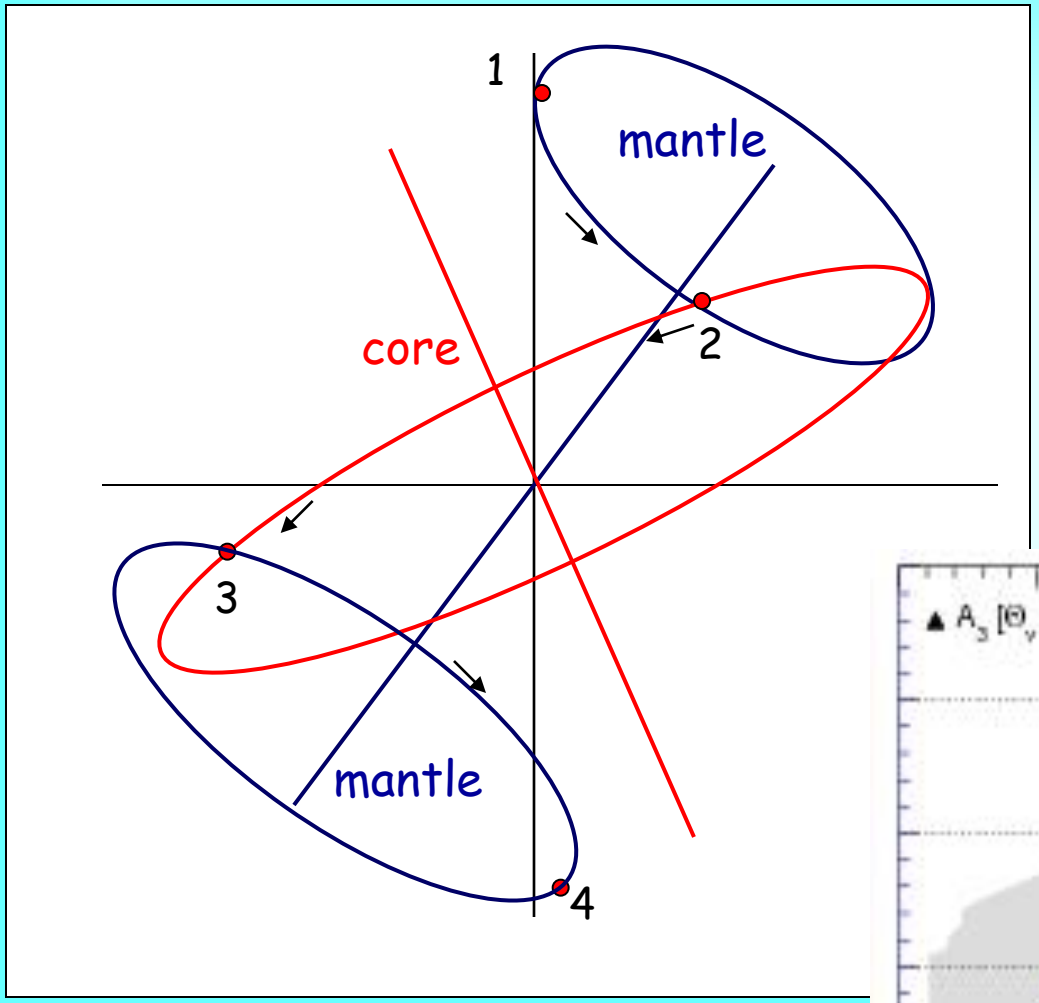
$$P_{ee} = \nu_e^+ \nu_e = P_Z + 1/2 = \cos^2\theta_Z/2$$

