Supernova Neutrino Oscillations

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3400 citations

Neutrino oscillations in matter

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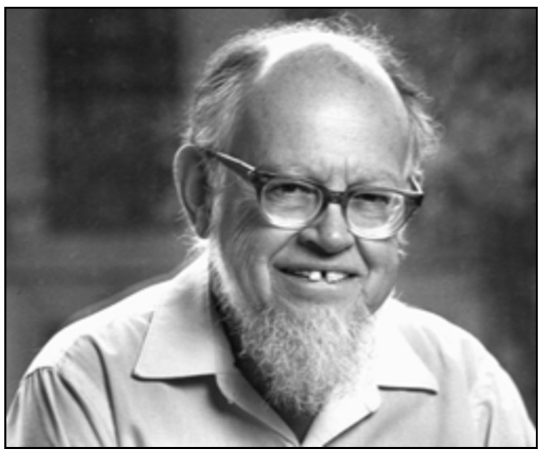
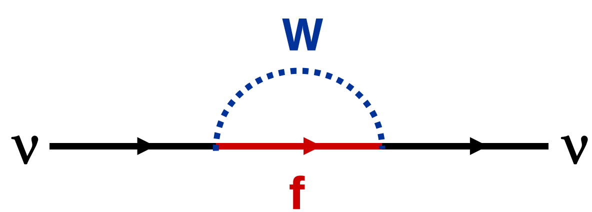
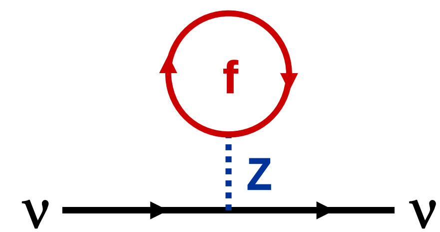
The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein

Neutrinos in a medium suffer flavor-dependent refraction





$$V_{\rm weak} = \sqrt{2} G_{\rm F} \times \begin{cases} N_{\rm e} - N_{\rm n}/2 & \text{for } \nu_{\rm e} \\ -N_{\rm n}/2 & \text{for } \nu_{\mu} \end{cases}$$

Typical density of Earth: 5 g/cm^3

$$\Delta V_{\rm weak} \approx 2 \times 10^{-13}\ {\rm eV} = 0.2\ {\rm peV}$$

Suppression of Oscillations in Supernova Core

Effective mixing angle in matter

$$\tan 2\theta_\mathrm{m} = \frac{\sin 2\theta}{\cos 2\theta - N_e 2E\sqrt{2}G_\mathrm{F}/\Delta m^2}$$

Supernova core

$$\rho = 3 \times 10^{14}\ \mathrm{g\ cm^{-3}}$$
$$Y_e = 0.35$$
$$N_e = 6 \times 10^{37}\ \mathrm{cm^{-3}}$$
$$E \sim 100\ \mathrm{MeV}$$

Solar mixing

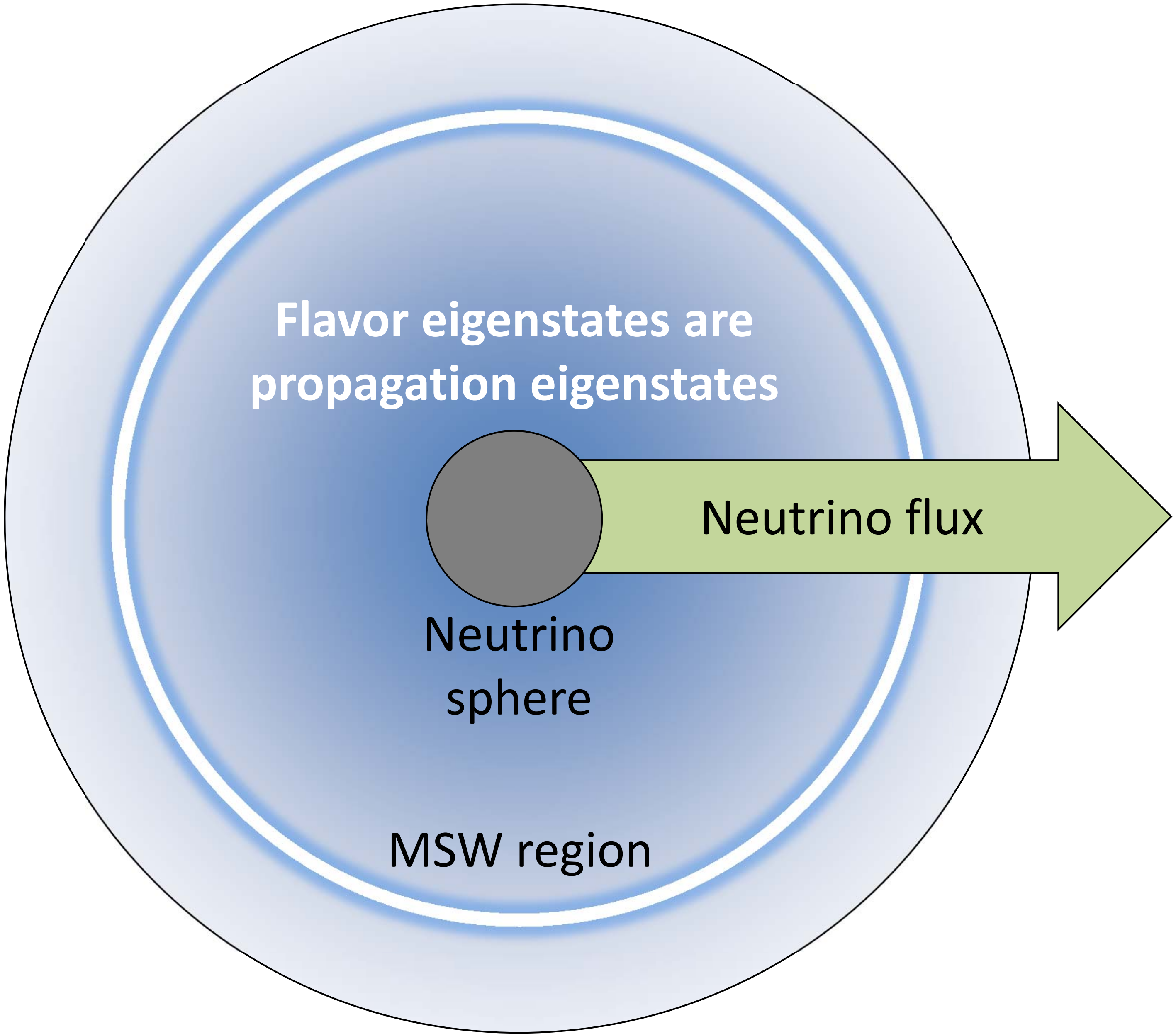
$$\Delta m^2 \sim 75\ \mathrm{meV}^2$$
$$\sin 2\theta \sim 0.94$$

Matter suppression effect

$$N_e 2E\sqrt{2}G_F/\Delta m^2 \sim 2 \times 10^{13}$$

- Inside a SN core, flavors are “de-mixed”
- Very small oscillation amplitude
- Trapped e-lepton number can only escape by diffusion

Flavor Oscillations in Core-Collapse Supernovae



Neutrino Oscillations in Matter

2-flavor neutrino evolution as an effective 2-level problem

$$\mathrm{i}\frac{\partial}{\partial z}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

With a 2×2 Hamiltonian matrix

Negative for $\overline{\nu}$

$$H = \frac{1}{2E}\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} \pm \sqrt{2}G_\mathrm{F}\begin{pmatrix}N_e - \frac{N_n}{2} & 0\\ 0 & -\frac{N_n}{2}\end{pmatrix}$$

Mass-squared matrix, rotated by mixing angle θ relative to interaction basis, drives oscillations

$$\frac{\Delta m^2}{2E} \sim \begin{cases} 4\ \mathrm{peV} & \text{for 12 mass splitting}\\ 120\ \mathrm{peV} & \text{for 13 mass splitting}\end{cases}$$

Solar, reactor and supernova neutrinos:
E ~ 10 MeV

Weak potential difference

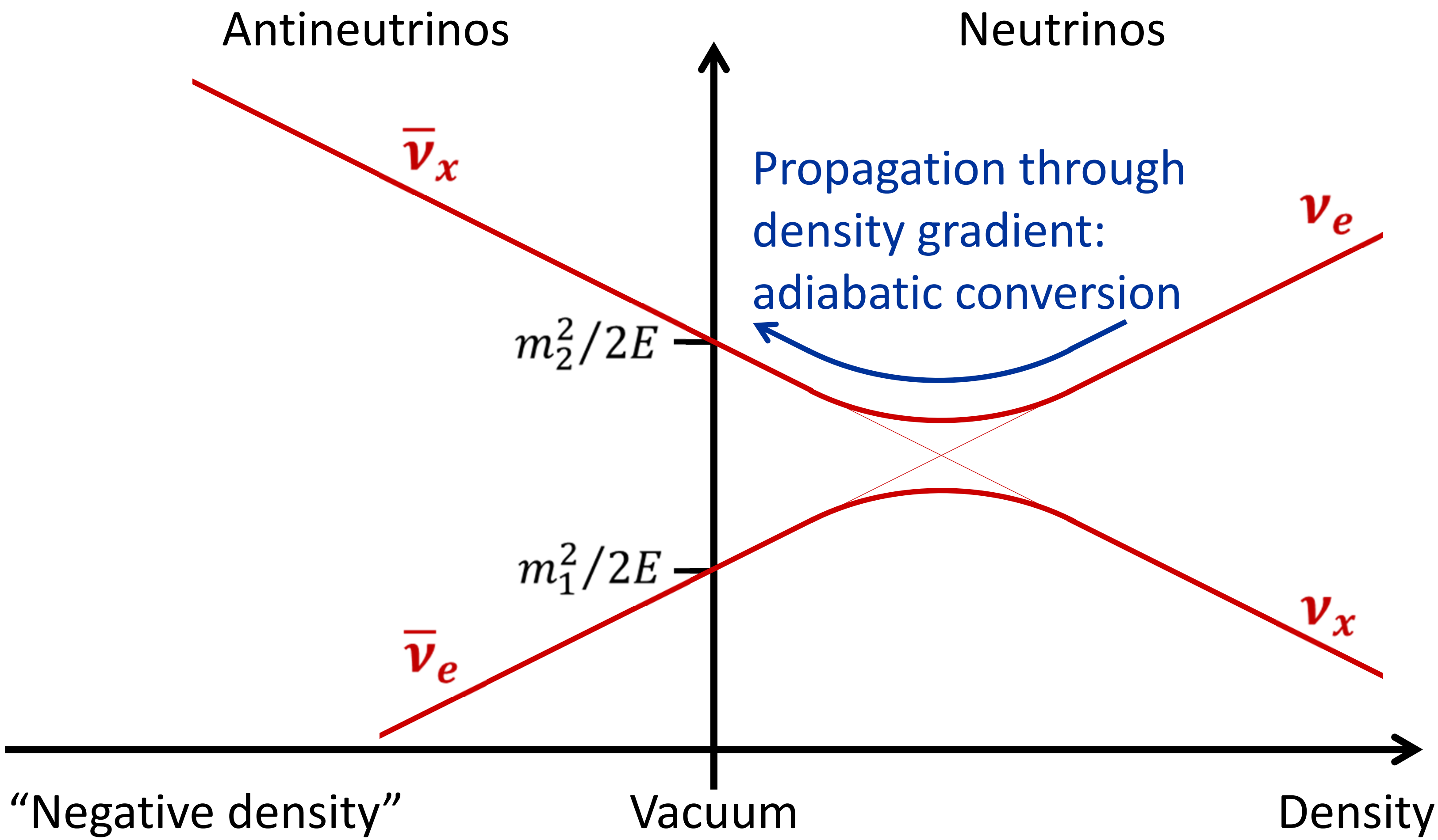
$$\Delta V_\mathrm{weak} = \sqrt{2}G_\mathrm{F}N_e \sim 0.2\ \mathrm{peV}$$

for normal Earth matter, but large effect in SN core (nuclear density $3\times10^{14}\ \mathrm{g/cm^3}$)

$$\Delta V_\mathrm{weak} \sim 10\ \mathrm{eV}$$

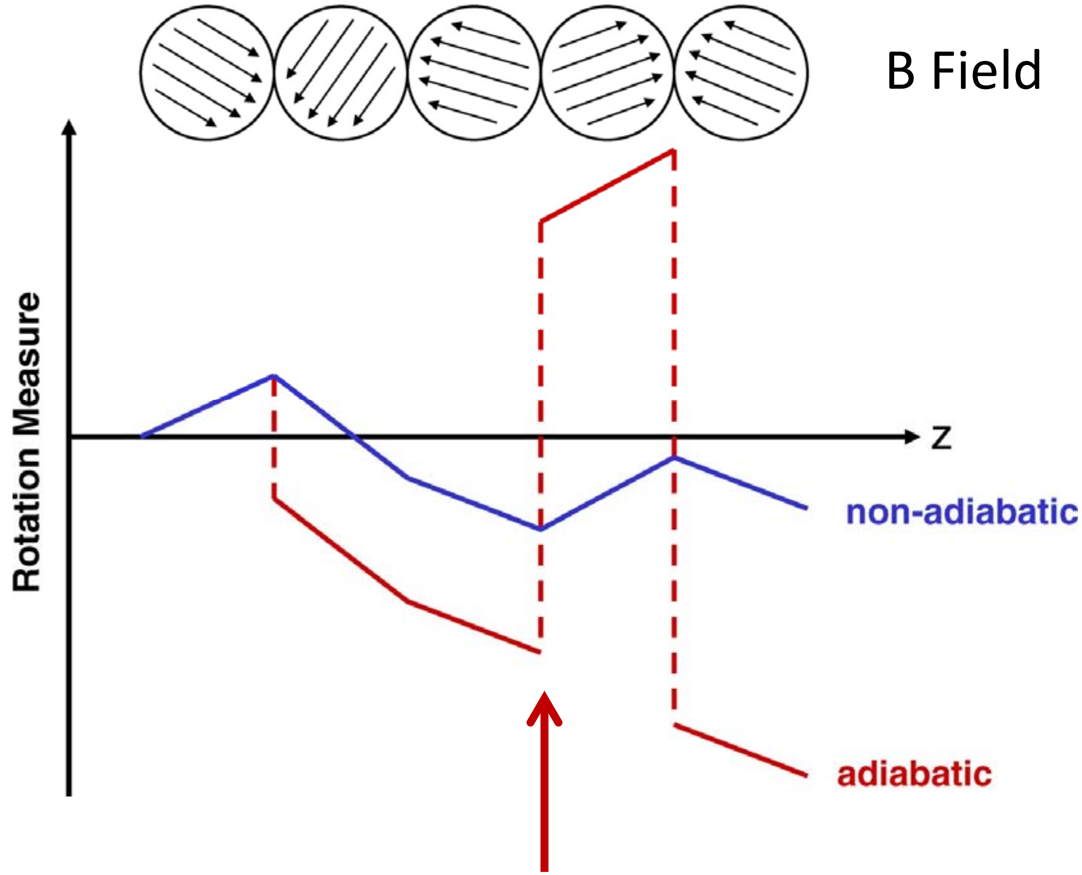
Mikheev-Smirnov-Wolfenstein (MSW) effect

Eigenvalue diagram of 2×2 Hamiltonian matrix for 2-flavor oscillations



"Negative density" represents antineutrinos in the same diagram

Adiabatic Faraday Effect in Analogy to MSW Effect



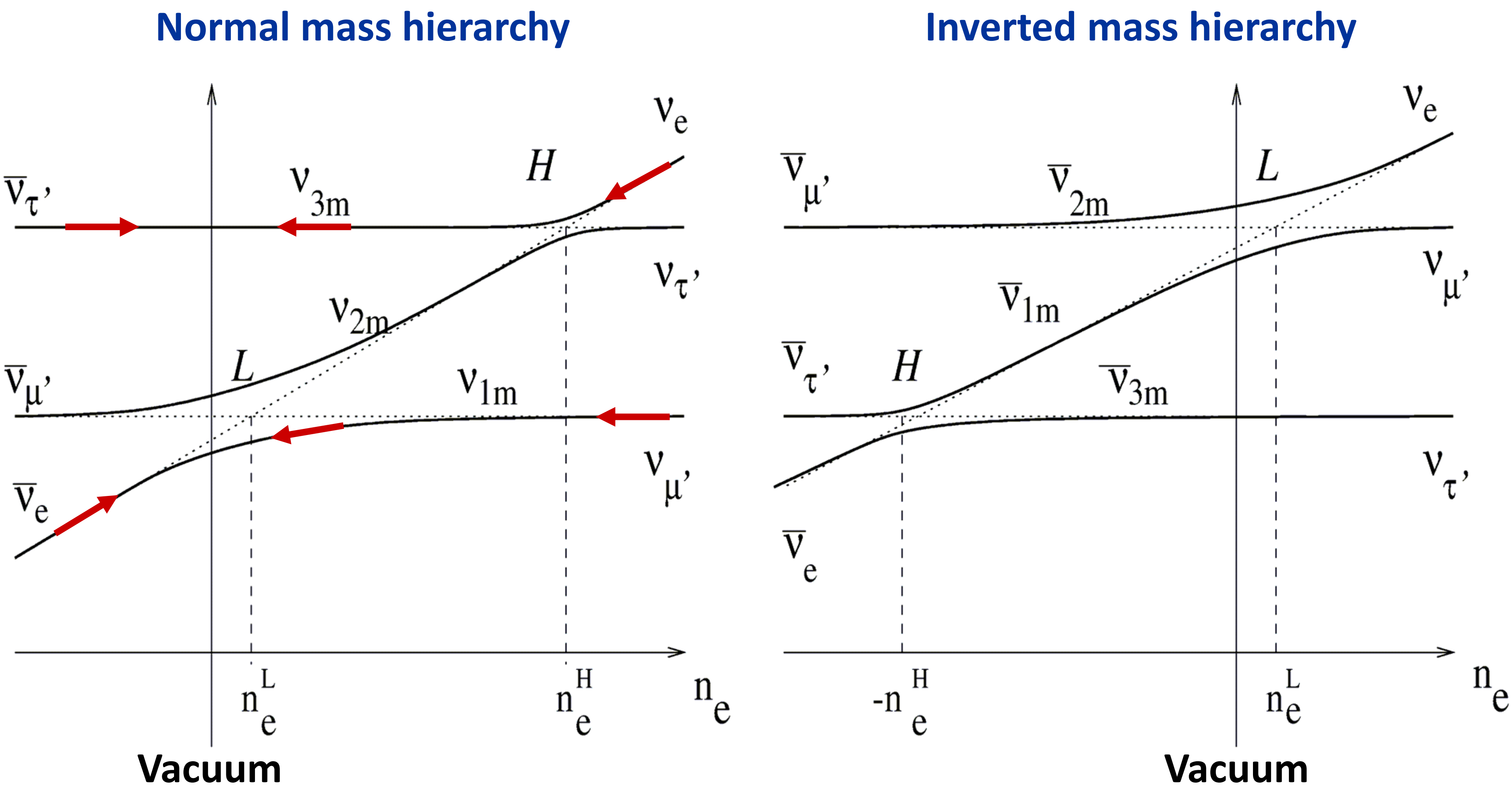
$\mathrm{RM} \propto \int B \cdot d\ell$

$\mathrm{RM} \propto \int |B \cdot d\ell|$

Photon helicity reversed adiabatically, reversing the rotation measure

Dasgupta & Raffelt, arXiv:1006.4158

Three-Flavor Eigenvalue Diagram



Dighe & Smirnov, Identifying the neutrino mass spectrum from a supernova neutrino burst, astro-ph/9907423

Signature of Flavor Oscillations (Accretion Phase)

	1-3-mixing scenarios		
	A	B	C
Mass ordering	Normal (NH)	Inverted (IH)	Any (NH/IH)
$\sin^2\theta_{13}$	$\gtrsim 10^{-3}$		$\lesssim 10^{-5}$
MSW conversion	adiabatic		non-adiabatic
ν_e survival prob.	0	$\sin^2\theta_{12} \approx 0.3$	$\sin^2\theta_{12} \approx 0.3$
$\bar{\nu}_e$ survival prob.	$\cos^2\theta_{12} \approx 0.7$	0	$\cos^2\theta_{12} \approx 0.7$
$\bar{\nu}_e$ Earth effects	Yes	No	Yes
	May distinguish mass ordering		

Assuming collective effects are not important during accretion phase (Chakraborty et al., arXiv:1105.1130, Sarikas et al. arXiv:1109.3601)

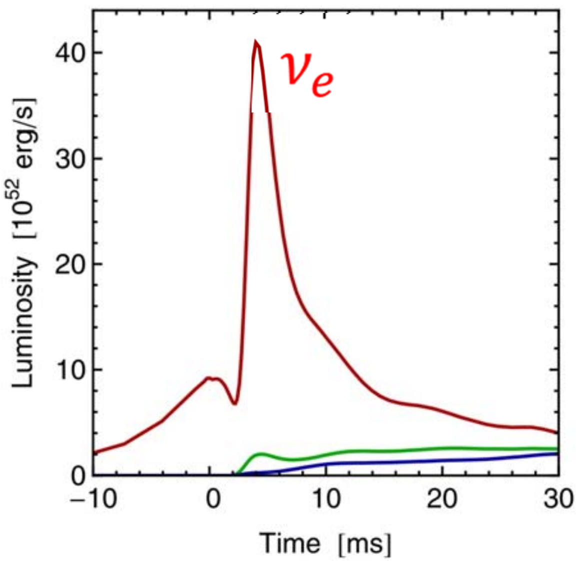
Signature of Flavor Oscillations (Accretion Phase)

	1-3-mixing scenarios		
	A	B	C
Mass ordering	Normal (NH)	Inverted (IH)	Any (NH/IH)
$\sin^2\theta_{13}$	$\Theta_{13} \approx 9°$		$\lesssim 10^{-5}$
MSW conversion	adiabatic		non-adiabatic
ν_e survival prob.	0	$\sin^2\theta_{12} \approx 0.3$	$\sin^2\theta_{12} \approx 0.3$
$\bar{\nu}_e$ survival prob.	$\cos^2\theta_{12} \approx 0.7$	0	$\cos^2\theta_{12} \approx 0.7$
$\bar{\nu}_e$ Earth effects	Yes	No	Yes
	May distinguish mass ordering		

Assuming collective effects are not important during accretion phase
(Chakraborty et al., arXiv:1105.1130, Sarikas et al. arXiv:1109.3601)

