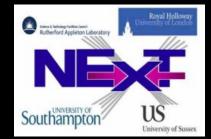
What is v? INVISIBLES 12 and Alexei Smirnov Fest GGI, Arcetri, 24–29 June 2012

# Leptogenesis confronting neutrino data

Pasquale Di Bari





# The double side of Leptogenesis

Cosmology (early Universe)

- <u>Cosmological Puzzles</u> :
- 1. Dark matter
- 2. Matter antimatter asymmetry
- 3. Inflation
- 4. Accelerating Universe
- <u>New stage in early Universe history</u>:

< 10<sup>14</sup> GeV — Inflation — Leptogenesis

100 GeV <mark>—</mark> EWSSB

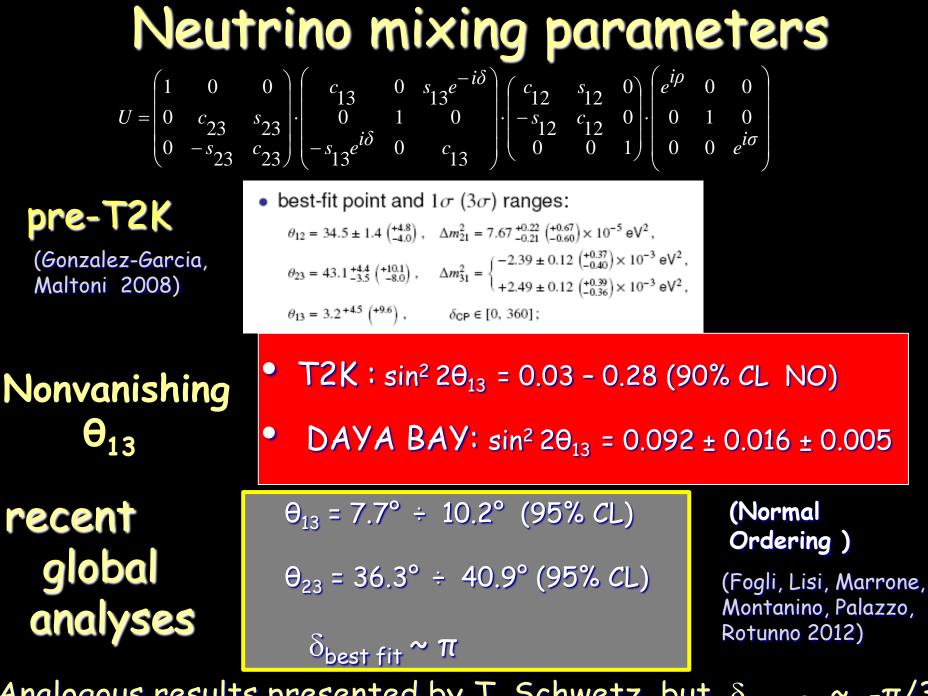
0.1-1 MeV - BBN

0.1-1 eV — Recombination

Neutrino Physics, New Physics

Leptogenesis complements low energy neutrino experiments testing the seesaw mechanism high energy parameters

Can Leptogenesis be useful to overconstrain the seesaw parameter space providing a way to understand the measured values of the neutrino parameters and even to make predictions on future measurements ?



Analogous results presented by T. Schwetz but  $\delta_{\text{best fit}} \sim -\pi/3$ 

# Neutrino masses: $m_1 < m_2 < m_3$

#### neutrino mixing data

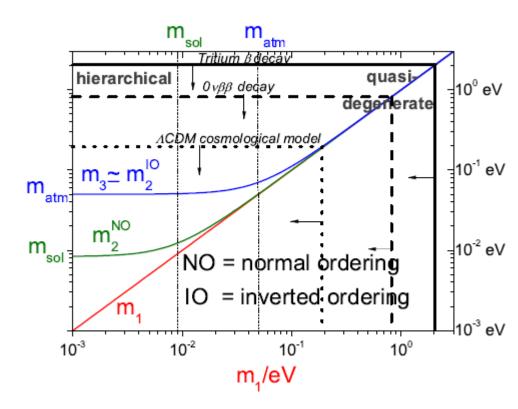
2 possible schemes: normal or inverted

$$m_3^2 - m_2^2 = \Delta m_{\text{atm}}^2 \text{ or } \Delta m_{\text{sol}}^2 \quad m_{\text{atm}} \equiv \sqrt{\Delta m_{\text{atm}}^2 + \Delta m_{\text{sol}}^2} \simeq 0.05 \,\text{eV}$$
$$m_2^2 - m_1^2 = \Delta m_{\text{sol}}^2 \text{ or } \Delta m_{\text{atm}}^2 \quad m_{\text{sol}} \equiv \sqrt{\Delta m_{\text{sol}}^2} \simeq 0.009 \,\text{eV}$$

Tritium  $\beta$  decay :  $m_e < 2 \text{ eV}$ (Mainz + Troitzk 95% CL)

 $\beta\beta 0\nu: m_{\beta\beta} < 0.34 - 0.78 eV$ (CUORICINO 95% CL, similar bound from Heidelberg-Moscow) NEW!:  $m_{\beta\beta} < 0.14 - 0.38 eV$ (EXO-200 90% CL)

 $\begin{array}{l} \textbf{CMB+BAO+HO}: \Sigma \ \textbf{m}_i < 0.58 \ \textbf{eV} \\ \textbf{(WMAP7+2dF+SDSS+HST, 95\%CL)} \\ \textbf{CMB+LSS} + \ \textbf{Ly} \alpha : \Sigma \ \textbf{m}_i < 0.17 \ \textbf{eV} \\ \textbf{(Seljak et al.)} \end{array}$ 



#### Minimal scenario of Leptogenesis (Fukugita, Yanagida '86)

#### •Type I seesaw

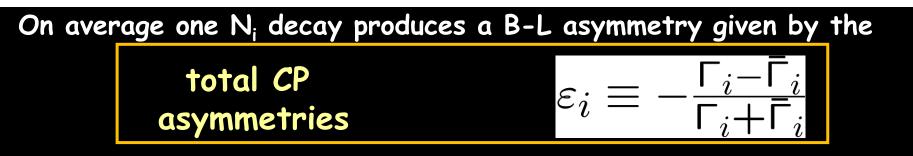
$$\mathcal{L}_{\rm mass}^{\nu} = -\frac{1}{2} \left[ \left( \bar{\nu}_L^c, \bar{\nu}_R \right) \left( \begin{array}{cc} 0 & \boldsymbol{m}_D^T \\ \boldsymbol{m}_D & \boldsymbol{M} \end{array} \right) \left( \begin{array}{c} \nu_L \\ \boldsymbol{\nu}_R^c \end{array} \right) \right] + h.c.$$

In the see-saw limit ( $M\gg m_D$ ) the spectrum of mass eigenstates splits in 2 sets:

• 3 light neutrinos  $u_1, \, 
u_2, \, 
u_3$  with masses

 $diag(m_1, m_2, m_3) = -U^{\dagger} m_D \frac{1}{M} m_D^T U^{\star}$ 

• 3 new heavy RH neutrinos  $N_1, N_2, N_3$  with masses  $M_3 > M_2 > M_1 \gg m_D$ 



•Thermal production of the RH neutrinos  $\Rightarrow$  T<sub>RH</sub>  $\gtrsim$  M<sub>i</sub> / (2÷10)

## Seesaw parameter space

Imposing  $\eta_B = \eta_B^{CMB}$  one would like to get information on U and  $m_i$ <u>Problem: too many parameters</u>

(Casas, Ibarra'01) 
$$m_{
u} = -m_D \frac{1}{M} m_D^T \Leftrightarrow \Omega^T \Omega = I$$

Orthogonal parameterisation

(in a basis where charged lepton and Majorana mass matrices are diagonal)

The 6 parameters in the orthogonal matrix  $\Omega$  encode the 3 life times and the 3 total CP asymmetries of the RH neutrinos and is an invariant <u>Parameter reduction from:</u> (King '07)

 $\begin{array}{ccc} m_{D} \\ m_{D} \end{array} = \begin{bmatrix} U \begin{pmatrix} \sqrt{m_{1}} & 0 & 0 \\ 0 & \sqrt{m_{2}} & 0 \\ 0 & 0 & \sqrt{m_{3}} \end{bmatrix} \Omega \begin{pmatrix} \sqrt{M_{1}} & 0 & 0 \\ 0 & \sqrt{M_{2}} & 0 \\ 0 & 0 & \sqrt{M_{3}} \end{bmatrix} \end{bmatrix} \begin{pmatrix} U^{\dagger} U \\ U^{\dagger} & m_{\nu} & U^{\star} \\ U^{\dagger} & m_{\nu} & U^{\star} \end{bmatrix}$ 

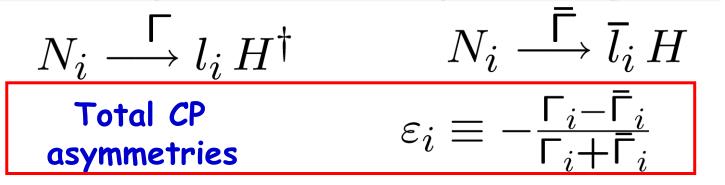
- Iso-asymmetry surfaces  $\eta_B(U, m_i; \lambda_1, ..., \lambda_N) = \eta_B^{CMB}$  (if they "close up" the leptogenesis bound can remove more than one parameter in this case)
- In the asymmetry calculation  $\eta_B = \eta_B (U, \mathbf{m}_i; \lambda_1, ..., \lambda_{M<9})$

 $\bullet$  Imposing some (model dependent) conditions on  $m_D$  one can reduce the number of parameters and arrive to a new parameterisation where

 $\Omega = \Omega (U, m_i; \lambda_1, ..., \lambda_{N < 9}) \text{ and } M_i = M_i (U, m_i; \lambda_1, ..., \lambda_{N \le M})$ 

## Vanilla leptogenesis

### 1) Flavor composition of final leptons is neglected



 $N_{B-L}^{\text{fin}} = \sum_{i} \varepsilon_{i} \kappa_{i}^{\text{fin}} \Rightarrow \eta_{B} = a_{\text{sph}} \frac{N_{B-L}^{\text{fin}}}{N_{\gamma}^{\text{rec}}} \stackrel{\text{baryon-to}}{\underset{\text{number ratio}}{\overset{\text{photon}}{\overset{\text{number ratio}}{\overset{\text{number ratio}}{\overset{\text{number ratio}}{\overset{\text{photon}}{\overset{\text{number ratio}}{\overset{\text{number ratio}}{\overset{number rati$ 

2) Hierarchical heavy RH neutrino spectrum:  $M_2 \stackrel{>}{\sim} 3 M_1$ 3) N<sub>3</sub> does not interfere with N<sub>2</sub>-decays:  $(m_D^{\dagger} m_D)_{23} = 0$ 

From the last two assumptions

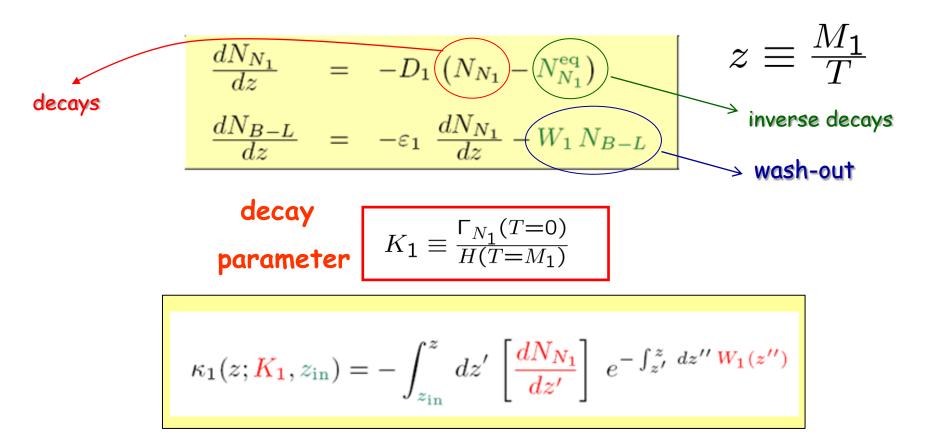
$$\Rightarrow N_{B-L}^{\text{fin}} = \sum_i \varepsilon_i \,\kappa_i^{\text{fin}} \simeq \varepsilon_1 \,\kappa_1^{\text{fin}}$$

