Coherence of Supernova Neutrinos

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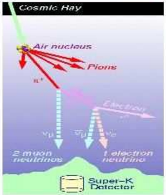
Based on work done in collaboration with A.Yu. Smirnov

Neutrino Sources

Sun

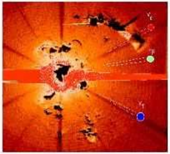
Earth

Supernovae



Atmosphere

Reactors



Big Bang

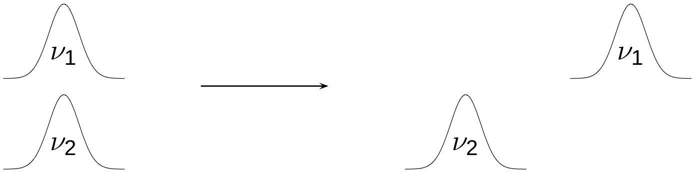
Accelerators & Laboratories

Neutrino Oscillations and Decoherence

- Normal (coherent) 2-flavor oscillation probability

$$P(\nu_e \to \nu_e) = |\cos^2\theta + \sin^2\theta e^{i\phi}|^2 = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

- Oscillation phase $\phi = -\frac{\Delta m^2 L}{2E}$
- Mass eigenstates have different velocities
  - $\leadsto$ Wave packets cease to overlap

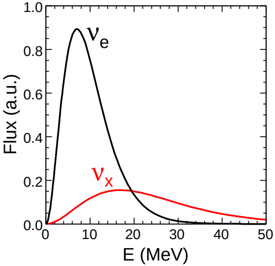


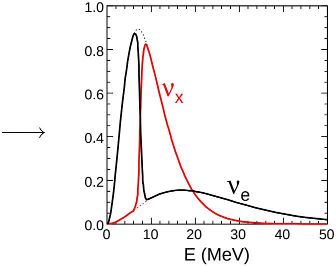
  - $\leadsto$ Probability is incoherent sum

$$P(\nu_e \to \nu_e) = |\cos^2\theta|^2 + |\sin^2\theta e^{i\phi}|^2 = \cos^4\theta + \sin^4\theta$$

  - $\leadsto$ No oscillations between supernova and Earth

- Oscillations inside the Earth
- MSW and collective effects inside the supernova





Is coherence preserved in these cases?

Coherence Length of Supernova Neutrinos

- Wave packets overlap up to coherence length $L_{\text{coh}} \sim \dfrac{\sigma}{\Delta v}$
- Depends on
  - Size of wave packets σ
  - Velocity difference Δv of mass eigenstates

Size of Wave Packets

- Determined by length of production process
- Electron capture $pe^- \to n\nu_e$
- Time scale $\sim$ time electron needs to cross proton, $\tau \sim \sigma_e / v_e$
- Temperature $\sim 5\,\text{MeV} \Rightarrow$ electron relativistic, $v_e \sim 1$
- Size of electron wave packet $\sigma_e \sim$ mean free path

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- Result: $\sigma \sim \sigma_e \sim \left(4\pi\alpha^2 n\right)^{-1/3} \sim 10^{-11}$ cm
- For comparison:
  - Atmospheric neutrinos: $\sigma \sim 1$ cm
  - Reactor neutrinos: $\sigma \sim 10^{-8}$ cm
- $\sigma_E \sim 1/\sigma \sim 1$ MeV not much smaller than $E \sim 10$ MeV

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Supernova neutrinos are special

Very short wave packets $\rightsquigarrow$ short coherence length

Sometimes Decoherence Is Irrelevant

$$P(\nu_e \to \nu_e) = |\cos^2\theta + \sin^2\theta e^{i\phi}|^2$$
$$= \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\cos\phi$$

- Measured probability: average over energy resolution ΔE of detector

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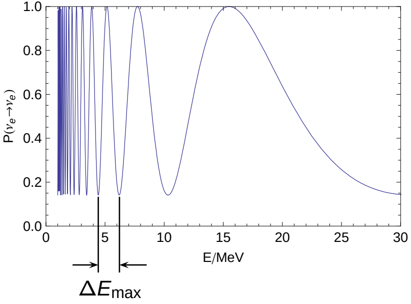

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$$
\begin{aligned}
P(\nu_e \to \nu_e) &= |\cos^2\theta + \sin^2\theta e^{i\phi}|^2 \\
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- Coherence preserved for wave packets arriving within Δt, even if they are spacially separated
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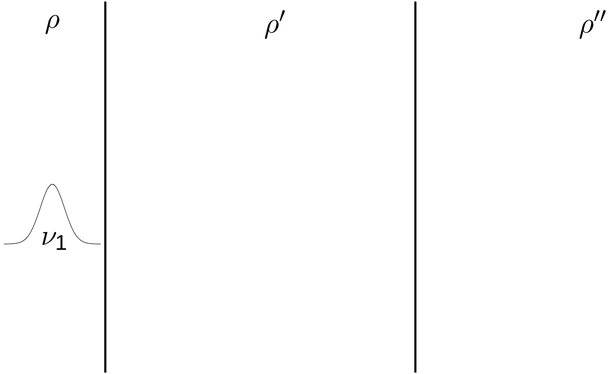
- Always the case in vacuum and matter with slowly changing density (adiabatic case)
- Does this change in non-adiabatic case?