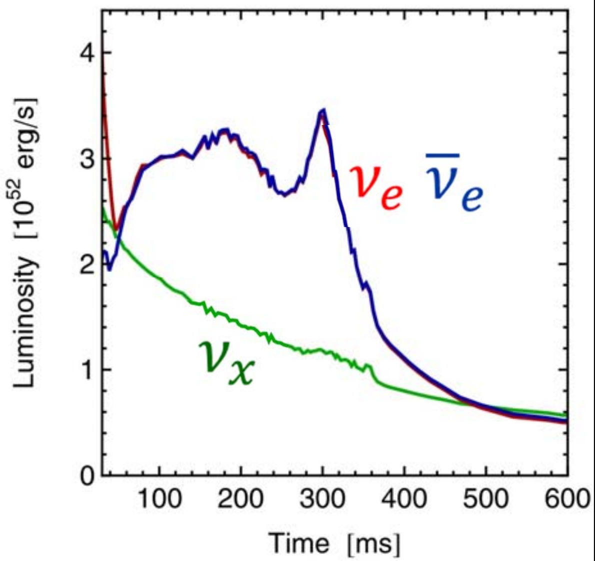
Three Phases of Neutrino Emission

Prompt ν_e burst



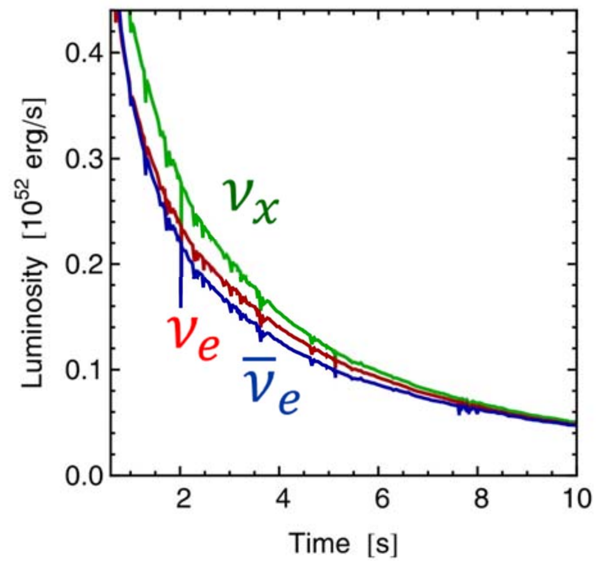
- Shock breakout
- De-leptonization of outer core layers

Accretion



- Shock stalls ~ 150 km
- Neutrinos powered by infalling matter

Cooling

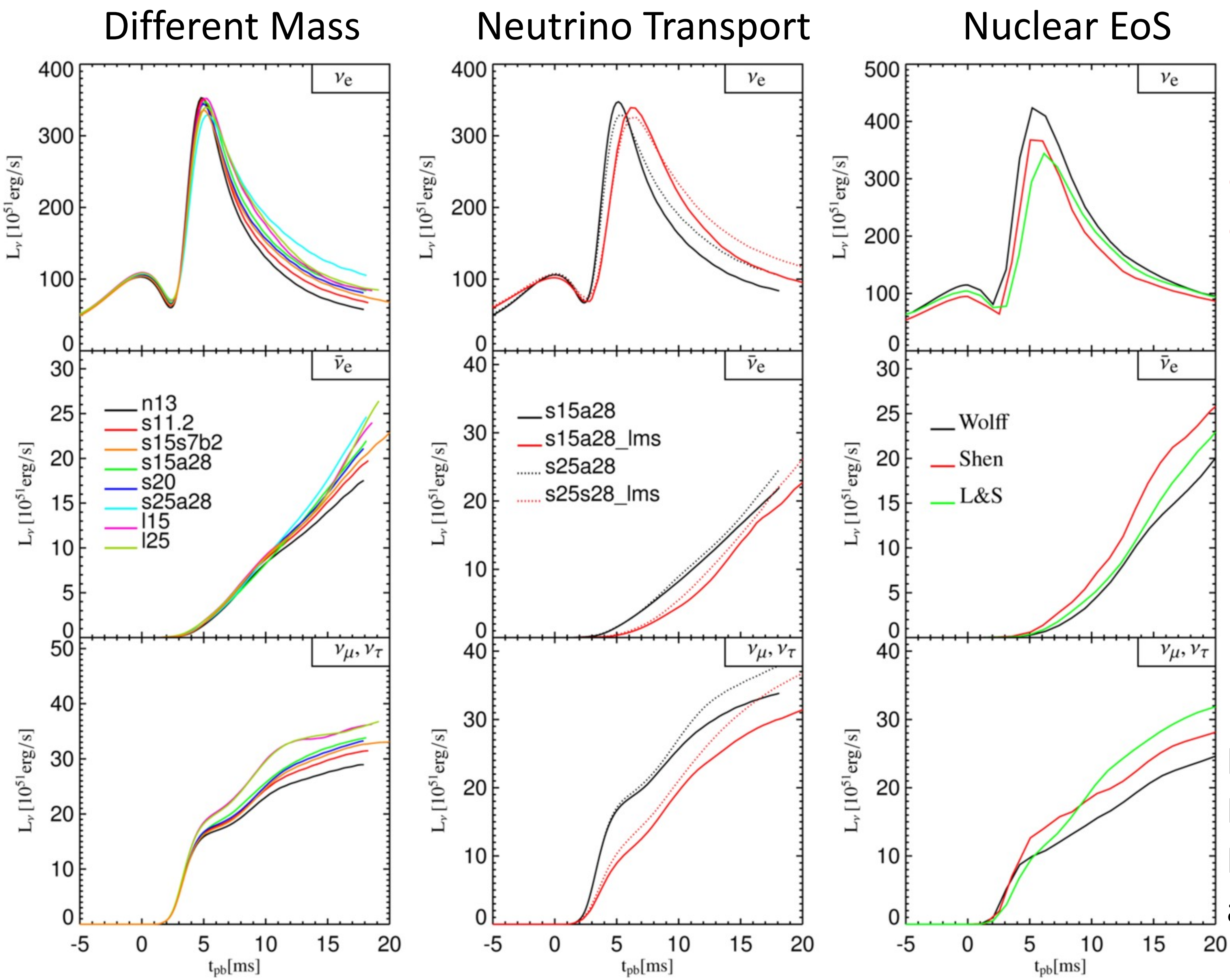


Cooling on neutrino diffusion time scale

- Spherically symmetric model (10.8 $M_\odot$) with Boltzmann neutrino transport
- Explosion manually triggered by enhanced CC interaction rate

Fischer et al. (Basel group), A&A 517:A80, 2010 [arxiv:0908.1871]

Neutronization Burst as a Standard Candle

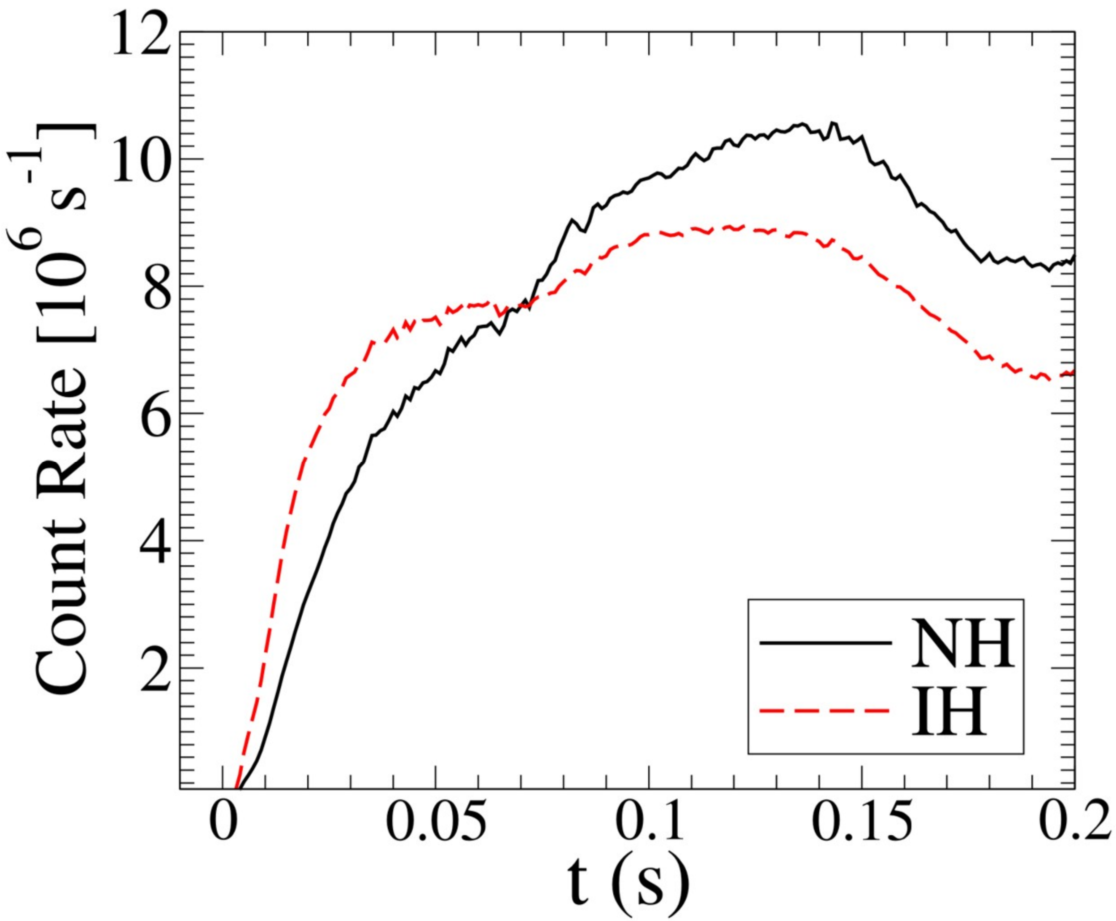


If mixing scenario is known, can determine SN distance (better than 5-10%)

Kachelriess, Tomàs, Buras, Janka, Marek & Rampp, astro-ph/0412082

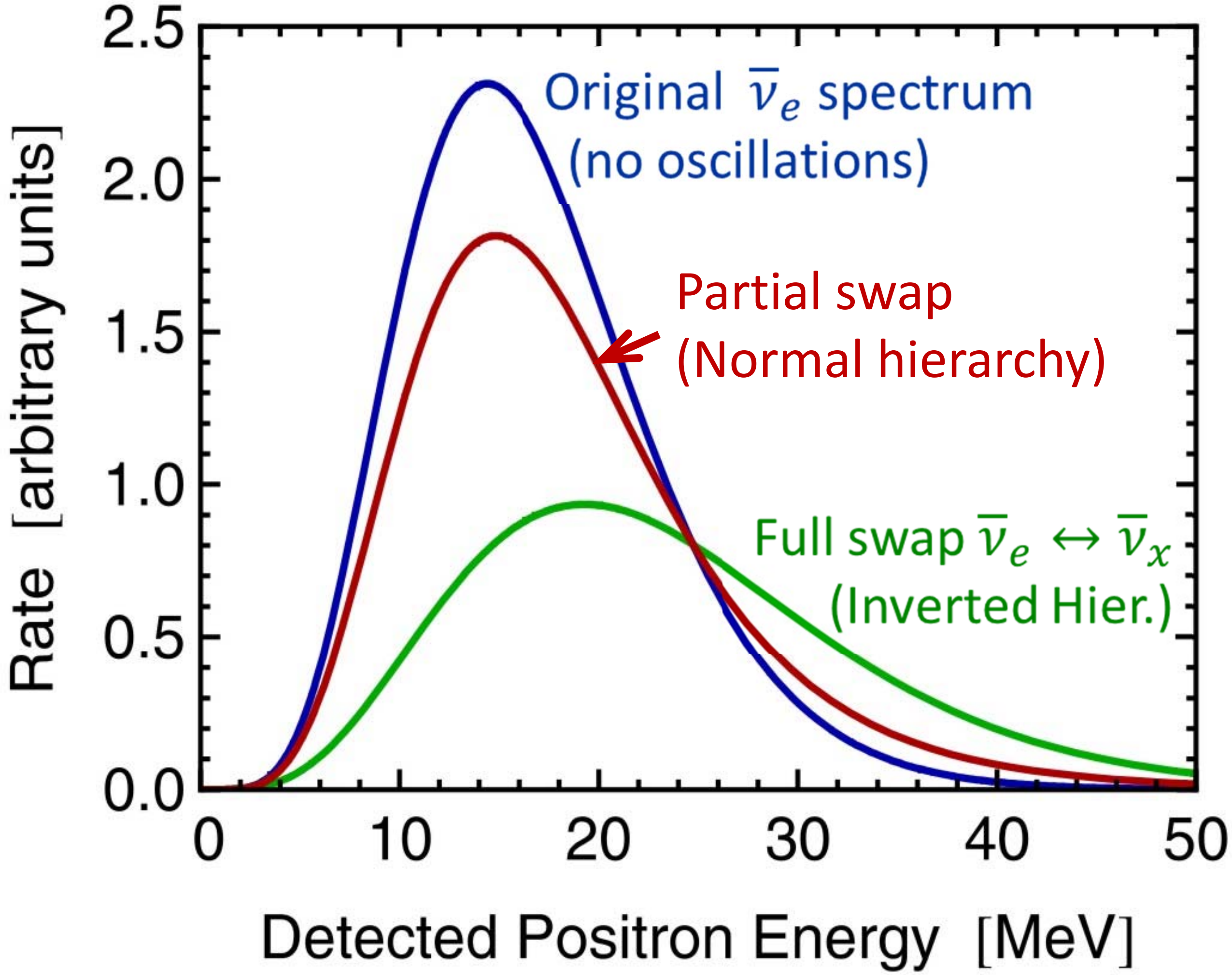
Rise Time as Hierarchy Discriminator

Rise time of counting rate in IceCube can distinguish hierarchy (for "large" θ_{13}), but depends on numerical model calibration



Chakraborty, Fischer, Hüdepohl, Janka, Mirizzi, Serpico, arXiv:1111.4483

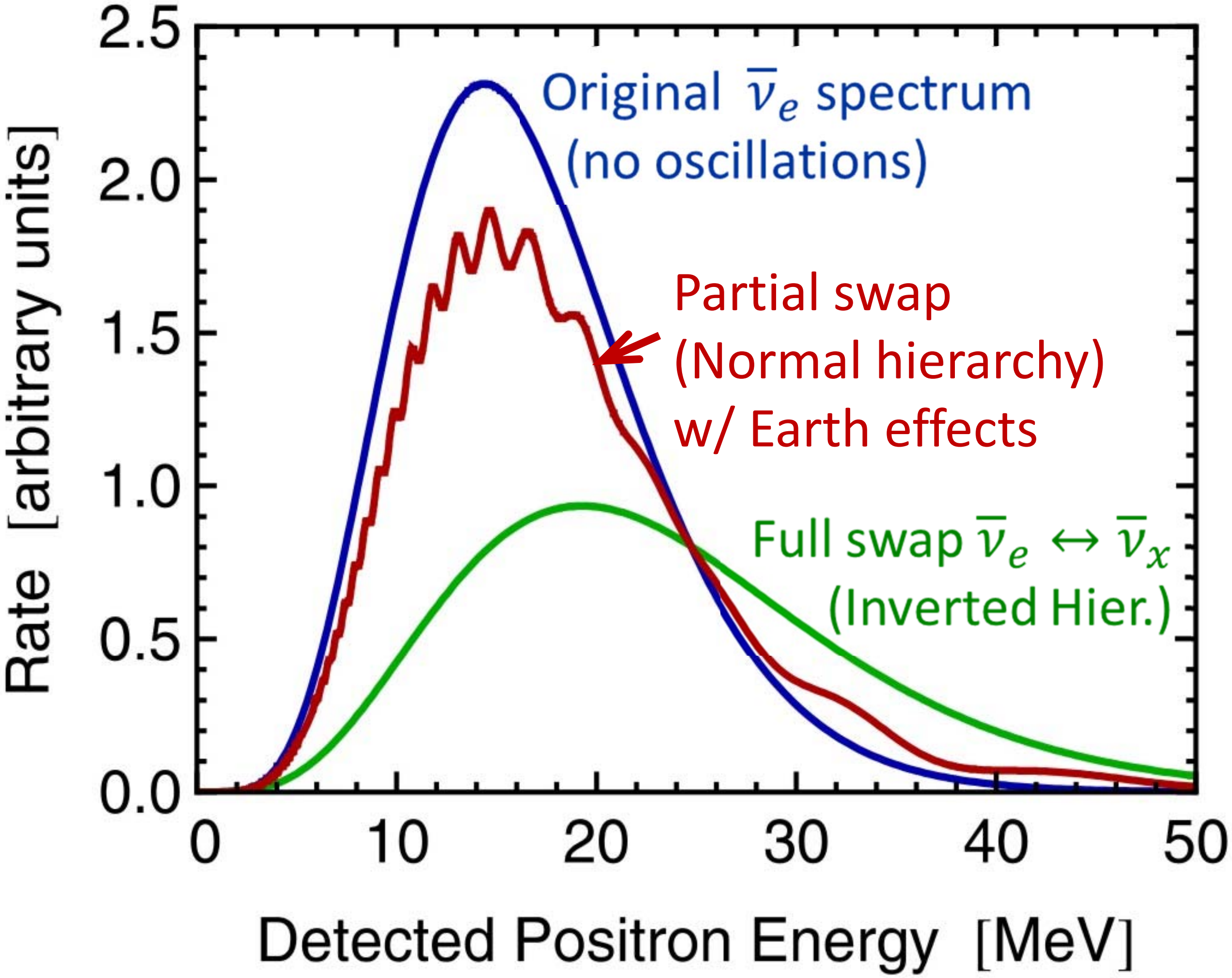
Oscillation of Supernova Anti-Neutrinos



Basel accretion phase model ($10.8\ M_\odot$)

Detection spectrum by $\overline{\nu}_e + p \to n + e^+$ (water Cherenkov or scintillator detectors)

Oscillation of Supernova Anti-Neutrinos



Basel accretion phase model (10.8 $M_\odot$)

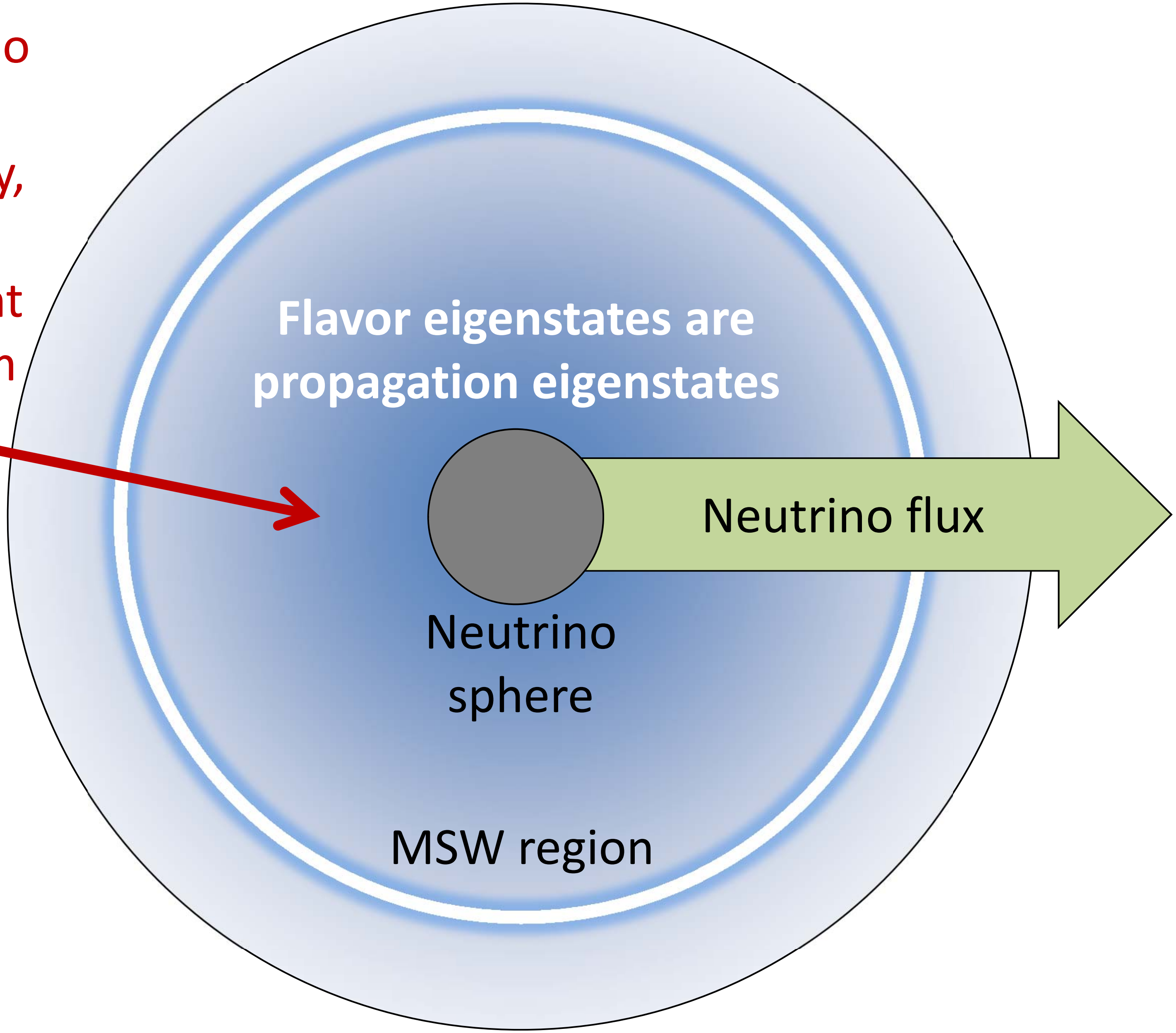
Detection spectrum by $\overline{\nu}_e + p \rightarrow n + e^+$ (water Cherenkov or scintillator detectors)

8000 km path length in Earth assumed

Detecting Earth effects requires good energy resolution
(Large scintillator detector, e.g. LENA, or megaton water Cherenkov)

Flavor Oscillations in Core-Collapse Supernovae

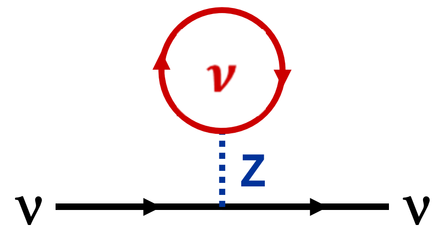
Neutrino-neutrino refraction causes a flavor instability, flavor exchange between different parts of spectrum