4) Barring fine-tuned mass cancellations in the seesaw

$$\Rightarrow \quad \varepsilon_1 \stackrel{<}{\sim} 10^{-6} \left(\frac{M_1}{10^{10} \,\mathrm{GeV}}\right) \frac{m_{\mathrm{atm}}}{m_1 + m_3}$$

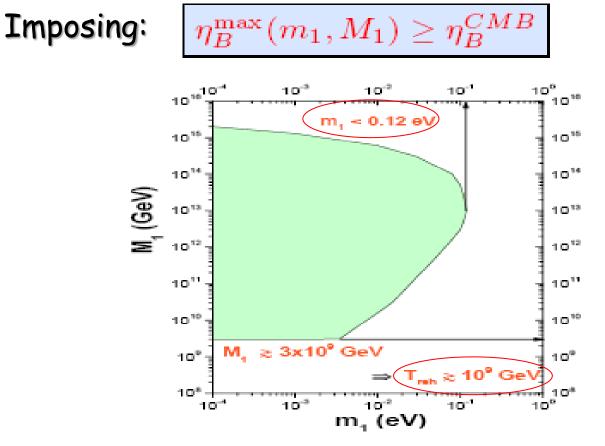
(Davidson, Ibarra '02)

#### 5) Efficiency factor from simple Boltzmann equations



# Neutrino mass bounds in vanilla leptog.

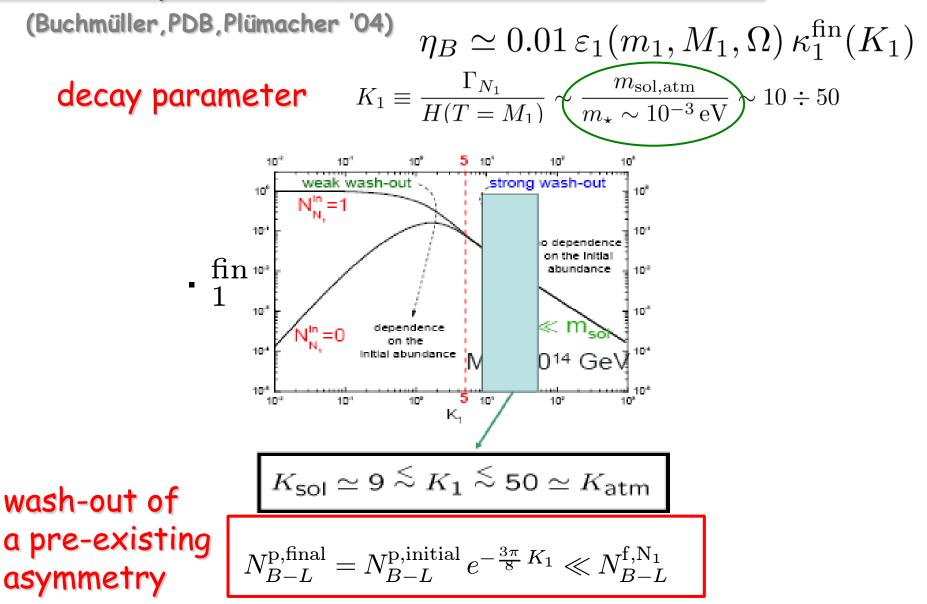
(Davidson, Ibarra '02; Buchmüller, PDB, Plümacher '02, '03, '04; Giudice et al. '04)  $\eta_B \simeq 0.01 \, \varepsilon_1(m_1, M_1, \Omega) \, \kappa_1^{\text{fin}}(K_1)$ 



No dipendence on the leptonic mixing matrix U

## Independence of the initial conditions

The early Universe "knows" neutrino masses ...



## SO(10)-inspired leptogenesis

(Branco et al. '02; Nezri, Orloff '02; Akhmedov, Frigerio, Smirnov '03)

Expressing the neutrino Dirac mass matrix  $m_D$  (in the basis where the Majorana mass and charged lepton mass matrices are diagonal) as:

$$m_D = V_L^{\dagger} D_{m_D} U_R$$

$$D_{m_D} = \operatorname{diag}\{\lambda_{D1}, \lambda_{D2}, \lambda_{D3}\}$$

SO(10) inspired conditions:

$$\lambda_{D1} = \alpha_1 \, m_u \,, \, \lambda_{D2} = \alpha_2 \, m_c \,, \, \lambda_{D3} = \alpha_3 \, m_t \,, \ (\alpha_i = \mathcal{O}(1))$$

$$V_L \simeq V_{CKM} \simeq I$$

One can express: 
$$\Omega = D_m^{-\frac{1}{2}} U^{\dagger} V_L^{\dagger} D_{m_D} U_R D_M^{-\frac{1}{2}}$$

and from the seesaw formula:  $U_R = U_R (U, m_i; \alpha_i, V_L), M_i = M_i (U, m_i; \alpha_i, V_L) \Longrightarrow \eta_B = \eta_B (U, m_i; \alpha_i, V_L)$ 

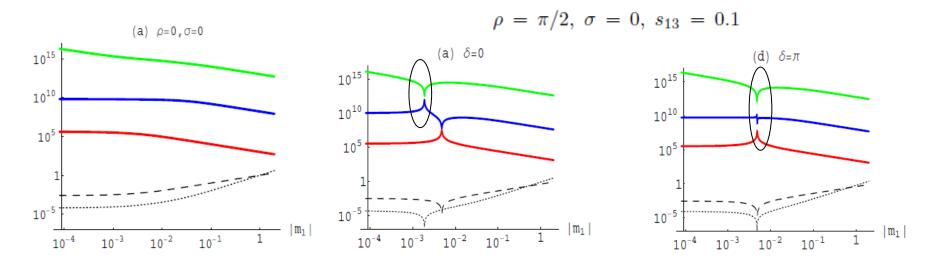
one typically obtains (barring fine-tuned 'crossing level' solutions):

$$M_1 \sim \alpha_1^2 \, 10^5 \text{GeV} \,, \ M_2 \sim \alpha_2^2 \, 10^{10} \, \text{GeV} \,, \ M_3 \sim \alpha_3^2 \, 10^{15} \, \text{GeV}$$

since  $M_1 \ll 10^9 \text{ GeV} \Rightarrow \eta_B(N_1) \ll \eta_B^{CMB}$ !  $\Rightarrow$  failure of the  $N_1$ -dominated scenario!

## **Crossing level solutions**

(Akhmedov, Frigerio, Smirnov '03)



At the crossing the CP asymmetries undergo a resonant enhancement (Covi,Roulet,Vissani '96; Pilaftsis '98; Pilaftsis,Underwood '04; ...)

The correct BAU can be attained for a fine tuned choice of parameters: many models have made use of these solutions but as we will see there is another option

# Beyond vanilla Leptogenesis

Degenerate limit and resonant leptogenesis

Vanilla Leptogenesis Non minimal Leptogenesis (in type II seesaw, non thermal,....)

Improved Kinetic description (momentum dependence, quantum kinetic effects,finite temperature effects,....., density matrix formalism)

Flavour Effects (heavy neutrino flavour effects, lepton flavour effects and their interplay)

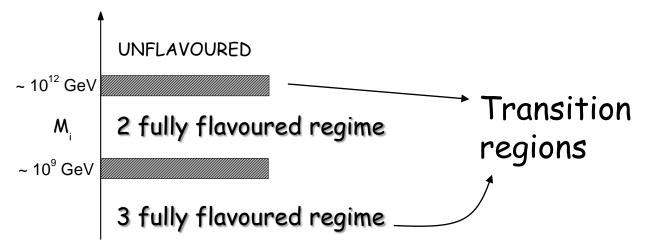
# Lepton flavour effects

(Abada, Davidson, Losada, Josse-Michaux, Riotto'06; Nardi, Nir, Roulet, Racker '06; Blanchet, PDB, Raffelt '06; Riotto, De Simone '06)

Flavor composition of lepton quantum states:

$$\begin{aligned} |l_1\rangle &= \sum_{\alpha} \langle l_{\alpha} | l_1 \rangle | l_{\alpha} \rangle \quad (\alpha = e, \mu, \tau) \qquad P_{1\alpha} \equiv |\langle l_{\alpha} | l_1 \rangle|^2 \\ |\overline{l}_1\rangle &= \sum_{\alpha} \langle l_{\alpha} | \overline{l}_1' \rangle | \overline{l}_{\alpha} \rangle \qquad \qquad \overline{P}_{1\alpha} \equiv |\langle \overline{l}_{\alpha} | \overline{l}_1' \rangle|^2 \end{aligned}$$

But for  $T \leq 10^{12} \text{ GeV} \implies \tau$ -Yukawa interactions  $(\bar{l}_{L\tau} \phi f_{\tau\tau} e_{R\tau})$ are fast enough to break the coherent evolution of  $|l_1\rangle$  and  $|\bar{l}_1'\rangle$  $\implies$  they become an incoherent mixture of a  $\tau$  and of a  $\mu$ +e component At  $T \leq 10^9$  GeV then also  $\mu$ - Yukawas in equilibrium  $\implies$  3-flavor regime



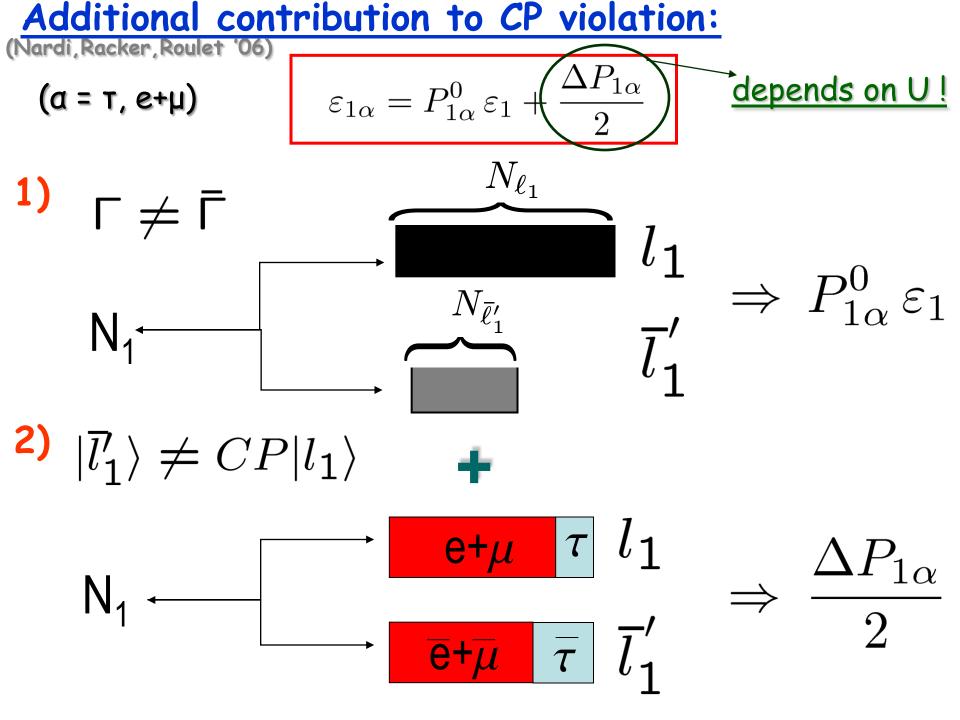
# Two fully flavoured regime

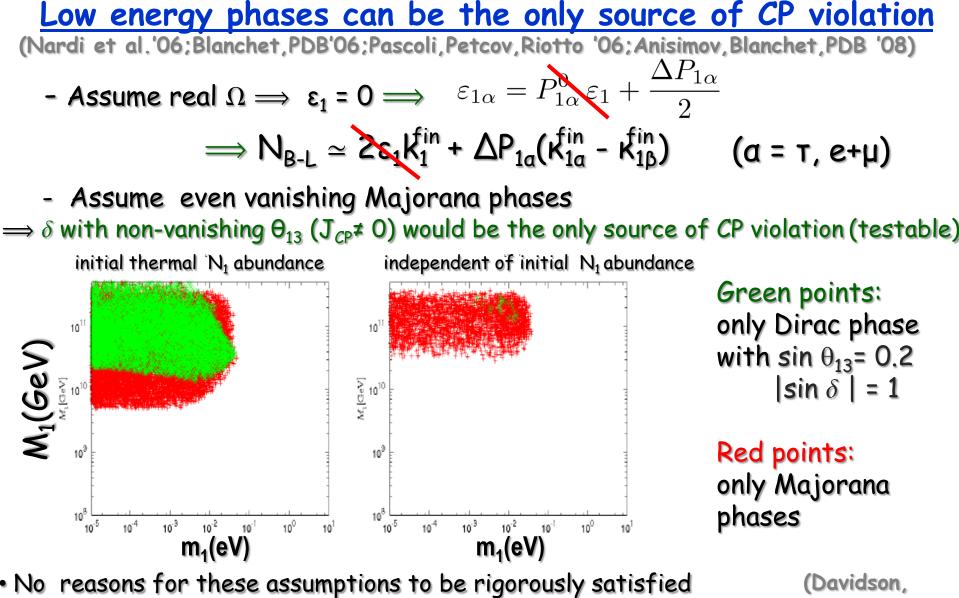
$$\begin{array}{l} \left(\mathbf{a}=\mathbf{T}, \mathbf{e}+\mathbf{\mu}\right) & P_{1\alpha} \equiv |\langle \bar{l}_{\alpha}|l_{1}\rangle|^{2} = P_{1\alpha}^{0} + \Delta P_{1\alpha}/2 & \left(\sum_{\alpha} P_{1\alpha}^{0} = 1\right) \\ \bar{P}_{1\alpha} \equiv |\langle \bar{l}_{\alpha}|\bar{l}_{1}'\rangle|^{2} = P_{1\alpha}^{0} - \Delta P_{1\alpha}/2 & \left(\sum_{\alpha} \Delta P_{1\alpha} = 0\right) \end{array}$$

$$\Rightarrow \quad \varepsilon_{1\alpha} \equiv -\frac{P_{1\alpha}\Gamma_1 - \bar{P}_{1\alpha}\bar{\Gamma}_1}{\Gamma_1 + \bar{\Gamma}_1} = P_{1\alpha}^0 \, \varepsilon_1 + \Delta P_{1\alpha}(\Omega, U)/2$$

