Physics of neutrino oscillations & flavor conversion

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Invisibles network INT Training lectures
June 25 - 29, 2012

Content:

1. Oscillations without Paradoxes
2. Matter effects: oscillations, flavor conversion
3. Neutrino propagation in the Earth
4. Advanced topics. Sterile neutrinos

Modern version

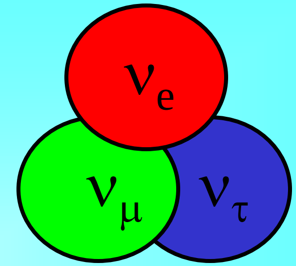
Oscillation and flavor conversion are consequence of

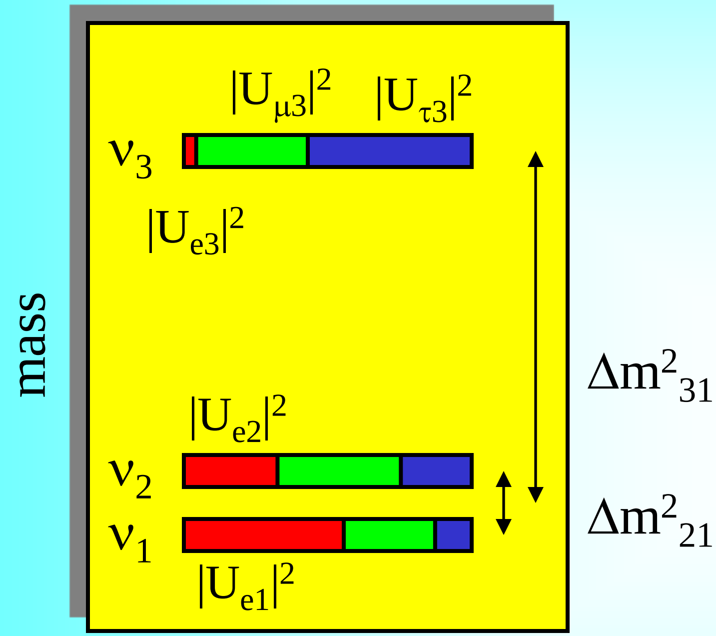
- the lepton mixing and
- production of mixed (flavor) states

Masses & mixing

also to set notations

Mixing





flavor

Normal mass hierarchy

$$\Delta m^2_{31} = m^2_3 - m^2_1$$

$$\Delta m^2_{21} = m^2_2 - m^2_1$$

Mixing parameters

$$\tan^2\theta_{12} = |U_{e2}|^2 \; / \; |U_{e1}|^2$$

$$\sin^2\theta_{13} = |U_{e3}|^2$$

$$\tan^2\theta_{23} = |U_{\mu3}|^2 \; / \; |U_{\tau3}|^2$$

Mass states can be enumerated by amount of electron flavor

Mixing matrix:

$$\nu_f = U_{PMNS}\, \nu_{mass}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{PMNS} = U_{23} I_\delta U_{13} I_{-\delta} U_{12}$$

Who mixes neutrinos?

Mixing in CC → mixing in produced states

Non-trivial interplay of

Charged current weak interactions

Kinematics of specific reactions

Difference of the charged lepton masses

Breaking of coherence

ν_e → $\beta-$ decays, energy conservation

ν_μ → $\pi-$ decays, chirality suppression

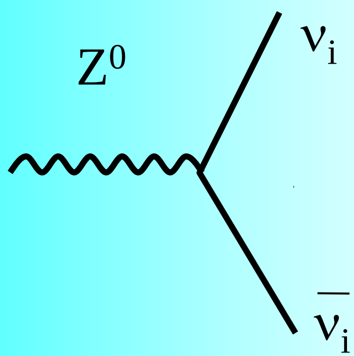
ν_τ → Beam dump, D - decay

What about neutral currents?

Can NC interactions prepare mixed state?

Z is flavor blind

What is the neutrino state produced in the Z-decay in the presence of mixing?

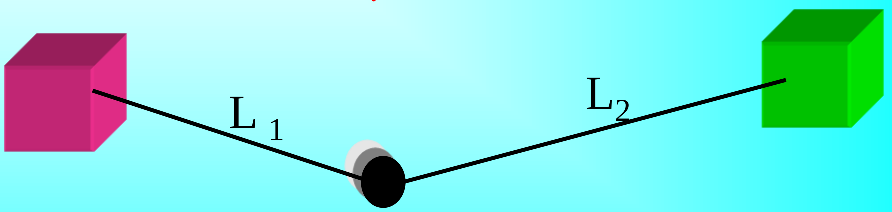


$$|f\rangle = \frac{1}{\sqrt{3}} [\, |\bar{\nu}_1 \nu_1\rangle + |\bar{\nu}_2 \nu_2\rangle + |\bar{\nu}_3 \nu_3\rangle \,]$$

$$|\langle f | H | Z\rangle|^2 = 3 |\langle \bar{\nu}_1 \nu_1 | H | Z\rangle|^2$$

Do neutrinos from Z^0- decay oscillate?

Two detectors experiment:
detection of both neutrinos



If the flavor of one of the neutrino is fixed, another neutrino oscillates

$$P = \sin^2 2\theta \; \sin^2 [\, \pi (L_1 + L_2)/ l_\nu]$$

Oscillations

Without paradoxes

Simple and straightforward

and still correct

Challenging theory of neutrino oscillations

New aspects

New experimental setups

- LBL long tunnels
- Long leaved parents

Based on

[1] Neutrino production coherence and oscillation experiments.
E. Akhmedov, D. Hernandez, A. Smirnov, JHEP 1204 (2012) 052,
arXiv:1201.4128 [hep-ph]

[2] Neutrino oscillations: Entanglement, energy-momentum conservation and QFT.
E.Kh. Akhmedov, A.Yu. Smirnov, Found. Phys. 41 (2011) 1279-1306
arXiv:1008.2077 [hep-ph]

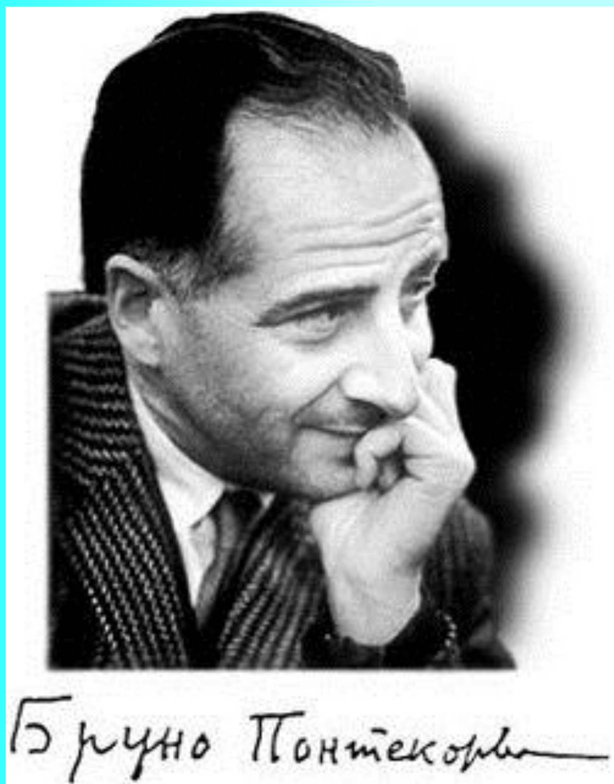
[3] Paradoxes of neutrino oscillations.
E. Kh. Akhmedov, A. Yu. Smirnov Phys. Atom. Nucl. 72 (2009) 1363-1381
arXiv:0905.1903 [hep-ph]

[4] Active to sterile neutrino oscillations: Coherence and MINOS results.
D. Hernandez, A.Yu. Smirnov, Phys.Lett. B706 (2012) 360-366
arXiv:1105.5946 [hep-ph]

[5] Neutrino oscillations: Quantum mechanics vs. quantum field theory.
E. Kh. Akhmedov, J. Kopp, JHEP 1004 (2010) 008
arXiv:1001.4815 [hep-ph]

55 years ago...

Pisa, 1913



B. Pontecorvo

``Mesonium and antimesonium"

Zh. Eksp.Teor. Fiz. 33, 549 (1957)
[Sov. Phys. JETP 6, 429 (1957)] translation

mentioned a possibility of neutrino mixing and oscillations

Oscillations imply non-zero masses (mass squared differences) and mixing

Proposal of neutrino oscillations by B Pontecorvo was motivated by rumor that Davis sees effect in Cl-Ar detector from atomic reactor

Computing oscillation effects

Lagrangian

$$\frac{g}{2\sqrt{2}}\,\bar{l}\,\gamma^{\mu}(1-\gamma_5)\nu_l\, W^{+}_{\mu}$$

$$-\,\tfrac{1}{2}\, m_L\, \nu_L{}^{T} C \nu_L$$

$$-\,\bar{l}_L\, m_l\, l_R \;+\; h.c.$$

Starting from
the first principles

QFT

QM

Amplitudes,
probabilities
of processes

Observables,
Number of
events, etc..

What is the problem?

Set-up

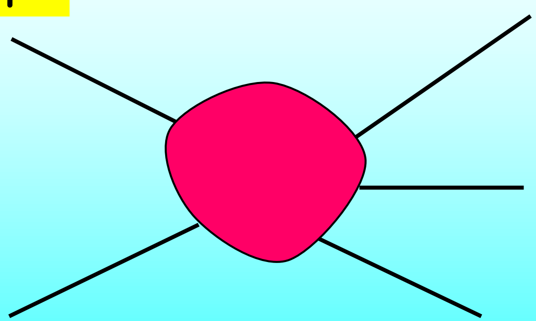
Formalism should be adjusted to specific physics situation

