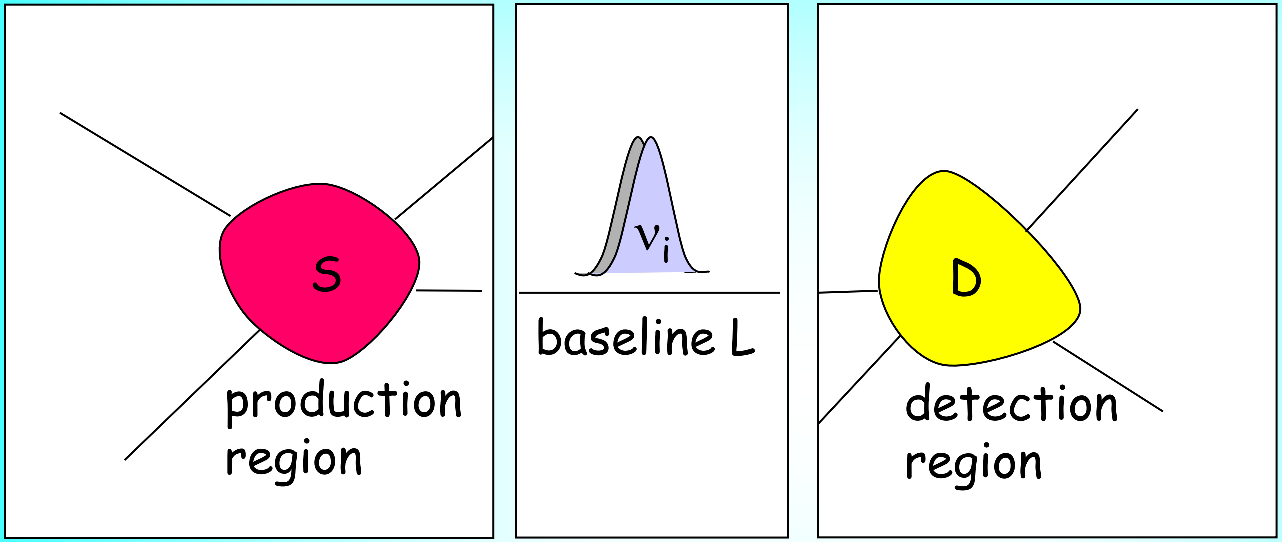
Physics of neutrino oscillations & flavor conversion

A. Yu. Smirnov

International Centre for Theoretical Physics, Trieste, Italy

Invisibles network INT Training lectures
June 25 - 29, 2012

Factorization



If oscillation effect in production/detection regions can be neglected ⇒ factorization

r_D , r_S << l_ν

Production propagation and detection can be considered as three independent processes

Oscillation probability

Amplitude of (survival) probability

$$A(\nu_e) = \langle \nu_e | \nu(x,t) \rangle = \cos^2\theta\, g_1(x - v_1 t) + \sin^2\theta\, g_2(x - v_2 t)\, e^{i\phi}$$

!

Probability in the moment of time t

interference

$$P(\nu_e) = \int_{-\infty}^{+\infty} dx\, |\langle \nu_e | \nu(x,t) \rangle|^2 =$$

$$= \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta \cos\phi \int dx\, g_1(x - v_1 t)\, g_2(x - v_2 t)$$

If $\int dx |g_k|^2 = 1$

!

If $g_1 = g_2$

$$P(\nu_e) = 1 - 2\sin^2\theta\cos^2\theta\,(1 - \cos\phi) = 1 - \sin^2 2\theta \sin^2 \tfrac{1}{2}\phi$$

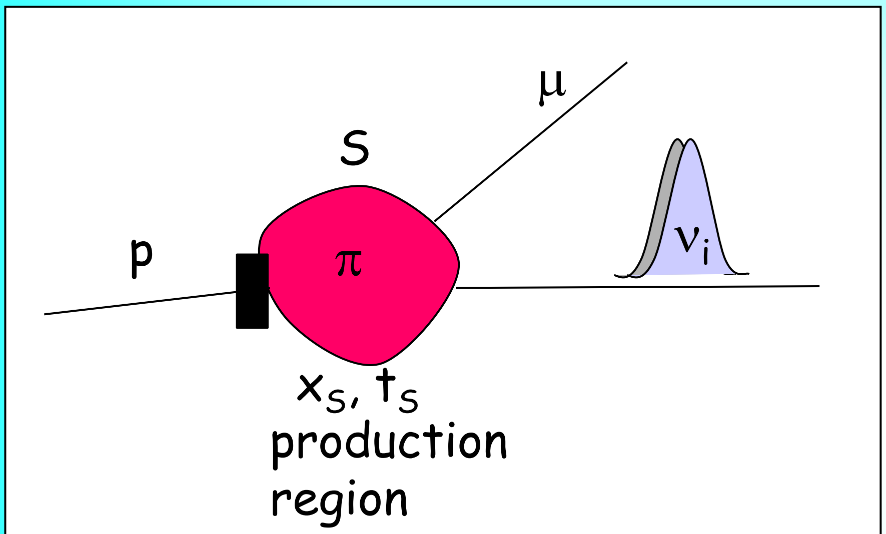
$$\phi = \frac{\Delta m^2 x}{2E} = \frac{2\pi x}{l_\nu}$$

depth of oscillations

$$l_\nu = \frac{4\pi E}{\Delta m^2}$$

Oscillation length

Formation of the wave packet



In most of the cases precise form of the shape factor and therefore details of its formation are not important

It is important in the cases of

of partial separation of wave packets

production region is comparable with oscillation length

$r_S \sim l_\nu$

Solving the wave problem: pion moves and emits neutrino waves

Integration of the neutrino waves emitted from space-time points where pion lives

Formation of the wave packet

Pion decay: !

E. Akhmedov, D. Hernandez, A.S.

$$g_i(x,t)\, e^{i\phi_i} = \int dp \int dx_S\, dt_S\, M\, \psi_\pi(x_S, t_S)\, \overline{\psi_\mu}(x_S, t_S)\, \exp[ip(x - x_S) - iE_i(t - t_S)]$$

integration over production region

part of matrix element

plane wave for neuutrino

Pion wave function:

$$\psi_\pi(x_S, t_S) = \exp[-\tfrac{1}{2}\Gamma t_S]\, g_\pi(x_S, t_S)\, \exp[-i\phi_\pi(x_S, t_S)]$$

usually: $g_\pi(x_S, t_S) \sim \delta(x_S - v_\pi t_S)$

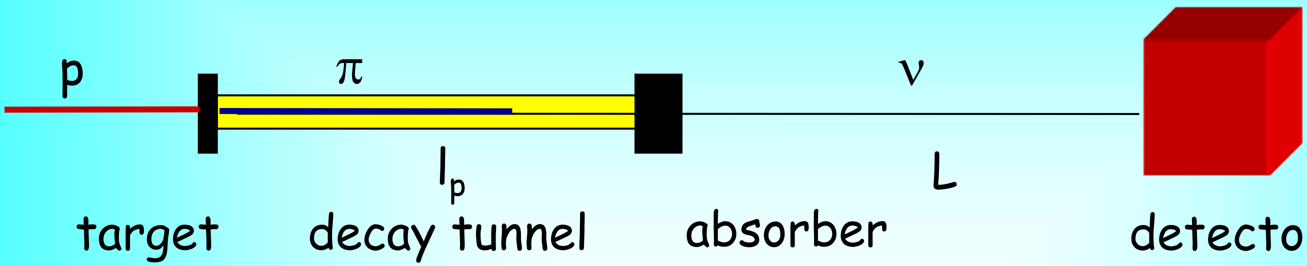
Muon wave function:

$$\psi_\mu(x_S, t_S) = g_\mu(x' - x_S, t' - t_S)\, \exp[i\phi_\mu(x' - x_S, t' - t_S)]$$

determined by detection of muon

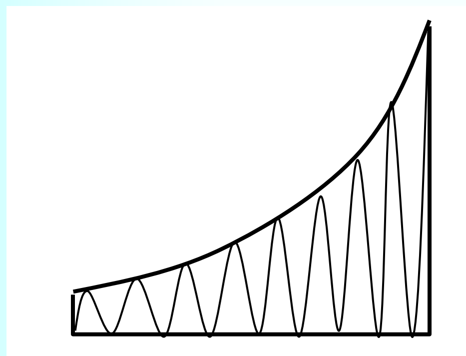
If muon is not detected: plane wave → phase factor → disappears from probability

Neutrino wave packets



D. Hernandez, AS
E. Kh Akhmedov,
D. Hernandez, AS
arXiv:1110.5453

ν wave packet



Doppler effect

The length of the ν wave packet emitted in the forward direction

$$\sigma = l_p \frac{v - v_\pi}{v_\pi}$$

-shorten

Shape factor

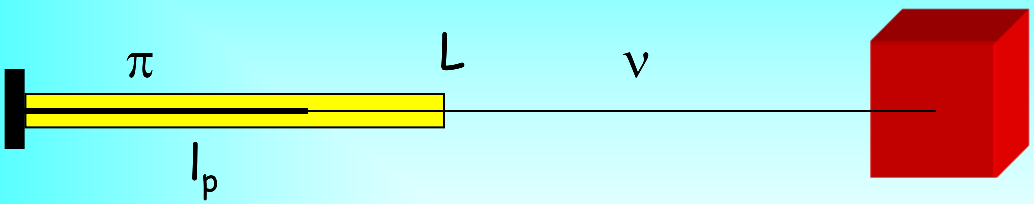
$$g = g_0 \exp\left(\frac{\Gamma}{2(v - v_\pi)}(x - \sigma)\right) \Pi(x, [0, \sigma])$$

box function

frequency increases

Decoherence at production

D. Hernandez, AS



$\Delta E_{ij} \sim \Gamma$

$$P = \bar{P} + \frac{\sin^2 2\theta}{2(1 + \xi^2)} \frac{1}{1 - e^{-\Gamma l_p}} [\cos \phi_L + K]$$

$\xi = \Delta m^2/2E\Gamma$

decoherence parameter

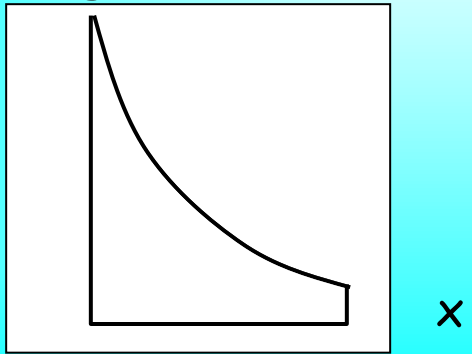
$$K = \xi \sin \phi_L - e^{-\Gamma l_p}[\cos(\phi_L - \phi_p) - \xi \sin(\phi_L - \phi_p)]$$

$$\phi_L = \Delta m^2 L/2E \qquad \phi_p = \Delta m^2 l_p/2E$$

MINOS: $\xi \sim 1$

β–beam ?