Adiabaticity Violation

- Simplest case: density step
- Each wave packet splits up into two



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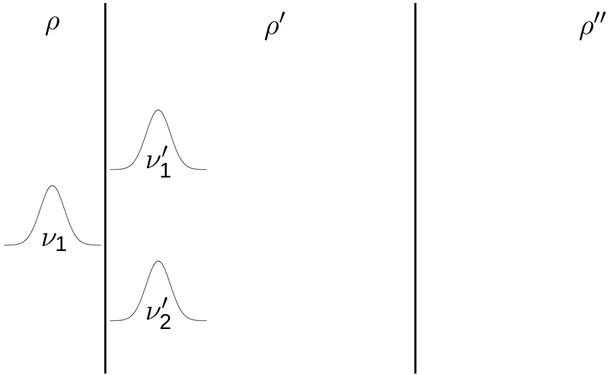

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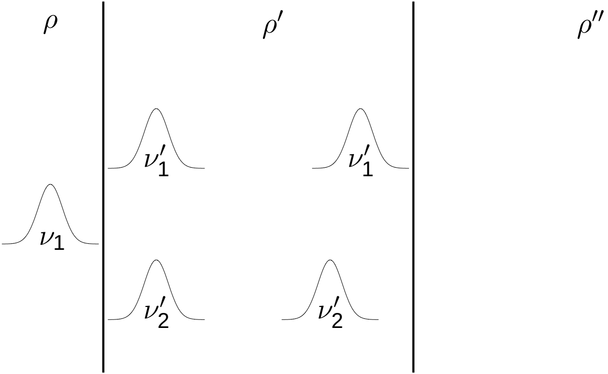

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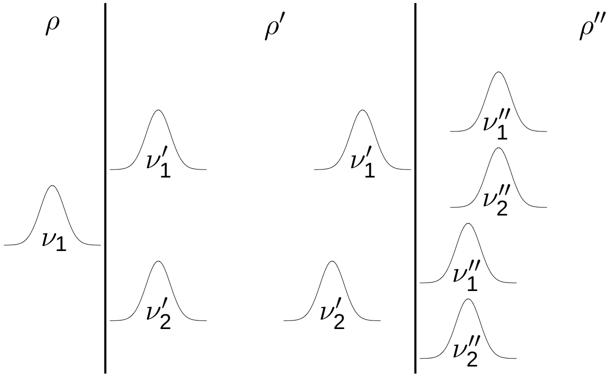

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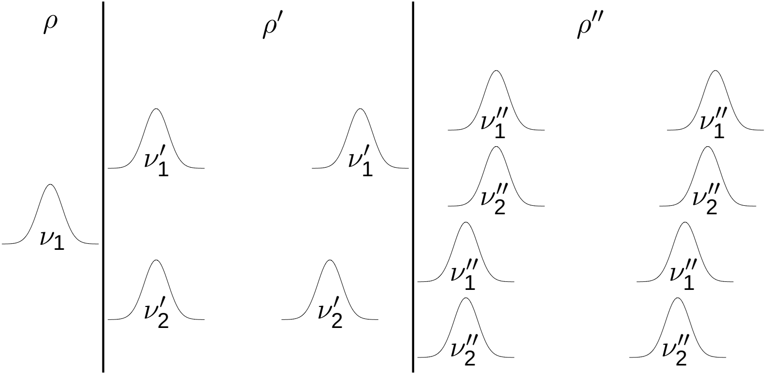

Adiabaticity Violation

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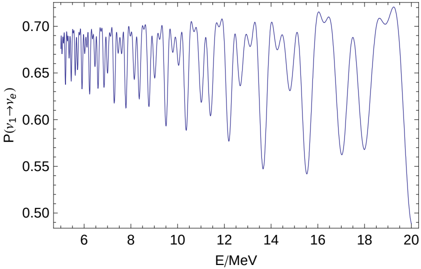
- Each wave packet splits up into two
- Different oscillation phase in each layer
- Complete coherence:

$$P = \left| a + b\,e^{i\phi_1} + c\,e^{i\phi_2} + \ldots \right|^2$$



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$$P = \left| a + b\, e^{i\phi_1} + c\, e^{i\phi_2} + \ldots \right|^2$$

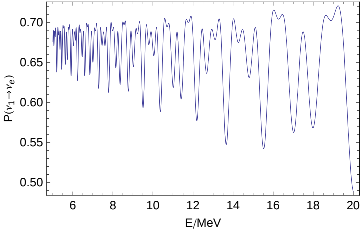
- Complete decoherence:

$$P = a^2 + b^2 + c^2 + \ldots$$

- Energy resolution good enough to resolve all oscillation features
  $\Rightarrow$ detector restores coherence as before

Incomplete Averaging

$$P = \left| a + b\, e^{i\phi_1} + c\, e^{i\phi_2} + \ldots \right|^2$$
$$= a^2 + b^2 + c^2 + \cdots + 2ab\cos\phi_1 + 2bc\cos(\phi_2 - \phi_1)$$