Initial conditions

Recall, the usual set-up

asymptotic states described by plane waves

- enormous simplification



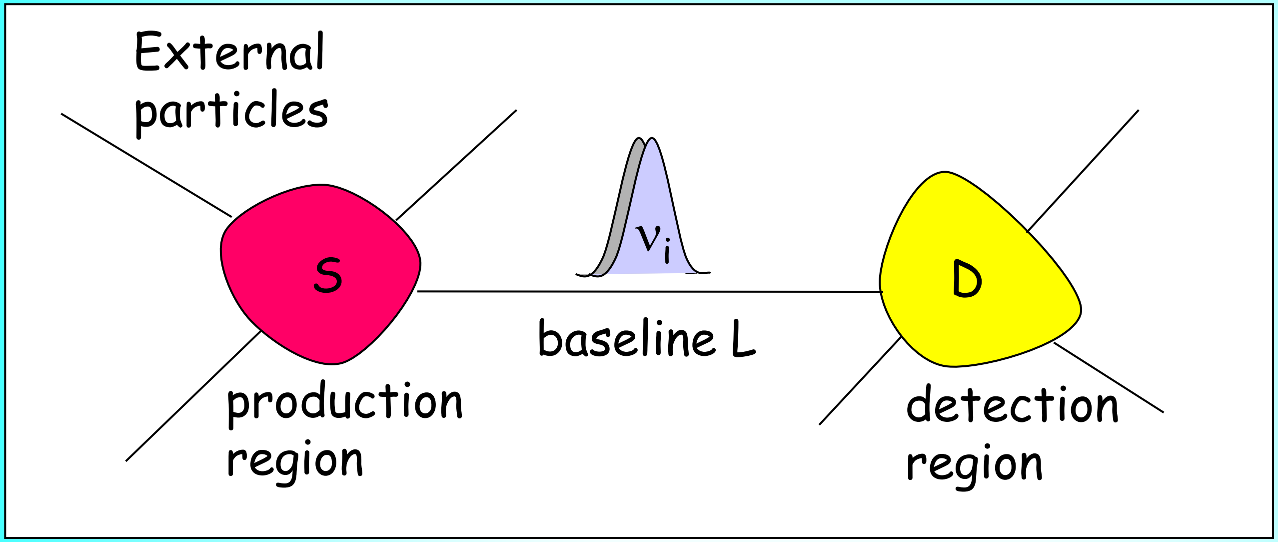
single interaction region

Approximations

Approximations, if one does not want to consider whole history of the Universe to compute signal in Daya Bay

Truncating the process

Oscillation set-up



E. Akhmedov, A.S.

QFT but formalism should be adjusted to these condition

Finite space and time phenomenon

Finite space-time integration limits

Two interaction regions in contrast to usual scattering problem

wave packets for external particles

encode info about finite production and detection regions

Neutrinos: propagator

Wave packets & oscillations

B. Kayser, Phys. Rev D 48 (1981) 110

Wave packet formalism.
Consistent description of oscillations requires consideration of wave packets of neutrino mass states.

31 years later, GGI lectures:

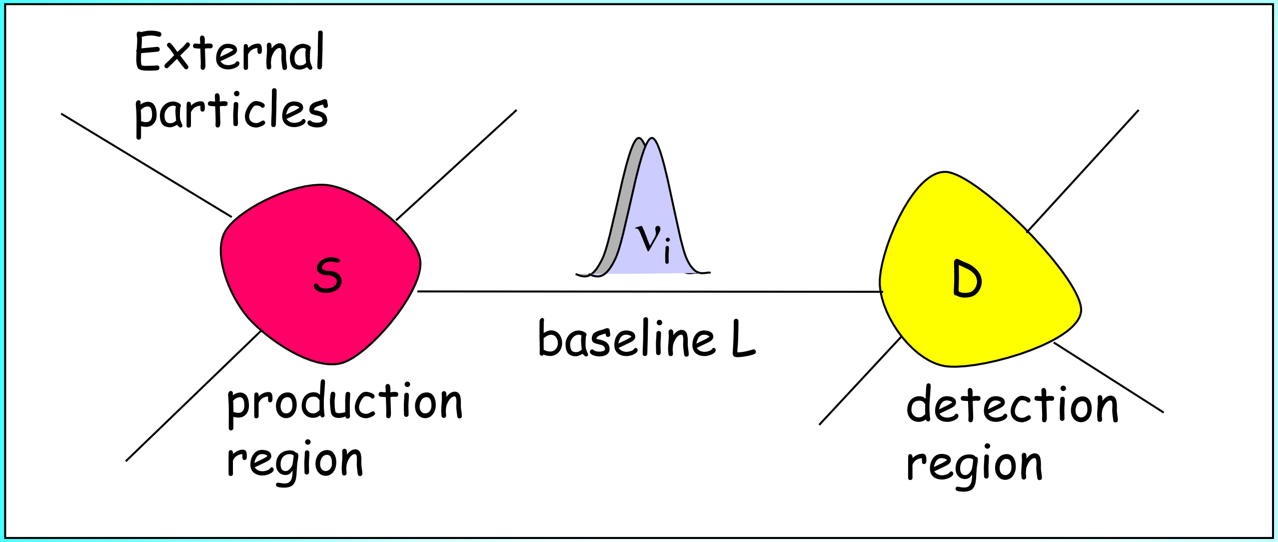
The highest level of sophistication:
to use proper time for neutrino mass and get correct result!

Key point: phases of mass eigenstates should be compared in the same space-time point

If not - factor of 2 in the oscillations phase

Where to truncate?

How external particles
Should be described?

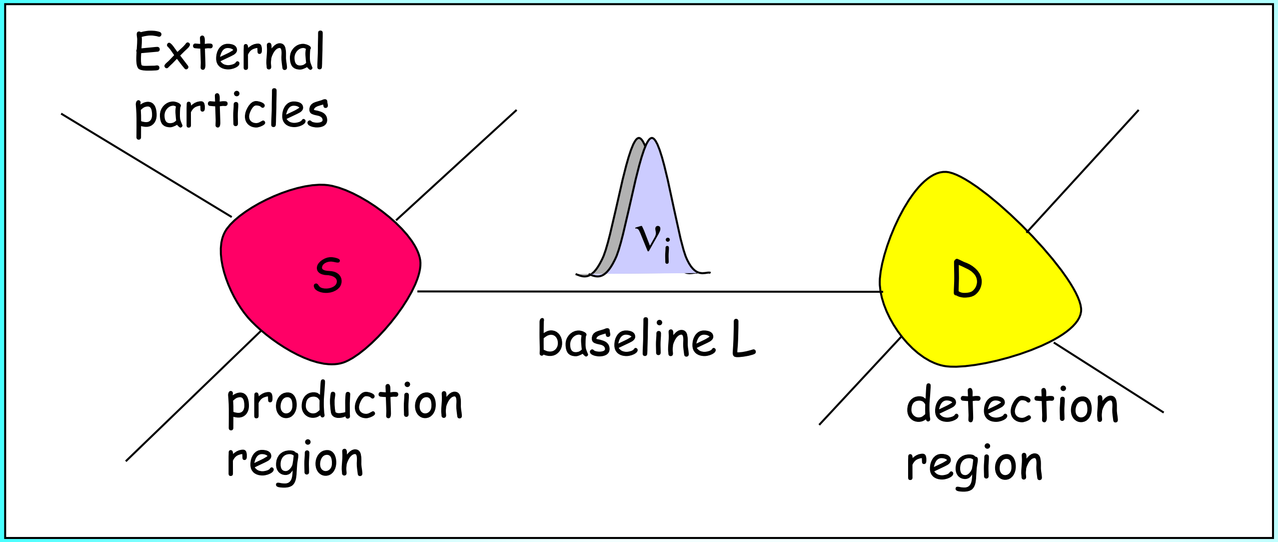


detection/production areas
are determined by localization
of particles involved in neutrino
production and detection
not source/detector volume
(still to integrate over)

→ wave packets for
external particles

→ Describe by plane waves
but introduce finite integration

How to treat neutrinos?



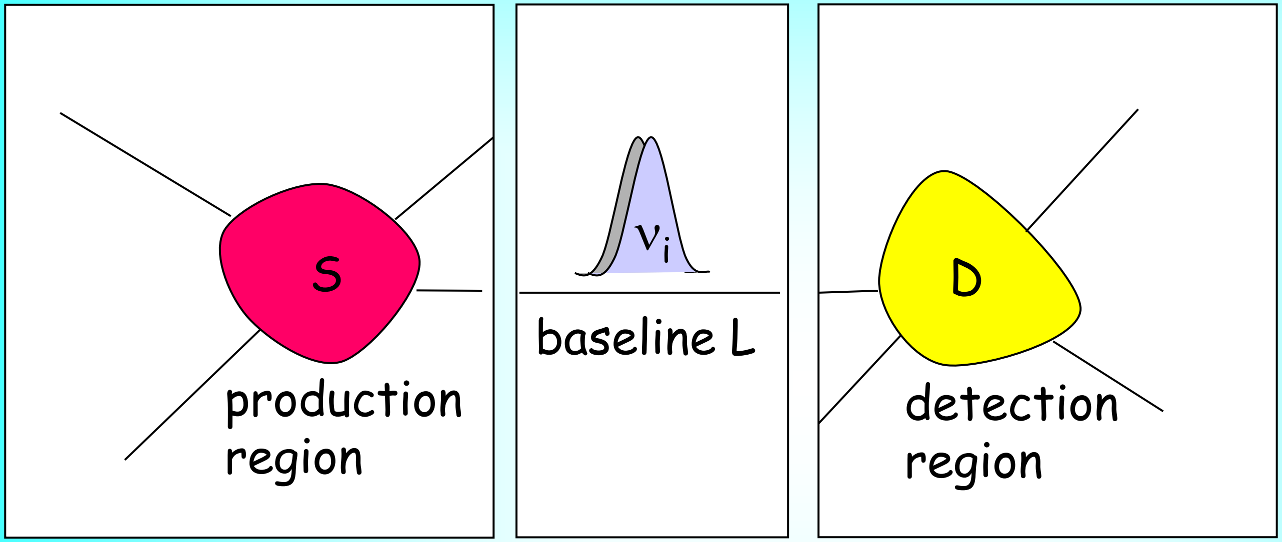
wave packets for
external particles

Unique process,
neutrinos with definite masses are described by propagators,
Oscillation pattern – result of interference of
amplitudes due to exchange of different mass eigenstates

Very quickly converge to mass shell

Real particles – described by wave packets

Factorization



If oscillation effect in Production/detection regions can be neglected → factorization

r_D , r_S << l_ν

Production propagation and Detection can be considered as three independent processes

In terms of mass eigenstates

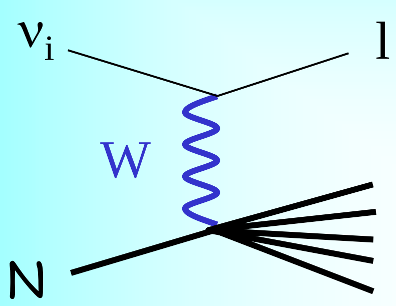
Without flavor states

$$\frac{g}{2\sqrt{2}}\, U_{li}\, \bar{l}\, \gamma^{\mu} (1 - \gamma_5)\, \nu_i\, W^{+}_{\mu} + h.c.$$

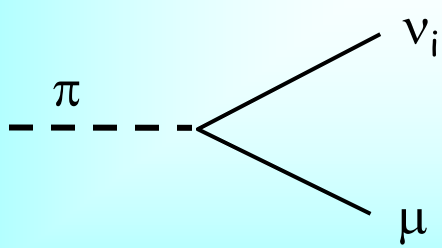
interaction constant

Eigenstates of the Hamiltonian in vacuum

Scattering



$\pi \to \mu\, \nu_i$



Lagrangian of interactions

wave functions of accompanying particles

compute the wave function of neutrino mass eigenstates

Wave packet

Wave packets and oscillations

Suppose ν_α be produced in the source centered at x = 0, t = 0

After formation of the wave packet (outside the production region)