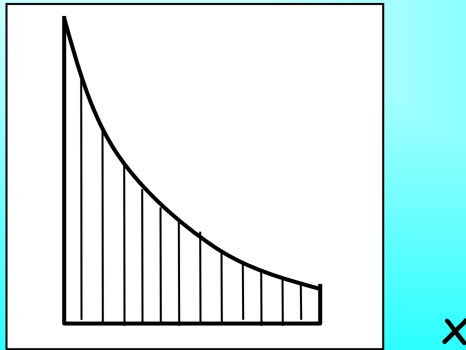
Coherent ν-emission
- long WP



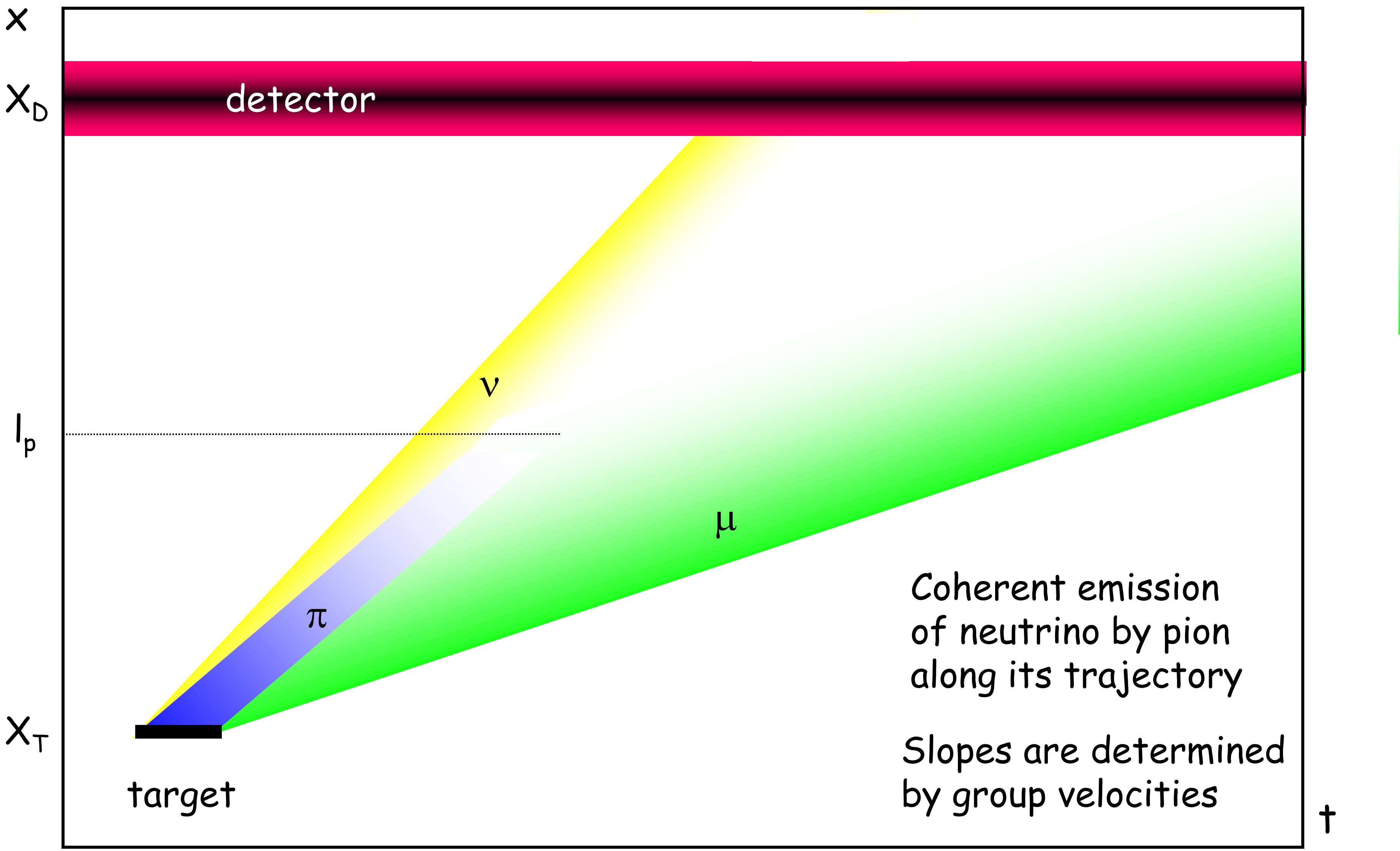
Equivalence

for point-like pion

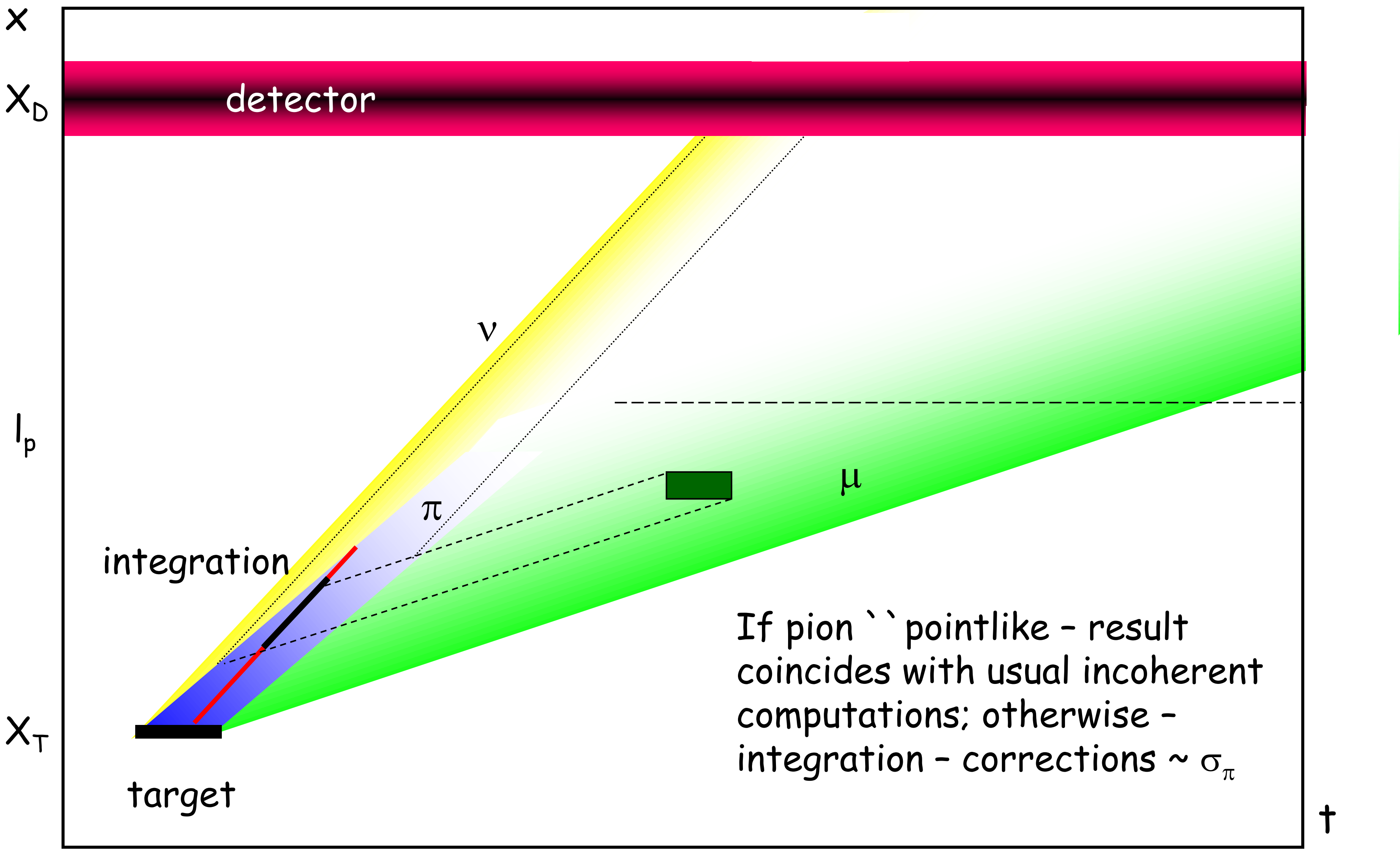
Incoherent ν-emission
- short WP



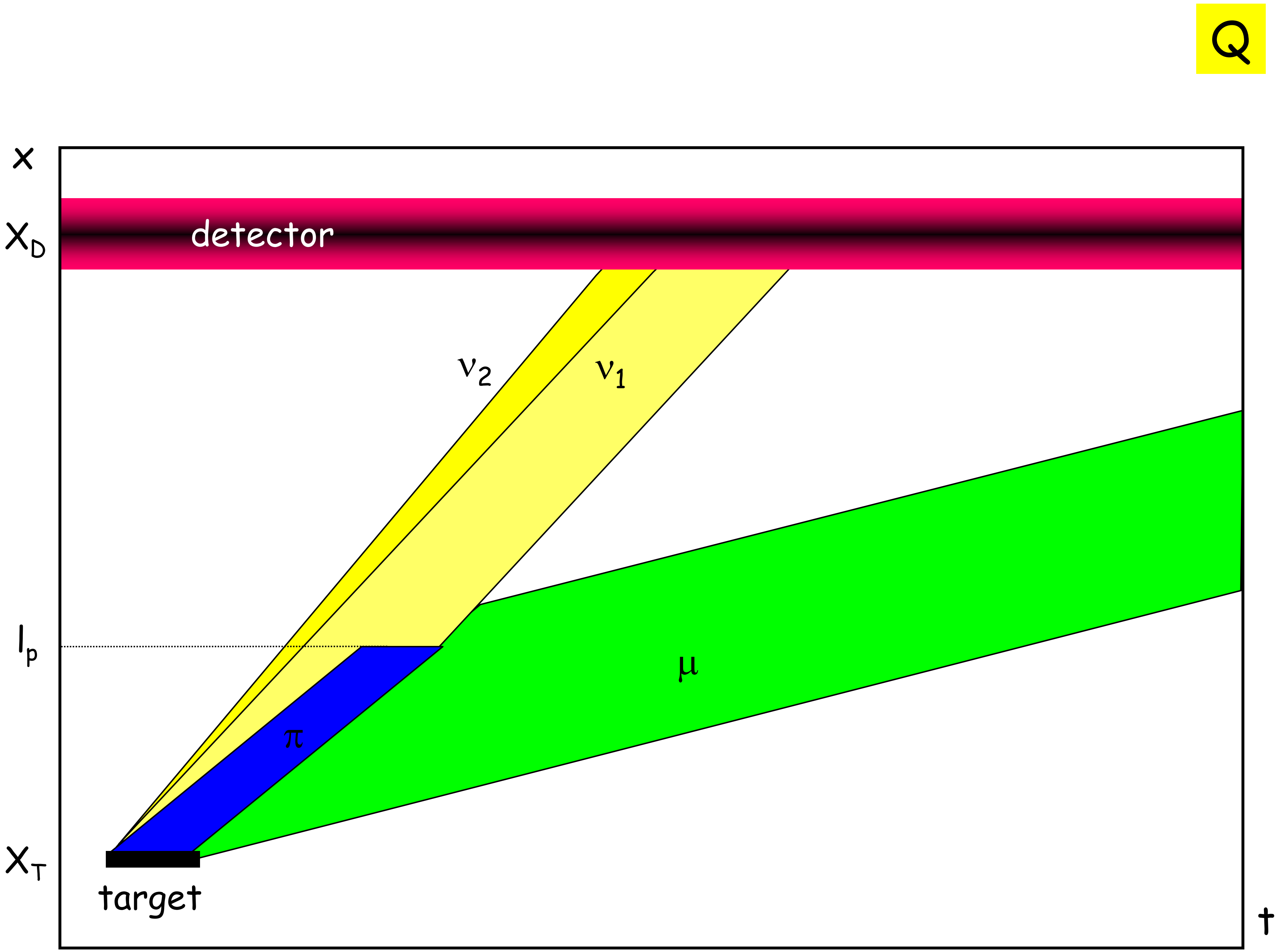
Space-time diagrams



Coherent and incoherent



If pion ``pointlike – result coincides with usual incoherent computations; otherwise – integration – corrections ~ σ_π


Q
x
X_D
detector
ν_2
ν_1
l_p
μ
π
X_T
target
t

Master equation

If loss of coherence and other complications related to WP picture are irrelevant – `` point-like" picture

$$i \frac{d\Psi}{dt} = H\Psi$$

$$\Psi = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix}$$

with mater effects

$$H = \frac{M M^+}{2E} + V(t)$$

generalization

$$E \sim p + \frac{m^2}{2E}$$

M is the mass matrix

V = diag (V_e , 0, 0) – effective potential

Mixing matrix in vacuum

$$M M^+ = U M_{diag}^2 U^+$$

$$M_{diag}^2 = \text{diag}\,(m_1^2,\ m_2^2,\ m_3^2)$$

Neutrino polarization vectors

$$\psi = \begin{pmatrix} \nu_e \\ \nu_\tau, \end{pmatrix} \Rightarrow$$

Polarization vector:

$$\mathbf{P} = \psi^+ \sigma/2 \, \psi$$

$$\mathbf{P} = \begin{pmatrix} \mathrm{Re}\, \nu_e^+ \nu_\tau, \\ \mathrm{Im}\, \nu_e^+ \nu_\tau, \\ \nu_e^+ \nu_e - 1/2 \end{pmatrix}$$

Evolution equation:

$$i \frac{d\Psi}{dt} = H \Psi \Rightarrow$$

$$i \frac{d\Psi}{dt} = (\mathbf{B}\sigma)\Psi$$

$$\mathbf{B} = \frac{2\pi}{l_m}(\sin 2\theta_m, \; 0, \; \cos 2\theta_m)$$

Differentiating **P** and using equation of motion

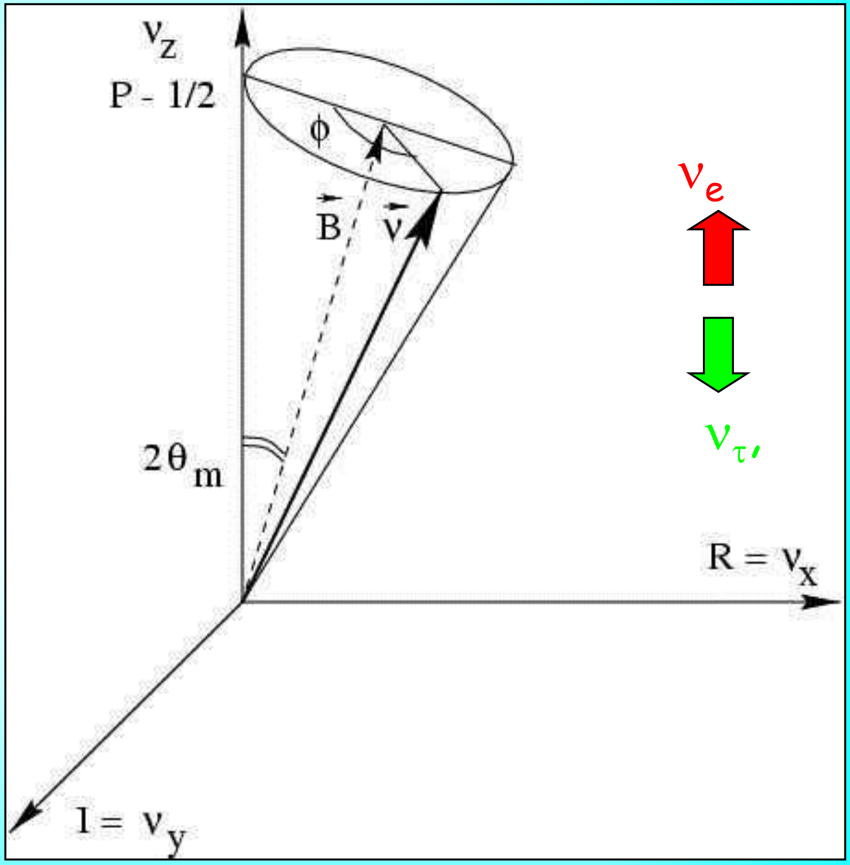
$$\frac{d\mathbf{P}}{dt} = (\mathbf{B} \times \mathbf{P})$$

Coincides with equation for the electron spin precession in the magnetic field

Graphical representation

$\vec{\nu} = \mathbf{P} =$
$(\mathrm{Re}\ \nu_e^+ \nu_\tau,\ \ \mathrm{Im}\ \nu_e^+ \nu_\tau,\ \ \nu_e^+ \nu_e - 1/2)$