Flavor-Off-Diagonal Refractive Index

2-flavor neutrino evolution as an effective 2-level problem

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$



Effective mixing Hamiltonian

$$H = \frac{M^2}{2E} + \sqrt{2}G_\mathrm{F}\begin{pmatrix} N_e - \frac{N_n}{2} & 0 \\ 0 & -\frac{N_n}{2}\end{pmatrix} + \sqrt{2}G_\mathrm{F}\begin{pmatrix} N_{\nu_e} & N_{\langle\nu_e|\nu_\mu\rangle} \\ N_{\langle\nu_\mu|\nu_e\rangle} & N_{\nu_\mu}\end{pmatrix}$$

Mass term in flavor basis: causes vacuum oscillations

Wolfenstein's weak potential, causes MSW "resonant" conversion together with vacuum term

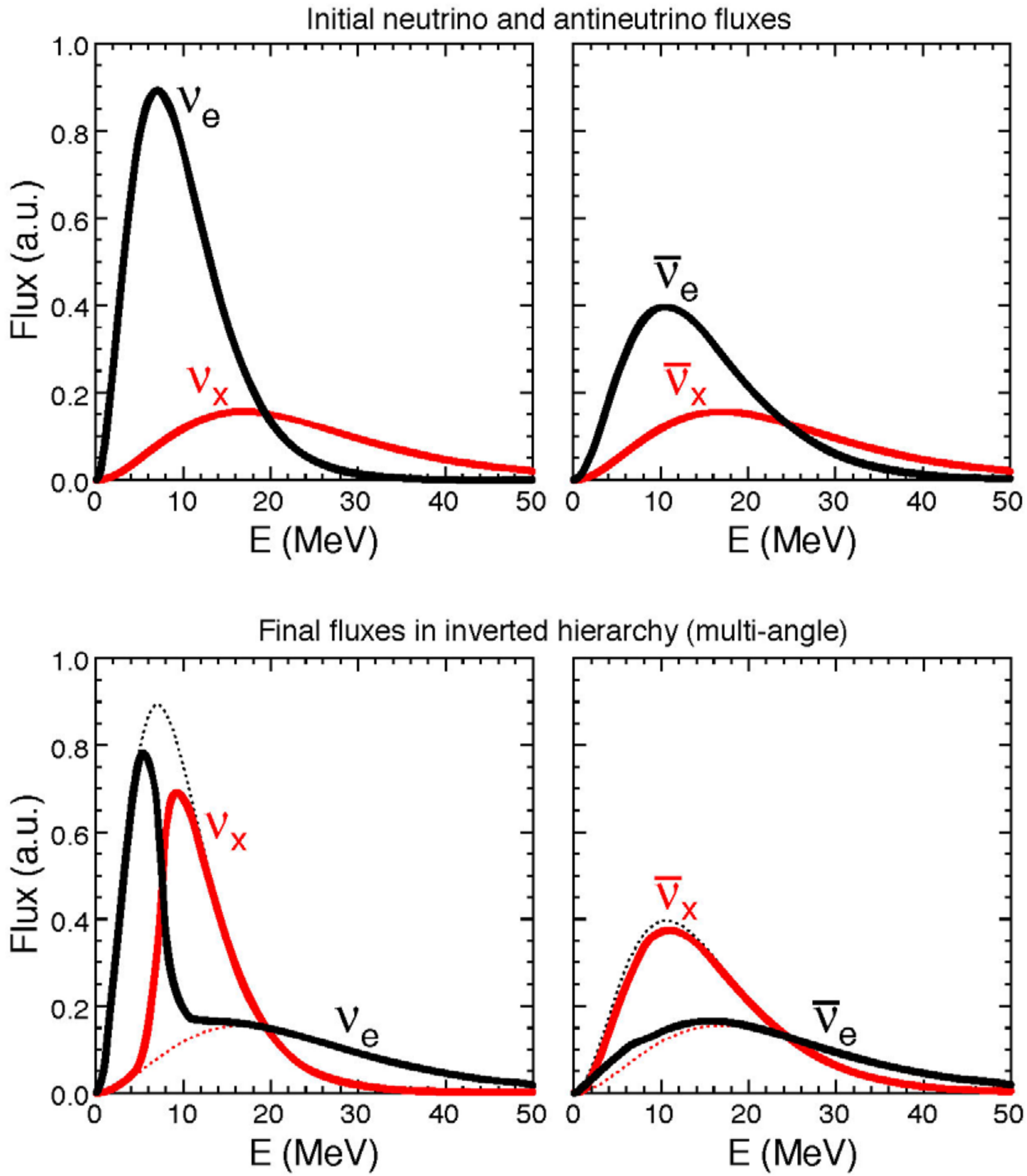
Flavor-off-diagonal potential, caused by flavor oscillations. (J.Pantaleone, PLB 287:128,1992)

Flavor oscillations feed back on the Hamiltonian: Nonlinear effects!

Spectral Split

Initial fluxes at neutrino sphere

After collective trans-formation



Figures from

Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998

Explanations in

Raffelt & Smirnov arXiv:0705.1830 and 0709.4641

Duan, Fuller, Carlson & Qian arXiv:0706.4293 and 0707.0290

Collective Supernova Nu Oscillations since 2006

Two seminal papers in 2006 triggered a torrent of activities
Duan, Fuller, Qian, astro-ph/0511275, Duan et al. astro-ph/0606616

Balantekin, Gava & Volpe [0710.3112]. Balantekin & Pehlivan [astro-ph/0607527]. Blennow, Mirizzi & Serpico [0810.2297]. Cherry, Fuller, Carlson, Duan & Qian [1006.2175, 1108.4064]. Cherry, Wu, Fuller, Carlson, Duan & Qian [1109.5195]. Cherry, Carlson, Friedland, Fuller & Vlasenko [1203.1607]. Chakraborty, Choubey, Dasgupta & Kar [0805.3131]. Chakraborty, Fischer, Mirizzi, Saviano, Tomàs [1104.4031, 1105.1130]. Choubey, Dasgupta, Dighe & Mirizzi [1008.0308]. Dasgupta & Dighe [0712.3798]. Dasgupta, Dighe & Mirizzi [0802.1481]. Dasgupta, Dighe, Raffelt & Smirnov [0904.3542]. Dasgupta, Dighe, Mirizzi & Raffelt [0801.1660, 0805.3300]. Dasgupta, Mirizzi, Tamborra & Tomàs [1002.2943]. Dasgupta, Raffelt & Tamborra [1001.5396]. Dasgupta, O'Connor & Ott [1106.1167]. Duan, Fuller, Carlson & Qian [astro-ph/0608050, 0703776, 0707.0290, 0710.1271]. Duan, Fuller & Qian [0706.4293, 0801.1363, 0808.2046, 1001.2799]. Duan, Fuller & Carlson [0803.3650]. Duan & Kneller [0904.0974]. Duan & Friedland [1006.2359]. Duan, Friedland, McLaughlin & Surman [1012.0532]. Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl [0807.0659]. Esteban-Pretel, Pastor, Tomàs, Raffelt & Sigl [0706.2498, 0712.1137]. Fogli, Lisi, Marrone & Mirizzi [0707.1998]. Fogli, Lisi, Marrone & Tamborra [0812.3031]. Friedland [1001.0996]. Gava & Jean-Louis [0907.3947]. Gava & Volpe [0807.3418]. Galais, Kneller & Volpe [1102.1471]. Galais & Volpe [1103.5302]. Gava, Kneller, Volpe & McLaughlin [0902.0317]. Hannestad, Raffelt, Sigl & Wong [astro-ph/0608695]. Wei Liao [0904.0075, 0904.2855]. Lunardini, Müller & Janka [0712.3000]. Mirizzi, Pozzorini, Raffelt & Serpico [0907.3674]. Mirizzi & Serpico [1111.4483]. Mirizzi & Tomàs [1012.1339]. Pehlivan, Balantekin, Kajino & Yoshida [1105.1182]. Pejcha, Dasgupta & Thompson [1106.5718]. Raffelt [0810.1407, 1103.2891]. Raffelt & Sigl [hep-ph/0701182]. Raffelt & Smirnov [0705.1830, 0709.4641]. Raffelt & Tamborra [1006.0002]. Sawyer [hep-ph/0408265, 0503013, 0803.4319, 1011.4585]. Sarikas, Raffelt, Hüdepohl & Janka [1109.3601]. Sarikas, Tamborra, Raffelt, Hüdepohl & Janka [1204.0971]. Saviano, Chakraborty, Fischer, Mirizzi [1203.1484]. Wu & Qian [1105.2068].

Three Ways to Describe Flavor Oscillations

Schrödinger equation in terms of "flavor spinor"

$$\mathrm{i}\partial_t\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = H\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \frac{\Delta m^2}{2E}\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

Neutrino flavor density matrix

$$\rho = \begin{pmatrix}\langle\nu_e|\nu_e\rangle & \langle\nu_e|\nu_\mu\rangle\\ \langle\nu_\mu|\nu_e\rangle & \langle\nu_\mu|\nu_\mu\rangle\end{pmatrix}$$

Equivalent commutator form of Schrödinger equation

$$\mathrm{i}\partial_t\rho = [H,\rho]$$

Expand 2×2 Hermitean matrices in terms of Pauli matrices

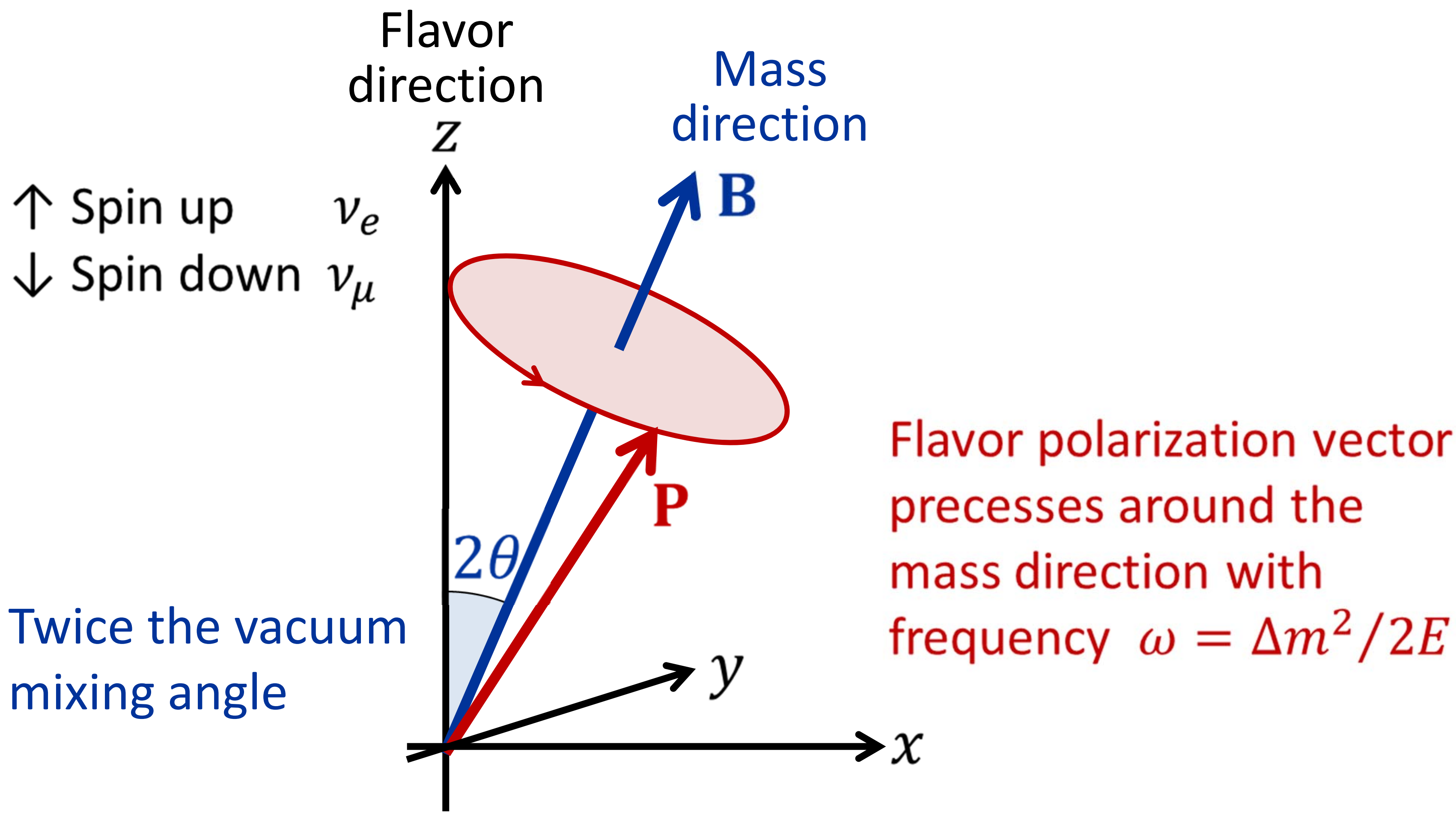
$\rho = \frac{1}{2}[\mathrm{Tr}(\rho) + \mathbf{P}\cdot\boldsymbol{\sigma}]$ and $H = \frac{\Delta m^2}{2E}\mathbf{B}\cdot\boldsymbol{\sigma}$ with $\mathbf{B} = (\sin 2\theta, 0, \cos 2\theta)$

Equivalent spin-precession form of equation of motion

$\dot{\mathbf{P}} = \omega\mathbf{B}\times\mathbf{P}$ with $\omega = \frac{\Delta m^2}{2E}$

$\mathbf{P}$ is "polarization vector" or "Bloch vector" or "flavor isospin vector"

Flavor Oscillation as Spin Precession



Flavor Matrices of Occupation Numbers

Neutrinos described by Dirac field

$$\Psi(t,x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[a_\mathbf{p}(t)u_\mathbf{p} + b^\dagger_{-\mathbf{p}}(t)\, v_{-\mathbf{p}}\right] e^{\mathrm{i}\mathbf{p}\cdot\mathbf{x}}$$

in terms of the spinors in flavor space, providing spinor of flavor amplitudes

$$\Psi = \begin{pmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\end{pmatrix},\quad a = \begin{pmatrix}a_1\\ a_2\\ a_3\end{pmatrix},\quad b = \begin{pmatrix}b_1\\ b_2\\ b_3\end{pmatrix} \quad \text{and} \quad \begin{vmatrix}\nu_1(t,\mathbf{p})\\ \nu_2(t,\mathbf{p})\\ \nu_3(t,\mathbf{p})\end{vmatrix}\Bigg\rangle = \begin{pmatrix}a_1^\dagger(t,\mathbf{p})\\ a_2^\dagger(t,\mathbf{p})\\ a_3^\dagger(t,\mathbf{p})\end{pmatrix}|0\rangle$$

Measurable quantities are expectation values of field bi-linears $\langle\Psi^\dagger\Psi\rangle$, therefore use "occupation number matrices" to describe the ensemble and its kinetic evolution (Boltzmann eqn for oscillations and collisions)

(ν) $\quad \rho_{ij} = \langle a_j^\dagger a_i\rangle$

$(\overline{\nu})$ $\quad \overline{\rho}_{ij} = \langle b_j b_i^\dagger\rangle = 1 - \langle b_i^\dagger b_j\rangle$

Drops out in commutators

Describe $\overline{\nu}$ with negative occupation numbers, reversed order of flavor indices (holes in Dirac sea)

Equations of Motion for Occupation Number Matrices

Mikheev-Smirnov-Wolfenstein term

Ordinary matter effect caused by matrix of charged lepton densities

$$L = \begin{pmatrix} N_e - N_{\overline{e}} & 0 & 0 \\ 0 & N_\mu - N_{\overline{\mu}} & 0 \\ 0 & 0 & N_\tau - N_{\overline{\tau}} \end{pmatrix}$$

Pantaleone term

Non-linear effect caused by nu-nu interactions has the same structure. In isotropic medium $1 - \cos\theta_{\mathbf{pq}} \to 1$ and $\sum_{\mathbf{q}}\left(\rho_{\mathbf{q}} + \overline{\rho}_{\mathbf{q}}\right)$ is matrix of net neutrino densities (not diagonal)

$$\dot{\rho}_{\mathbf{p}} = +\frac{1}{2E}\left[M^2, \rho_{\mathbf{p}}\right] + \sqrt{2}G_{\mathrm{F}}\left[L, \rho_{\mathbf{p}}\right] + \sqrt{2}G_{\mathrm{F}}\sum_{\mathbf{q}}(1 - \cos\theta_{\mathbf{pq}})\left[(\rho_{\mathbf{q}} + \overline{\rho}_{\mathbf{q}}), \rho_{\mathbf{p}}\right]$$

$$\dot{\overline{\rho}}_{\mathbf{p}} = -\frac{1}{2E}\left[M^2, \overline{\rho}_{\mathbf{p}}\right] + \sqrt{2}G_{\mathrm{F}}\left[L, \overline{\rho}_{\mathbf{p}}\right] + \sqrt{2}G_{\mathrm{F}}\sum_{\mathbf{q}}(1 - \cos\theta_{\mathbf{pq}})\left[(\rho_{\mathbf{q}} + \overline{\rho}_{\mathbf{q}}), \overline{\rho}_{\mathbf{p}}\right]$$

- Vacuum oscillations driven by mass-squared matrix in flavor basis
- Opposite sign for ν and $\overline{\nu}$
- Treat $\overline{\nu}$ modes as ν modes with negative energy: $\omega = \Delta m^2/2E \to -\omega$ for same momentum $\mathbf{p}$

Pontecorvo term

Collective Nu Oscillations as a Many-Body Problem

Hamiltonian for interacting "flavor spins" (*classical* in mean-field approach)

$$H = \sum_{i=1}^{N} \omega_i \mathbf{B} \cdot \mathbf{P}_i + \lambda \mathbf{L} \cdot \sum_{i=1}^{N} \mathbf{P}_i + \mu \sum_{i,j=1}^{N} (1 - \cos\theta_{ij})\, \mathbf{P}_i \cdot \mathbf{P}_j$$

Unit vector in mass direction

Unit vector in flavor direction

Multi-angle effects from current-current structure

"Spin-pairing H" for isotropic system (or single angle), ignoring matter effect

$$H = \sum_{i=1}^{N} \omega_i \mathbf{B} \cdot \mathbf{P}_i + \mu \mathbf{P}_{\rm tot}^2$$

BCS theory (using Anderson's pseudo-spin), nuclear physics, ...

Integrable system (as many "Gaudin invariants" as spins)

→ Pehlivan, Balantekin, Kajino & Yoshida [arxiv:1105.1182] for introduction

N-mode coherent solutions ("Normal and anomalous solitons")

- Emil Yuzbashian, Phys. Rev. **B** 78, 184507 (2008) Super-conductivity (BCS)
- Georg Raffelt, Phys. Rev. **D** 83, 105022 (2011) Collective Nus

Adding Matter

Schrödinger equation including matter

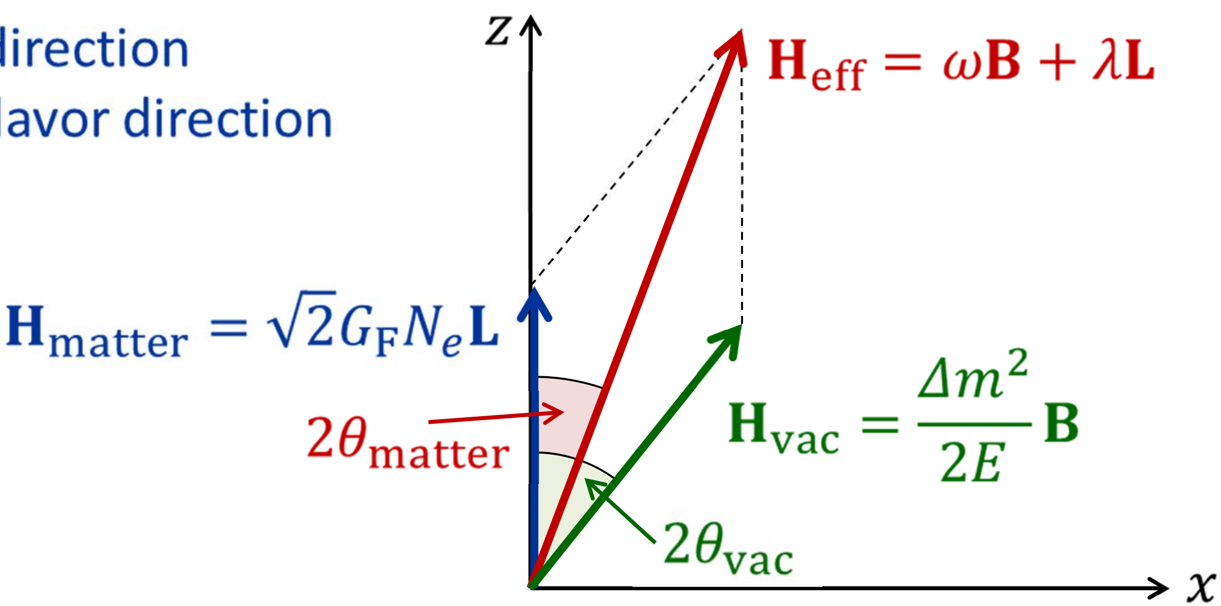
$$\mathrm{i}\partial_t \begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \left[\frac{\Delta m^2}{2E}\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix} + \sqrt{2}G_F\begin{pmatrix}N_e - \frac{N_n}{2} & 0\\ 0 & -\frac{N_n}{2}\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

Corresponding spin-precession equation

$\dot{\mathbf{P}} = \underbrace{(\omega\mathbf{B} + \lambda\mathbf{L})}_{\mathbf{H}_\mathrm{eff}} \times \mathbf{P}$ with $\omega = \Delta m^2/2E$ and $\lambda = \sqrt{2}G_\mathrm{F}N_e$

