Perturbations to $\mu - \tau$ symmetry in neutrino mixing

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$\mu - \tau$ symmetry/universality

In the diagonal charged lepton mass basis

$$M = U^* M^{\rm diag} U^\dagger$$

$$|U_{\mu i}| = |U_{\tau i}| \,, \text{ for } i = 1, 2, 3.$$

$$\Longrightarrow \theta_{23} = 45^\circ \,, \quad {\rm Re}(\cos\theta_{12} \sin\theta_{12} \sin\theta_{13} e^{i\delta}) = 0$$

4 possibilities:

- $\theta_{23} = 45^\circ, \theta_{13} = 0$ most studied
- $\theta_{23} = 45^\circ, \theta_{12} = 0$ not studied before
- $\theta_{23} = 45^\circ, \theta_{12} = 90^\circ$ not studied before
- $\theta_{23} = 45^\circ, \delta = \pm 90^\circ$ seldom studied

Class a) $\theta_{23}^0 = 45^\circ,\, \theta_{13}^0 = 0$

Unperturbed:

$$U_0 = \begin{pmatrix} \cos\theta_{12}^0 & \sin\theta_{12}^0 & 0 \\ -\frac{\sin\theta_{12}^0}{\sqrt{2}} & \frac{\cos\theta_{12}^0}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}^0}{\sqrt{2}} & -\frac{\cos\theta_{12}^0}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$M_0 = U_0^* M_0^{\rm diag} U_0^\dagger = \begin{pmatrix} m_1^0 c_{12}^2 + m_2^0 s_{12}^2 & \frac{(m_2^0 - m_1^0) s_{12} c_{12}}{\sqrt{2}} & \frac{(m_1^0 - m_2^0) s_{12} c_{12}}{\sqrt{2}} \\ \frac{(m_2^0 - m_1^0) s_{12} c_{12}}{\sqrt{2}} & \frac{1}{2}(m_3^0 + m_2^0 c_{12}^2 + m_1^0 s_{12}^2) & \frac{1}{2}(m_3^0 - m_2^0 c_{12}^2 - m_1^0 s_{12}^2) \\ \frac{(m_1^0 - m_2^0) s_{12} c_{12}}{\sqrt{2}} & \frac{1}{2}(m_3^0 - m_2^0 c_{12}^2 - m_1^0 s_{12}^2) & \frac{1}{2}(m_3^0 + m_2^0 c_{12}^2 + m_1^0 s_{12}^2) \end{pmatrix}$$

Include real perturbation:

$$M = U_0^* M_0^{\mathrm{diag}} U_0^\dagger + E$$

$$E = M - M_0 = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{pmatrix}$$

$$\epsilon_1 = \epsilon_{11}\,, \quad \epsilon_2 = \epsilon_{12} + \epsilon_{13}\,, \quad \epsilon_3 = \epsilon_{12} - \epsilon_{13}\,, \quad \epsilon_4 = \epsilon_{22} + \epsilon_{33} + 2\epsilon_{23}\,,$$
$$\epsilon_5 = \epsilon_{22} - \epsilon_{33}\,, \quad \epsilon_6 = \epsilon_{22} + \epsilon_{33} - 2\epsilon_{23} - 2\epsilon_{11}$$

To first order in (degenerate) perturbation theory

$$\delta\theta_{23}^{(1)} \simeq \frac{\epsilon_5}{2\delta m_{31}^0}\,, \quad \delta\theta_{13}^{(1)} \simeq \frac{\sqrt{2}\epsilon_2}{2\delta m_{31}^0}$$

$$\delta m_{ji}^0 = m_j^0 - m_i^0$$

suppressed by $\epsilon_j/\delta m_{31}^0$

$$\delta\theta_{12}^{(1)} = \frac{1}{2}\arctan\frac{2\sqrt{2}\epsilon_3\cos 2\theta_{12}^0 - \epsilon_6\sin 2\theta_{12}^0}{2\sqrt{2}\epsilon_3\sin 2\theta_{12}^0 + \epsilon_6\cos 2\theta_{12}^0 + 2\delta m_{21}^0}$$

depends on ratios of linear combinations of ϵ_3 and ϵ_6

Large corrections to θ_{12} are possible even for small corrections to the other mixing angles

$$\delta m_i^{(1)} = \frac{1}{4}\Big[4\epsilon_1 + \epsilon_6 \pm \left(2\delta m_{21}^0 - \sqrt{8\epsilon_3^2 + \epsilon_6^2 + 4(\delta m_{21}^0)^2 + 4\delta m_{21}^0 (2\sqrt{2}\epsilon_3 \sin 2\theta_{12}^0 + \epsilon_6 \cos 2\theta_{12}^0)}\right)\Big]$$

$\pm$ for $i = 1, 2$

$$\delta m_3^{(1)} = \frac{1}{2}\epsilon_4$$

Best-fit values and 2σ ranges of the oscillation parameters used to find the ϵ_{ij}, with $\delta m^2 \equiv |m_2|^2 - m_1^2$ and $\Delta m^2 \equiv |m_3|^2 - (m_1^2 + |m_2|^2)/2$.