$$\mathbf{B} = \frac{2\pi}{l_m}(\sin 2\theta_m,\ 0,\ \cos 2\theta_m)$$



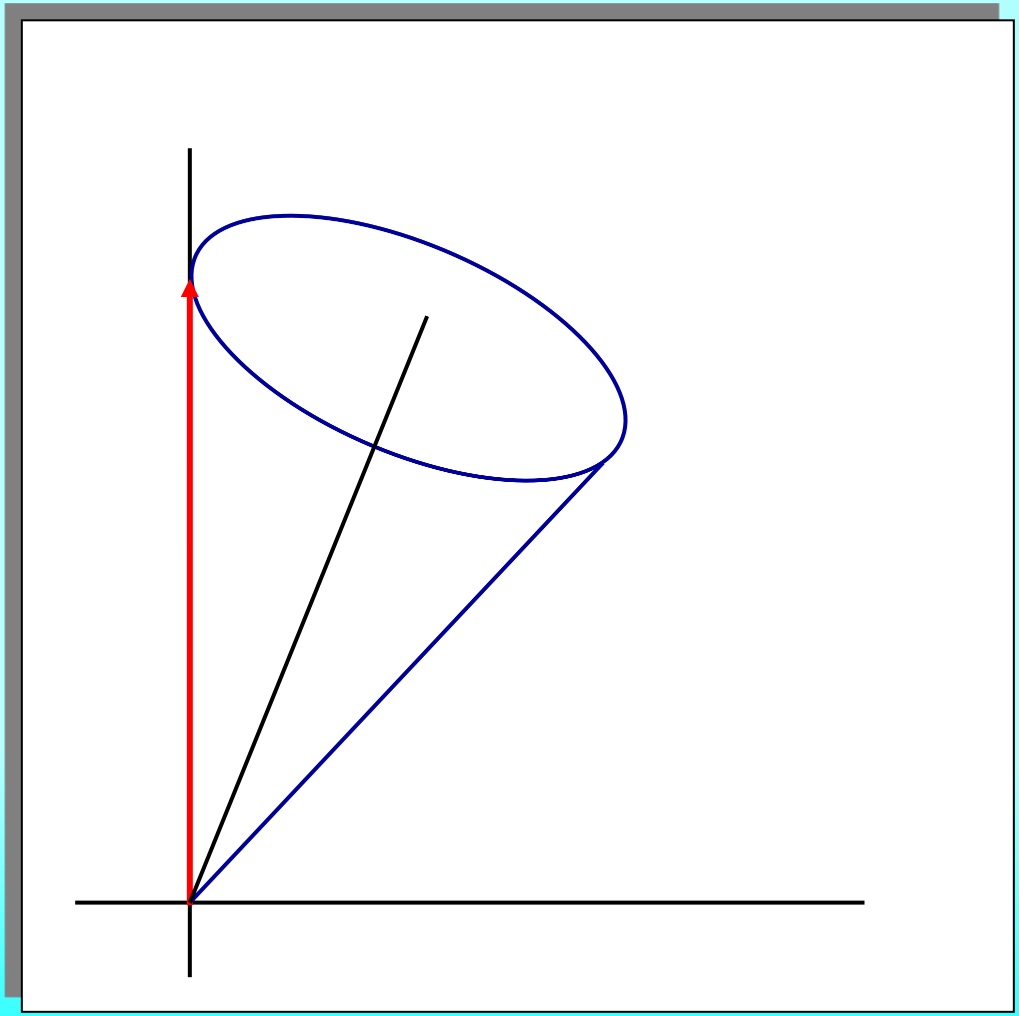
Evolution equation

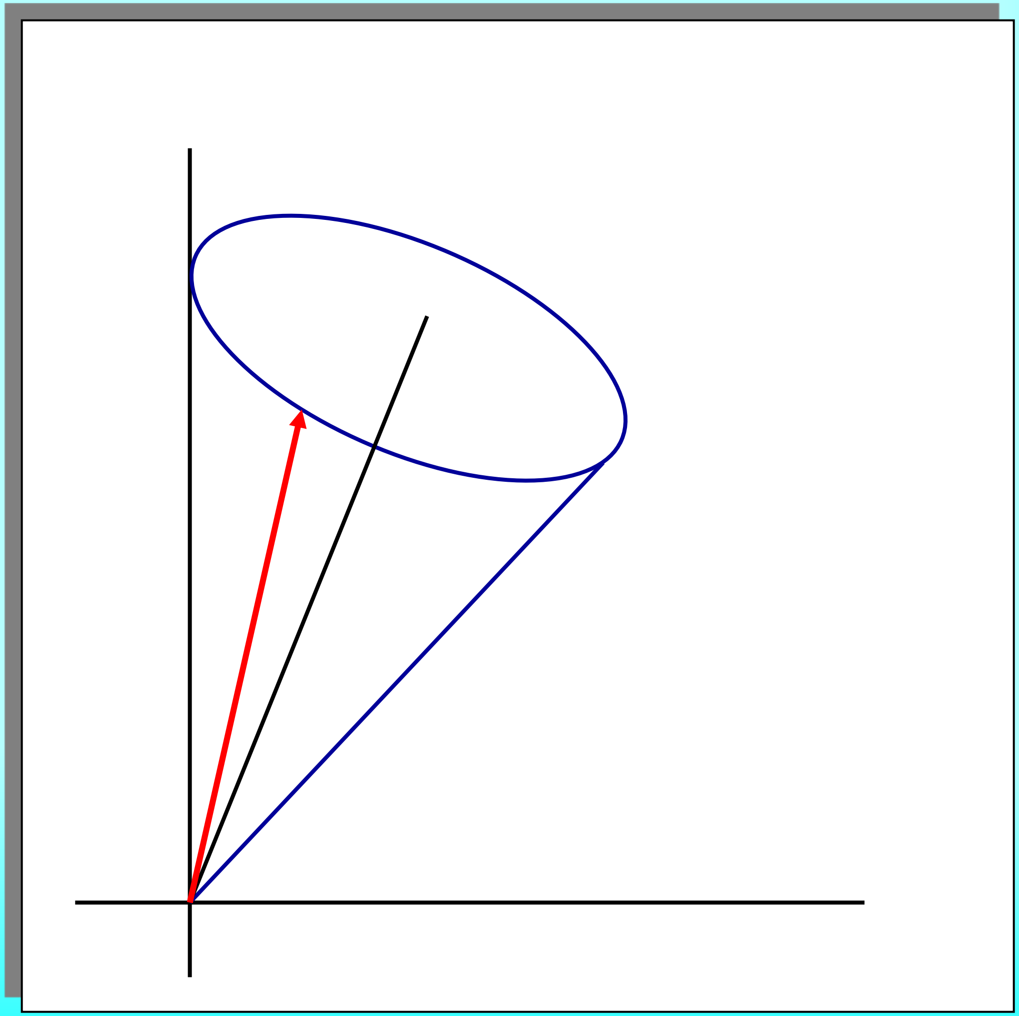
$$\frac{d\vec{\nu}}{dt} = (\vec{B} \times \vec{\nu})$$

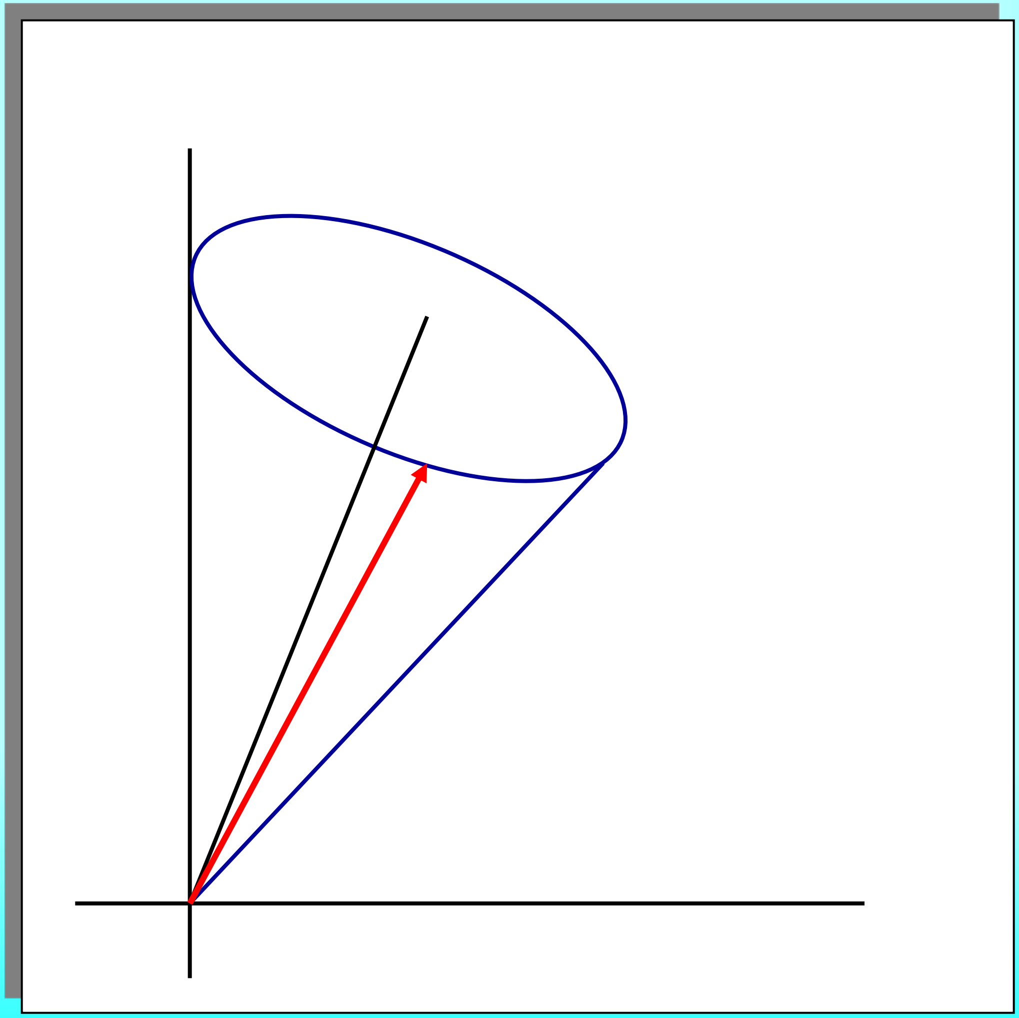
$\phi = 2\pi t / l_m$ - phase of oscillations

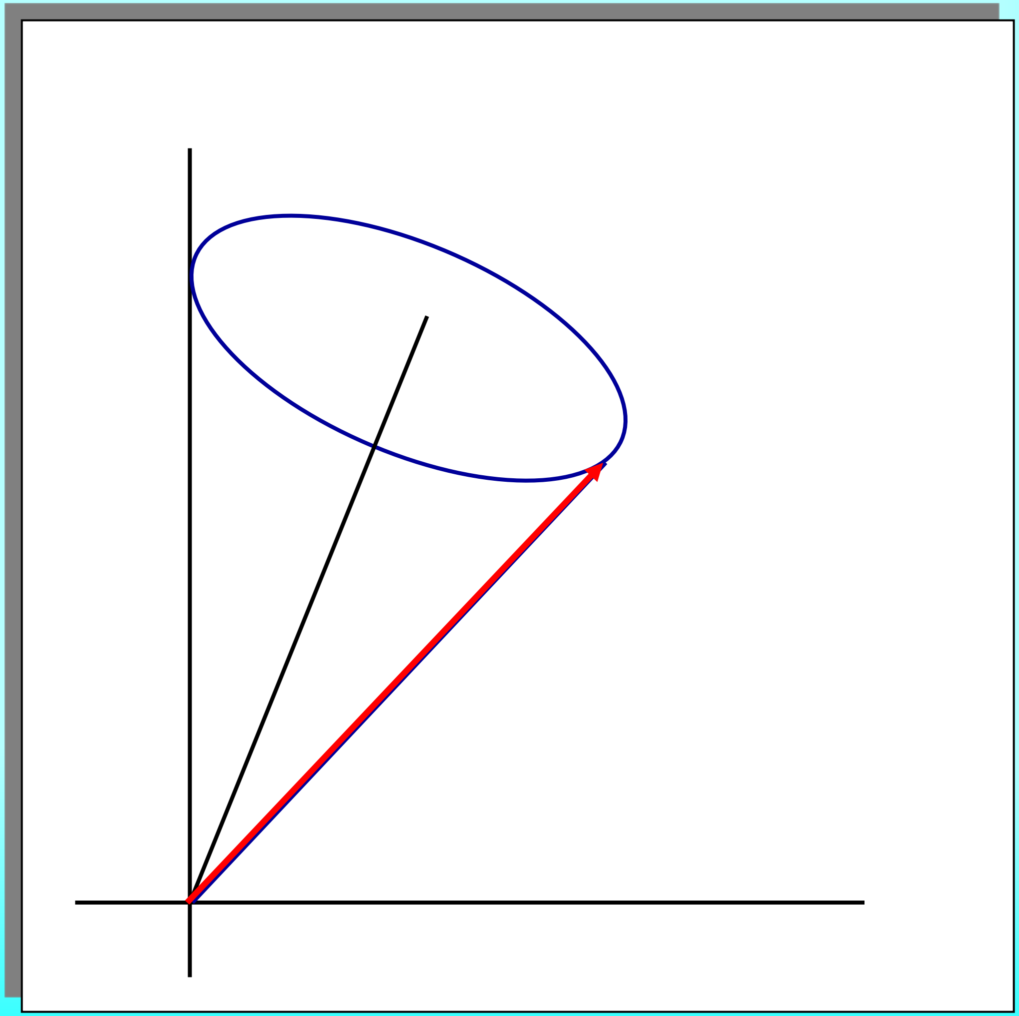
$P = \nu_e^+ \nu_e = \nu_Z + 1/2 = \cos^2\theta_Z/2$ probability to find ν_e

