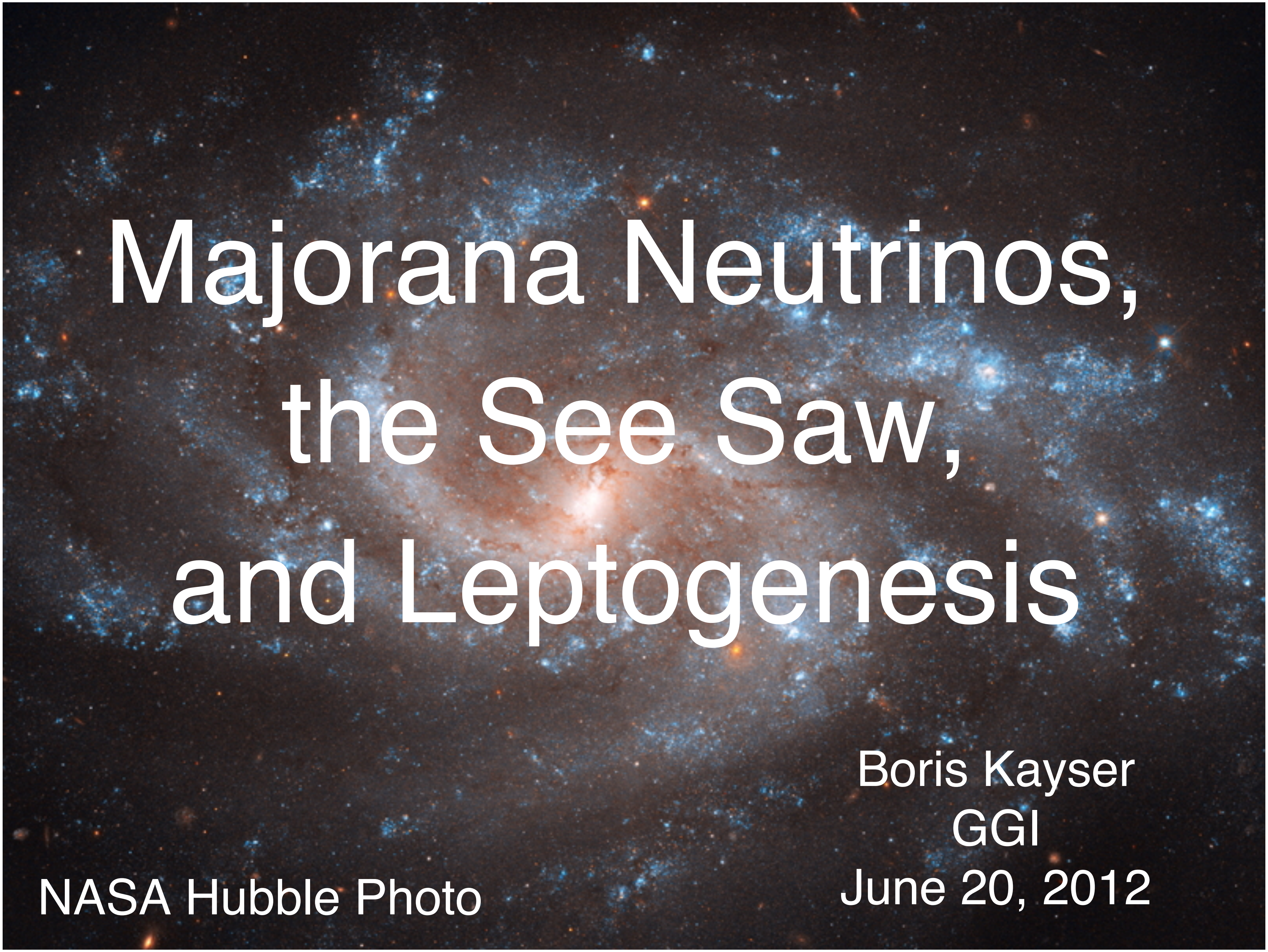

Majorana Neutrinos, the See Saw, and Leptogenesis

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GGI
June 20, 2012

NASA Hubble Photo

For Discussion of Mixing

The Mixing Matrix *U*

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\times \underbrace{\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Majorana phases}}$$

$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

$\theta_{12} \approx 34°$, $\theta_{23} \approx 39\text{-}51°$, $\theta_{13} \approx 8\text{-}10°$ *Very new!*

δ would lead to $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$. *CP violation*

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for $\cancel{CP}$ in oscillation.

For example —

$$P\left(\bar{\nu}_\mu \rightarrow \bar{\nu}_e\right) - P\left(\nu_\mu \rightarrow \nu_e\right) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put δ next to θ_{12} instead of θ_{13}.

Does $\bar{\nu} = \nu$?

What Is the Question?

For each ***mass eigenstate*** ν_i , and ***given helicty*** h, does —

- $\overline{\nu_i}(h) = \nu_i(h)$ (Majorana neutrinos)

or

- $\overline{\nu_i}(h) \neq \nu_i(h)$ (Dirac neutrinos) ?

Equivalently, do neutrinos have *Majorana masses*? If they do, then the mass eigenstates are *Majorana neutrinos*.

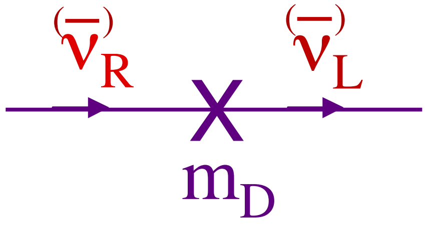
Dirac Masses

Dirac neutrino masses are the neutrino analogues of the SM quark and charged lepton masses.

To build a Dirac mass for the neutrino ν, we require not only the left-handed field ν_L in the Standard Model, but also a right-handed neutrino field ν_R.

The Dirac neutrino mass term is —

$$m_D \overline{\nu_L} \, \nu_R$$

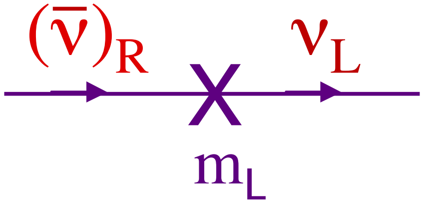


Dirac neutrino masses do not mix neutrinos and antineutrinos.

Majorana Masses

Out of, say, a left-handed neutrino field, ν_L, and its charge-conjugate, $\nu_L^{\,c}$, we can build a Left-Handed Majorana mass term —

$$m_L \overline{\nu_L}\, \nu_L^{\,c}$$



Majorana masses do mix ν and $\bar{\nu}$, so they do not conserve the Lepton Number L defined by —

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1.$$

A Majorana mass for any fermion f causes $f \leftrightarrow \bar{f}$.