In general

$$|\nu_\alpha(x,t)\rangle = \Sigma_k U_{\alpha k}^* \Psi_k(x, t)|\nu_k\rangle$$

$$\Psi_k \sim \int dp\, f_k(p - p_k)\, e^{ipx - iE_k(p)t}$$

$$E_k(p) = \sqrt{p^2 + m_k^2}$$ - dispersion relation

$f_k(p - p_k)$ - the momentum distribution function peaked at

p_k - the mean momentum

Expanding around mean momentun

describes spread of the wave packets

$$E_k(p) = E_k(p_k) + (dE_k/dp)\Big|_{p_k}(p - p_k) + (dE_k^2/dp^2)\Big|_{p_k}(p - p_k)^2 + \dots$$

$$v_k = (dE_k/dp)\Big|_{p_k} = (p/E_k)\Big|_{p_k}$$ - *group velocity of* ν_k

Shape factor and phase factor

$E_k(p) = E_k(p_k) + v_k(p - p_k)$

(neglecting spread of the wave packets)

Inserting into $\Psi_k \sim \int dp\, f_k(p - p_k)\, e^{ipx - iE_k(p)t}$

$$\Psi_k \sim e^{ip_k x - iE_k(p_k)t}\, g_k(x - v_k t)$$

Phase factor

$$e^{i\phi_k}$$

$$\phi_k = p_k x - E_k t$$

Depends on mean characteristics p_k and corresponding energy:

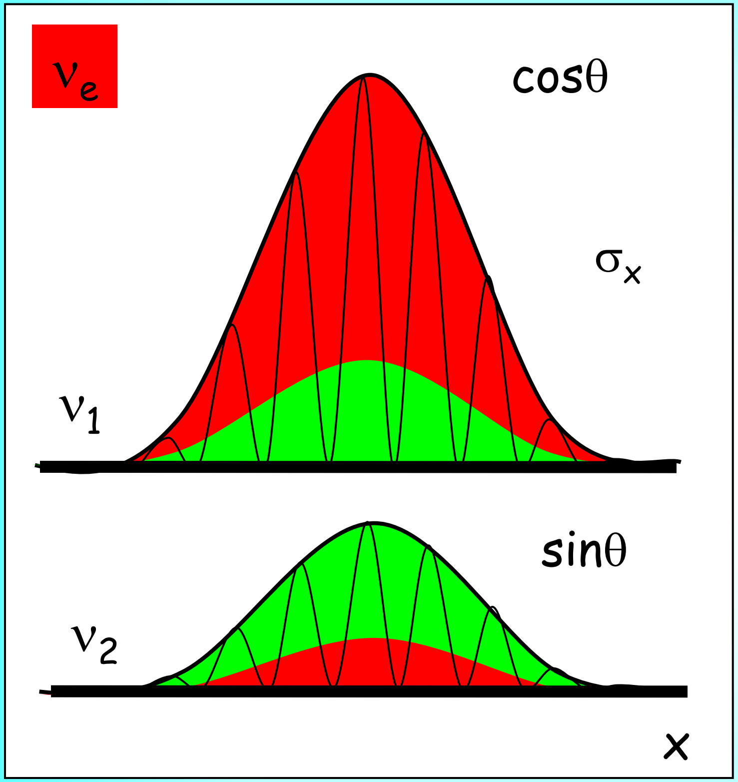
$$E_k(p_k) = \sqrt{p_k^2 + m_k^2}$$

Shape factor

$$g_k(x - v_k t) = \int dp\, f_k(p)\, e^{ip(x - v_k t)}$$

Depends on x and t only in combination $(x - v_k t)$ and therefore Describes propagation of the wave packet with group velocity v_k without change of the shape

Wave packet picture



$\nu_e = \cos\theta\, \nu_1 + \sin\theta\, \nu_2$

$\nu_\mu = - \sin\theta\, \nu_1 + \cos\theta\, \nu_2$

opposite phase

$\nu_1 = \cos\theta\, \nu_e - \sin\theta\, \nu_\mu$

$\nu_2 = \cos\theta\, \nu_\mu + \sin\theta\, \nu_\varepsilon$

Interference of the same flavor parts

$\phi = 0$

Main, effective frequency

$$|\nu (x,t)\rangle = \cos\theta\, g_1(x - v_1 t) e^{i\phi_1} |\nu_1\rangle + \sin\theta\, g_2(x - v_2 t) e^{i\phi_2} |\nu_2\rangle$$

$\phi = \phi_2 - \phi_1$

Oscillation phase

Mixing & mixed states

One needs to compute the state which is produced
i.e. compute

the shape factors

$g_k(x - v_k t)$

mean momenta p_k

- Fundamental interactions
- Kinematics
- characteristics of parent and accompanying particles

Process dependent

If heavy neutrinos are present but can not be produced for kinematical reasons, flavor states in Lagrangian differe from the produced states, etc..

Propagation of wave packets

What happens?

Phase difference change	Due to different masses (dispersion relations) → phase velocities	Oscillations
Separation of wave packets	Due to different group velocities	Loss of coherence
Spread of individual wave packets	Due to presence of waves with different momenta and energy in the packet	...homework

Oscillation phase

$\phi = \phi_2 - \phi_1$ $\phi_i = - E_i t + p_i x$ $p_i = \sqrt{E_i^2 - m_i^2}$

Dispersion relation enters here

$\phi = \Delta E t - \Delta p x$

$\Delta p = (dp/dE)\, \Delta E + (dp/dm^2)\, \Delta m^2 \quad = \quad 1/v_g\, \Delta E + (1/2p)\, \Delta m^2$

group velocity

$$\phi \; = \; \Delta E/v_g\, (v_g t - x) + \frac{\Delta m^2}{2E}\, x$$

standard oscillation phase

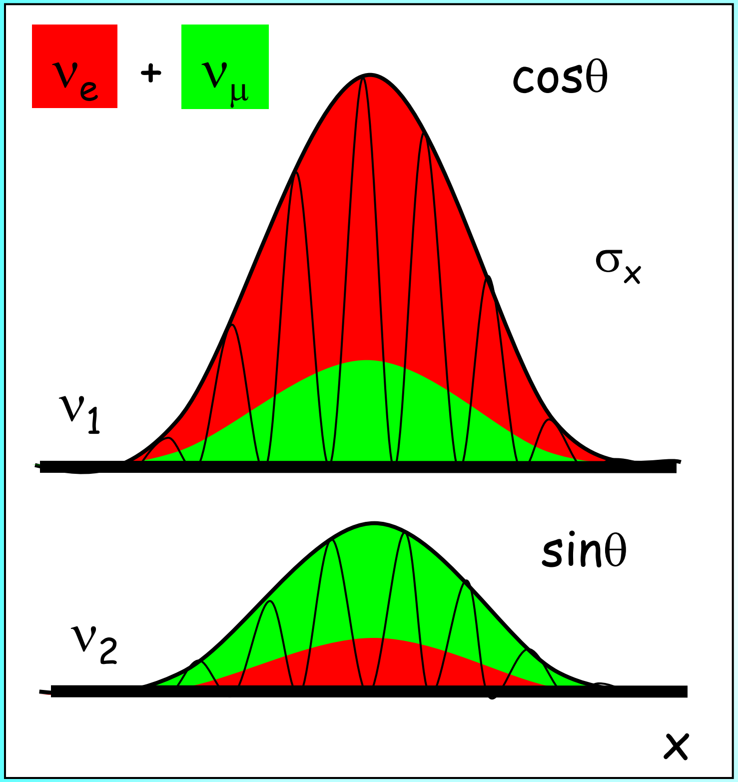
$\Delta E^0 \sim \Delta m^2/2E$ $< \sigma_x$

$\Downarrow$

$\sigma_x \Delta m^2/2E$

Effect of averaging over size of the wave packet usually- small

Oscillations



ν_e = cosθ ν_1 + sinθ ν_2

ν_μ = - sinθ ν_1 + cosθ ν_2

opposite phase

Interference pattern depends on relative (oscillation) phase

$\phi = \pi$

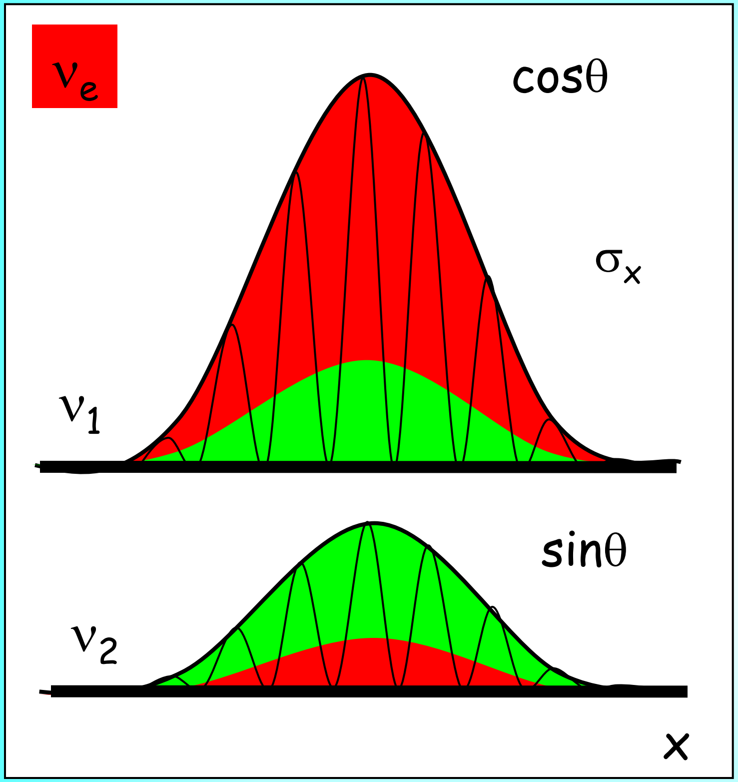
Main, effective frequency

$$|\nu(x,t)\rangle = \cos\theta \, g_1(x - v_1 t)|\nu_1\rangle + \sin\theta \, g_2(x - v_2 t)e^{i\phi} \, |\nu_2\rangle$$