Energy resolution may be

- too bad to observe $\cos\phi_1$ term
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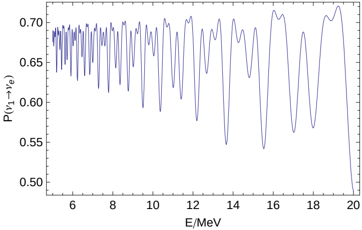

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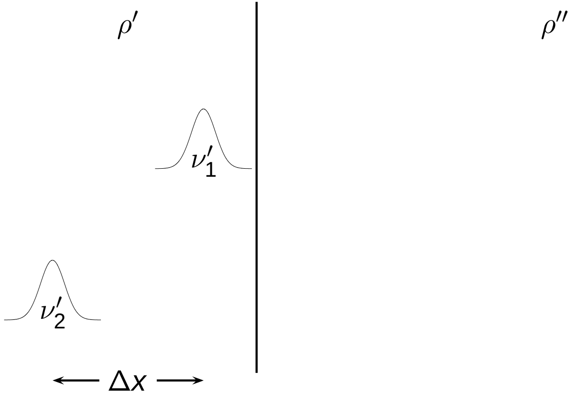
But only if wave packets overlap (coherent case)?

Wave Packet Catch-Up

- Wave packets don't need to separate forever:



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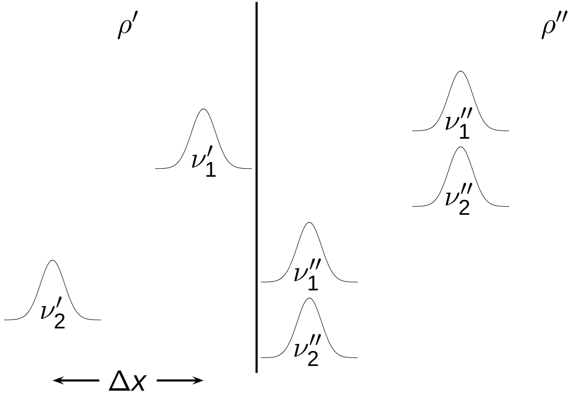

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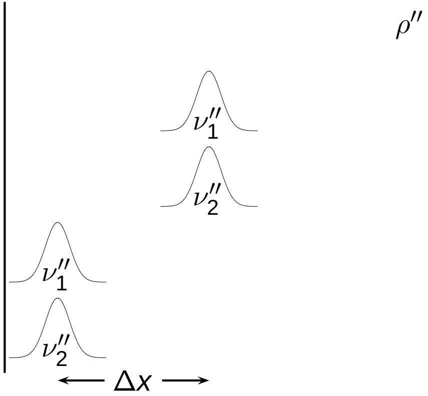

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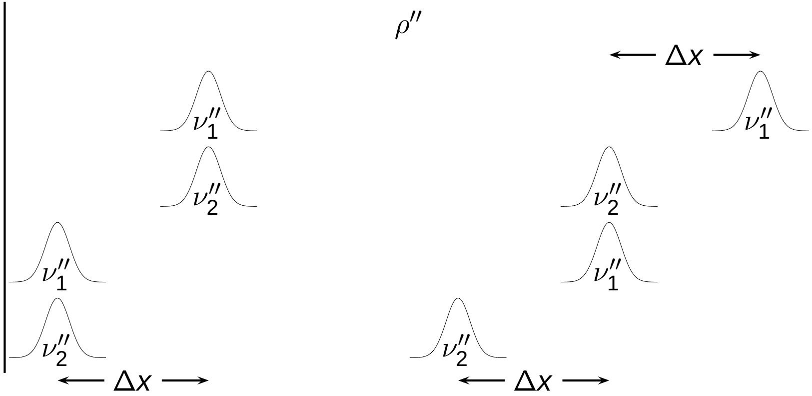

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- Wave packets don't need to separate forever:
- Wave packets catch up $\Longleftrightarrow$ ΔE good enough to observe $\cos(\phi_2 - \phi_1)$ term
- Decoherence and energy averaging are equivalent
- Open questions:
  - How general is this?
  - Are collective effects in a supernova different?

Increase of the Coherence Length

- Vacuum or constant matter density:

$$L_{\text{coh}} \sim \frac{E}{\Delta E}$$

- Matter with density jumps: For some interference terms

$$L_{\text{coh}} \sim \left(\frac{E}{\Delta E}\right)^3$$

- Coherence length increases by $\sim$ 2 orders of magnitude
- Only valid in certain energy range $\rightsquigarrow$ no complete oscillation pattern restored
- Effect small for small density jumps $\rightsquigarrow$ below per cent level in the Earth

Conclusions

Supernova Neutrinos

- Extremely small size of wave packets $\sigma \sim 10^{-11}$ cm
- No experimental consequences in simple examples
- Possibly relevant for
  - collective effects in a supernova
  - energy spectrum
- Matter, non-adiabaticity $\rightsquigarrow$ increase of coherence length