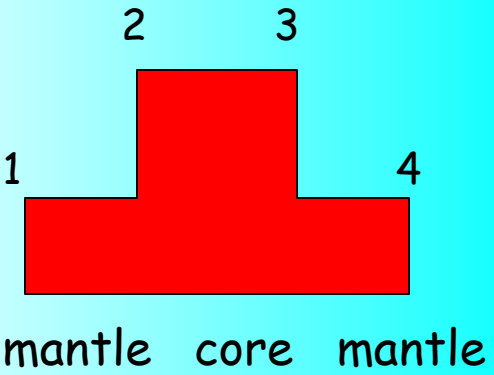
Resonance enhancement in mantle

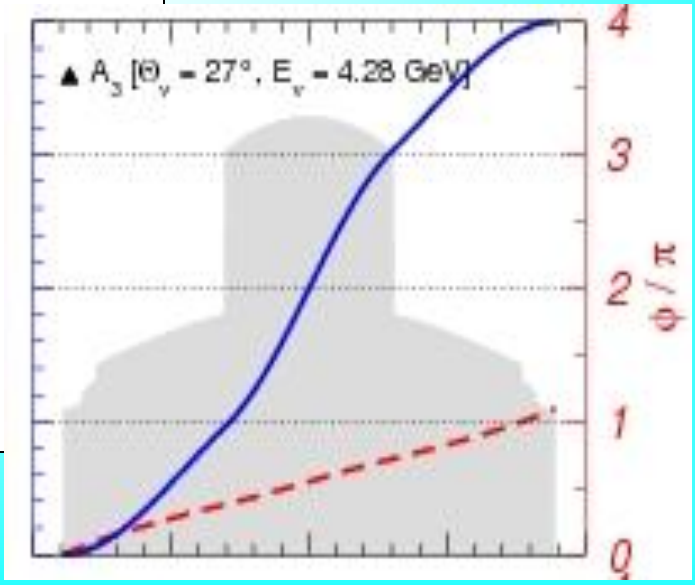




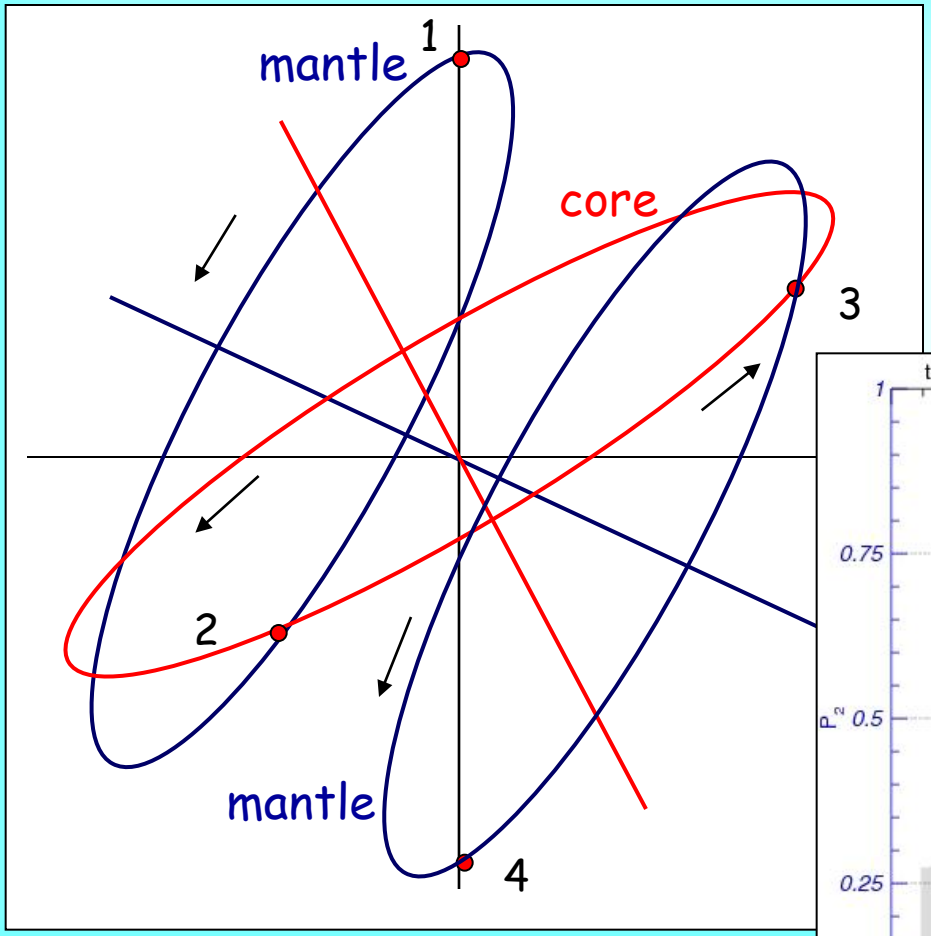
Parametric enhancement

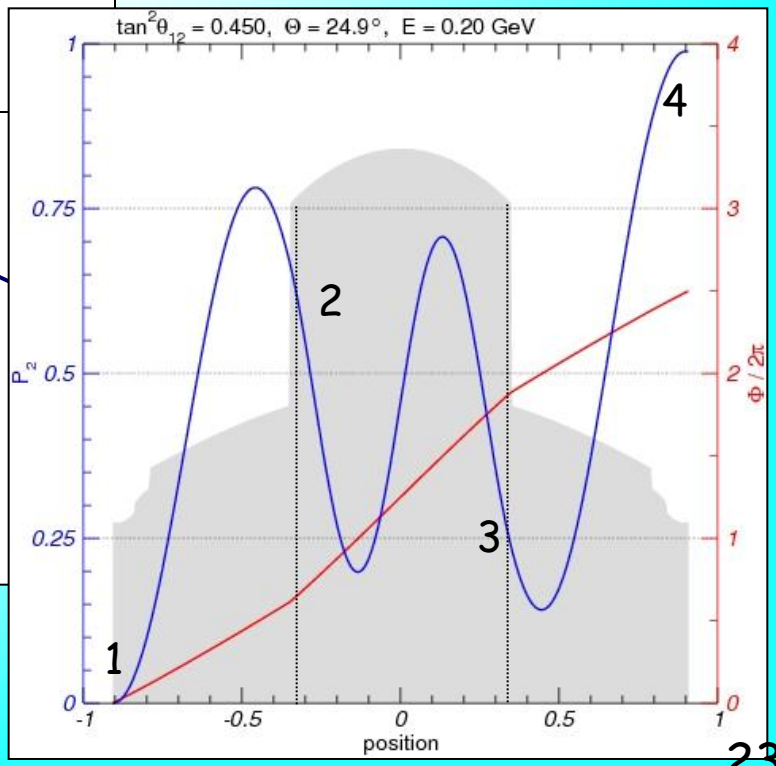






Parametric enhancement of 1-2 mode





Graphical representation

a). Resonance in the mantle

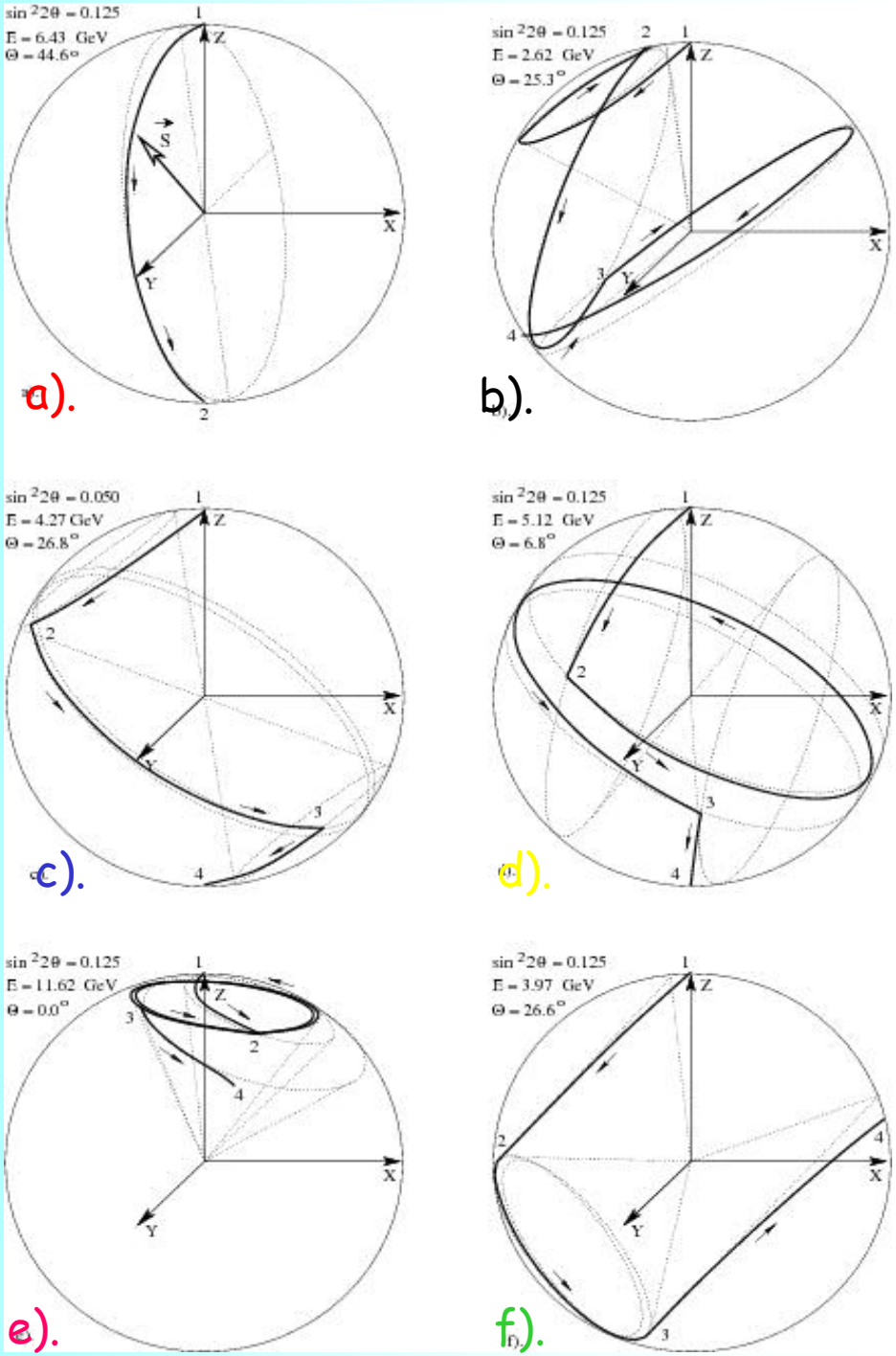
b). Resonance in the core

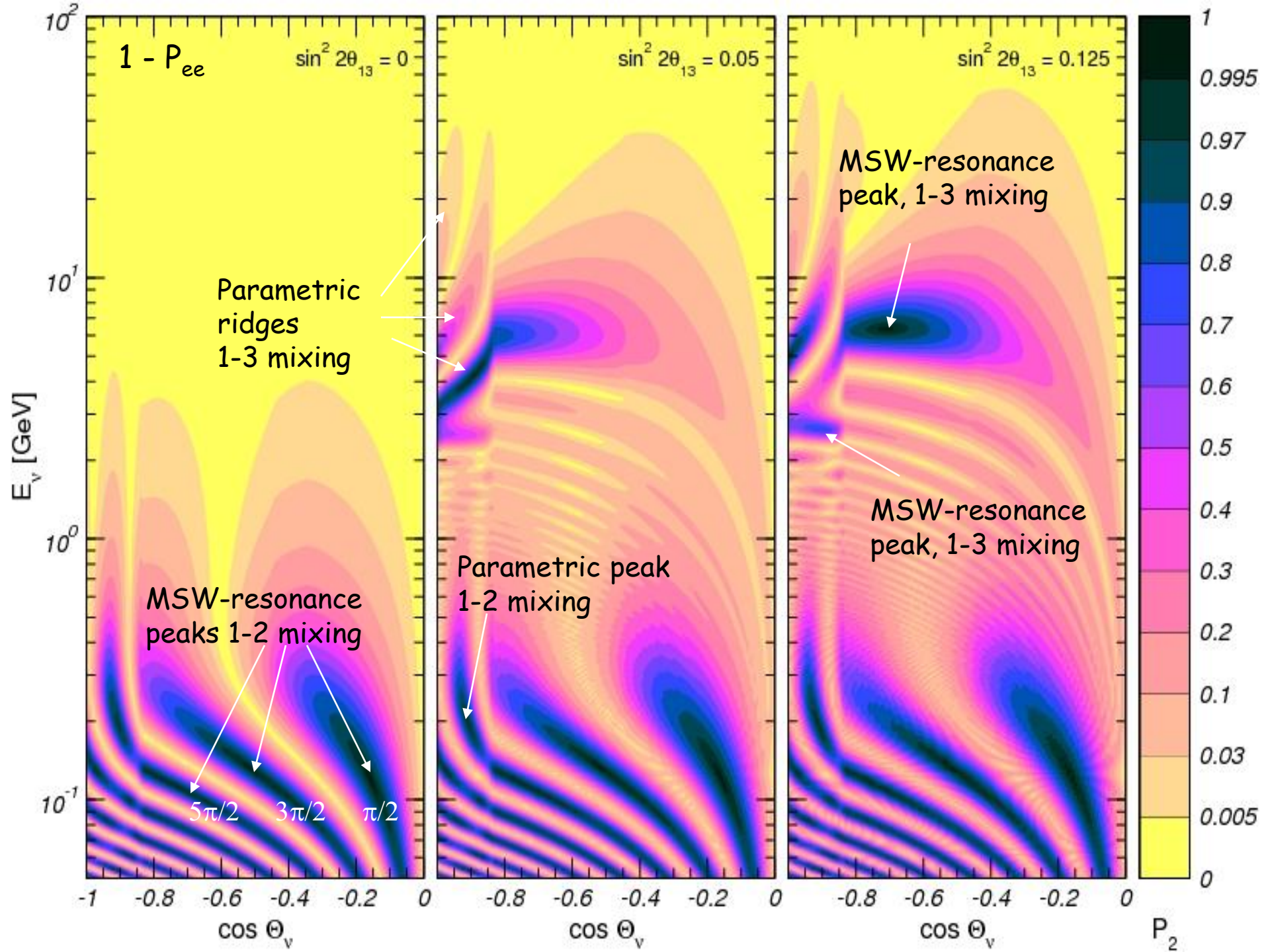
c). Parametric ridge A

d). Parametric ridge B

e). Parametric ridge C

f). Saddle point




1 - P_ee
sin² 2θ13 = 0
sin² 2θ13 = 0.05
sin² 2θ13 = 0.125
Parametric ridges 1-3 mixing
MSW-resonance peak, 1-3 mixing
MSW-resonance peak, 1-3 mixing
Parametric peak 1-2 mixing
MSW-resonance peaks 1-2 mixing
5π/2
3π/2
π/2
E_ν [GeV]
cos Θ_ν
P_2

Evolution

For $E > 0.1$ GeV

Propagation basis

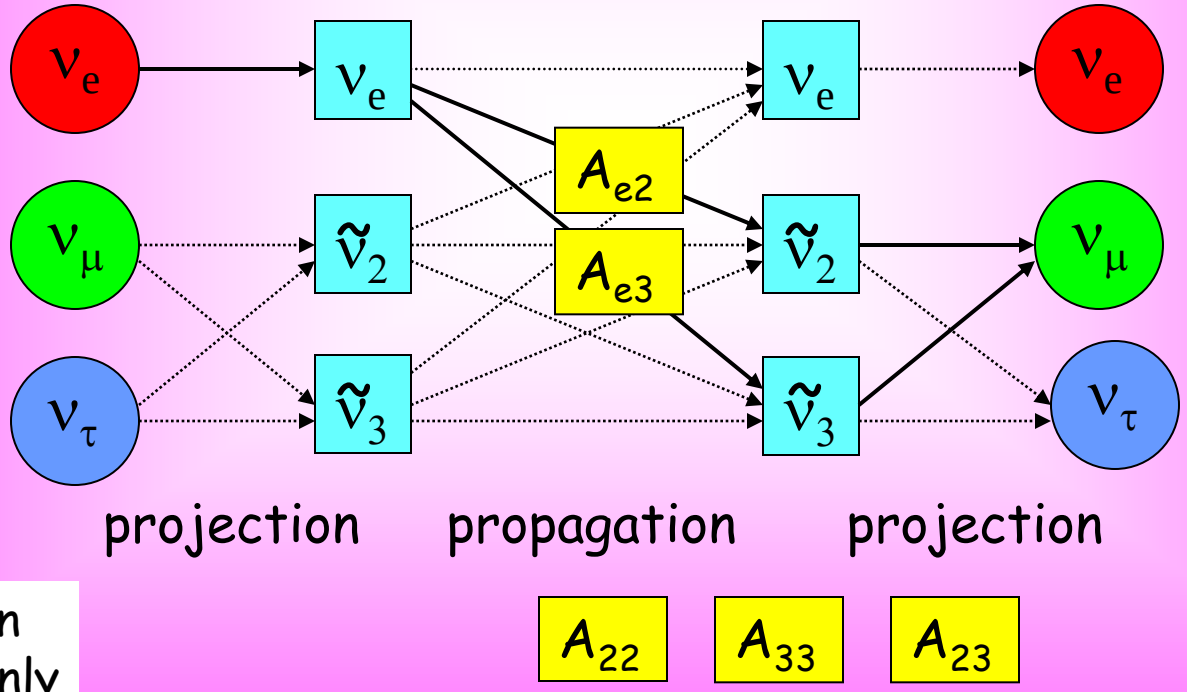
$$\nu_f = U_{23} I_\delta \tilde{\nu}$$

$$I_\delta = \mathrm{diag}\,(1, 1, e^{i\delta})$$

$$\tilde{H} = U_{13}^T U_{12}^T H^{diag} U_{12} U_{13}$$

$$H^{diag} = \mathrm{diag}\,(H_{1m}, H_{2m}, H_{3m})$$

CP-violation and 2-3 mixing - excluded from dynamics of propagation



CP appears in projection only

For instance: $A(\nu_e \to \nu_\mu) = \cos\theta_{23} A_{e2} e^{i\delta} + \sin\theta_{23} A_{e3}$

Probabilities

E Kh Akhmedov,
S Razzaque,
A. Y.S.

for hierarchy determination,
neglect 1-2 mixing effects

$$P(\nu_e \rightarrow \nu_\mu) = s_{23}^2 |A_{e3}|^2$$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \tfrac{1}{2} \sin^2 2\theta_{23} - s_{23}^4 |A_{e3}|^2 + \tfrac{1}{2} \sin^2 2\theta_{23} (1 - |A_{e3}|^2)^{\frac{1}{2}} \cos \phi$$

Reduces the average probability

Reduces the depth of oscillations interference

Modifies phase

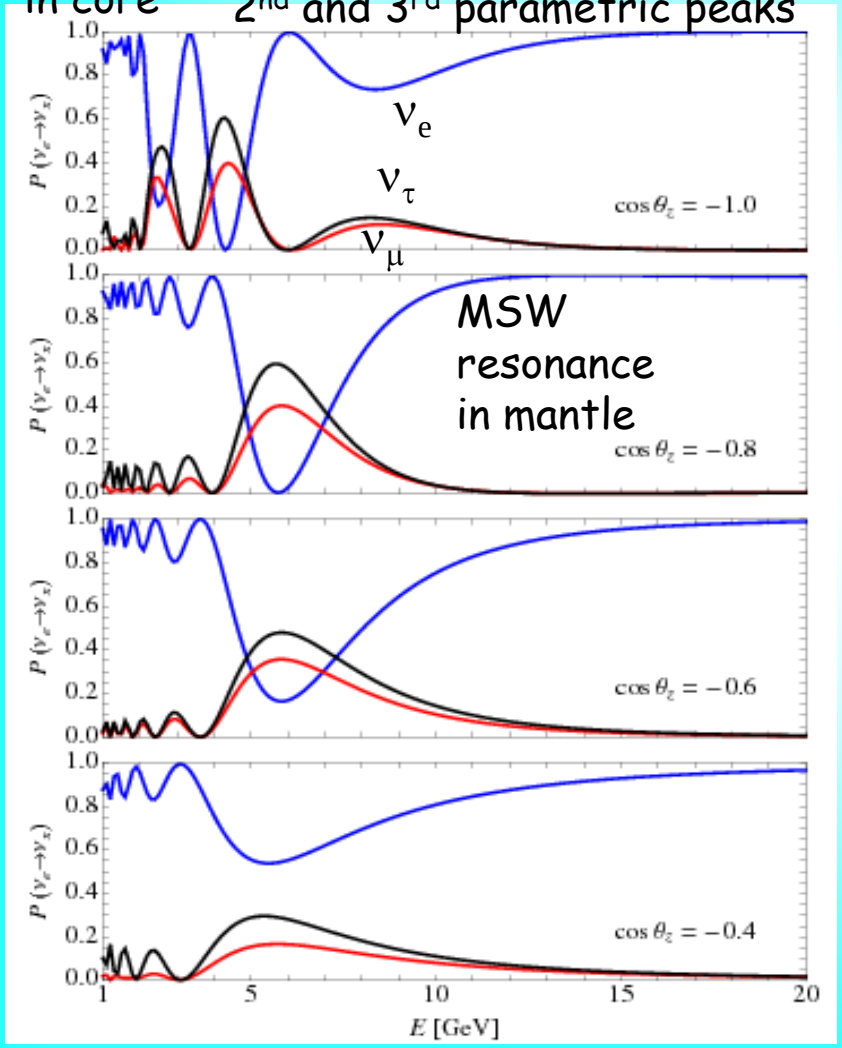
$$\phi = \arg (A_{22} A_{33}^*)$$

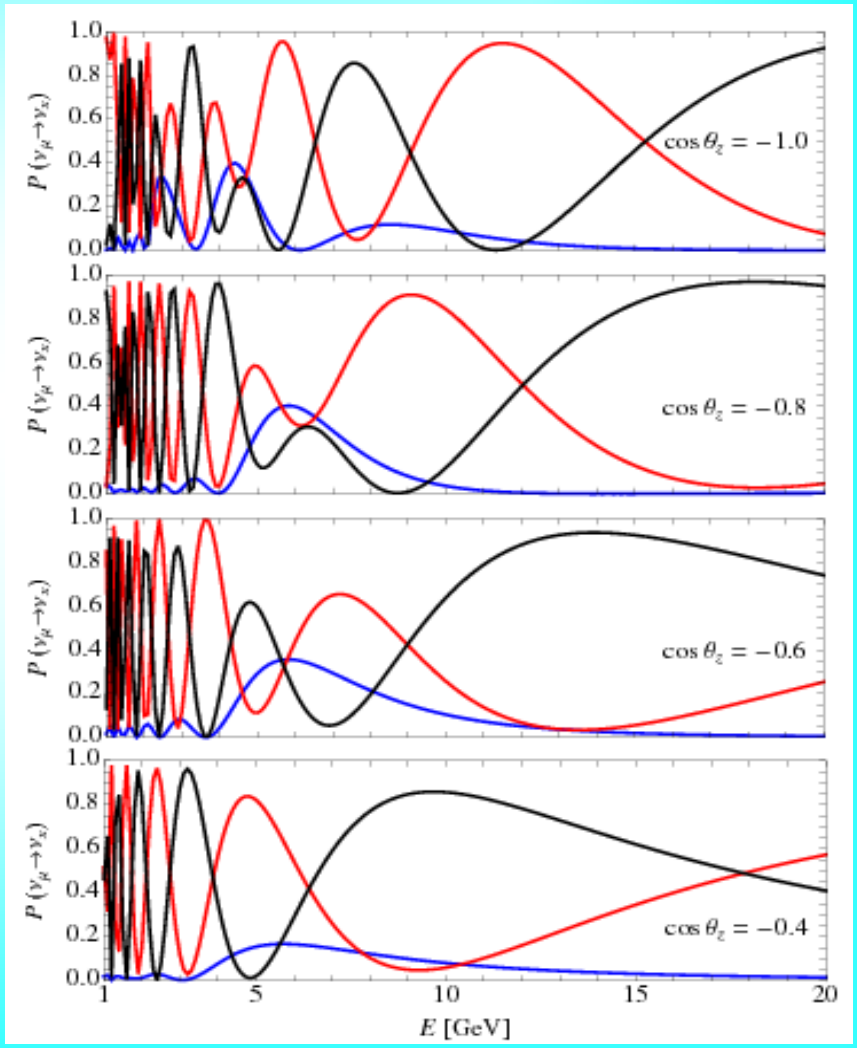
$$P(\nu_\mu \rightarrow \nu_\tau) = \tfrac{1}{2} \sin^2 2\theta_{23} - s_{23}^2 c_{23}^2 |A_{e3}|^2 - \tfrac{1}{2} \sin^2 2\theta_{23} (1 - |A_{e3}|^2)^{\frac{1}{2}} \cos \phi$$