Quark and *charged-lepton* Majorana masses are forbidden by electric charge conservation.

***Neutrino* Majorana masses would make the neutrinos *very* distinctive.**

SM Higgs

Majorana ν masses cannot come from $H_{SM}\bar{\nu}_L\nu_R$, the progenitor of the Dirac mass term, and the ν analogue of the Higgs coupling that leads to the q and ℓ masses.

Possible progenitors of Majorana mass terms:

$$\underbrace{H_{SM} H_{SM} \overline{\nu_L^c} \nu_L}_{\text{Not renormalizable}}, \quad \underbrace{H_{I_W=1}}_{\text{This Higgs not in SM}} \overline{\nu_L^c} \nu_L, \quad \underbrace{m_R \overline{\nu_R^c} \nu_R}_{\text{No Higgs}}$$

Majorana neutrino masses must have a different origin than the masses of quarks and charged leptons.

Why Majorana Masses ➡ Majorana Neutrinos

The objects ν_L and $\nu_L{}^c$ in $m_L \overline{\nu_L}\, \nu_L{}^c$ are not the mass eigenstates, but just the neutrinos in terms of which the model is constructed.

$m_L \overline{\nu_L}\, \nu_L{}^c$ induces $\nu_L \leftrightarrow \nu_L{}^c$ mixing.

As a result of $K^0 \longleftrightarrow \overline{K^0}$ mixing, the neutral K mass eigenstates are —

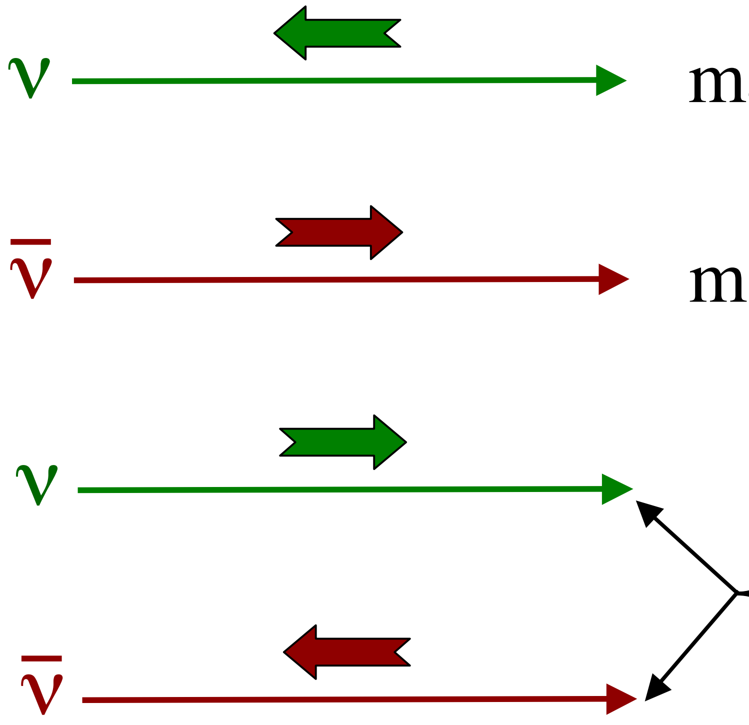
$$K_{S,L} \cong (K^0 \pm \overline{K^0})/\sqrt{2}\,. \qquad \overline{K_{S,L}} = K_{S,L}\,.$$

As a result of $\nu_L \leftrightarrow \nu_L{}^c$ mixing, the neutrino mass eigenstate is —

$$\nu_i = \nu_L + \nu_L{}^c = \text{“}\, \nu + \overline{\nu}\, \text{”}. \quad \overline{\nu_i} = \nu_i\,.$$

SM Interactions Of A Dirac Neutrino

We have 4 mass-degenerate states:



Conserved L

ν makes ℓ^- +1

$\bar{\nu}$ makes ℓ^+ –1

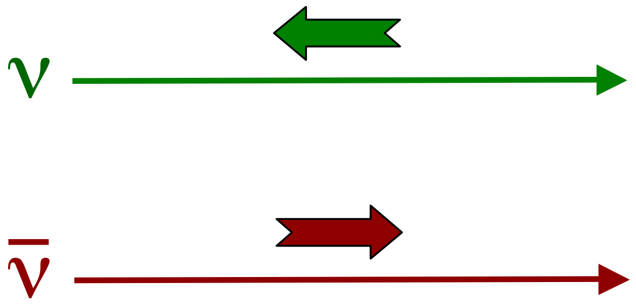
ν

$\bar{\nu}$

These states, when Ultra Rel., do not interact. $\left(\text{The weak interaction is Left Handed.}\right)$

SM Interactions Of A Majorana Neutrino

We have only 2 mass-degenerate states:



ν makes ℓ^-

$\bar{\nu}$ makes ℓ^+

The weak interactions violate *parity*.

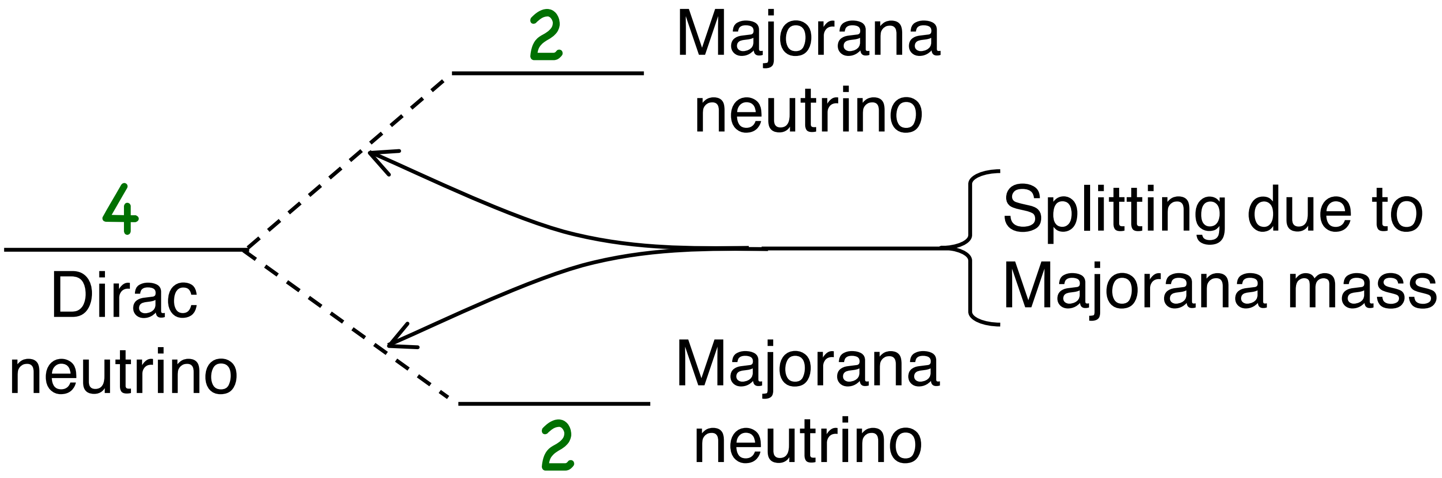
(They can tell *Left* from *Right*.)