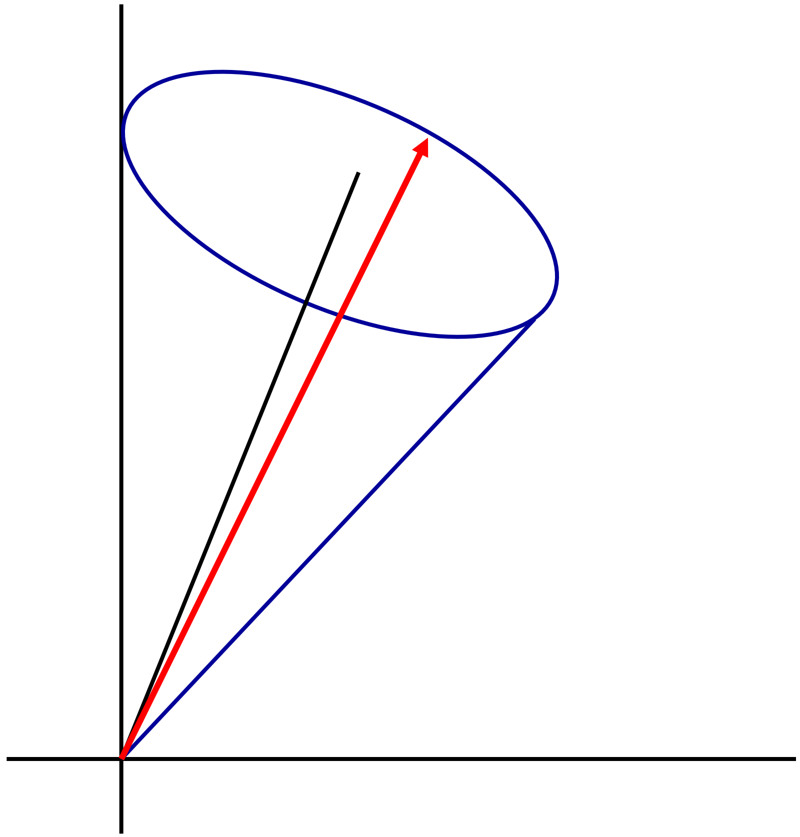
Oscillations

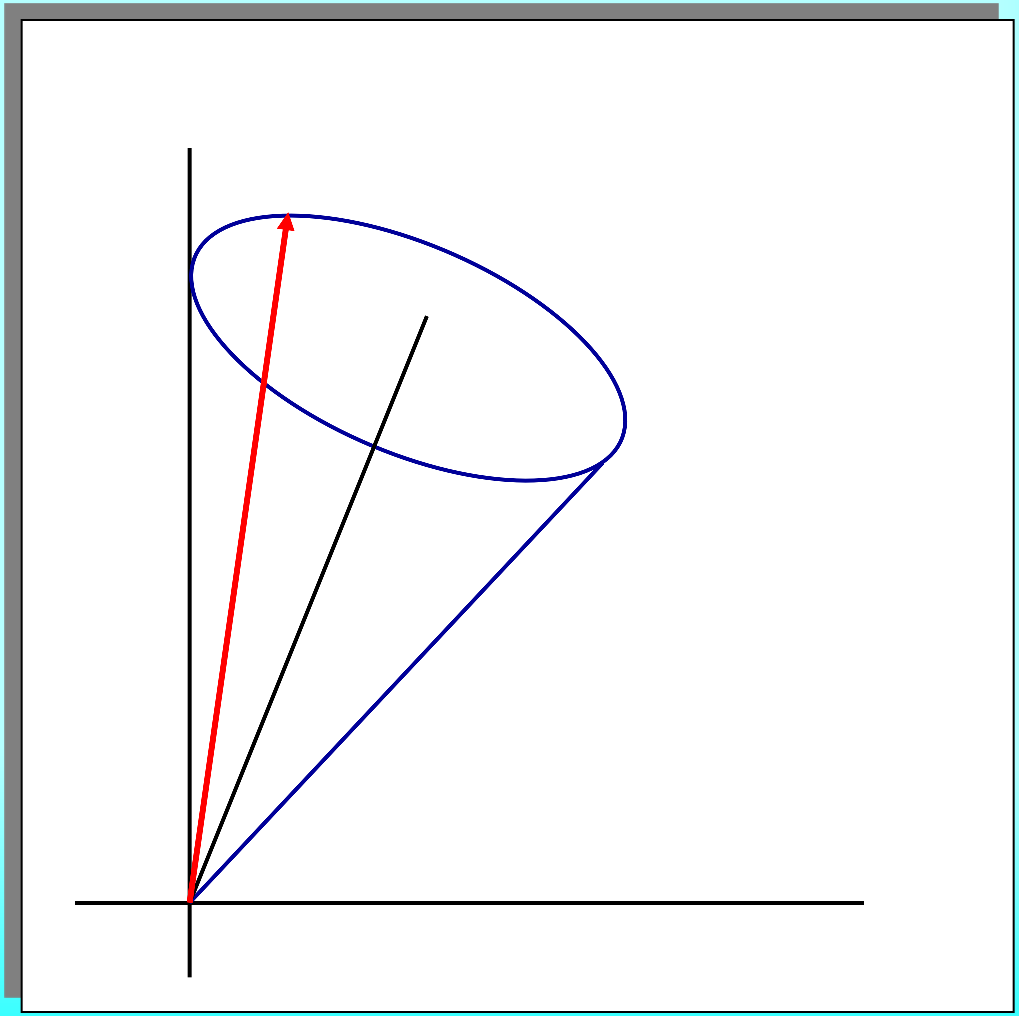


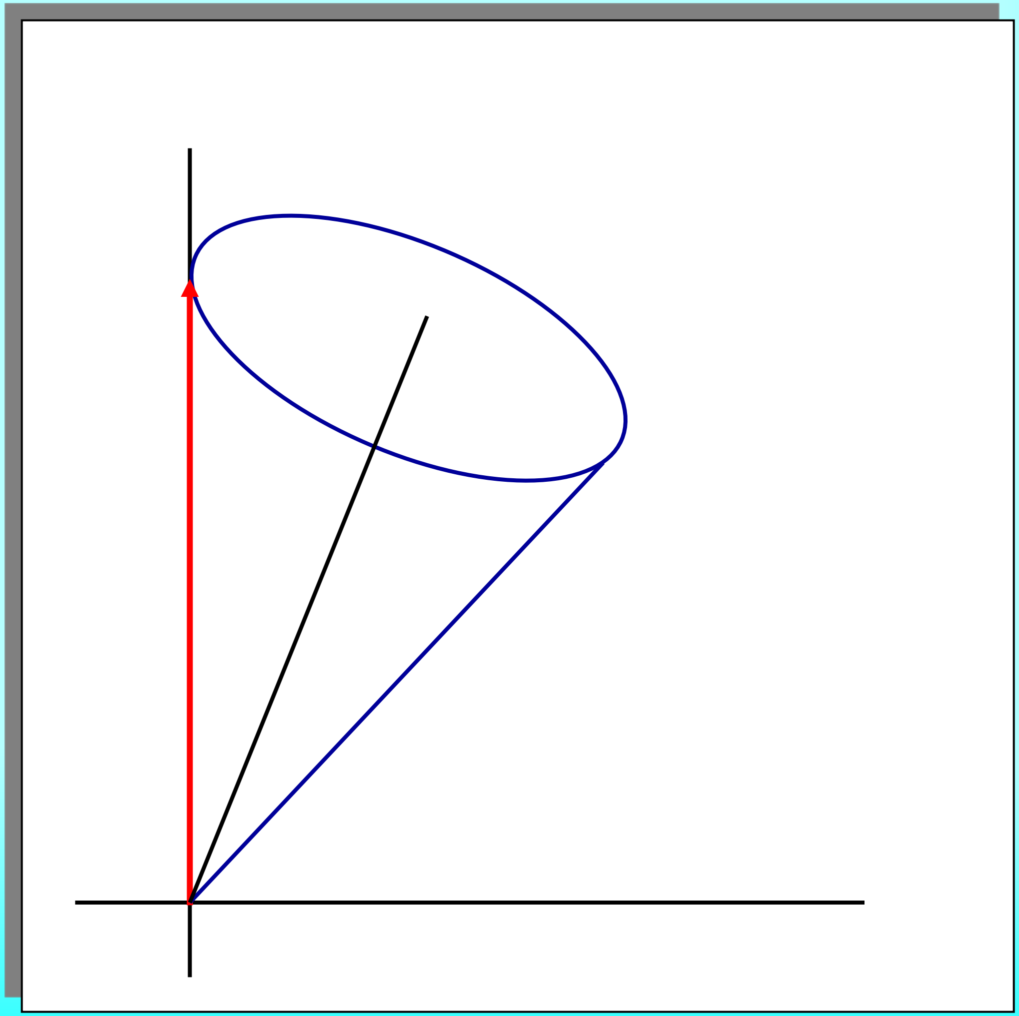












Conclusion:

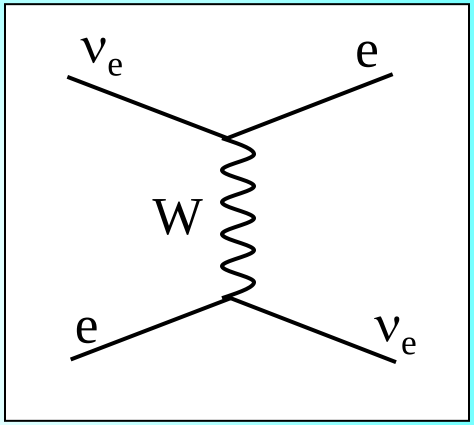
Oscillations is effect of
monotonous increase of phase difference
between eigenstates of propagation (mass eigenstates)
In course of propagation in space-time

Matter effects:
Oscillations & flavor conversion

Matter potential

L. Wolfenstein, 1978

for ν_e ν_μ



at low energies Re A >> Im A
inelelastic interactions can be neglected

Elastic forward scattering ⇒ V_e, V_μ

potentials

difference of potentials

$$V = V_e - V_\mu = \sqrt{2}\, G_F n_e$$

$V \sim 10^{-13}$ eV inside the Earth

Refraction index:

$$n - 1 = V / p$$

for E = 10 MeV

$$n - 1 = \begin{cases} \sim 10^{-20} & \text{inside the Earth} \\ < 10^{-18} & \text{inside the Sun} \end{cases}$$

Refraction length:

$$l_0 = \frac{2\pi}{V}$$

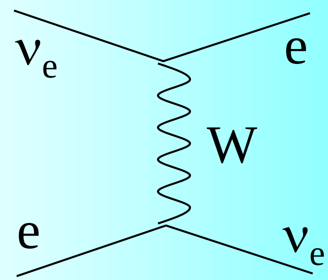
Matter potential

derivation

At low energies: neglect the inelastic scattering and absorption effect is reduced to the elastic forward scattering (refraction) described by the potential V:

$$H_{int}(\nu) = \langle \psi \mid H_{int} \mid \psi \rangle = V \,\bar{\nu}\, \nu$$

ψ is the wave function of the medium



CC interactions with electrons

$$H_{int} = \frac{G_F}{\sqrt{2}}\, \bar{\nu}\gamma^{\mu}(1-\gamma_5)\,\nu\; \bar{e}\gamma_{\mu}(1-\gamma_5)\, e$$

$\langle \bar{e}\gamma_0 (1-\gamma_5)\, e \rangle = n_e$ - the electron number density

$\langle \bar{e}\vec{\gamma}\, e \rangle = n_e \vec{v}$

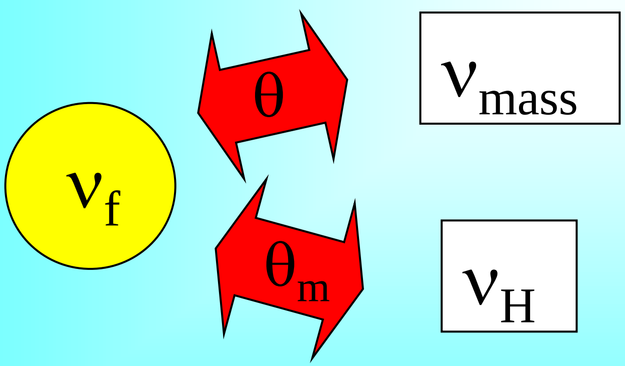
$\langle \bar{e}\vec{\gamma}\gamma_5\, e \rangle = n_e \vec{\lambda}_e$ - averaged polarization vector of e

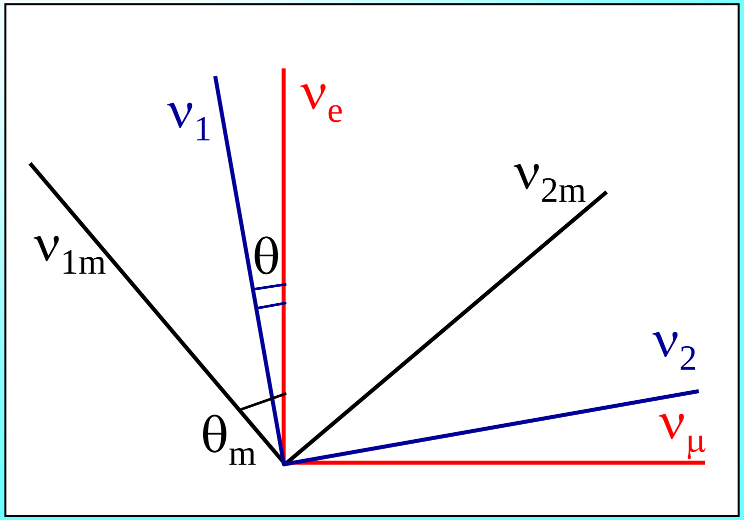
For unpolarized medium at rest:

$$V = \sqrt{2}\, G_F n_e$$

Mixing in matter

	in vacuum:		*in matter:*	
Effective Hamiltonian	H_0	⇨	$H = H_0 + V$	
Eigenstates	ν_1, ν_2	⇨	ν_{1m}, ν_{2m}	depend on n_e, E
Eigenvalues	$m_1^2/2E, m_2^2/2E$	⇨	H_{1m}, H_{2m}	instantaneous





Mixing angle determines flavors (flavor composition) of the eigenstates

Evolution equation

$$i \frac{d\nu_f}{dt} = H_{tot}\, \nu_f \qquad \nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$H_{tot} = H_{vac} + V$ is the total Hamiltonian

$$H_{vac} = \frac{M^2}{2E}$$ is the vacuum (kinetic) part

$$V = \begin{pmatrix} V_e & 0 \\ 0 & 0 \end{pmatrix}$$ matter part $\qquad V_e = \sqrt{2}\, G_F n_e$

H_{tot}

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{2E}\cos 2\theta + V_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Mixing in matter

Diagonalization of the Hamiltonian:

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - 2EV/\Delta m^2)^2 + \sin^2 2\theta}$$

$$V = \sqrt{2}\, G_F\, n_e$$

Mixing is maximal if

$$V = \frac{\Delta m^2}{2E} \cos 2\theta$$

Resonance condition

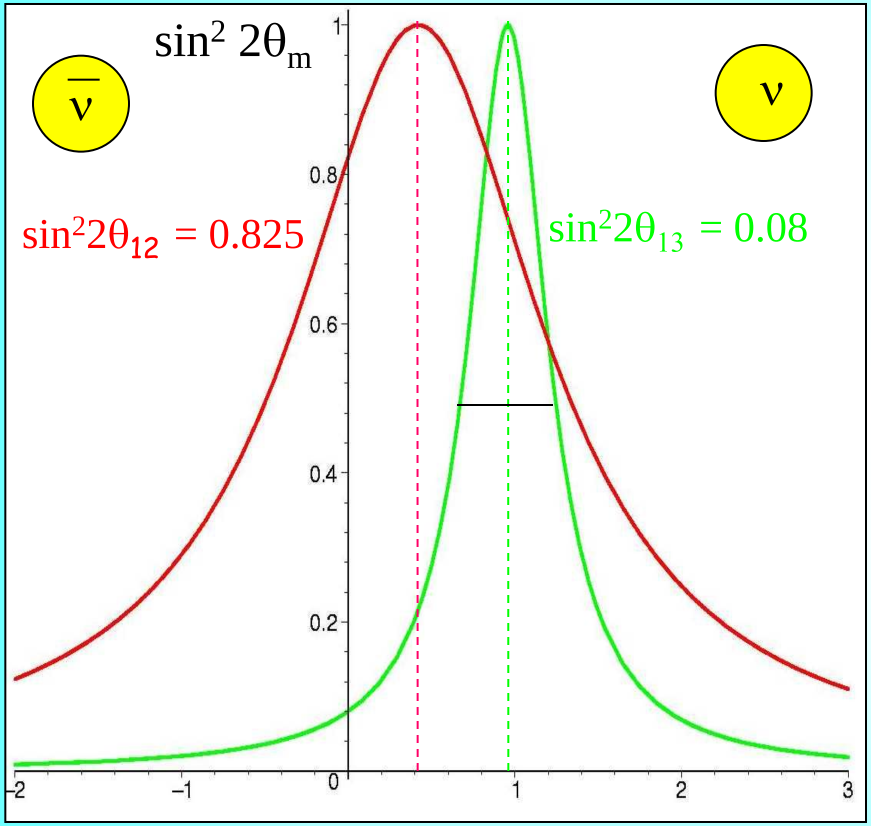
$$H_e = H_\mu$$

$$\sin^2 2\theta_m = 1$$

Difference of the eigenvalues

$$H_{2m} - H_{1m} = \frac{\Delta m^2}{2E} \sqrt{(\cos 2\theta - 2EV/\Delta m^2)^2 + \sin^2 2\theta}$$