Oscillation probabilities

MSW resonance in core

2nd and 3rd parametric peaks

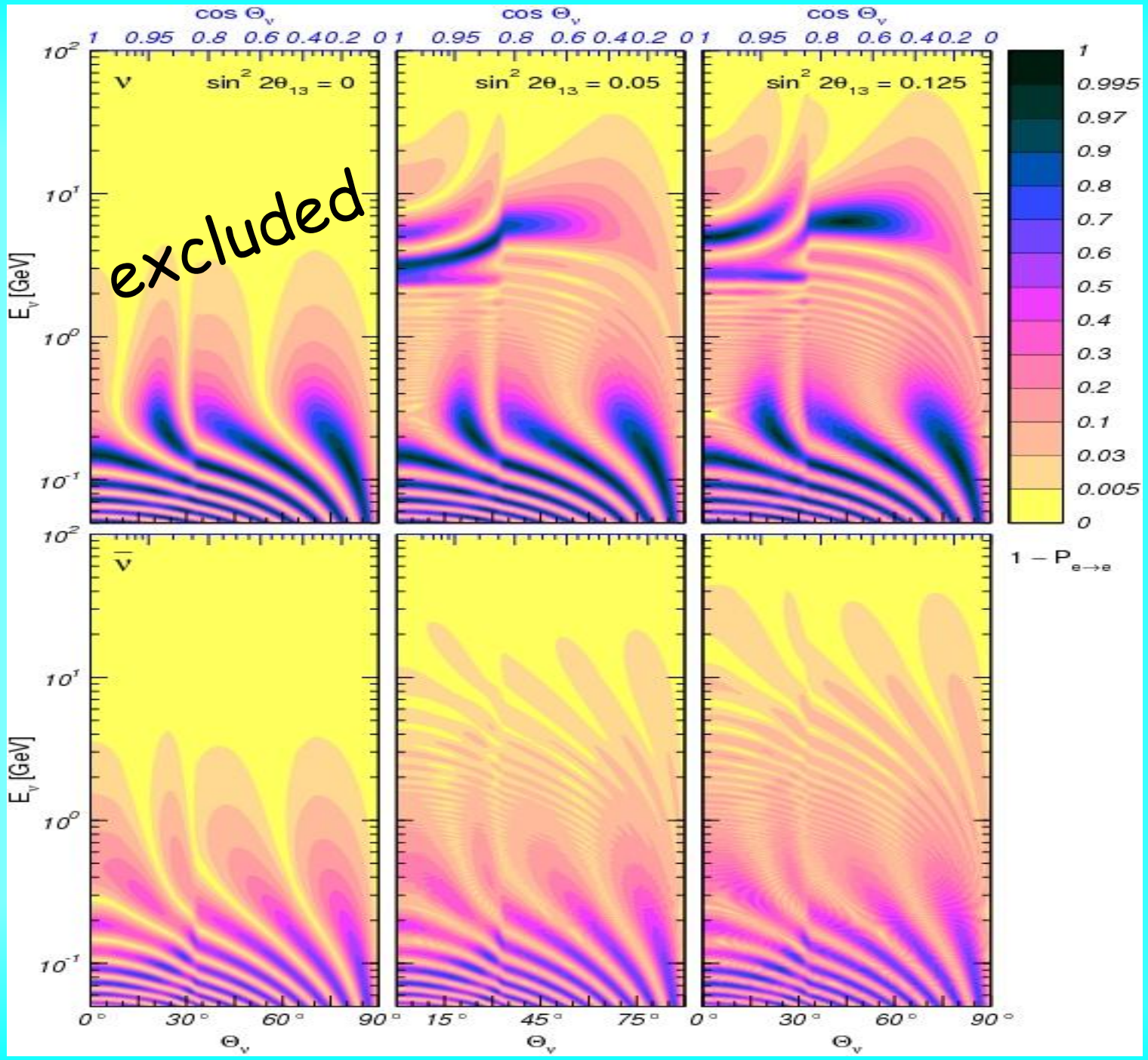


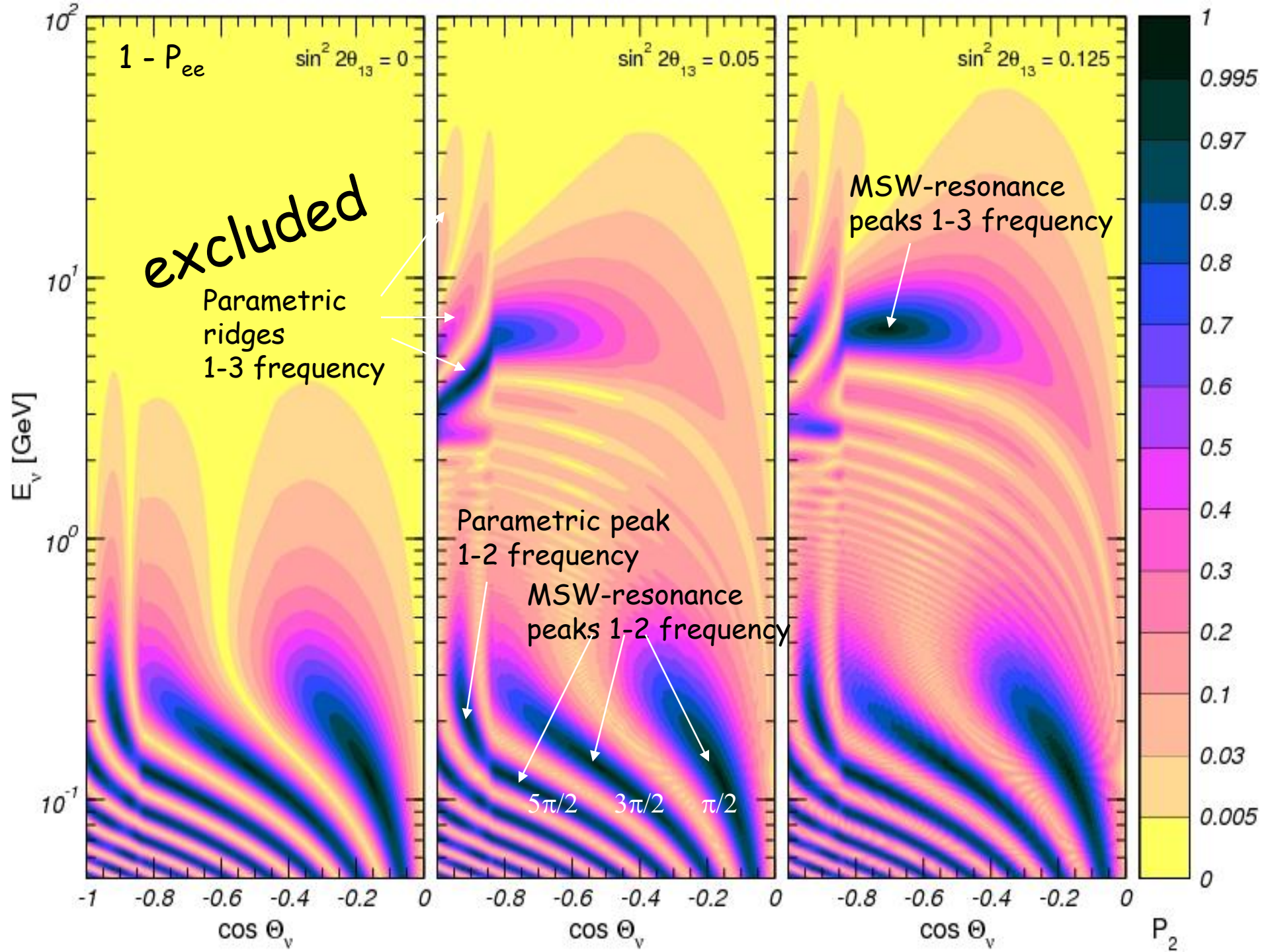


Oscillograms

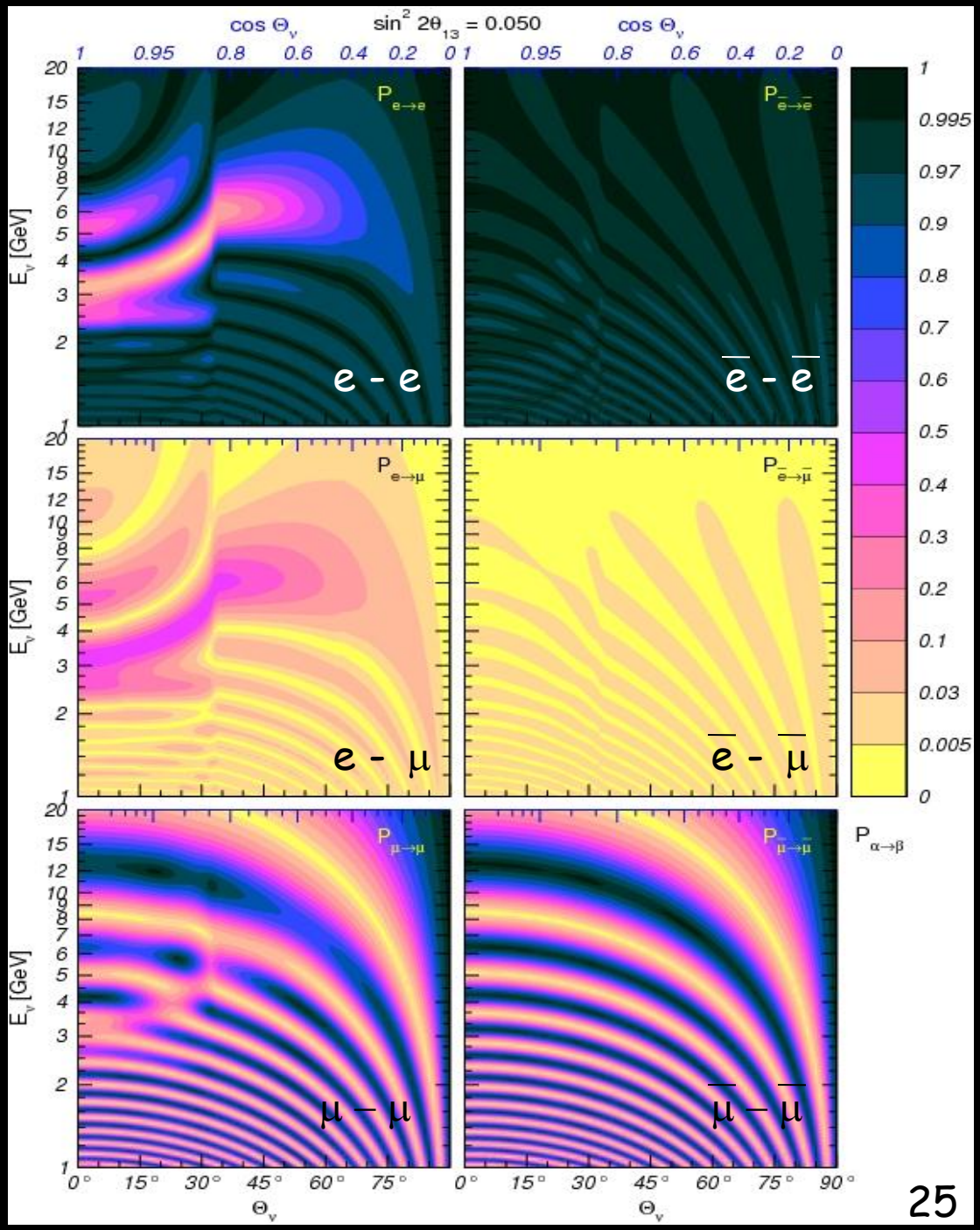
and physics of oscillations

P. Lipari ,
T. Ohlsson
M. Blennow
M. Chizhov,
M. Maris,
S .Petcov
T. Kajita
…


cos Θν
1 0.95 0.8 0.6 0.4 0.2 0
ν
sin² 2θ13 = 0
sin² 2θ13 = 0.05
sin² 2θ13 = 0.125
excluded
ν̄
Eν [GeV]
10²
10¹
10⁰
10⁻¹
0° 30° 60° 90°
15° 45° 75°
Θν
1
0.995
0.97
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.03
0.005
0
1 − Pe→e


1 - P_ee
sin² 2θ13 = 0
sin² 2θ13 = 0.05
sin² 2θ13 = 0.125
excluded
Parametric ridges 1-3 frequency
MSW-resonance peaks 1-3 frequency
Parametric peak 1-2 frequency
MSW-resonance peaks 1-2 frequency
5π/2
3π/2
π/2
E_ν [GeV]
cos Θ_ν
P_2

Other channels mass hierarchy



For 2ν system

normal → inverted

neutrino → antineutrino

CP-violation domains

Three grids of lines:

Solar magic lines

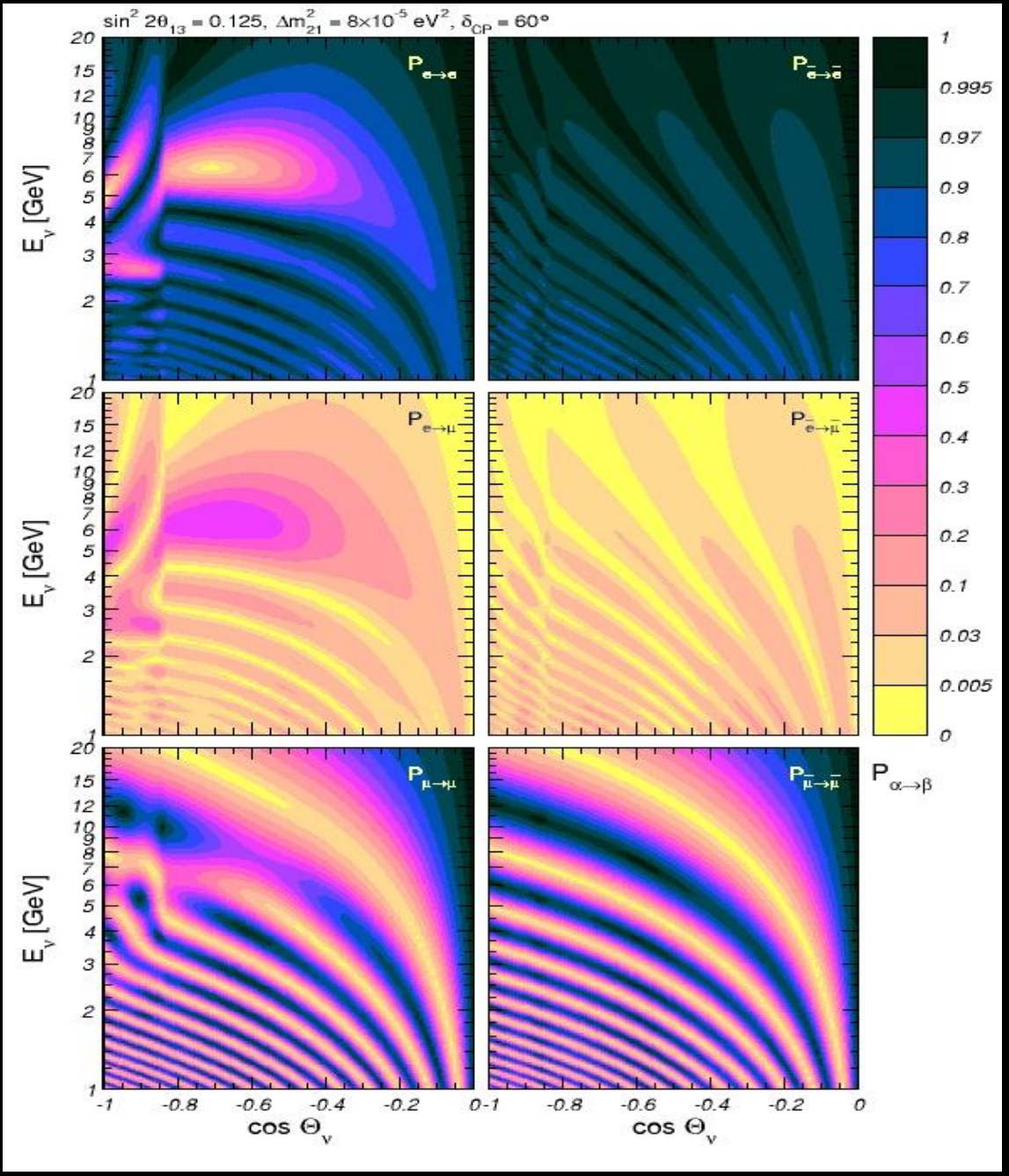
Atmospheric magic lines

Interference phase lines

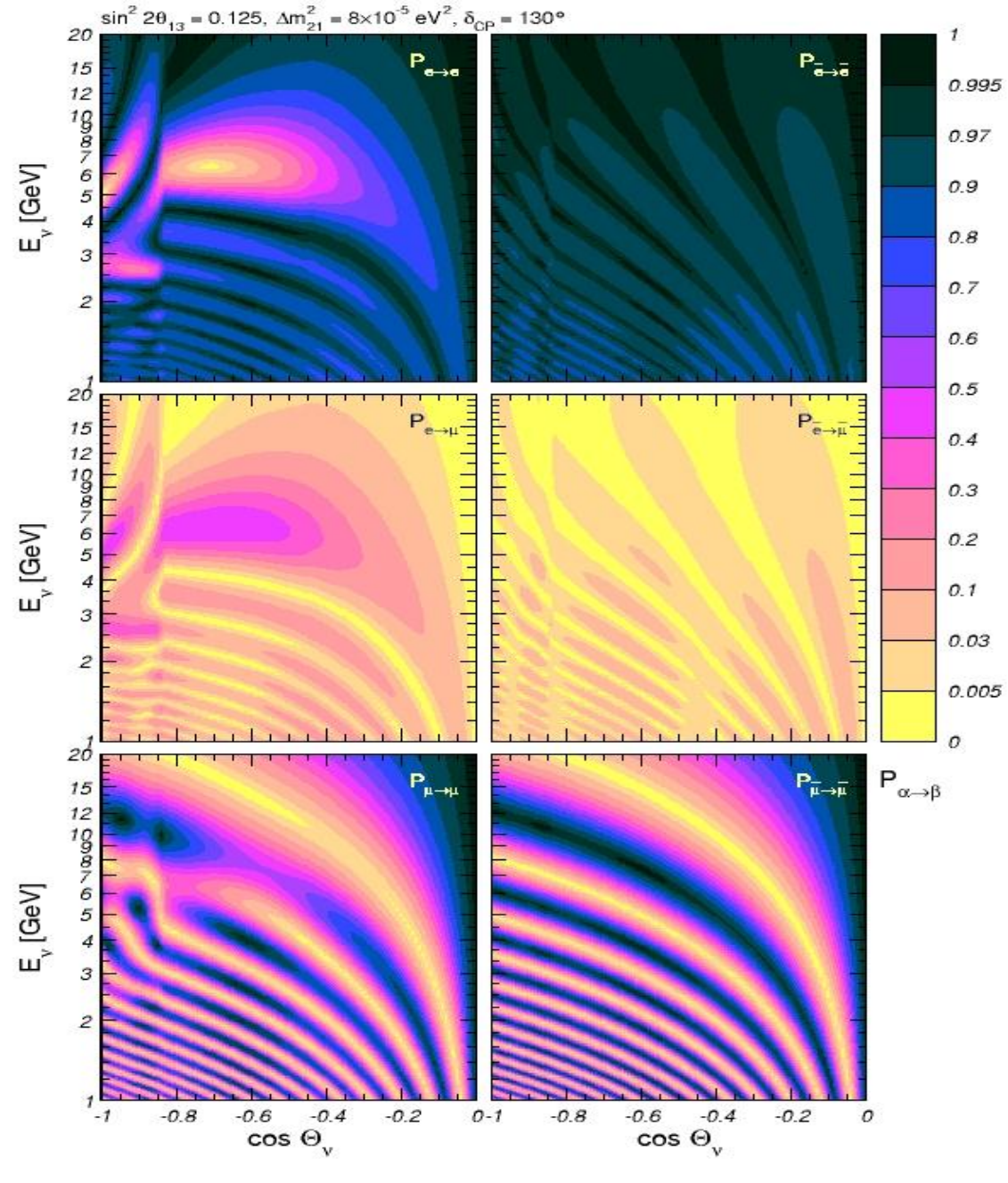
CP-violation

$\delta = 60^o$

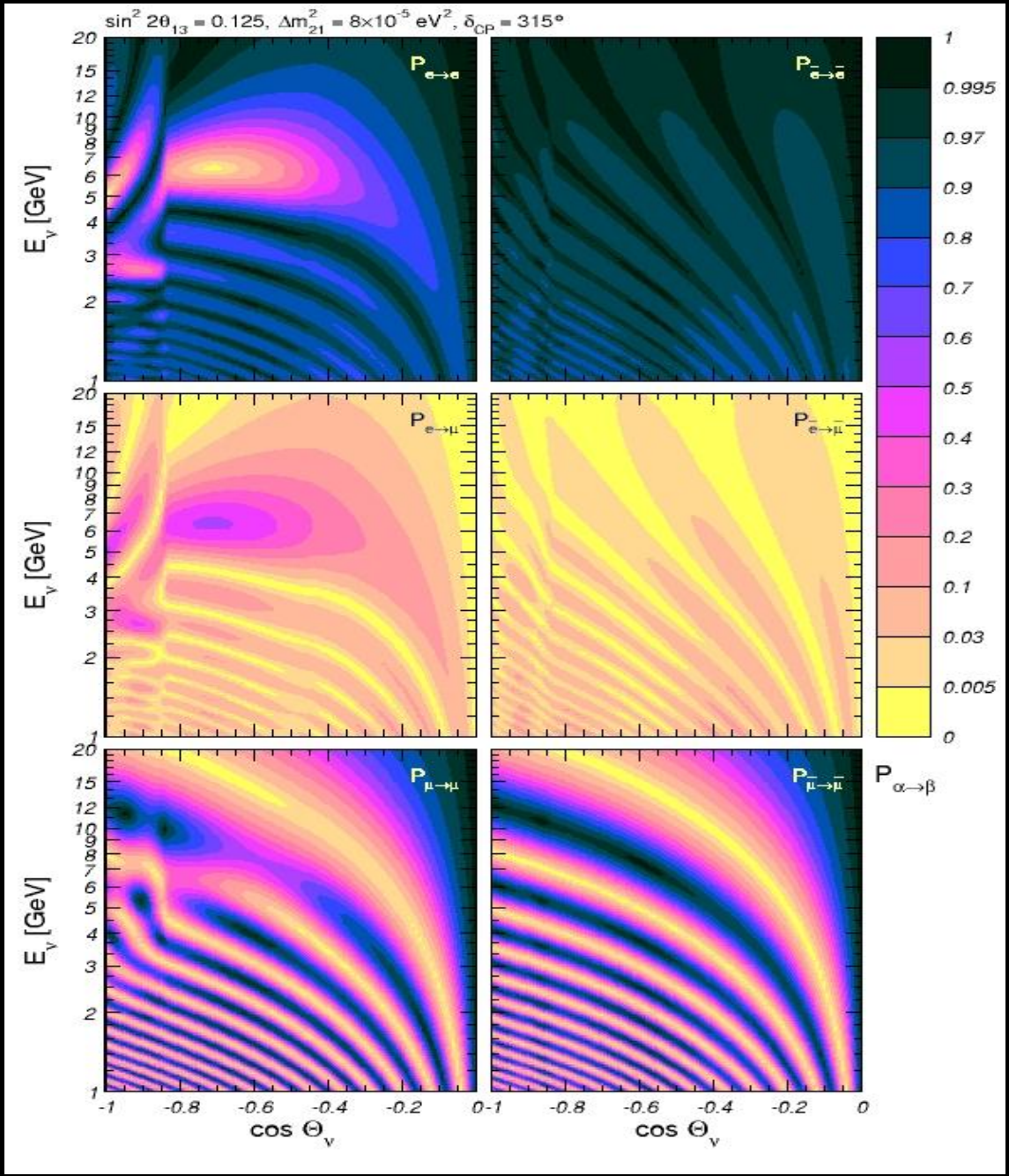
Standard parameterization



$\delta = 130^o$


sin² 2θ13 = 0.125, Δm²21 = 8×10⁻⁵ eV², δCP = 130°
Pe→e
Pē→ē
Pe→μ
Pē→μ̄
Pμ→μ
Pμ̄→μ̄
Pα→β
Eν [GeV]
cos Θν
1
0.995
0.97
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.03
0.005
0

$\delta = 315^{o}$


sin² 2θ13 = 0.125, Δm²21 = 8×10⁻⁵ eV², δCP = 315°
Pe→e
Pē→ē
Pe→μ
Pē→μ̄
Pμ→μ
Pμ̄→μ̄
Pα→β
Eν [GeV]
cos Θν
1
0.995
0.97
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.03
0.005
0

CP-interference

Due to specific form of matter potential matrix (only $V_{ee} \neq 0$)

$$P(\nu_e \rightarrow \nu_\mu) = |\cos\theta_{23} A_{e2} e^{i\delta} + \sin\theta_{23} A_{e3}|^2$$

``solar'' amplitude

``atmospheric'' amplitude

dependence on δ and θ_{23} is explicit

<u>For maximal 2-3 mixing</u>

$$P(\nu_e \rightarrow \nu_\mu)^\delta = |A_{e2} A_{e3}| \cos(\phi - \delta)$$

$$\phi = \arg(A_{e2}^* A_{e3})$$

$$P(\nu_\mu \rightarrow \nu_\mu)^\delta = - |A_{e2} A_{e3}| \cos\phi \cos\delta$$

$$P(\nu_\mu \rightarrow \nu_\tau)^\delta = - |A_{e2} A_{e3}| \sin\phi \sin\delta$$

$$\Sigma = 0$$

"Magic lines"

P. Huber, W. Winter
V. Barger, D. Marfatia,
K Whisnant, A.S.

Explicitly

$$P(\nu_e \rightarrow \nu_\mu) = c_{23}^2|A_S|^2 + s_{23}^2|A_A|^2 + 2s_{23}c_{23}|A_S||A_A|\cos(\phi + \delta)$$

$$\phi = \arg(A_S A_A^*)$$

$$P_{int} = 2s_{23}c_{23}|A_S||A_A|\cos(\phi + \delta)$$

Dependence on δ disappears, interference term is zero if

$P_{int} = 0$

- $A_S = 0$ - solar magic lines
- $A_A = 0$ - atmospheric magic lines
- $(\phi + \delta) = \pi/2 + 2\pi k$ - interference phase condition

$$\phi(E, L) = -\delta + \pi/2 + \pi k$$

depends on δ

"Magic lines"

For $\nu_\mu \rightarrow \nu_\mu$ channel

$$P_{int} \sim 2 s_{23} c_{23} |A_S| |A_A| \cos\phi \cos\delta$$

- The survival probabilities is CP-even functions of δ
- no CP-violation
- dependences on phases factorize

Dependence on δ disappears

$P_{int} = 0$ ⇒ $A_S = 0$, $A_A = 0$, $\phi = \pi/2 + \pi k$

interference phase does not depends on δ

Form the phase line grid

Sensitivity to CP phase

δ - true (experimental) value of phase
δ_f - fit value

Interference term: $\Delta P = P(\delta) - P(\delta_f)$ $= P_{int}(\delta) - P_{int}(\delta_f)$

For $\nu_e \rightarrow \nu_\mu$ channel:

$$\Delta P = 2\, s_{23}\, c_{23}\, |A_S|\, |A_A|\, [\cos(\phi + \delta) - \cos(\phi + \delta_f)]$$

$\Delta P = 0$ ⇒

$A_S = 0$ (along the magic lines)

$A_A = 0$

$(\phi + \delta) = -(\phi + \delta_f) + 2\pi k$

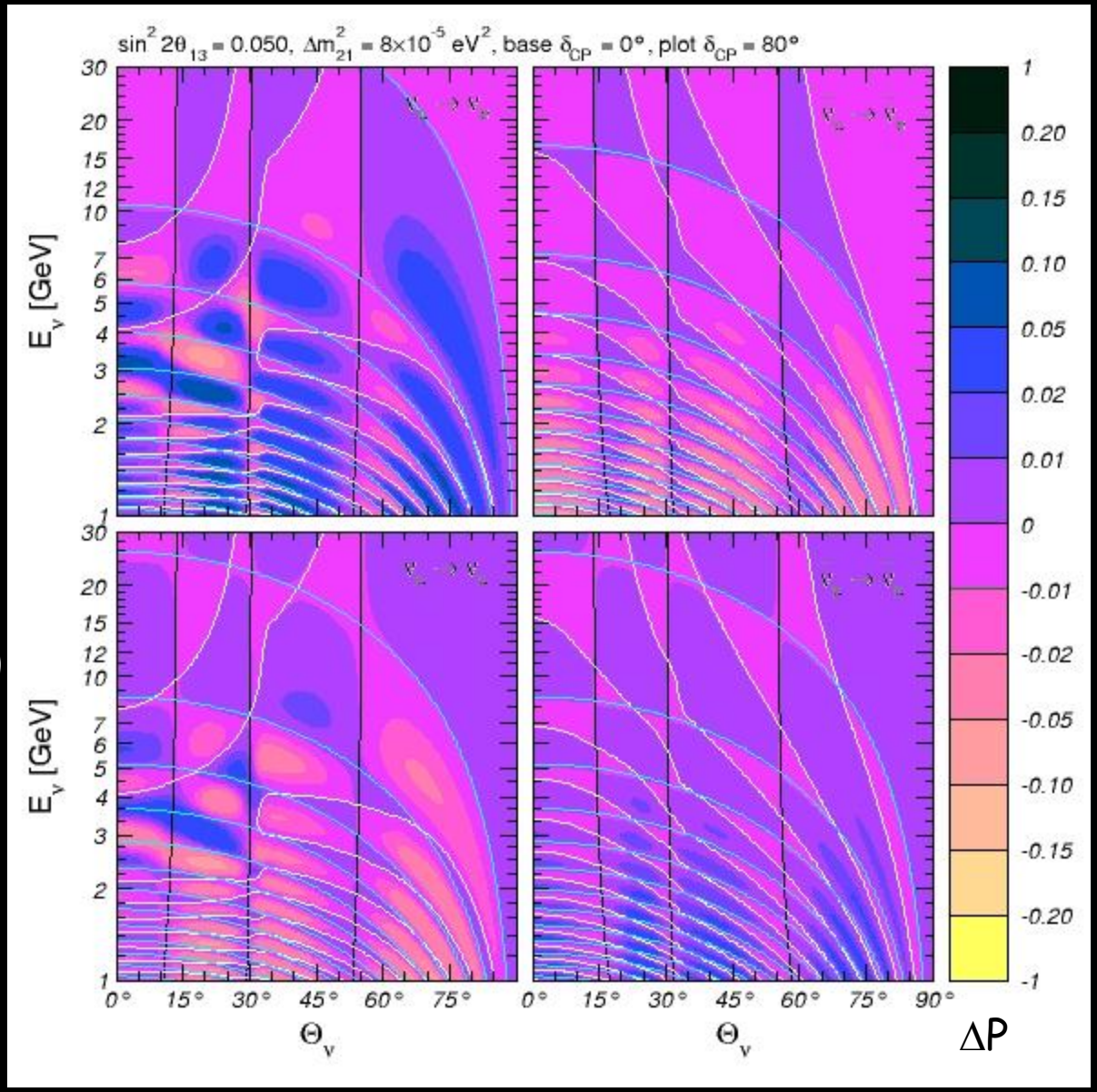
$$\phi(E, L) = -(\delta + \delta_f)/2 + \pi k$$