An incoming left-handed neutral lepton makes ℓ^-.

An incoming right-handed neutral lepton makes ℓ^+.

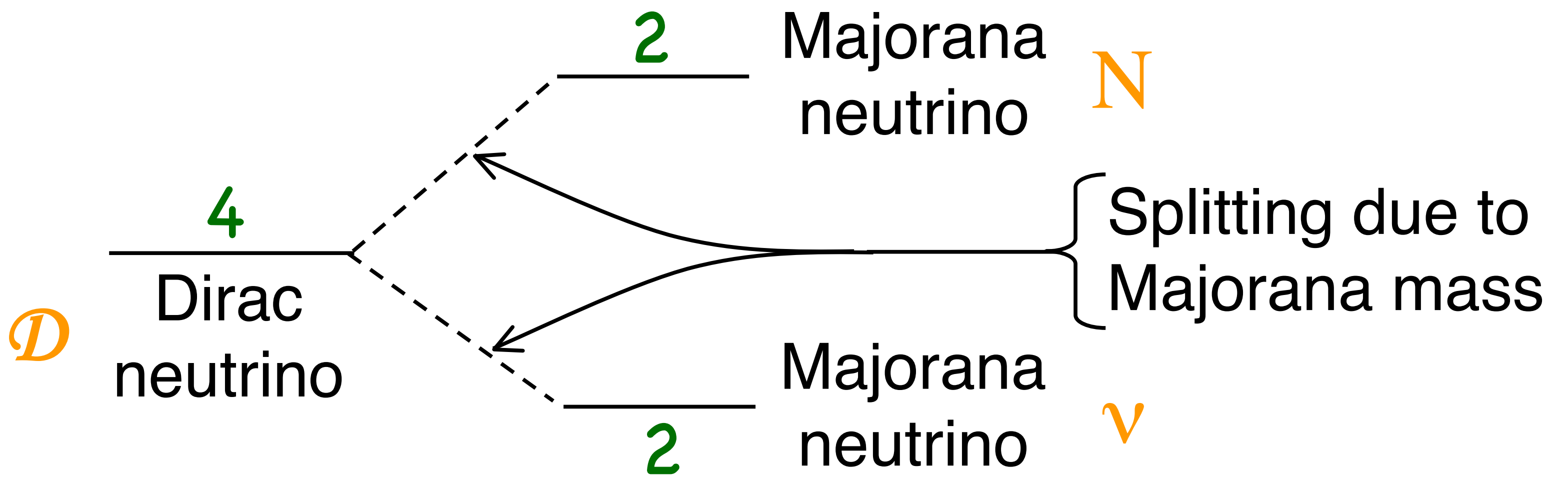
Majorana Masses Split Dirac Neutrinos

A Majorana mass term splits a Dirac neutrino into two Majorana neutrinos.



What Happens In the See-Saw

A ***BIG*** Majorana mass term splits a Dirac neutrino into two ***widely-spaced*** Majorana neutrinos.



$m_\nu m_N \approx m_{\mathcal{D}}^2$ ***The See-Saw Relation***

If $m_{\mathcal{D}}$ *is a typical fermion mass,* m_N *will be very large.*

Signature Predictions of the See-Saw

- The light neutrinos have heavy partners N_i

- Both light and heavy neutrinos are their own antiparticles (Majorana neutrinos)

Why Most Theorists Expect Majorana Masses

The Standard Model (SM) is defined by the fields it contains, its symmetries (notably weak isospin invariance), and its renormalizability.

Leaving neutrino masses aside, anything allowed by the SM symmetries occurs in nature.

Right-Handed Majorana mass terms
are allowed by the SM symmetries.

Then quite likely *Majorana masses*
occur in nature too.

To Determine Whether Majorana Masses Occur in Nature

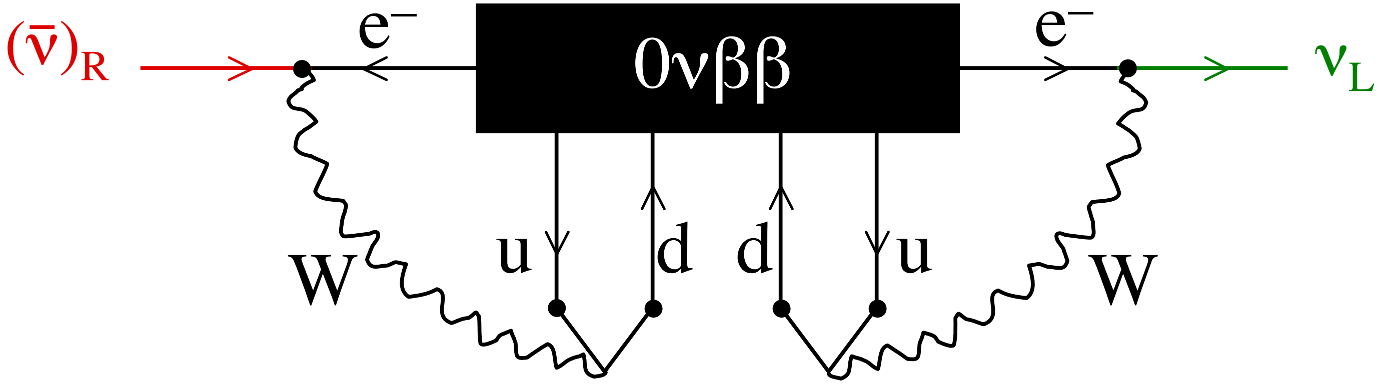
The Promising Approach — Seek

Neutrinoless Double Beta Decay [0νββ]

We are looking for a ***small*** Majorana neutrino mass. Thus, we will need ***a lot*** of parent nuclei (say, one ton of them).