Classic Kinetic Equations (in their simplest form)

$$\frac{dN_{N_1}}{dz} = -D_1 \left( N_{N_1} - N_{N_1}^{eq} \right)$$
$$\frac{dN_{\Delta_{\alpha}}}{dz} = -\varepsilon_{1\alpha} \frac{dN_{N_1}}{dz} - P_{1\alpha}^0 W_1 N_{\Delta_{\alpha}}$$
$$\Rightarrow N_{B-L} = \sum_{\alpha} N_{\Delta_{\alpha}} \qquad (\Delta_{\alpha} \equiv B/3 - L_{\alpha})$$
$$\Rightarrow N_{B-L}^{fin} = \sum_{\alpha} \varepsilon_{1\alpha} \kappa_{1\alpha}^{fin} \simeq 2 \varepsilon_1 \kappa_1^{fin} + \frac{\Delta P_{1\alpha}}{2} \left[ \kappa_{1\alpha}^{fin} - \kappa_{1\beta}^{fin} \right]$$

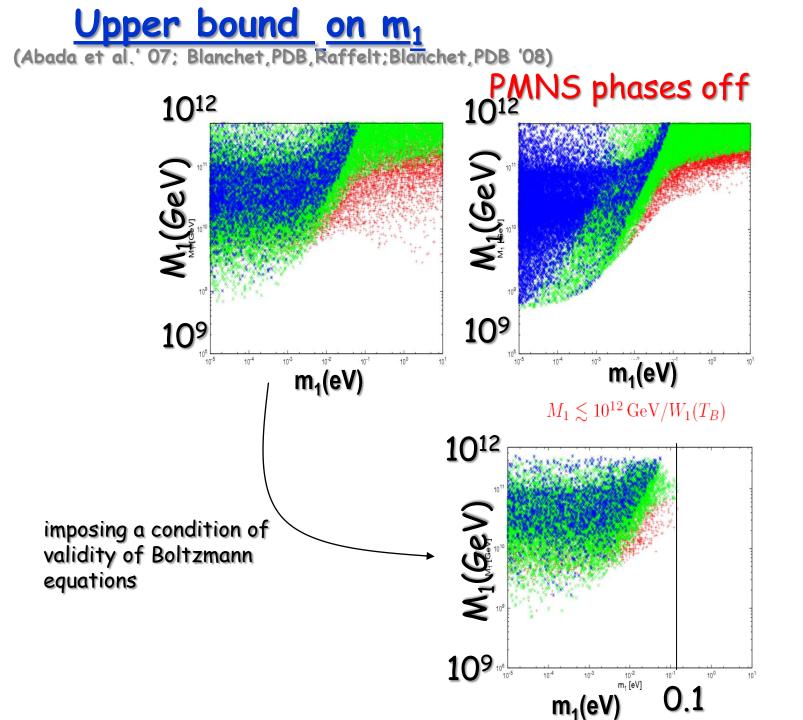




• In general this contribution is *overwhelmed* by the high energy phases Rius et al. '07)

But they can be approximately satisfied in specific scenarios for some regions

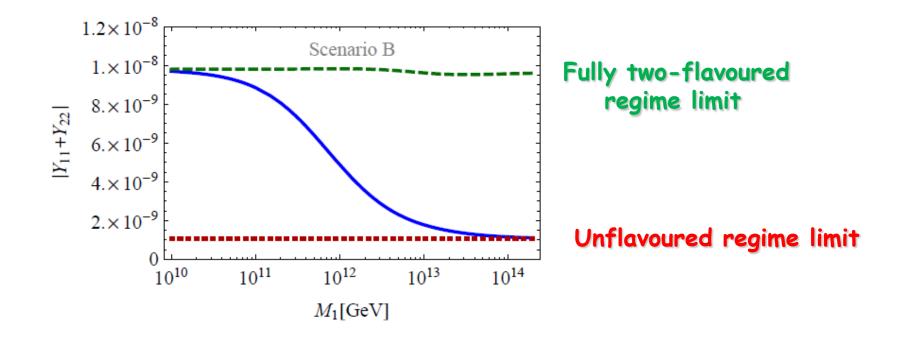
 It is in any case by itself interesting that CP violation in neutrino mixing could be sufficient to reproduce the observed BAU



# Density matrix and CTP formalism to describe the transition regimes

(De Simone, Riotto '06; Beneke, Gabrecht, Fidler, Herranen, Schwaller '10)

$$\frac{\mathrm{d}Y_{\alpha\beta}}{\mathrm{d}z} = \frac{1}{szH(z)} \left[ (\gamma_D + \gamma_{\Delta L=1}) \left( \frac{Y_{N_1}}{Y_{N_1}^{\mathrm{eq}}} - 1 \right) \epsilon_{\alpha\beta} - \frac{1}{2Y_{\ell}^{\mathrm{eq}}} \left\{ \gamma_D + \gamma_{\Delta L=1}, Y \right\}_{\alpha\beta} \right] - \left[ \sigma_2 \mathrm{Re}(\Lambda) + \sigma_1 |\mathrm{Im}(\Lambda)| \right] Y_{\alpha\beta}$$



## Heavy neutrino flavour effects: N<sub>2</sub>-dominated scenario

#### ( PDB '05)

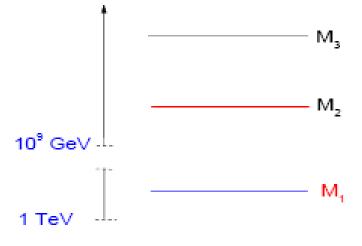
If light flavour effects are neglected the asymmetry from the next-to-lightest ( $N_2$ ) RH neutrinos is typically negligible:

$$N_{B-L}^{\mathrm{f},\mathrm{N}_2} = \varepsilon_2 \cdot (K_2) \, e^{-\frac{3\pi}{8} K_1} \ll N_{B-L}^{\mathrm{f},\mathrm{N}_1} = \varepsilon_1 \cdot (K_1)$$

...except for a special choice of  $\Omega = R_{23}$  when  $K_1 = m_1/m_* \ll 1$  and  $\varepsilon_1 = 0$ :

$$\Rightarrow \boxed{N_{B-L}^{\rm fin} = \sum_i \, \varepsilon_i \, \kappa_i^{\rm fin} \simeq \varepsilon_2 \, \kappa_2^{\rm fin}}_{2} \qquad \varepsilon_2 \stackrel{<}{\sim} 10^{-6} \left(\frac{M_2}{10^{10} \, {\rm GeV}}\right)$$

The lower bound on  $M_1$  disappears and is replaced by a lower bound on  $M_2$  ... that however still implies a lower bound on  $T_{reh}$ !



# Interplay between lepton and heavy neutrino flavour effects:

- N<sub>2</sub> flavoured leptogenesis
   (Vives '05; Blanchet, PDB '06; Blanchet, PDB '08)
- Flavour projection
   (Barbieri, Creminelli, Stumia, Tetradis '00; Engelhard, Grossman, Nardi, Nir '07)
- Phantom leptogenesis

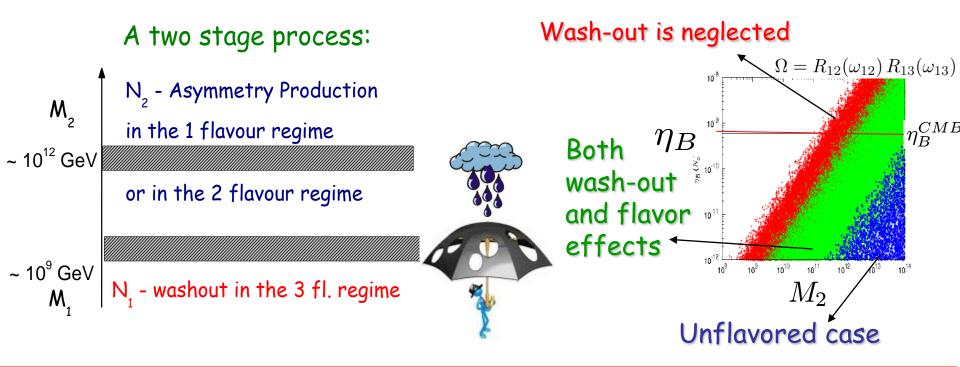
(Antusch, PDB, King, Jones '10; Blanchet, PDB, Jones, Marzola '11)

 Flavour coupling (Abada, Josse Michaux '07, Antusch, PDB, King, Jones '10)

# N<sub>2</sub>-flavored leptogenesis

(Vives '05; Blanchet, PDB '06; Blanchet, PDB '08)

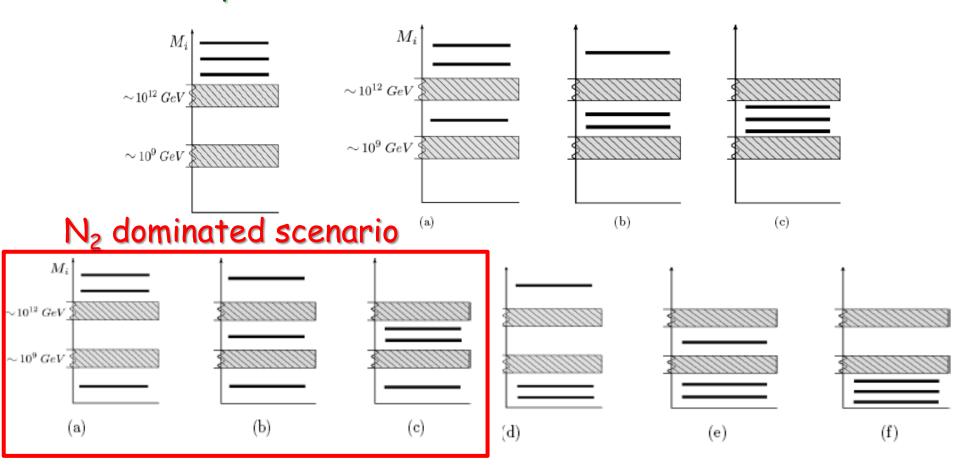
Combining together lepton and heavy neutrino flavour effects one has



$$N_{B-L}^{\rm f}(N_2) = P_{2e}^0 \,\varepsilon_2 \cdot (K_2) \, e^{-\frac{3\pi}{8} \, K_{1e}} + P_{2\mu}^0 \,\varepsilon_2 \cdot (K_2) \, e^{-\frac{3\pi}{8} \, K_{1\mu}} + P_{2\tau}^0 \,\varepsilon_2 \cdot (K_2) \, e^{-\frac{3\pi}{8} \, K_{1\tau}}$$