Resonance



$l_\nu / l_0 \sim n E$

In resonance:

$\sin^2 2\theta_m = 1$

Flavor mixing is maximal

$l_\nu = l_0 \cos 2\theta$

Vacuum oscillation length $\approx$ Refraction length

Resonance width: $\Delta n_R = 2 n_R \tan 2\theta$

Resonance layer: $n = n_R +/- \Delta n_R$

Level crossing

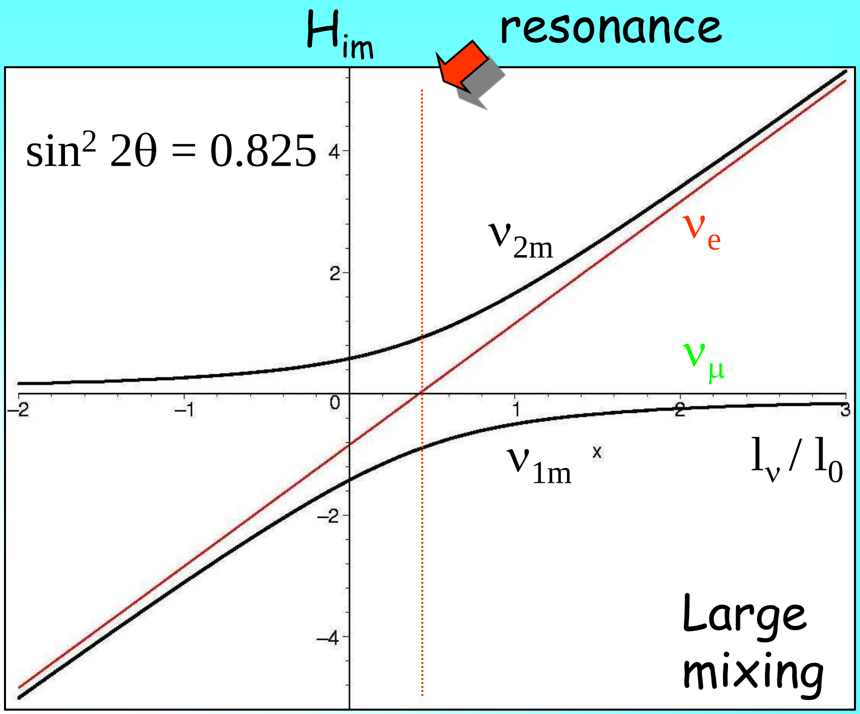
V. Rubakov, private comm.
N. Cabibbo, Savonlinna 1985
H. Bethe, PRL 57 (1986) 1271

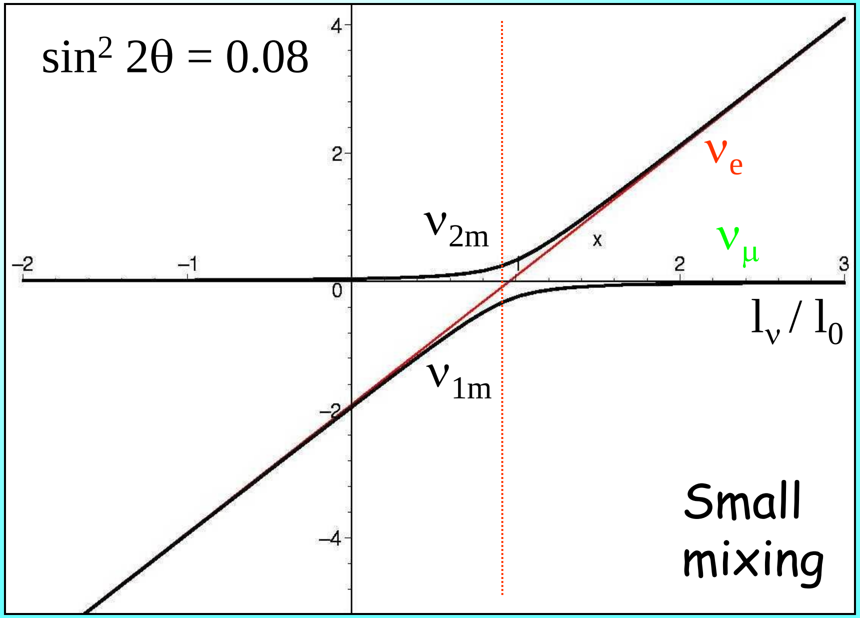
Dependence of the neutrino eigenvalues on the matter potential (density)

$$\frac{l_\nu}{l_0} = \frac{2E\,V}{\Delta m^2}$$

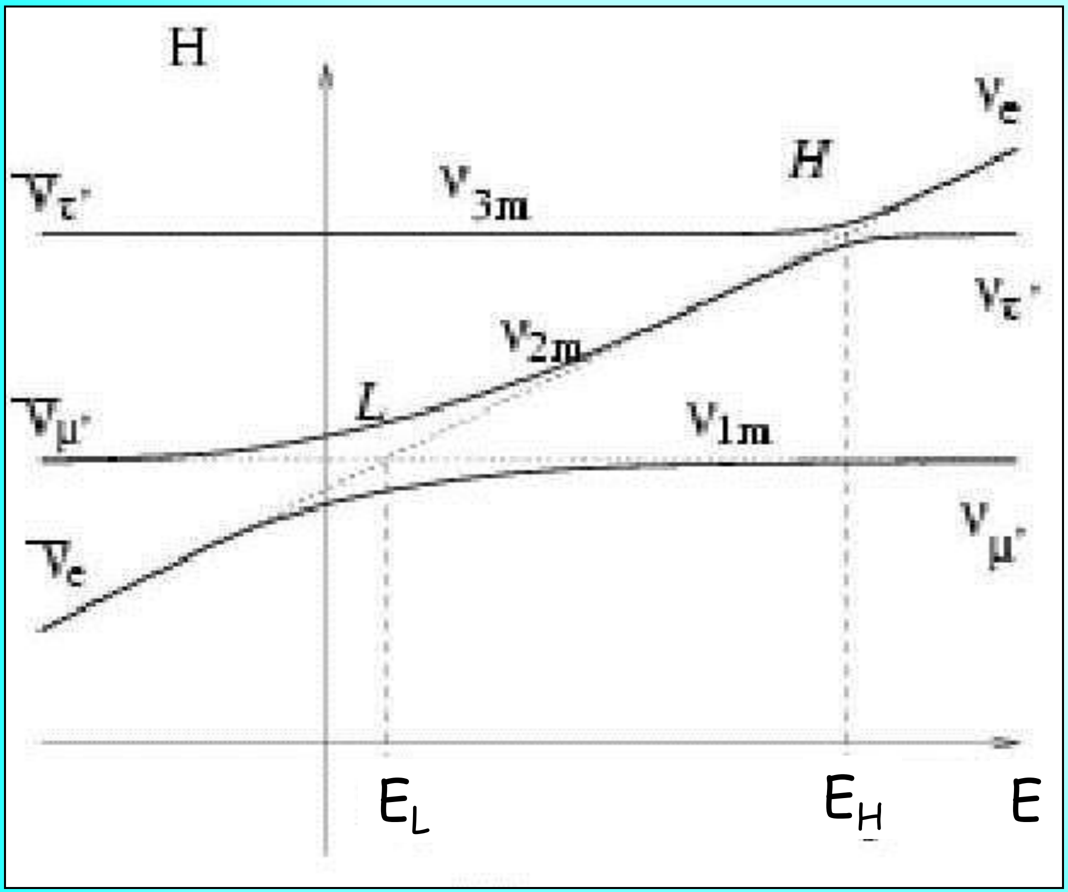
$$\frac{l_\nu}{l_0} = \cos 2\theta$$

Crossing point - resonance
- the level split is minimal
- the oscillation length is maximal





Level crossings



Normal mass hierarchy

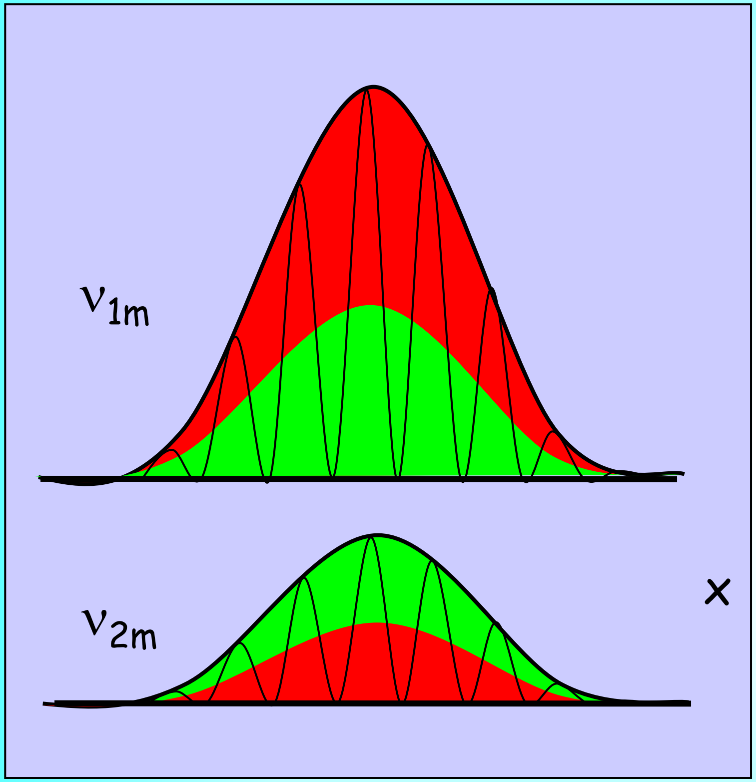
0.1 GeV

6 GeV

Resonance region

High energy range

Oscillations in matter



Constant density medium

$H_0 \rightarrow H = H_0 + V$

$\nu_k \rightarrow \nu_{mk}$

eigenstates of H_0 — eigenstates of H

$\theta \rightarrow \theta_m(n)$

Resonance - maximal mixing in matter - oscillations with maximal depth

$\theta_m = \pi/4$

Mixing changed

phase difference changed

Resonance condition:

$$V = \cos 2\theta \frac{\Delta m^2}{2E}$$

Oscillations in matter

Oscillation probability constant density

$$P(\nu_e \to \nu_a) = \sin^2 2\theta_m \sin^2\left(\frac{\pi L}{l_m}\right)$$

half-phase ϕ

Amplitude of oscillations

oscillatory factor

$\theta_m(E, n)$ - mixing angle in matter

$l_m(E, n)$ – oscillation length in matter

$$l_m = 2\pi/(H_{2m} - H_{1m})$$

In vacuum:

$\theta_m \rightarrow \theta$

$l_m \rightarrow l_\nu$

Maximal effect:

$\sin^2 2\theta_m = 1$ → MSW resonance condition

$\phi = \pi/2 + \pi k$

Oscillation length in matter

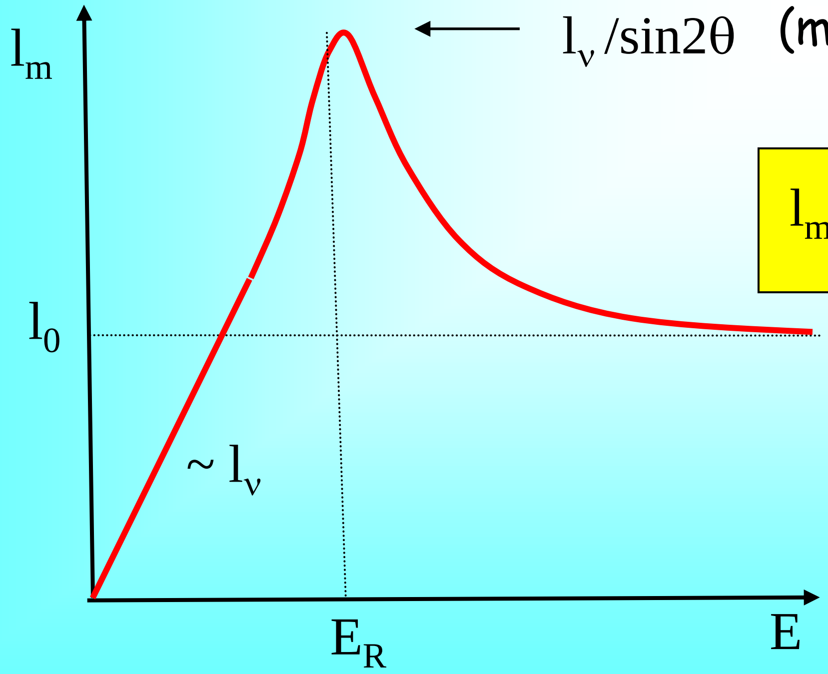
Oscillation length in vacuum

$$l_\nu = \frac{4\pi E}{\Delta m^2}$$

Refraction length

$$l_0 = \frac{2\pi}{\sqrt{2} G_F n_e}$$

- determines the phase produced by interaction with matter



(maximum at $l_\nu = l_0 / \cos 2\theta$)

$$l_m = \frac{2\pi}{H_2 - H_1}$$

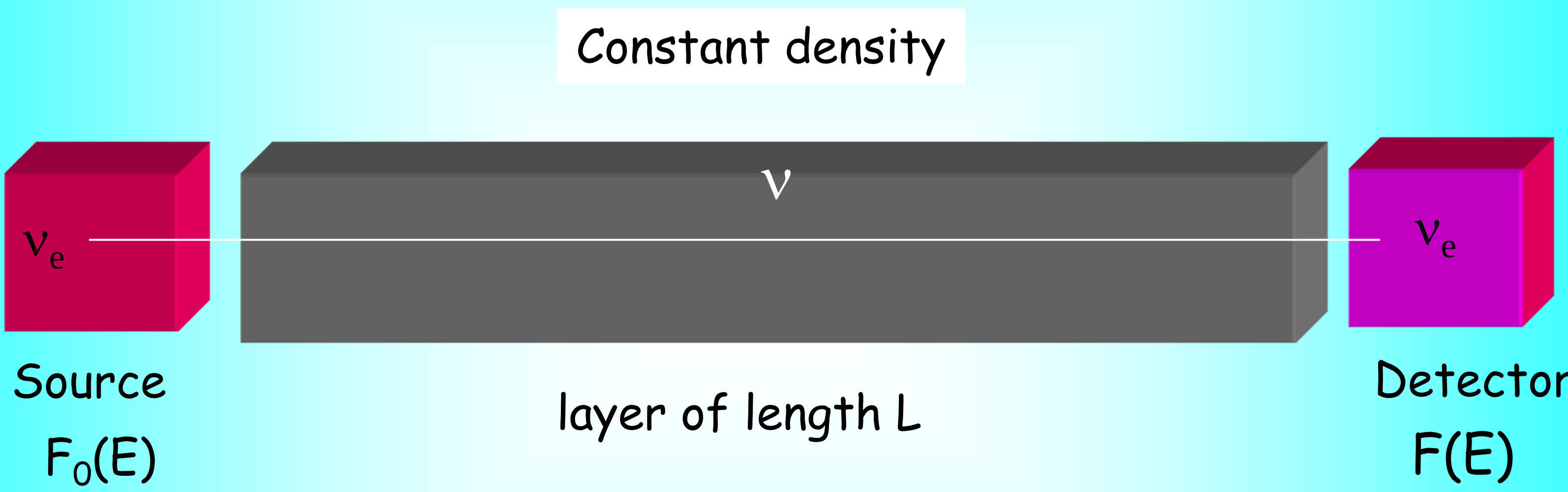
Resonance energy:

$$l_\nu (E_R) = l_0 \cos 2\theta$$

Resonance enhancement of oscillations

Constant density

Resonance enhancement



Depth of oscillations determined by $\sin^2 2\theta_m$
as well as the oscillation length, l_m
depend on neutrino energy