Whatever diagrams cause 0νββ, its observation would imply the existence of a Majorana mass term:

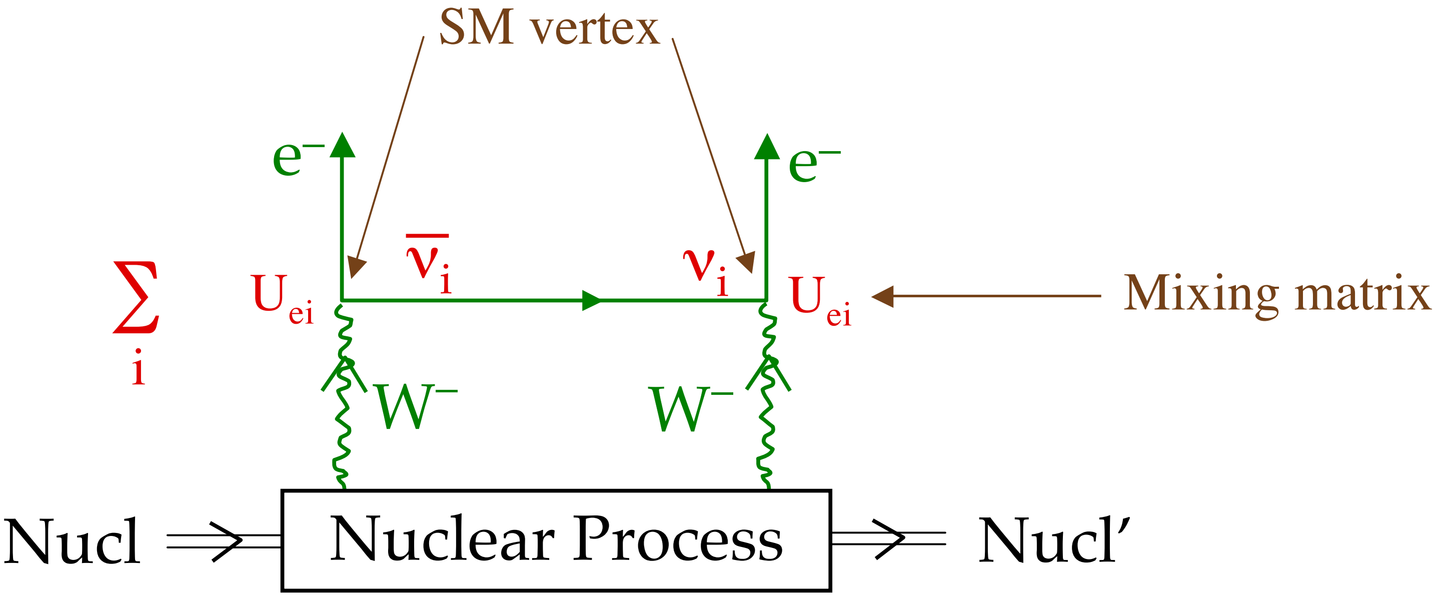
(Schechter and Valle)



$(\bar{\nu})_R \to \nu_L$: A (tiny) Majorana mass term

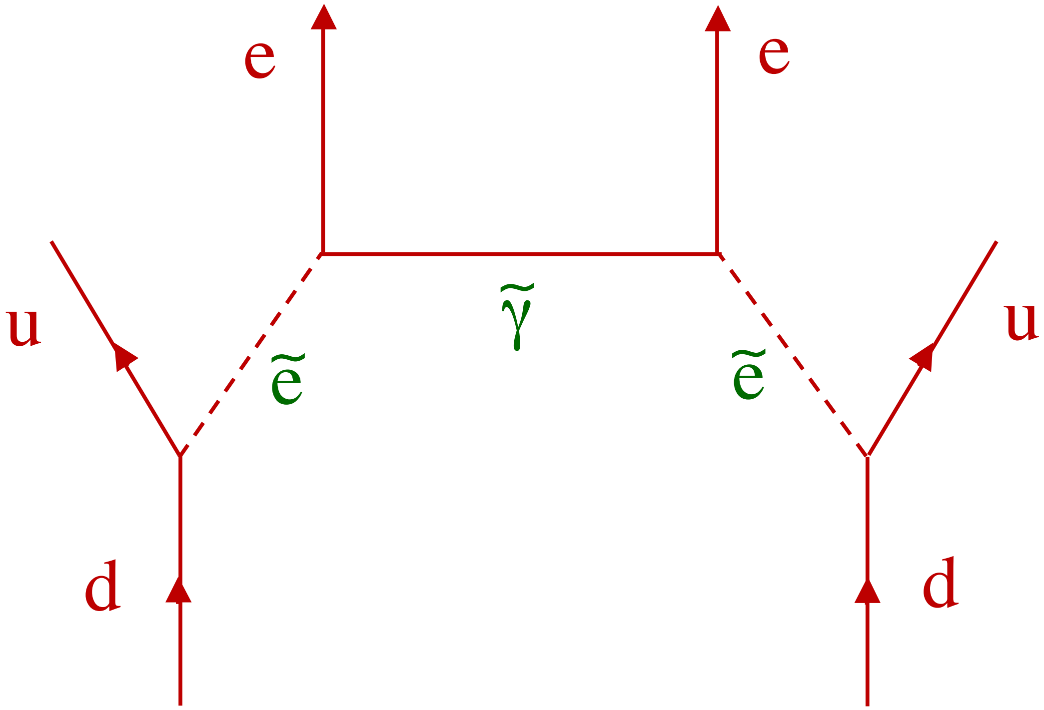
$\therefore$ 0νββ ➡ $\bar{\nu}_i = \nu_i$

We anticipate that 0νββ is dominated by a diagram with Standard Model vertices:

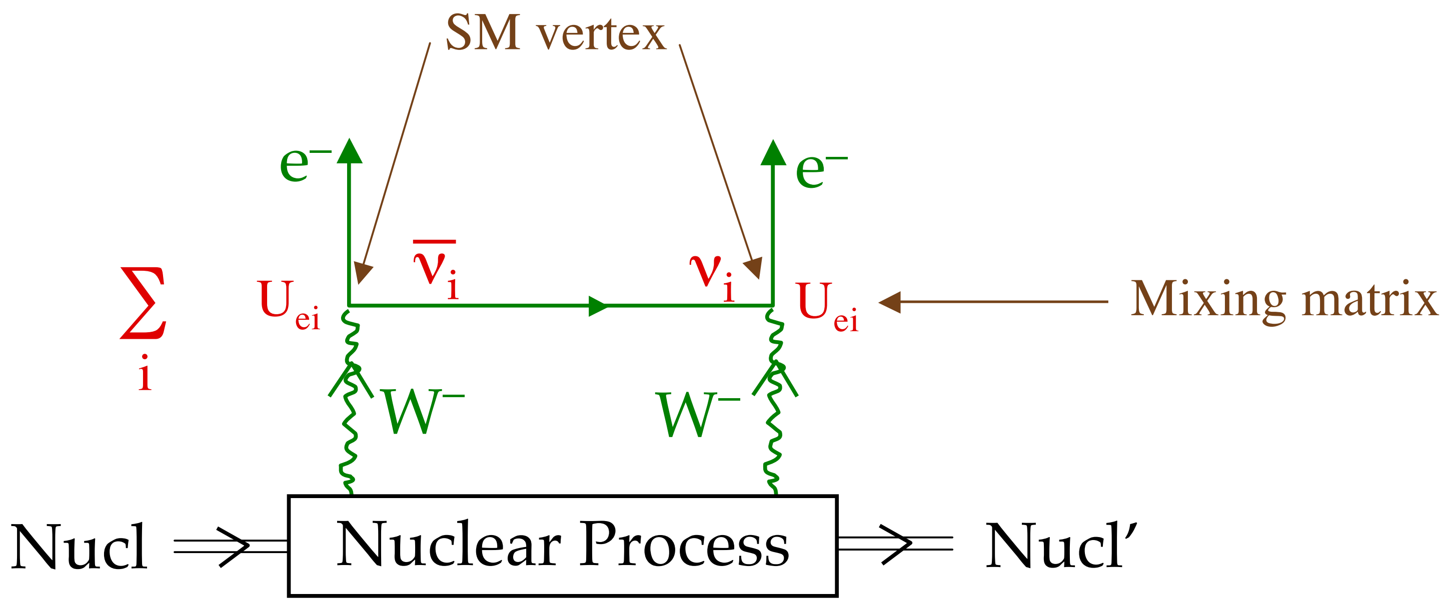


But there could be other contributions to 0νββ, which at the quark level is the process dd → uuee.

An example from Supersymmetry:



Assume the dominant mechanism is —



Mass (ν_i)

The $\bar{\nu}_i$ is emitted [RH + O{m_i/E}LH].

Thus, Amp [ν_i contribution] ∝ m_i

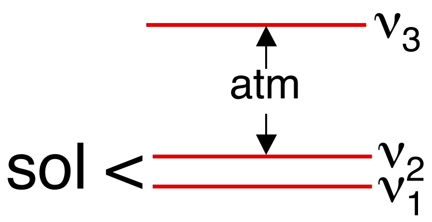
$$\mathrm{Amp}[0\nu\beta\beta] \propto \left|\sum m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$

How Large is $m_{\beta\beta}$?

How sensitive need an experiment be?

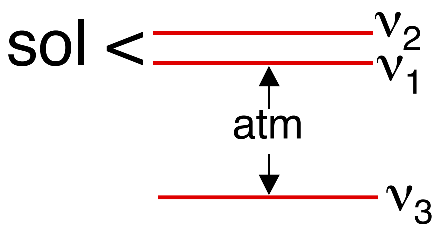
Suppose there are only 3 neutrino mass eigenstates. (More might help.)