Notice that  $K_1 = K_{1e} + K_{1\mu} + K_{1\tau}$ 

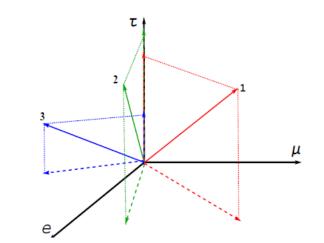
With flavor effects the domain of applicability goes much beyond the choice  $\Omega = R_{23}$ The existence of the heaviest RH neutrino N<sub>3</sub> is necessary for the  $\varepsilon_{2a}$  not to be negligible More generally one has to distinguish 10 different RH neutrino mass patterns (Bertuzzo,PDB,Marzola '10)



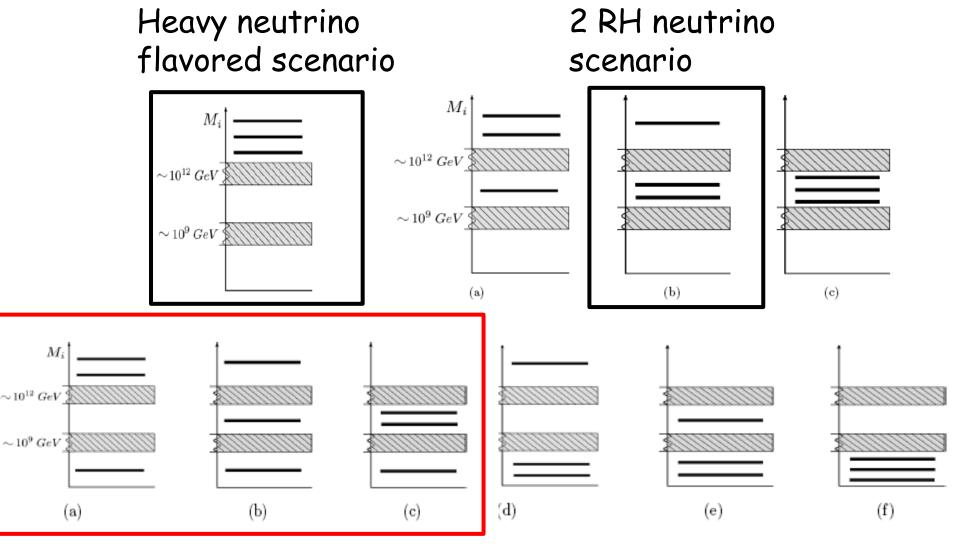
For each pattern a specific set of Boltzmann equations has to be considered !

# Density matrix formalism with heavy neutrino flavours

(Blanchet, PDB, Jones, Marzola '11) For a thorough description of all neutrino mass patterns including transition regions and all effects (flavour projection, phantom leptogenesis,...) one needs a description in Terms of a density matrix formalism The result is a "monster" equation:



$$\frac{dN_{\alpha\beta}^{B-L}}{dz} = \varepsilon_{\alpha\beta}^{(1)} D_1 \left( N_{N_1} - N_{N_1}^{eq} \right) - \frac{1}{2} W_1 \left\{ \mathcal{P}^{0(1)}, N^{B-L} \right\}_{\alpha\beta} \tag{80} 
+ \varepsilon_{\alpha\beta}^{(2)} D_2 \left( N_{N_2} - N_{N_2}^{eq} \right) - \frac{1}{2} W_2 \left\{ \mathcal{P}^{0(2)}, N^{B-L} \right\}_{\alpha\beta} 
+ \varepsilon_{\alpha\beta}^{(3)} D_3 \left( N_{N_3} - N_{N_3}^{eq} \right) - \frac{1}{2} W_3 \left\{ \mathcal{P}^{0(3)}, N^{B-L} \right\}_{\alpha\beta} 
+ i \operatorname{Re}(\Lambda_{\tau}) \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{\ell+\bar{\ell}} \right]_{\alpha\beta} - \operatorname{Im}(\Lambda_{\tau}) \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L} \right]_{\alpha\beta} 
+ i \operatorname{Re}(\Lambda_{\mu}) \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{\ell+\bar{\ell}} \right]_{\alpha\beta} - \operatorname{Im}(\Lambda_{\mu}) \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta} .$$

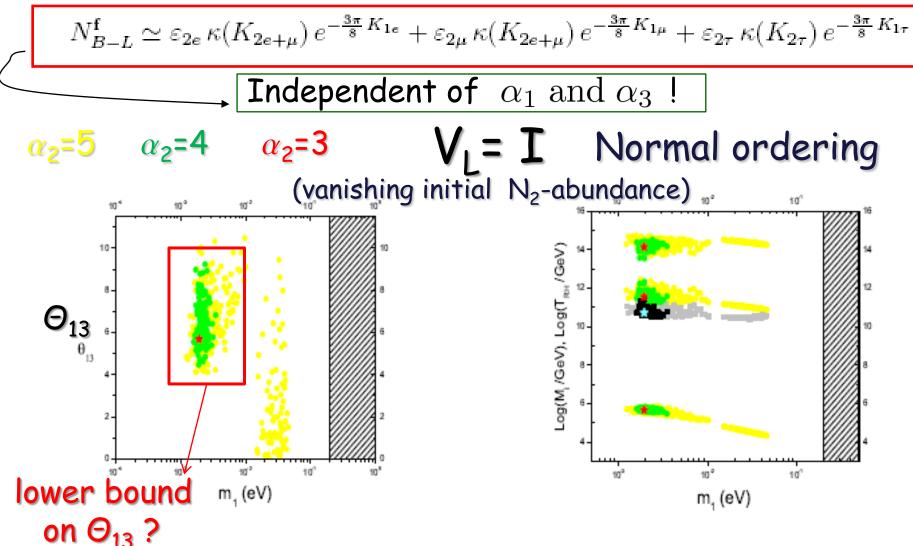


#### N<sub>2</sub>-dominated <u>Particularly attractive</u> scenario <u>For two reasons</u>

1) It is just that one realised in SO(10) inspired models! Can they be reconciled with leptogenesis?

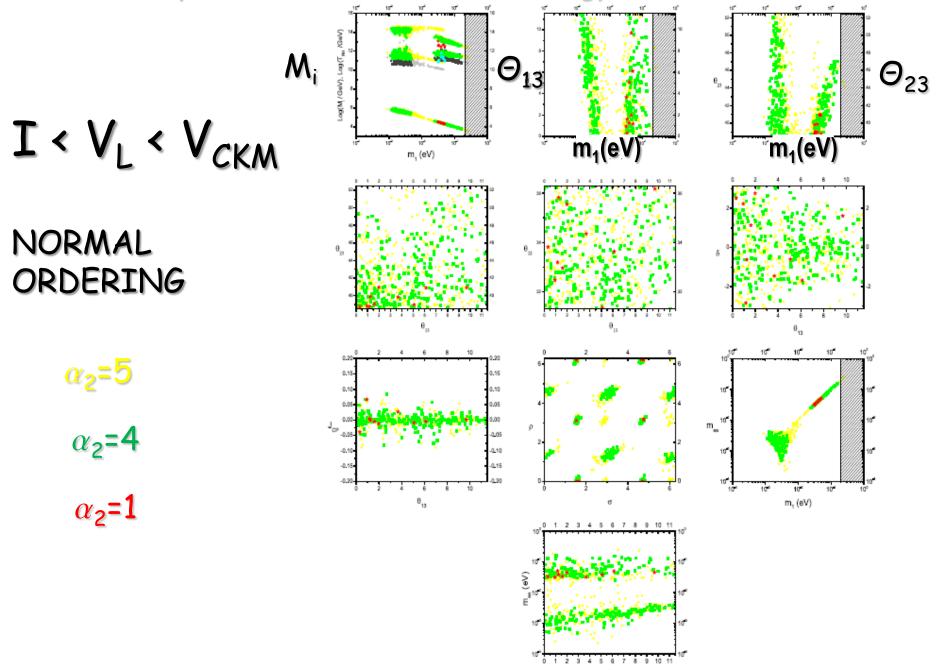
### The $N_2$ -dominated scenario rescues SO(10) inspired models

(PDB, Riotto '08)



Another way to rescue SO(10) inspired models is by considering a left-right symmetric seesaw (Abada, Hosteins, Josse-Michaux, Lavignac'08)

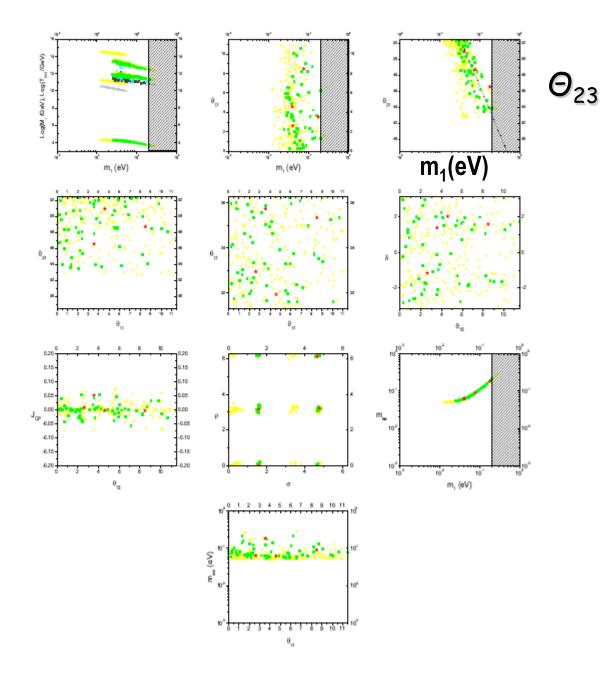
The model yields constraints on all low energy neutrino observables !



# $I < V_L < V_{CKM}$

### INVERTED ORDERING

α<sub>2</sub>=5 α<sub>2</sub>=4 α<sub>2</sub>=1.5

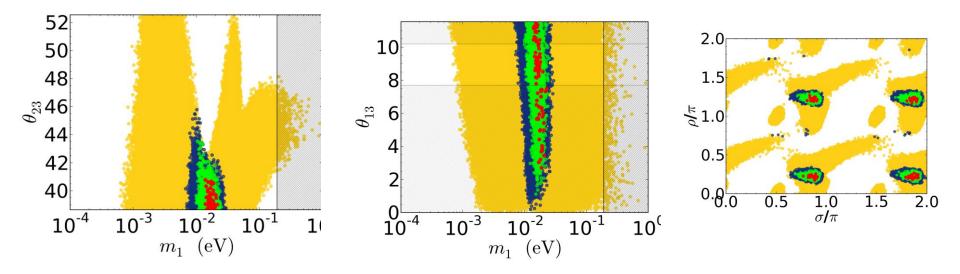


## An improved analysis

(PDB, Marzola '11-'12)

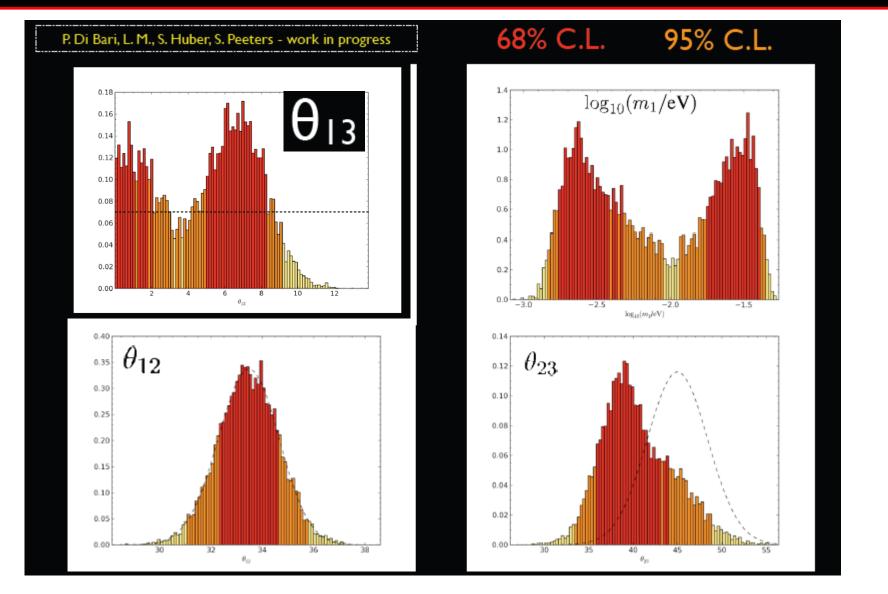
We optimised the procedure increasing of two orders of magnitudes the number of solutions (focus on yellow points for the time being):