For neutrinos propagating
in the mantle of the Earth

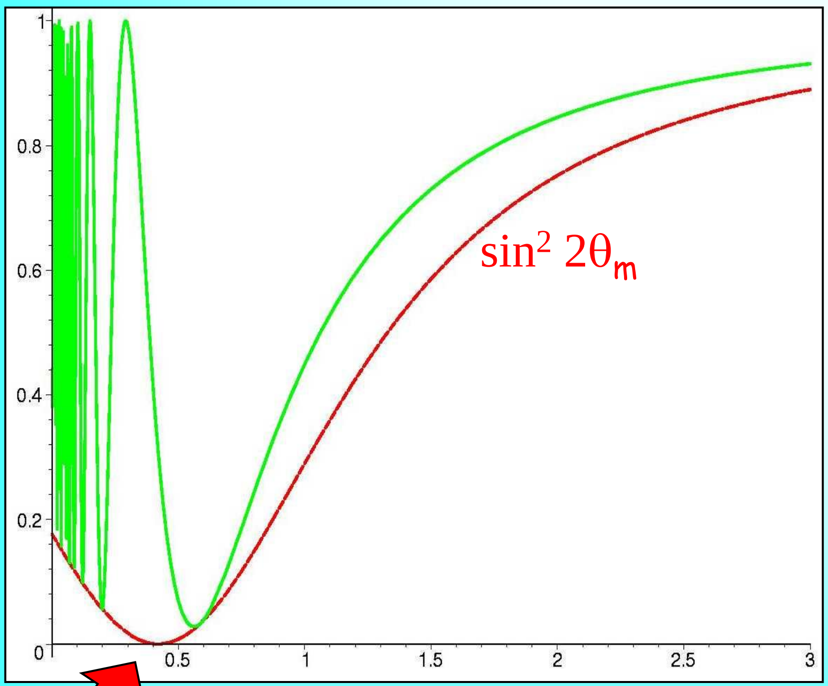
Large mixing $\sin^2 2\theta = 0.824$

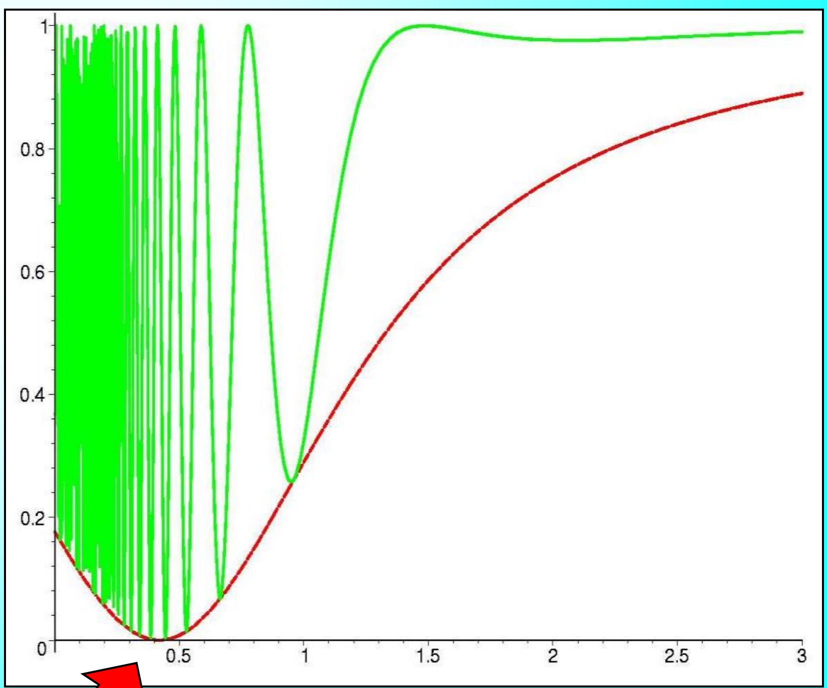
Layer of length L_ν $k = \pi L / l_0$

thin layer $k = 1$

thick layer $k = 10$

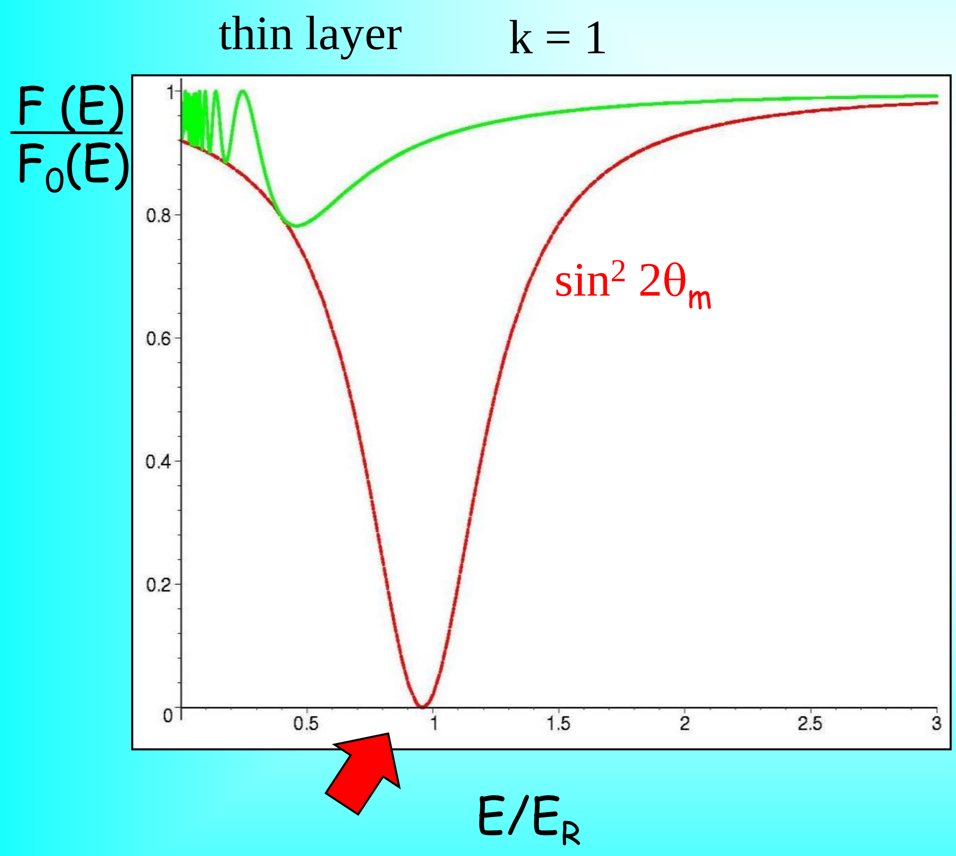
$\frac{F(E)}{F_0(E)}$

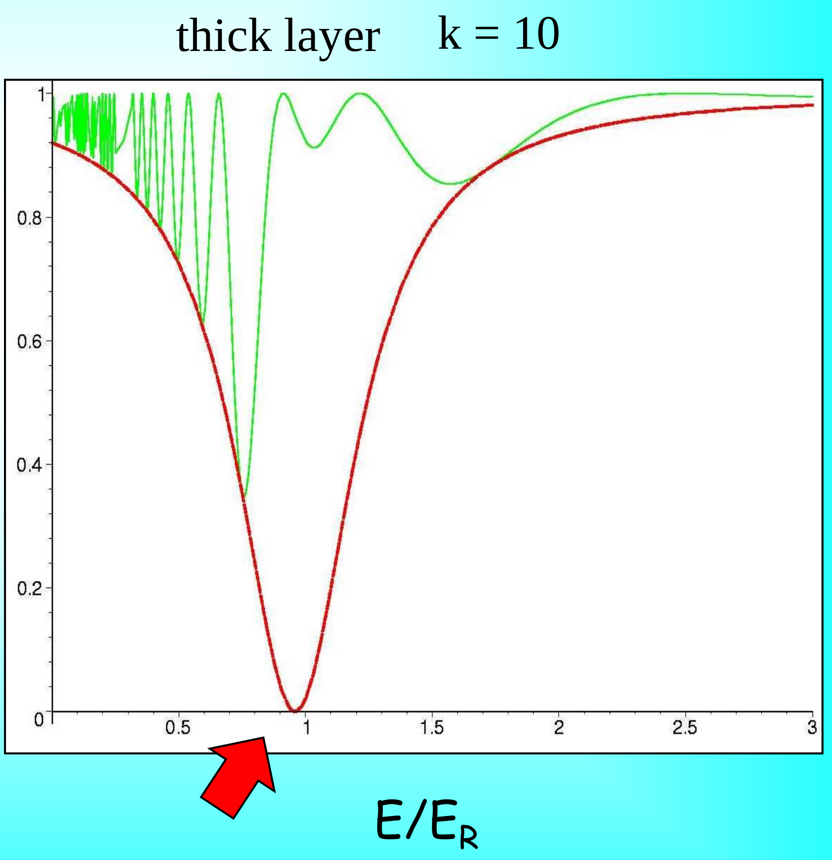




E/E_R E/E_R

Small mixing $\sin^2 2\theta = 0.08$





Adiabatic conversion

Varying density

Evolution equation for eigenstates

In non-uniform medium the Hamiltonian depends on time:

$$H_{tot} = H_{tot}(n_e(t))$$

$$i \frac{d\nu_f}{dt} = H_{tot}\, \nu_f$$

$$\nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$\nu_m = \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

Inserting $\nu_f = U(\theta_m)\, \nu_m$

$$\theta_m = \theta_m(n_e(t))$$

$$i \frac{d}{dt} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} 0 & i \frac{d\theta_m}{dt} \\ -i \frac{d\theta_m}{dt} & H_{2m} - H_{1m} \end{pmatrix} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

off=diagonal terms imply transitios

$\nu_{1m} \Leftrightarrow \nu_{2m}$

However

if $\left| \frac{d\theta_m}{dt} \right| \ll H_{2m} - H_{1m}$

off-diagonal elements can be neglected
no transitions between eigenstates
propagate independently

Adiabaticity

Adiabaticity condition $\left| \frac{d\theta_m}{dt} \right| \ll H_{2m} - H_{1m}$

External conditions (density) change slowly the system has time to adjust them

transitions between the neutrino eigenstates can be neglected $\nu_{1m} \not\leftrightarrow \nu_{2m}$ → The eigenstates propagate independently

Shape factors of the eigenstates do not change

Crucial in the resonance layer:
- the mixing changes fast
- level splitting is minimal

$\Delta r_R > l_R$ if vacuum mixing is small

$l_R = l_\nu / \sin 2\theta$ oscillation length in resonance

$\Delta r_R = n_R / (dn/dx)_R \tan 2\theta$ width of the res. layer

If vacuum mixing is large, the point of maximal adiabaticity violation is shifted to larger densities

$n(a.v.) \rightarrow n_R^0 > n_R$

$n_R^0 = \Delta m^2 / 2\sqrt{2} G_F E$

Adiabatic parameter

$$\kappa = \frac{H_{2m} - H_{1m}}{\left| \frac{d\theta_m}{dt} \right|}$$

Adiabaticity condition:
$\kappa > 1$

most crucial in the resonance where
the mixing angle in matter changes fast

$$\kappa_R = \frac{\Delta r_R}{l_R}$$

$\Delta r_R = h_n \tan 2\theta$ is the width of the resonance layer

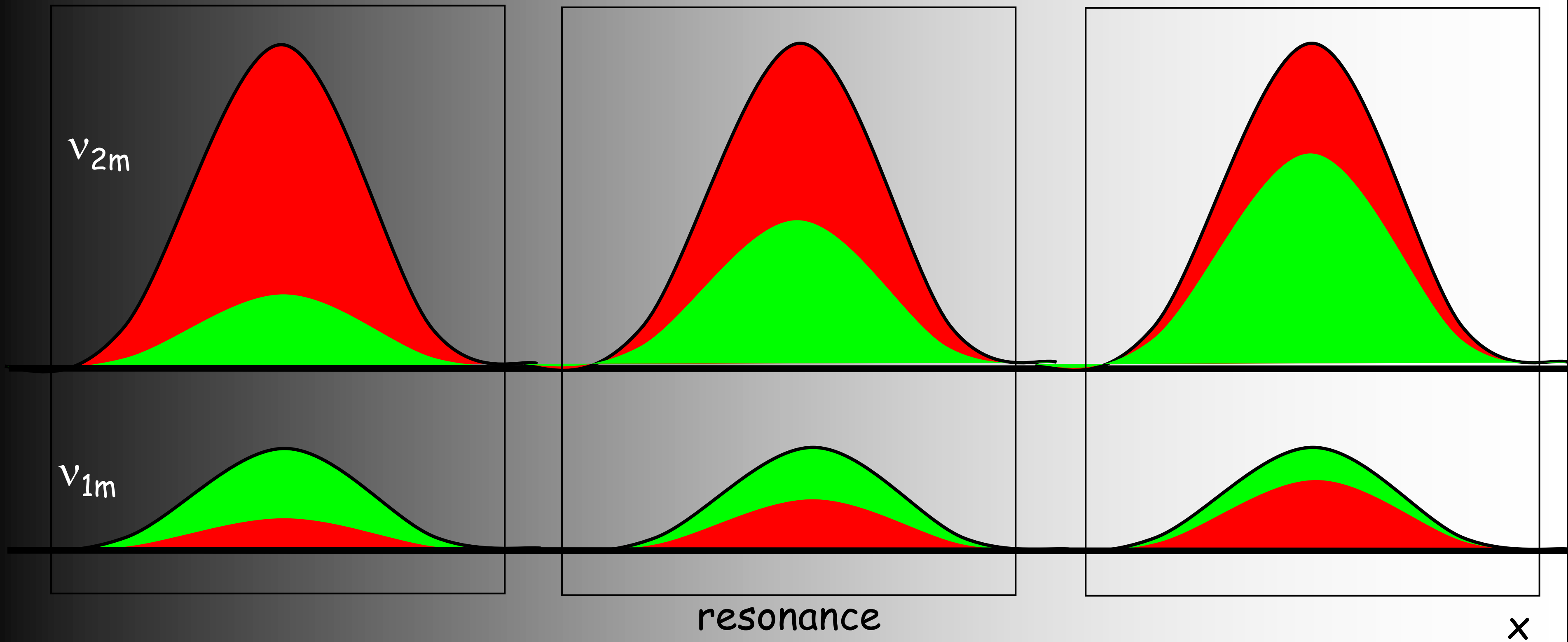
$h_n = \frac{n}{dn/dx}$ is the scale of density change