Then the spectrum looks like —



Normal hierarchy

or

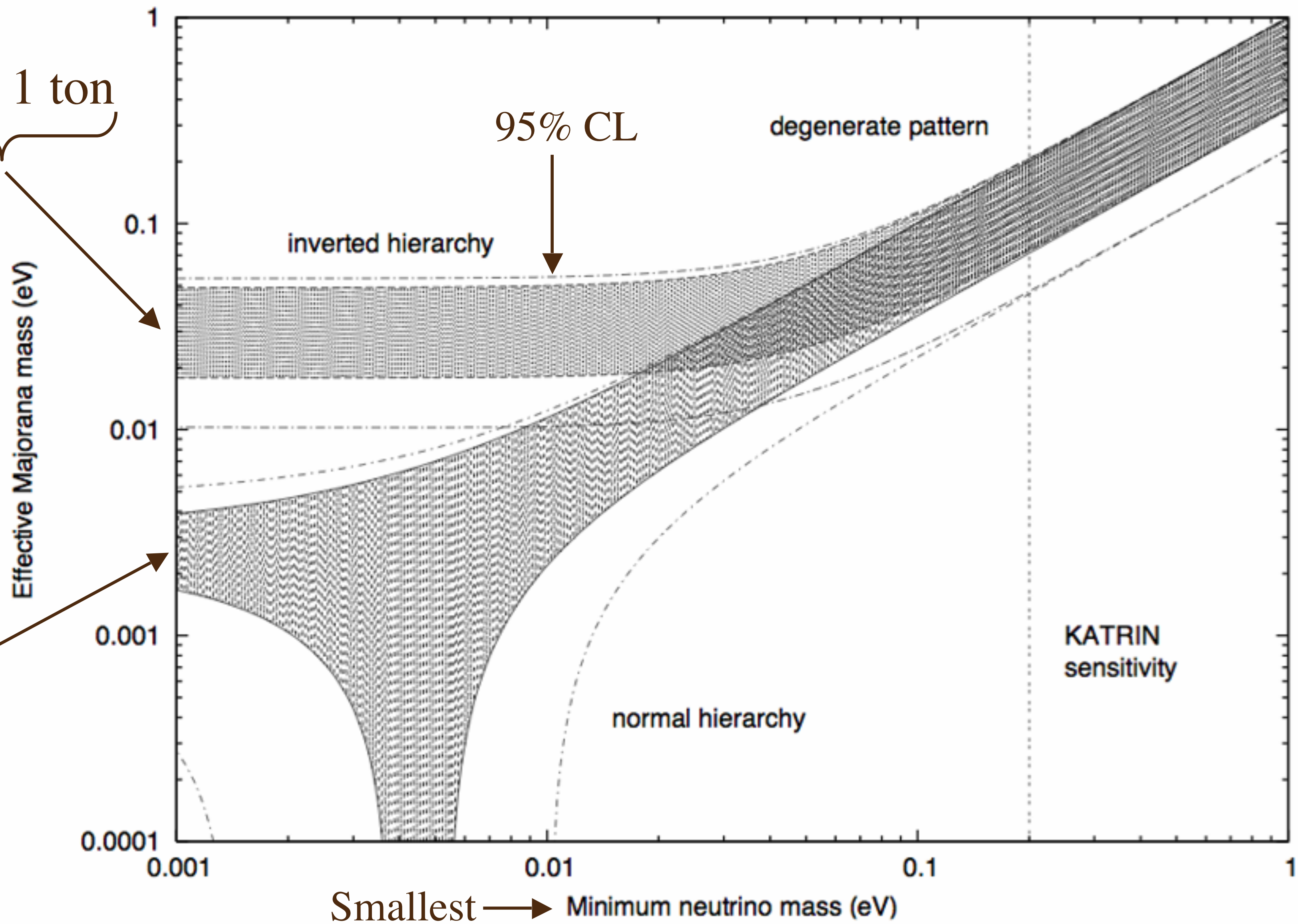


Inverted hierarchy

Takes 1 ton

95% CL

$m_{\beta\beta}$



Smallest →

Takes
100 tons

$m_{\beta\beta}$ *For Each Hierarchy*

There is no clear theoretical preference
for either hierarchy.

If the hierarchy is **inverted**—

then $0\nu\beta\beta$ searches with sensitivity
to $m_{\beta\beta} = 0.01$ eV have
a very good chance to see a signal.

Sensitivity in this range is the target
for the next generation of experiments.

Are we descended, via Leptogenesis, from heavy neutrinos?

The Challenge —
A Cosmic Broken Symmetry

Today: $B \equiv \#(\text{Baryons}) - \#(\text{Antibaryons}) \neq 0$.

Standard cosmology: Right after the Big Bang, $B = 0$.

How did $B = 0 \Rightarrow B \neq 0$?

Sakharov: $B = 0$ ⇒ $B \neq 0$ requires $\cancel{CP}$.

The $\cancel{CP}$ in the quark mixing matrix,
seen in B and K decays, leads to
much too small a Baryon Number B.

Leptogenesis can explain the observed
Baryon Number through CP-violating decays of
the heavy neutrinos in the See-Saw picture.

(Fukugita, Yanagida)

Leptogenesis is a very natural consequence
of the See-Saw picture.

Leptogenesis –

A Two-Step Process

Here Is How It Works

The straightforward (type-I) see-saw model adds to the Standard Model (SM) just 3 additional neutrinos N_i, to match the 3 light lepton families $(\nu_\alpha, \ell_\alpha)$.

The neutrinos N_i are given large Majorana masses, making them very heavy.

The heavy neutrinos are coupled to the rest of the world only through the "Yukawa" interaction —

$$\mathcal{L}_{\text{new}} = \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} y_{\alpha i}\left[\overline{\nu_{L\alpha}}\,\overline{H^0} - \overline{\ell_{L\alpha}}H^-\right]N_{Ri} + h.c.$$

RH component

Yukawa coupling matrix

SM Higgs doublet

This "new" interaction simply gives leptons the same Yukawa interaction as the quarks have in the SM.

The See-Saw Relation That Follows, In Full Detail

Leptonic mixing matrix

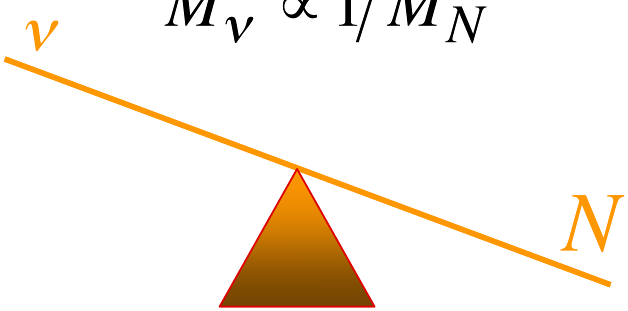
Heavy N mass eigenvalues

$$UM_{\nu}U^{T} = -v^{2}\left(y\,M_{N}^{-1}y^{T}\right)$$

Light ν mass eigenvalues

The Higgs vev, 174 GeV

$$M_{\nu} \propto 1/M_{N}$$



Yanagida;
Gell-Mann, Ramond, Slansky;
Mohapatra, Senjanovic;
Minkowski

In the very hot early universe, the N_i are produced.

Then the Yukawa interaction —

$$\mathcal{L}_{\text{new}} = \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} y_{\alpha i} \left[\overline{\nu_{L\alpha}}\, \overline{H^0} - \overline{\ell_{L\alpha}}\, H^- \right] N_{Ri} + h.c.$$

causes the decays —

$$N \to \ell^{\mp} + H^{\pm} \quad \text{and} \quad N \to \overset{(-)}{\nu} + \overset{(\overline{\quad})}{H^0}$$

SM Higgs particle

$\cancel{CP}$ phases in the matrix y will lead to —

$$\Gamma\left(N \to \ell^- + H^+\right) \neq \Gamma\left(N \to \ell^+ + H^-\right)$$

and

$$\Gamma\left(N \to \nu + H^0\right) \neq \Gamma\left(N \to \overline{\nu} + \overline{H^0}\right)$$

How Do Such $\cancel{CP}$ Inequalities Come About?

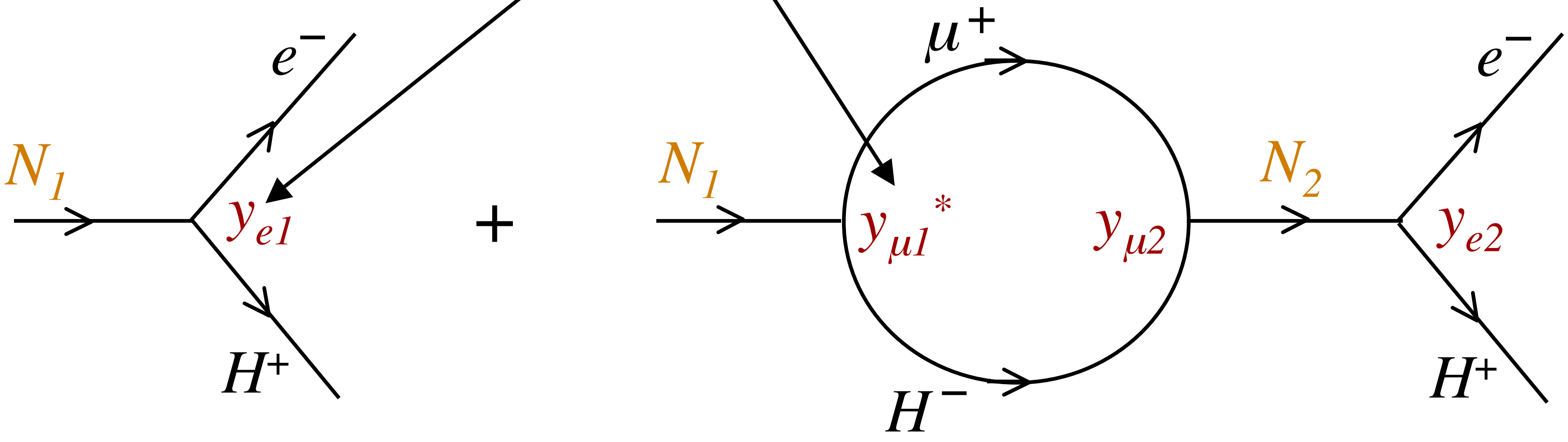
$\cancel{CP}$ always comes from *phases*.