 $\alpha_2=5$  NORMAL ORDERING  $I < V_L < V_{CKM}$ 



Why? Just to have sharper borders ? NO, two important reasons: i) statistical analysis ii) to obtain the blue green and red points

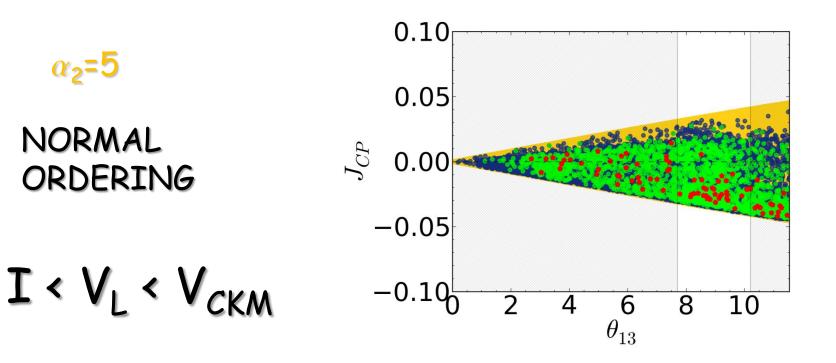
## A statistical analysis



Talk by Luca Marzola at the DESY theory workshop 28/9/11

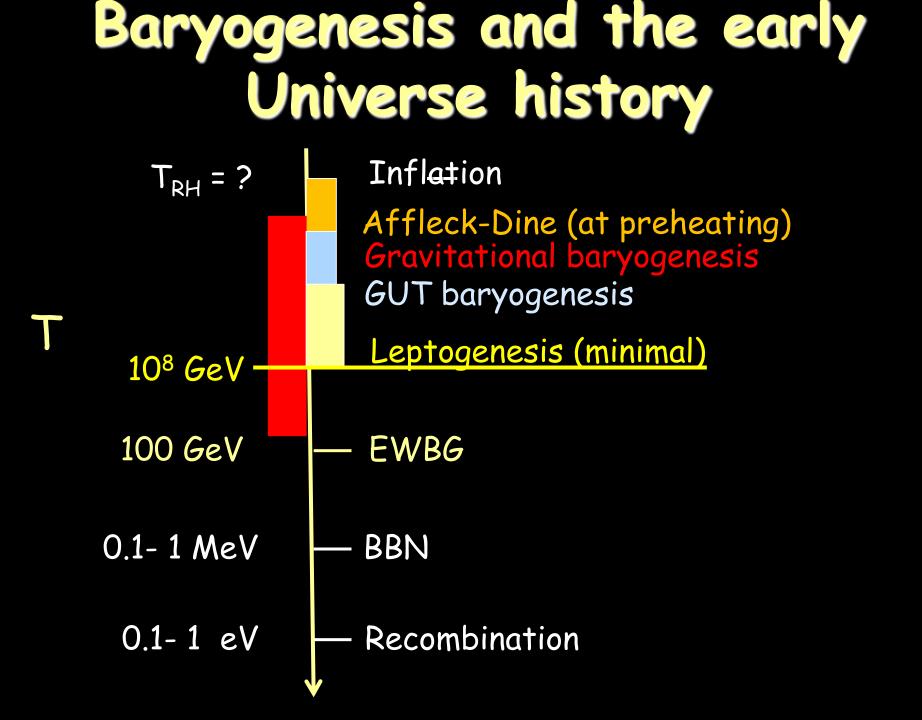
No link between the sign of the asymmetry and  $\mathbf{J}_{CP}$ 

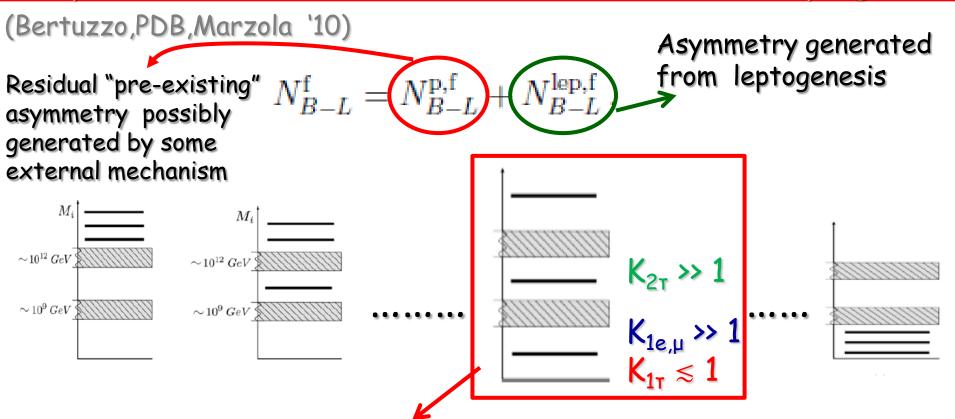
(PDB, Marzola '11-'12)



It is confirmed that there is no link between the matter-antimatter asymmetry and CP violation in neutrino mixing......for the yellow points

#### WHAT ARE THEN THE NON-YELLOW POINTS?





The conditions for the wash-out of a pre-existing asymmetry ('strong thermal leptogenesis') can be realised only within a  $N_2$ -dominated scenario where the final asymmetry is dominantly produced in the tauon flavour

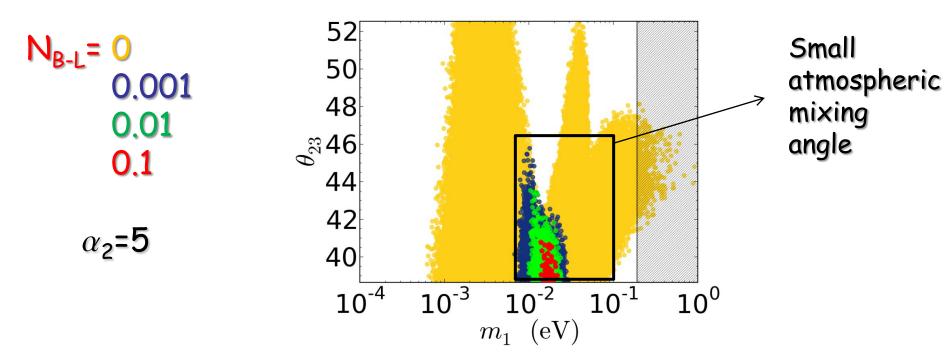
This mass pattern is just that one realized in the SO(10) inspired models: can they realise strong thermal leptogenesis?

## SO(10)-inspired+strong thermal leptogenesis

(PDB, Marzola '11-'12)  $N_{B-L}^{\rm f} = N_{B-L}^{\rm p,f} + N_{B-L}^{\rm lep,f}$ ,

Imposing both successful SO(10)-inspired leptogenesis  $\eta_{B} = \eta_{B}^{CMB} = (6.2 \pm 0.15) \times 10^{-10}$  and  $N_{B-L}^{P,f} \leftrightarrow N_{B-L}^{leP,f}$ 

## NO Solutions for Inverted Ordering ! But for Normal Ordering there is a subset with definite predictions UPPER BOUND ON THE ATMOSPHERIC MIXING ANGLE



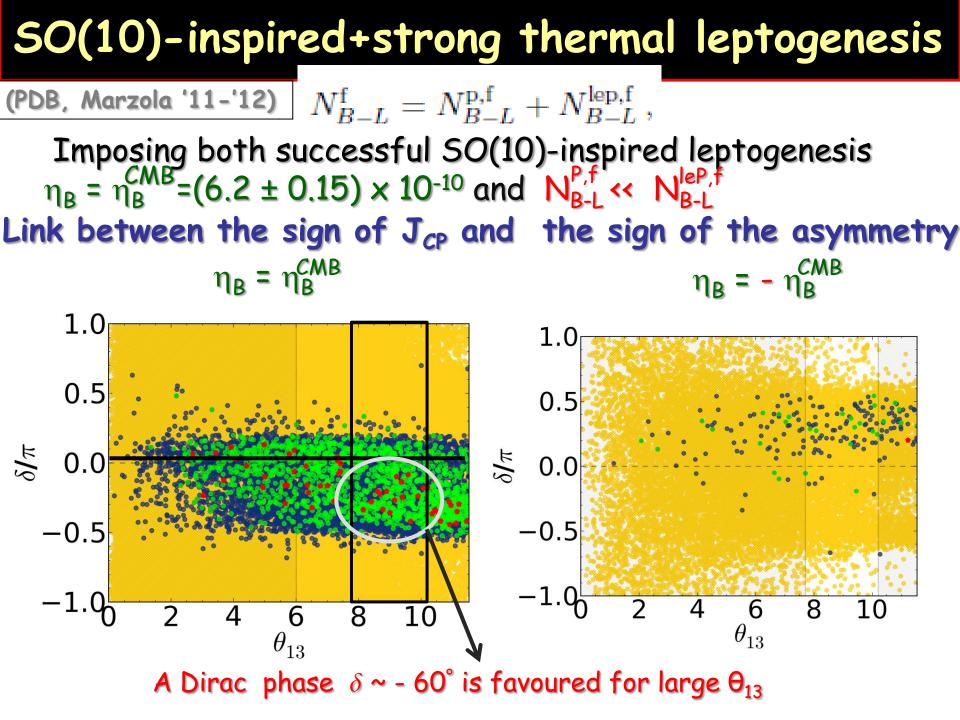
## SO(10)-inspired+strong thermal leptogenesis

(PDB, Marzola '11-'12)  $N_{B-L}^{\rm f} = N_{B-L}^{\rm p,f} + N_{B-L}^{\rm lep,f}$ ,

Imposing both successful SO(10)-inspired leptogenesis  $\eta_{B} = \eta_{B}^{CMB} = (6.2 \pm 0.15) \times 10^{-10}$  and  $N_{B-L}^{P,f} \leftrightarrow N_{B-L}^{leP,f}$ 

NON-VANISHING REACTOR MIXING ANGLE

10 N<sub>B-L</sub>= 0 0.001 8 non-0.01  $\theta_{13}$ 6 vanishing 0.1  $\Theta_{13}$ (green and 2 red points) 0 10<sup>-4</sup>  $10^{-3}$  $10^{-1}$  $\alpha_2=5$  $10^{-2}$  $10^{0}$  $m_1$  (eV)

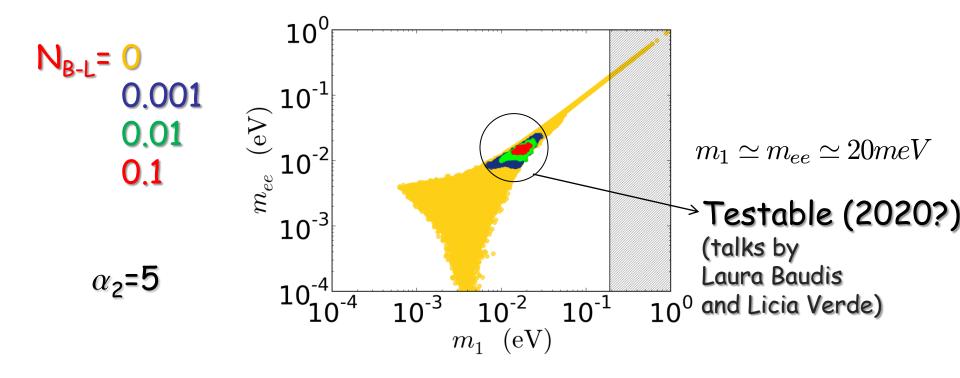


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Sharp prediction on the absolute neutrino mass scales

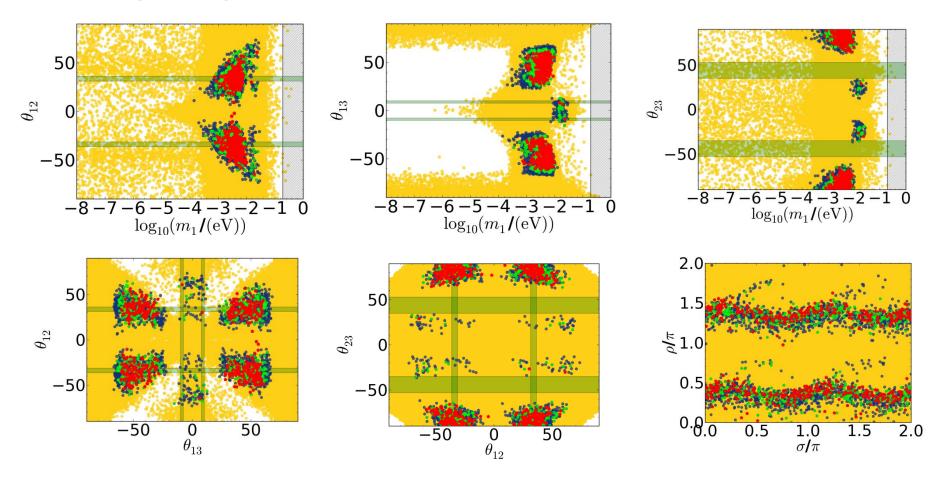


Strong thermal SO(10)-inspired leptogenesis:

#### Is it on the right track?

(PDB, Riotto '08; PDB, Marzola '12)

If we do not plug any experimental information (mixing angles left completely free, PRELIMINARY RESULTS):



#### Strong thermal SO(10) inspired leptogenesis: summary

 SO(10)-inspired leptogenesis is not only alive but it produces a set of solutions able to satisfy a very difficult condition when flavour effects are taken into account: *independence of the initial conditions (strong thermal leptogenesis)*

ORDERING	NORMAL		
θ <sub>13</sub>	$\gtrsim$ 3°		
θ <sub>23</sub>	$\lesssim$ 41°		
δ	~ −60°		
$m_1 \simeq m_{ee}$	~ 20 meV		

- It provides an example of how (minimal) leptogenesis within a reasonable set of assumptions can yield testable predictions
- Corrections: flavour coupling, RGE effects,...
- Statistical analysis

# Interplay between lepton and heavy neutrino flavour effects:

- N<sub>2</sub> flavoured leptogenesis
   (Vives '05; Blanchet, PDB '06; Blanchet, PDB '08)
- Flavour projection
   (Barbieri, Creminelli, Stumia, Tetradis '00; Engelhard, Grossman, Nardi, Nir '07)
- Phantom leptogenesis

(Antusch, PDB, King, Jones '10; Blanchet, PDB, Jones, Marzola '11)

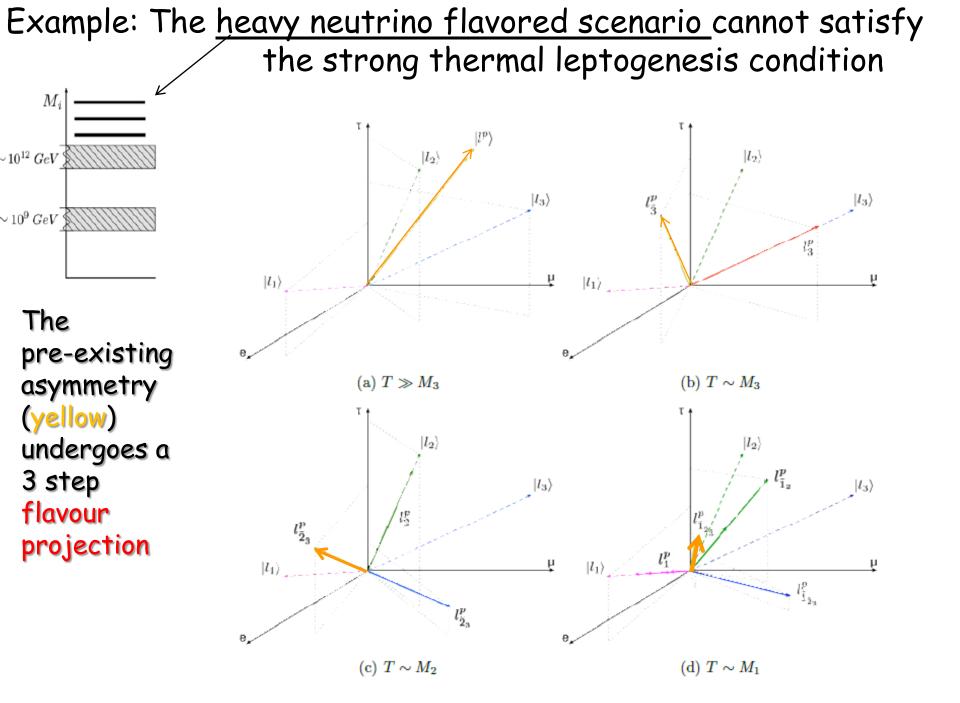
 Flavour coupling (Abada, Josse Michaux '07, Antusch, PDB, King, Jones '10)

# Flavour projection

(Engelhard, Nir, Nardi '08 , Bertuzzo, PDB, Marzola '10)

Assume  $M_{i+1} \gtrsim 3M_i$  (i=1,2)

The heavy neutrino flavour basis cannot be orthonormal otherwise the CP asymmetries would vanish: this complicates the calculation of the final asymmetry μ  $p_{ij} = |\langle \ell_i | \ell_j \rangle|^2 \qquad p_{ij} = \frac{\left| (m_D^{\dagger} m_D)_{ij} \right|^2}{(m_D^{\dagger} m_D)_{ii} (m_D^{\dagger} m_D)_{ii}}.$ 10c (1-P12)  $N_{B-L}^{(N_2)}(T \ll M_1) = N_{\Delta_1}^{(N_2)}(T \ll M_1) + N_{\Delta_{1\perp}}^{(N_2)}(T \ll M_1)$ Component from heavier RH neutrinos Contribution from heavier RH **parallel** to  $I_1$  and washed-out by  $N_1$ neutrinos orthogonal to  $I_1$  and escaping inverse decays N<sub>1</sub> wash-out  $N_{\Delta_1}^{(N_2)}(T \ll M_1) = p_{12} \left( e^{-\frac{3\pi}{8}K_1} N_{B-L}^{(N_2)}(T \sim M_2) \right)$ 



#### Phantom Leptogenesis

(Antusch, PDB, King, Jones '10)

Consider this situation

 $M_2$   $\sim 10^{12} \text{ GeV}$   $M_1$   $\sim 10^9 \text{ GeV}$  $N_1 - \text{washout in the 2 fl. regime}$ 

What happens to  $N_{B-L}$  at  $T\sim 10^{12}$  GeV? How does it split into a  $N_{\Delta\tau}$  component and into a  $N_{\Delta e^{+\mu}}$  component? One could think:

$$N_{\Delta \tau} = p_{2\tau} N_{B-L},$$

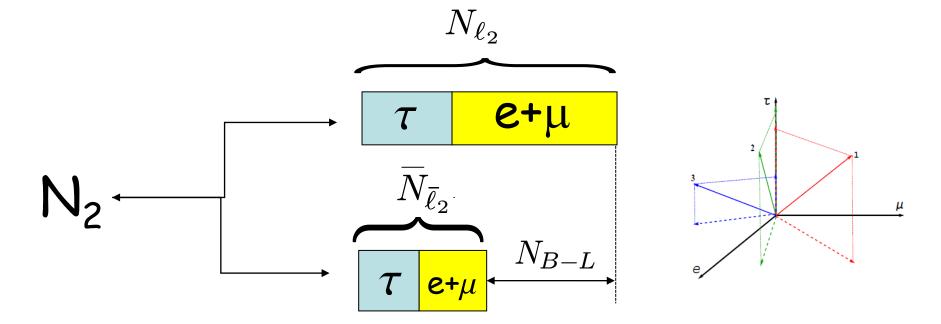
' Δe+μ - μ2 e+μ B-L

#### Phantom terms

However one has to consider that in the unflavoured case there are contributions to  $N_{\Delta\tau}$  and  $N_{\Delta e^+\mu}$  that are not just proportional to  $N_{B-L}$ 

Remember that: 
$$\varepsilon_{1\alpha} = P_{1\alpha}^0 \varepsilon_1 + \frac{\Delta P_{1\alpha}}{2}$$

Assume an initial thermal  $N_2$ -abundance at T~  $M_2$  >>  $10^{12}$  GeV



#### Phantom Leptogenesis

(Antusch, PDB, King, Jones '10)

Let us then consider a situation where K<sub>2</sub>>> 1 so that at the end of the N<sub>2</sub> washout the total asymmetry is negligible: 1) T ~ M<sub>2</sub> : unflavoured regime

$$\begin{array}{c|c} \tau & e^{+}\mu \\ \hline \overline{\tau} & e^{+}\mu \end{array}$$

$$\Rightarrow N_{B-L}^{T \sim M_2} \simeq 0 !$$

2)  $10^{12}$  GeV  $\gtrsim$  T >> M<sub>1</sub> :decoherence  $\implies$  2 flavoured regime

 $N_{B-L}^{T \sim M_2} = N_{\Delta \tau}^{T \sim M_2} + N_{\Delta_{e+\mu}}^{T \sim M_2} \simeq 0 !$ 3) T  $\simeq M_1$ : asymmetric washout from lightest RH neutrino Assume K<sub>1t</sub>  $\leq 1$  and K<sub>1e+µ</sub> >> 1  $N_{B-L}^{f} \simeq N_{\Delta_{\tau}}^{T \sim M_2} !$ The N<sub>1</sub> wash-out un-reveal the phantom term and effectively it

creates a N<sub>B-L</sub> asymmetry. Fully confirmed within a density matrix formalism (Blanchet, PDB, Marzola, Jones '11)

#### Remarks on phantom Leptogenesis

We assumed an initial  $N_2$  thermal abundance but if we were assuming An initial vanishing  $N_2$  abundance the phantom terms were just zero !

$$N_{\Delta_{\tau}}^{\text{phantom}} = \frac{\Delta p_{2\tau}}{2} N_{N_2}^{\text{in}}$$

The reason is that if one starts from a vanishing abundance during the  $N_2$  production one creates a contribution to the phantom term by inverse decays with opposite sign and exactly cancelling with what is created in the decays

In conclusion ....phantom leptogenesis introduces additional strong dependence on the initial conditions

NOTE: in strong thermal leptogenesis phantom terms are also washed out: full independence of the initial conditions!

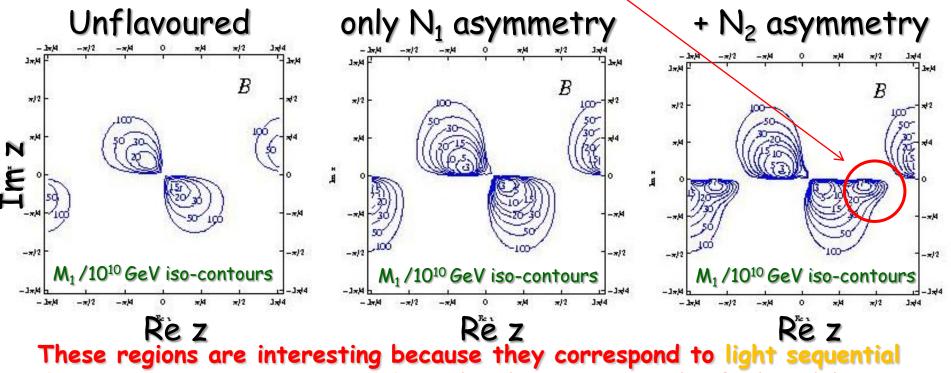
Phantom terms cannot contribute to the final asymmetry in N<sub>1</sub> leptogenesis but (canceling) flavoured asymmetries can be much bigger than the baryon asymmetry and have implications in active-sterile neutrino oscillations

# 2 RH neutrino scenario revisited

(King 2000:Frampton, Yanagida, Glashow '01, Ibarra, Ross 2003; Antusch, PDB, Jones, King '11) In the 2 RH neutrino scenario the N<sub>2</sub> production has been so far considered to be safely negligible because ε<sub>2a</sub> were supposed to be strongly suppressed and very strong N<sub>1</sub> wash-out. But taking into account:

- the  $N_2$  asymmetry  $N_1$ -orthogonal component
- an additional unsuppressed term to  $\epsilon_{2\alpha}$

New allowed  $N_2$  dominated regions appear



dominated neutrino mass models realized in some grandunified models

#### Conclusion

The interplay between heavy neutrino and charged lepton flavour effects introduces many new ingredients in the calculation of the final asymmetry and a density matrix formalism becomes much more useful in calculating correctly the final asymmetry

All this finds a nice applications for example - in a 2 RH neutrino model -In SO(10) inspired models