$\phi = \phi_2 - \phi_1$

Oscillation phase

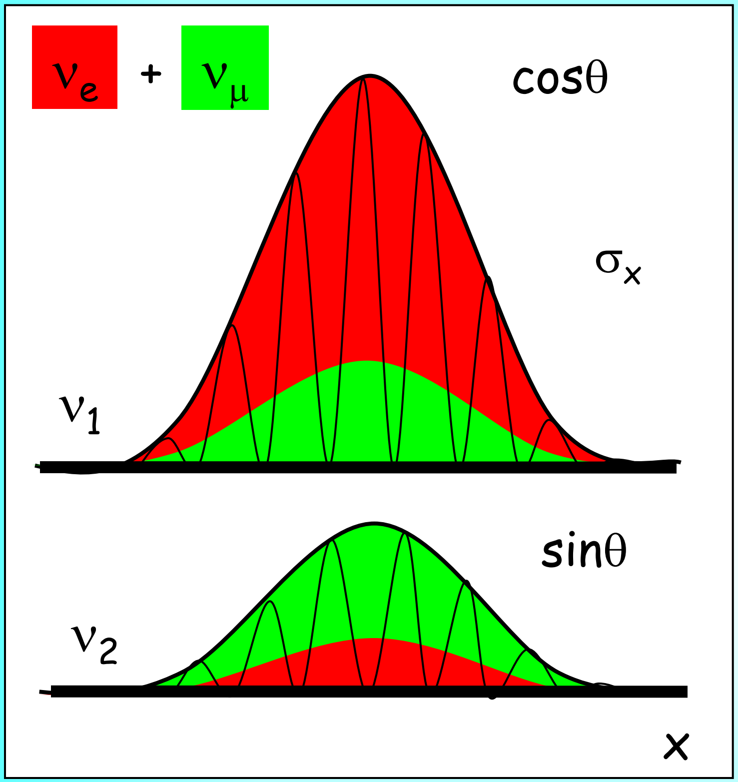
Oscillations



$\phi = 0$

- Destructive interference of the muon parts
- Constructive interference of electron parts

Oscillations



$\phi = \pi$

- Destructive interference of the electron parts
- Constructive interference of muon parts

Detection:

As important as production
Should be considered symmetrically with production

Detection effect can be included in
the generalized shape factors

$$g_k(x - v_k t) \rightarrow G_k(L - v_k t)$$

$x \rightarrow L$ - distance between central points of the production and detection regions

HOMEWORK...

Oscillation probability

Amplitude of (survival) probability

$$A(\nu_e) = \langle \nu_e | \nu (x,t) \rangle = \cos^2\theta\, g_1(x - v_1 t) + \sin^2\theta\, g_2(x - v_2 t)\, e^{i\phi}$$

Probability

inteference

$$P(\nu_e) = \int dx\, |\langle \nu_e | \nu (x,t) \rangle|^2 =$$

(integration over the detection area)

$$= \cos^4\theta + \sin^4\theta + 2\sin^2\theta \cos^2\theta \cos\phi \int dx\, g_1(x - v_1 t)\, g_2(x - v_2 t)$$

If $g_1 = g_2$

$$P(\nu_e) = 1 - 2\sin^2\theta \cos^2\theta\, (1 - \cos\phi) = 1 - \sin^2 2\theta \sin^2 \tfrac{1}{2}\phi$$

$$\phi = \frac{\Delta m^2\, x}{2E} = \frac{2\pi x}{l_\nu}$$

depth of oscillations

$$l_\nu = \frac{4\pi E}{\Delta m^2}$$

Oscillation length

Appearance probability

$$P(\nu_\mu) = \sin^2 2\theta \, \sin^2 \frac{\pi x}{l_\nu}$$

All complications – absorbed in normalization
or reduced to partial averaging of oscillations
or lead to negligible corrections of order $m/E \ll 1$

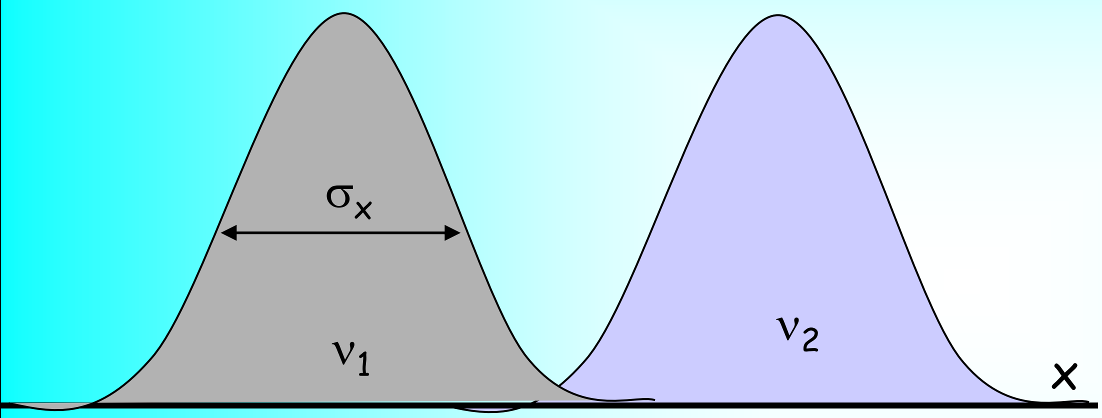
Oscillations - effect of the phase difference increase between mass eigenstates

Admixtures of the mass eigenstates ν_i in a given neutrino state do not change during propagation

Flavors (flavor composition) of the eigenstates are fixed by the vacuum mixing angle

Coherence in propagation

In the configuration space: separation of the wave packets due to difference of group velocities



$\Delta v_{gr} = \Delta m^2 / 2E^2$

separation: $\Delta v_{gr} L = \Delta m^2 L / 2E^2$

no overlap: $\Delta v_{gr} L > \sigma_x$

coherence length:

$L_{coh} = \sigma_x \; E^2/\Delta m^2$

In the energy space: period of oscillations

$\Delta E = 4\pi E^2/(\Delta m^2 L)$

Averaging (loss of coherence) if energy resolution σ_E is bad:

$\Delta E < \sigma_E$

Coherence is determined by conditions of detection

If $\Delta E > \sigma_E$ - restoration of coherence even if the wave packets separated

Formation of the wave packet

E. Akhmedov, D. Hernandez, A.S.

Pion decay:

$$g_i(x,t)\, e^{i\phi_i} = \int dx_S\, dt_S\; M\, \psi_\pi(x_S, t_S)\, \overline{\psi_\mu}(x_S, t_S)\, \exp[\, ip_i(x - x_S) - iE_i(t - t_S)]$$

integration over production region

part of matrix element

plane wave for neuutrino

Pion wave function:

$$\psi_\pi(x_S, t_S) = \exp[-\tfrac{1}{2}\Gamma t_S]\, g_\pi(x_S, t_S)\, \exp[-i\phi_\pi(x_S, t_S)]$$

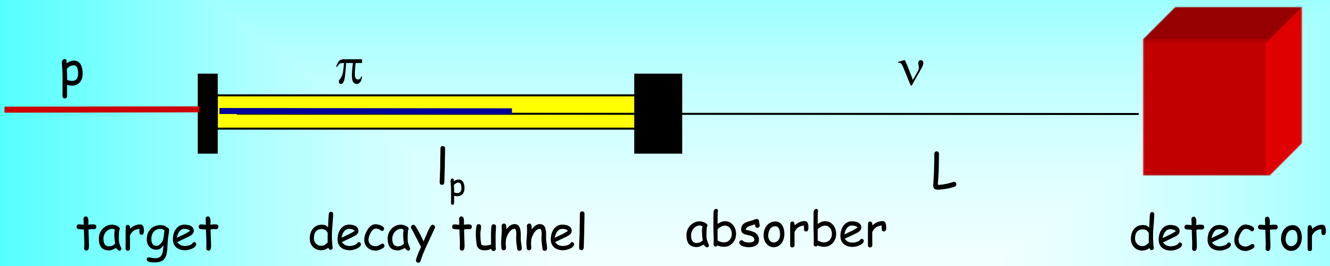
usually: $g_\pi(x_S, t_S) \sim \delta(x_S - v_\pi t_S)$

Muon wave function:

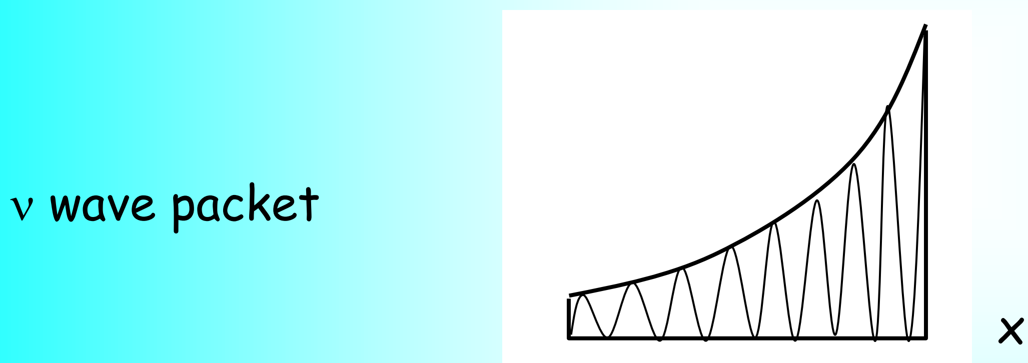
$$\psi_\mu(x_S, t_S) = g_\mu(x' - x_S, t' - t_S)\, \exp[\, i\phi_\mu(x' - x_S\, , t' - t_S)]$$

determined by detection of muon

Neutrino wave packets



D. Hernandez, AS



E. Kh Akhmedov, D. Hernandez, AS arXiv:1110.5453

The length of the ν wave packet emitted in the forward direction

$$\sigma = l_p \frac{v - v_\pi}{v_\pi}$$

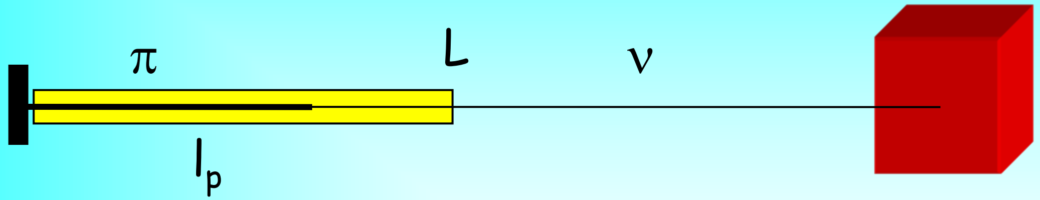
Shape factor

$$g = g_0 \exp\left(\frac{\Gamma}{2(v - v_\pi)}(x - \sigma)\right) \Pi(x, [0, \sigma])$$

box function

Doppler effect

Decoherence at production



D. Hernandez, AS

$$P = \bar{P} + \frac{\sin^2 2\theta}{2(1 + \xi^2)} \frac{1}{1 - e^{-\Gamma l_p}} [\cos \phi_L + K]$$

$\xi = \Delta m^2/2E\Gamma$

decoherence parameter

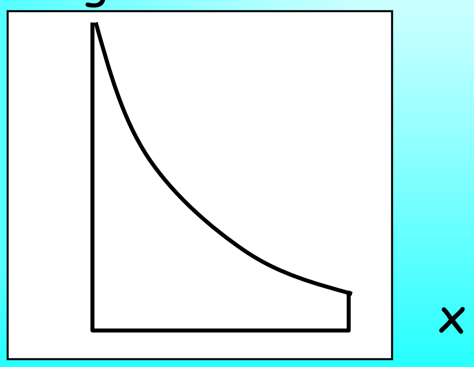
$$K = \xi \sin \phi_L - e^{-\Gamma l_p}[\cos(\phi_L - \phi_p) - \xi \sin(\phi_L - \phi_p)]$$

$\phi_L = \Delta m^2 L/2E \quad \phi_p = \Delta m^2 l_p/2E$

MINOS: $\xi \sim 1$

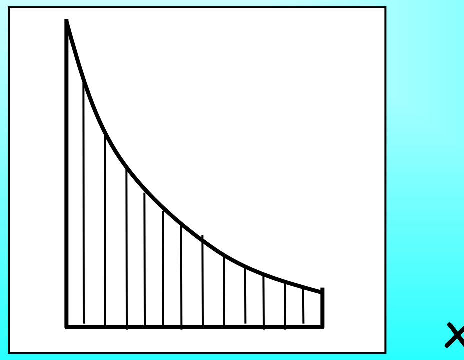
β–beam ?