Phases never matter except in *interferences* between coherent amplitudes.

∴ These decays must involve interfering amplitudes.

In addition, $\cancel{CP}$ in ***any*** decay always involves amplitudes ***beyond*** those of lowest order in the Hamiltonian.

An example:



From $y_{e1}\overline{\ell_{Le}}H^- N_{R1}$

From $y_{\mu 1}^*\overline{N_{R1}}H^+\ell_{L\mu}$

Tree

Loop

$$\Gamma\left(N_1 \rightarrow e^- + H^+\right) = \left| y_{e1}K_{\text{Tree}} + y_{\mu 1}^* y_{\mu 2} y_{e2} K_{\text{Loop}} \right|^2$$

Kinematical factors

$$\Gamma\left(N_1 \to e^- + H^+\right) = \left| y_{e1} K_{\text{Tree}} + y_{\mu 1}^* y_{\mu 2} y_{e2} K_{\text{Loop}} \right|^2$$

When we go to the CP-mirror-image decay, $N_1 \to e^+ + H^-$, all the coupling constants get complex conjugated, but the kinematical factors do not change.

$$\Gamma\left(N_1 \to e^+ + H^-\right) = \left| y_{e1}^* K_{\text{Tree}} + y_{\mu 1} y_{\mu 2}^* y_{e2}^* K_{\text{Loop}} \right|^2$$

Then —

$$\boxed{\begin{aligned} &\Gamma\left(N_1 \to e^- + H^+\right) - \Gamma\left(N_1 \to e^+ + H^-\right) \\ &= 4\,\mathrm{Im}\left(y_{e1}^* y_{\mu 1}^* y_{e2} y_{\mu 2}\right) \mathrm{Im}\left(K_{\text{Tree}} K_{\text{Loop}}^*\right) \end{aligned}}$$

The $\cancel{CP}$ inequalities —

$$\Gamma\left(N \to \ell^- + H^+\right) \neq \Gamma\left(N \to \ell^+ + H^-\right)$$

and

$$\Gamma\left(N \to \nu + H^0\right) \neq \Gamma\left(N \to \bar{\nu} + \overline{H^0}\right)$$

will produce a universe with unequal numbers of leptons (ℓ^- and ν) and antileptons (ℓ^+ and $\bar{\nu}$).

In this universe the lepton number L, defined by $L\left(\ell^-\right) = L(\nu) = -L\left(\ell^+\right) = -L(\bar{\nu}) = 1$, is not zero.

This is Leptogenesis — Step 1

Leptogenesis — Step 2

The Standard-Model ***Sphaleron*** process,
which does not conserve Baryon Number B,
or Lepton Number L, but does conserve $B - L$, acts.

$$B_i = 0$$

$$L_i \neq 0$$

Initial state
from N decays

Sphaleron
Process

$$B_f \cong -\frac{1}{3} L_i$$

$$L_f \cong \frac{2}{3} L_i \cong -2B_f$$

Final state

There is now a nonzero *Baryon Number*.

There are baryons, but ~ no antibaryons.

Reasonable parameters give the observed value of B.

What N masses are required?

$$U M_\nu U^T = -v^2 \left(y\, M_N^{-1} y^T \right) \quad \Longrightarrow \quad M_\nu \sim \frac{v^2 y^2}{M_N}$$

The light neutrino masses $M_\nu \sim 0.1$ eV.

$v = 174$ GeV.

y^2 is constrained by the observed Baryon Number.

The CP-violating asymmetry between the N decay rates,

$$\varepsilon_{CP} \equiv \frac{\Gamma\left(N \to \overset{\substack{\nu \text{ or } \ell^- \\ \downarrow}}{L}\,\overset{\substack{H^0 \text{ or } H^+ \\ \downarrow}}{H}\right) - \Gamma\left(N \to \overline{L}\,\overline{H}\right)}{\Gamma\left(N \to LH\right) + \Gamma\left(N \to \overline{L}\,\overline{H}\right)} \quad ,$$

which produces the nonzero Lepton Number,

will be $\propto (y^4/y^2) = y^2$.

Getting the observed Baryon Number requires $y^2 \sim 10^{-5}$.

Then the see-saw relation —

$$M_\nu \sim \frac{v^2 y^2}{M_N}$$

⇒ $M_N \sim 10^{(9-10)}$ GeV.

This places the heavy neutrinos N far out of reach of the LHC.

⇒ ***The possibility of Leptogenesis must be explored without producing the heavy neutrinos.***

Number of leptonic parameters in the See-Saw picture: 21

Number of these parameters that can be measured
without producing the heavy neutrinos N: 12

Since 21 > 12, laboratory measurements today
cannot pin down what happened in the early universe.

Can there be $\cancel{CP}$ in ν oscillation but no leptogenesis? Yes.