$l_R = l_\nu / \sin 2\theta$ is the oscillation length in resonance

Explicitly:

$$\kappa_R = \frac{\Delta m^2 \sin^2 2\theta \, h_n}{2E \cos 2\theta}$$

Adiabatic conversion



if density changes slowly

- the amplitudes of the wave packets do not change
- flavors of the eigenstates follow the density change

Adiabatic conversion probability

Sun, Supernova

From high to low densities

Initial state: $\nu(0) = \nu_e = \cos\theta_m^0\, \nu_{1m}(0) + \sin\theta_m^0\, \nu_{2m}(0)$

Mixing angle in matter in initial state

Adiabatic evolution to the surface of the Sun (zero density):

$\nu_{1m}(0) \rightarrow \nu_1$

$\nu_{2m}(0) \rightarrow \nu_2$

Final state: $\nu(f) = \cos\theta_m^0\, \nu_1 + \sin\theta_m^0\, \nu_2\, e^{-i\phi}$

Probability to find ν_e averaged over oscillations

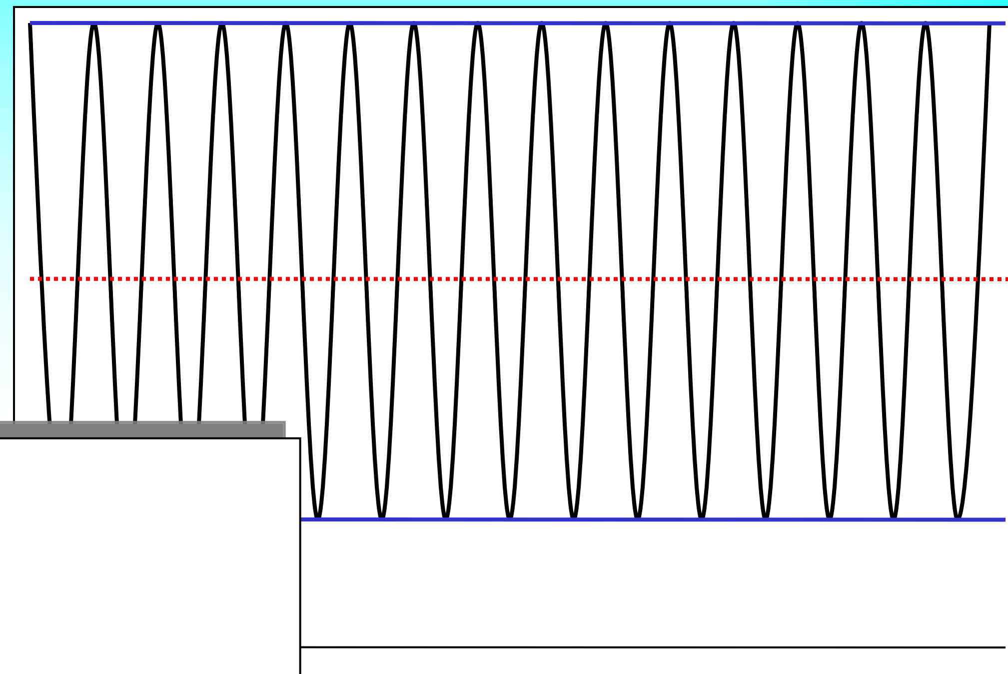
$P = |\langle \nu_e | \nu(f) \rangle|^2 = (\cos\theta \cos\theta_m^0)^2 + (\sin\theta \sin\theta_m^0)^2$

$= 0.5[\, 1 + \cos 2\theta_m^0 \cos 2\theta \,]$

$P = \sin^2\theta + \cos 2\theta \cos^2\theta_m^0$

Spatial picture

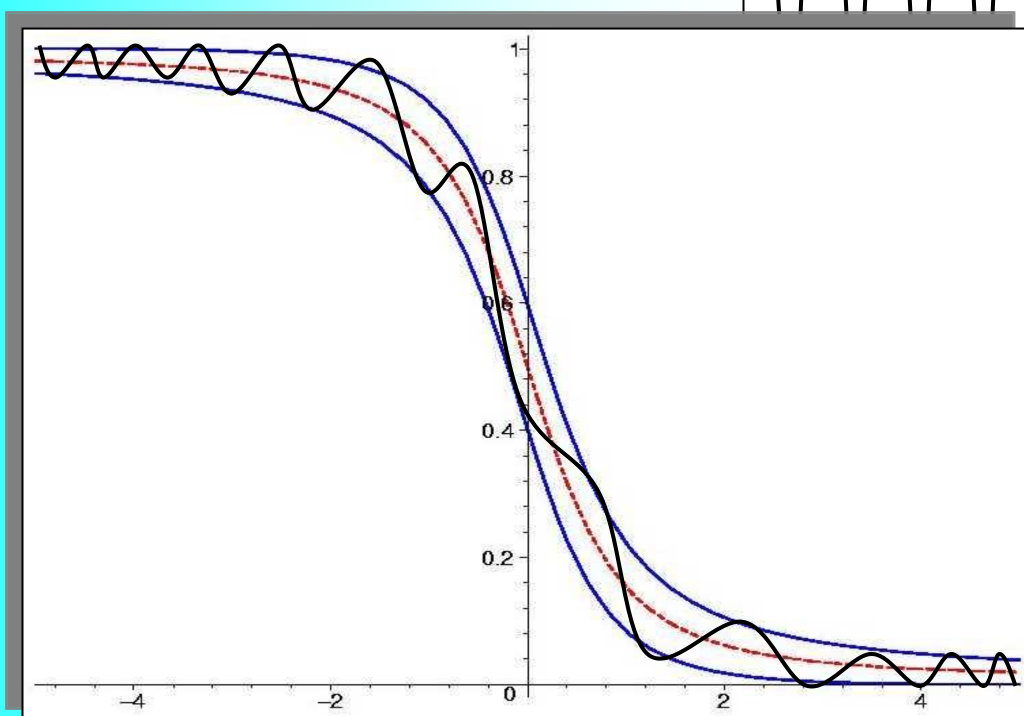
Oscillations



distance

Adiabatic conversion

survival probability



distance

A Yu Smirnov

Adiabaticity violation

If density $n_e(t)$ changes fast

SN shock waves

$\kappa \sim 1$ $\qquad \left| \frac{d\theta_m}{dt} \right| \sim |H_{2m} - H_{1m}|$

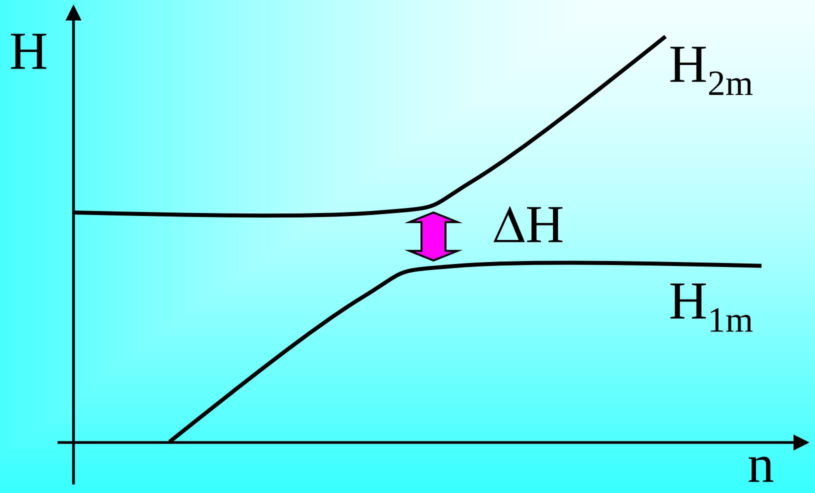
the off-diagonal terms in the Hamiltonian can not be neglected

transitions $\qquad \nu_{1m} \Leftrightarrow \nu_{2m}$

Admixtures of ν_{1m} ν_{2m} in a given neutrino state change

``Jump probability" penetration under barrier:

$$P_{12} = e^{-\frac{\Delta H}{E_n}}$$



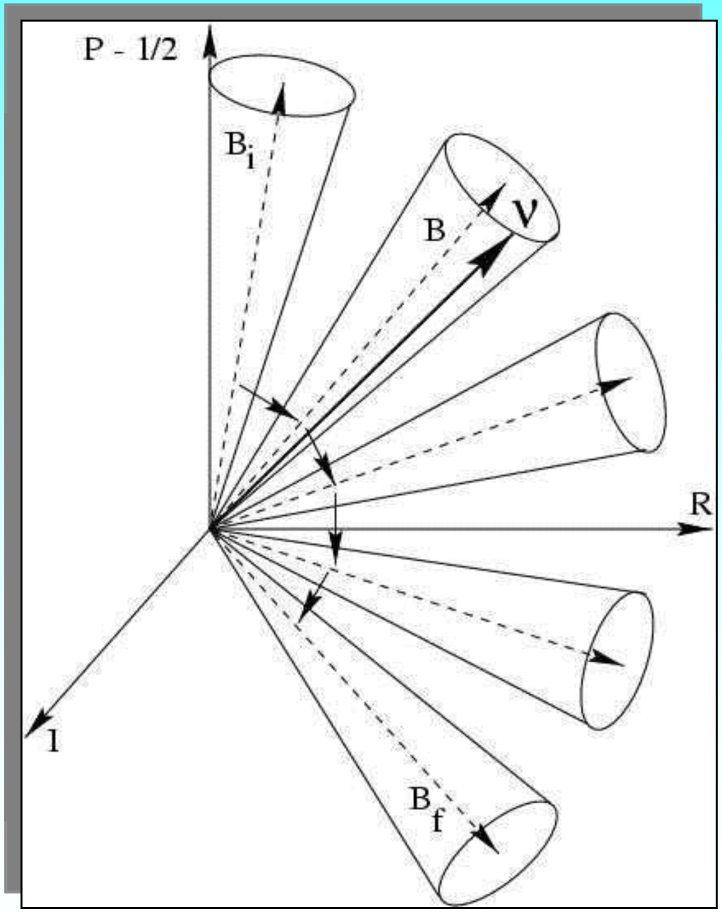
$E_n \sim 1/h_n$ is the energy associated to change of parameter (density)

$$P_{12} = e^{-\pi\kappa_R/2}$$

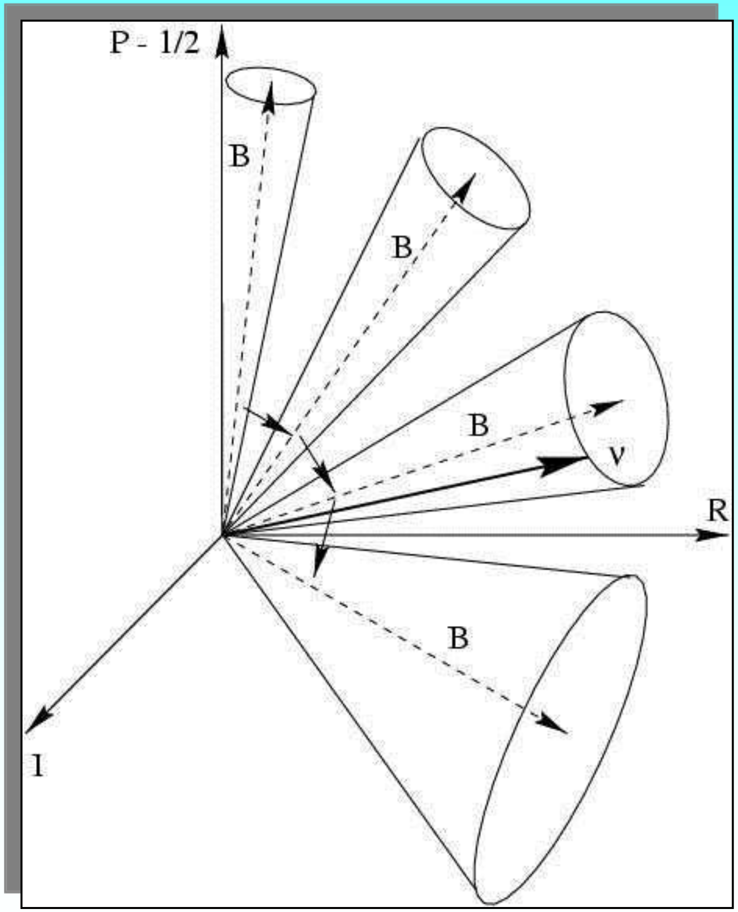
Landau-Zener

Adiabatic conversion

Pure adiabatic conversion



Partialy adiabatic conversion



Oscillations versus MSW

Different degrees of freedom

Oscillations

Vacuum or uniform medium with constant parameters

Phase difference increase between the eigenstates

ϕ

Adiabatic conversion

Non-uniform medium or/and medium with varying in time parameters

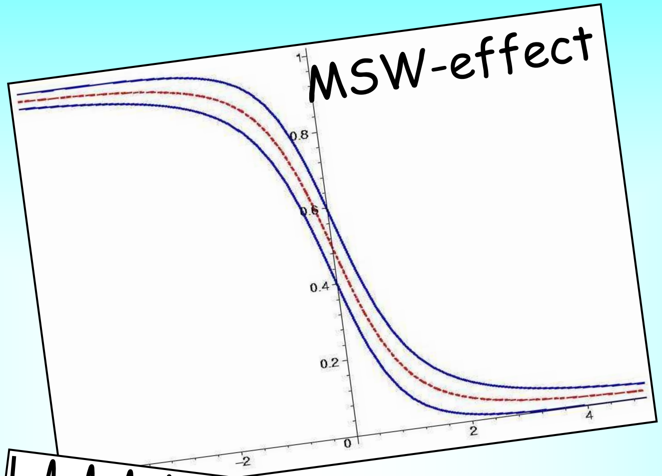
Change of mixing in medium = change of flavor of the eigenstates