Parameter	$\theta_{12}(^\circ)$	$\theta_{23}(^\circ)$	$\theta_{13}(^\circ)$	$\delta m^2(10^{-5}\mathrm{eV}^2)$	$\lvert\Delta m^2\rvert(10^{-3}\mathrm{eV}^2)$
Normal hierarchy	$33.6^{+2.1}_{-2.0}$	$39.1^{+4.5}_{-2.8}$	$9.0^{+1.2}_{-1.3}$	$7.54^{+0.46}_{-0.39}$	$2.43^{+0.15}_{-0.17}$
Inverted hierarchy	$33.6^{+2.1}_{-2.0}$	$39.7^{+12.7}_{-3.1}$	$9.0^{+1.2}_{-1.2}$	$7.54^{+0.46}_{-0.39}$	$2.42^{+0.14}_{-0.17}$

Seek perturbed mass matrices that give the oscillation parameters and which have small perturbations

Characterize size of perturbation by

$$\epsilon_{RMS} = \sqrt{\frac{\sum_{i,j=1}^{3} |M_{ij} - M_{0ij}|^2}{9}}$$

Top half: values of the perturbations (in 10^{-3} eV) that give the best-fit parameters *and* have the minimum ϵ_{RMS} for the given θ^0_{12}, for the normal hierarchy and $m_1 = 0$. Bottom half: representative values that fit the experimental data within 2σ and for which all ϵ_{ij} have similar magnitude (with $m^0_1 = 0$, $m^0_2 = 0.0054$ eV, $m^0_3 = 0.0595$ eV).

$\theta^0_{12}(^\circ)$	ϵ_{11}	ϵ_{12}	ϵ_{13}	ϵ_{22}	ϵ_{23}	ϵ_{33}	ϵ_{RMS}
60	-2.97	-3.61	-6.04	-2.24	-1.49	5.21	4.01
45 (BM)	-1.26	-4.83	-4.83	-3.09	-0.63	4.35	3.71
35.3 (TBM)	0.35	-4.74	-4.91	-3.89	0.17	3.55	3.67
30 (HM)	1.08	-4.38	-5.27	-4.26	0.54	3.18	3.71
0	0.00	-1.52	-8.13	-3.72	0.00	3.72	4.28
60	5.77	-4.30	-4.45	-4.33	-9.97	3.05	6.12
45 (BM)	7.12	-4.56	-4.19	-5.01	-9.30	2.37	6.07
35.3 (TBM)	8.02	-4.45	-4.30	-5.45	-8.85	1.93	6.06
30 (HM)	8.47	-4.30	-4.45	-5.68	-8.62	1.70	6.08
0	9.82	-2.65	-6.10	-6.36	-7.95	1.02	6.26

Perturbations have similar size irrespective of θ^0_{12}

$$\epsilon_{RMS} = \frac{\sqrt{\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \frac{1}{2}\epsilon_5^2 + \frac{1}{4}\epsilon_4^2 + \frac{1}{4}(2\epsilon_1 + \epsilon_6)^2}}{3}$$

- $\delta m_{31}^0 \approx m_3^0 \approx m_3 \approx \sqrt{\delta m_{31}^2} = 0.0493\,\mathrm{eV}$
- To get $\delta\theta_{23} = -5.9^\circ$ and $\delta\theta_{13} = 9.0^\circ$, need

$$\epsilon_5 = -0.010 \text{ eV and } \epsilon_2 = 0.011 \text{ eV}$$

- $\frac{\sqrt{\epsilon_2^2 + \epsilon_5^2/2}}{3} = 0.00437 \text{ eV}$

Size of perturbation determined by corrections to θ_{13} and θ_{23}

Top half: same as previous table, except for the inverted hierarchy and $m_3 = 0$. Bottom half: same as previous table, except for the inverted hierarchy and $m_1^0 = 0.05$ eV, $m_2^0 = 0.052$ eV, $m_3^0 = 0$.