Coherent ν-emission
- long WP

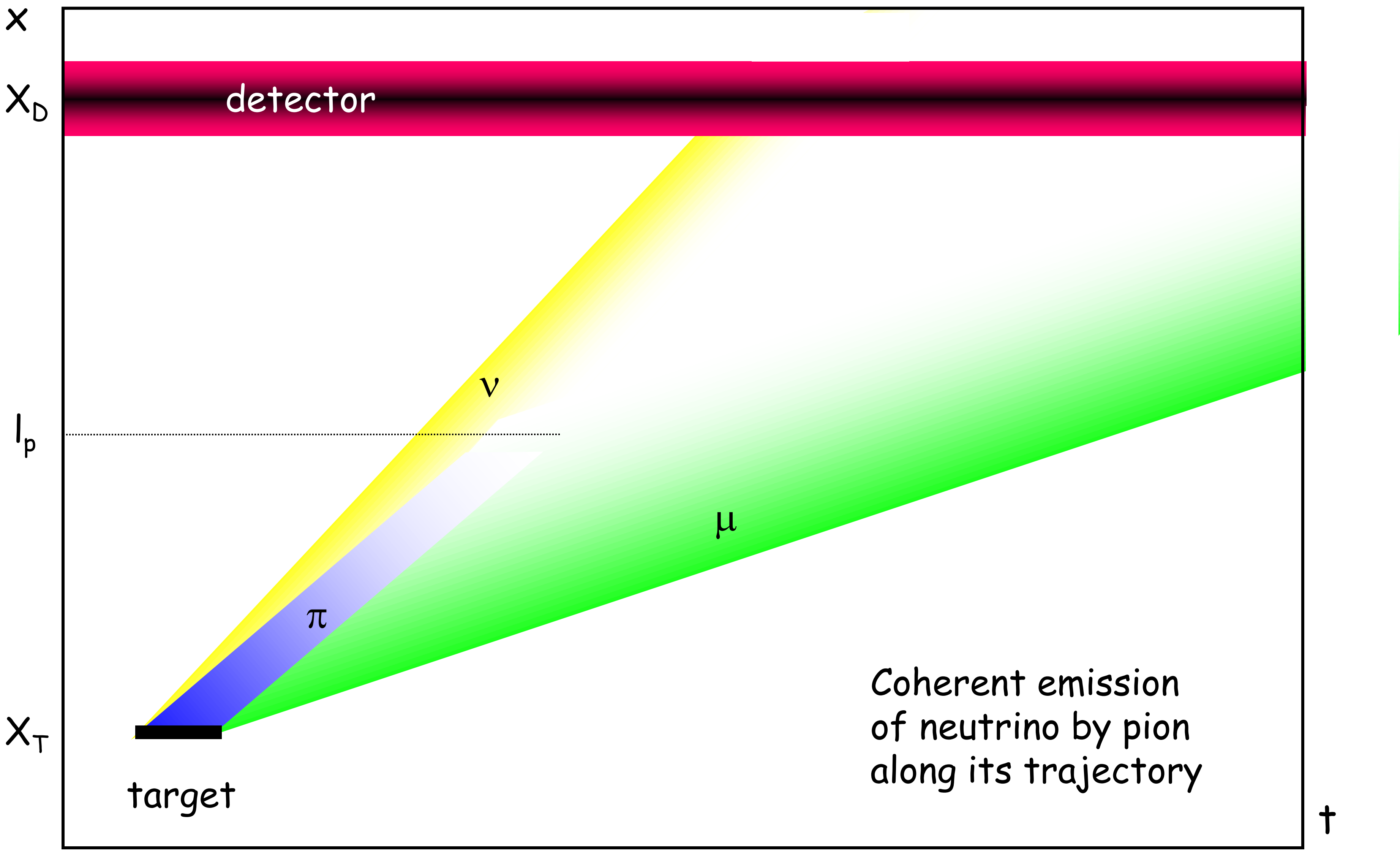


Equivalence

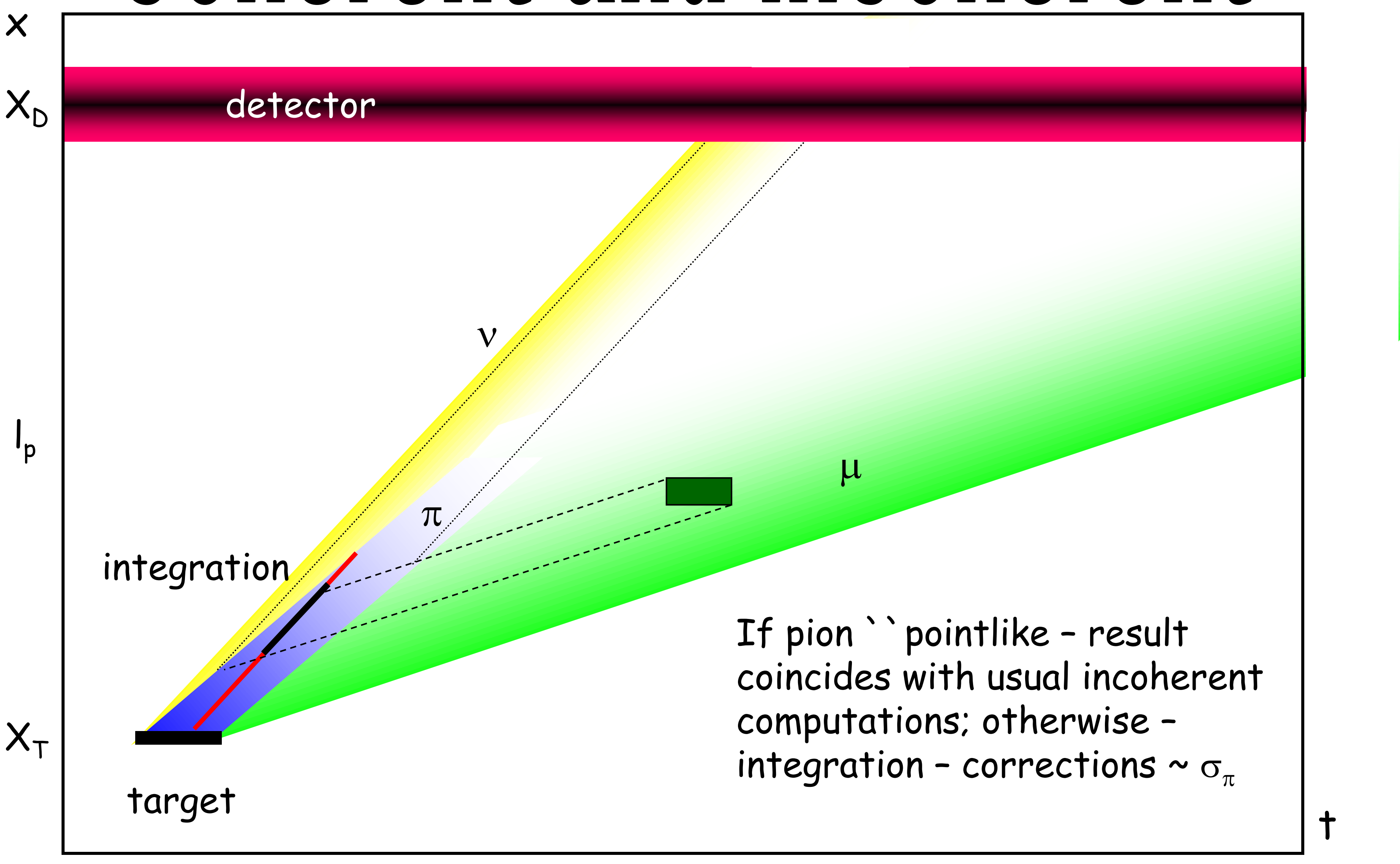
Incoherent ν-emission
- short WP



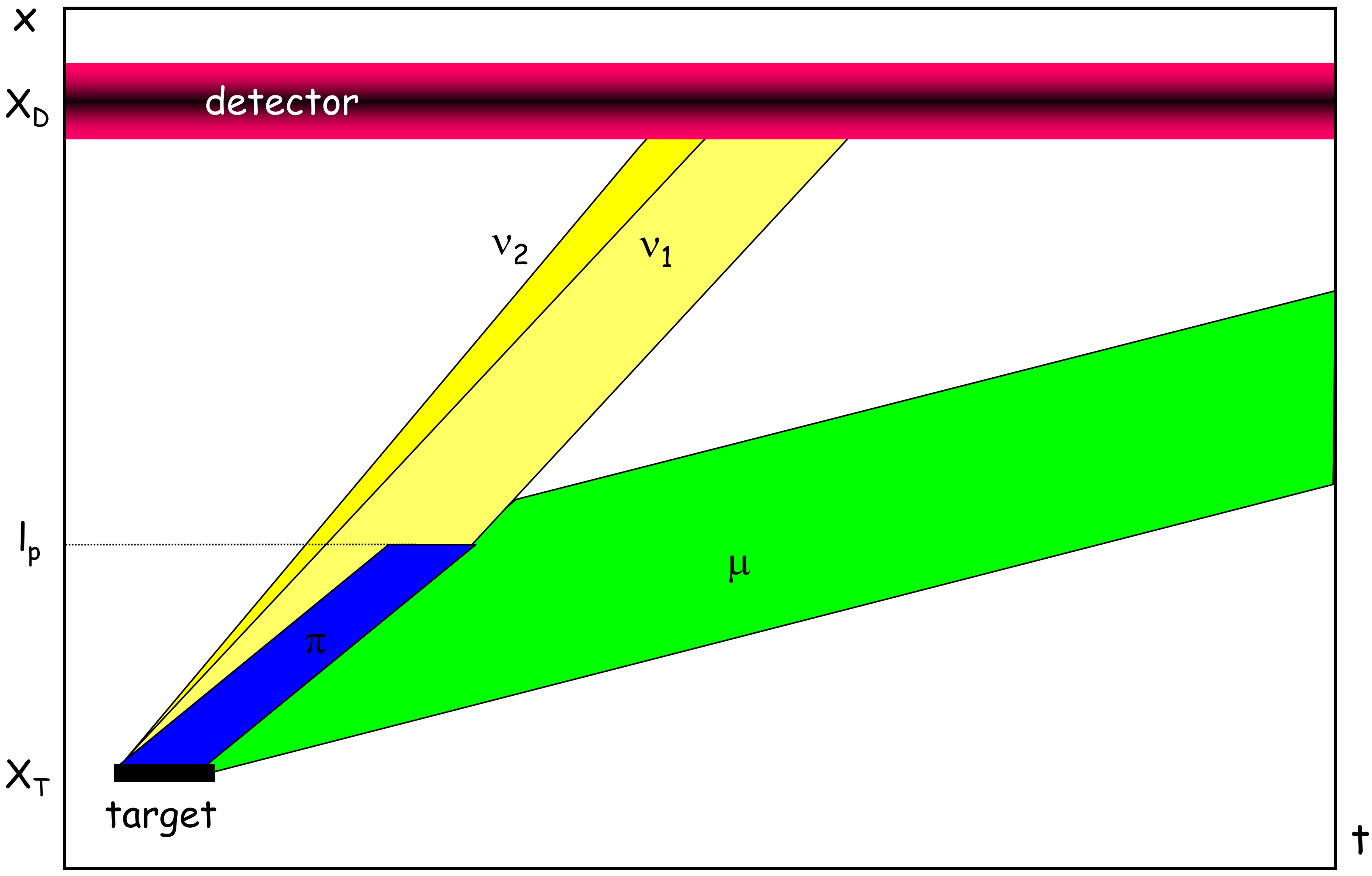
Space-time diagrams



Coherent and incoherent



If pion ``pointlike – result coincides with usual incoherent computations; otherwise – integration – corrections ~ σ_π



x
X_D
detector
ν_2
ν_1
l_p
μ
π
X_T
target
t

Master equation

If loss of coherence and other complications related to WP picture are irrelevant – point-like picture

$$i \frac{d\Psi}{dt} = H\Psi$$

$$\Psi = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix}$$

$$H = \frac{M M^+}{2E} + V(t)$$

M is the mass matrix

V = diag (V_e , 0, 0) – effective potential

Mixing matrix in vacuum

$$M M^+ = U M_{diag}^2 U^+$$

$$M_{diag}^2 = diag(m_1^2, m_2^2, m_3^2)$$

Neutrino polarization vectors