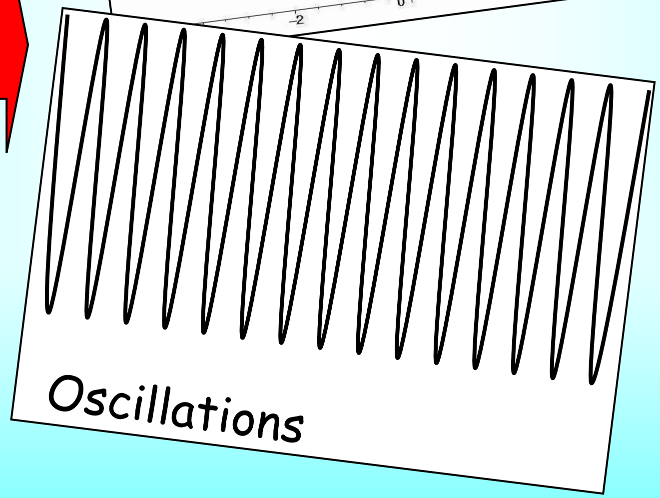
θ_m

In non-uniform medium: interplay of both processes

Two effects

Solar neutrinos
KamLAND
Atmospheric neutrinos
Double Chooz
Daya Bay
MINOS
K2K
RENO
Antares
T2K
DeepCore





Can be resonantly enhanced in matter

Δm^2

θ

Conclusion:

Adiabatic conversion is effect of change of mixing angle in matter in medium with slowly enough density change on the way of neutrino propagation

Resonance enhancement of oscillations occures in certain energy range in matter with constant density

nearly constant

Evolution equation

$$i \frac{d\Psi}{dx} = H\Psi$$

$$H = \frac{M M^+}{2E} + V(x) + \ldots$$

$$\Psi = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix}$$

M is the mass matrix

V = diag (V_e , 0, 0) – effective potential due to scattering on electrons

mixing matrix in vacuum

$$M M^+ = U M_{diag}^2 U^+$$

$$M_{diag}^2 = \text{diag}(m_1^2, m_2^2, m_3^2)$$

Adiabatic approximation perturbation theory

Small/large density limits

Approximate decoupling of some states

Neutrinos and antineutrinos

- Continuity:
  neutrino and antineutrino semiplanes
  normal and inverted hierarchy

- Oscillations (amplitude of oscillations) are enhanced in the resonance layer

$$E = (E_R - \Delta E_R) \; \text{--} \; (E_R + \Delta E_R)$$

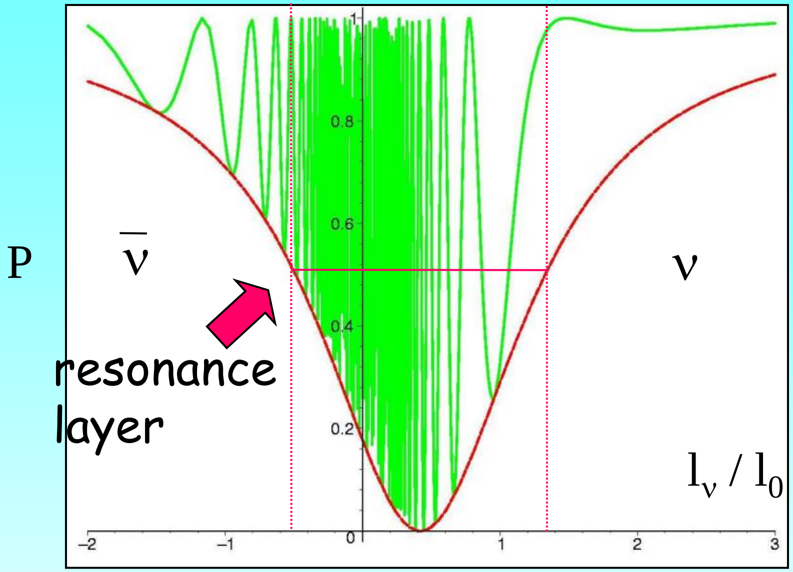
$$\Delta E_R = E_R \tan 2\theta = E_R^0 \sin 2\theta$$

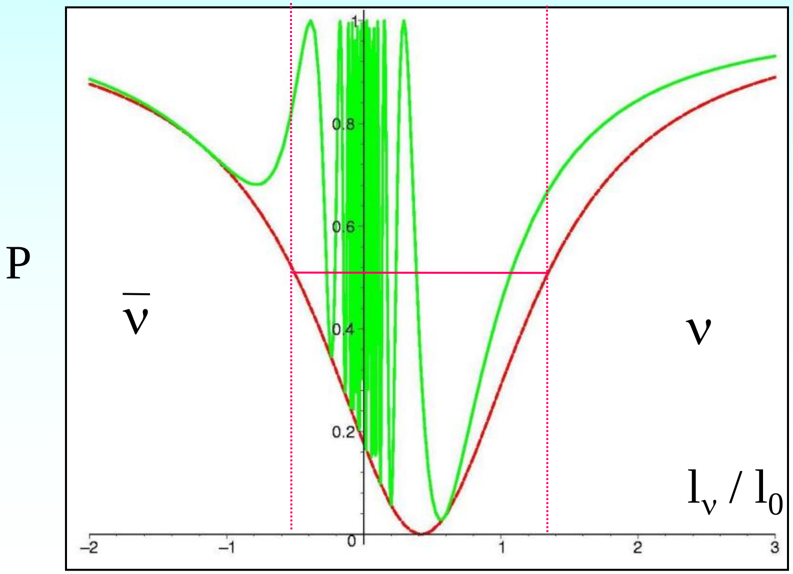
$$E_R^0 = \Delta m^2 / 2V$$

- With increase of mixing: $\theta \to \pi/4$

$$E_R \to 0$$

$$\Delta E_R \to E_R^0$$





A Yu Smirnov

Neutrinos and antineutrinos

- Continuity:
  neutrino and antineutrino semiplanes
  normal and inverted hierarchy

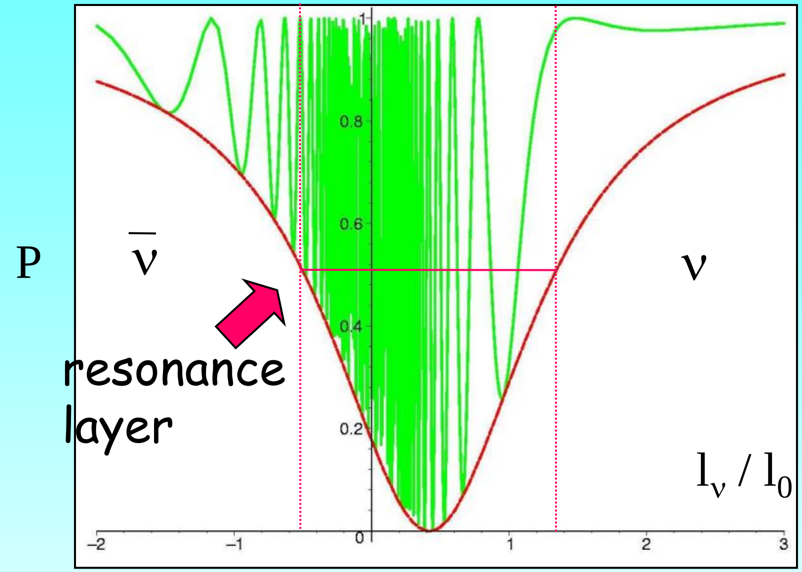
- Oscillations (amplitude of oscillations)
  are enhanced in the resonance layer

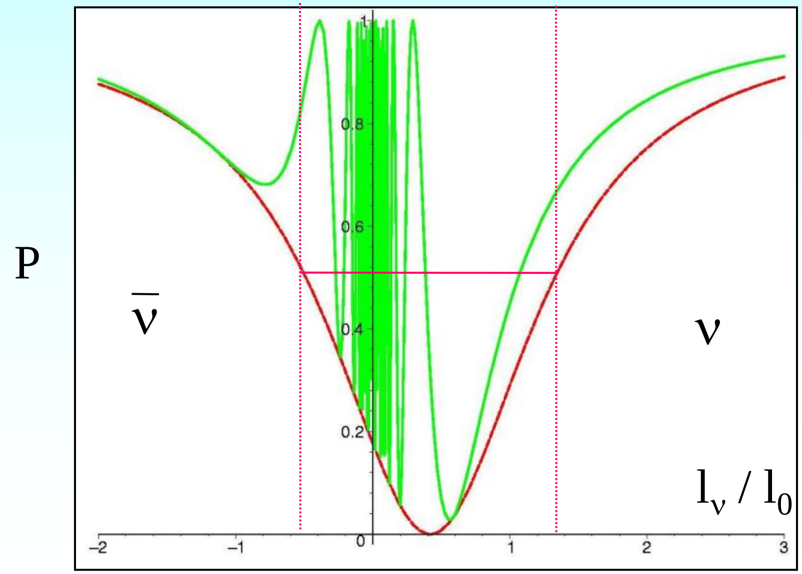
$$E = (E_R - \Delta E_R) \text{ -- } (E_R + \Delta E_R)$$
$$\Delta E_R = E_R \tan 2\theta = E_R^0 \sin 2\theta$$

$$E_R^0 = \Delta m^2 / 2V$$

- With increase of mixing: $\theta \to \pi/4$

$$E_R \to 0$$
$$\Delta E_R \to E_R^0$$





A Yu Smirnov

Evolution equation

`Physics derivation'

Input
- neutrinos are ultrarelativistic $E \sim p + m^2/2E$
- no spin-flip, no change of the spinor structure
- lowest order in m/E

In vacuum the mass states are the eigenstates of Hamiltonian

$$i \frac{d\nu_{mass}}{dt} = \left(p\,I + \frac{1}{2E} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \right) \nu_{mass} \qquad \nu_{mass} = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Using relation $\nu_{mass} = U^+ \nu_f$ find equation for the flavor states:

$$i \frac{d\nu_f}{dt} = \frac{M^2}{2E} \nu_f \qquad \nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

the term pI proportional to unit matrix is omitted

where

$$M^2 = U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^+$$

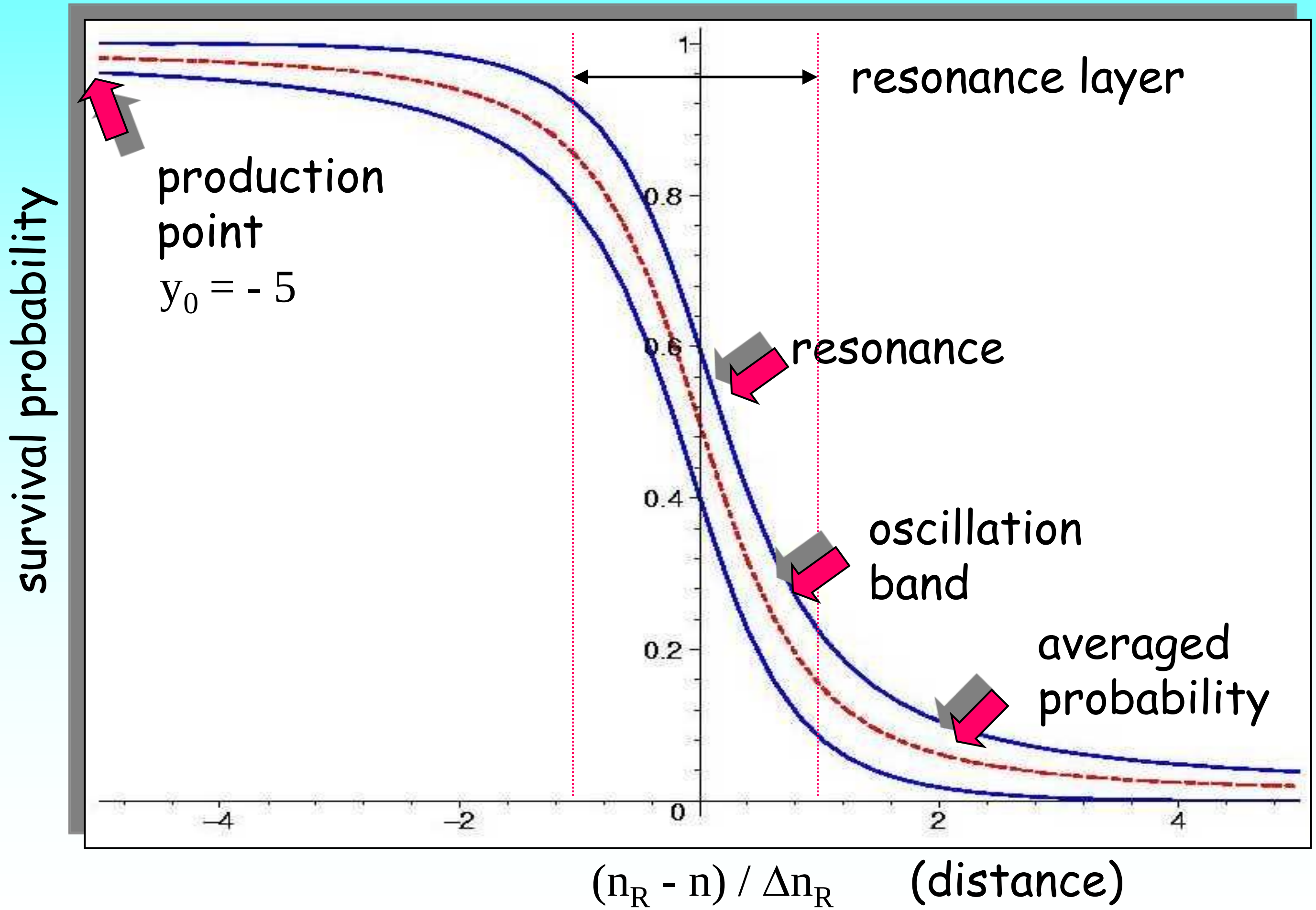
mass matrix in flavor basis

Spatial picture

The picture is universal in terms of variable $y = (n_R - n) / \Delta n_R$

no explicit dependence on oscillation parameters, density distribution, etc.

only initial value y_0 matters



A Yu Smirnov

Degrees of freedom

Arbitrary state:

$$\nu(t) = \cos\theta_a\, \nu_{1m} + \sin\theta_a\, \nu_{2m}\, e^{-i\phi(t)}$$

Effects associated to different degrees of freedom

- $\theta_a = \theta_a(t)$ - determines the admixtures of the eigenstates
- $\phi(t)$ is the phase difference between the two eigenstates

$$\phi(t) = \int_0^t H\, dt`$$

Adiabaticity violation

Oscillations

Adiabatic conversion

- Flavors (flavor composition) of the eigenstates are determined by the mixing angle in matter

$$\langle\nu_e|\nu_{1m}\rangle = \cos\theta_m \qquad \langle\nu_\mu|\nu_{1m}\rangle = -\sin\theta_m$$

- Combination of effects

$\theta_a(t) + \phi(t)$ -> parametric effects, etc.

$\theta_m(t) + \phi(t)$ -> ad. conv. + oscillations

Survival Probability

Non-uniform medium