$\mathbf{B}$ unit vector in mass direction

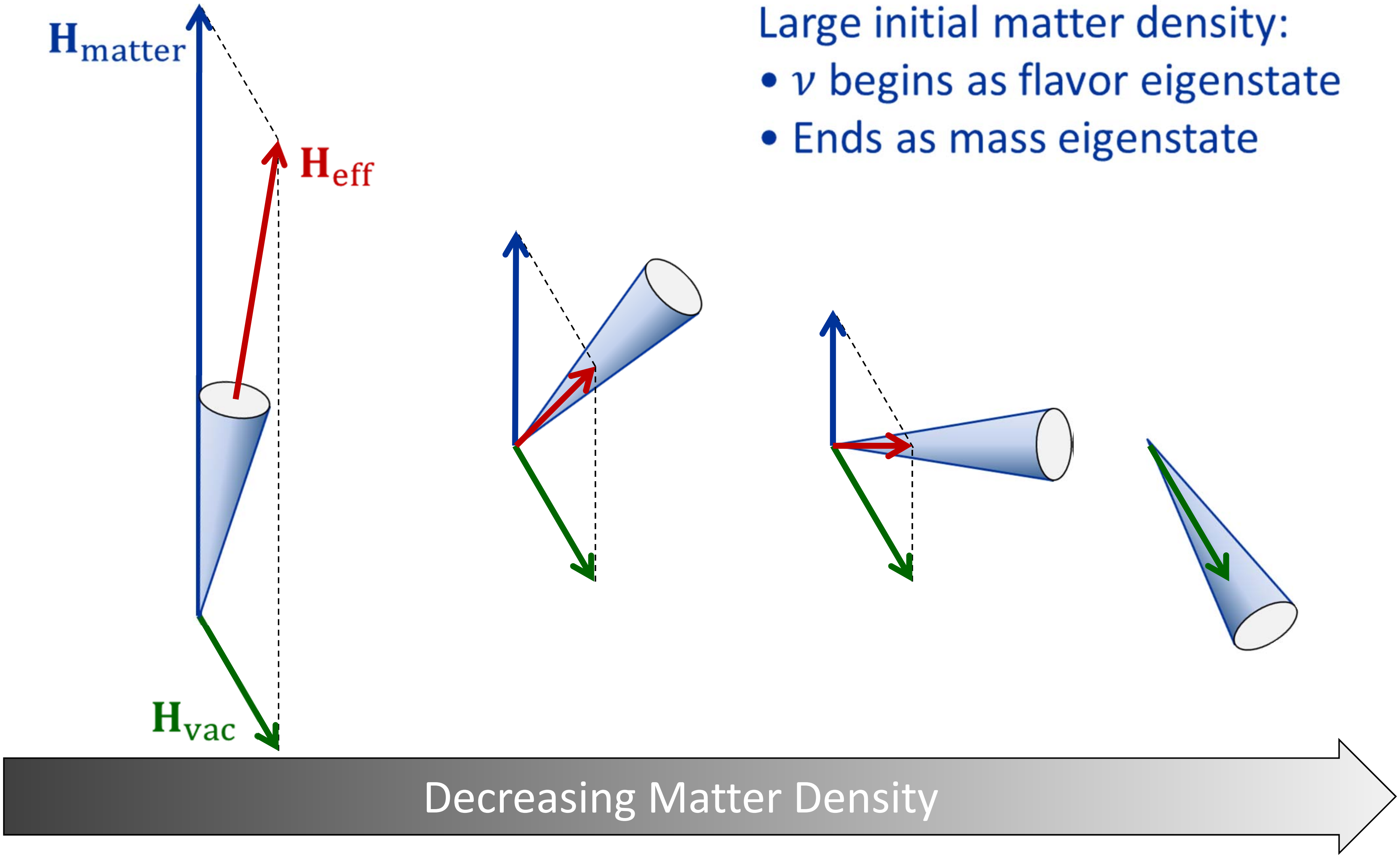
$\mathbf{L} = \mathbf{e}_z$ unit vector in flavor direction



MSW Effect

Adiabatically decreasing density: Precession cone follows $\mathbf{H}_{\mathrm{eff}}$

Large initial matter density:
- ν begins as flavor eigenstate
- Ends as mass eigenstate



Adding Neutrino-Neutrino Interactions

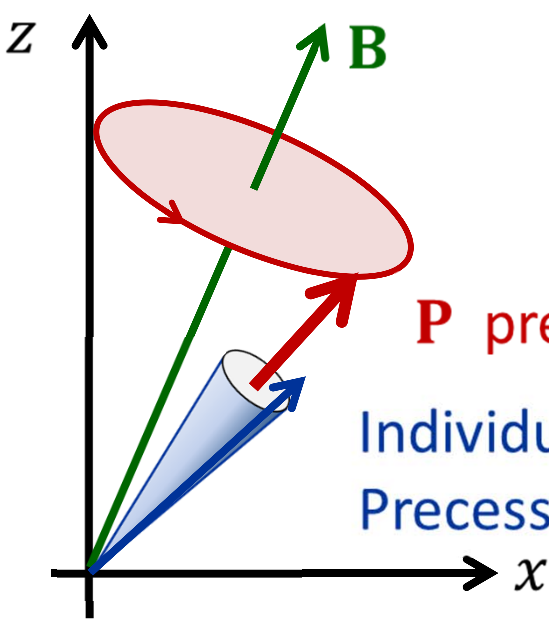
Precession equation for each ν mode with energy E, i.e. $\omega = \Delta m^2/2E$

$$\dot{\mathbf{P}}_\omega = \underbrace{(\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P})}_{\mathbf{H}_{\rm eff}} \times \mathbf{P}_\omega \quad \text{with} \quad \lambda = \sqrt{2}G_{\rm F}N_e \quad \text{and} \quad \mu = \sqrt{2}G_{\rm F}N_\nu$$

Total flavor spin of entire ensemble

$$\mathbf{P} = \textstyle\sum_\omega \mathbf{P}_\omega \qquad \text{normalize} \qquad |\mathbf{P}_{\mathbf{t=0}}| = 1$$

Individual spins do not remain aligned – feel "internal" field $\mathbf{H}_{\nu\nu} = \mu\mathbf{P}$



Synchronized oscillations for large neutrino density $\mu \gg \delta\omega$

$\mathbf{P}$ precesses with $\omega_{\rm sync}$ for large ν density

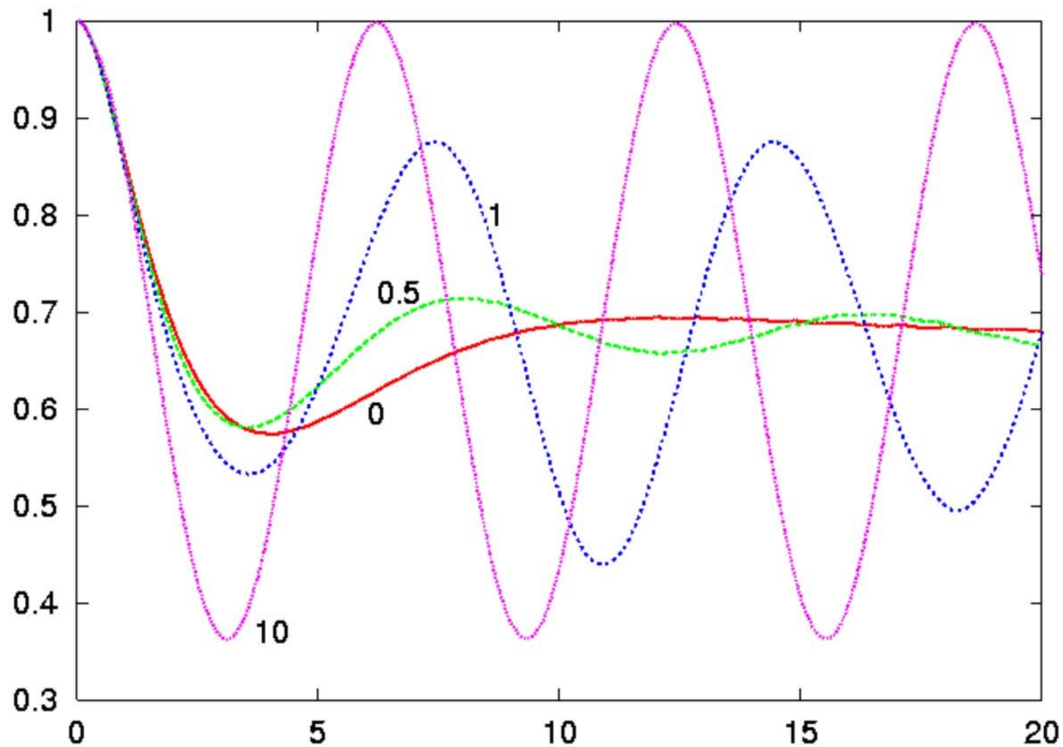
Individual $\mathbf{P}_\omega$ "trapped" on precession cones

Precess around $\mathbf{P}$ with frequency $\sim \mu$

Synchronizing Oscillations by Neutrino Interactions

- Vacuum oscillation frequency depends on energy $\omega = \Delta m^2/2E$
- Ensemble with broad spectrum quickly decoheres kinematically
- ν-ν interactions "synchronize" the oscillations: $\omega_{\rm sync} = \langle \Delta m^2/2E \rangle$

Average e-flavor component of polarization vector



Time

Pastor, Raffelt & Semikoz, hep-ph/0109035

Two Spins Interacting with a Dipole Force

Simplest system showing ν-ν effects:
Isotropic neutrino gas with 2 energies E_1 and E_2, no ordinary matter

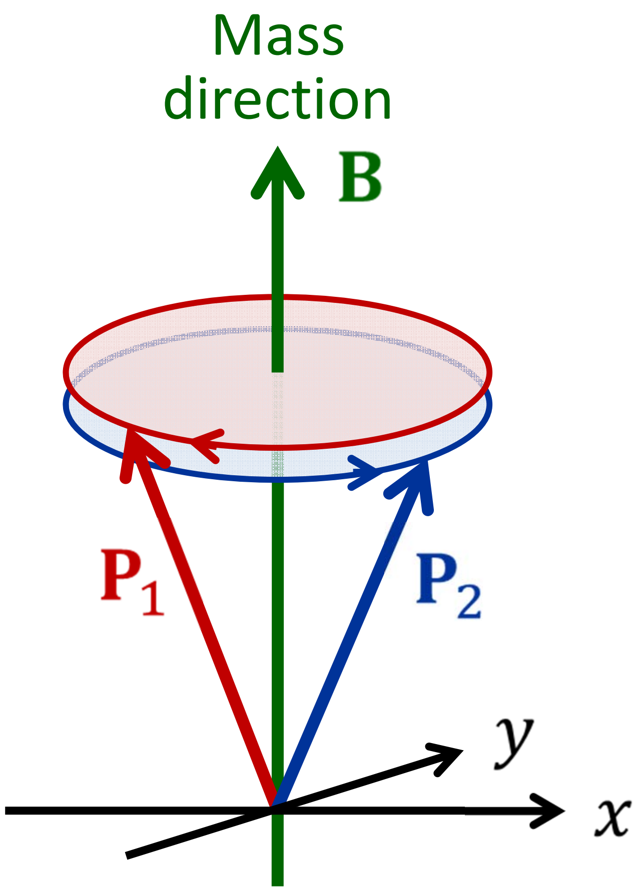
$$\dot{\mathbf{P}}_1 = (\omega_1 \mathbf{B} + \mu \mathbf{P}) \times \mathbf{P}_1 \quad \text{with} \quad \mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2 \quad \text{and} \quad \omega_{1,2} = \Delta m^2/2E$$
$$\dot{\mathbf{P}}_2 = (\omega_2 \mathbf{B} + \mu \mathbf{P}) \times \mathbf{P}_2$$

Go to "co-rotating frame" around $\mathbf{B}$ direction

$$\dot{\mathbf{P}}_1 = (\omega_c \mathbf{B} - \omega \mathbf{B} + \mu \mathbf{P}) \times \mathbf{P}_1$$
$$\dot{\mathbf{P}}_2 = (\omega_c \mathbf{B} + \omega \mathbf{B} + \mu \mathbf{P}) \times \mathbf{P}_2$$

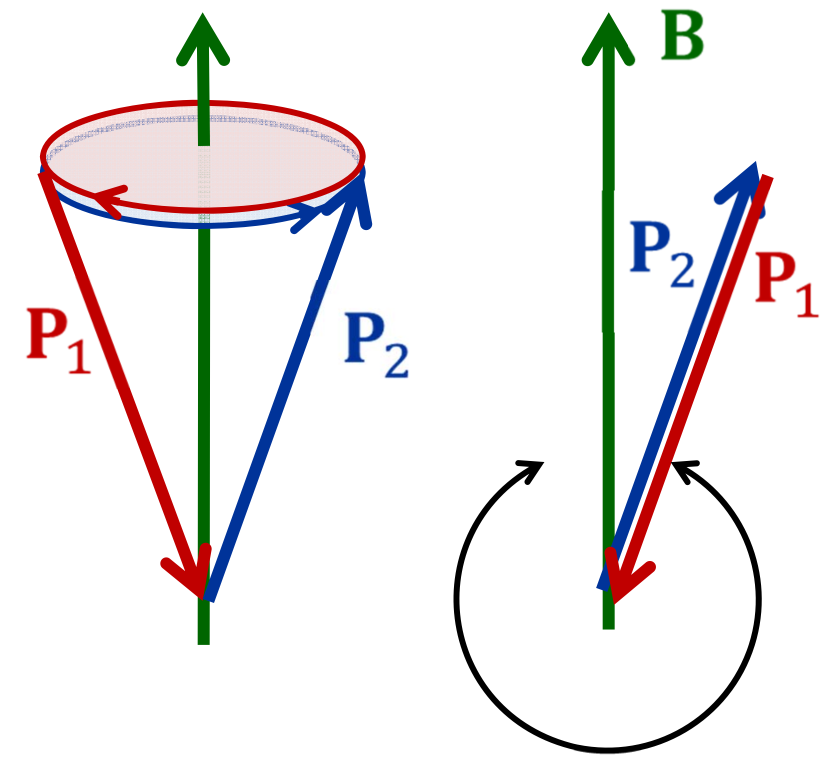
with $\omega_c = \frac{1}{2}(\omega_2 + \omega_1)$ and $\omega = \frac{1}{2}(\omega_2 - \omega_1)$

No interaction ($\mu = 0$)
$P_{1,2}$ precess in opposite directions
Strong interactions ($\mu \to \infty$)
$P_{1,2}$ stuck to each other
(no motion in co-rotating frame, perfectly synchronized in lab frame)



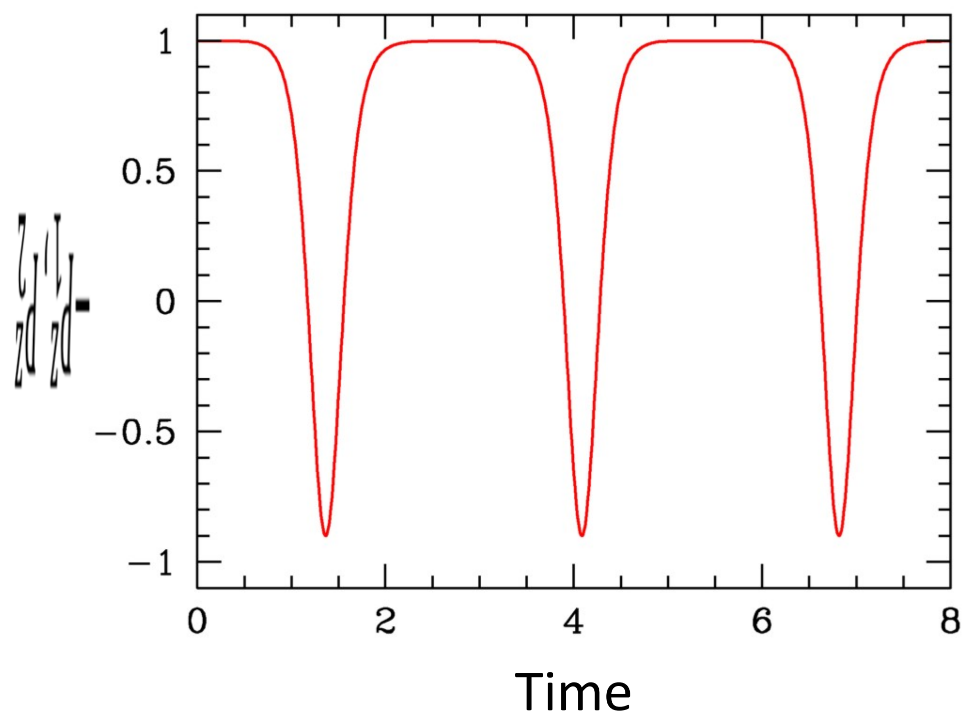
Two Spins with Opposite Initial Orientation

No interaction ($\mu = 0$)
Free precession in opposite directions



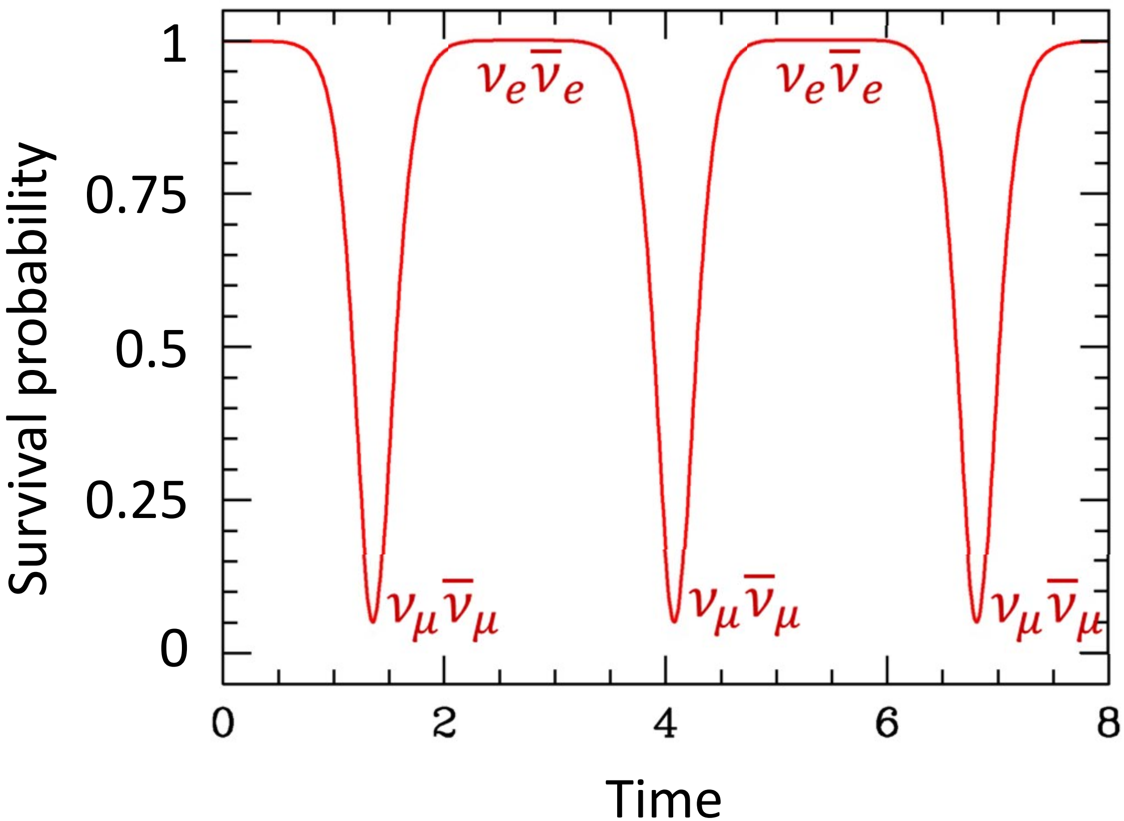
Strong interaction ($\mu \to \infty$)
Pendular motion

Even for very small mixing angle, large-amplitude flavor oscillations



Collective Pair Conversion

Gas of equal abundances of ν_e and $\bar{\nu}_e$, inverted mass hierarchy
Small effective mixing angle (e.g. made small by ordinary matter)

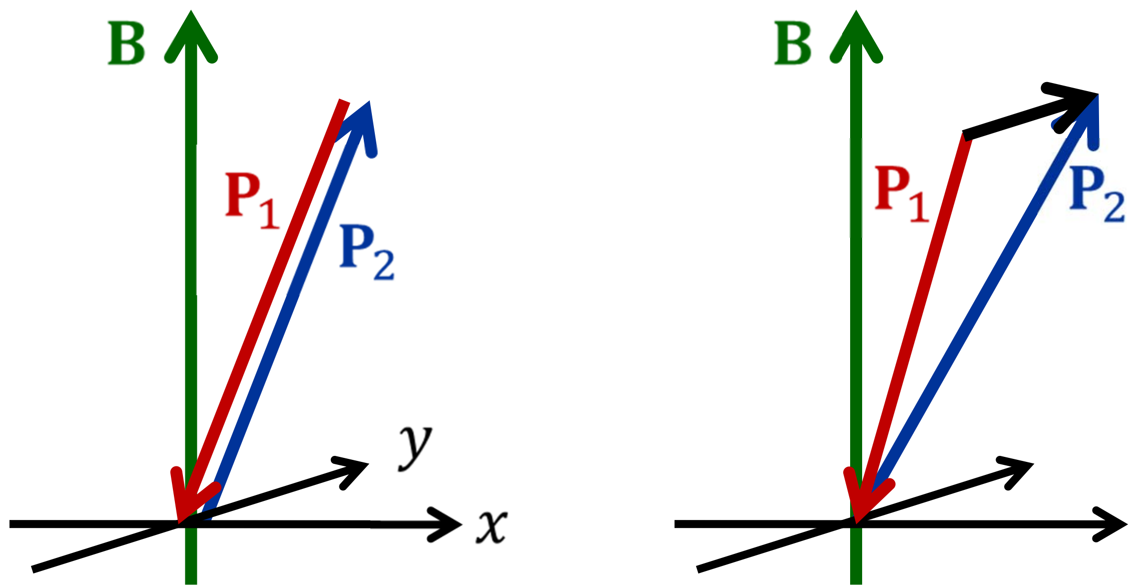


Dense neutrino gas unstable in flavor space: $\nu_e\bar{\nu}_e \leftrightarrow \nu_\mu\bar{\nu}_\mu$
Complete pair conversion even for a small mixing angle

Instability in Flavor Space

Two-mode example in co-rotating frame, initially $\mathbf{P}_1 = \downarrow$, $\mathbf{P}_2 = \uparrow$ (flavor basis)

$$\dot{\mathbf{P}}_1 = [-\omega\mathbf{B} + \mu\,(\mathbf{P_1} + \mathbf{P_2})] \times \mathbf{P_1}$$
$$\dot{\mathbf{P}}_2 = [+\omega\mathbf{B} + \mu\,\underbrace{(\mathbf{P_1} + \mathbf{P_2})}_{\text{0 initially}}] \times \mathbf{P_2}$$



$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$

Matter effect transverse to mass and flavor directions

Both $\mathbf{P}_1$ and $\mathbf{P}_2$ tilt around $\mathbf{P}$ if μ is large

- Initially aligned in flavor direction and $\mathbf{P} = 0$
- Free precession $\pm\omega$