$$\psi = \begin{pmatrix} \nu_e \\ \nu_\tau, \end{pmatrix} \Rightarrow$$

Polarization vector:

$$\mathbf{P} = \psi^+ \sigma/2\, \psi$$

$$\mathbf{P} = \begin{pmatrix} \mathrm{Re}\, \nu_e^+ \nu_\tau, \\ \mathrm{Im}\, \nu_e^+ \nu_\tau, \\ \nu_e^+ \nu_e - 1/2 \end{pmatrix}$$

Evolution equation:

$$i \frac{d\Psi}{dt} = H\Psi \Rightarrow$$

$$i \frac{d\Psi}{dt} = (\mathbf{B}\sigma)\Psi$$

$$\mathbf{B} = \frac{2\pi}{l_m}(\sin 2\theta_m,\ 0,\ \cos 2\theta_m)$$

Differentiating **P** and using equation of motion

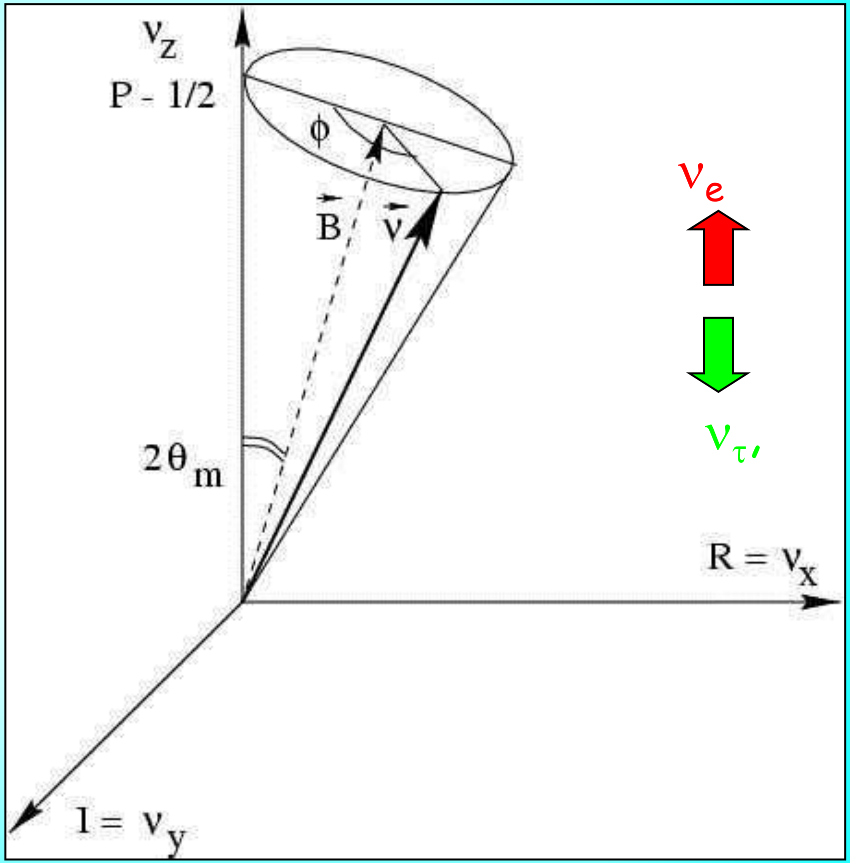
$$\frac{d\mathbf{P}}{dt} = (\mathbf{B} \times \mathbf{P})$$

Coincides with equation for the electron spin precession in the magnetic field

Graphical representation

$\vec{\nu} = \mathbf{P} =$
$(\mathrm{Re}\ \nu_e^+ \nu_\tau,\quad \mathrm{Im}\ \nu_e^+ \nu_\tau,\quad \nu_e^+ \nu_e - 1/2)$