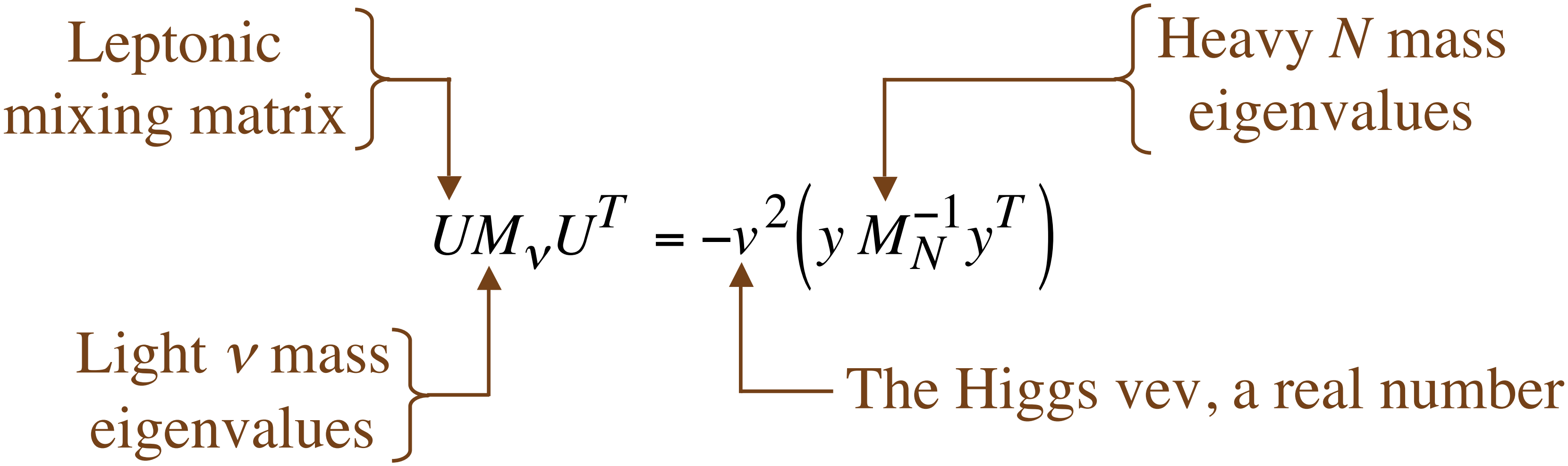
Can there be leptogenesis but no $\cancel{CP}$ in ν oscillation? Yes.

Is either of these possibilities likely? ***NO!***

An Argument

(BK, arXiv:1012.4469)

The See-Saw Relation



Leptonic mixing matrix

Heavy N mass eigenvalues

$$UM_\nu U^T = -v^2\left(y\,M_N^{-1}y^T\right)$$

Light ν mass eigenvalues

The Higgs vev, a real number

$$\left(\underbrace{UM_\nu U^T}_{\text{Outputs}} = -v^2\underbrace{\left(y\,M_N^{-1}y^T\right)}_{\text{Inputs, in }\mathcal{L}}\right)$$

Through ***U***, the phases in ***y*** lead to
$\not{C}\not{P}$ in light neutrino oscillation.

$$P(\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta) =$$

e, μ, or τ

Distance

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin^2(\Delta m^2_{ij}\frac{L}{4E})$$

$$\underset{(-)}{+}2\sum_{i>j} \Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin(\Delta m^2_{ij}\frac{L}{2E})$$

Neutrino (Mass)^2 splitting

Energy

Generically, leptogenesis and light-neutrino ~~CP~~ imply each other.

Motivated partly by the connection to
Leptogenesis *—*

Seeking CP violation in neutrino oscillation is now a worldwide goal.

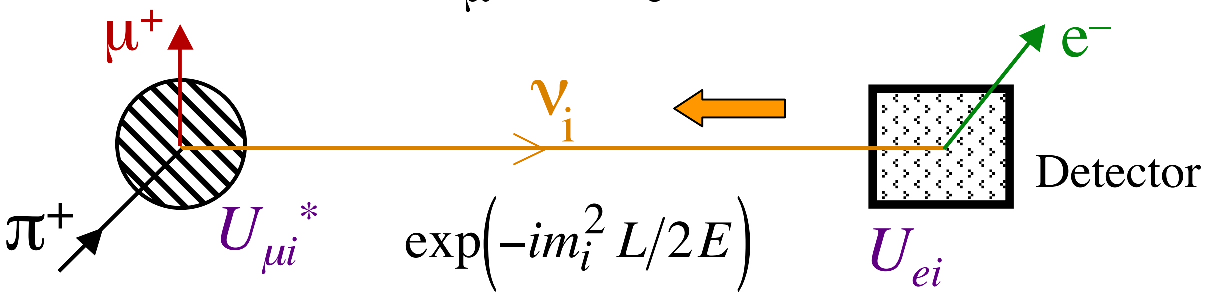
The search will use long-baseline accelerator neutrino beams to study $\nu_\mu \rightarrow \nu_e$ ***and*** $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$***, or their inverses.***

Q : *Can CP violation still lead to $\mathcal{P}(\overline{\nu}_\mu \to \overline{\nu}_e) \neq \mathcal{P}(\nu_\mu \to \nu_e)$ when $\overline{\nu} = \nu$?*

A : *Certainly!*

Compare $\nu_\mu \to \nu_e$

$\sum_i$



μ^+

π^+

ν_i

$U_{\mu i}^*$

$\exp\left(-im_i^2 L/2E\right)$

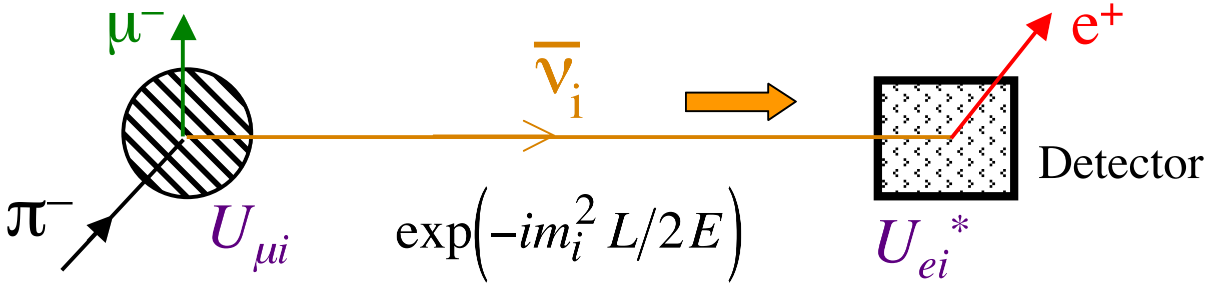
e^-

Detector

U_{ei}

with "$\overline{\nu}_\mu \to \overline{\nu}_e$"

$\sum_i$



μ^-

π^-

$\overline{\nu}_i$

$U_{\mu i}$

$\exp\left(-im_i^2 L/2E\right)$

e^+

Detector

U_{ei}^*

Enjoy the Rest of the Workshop!