After a short time, transverse $\mathbf{P}$ develops by free precession

Inverse-Energy Spectrum

Fermi-Dirac energy spectrum

$$\frac{dN}{dE} \propto \frac{E^2}{e^{E/T-\eta}+1}$$

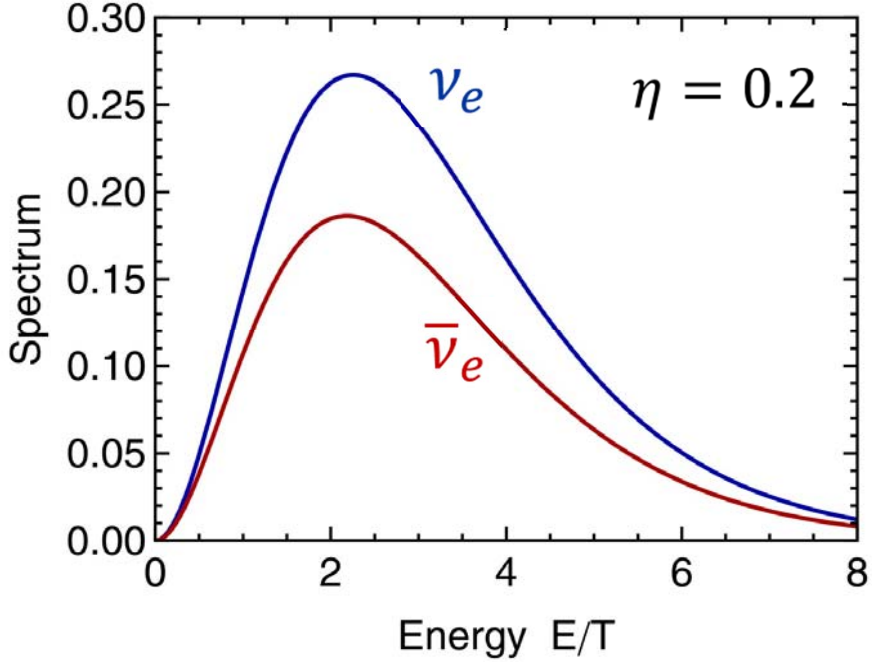
η degeneracy parameter, $-\eta$ for $\overline{\nu}$

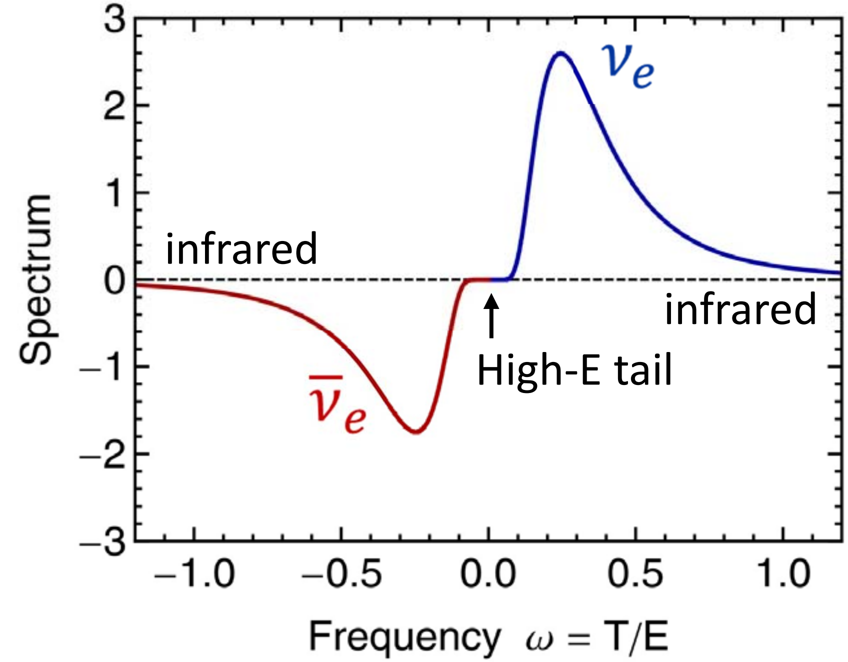
Spectrum in terms of $\omega = T/E$

- Antineutrinos $E \to -E$
- and dN/dE negative (flavor isospin convention)

$\omega > 0$: $\nu_e = \uparrow$ and $\nu_\mu = \downarrow$

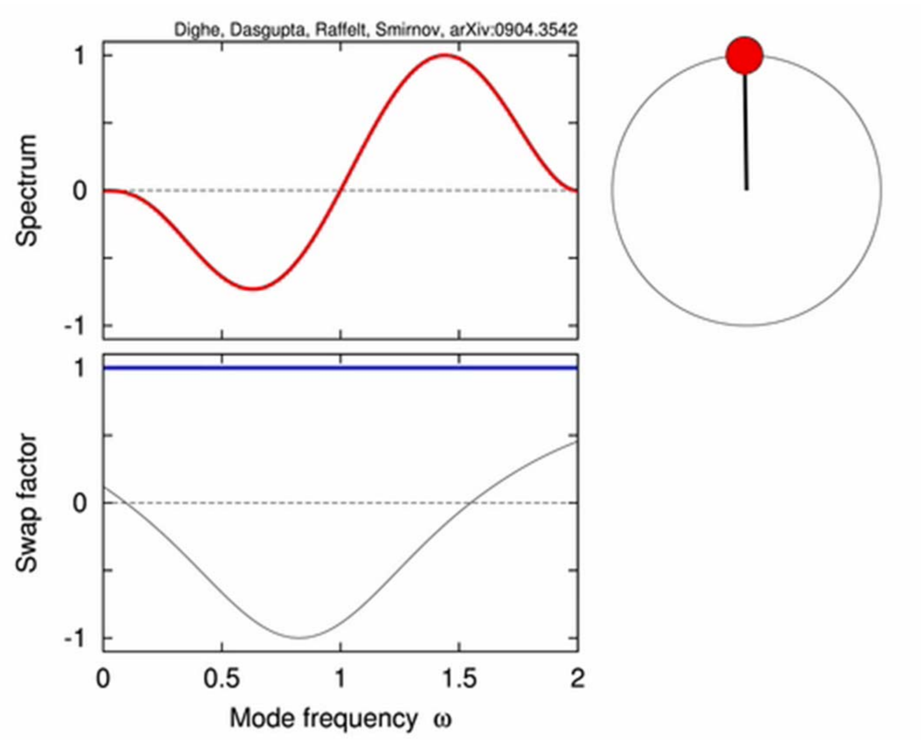
$\omega < 0$: $\overline{\nu}_e = \downarrow$ and $\overline{\nu}_\mu = \uparrow$



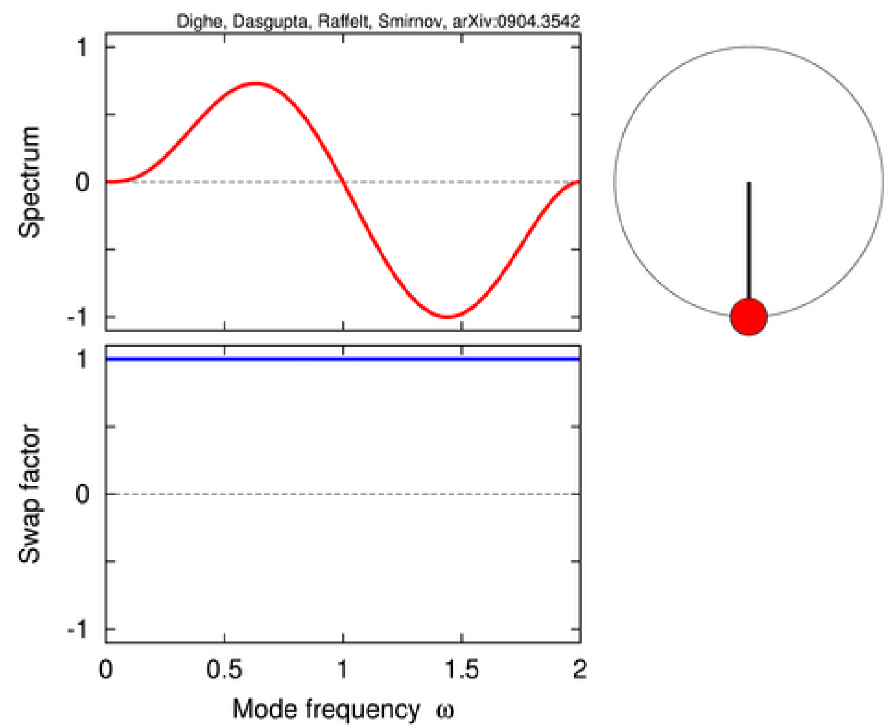


Flavor Pendulum

Single "positive" crossing (IH)
(potential energy at a maximum)



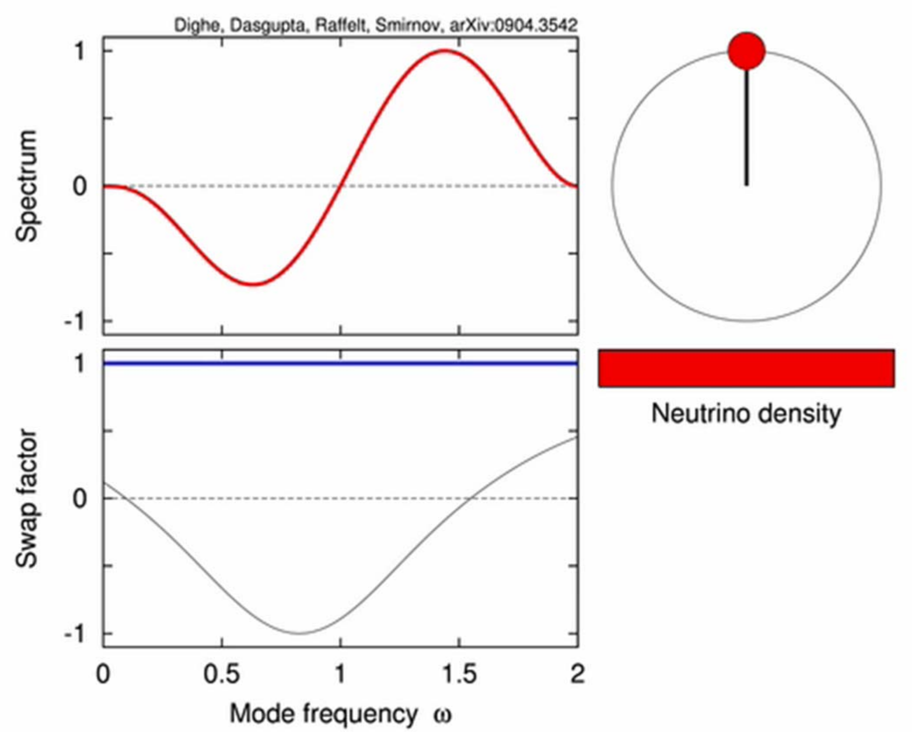
Single "negative" crossing (NH)
(potential energy at a minimum)



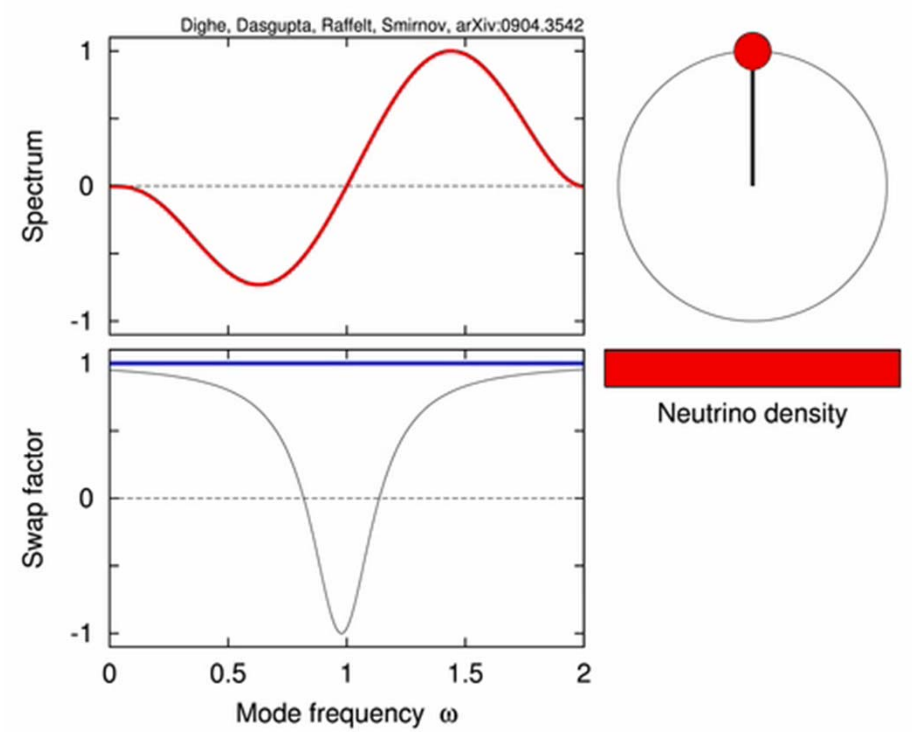
Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542
For movies see http://www.mppmu.mpg.de/supernova/multisplits

Decreasing Neutrino Density

Certain initial neutrino density



Four times smaller initial neutrino density

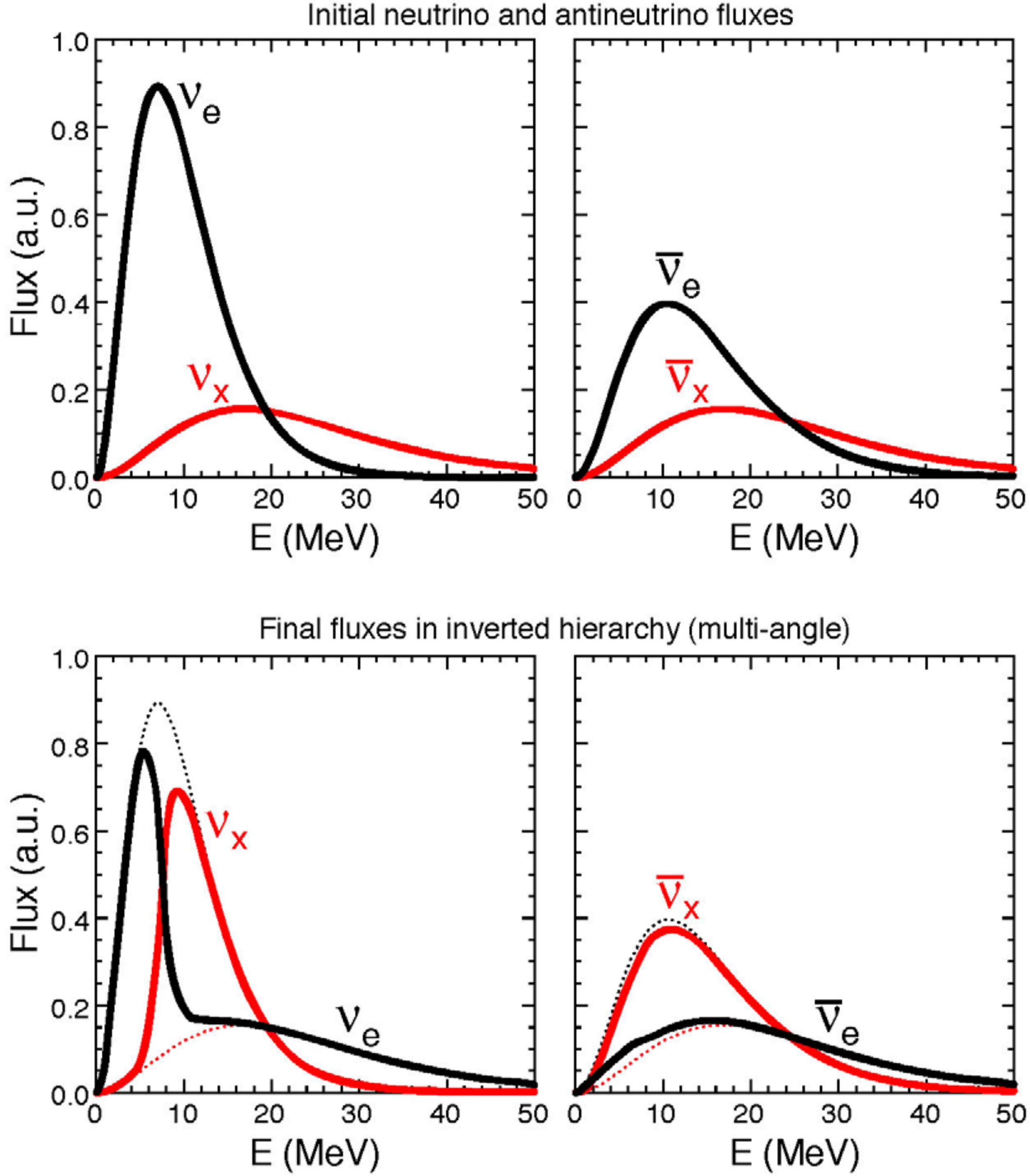


Dasgupta, Dighe, Raffelt & Smirnov, arXiv:0904.3542
For movies see http://www.mppmu.mpg.de/supernova/multisplits

Spectral Split

Initial fluxes at neutrino sphere

After collective trans-formation



Figures from

Fogli, Lisi, Marrone & Mirizzi, arXiv:0707.1998

Explanations in

Raffelt & Smirnov arXiv:0705.1830 and 0709.4641

Duan, Fuller, Carlson & Qian arXiv:0706.4293 and 0707.0290

Linearized Stability Analysis

Schrödinger equation for flavor matrices of neutrino fluxes $\Phi_{\omega,u}$

$\omega = \pm\Delta m^2/2E$ $\quad$ $u = \sin^2(\text{emission angle})$ $\quad$ $v_u = $ radial velocity at r

$$\mathrm{i}\partial_r \Phi_{\omega,u} = \left[\frac{\omega + \sqrt{2}G_\mathrm{F}N_\ell}{v_u} + \frac{\sqrt{2}G_\mathrm{F}}{4\pi r^2}\int d\omega' du'\, \Phi_{\omega',u'} \frac{1 - v_u v_{u'}}{v_u v_{u'}}, \Phi_{\omega,u}\right]$$

Linearize in small off-diagonal flux terms and Fourier transform

$$\Phi_{\omega,u} = \frac{g_{\omega,u}}{2}\begin{pmatrix} 1 & Q_{\omega,u}\, e^{-\mathrm{i}\Omega r} \\ Q^*_{\omega,u}\, e^{\mathrm{i}\Omega r} & -1 \end{pmatrix}$$

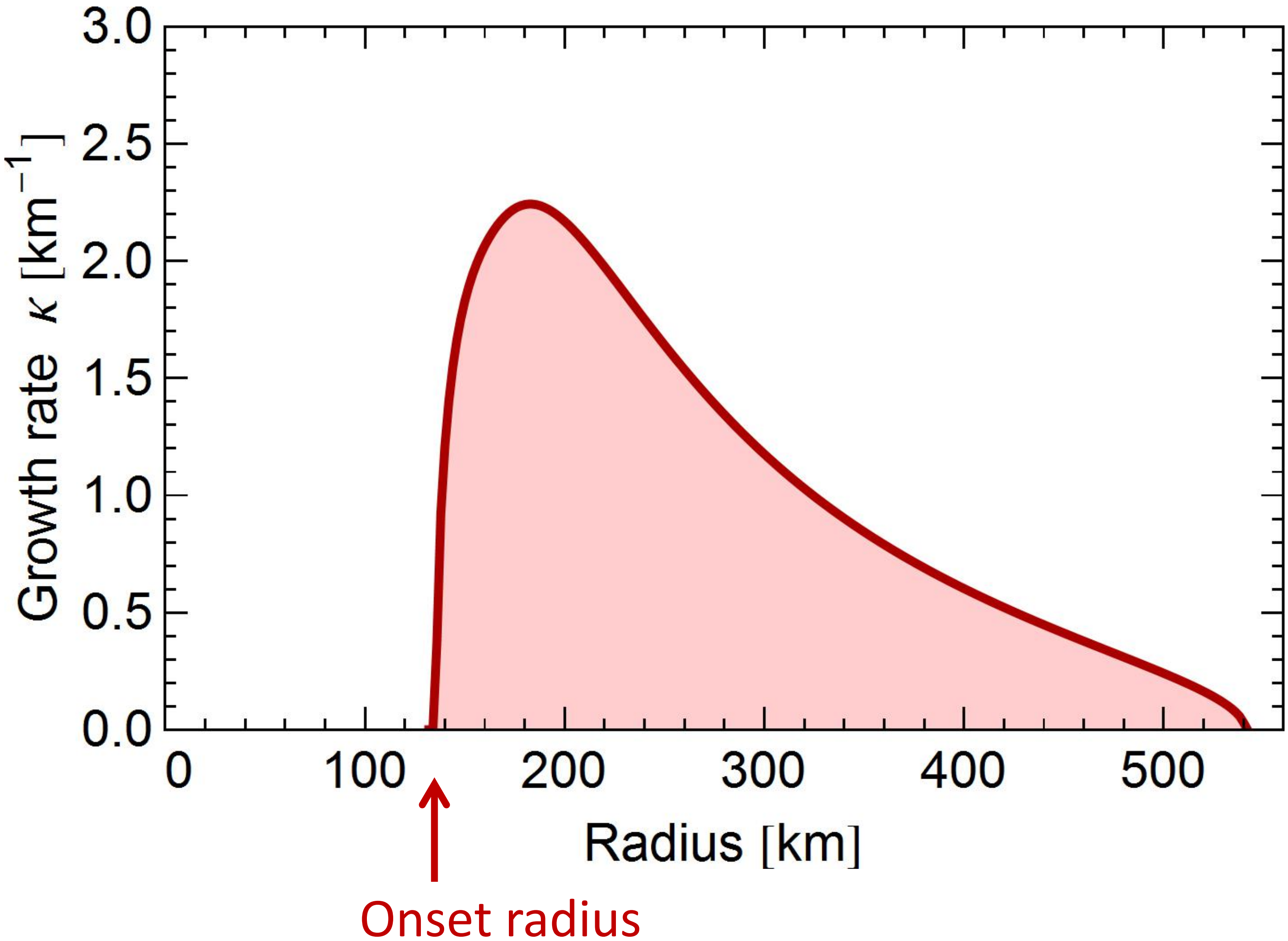
Eigenvalue equation for $Q_{\omega,u}$ in terms of eigenfrequency $\Omega = \gamma + \mathrm{i}\,\kappa$, where κ is the exponential growth rate

$$\left[\omega + u\left(\lambda + \int d\omega' du'\, g_{\omega',u'}\right) - \Omega\right] Q_{\omega,u} = \mu \int d\omega' du'\, (u + u') g_{\omega',u'} Q_{\omega',u'}$$