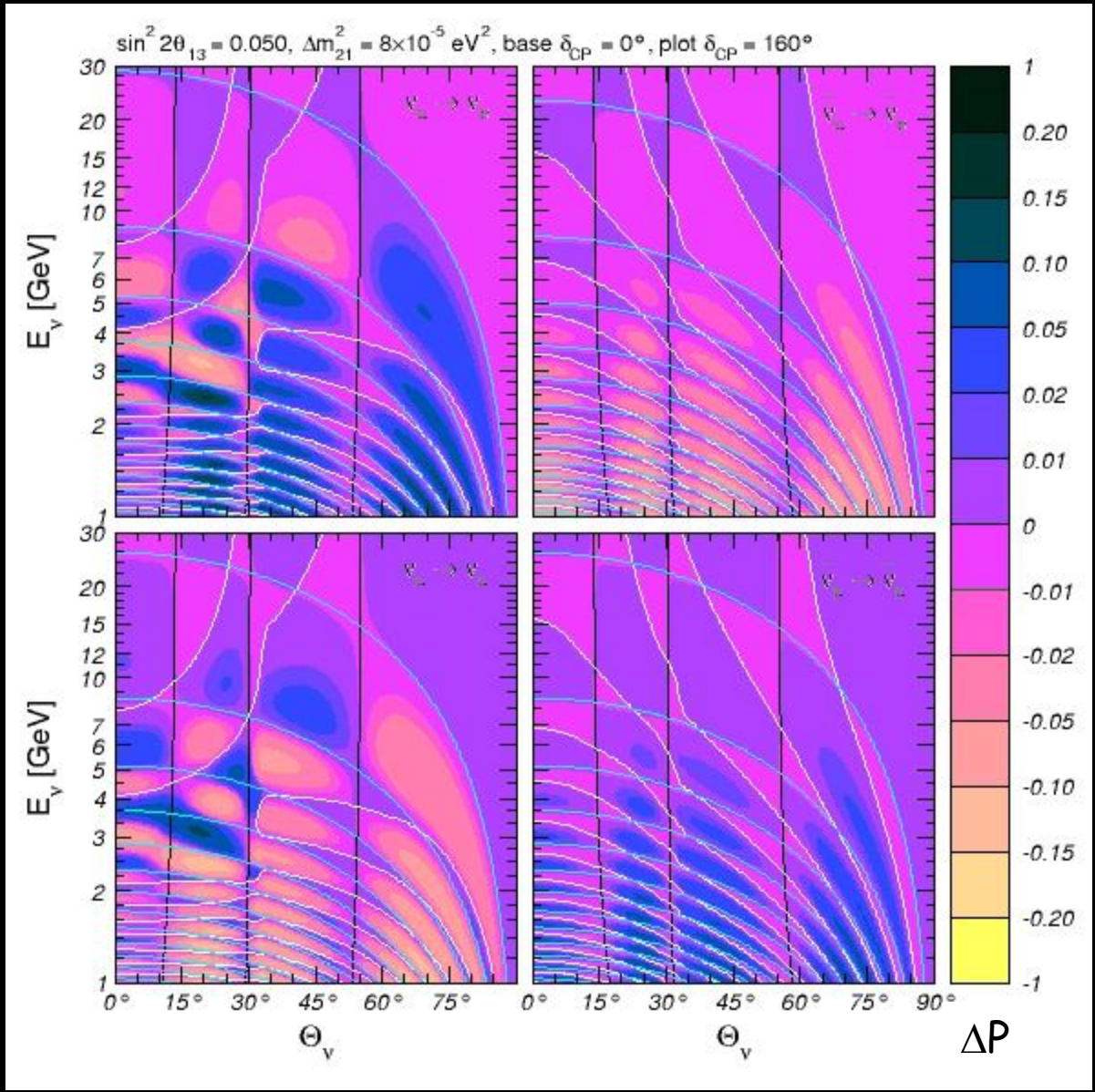
int. phase condition

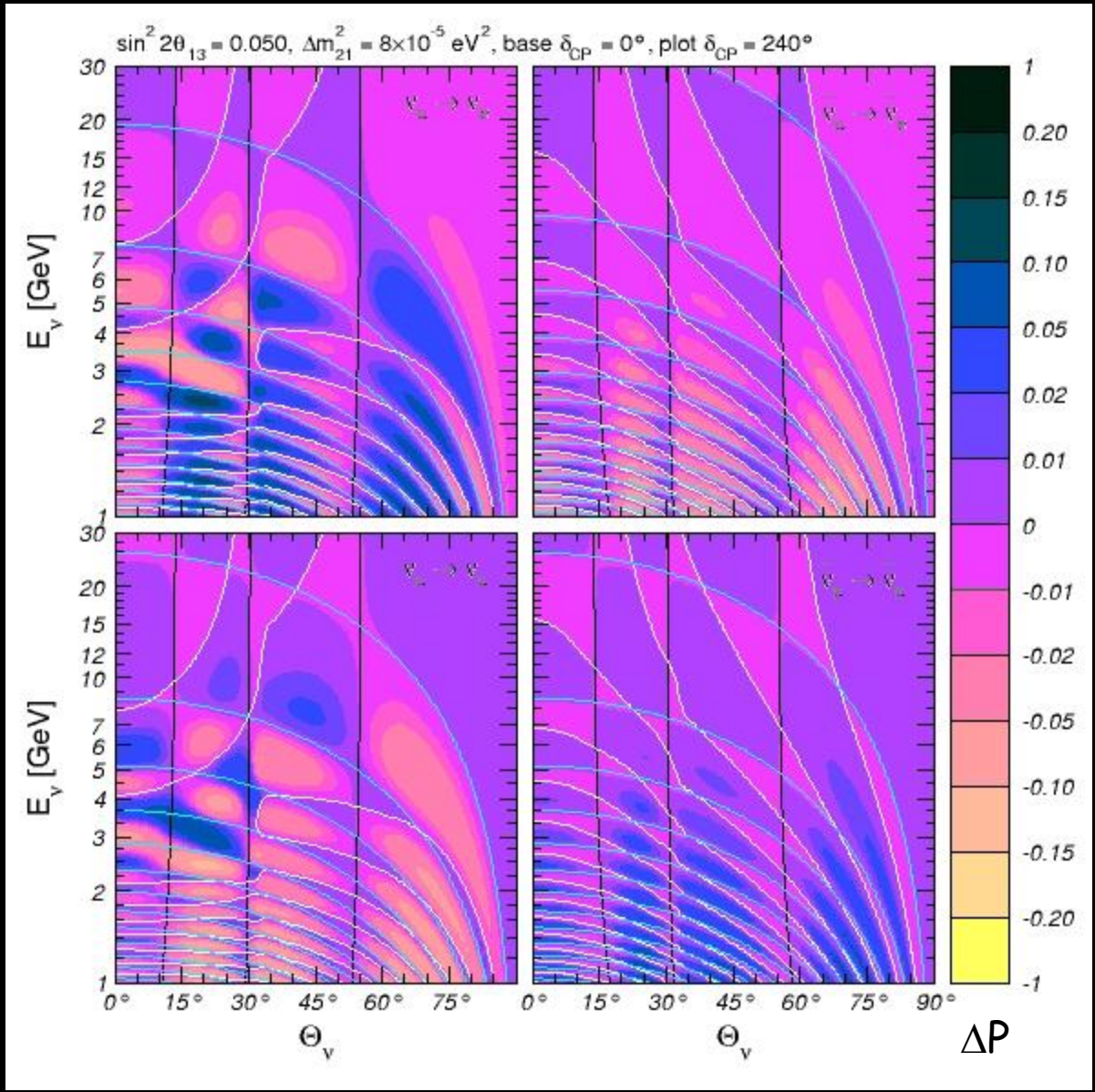
depends on δ

Int. phase line moves with δ-change

Grid (domains) does not change with δ



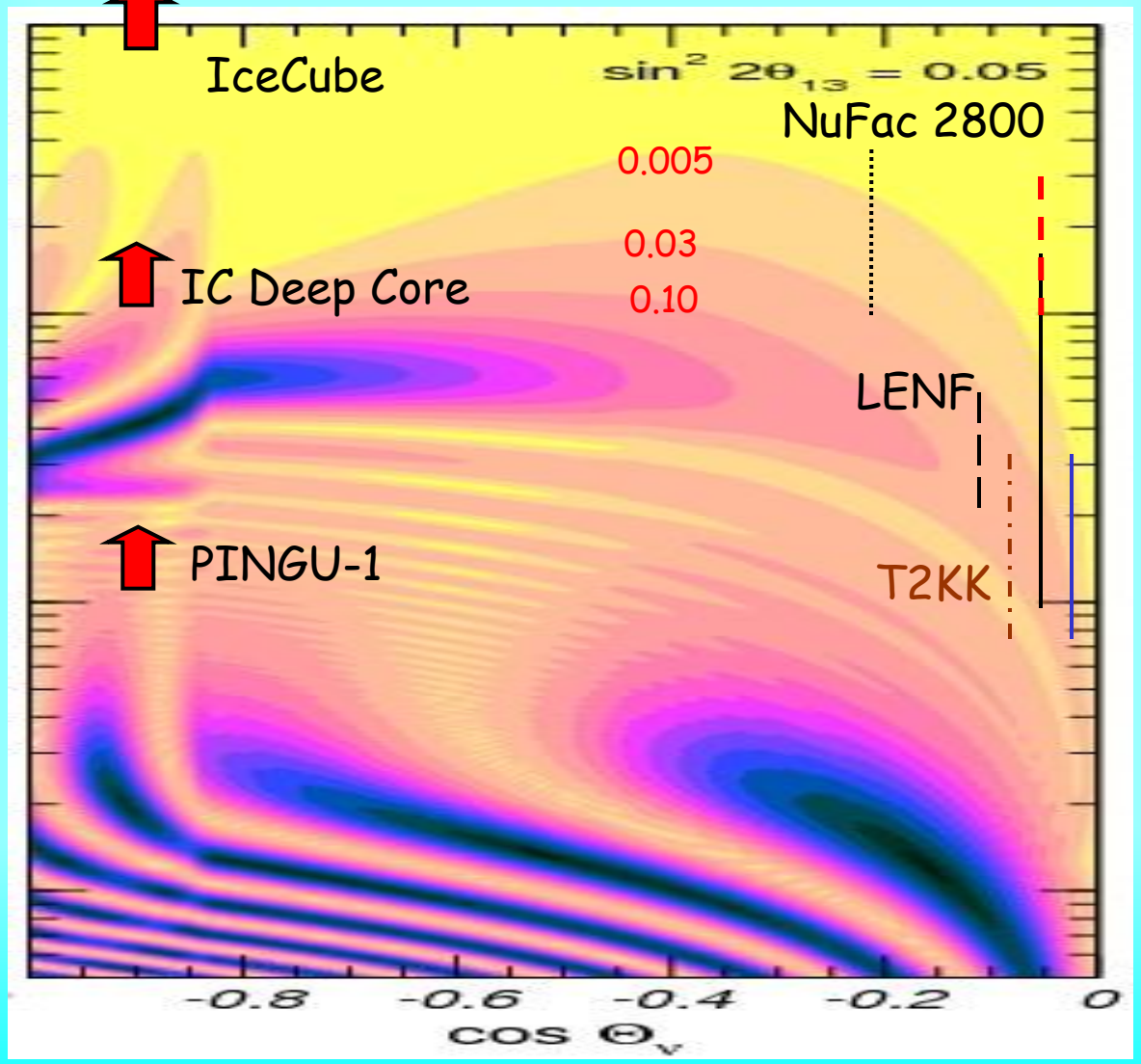

sin² 2θ13 = 0.050, Δm²21 = 8×10⁻⁵ eV², base δCP = 0°, plot δCP = 160°
Eν [GeV]
Θν
ΔP


sin² 2θ13 = 0.050, Δm²21 = 8×10⁻⁵ eV², base δCP = 0°, plot δCP = 240°
Eν [GeV]
Θν
ΔP

Oscillograms

contours of constant oscillation probability in energy- nadir (or zenith) angle plane

$\nu_e \rightarrow \nu_\mu , \nu_\tau$



CNGS

MINOS

NOvA

T2K HyperK

OK for CP

ICAL, INO

LAr

Physics with HAND's

Enormous physics potential

which is not completely explored and largely unused

Energy range: 0.01 – 10^5 GeV

Baselines: 0 – 13000 km

Matter effects: 3 – 15 g/cm^3

Flavor content nue, numu

Lepton number nu - antinu

which change with energy and zenith angle

Achievements:

Discovery of neutrino oscillations

Measurements of 2-3 mixing and mass splitting

Bounds on new physics
- sterile neutrinos
- non-standards interaction
- violation of fundamental symmetries, CPT

Limitations:

Uncertainties of original fluxes

Flavor identification

Reconstruction of direction

Statistics

Energy resolution

High statistics solve the problems

from LAND to HAND

TITAND?

Y. Suzuki

E. Kh Akhmedov M. Maltoni A.Y.S.
JHEP 05, (2007) 077 [hep-ph/0612285]
JHEP 06 (2008) 072 [arXiv:0804.1466]
PRL 95 (2005) 211801 arXiv:0506064
unpublished, see M Maltoni talks

A.Y.S. , hep-ph/0610198.

E. Kh Akhmedov, S Razzaque, A.S. in preparation

E Kh Akhmedov, A Dighe, P. Lipari, A Y. Smirnov , Nucl. Phys. B542 (1999) 3-30 hep-ph/9808270

Developments of new detection methods?

Suppression of effects

Original fluxes

different flavors: ν_e and ν_μ

Screening factors $(1 - r\, s_{23}^2)$

neutrinos and antineutrinos

$(1 - \kappa_e)$

$(1 - \kappa_\mu)$

Reduces CP-asymmetry

Integration averaging

averaging and smoothing effects
reconstruction of neutrino energy and direction

Detection

identification of flavor

Numbers of events

Triple suppression

$$N_e^{IH} - N_e^{NH} \sim (\bar{P}_A - P_A)\,(1 - \kappa_\mu)\,[r\, s_{23}^2 - (1 - \kappa_e)/(1 - \kappa_\mu)]$$

CP asymm etry

Neutrino - antineutrino factor

can be avoided

Flavor suppression (screening factors)

unavoidable

$$P_A = |A_{e3}|^2$$

$$\kappa_\alpha = (\bar{\sigma}\,\bar{\Phi}_\alpha)/(\sigma\,\Phi_\alpha)$$

$$N_\mu^{IH} - N_\mu^{NH} \sim (\bar{P}_{\mu\mu} - P_{\mu\mu})\,(1 - \kappa_\mu) - r^{-1}(1 - \bar{\kappa}_e)\,(P_{e\mu} - P_{e\mu})]$$