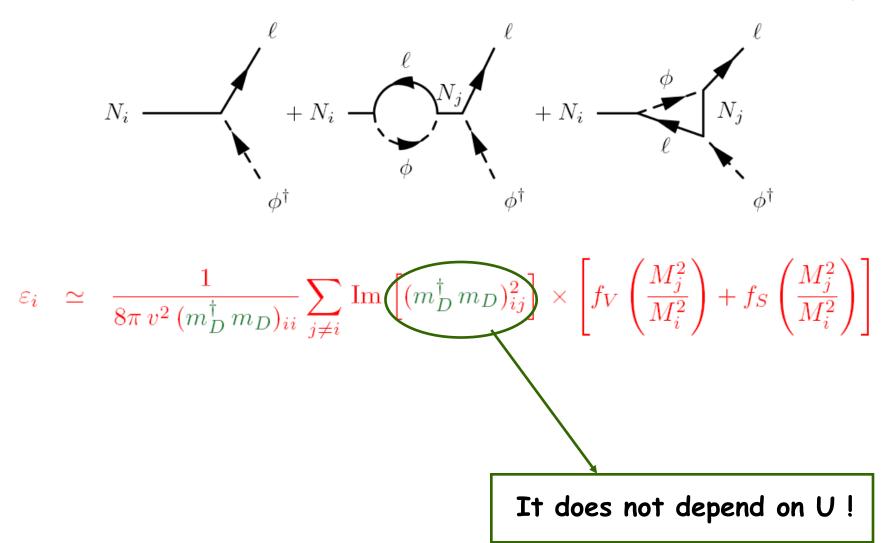
These are able to produce a scenario of leptogenesis with definite predictions on low energy neutrino parameters and with the next experimental developments all this could become extremely exciting

Strong thermal SO(10) inspired leptogenesis solution

ORDERING	NORMAL		
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θ <sub>23</sub>	$\lesssim$ 41°		
δ	~ -60°		
$\mathbf{m}_1 \simeq \mathbf{m}_{ee}$	~ 20 meV		

#### Total CP asymmetries

(Flanz, Paschos, Sarkar'95; Covi, Roulet, Vissani'96; Buchmüller, Plümacher'98)



# Flavour projection

(Engelhard, Nir, Nardi '08 , Bertuzzo, PDB, Marzola '10)

Assume  $M_{i+1} \gtrsim 3M_i$  (i=1,2)

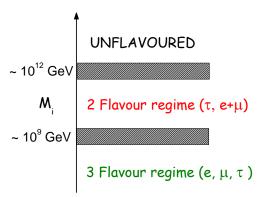
The heavy neutrino flavour basis cannot be orthonormal otherwise the CP asymmetries would vanish: this complicates the calculation of the final asymmetry μ  $p_{ij} = |\langle \ell_i | \ell_j \rangle|^2 \qquad p_{ij} = \frac{\left| (m_D^{\dagger} m_D)_{ij} \right|^2}{(m_D^{\dagger} m_D)_{ii} (m_D^{\dagger} m_D)_{ii}}.$ 10c (1-P12)  $N_{B-L}^{(N_2)}(T \ll M_1) = N_{\Delta_1}^{(N_2)}(T \ll M_1) + N_{\Delta_1 \perp}^{(N_2)}(T \ll M_1)$ Component from heavier RH neutrinos Contribution from heavier RH **parallel** to  $I_1$  and washed-out by  $N_1$ neutrinos orthogonal to  $I_1$  and escaping inverse decays N<sub>1</sub> wash-out  $N_{\Delta_1}^{(N_2)}(T \ll M_1) = p_{12} \left( e^{-\frac{3\pi}{8}K_1} \right) N_{B-L}^{(N_2)}(T \sim M_2)$ 

# Limitations of Boltzmann equations

All results have been obtained within Boltzmann kinetic formalism assuming that leptons are either pure states or a full incoherent admixture of lepton flavour eigenstates (mixed states)

Limitations:

• Asymmetry cannot be calculated when masses fall in transition regions

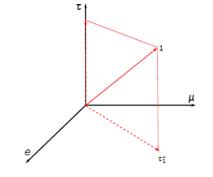


- Even in the fully flavoured regimes, the simultaneous occurrence of many effects makes the calculation quite contrived and one should worry whether everything is consistently taken into account
- More insight is certainly needed!

Within a density matrix formalism it is possible to describe consistently a system that is a statistical ensemble of several elementary quantum states that are either pure states or mixed states.

Consider our leptons  $\ell_1$  produced by the decays of N<sub>1</sub>

$$\begin{aligned} |1\rangle &= \mathcal{C}_{1\tau} |\tau\rangle + \mathcal{C}_{1\tau_1^{\perp}} |\tau_1^{\perp}\rangle, \quad \mathcal{C}_{1\alpha} \equiv \langle \alpha | 1 \rangle \\ (\alpha = \tau, \tau_1^{\perp}) \end{aligned}$$



Density  $\hat{
ho}^{\ell_1}\equiv|1
angle\langle 1|=\sum_{lpha,eta}\,
ho_{lphaeta}|lpha
angle\langleeta|$  operator

For a pure state  $\hat{
ho}^2 = \hat{
ho}$  Moreover since  $ho = 
ho^{\dagger}$  there is always a basis where is diagonal, in this case the basis is simply  $|1\rangle$ ,  $|1^{\perp}\rangle$ 

$$\rho_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (i, j = 1, 1^{\perp})$$

When the  $\ell_1$  start to interact with the thermal bath, there will be

the early Universe starts to be populated both with pure states  $~|1\rangle$  and with mixed states  $~|\tau\rangle, |\tau_1^{\perp}\rangle$ 

I can still find a basis  $|A
angle, |B
angle\,$  where the density matrix is diagonal:

 $\rho_{AB} = \text{diag}(p_A, p_B), \text{ where } p_A + p_B = 1 \text{ but now } \rho \neq \rho^2$ 

- When all states are pure simply  $|A
  angle=|1
  angle,|B
  angle=|1^{\perp}
  angle$
- When all states are mixed  $|A
  angle=| au
  angle, |B
  angle=| au_1^{\perp}
  angle$  but this time

$$\rho_{\tau\tau_1^{\perp}} = \operatorname{diag}(p_{1\tau}, 1 - p_{1\tau})$$

We can also introduce the lepton number density matrix simply as

$$N_{ij}^{\ell} = N_{\ell_1} \,\rho_{ij}^{\ell}$$

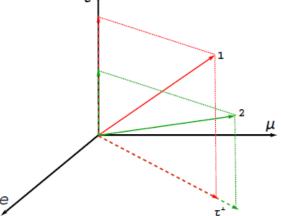
In the charged lepton flavour basis  $|\tau\rangle$ ,  $|\tau_1^{\perp}\rangle$  one has a transition from a matrix with off-diaagonal elements to a diagonal matrix. This evolution can be described with kinetic equations introducing decoherence due to the scatterings with the thermal bath

The result (subtracting density matrix for leptons and anti-leptons) for the B-L asymmetry matrix is

$$\frac{dN_{\alpha\beta}^{B-L}}{dz} = \varepsilon_{\alpha\beta}^{(1)} D_1 \left( N_{N_1} - N_{N_1}^{eq} \right) - \frac{1}{2} W_1 \left\{ \mathcal{P}^{0(1)}, N^{B-L} \right\}_{\alpha\beta}$$

$$+ i \frac{\operatorname{Re}(\Lambda_{\tau})}{H z} \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, N^{\ell+\bar{\ell}} \right]_{\alpha\beta} - \frac{\operatorname{Im}(\Lambda_{\tau})}{H z} \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta}$$
(45)

When more than 1 heavy neutrino flavour is included but still one has only 2 lepton flavours  $|\tau\rangle, |\tau_1^{\perp}\rangle$ The equation includes 2 source terms  $\tau$ 

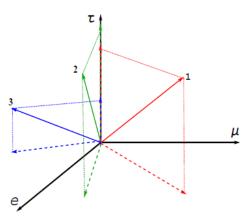


$$\frac{dN_{\alpha\beta}^{B-L}}{dz} = \varepsilon_{\alpha\beta}^{(1)} D_1 \left( N_{N_1} - N_{N_1}^{eq} \right) - \frac{1}{2} W_1 \left\{ \mathcal{P}^{0(1)}, N^{B-L} \right\}_{\alpha\beta}$$

$$+ \varepsilon_{\alpha\beta}^{(2)} D_2 \left( N_{N_2} - N_{N_2}^{eq} \right) - \frac{1}{2} W_2 \left\{ \mathcal{P}^{0(2)}, N^{B-L} \right\}_{\alpha\beta}$$

$$+ i \operatorname{Re}(\Lambda_{\tau}) \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, N^{\ell+\ell} \right]_{\alpha\beta} - \operatorname{Im}(\Lambda_{\tau}) \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, N^{B-L} \right]_{\alpha\beta} \right]_{\alpha\beta}$$
(50)

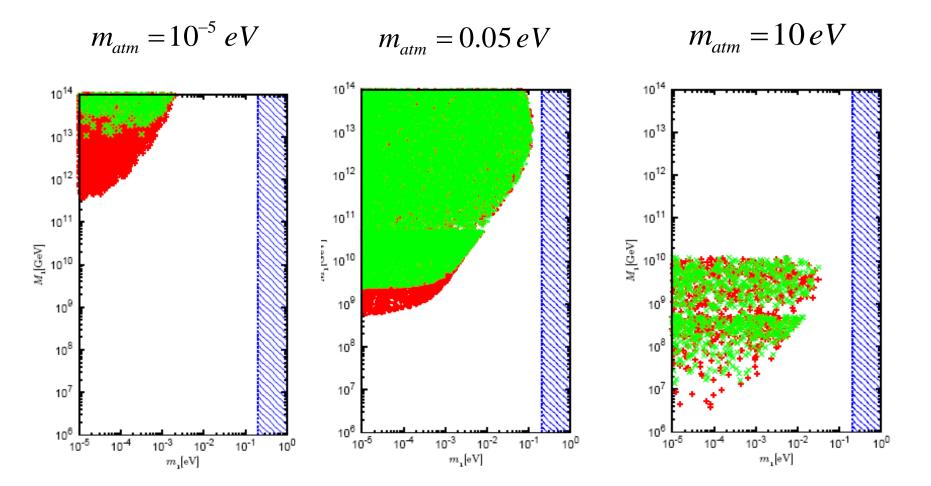
When the whole 3 flavour structure is taken into account



The result is a monster equation:

$$\frac{dN_{\alpha\beta}^{B-L}}{dz} = \varepsilon_{\alpha\beta}^{(1)} D_1 \left( N_{N_1} - N_{N_1}^{eq} \right) - \frac{1}{2} W_1 \left\{ \mathcal{P}^{0(1)}, N^{B-L} \right\}_{\alpha\beta} \tag{80} 
+ \varepsilon_{\alpha\beta}^{(2)} D_2 \left( N_{N_2} - N_{N_2}^{eq} \right) - \frac{1}{2} W_2 \left\{ \mathcal{P}^{0(2)}, N^{B-L} \right\}_{\alpha\beta} 
+ \varepsilon_{\alpha\beta}^{(3)} D_3 \left( N_{N_3} - N_{N_3}^{eq} \right) - \frac{1}{2} W_3 \left\{ \mathcal{P}^{0(3)}, N^{B-L} \right\}_{\alpha\beta} 
+ i \operatorname{Re}(\Lambda_{\tau}) \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{\ell+\bar{\ell}} \right]_{\alpha\beta} - \operatorname{Im}(\Lambda_{\tau}) \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta} 
+ i \operatorname{Re}(\Lambda_{\mu}) \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{\ell+\bar{\ell}} \right]_{\alpha\beta} - \operatorname{Im}(\Lambda_{\mu}) \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L} \right] \right]_{\alpha\beta}.$$

#### A first encouraging coincidence



#### Green points: Unflavored

Red points: Flavored

# SO(10)-inspired leptogenesis

(Branco et al. '02; Nezri, Orloff '02; Akhmedov, Frigerio, Smirnov '03)

Expressing the neutrino Dirac mass matrix  $m_D$  (in the basis where the Majorana mass and charged lepton mass matrices are diagonal):

$$m_D = V_L^{\dagger} D_{m_D} U_R \quad \text{(bi-unitary parametrization)} *$$
where  $D_{m_D} = \text{diag}\{\lambda_{D1}, \lambda_{D2}, \lambda_{D3}\}$ 
and
assuming: 1)  $\lambda_{D1} = \alpha_1 m_u, \lambda_{D2} = \alpha_2 m_c, \lambda_{D3} = \alpha_3 m_t, \quad (\alpha_i = \mathcal{O}(1))$ 
2)  $V_L \simeq V_{CKM} \simeq I$ 

one typically obtains (barring fine-tuned exceptions):

 $M_1 \sim \alpha_1^2 \, 10^5 {
m GeV} \,, \ M_2 \sim \alpha_2^2 \, 10^{10} \, {
m GeV} \,, \ M_3 \sim \alpha_3^2 \, 10^{15} \, {
m GeV}$ 

since  $M_1 \leftrightarrow 10^9 \text{ GeV} \Rightarrow \eta_B(N_1) \leftrightarrow \eta_B^{CMB}$ !