Straightforward to solve for eigenvalue Ω and eigenfunction $Q_{\omega,u}$

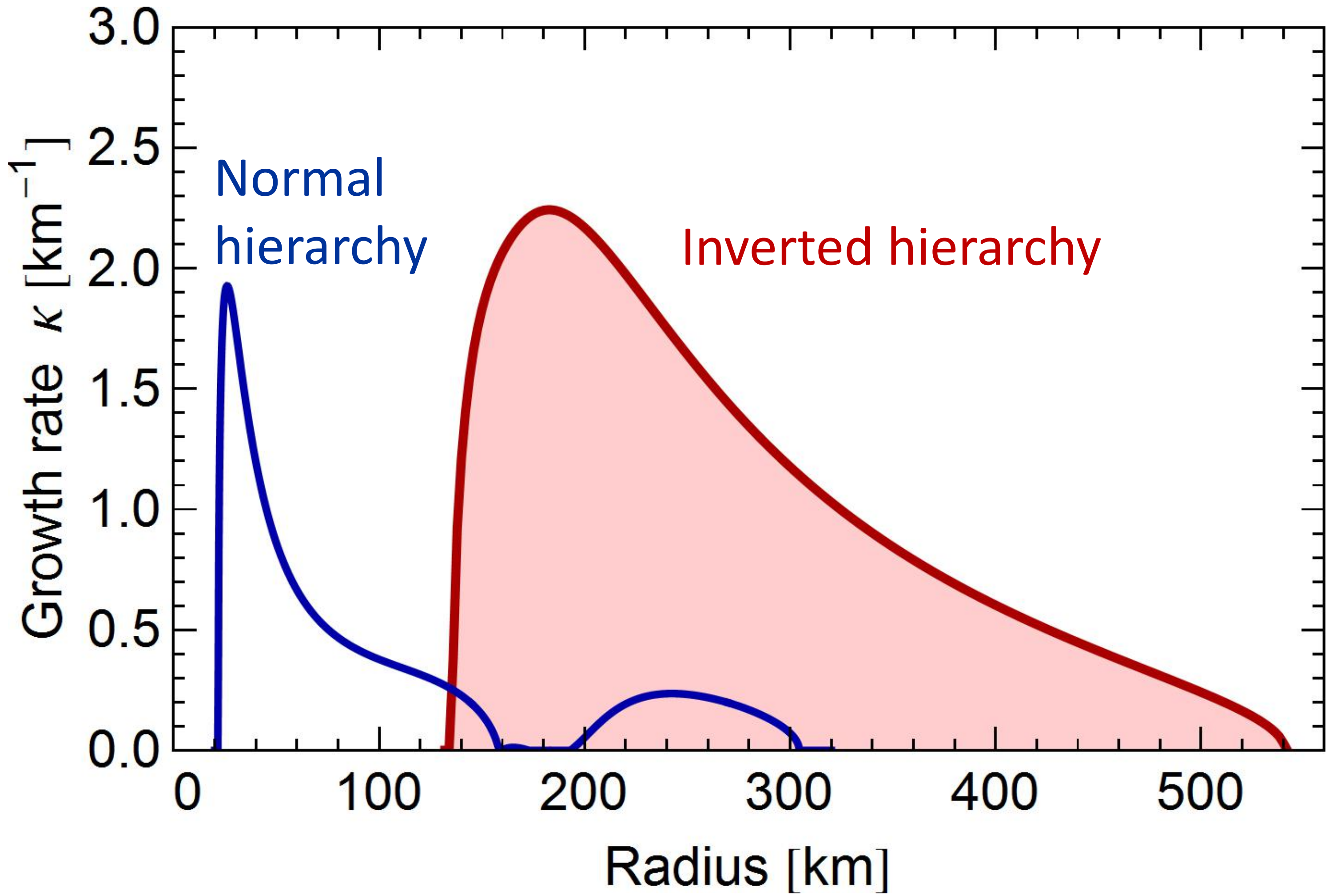
Banerjee, Dighe & Raffelt, arXiv:1107.2308

Stability Analysis for Simplified SN Example



Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Normal vs Inverted Hierarchy



Continuous angle distribution, NH stable for single-angle case

Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Effect

Precession equation in a homogeneous ensemble

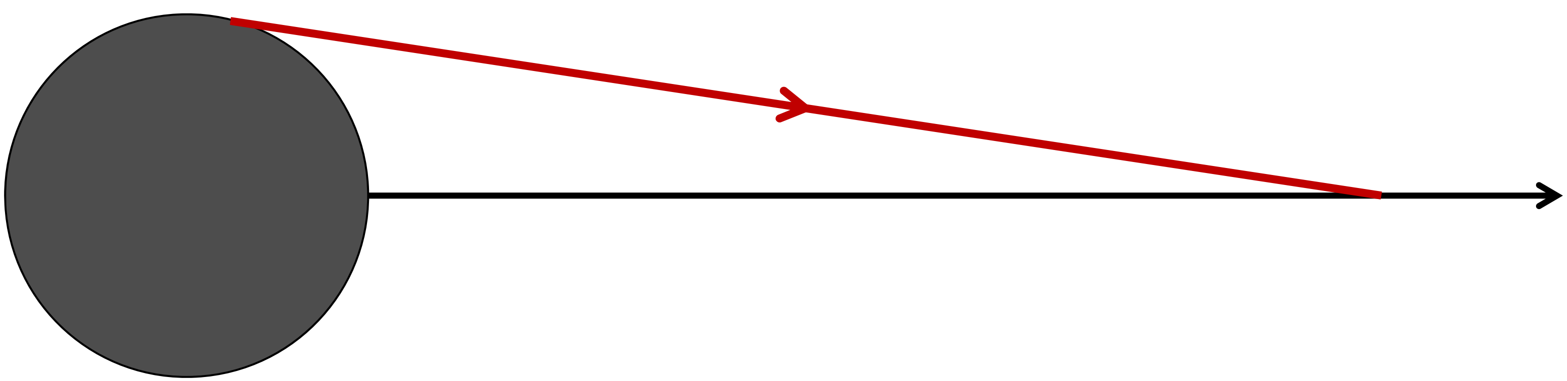
$$\partial_t \mathbf{P}_{\omega,\mathbf{v}} = (\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P}) \times \mathbf{P}_{\omega,\mathbf{v}},$$ where $\lambda = \sqrt{2}G_\mathrm{F}N_e$ and $\mu = \sqrt{2}G_\mathrm{F}N_\nu$

Matter term is "achromatic", disappears in a rotating frame

Neutrinos streaming from a SN core, evolution along radial direction

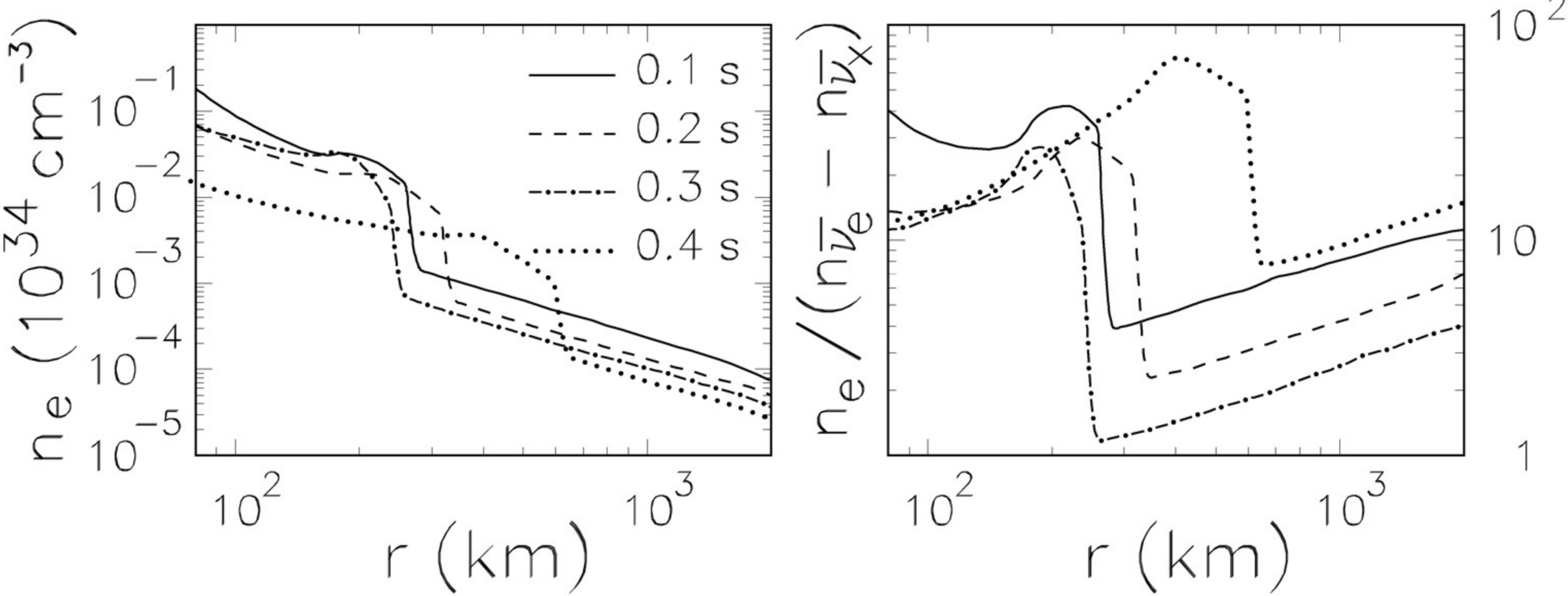
$$(\mathbf{v}\cdot\boldsymbol{\nabla}_r)\,\mathbf{P}_{\omega,\mathbf{v}} = (\omega\mathbf{B} + \lambda\mathbf{L} + \mu\mathbf{P}) \times \mathbf{P}_{\omega,\mathbf{v}}$$

Projected on the radial direction, oscillation pattern compressed:
Accrues vacuum and matter phase faster than on radial trajectory
Matter effect can suppress collective conversion unless $N_\nu \gtrsim N_e$



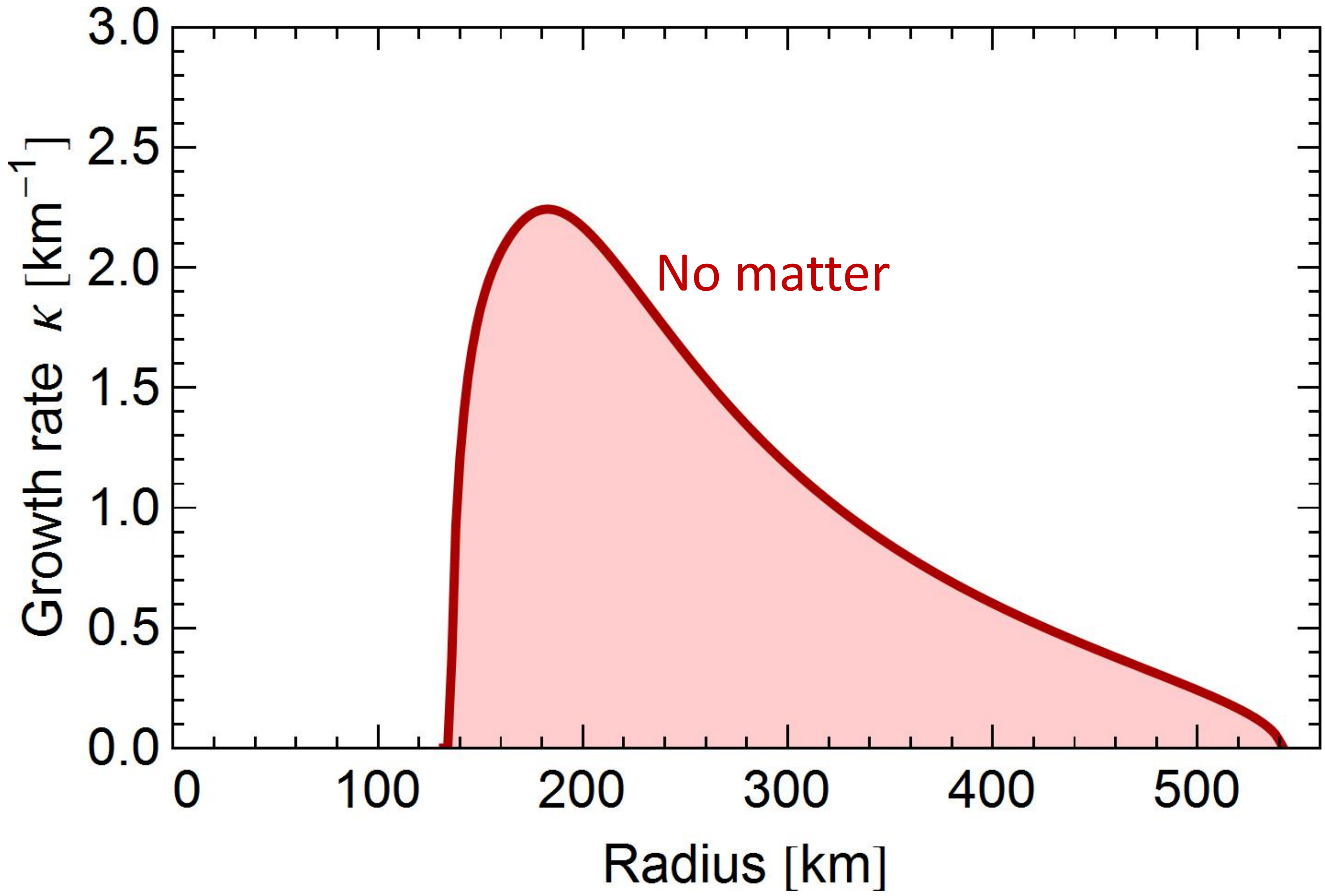
Esteban-Pretel, Mirizzi, Pastor, Tomàs, Raffelt, Serpico & Sigl, arXiv:0807.0659

Accretion-Phase Matter Profiles (Basel 10.8 M_{sun})



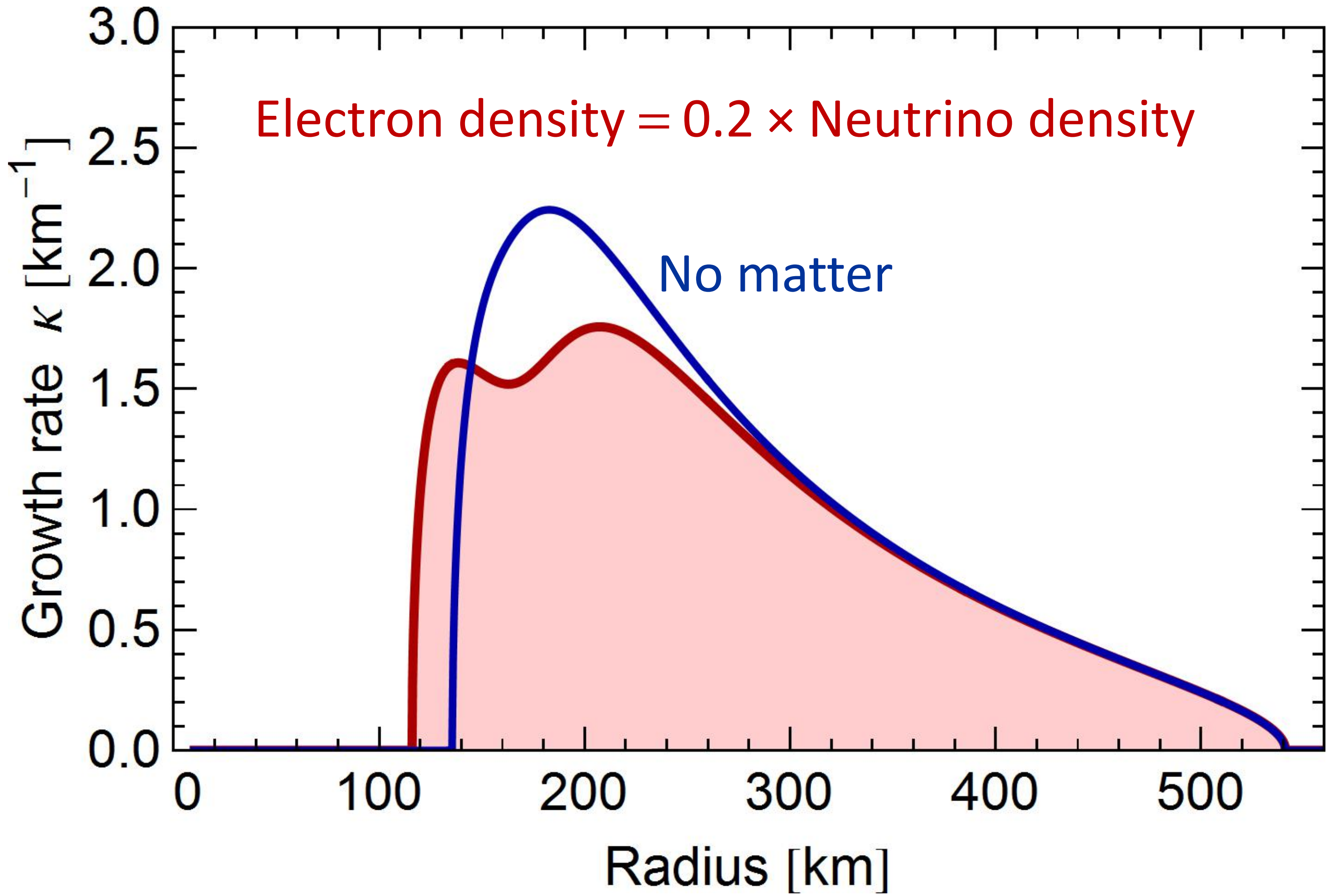
Chakraborty, Fischer, Mirizzi, Saviano & Tomàs, arXiv:1105.1130

Multi-Angle Matter Suppression



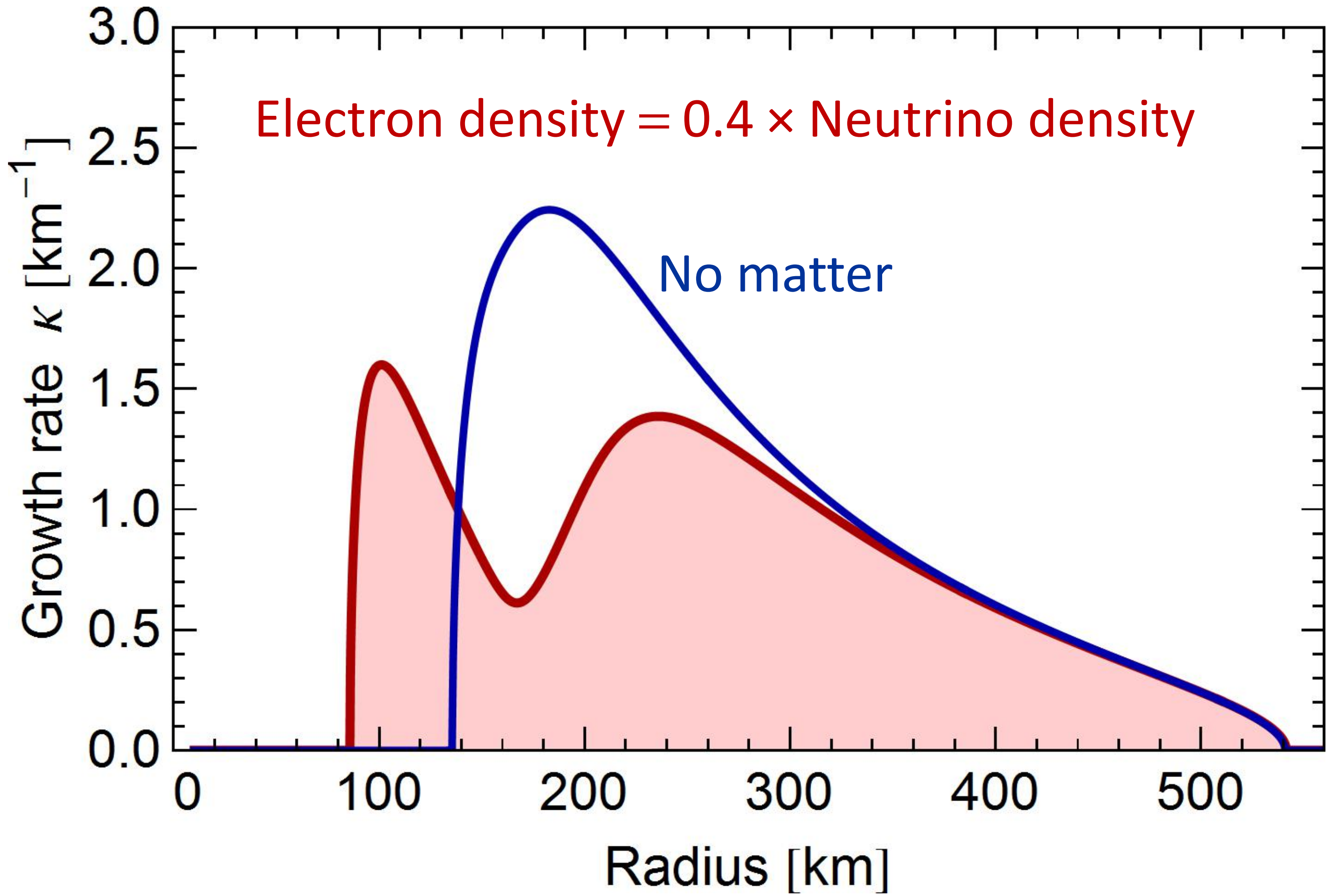
Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression



Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression



Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

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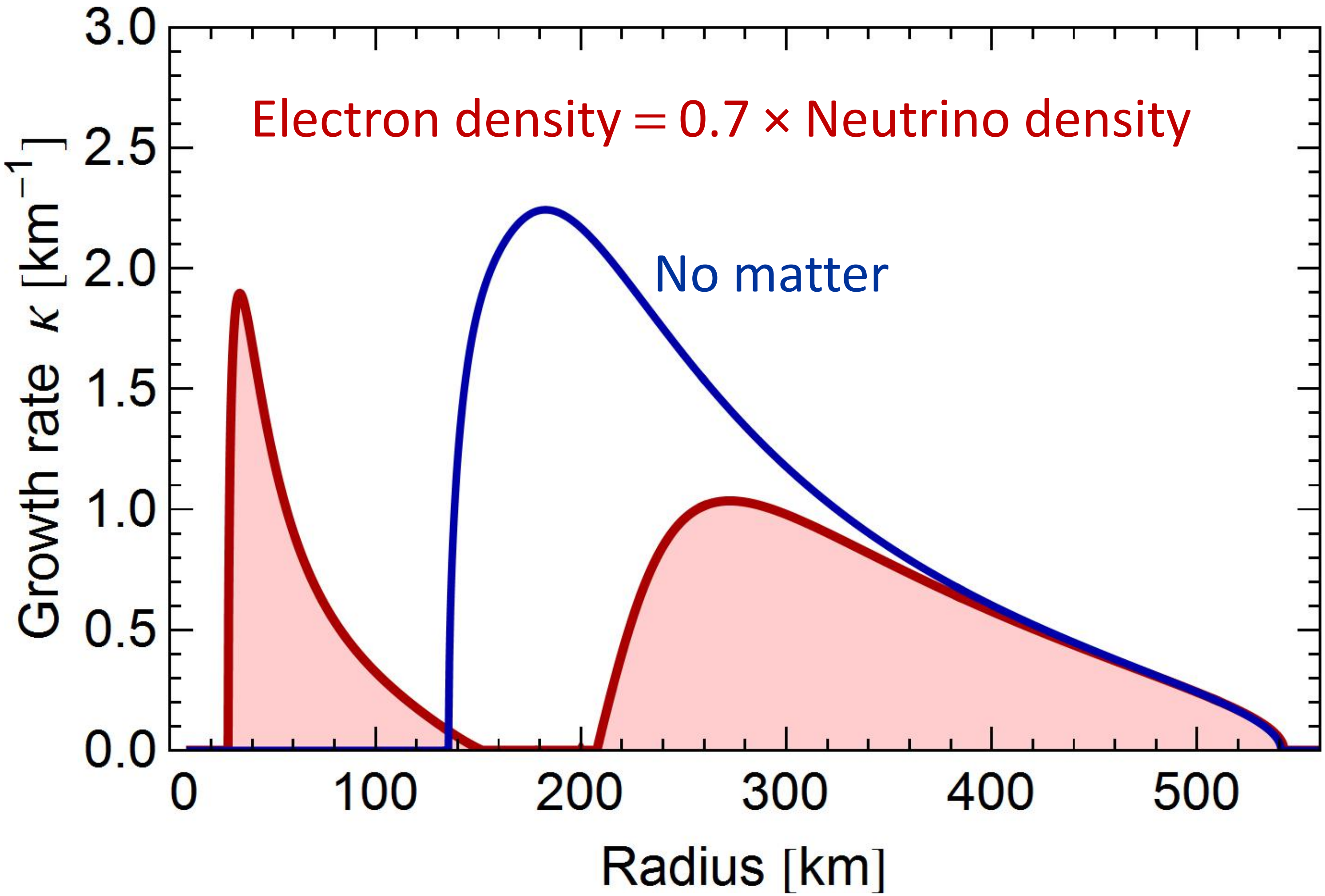
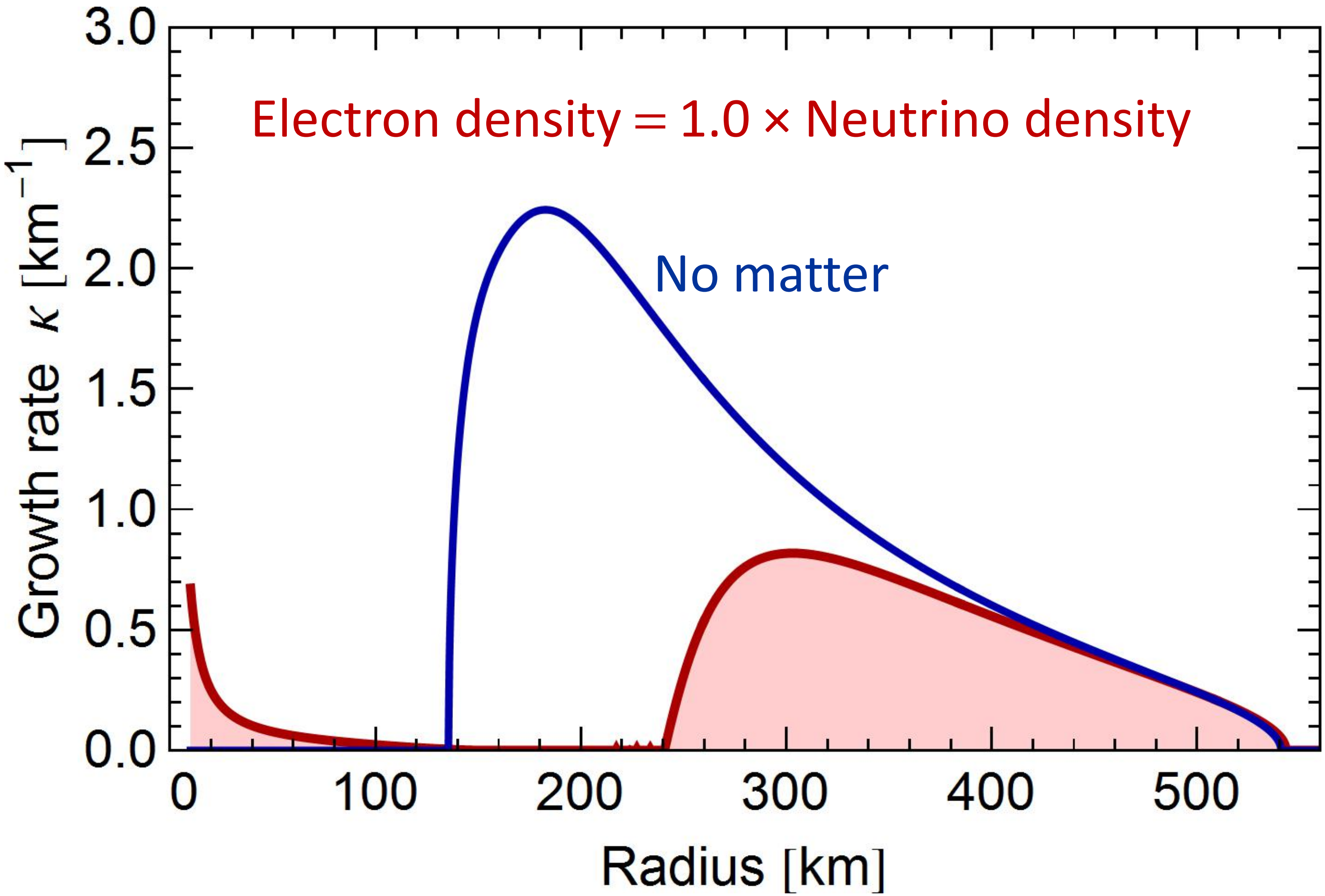


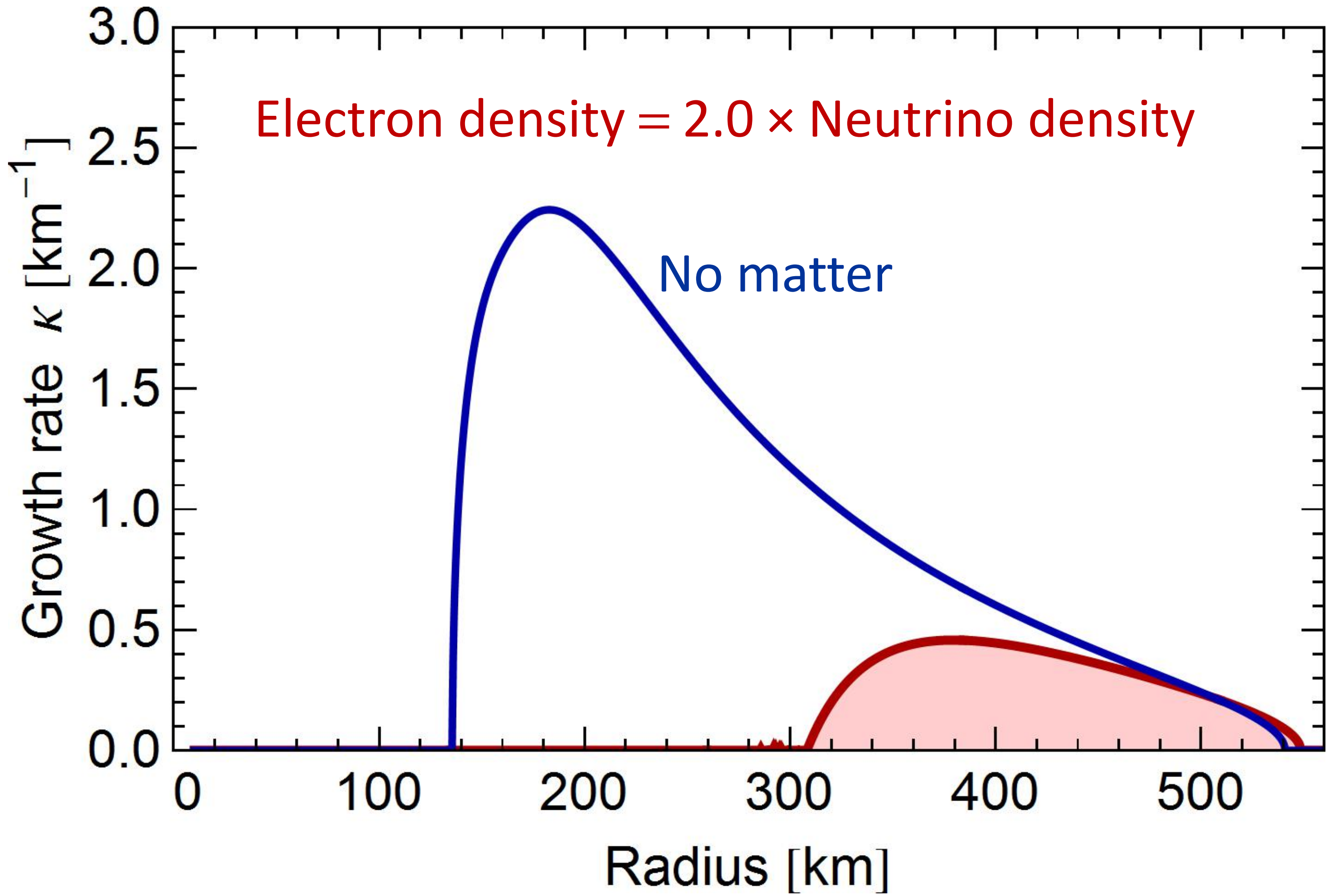
Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression



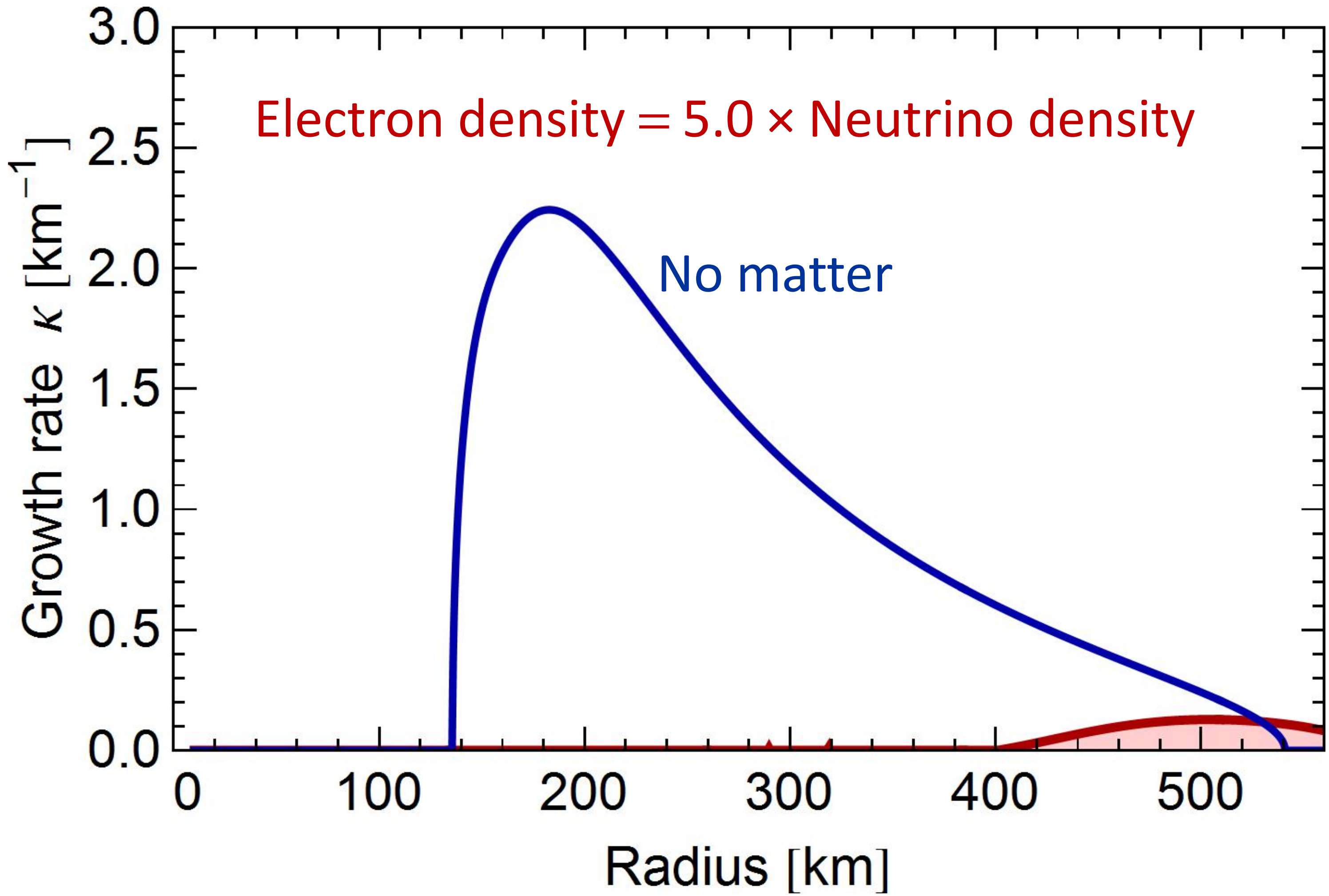
Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression



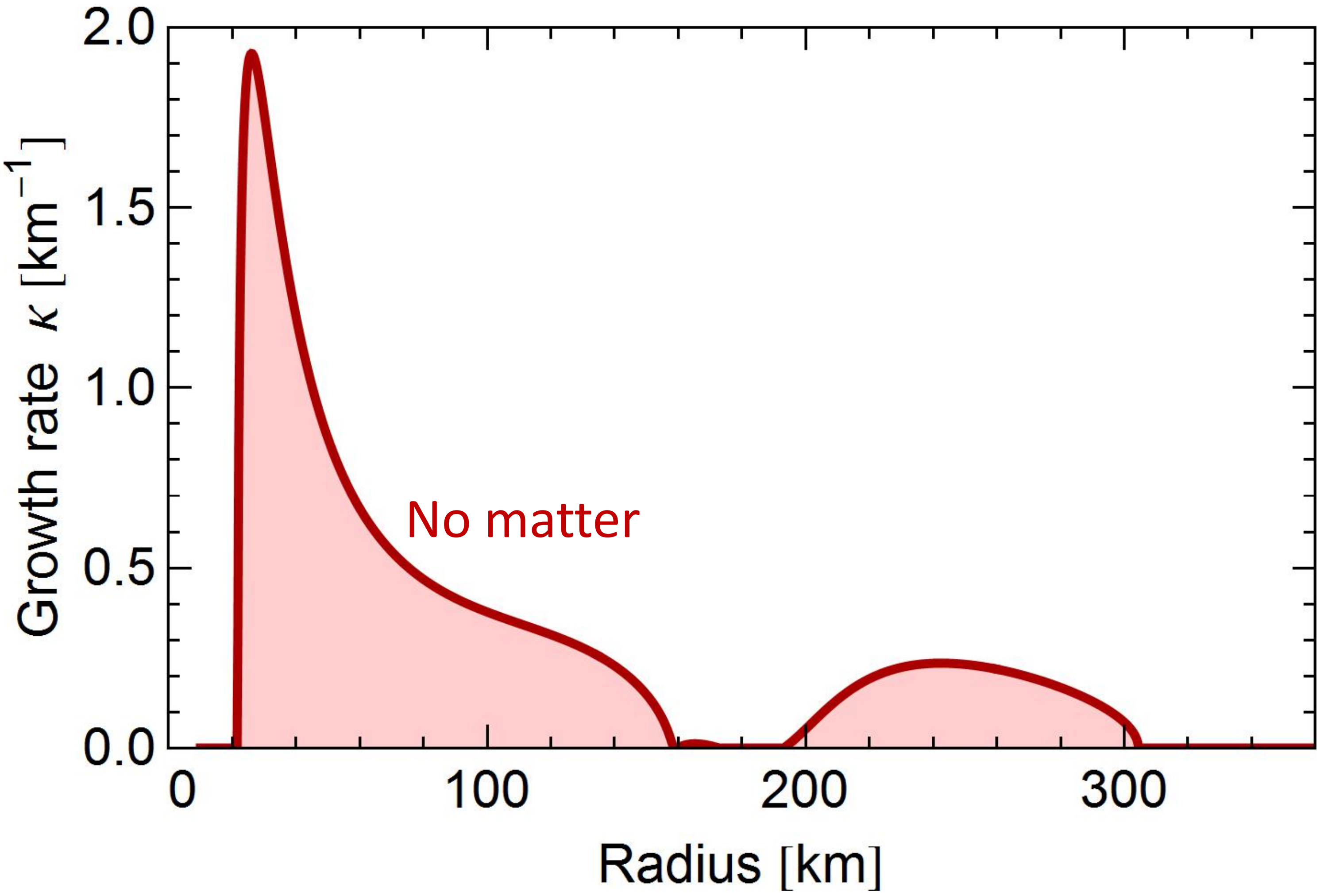
Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression



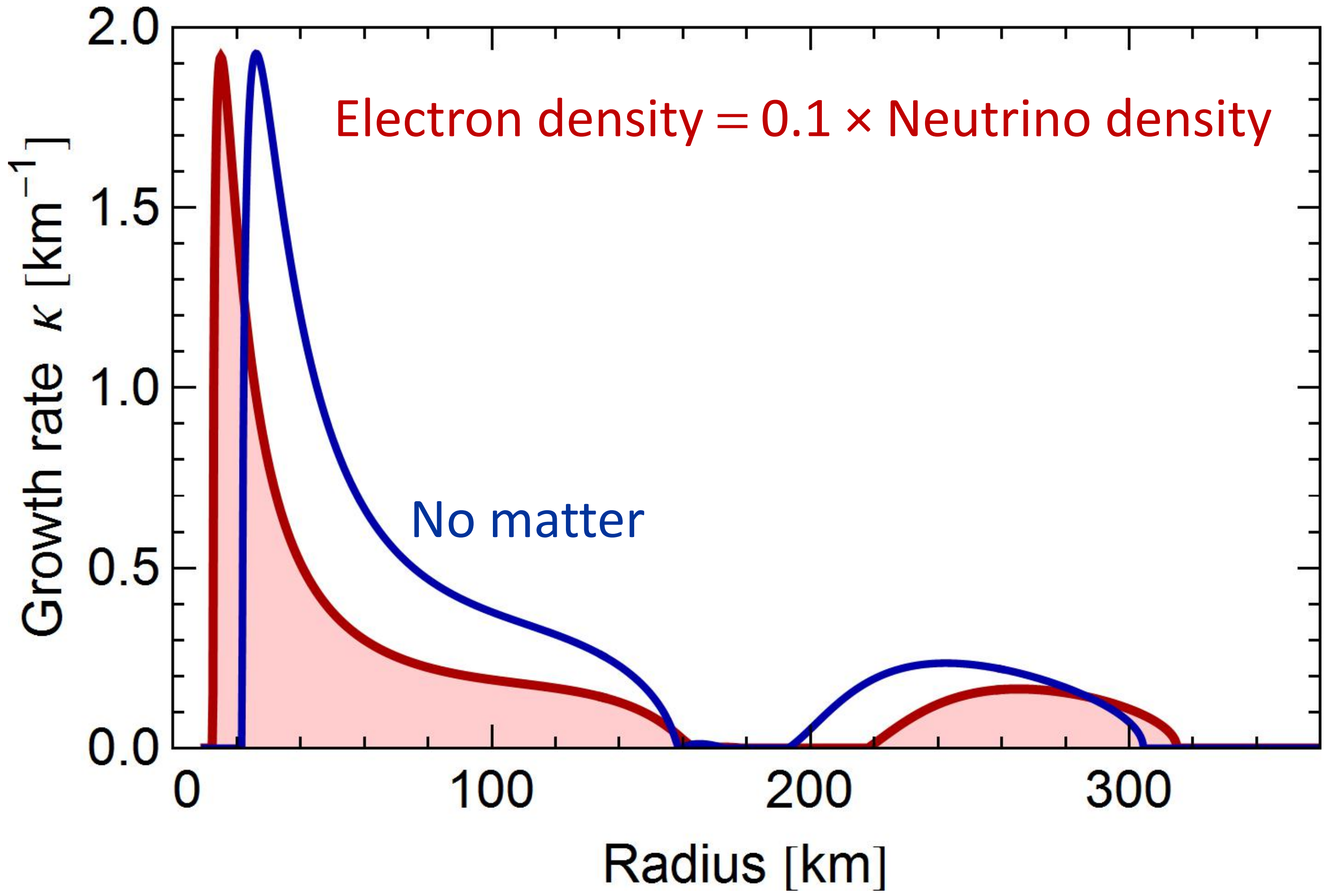
Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression (Normal Hierarchy)