PINGU

Precision IceCube Next Generation Upgrade

Mass hierarchy,
2-3 mixing,
CP

IC, DeepCore and PINGU

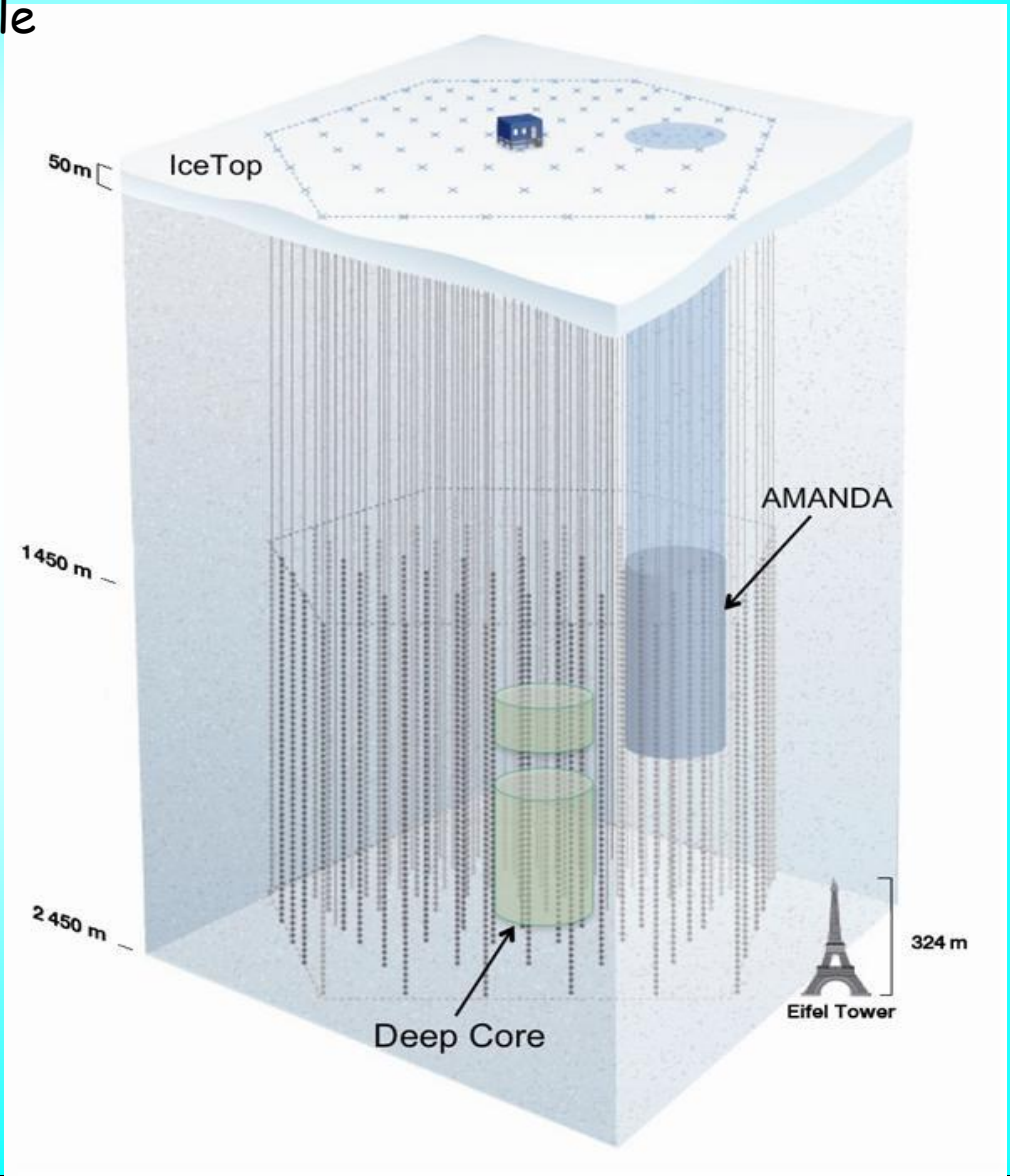
Digital Optical Module

IceCube :
86 strings (x 60 DOM)
100 GeV threshold
Gton volume

Deep Core IC :
- 8 more strings (480 DOMs)
- 10 GeV threshold
- 30 Mton volume

PINGU:
18, 20, 25 ? new strings
(~1000 DOMs)
in DeepCore volume

- Existing IceCube strings
- Existing DeepCore strings
- New PINGU strings



PINGU Geometry

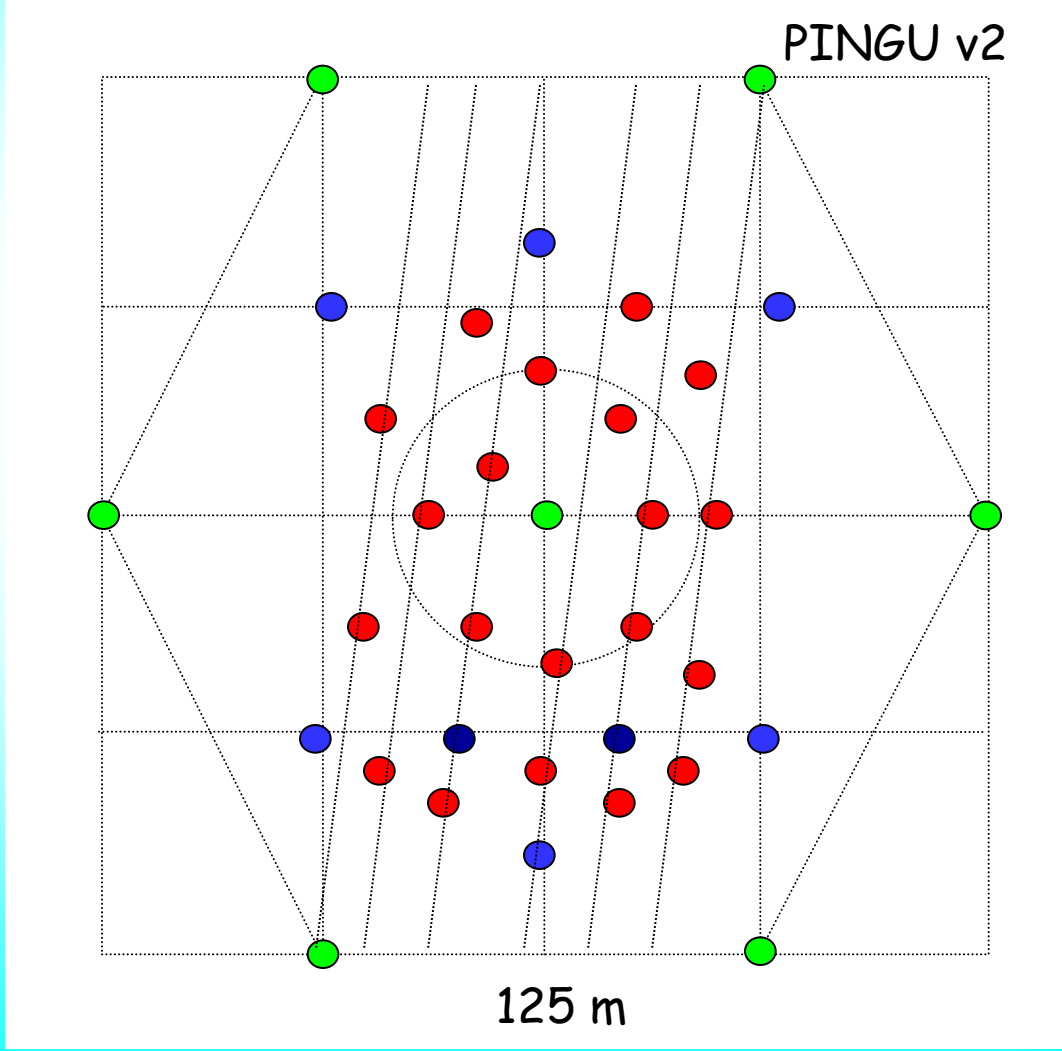
Denser array

20 new strings (~60 DOMs each)
in 30 MTon DeepCore volume

Few GeV threshold in inner
10 Mton volume

Energy resolution ~ 3 GeV

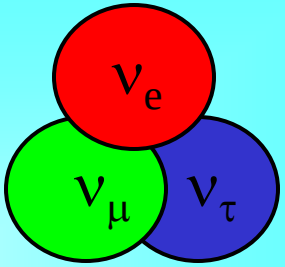
- Existing IceCube strings
- Existing DeepCore strings
- New PINGU-I strings

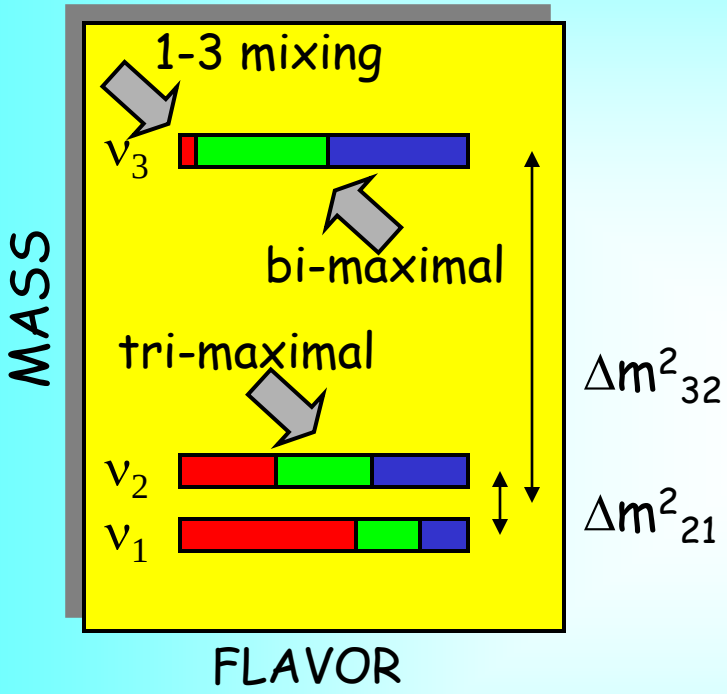


Mass hierarchy

Effective area, effective volume

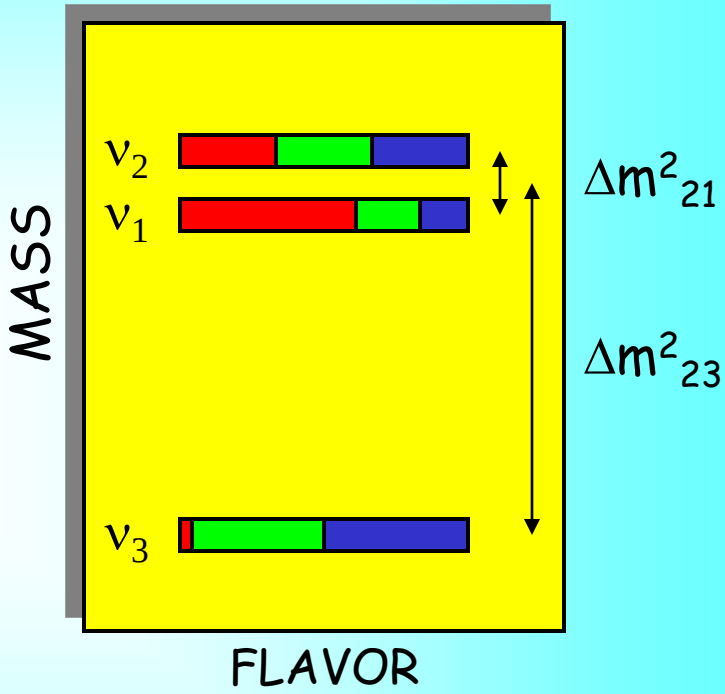
Mixing & masses





Normal mass hierarchy

?



Inverted mass hierarchy

Two large mixings

~ Tri-bimaximal mixing

Symmetry?

ν_μ - ν_τ symmetry

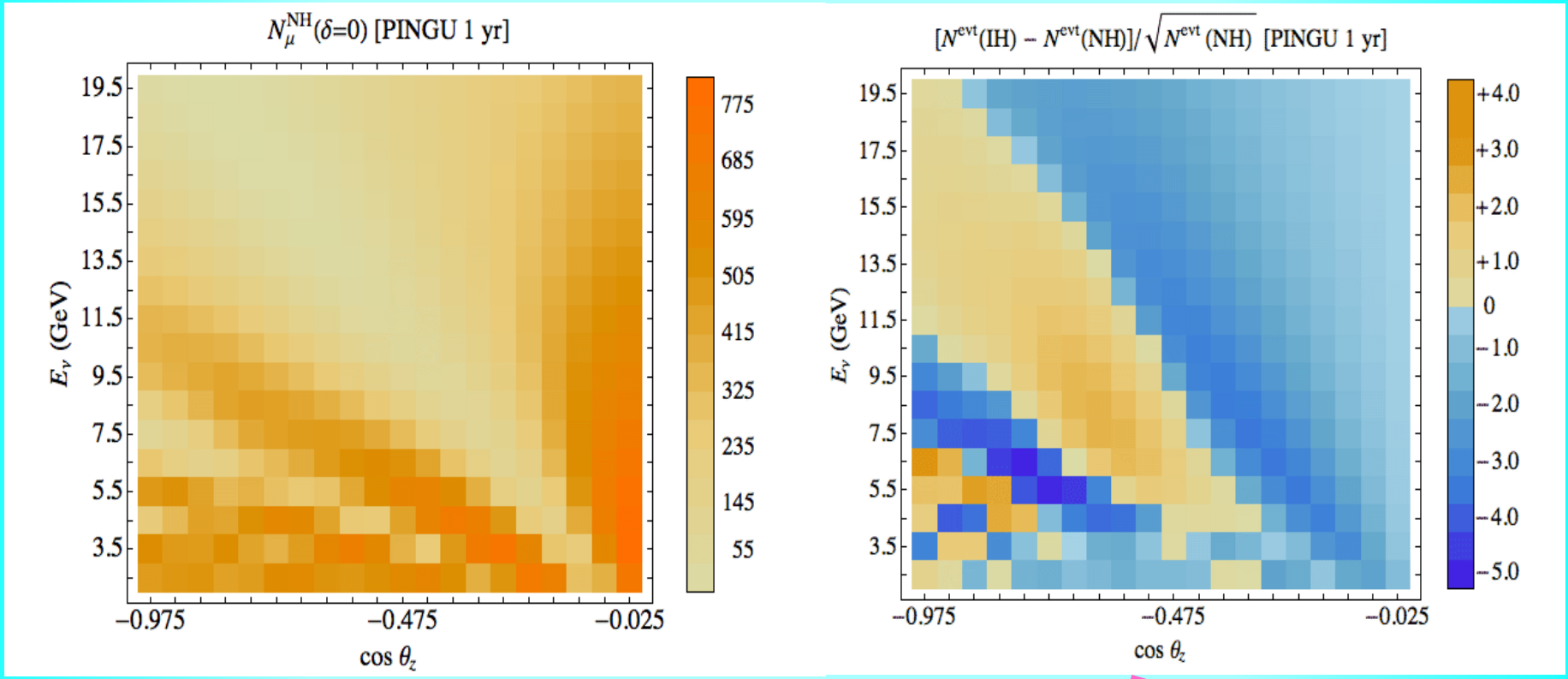
$\Delta m^2_{32} = 2.3 \times 10^{-3}\ eV^2$

$\Delta m^2_{21} = 8 \times 10^{-5}\ eV^2$

PINGU: Tracking events

E. Kh Akhmedov,
S Razzaque,
A. Y. S.

Asymmetry, statistical significance



$S_{tot} \sim s\, n^{1/2}$

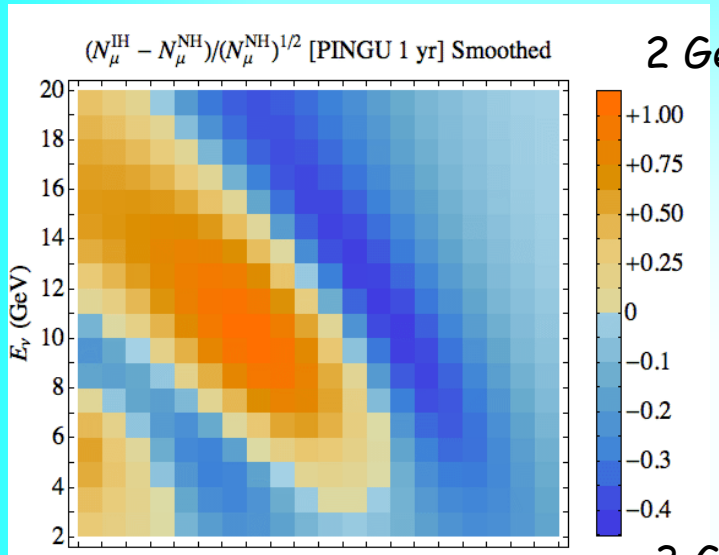
Quick estimation of significance

Effective average significance in individual bin

Number of bins in resolution domains

Systematics reduces significance by factor 2

PINGU and mass hierarchy

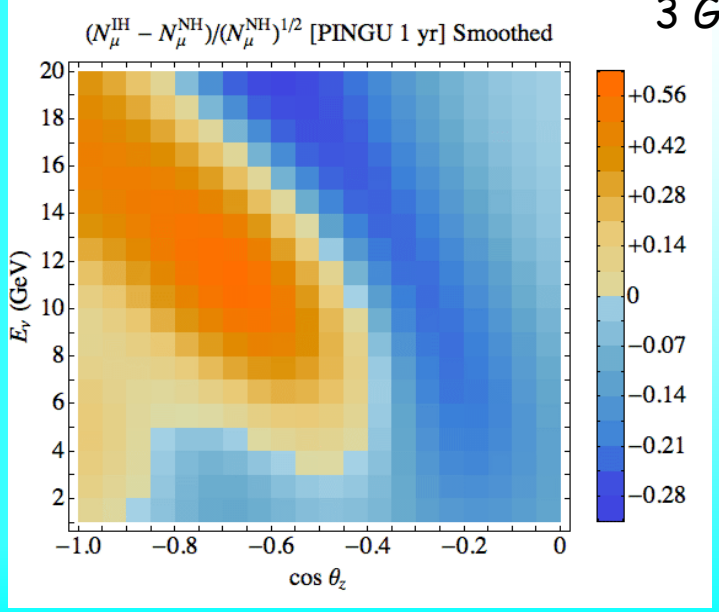


2 GeV, 11.25^0

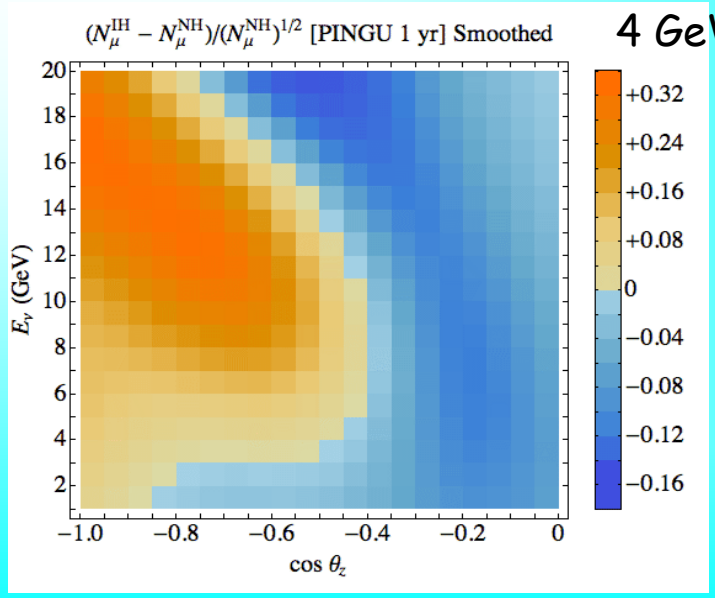
E. Akhmedov, S. Razzaque, A. Y. Smirnov
arXiv: 1205.7071