$\theta_{12}^0(^\circ)$	ϵ_{11}	ϵ_{12}	ϵ_{13}	ϵ_{22}	ϵ_{23}	ϵ_{33}	ϵ_{RMS}
60	-0.87	-4.97	-5.68	4.81	-0.43	-3.94	4.13
45 (BM)	-0.55	-5.33	-5.33	4.65	-0.28	-4.10	4.11
35.3 (TBM)	-0.25	-5.37	-5.29	4.45	0.08	-4.29	4.11
30 (HM)	0.04	-5.31	-5.34	4.35	0.02	-4.39	4.11
0	0.00	-4.58	-6.07	4.37	0.00	-4.37	4.14
60	-3.57	-5.00	-5.22	5.22	2.52	-3.18	4.31
45 (BM)	-3.07	-5.10	-5.13	4.97	2.77	-3.43	4.29
35.3 (TBM)	-2.74	-5.06	-5.17	4.80	2.94	-3.59	4.29
30 (HM)	-2.57	-5.00	-5.22	4.72	3.02	-3.68	4.29
0	-2.07	-4.39	-5.83	4.47	3.27	-3.93	4.32

Variation in size of perturbation is only about 1%

For the quasi-degenerate spectrum,

$$\delta m_{31}^0 \approx m_3 - m_1 = \frac{\delta m_{31}^2}{m_3 + m_1}$$

decreases as the neutrino mass scale increases

Therefore, to get the same corrections to θ_{13} and θ_{23} the size of the perturbation must decrease

Class b) $\theta_{23}^0 = 45^\circ, \theta_{12}^0 = 0$

$$U_0 = \begin{pmatrix} \cos\theta_{13}^0 & 0 & \sin\theta_{13}^0 \\ -\frac{\sin\theta_{13}^0}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\cos\theta_{13}^0}{\sqrt{2}} \\ -\frac{\sin\theta_{13}^0}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\cos\theta_{13}^0}{\sqrt{2}} \end{pmatrix}$$

$$M_0 = U_0^* M_0^{\rm diag} U_0^\dagger = \begin{pmatrix} m_1^0 c_{13}^2 + m_3^0 s_{13}^2 & \frac{(m_3^0 - m_1^0) s_{13} c_{13}}{\sqrt{2}} & \frac{(m_3^0 - m_1^0) s_{13} c_{13}}{\sqrt{2}} \\ \frac{(m_3^0 - m_1^0) s_{13} c_{13}}{\sqrt{2}} & \frac{1}{2}(m_2^0 + m_3^0 c_{13}^2 + m_1^0 s_{13}^2) & \frac{1}{2}(-m_2^0 + m_3^0 c_{13}^2 + m_1^0 s_{13}^2) \\ \frac{(m_3^0 - m_1^0) s_{13} c_{13}}{\sqrt{2}} & \frac{1}{2}(-m_2^0 + m_3^0 c_{13}^2 + m_1^0 s_{13}^2) & \frac{1}{2}(m_2^0 + m_3^0 c_{13}^2 + m_1^0 s_{13}^2) \end{pmatrix}$$

Top half: for class (b) ($\theta_{12}^0 = 0$) and normal hierarchy. Bottom half: for class (b) and normal hierarchy with $m_1^0 = 0$, $m_2^0 = 0.0054$ eV, $m_3^0 = 0.0595$ eV.

$\theta_{13}^0(^\circ)$	ϵ_{11}	ϵ_{12}	ϵ_{13}	ϵ_{22}	ϵ_{23}	ϵ_{33}	ϵ_{RMS}
0	0.00	-1.52	-8.13	-3.72	0.00	3.72	4.28
5	0.50	1.31	-5.31	-3.97	-0.25	3.47	3.13
10	-0.40	4.09	-2.53	-3.52	0.20	3.92	2.87
15	-2.59	6.49	-0.13	-2.43	1.30	5.02	3.73
20	-5.81	8.21	1.59	-0.82	2.90	6.63	5.11
0	9.82	-2.65	-6.10	-6.36	-7.95	1.02	6.26
5	9.37	1.00	-2.45	-6.13	-7.72	1.25	5.38
10	8.03	4.55	1.09	-5.46	-7.05	1.92	5.18
15	5.84	7.87	4.41	-4.36	-5.95	3.02	5.73
20	2.86	10.87	7.42	-2.88	-4.47	4.50	6.86

For $\theta_{13}^0 < 20^\circ$ can explain data for same size perturbation as for class (a)

Class c) $\theta_{23}^0 = 45°, \theta_{12}^0 = 90°$

Unperturbed mass matrix is same as for class (b) with $m_1^0 \leftrightarrow m_2^0$, so perturbation size is same as for class (b)

Class d) $\theta_{23}^0 = 45°, \delta^0 = \pm 90°$

Small perturbations work for a wide range of θ_{12}^0 and θ_{13}^0

Summary

- Perturbed neutrino mass matrix with $\mu - \tau$ symmetry
- Perturbation size determined by corrections to θ_{13} and θ_{23}
- Small perturbations can give a large correction to θ_{12} because the correction depends only on the ratio of perturbation terms, not on their absolute sizes
- Most mixing scenarios with $\mu - \tau$ symmetry can explain the data with perturbations of similar size (with < 20% variation); nothing special about tribimaximal mixing
- 2 new viable classes of models with θ_{12} equal to 0 or pi/2