$$\mathbf{B} = \frac{2\pi}{l_m}(\sin 2\theta_m,\ 0,\ \cos 2\theta_m)$$



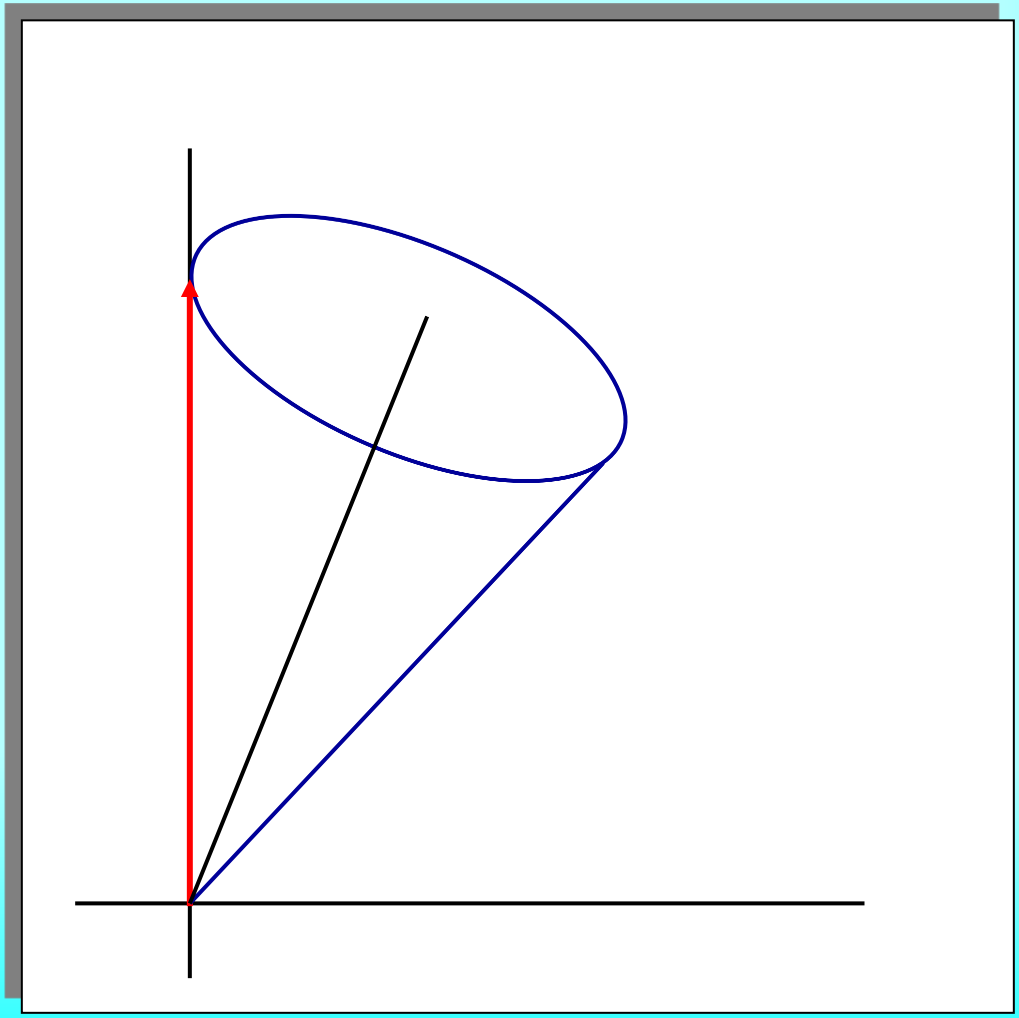
Evolution equation

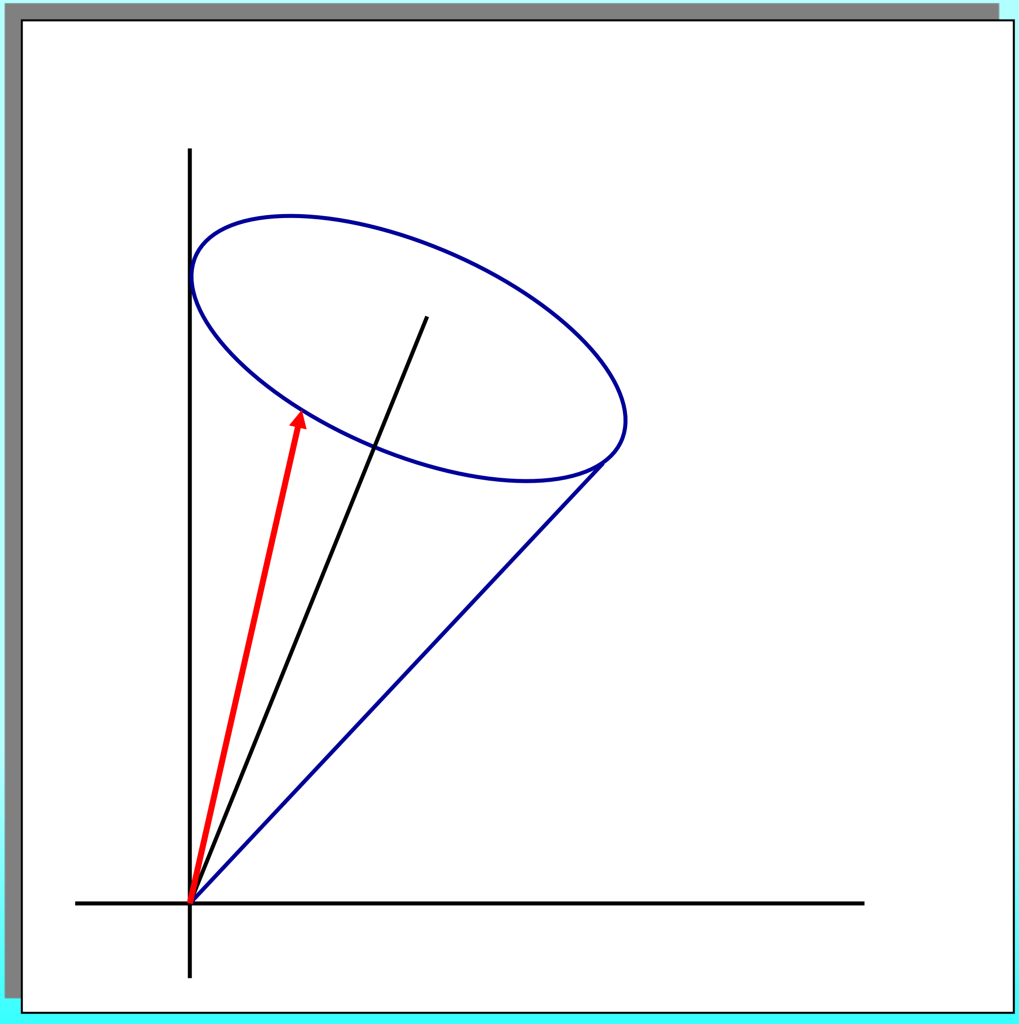
$$\frac{d\vec{\nu}}{dt} = (\vec{B} \times \vec{\nu})$$

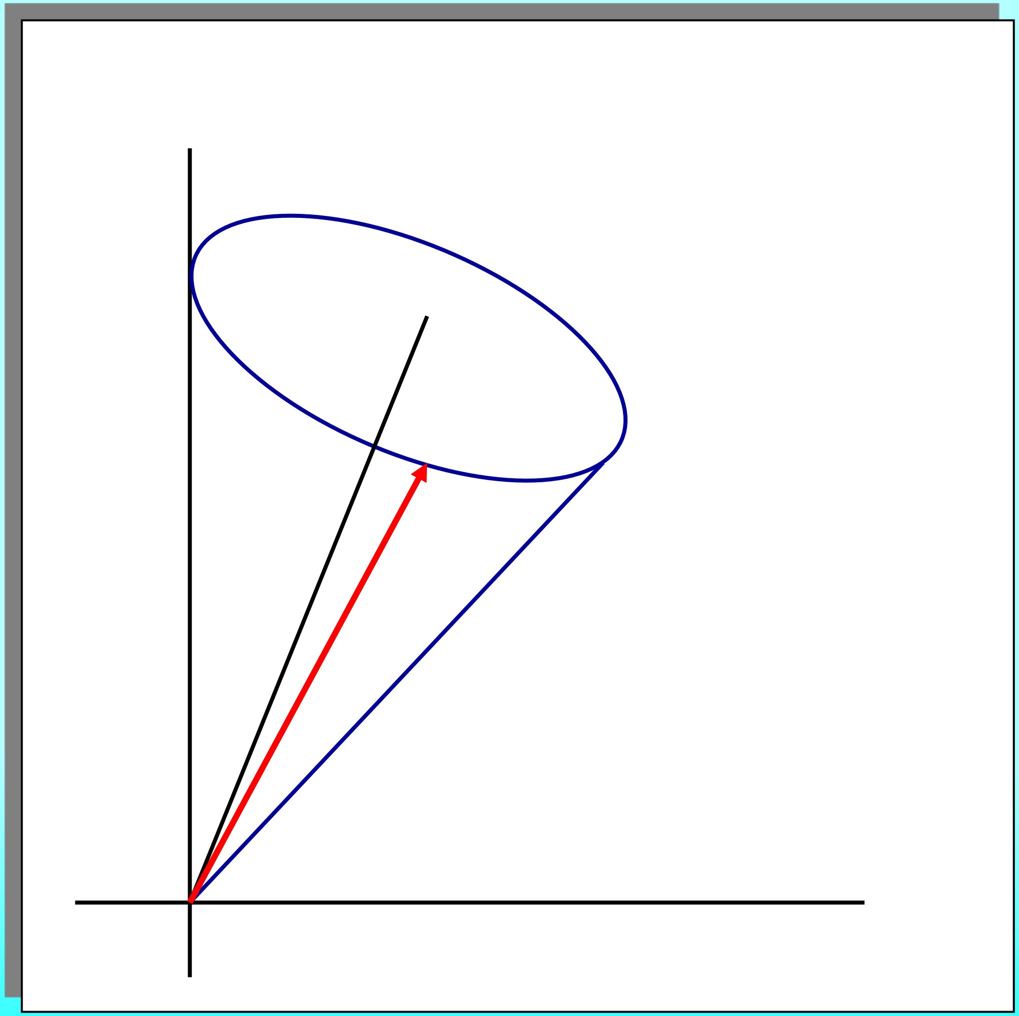
$\phi = 2\pi t / l_m$ - phase of oscillations

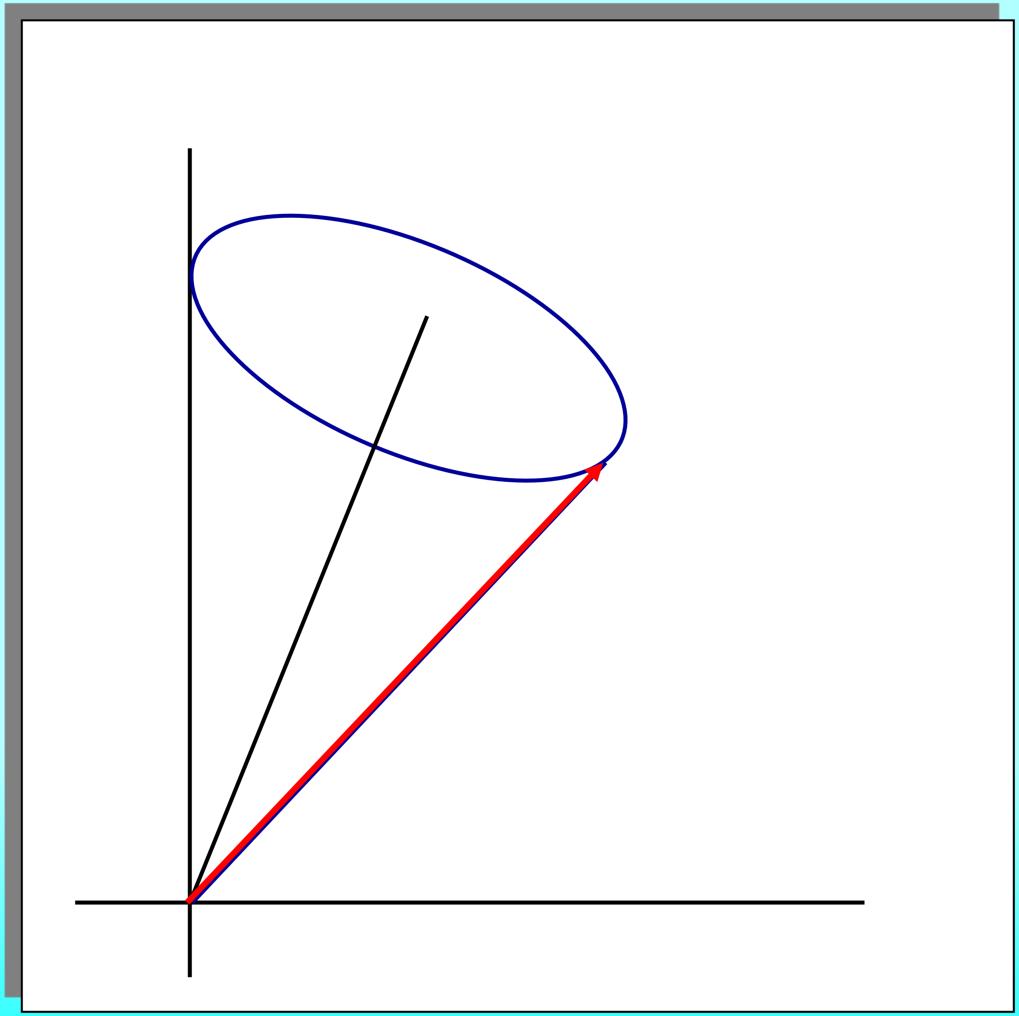
$P = \nu_e^+ \nu_e = \nu_Z + 1/2 = \cos^2\theta_Z/2$ probability to find ν_e