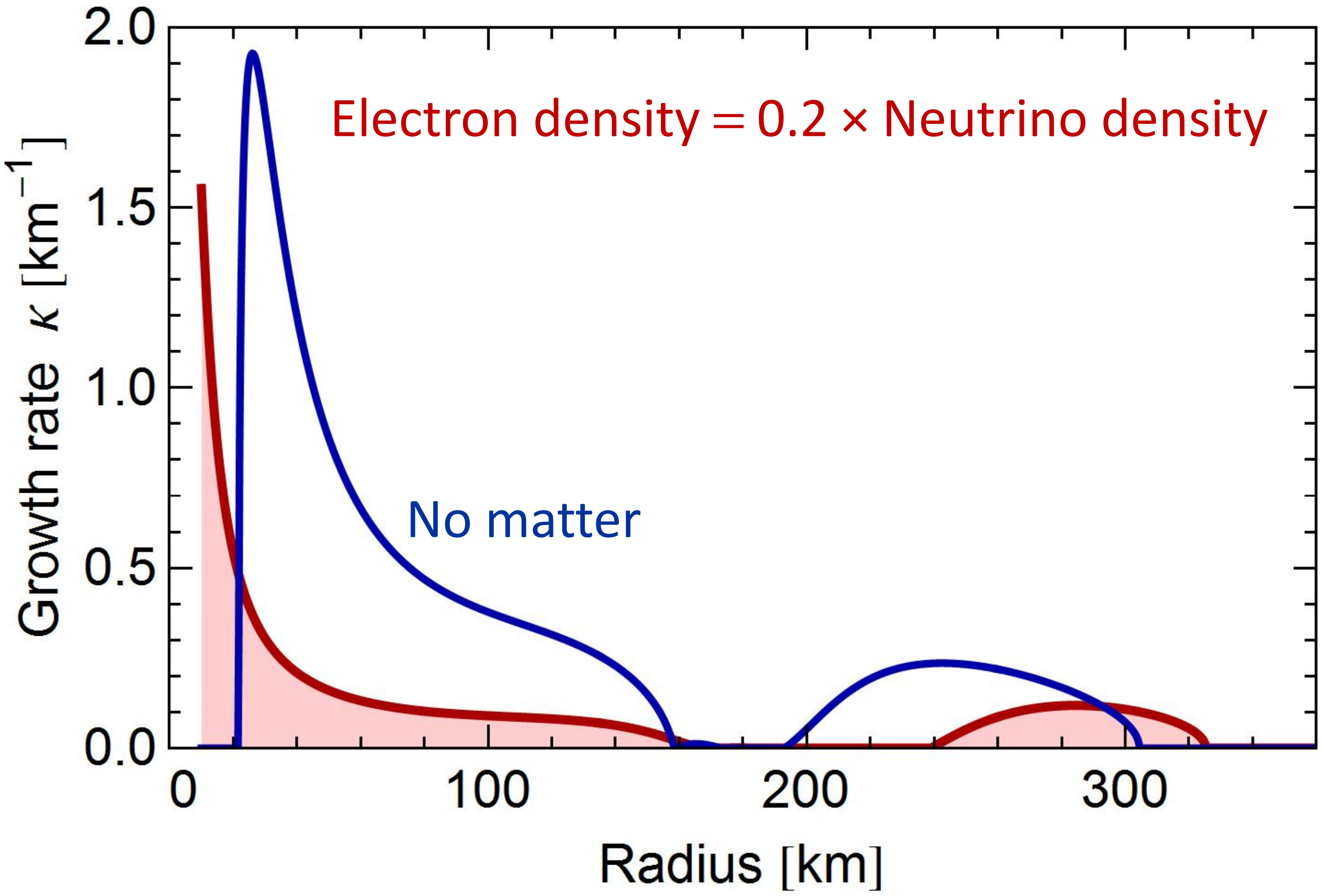
Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression (Normal Hierarchy)



Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression (Normal Hierarchy)



Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

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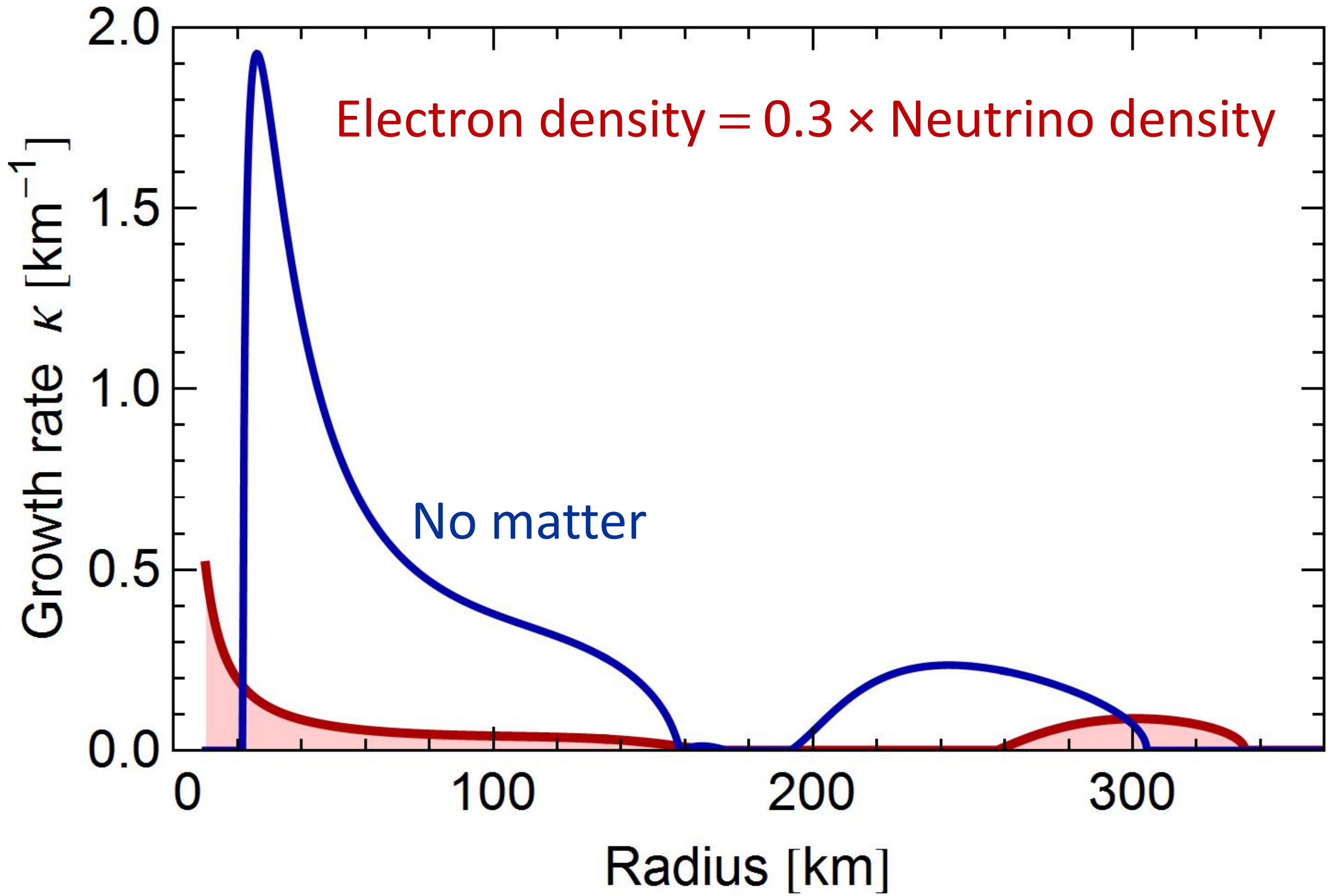
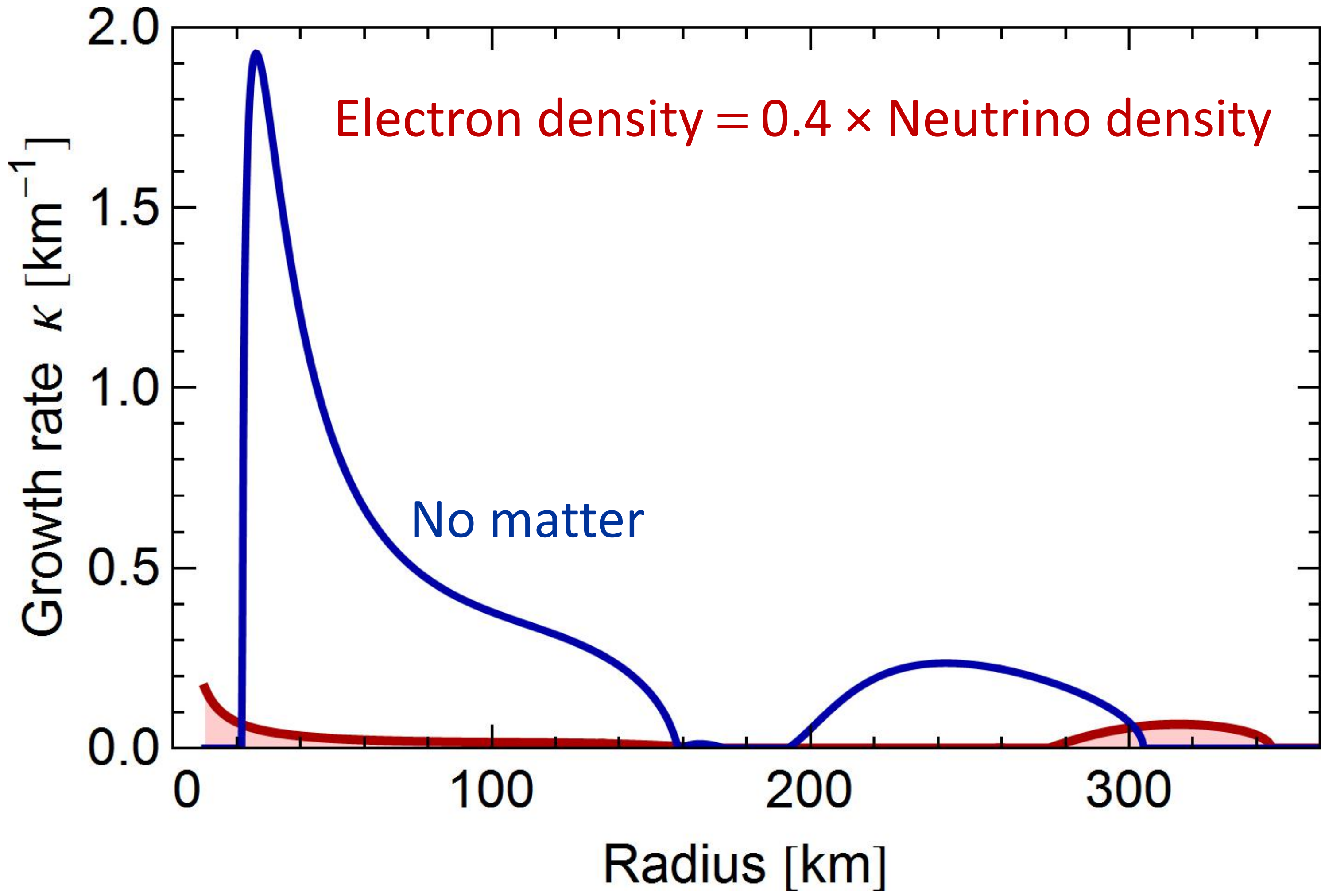


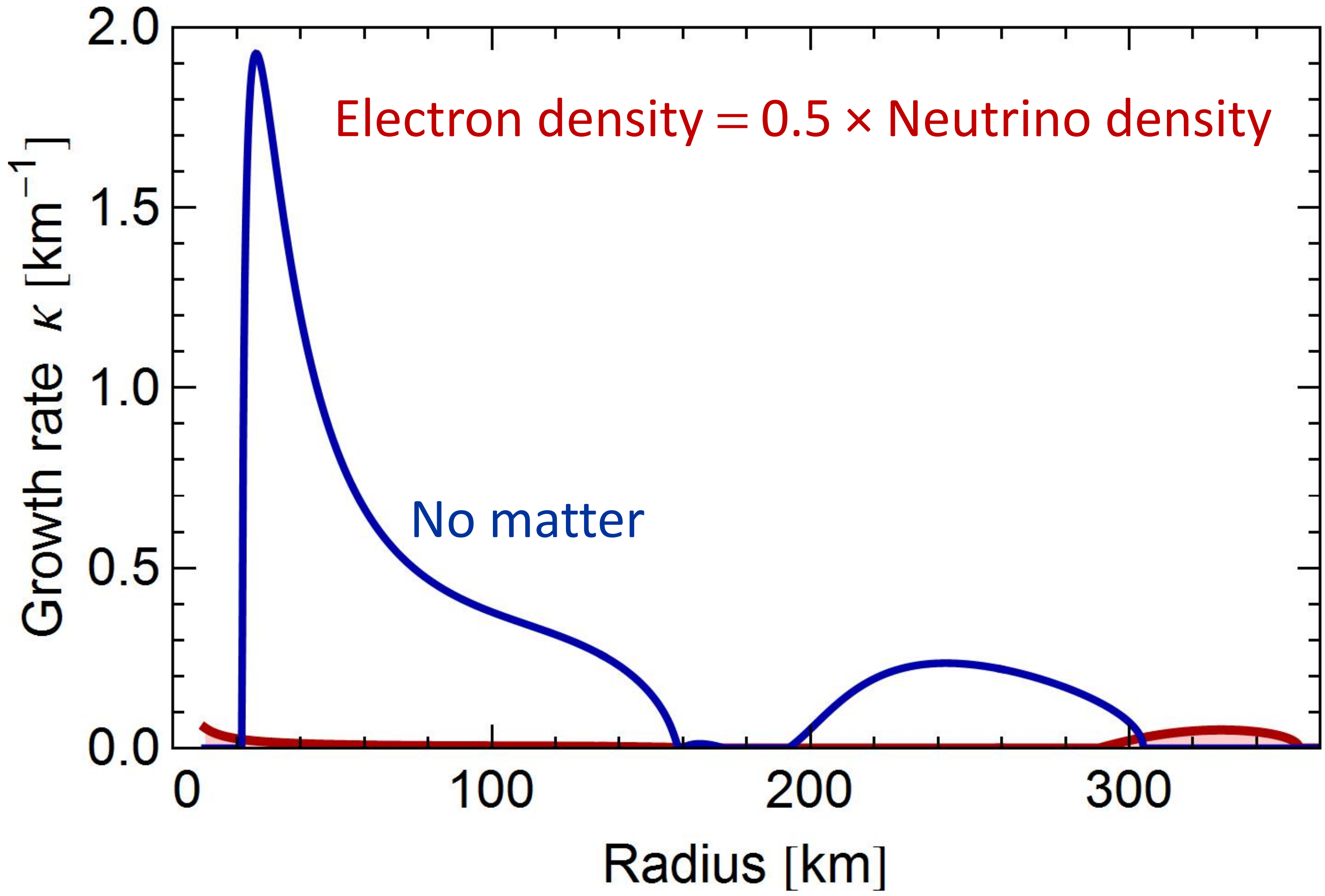
Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression (Normal Hierarchy)



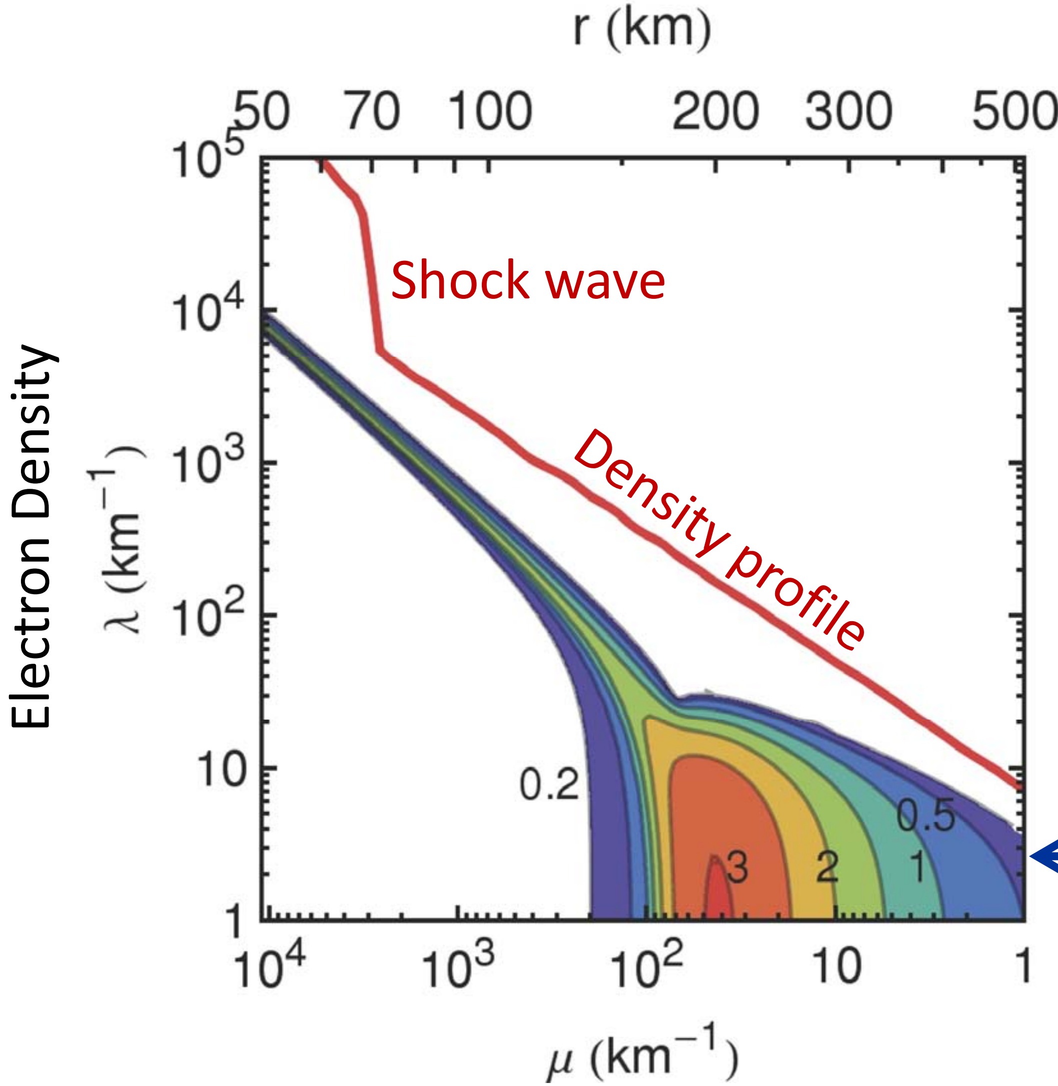
Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Matter Suppression (Normal Hierarchy)



Sarikas, Seixas, Tamborra & Raffelt, work in progress (2012)

Multi-Angle Multi-Energy Stability Analysis



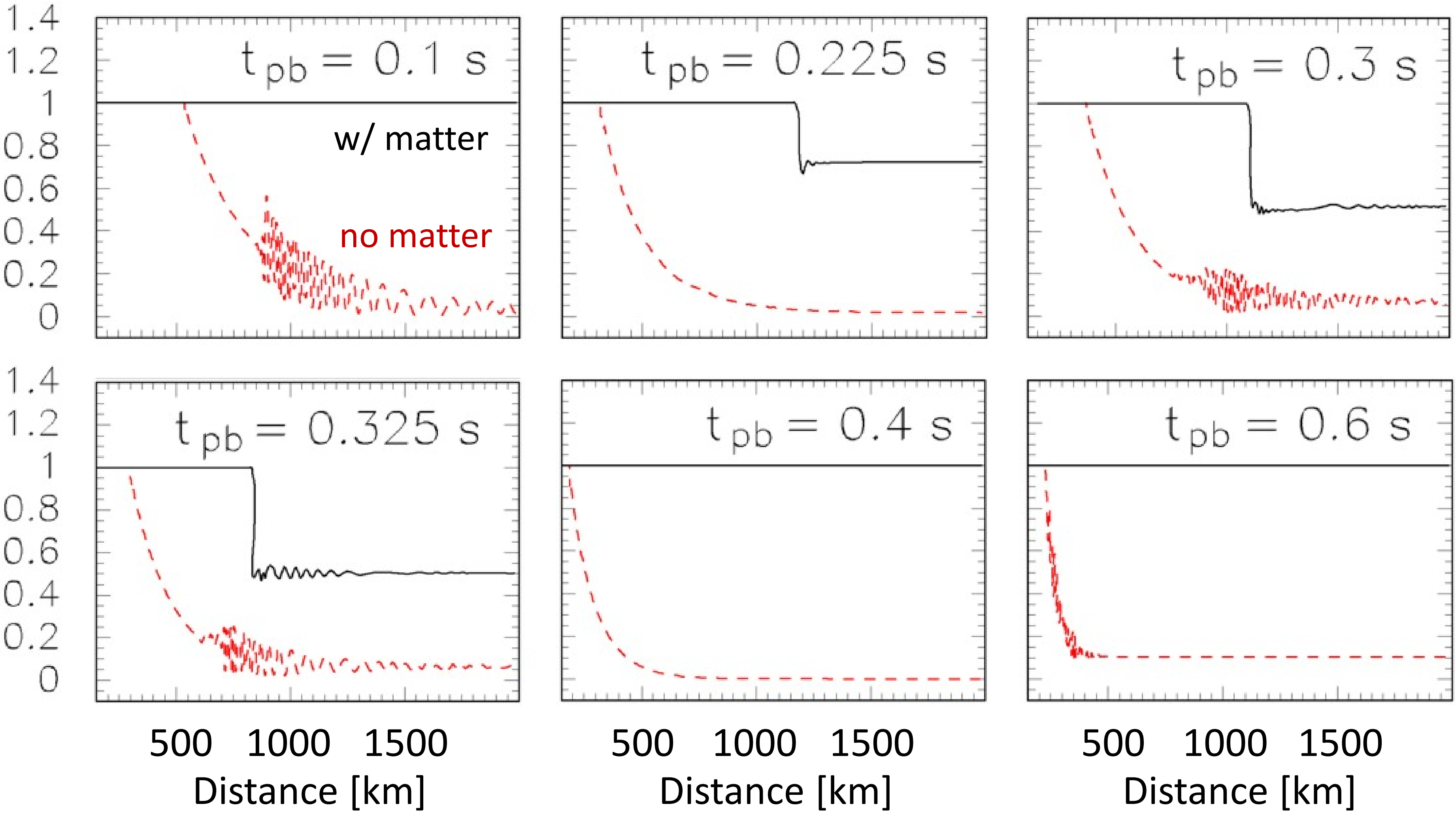
Tested 15 $M_\odot$ accretion-phase models (Garching) are stable against collective flavor conversion

Contours of growth rate κ $[\mathrm{km}^{-1}]$

Sarikas, Raffelt, Hüdepohl & Janka, arXiv:1109.3601

Multi-Angle Matter Effect (Basel Model 10.8 M_{sun})

Schematic single-energy, multi-angle simulations with realistic density profile



Chakraborty, Fischer, Mirizzi, Saviano & Tomàs, arXiv:1105.1130


Looking forward to the next galactic supernova
http://antwrp.gsfc.nasa.gov/apod/ap060430.html