 $\Rightarrow$  failure of the N<sub>1</sub>-dominated scenario !

\* Note that:  $\Omega = D_m^{-\frac{1}{2}} U^{\dagger} V_L^{\dagger} D_{m_D} U_R D_M^{-\frac{1}{2}}$ 

# Heavy neutrino flavored scenario

(Engelhard, Nir, Nardi '08 , Bertuzzo, PDB, Marzola '10)

Assume  $M_{i+1} \gtrsim 3M_i$  (i=1,2)

The heavy neutrino flavour basis is not orthogonal in general and this complicates the calculation of the final asymmetry

$$p_{ij} = |\langle \ell_i | \ell_j \rangle|^2 \qquad p_{ij} = \frac{\left| (m_D^{\dagger} m_D)_{ij} \right|^2}{(m_D^{\dagger} m_D)_{ii} (m_D^{\dagger} m_D)_{jj}}.$$

$$N_{B-L}^{\text{lep}}(T_{B1}) = N_{\Delta_1}^{\text{lep}}(T_{B1}) + N_{\Delta_2}^{\text{lep}}(T_{B1}),$$

$$N_{\Delta_{1}}^{\text{lep}}(T_{B1}) = p_{21} p_{32} \varepsilon_{3} \kappa(K_{3}) e^{-\frac{3\pi}{8}(K_{1}+K_{2})} + p_{21} \varepsilon_{2} \kappa(K_{2}) e^{-\frac{3\pi}{8}K_{1}} + p_{\tilde{2}_{31}} (1-p_{32}) \varepsilon_{3} \kappa(K_{3}) e^{-\frac{3\pi}{8}K_{1}} + \varepsilon_{1} \kappa(K_{1})$$

Contribution from heavier RH neutrinos orthogonal to  ${\sf I}_1$  and escaping  ${\sf N}_1$  wash-out

Notice that some deviation from orthogonality is typically necessary since otherwise (e.g. with tri-bimaximal mixing) one would have vanishing CP asymmetries and therefore no asymmetry produced from leptogenesis (Antusch, King, Riotto '08; Aristizabal, Bazzocchi, Merlo, Morisi '09)

#### A recent global analysis

#### Global analysis of neutrino masses, mixings and phases: entering the era of leptonic CP violation searches

G.L. Fogli,<sup>1,2</sup> E. Lisi,<sup>2</sup> A. Marrone,<sup>1,2</sup> D. Montanino,<sup>3,4</sup> A. Palazzo,<sup>5</sup> and A.M. Rotunno<sup>1</sup>

arXiv:1205.5254v2 [hep-ph] 25 May 2012

TABLE I: Results of the global  $3\nu$  oscillation analysis, in terms of best-fit values and allowed 1, 2 and  $3\sigma$  ranges for the  $3\nu$  mass-mixing parameters. We remind that  $\Delta m^2$  is defined herein as  $m_3^2 - (m_1^2 + m_2^2)/2$ , with  $+\Delta m^2$  for NH and  $-\Delta m^2$  for IH.

Parameter	Best fit	$1\sigma$ range	$2\sigma$ range	$3\sigma$ range
$\delta m^2/10^{-5} \text{ eV}^2$ (NH or IH)	7.54	7.32 - 7.80	7.15 - 8.00	6.99 - 8.18
$\sin^2 \theta_{12} / 10^{-1}$ (NH or IH)	3.07	2.91 - 3.25	2.75 - 3.42	2.59 - 3.59
$\Delta m^2 / 10^{-3} \text{ eV}^2 \text{ (NH)}$	2.43	2.34 - 2.50	2.26 - 2.58	2.15 - 2.66
$\Delta m^2/10^{-3} \text{ eV}^2$ (IH)	2.42	2.32 - 2.49	2.25 - 2.56	2.14 - 2.65
$\sin^2 \theta_{13} / 10^{-2}$ (NH)	2.45	2.14 - 2.79	1.81 - 3.11	1.49 - 3.44
$\sin^2 \theta_{13} / 10^{-2}$ (IH)	2.46	2.15 - 2.80	1.83 - 3.13	1.50 - 3.47
$\sin^2 \theta_{23}/10^{-1}$ (NH)	3.98	3.72 - 4.28	3.50 - 4.75	3.30 - 6.38
$\sin^2 \theta_{23} / 10^{-1}$ (IH)	4.08	3.78 - 4.43	3.55 - 6.27	3.35 - 6.58
$\delta/\pi$ (NH)	0.89	0.45 - 1.18		
$\delta/\pi$ (IH)	0.90	0.47 - 1.22	—	_

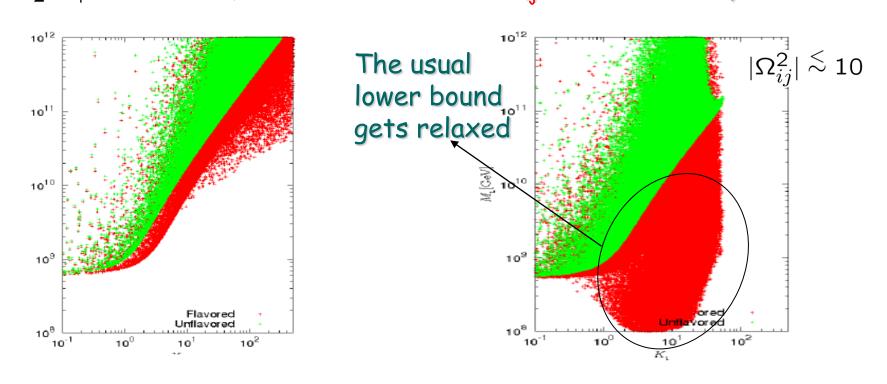
#### 2) The lower bounds on M<sub>1</sub> and on T<sub>reh</sub> get relaxed: (Blanchet, PDB '08)

$$\frac{\Delta P_{i\alpha}}{2} \simeq \frac{1}{8 \pi (h^{\dagger} h)_{ii}} \sum_{j \neq i} \left\{ \operatorname{Im} \left[ h_{\alpha i}^{\star} h_{\alpha j} \left( \frac{3}{2 \sqrt{x_j}} (h^{\dagger} h)_{ij} \right) + \left( \begin{array}{c} \\ \end{array} \right) \right\} \quad \left[ x_j = \frac{M_j^2}{M_1^2} \right] \right\}$$

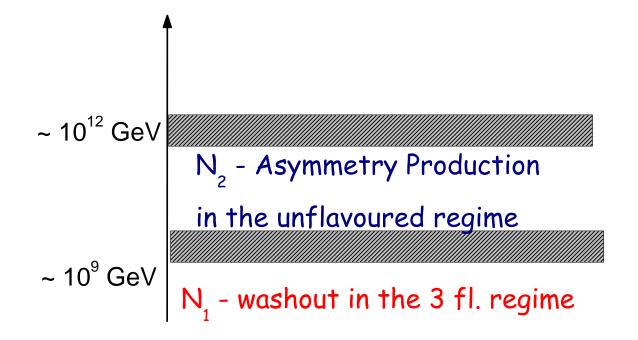
It dominates for  $|\Omega_{ij}| \leq 1$  but is upper bounded because of  $\Omega$  orthogonality:

 $\left|\frac{\Delta P_{1\alpha}}{2}\right| < \overline{\varepsilon}(M_1) \sqrt{P_{1\alpha}^0}$ 

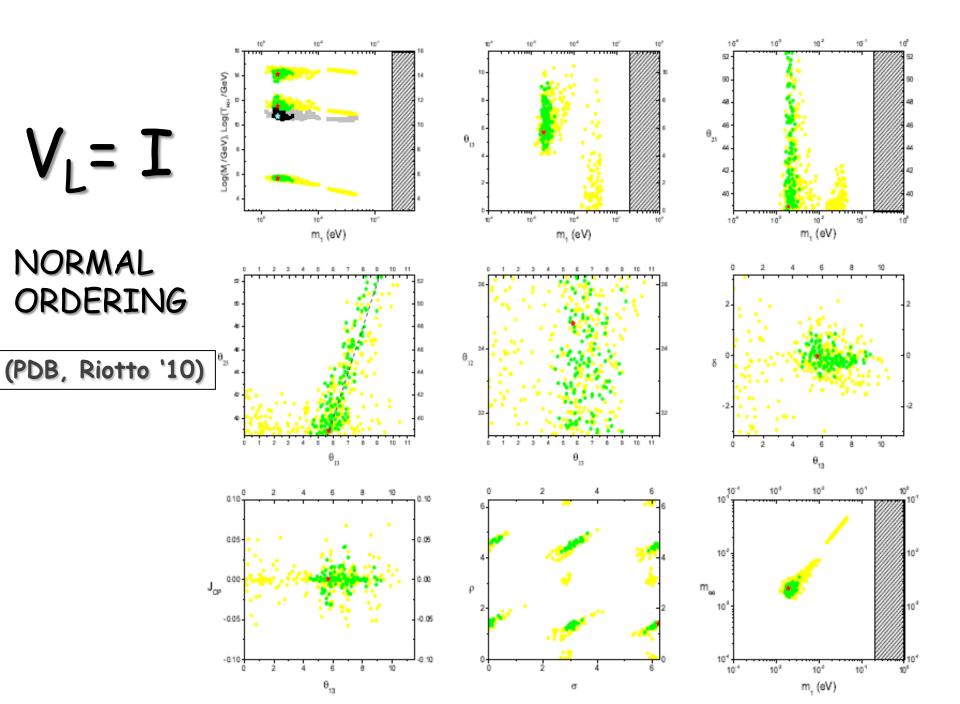
It is usually neglected but since it is not upper bounded by orthogonality, for  $|\Omega_{ii}| \ge 1$  it can be important



Analogous results hold in the case when the production occurs in the 2 flavour regime for  $10^{12}~GeV \gtrsim M_2 \gtrsim 10^9~GeV$ :



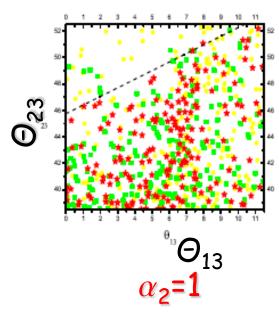
$$N_{B-L}^{\rm f} \simeq \varepsilon_{2e} \,\kappa(K_{2e+\mu}) \, e^{-\frac{3\pi}{8}K_{1e}} + \varepsilon_{2\mu} \,\kappa(K_{2e+\mu}) \, e^{-\frac{3\pi}{8}K_{1\mu}} + \varepsilon_{2\tau} \,\kappa(K_{2\tau}) \, e^{-\frac{3\pi}{8}K_{1\tau}} \,.$$



 $I < V_L < V_{CKM}$  NORMAL ORDERING  $\alpha_2=5 \alpha_2=4$ 

10

*α*<sub>2</sub>=3.7

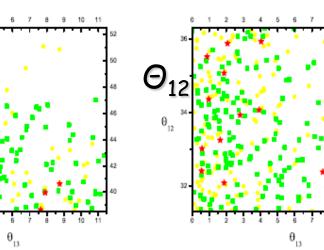


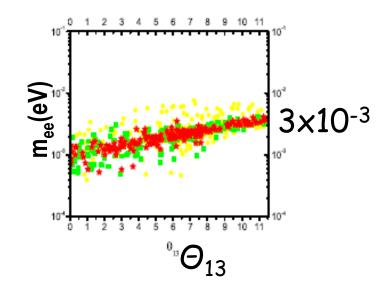
 $\Theta_{23}$ 

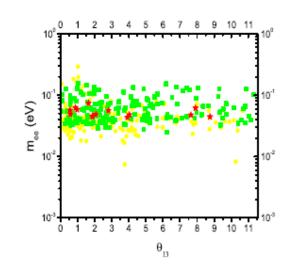
θ\_23

**O**12  $\Theta_{13}$  $m_1 > 0.01 eV$ 

 $m_1 < 0.01 eV$ 







#### Are the data pointing in the right direction?

(PDB, Riotto '10)

Blue points:  $\alpha_2$ =4 and mixing angles let free in (0,180°) Green points:  $\alpha_2$ =4 and current experimental constraints imposed on mixing angles