Smearing with Gaussian reconstruction functions characterized by (half) widths

$(\sigma_E, \sigma_\theta)$

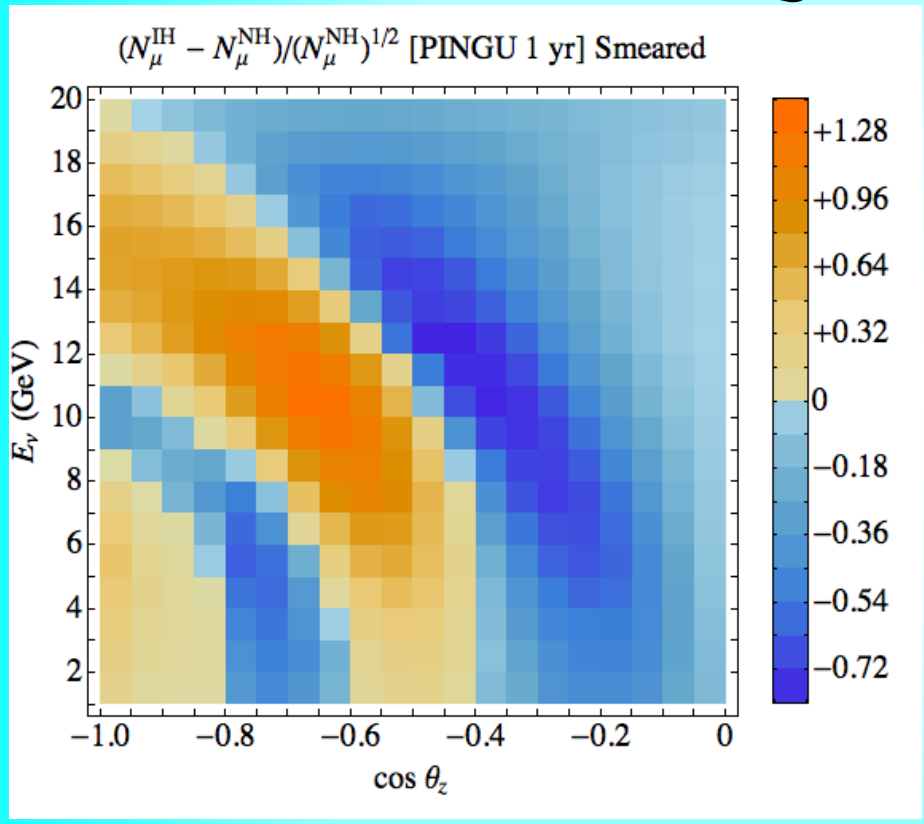


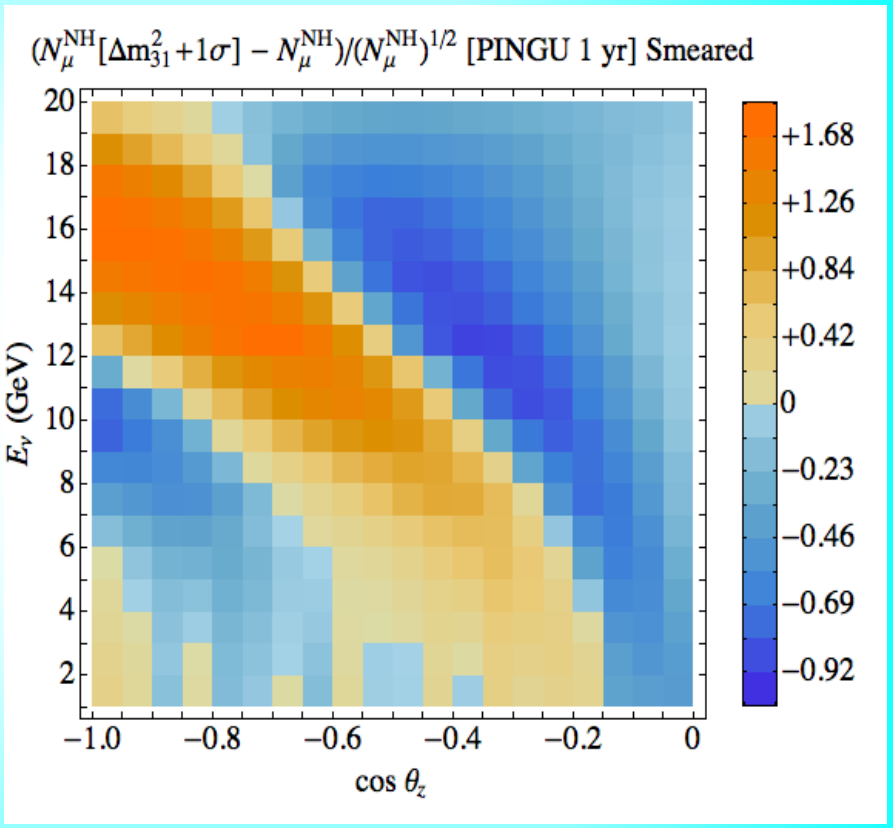
3 GeV,15^0



4 GeV, 22.5^0

Hierarchy with PINGU



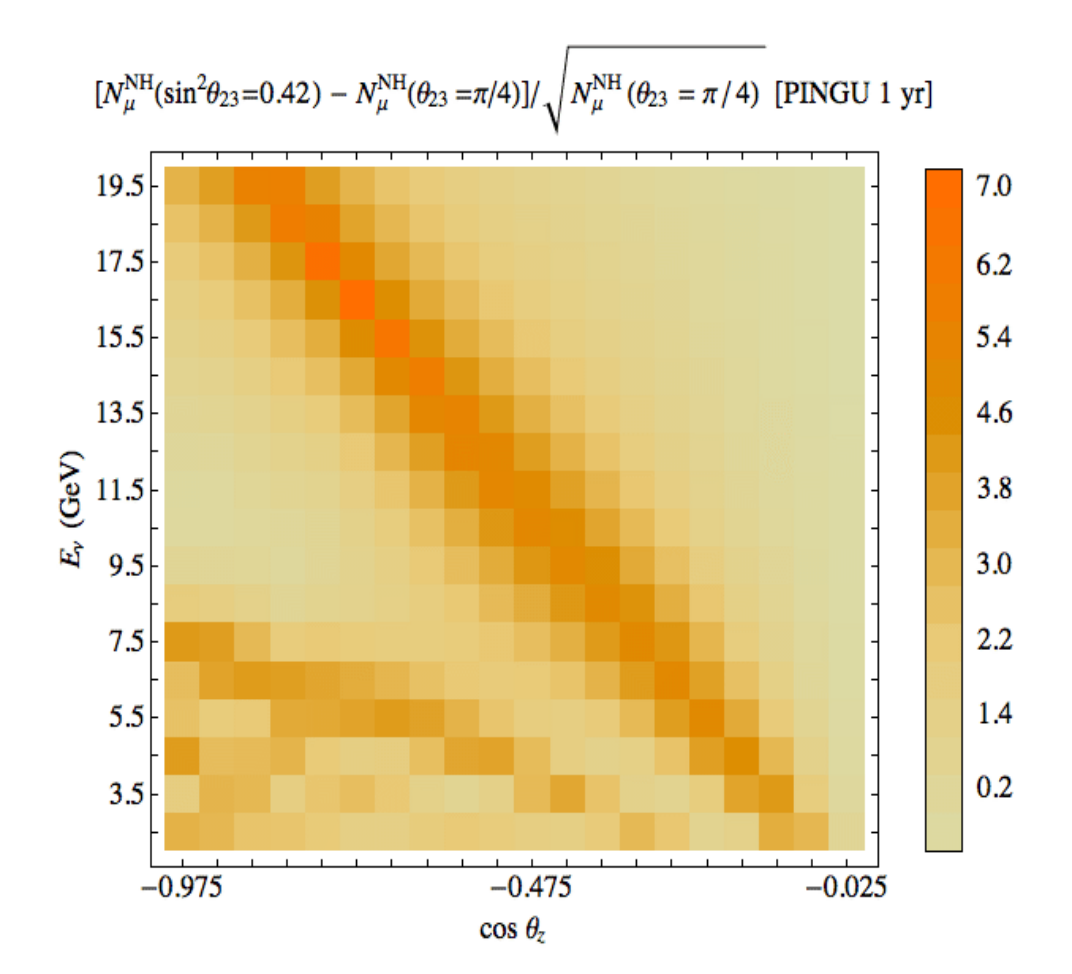


$\sigma_E = 0.2E$

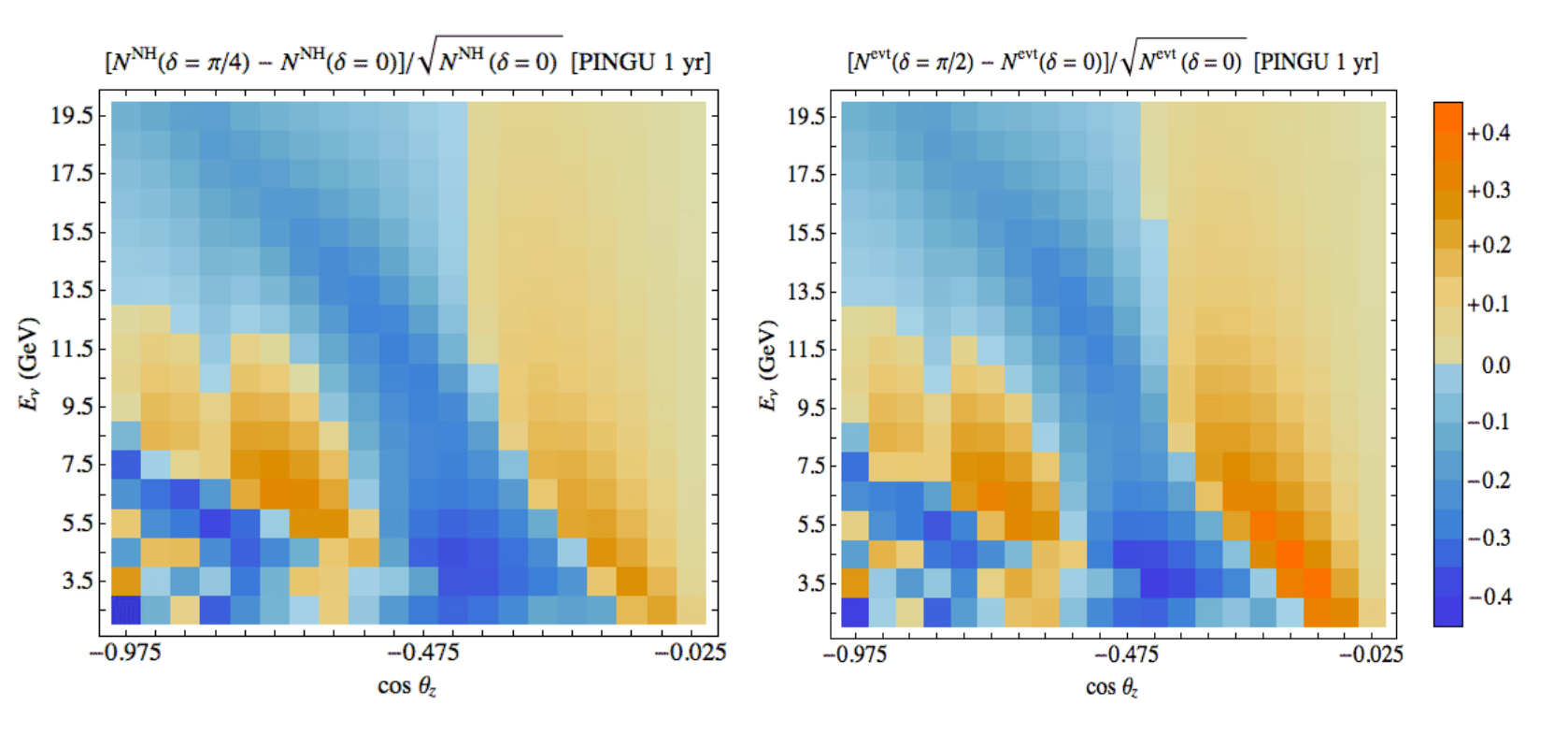
$\sigma_\theta \sim 1/E^{0.5}$

Degeneracy

Tracking events



CP asymmetry



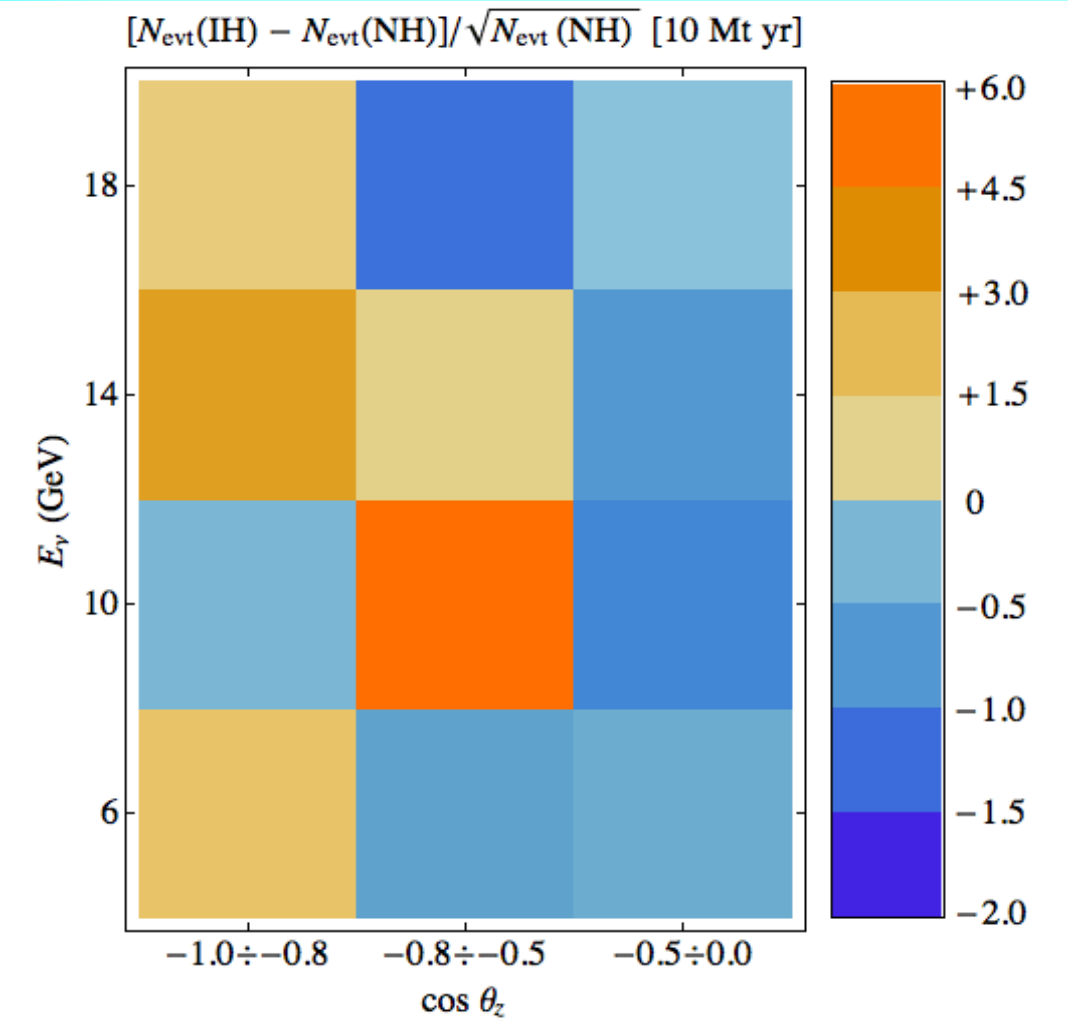
Summary

Flavor mixing in neutrino-neutrino scattering
from flavor diagonal interactions

Propagation in the Earth - neutrino image of the Earth:
resonance enhancement of oscillations, parametric effects
Magic lines, CP-violation domains

Determination of neutrino parameters with huge
atmospheric neutrino detectors

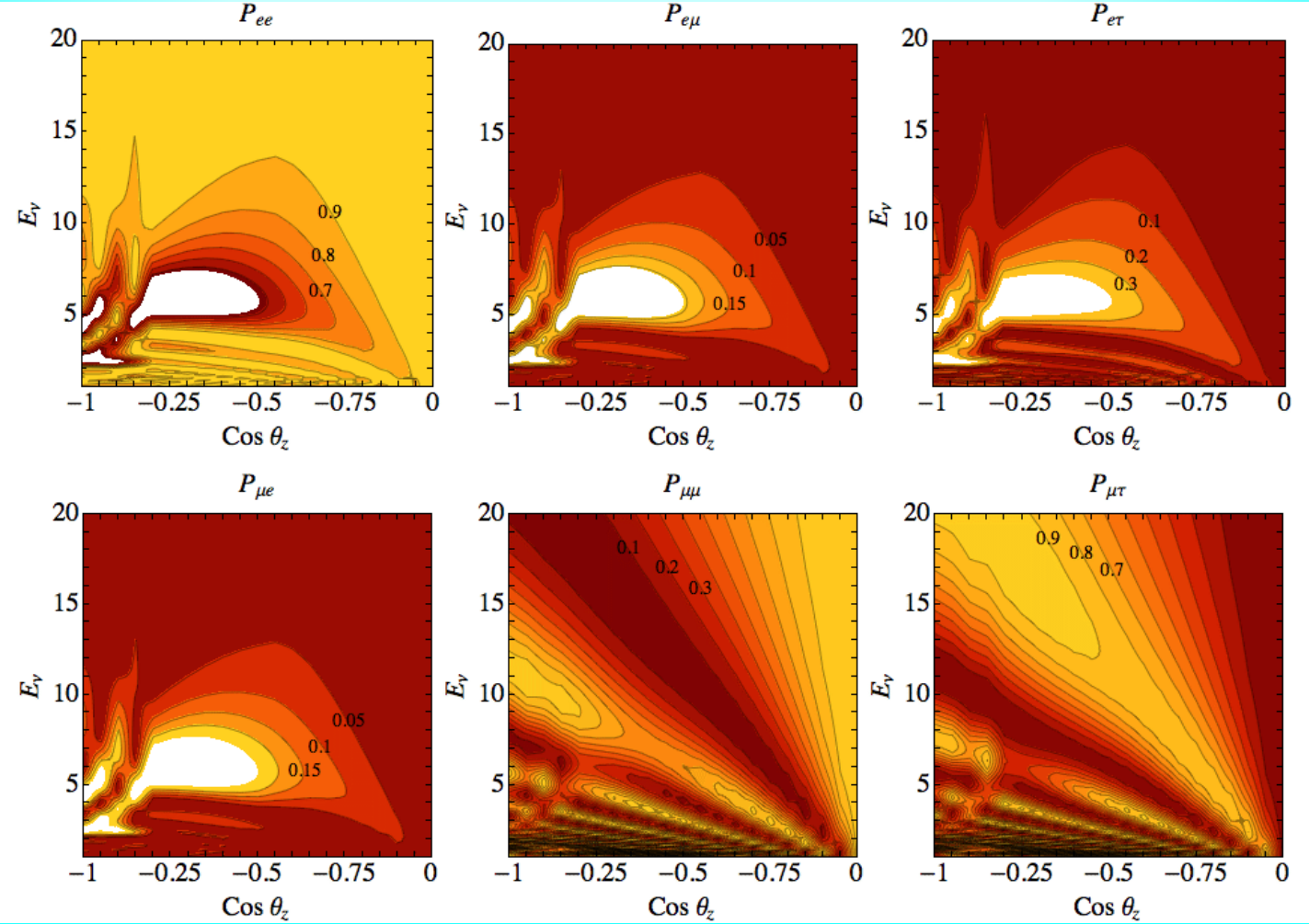
Significance plot



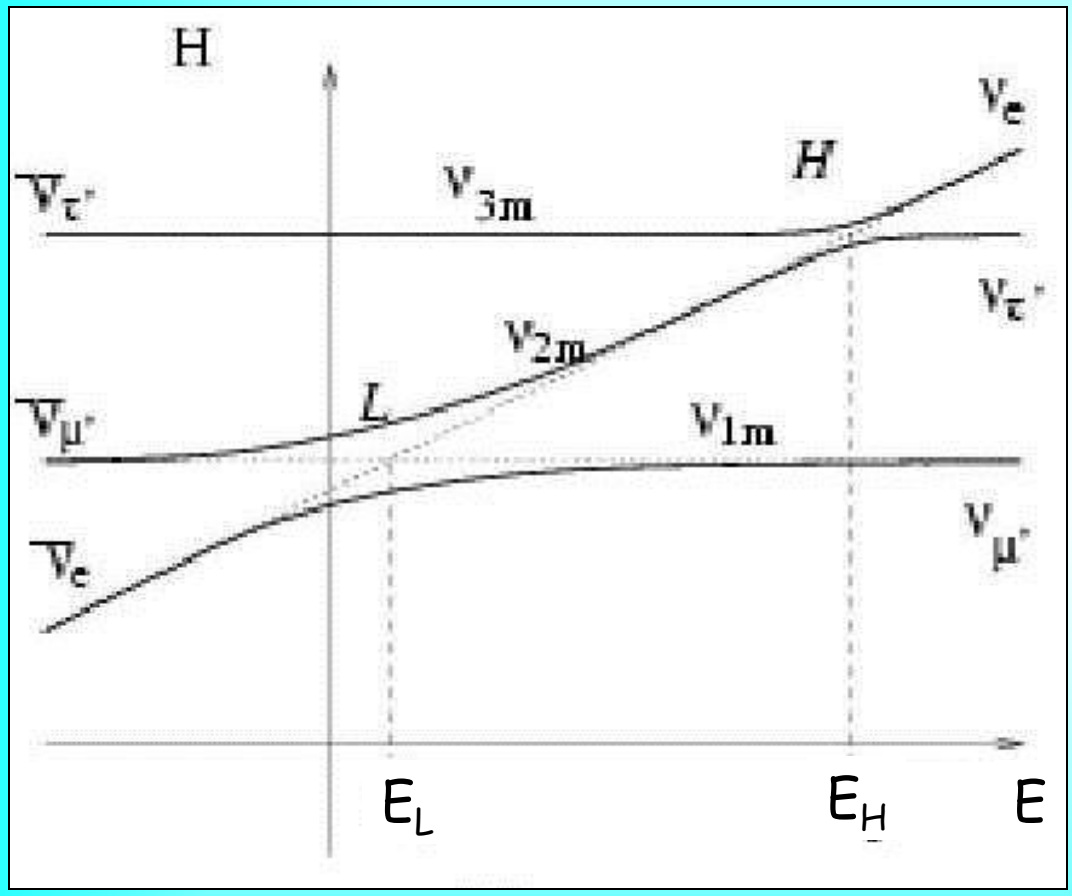
Integration over resolution (reconstruction domain) tracks

Oscillograms

For the best fit values of parameters



Level crossings



Normal mass hierarchy

0.1 GeV

6 GeV

Resonance region

High energy range

Neutrino density

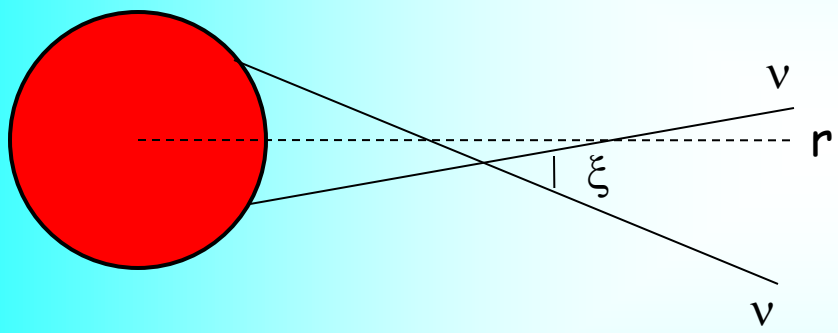
neutrinosphere

R = 20 – 50 km

usual matter potential: $\lambda = V = \sqrt{2}\, G_F\, n_e$

neutrino potential:

$\mu = \sqrt{2}\, G_F\, (1 - \cos\xi)\, n_\nu$



$n_\nu \sim 1/r^2$

$\xi \sim 1/r$ for large r

$\mu \sim 1/r^4$

in neutrinosphere
in all neutrino species: $n_\nu \sim 10^{33}\ \mathrm{cm}^{-3}$

electron density: $n_e \sim 10^{35}\ \mathrm{cm}^{-3}$

$\lambda \gg \mu$

Combining neutrinos and antineutrinos

Introducing negative frequencies for antineutrinos $\bar{\mathbf{P}}_\omega = \mathbf{P}_{-\omega}$ $\omega > 0$

$$d_t \mathbf{P}_\omega = (\omega \mathbf{B} + \mu \mathbf{D}) \times \mathbf{P}_\omega$$

$$\mathbf{D} = \int_{-\inf}^{+\inf} d\omega \; s_\omega \, \mathbf{P}_\omega$$

where $s_\omega = \mathrm{sign}(\omega)$

Equation of motion for **D**: integrating equation of motion with s_w

$$d_t \mathbf{D} = \mathbf{B} \times \mathbf{M}$$

where $\mathbf{M} = \int_{-\inf}^{\inf} d\omega \; s_\omega \, \omega \, \mathbf{P}_\omega$

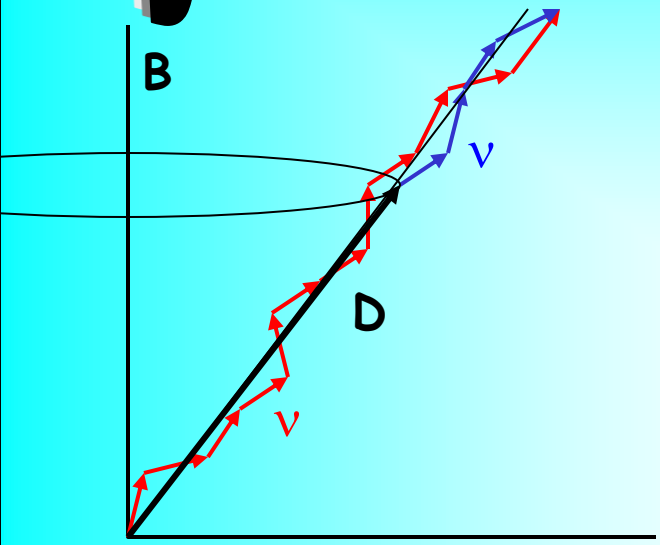
In another form:

$$d_t \mathbf{P}_\omega = \mathbf{H}_\omega(\mu) \times \mathbf{P}_\omega$$

where $\mathbf{H}_\omega = (\omega \mathbf{B} + \mu \mathbf{D})$

Synchronization

Kostelecky & Samuel
Pastor, Raffelt, Semikoz



If $\mu\,|D| \gg \omega$ - the individual vectors form large
the self-interaction term dominates

$$d_t \mathbf{P}_\omega \sim \mu\, \mathbf{D} \times \mathbf{P}_\omega$$

does not depend on ω

- evolution is the same for all modes
- $\mathbf{P}_\omega$ are pinned to each other

$$\mathbf{M} = \omega_{syn}\, \mathbf{D}$$

synchronization frequency

$$\omega_{syn} = \frac{\int d\omega\, s_\omega\, \omega P_\omega}{\int d\omega\, s_\omega P_\omega}$$

$$d_t \mathbf{D} = \omega_{syn}\, \mathbf{B} \times \mathbf{D}$$

D - precesses around **B** with synchronization frequency

Two aspects of mixing

$$\nu_e = \cos\theta_m\, \nu_{1m} + \sin\theta_m \nu_{2m}$$
$$\nu_\mu = -\sin\theta_m \nu_{1m} + \cos\theta_m \nu_{2m}$$

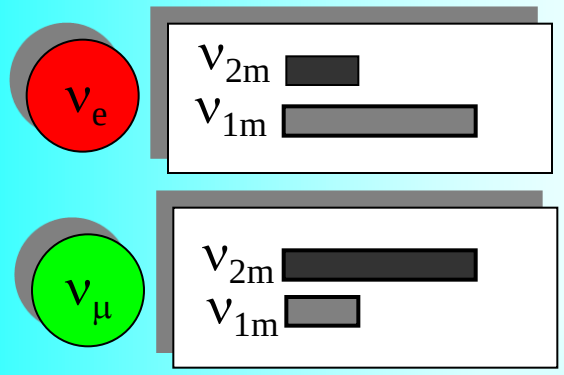
inversely

$$\nu_{2m} = \sin\theta_m \nu_e + \cos\theta_m\, \nu_\mu$$
$$\nu_{1m} = \cos\theta_m \nu_e - \sin\theta\, \nu_\mu\, \theta_m$$

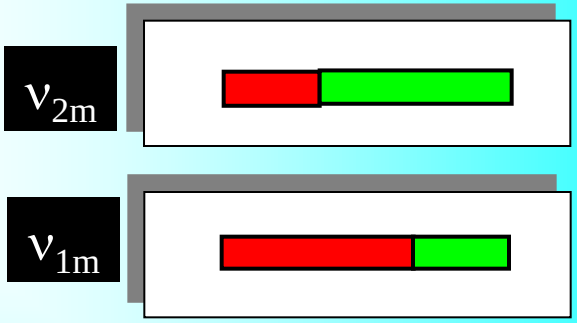
coherent mixtures of mass eigenstates

Vacuum $\theta_m \rightarrow \theta$

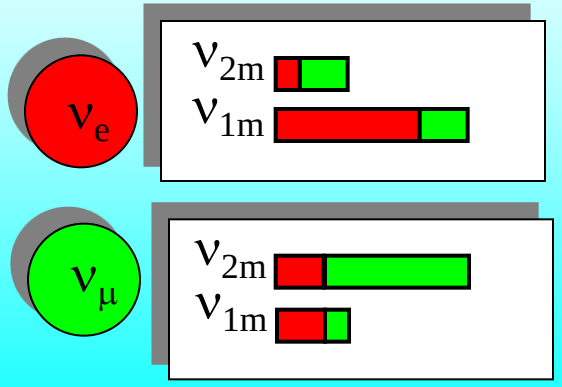
flavor composition of the mass eigenstates



wave packets



flavors of eigenstates



The relative phases of the mass states in ν_e and ν_μ are opposite

Interference of the parts of wave packets with the same flavor depends on the phase difference $\Delta\phi$ between ν_1 and ν_2