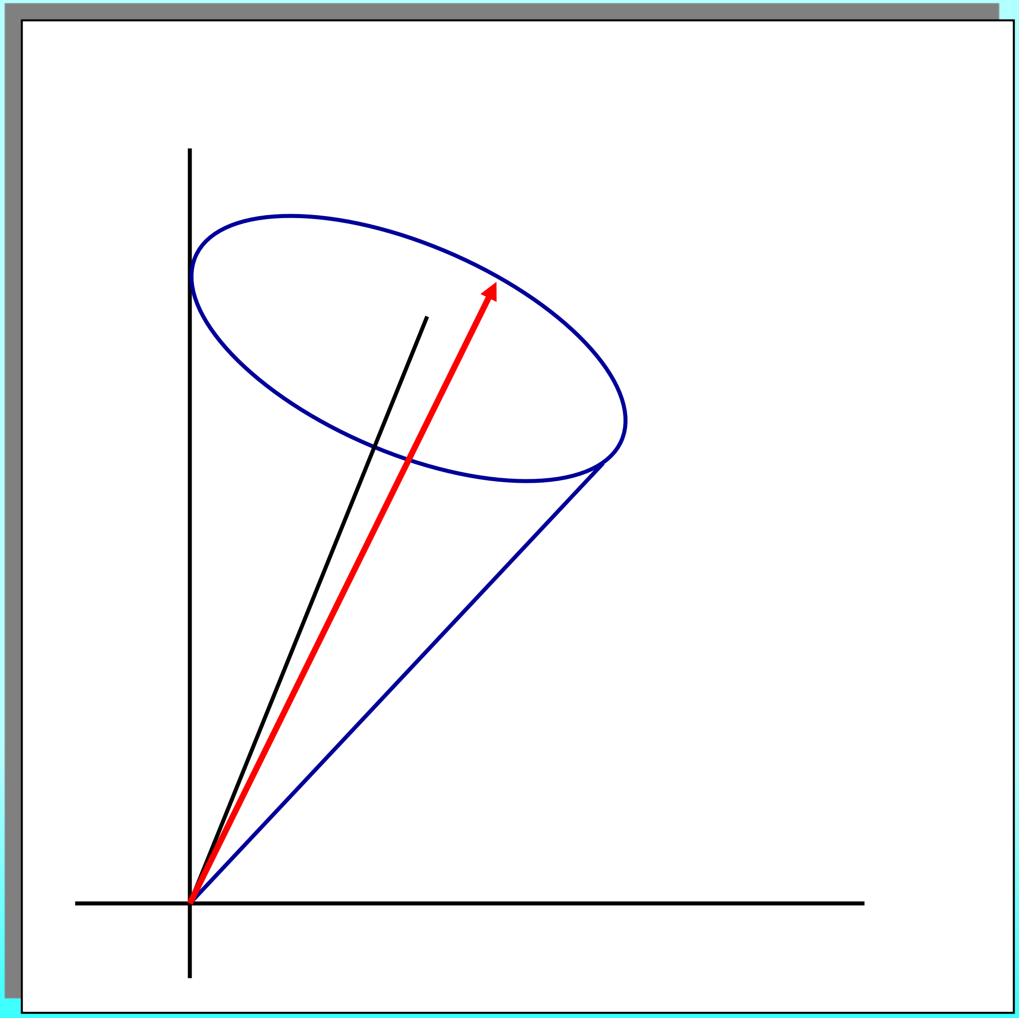
Oscillations

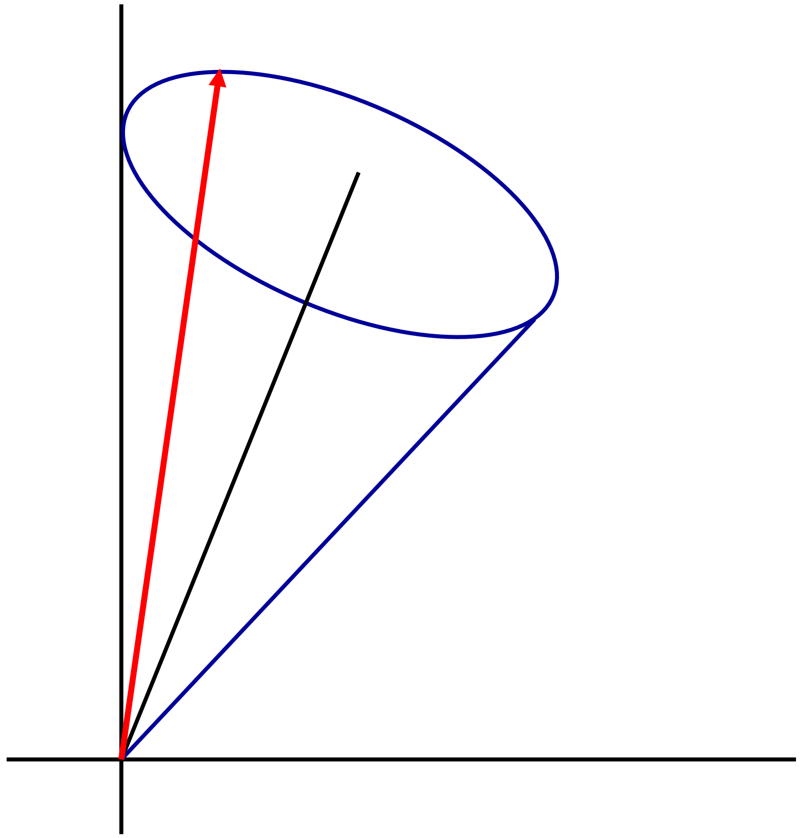


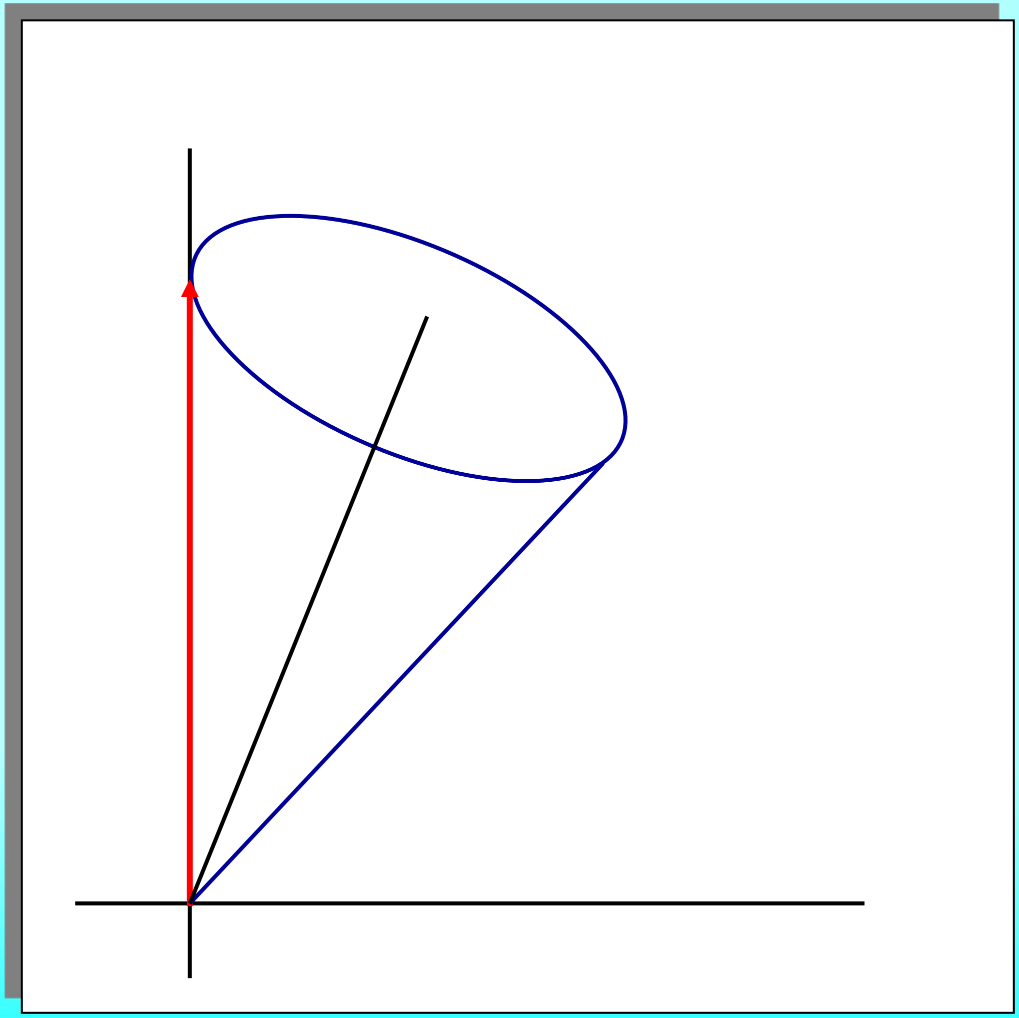










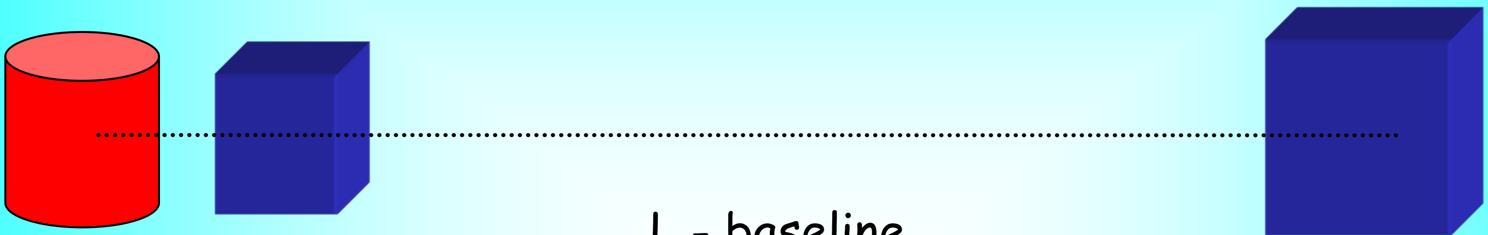


Conclusion:

Oscillations is effect of monotonous phase difference increase between eigenstates of propagation (mass eigenstates)

In course of propagation in space-time

Experimental set-up



ν_α

ν_α disappearance

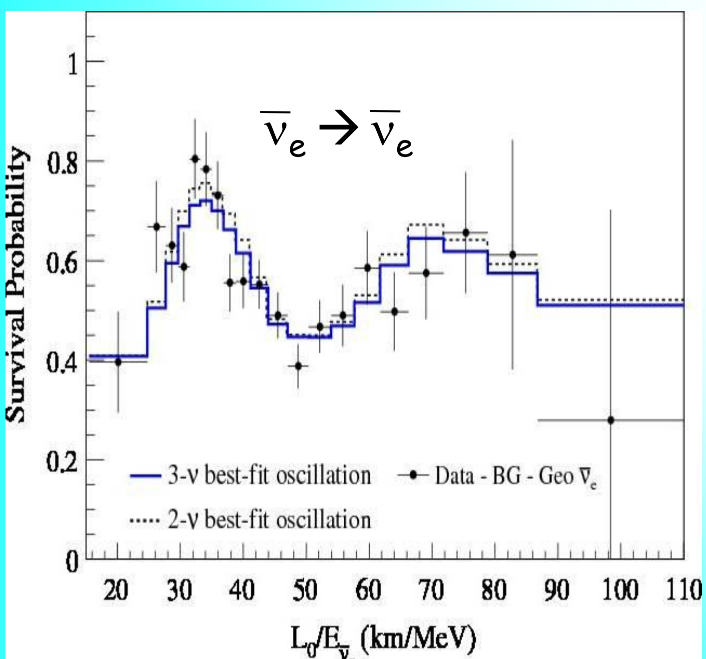
ν_β appearance

Oscillation probability – periodic function of

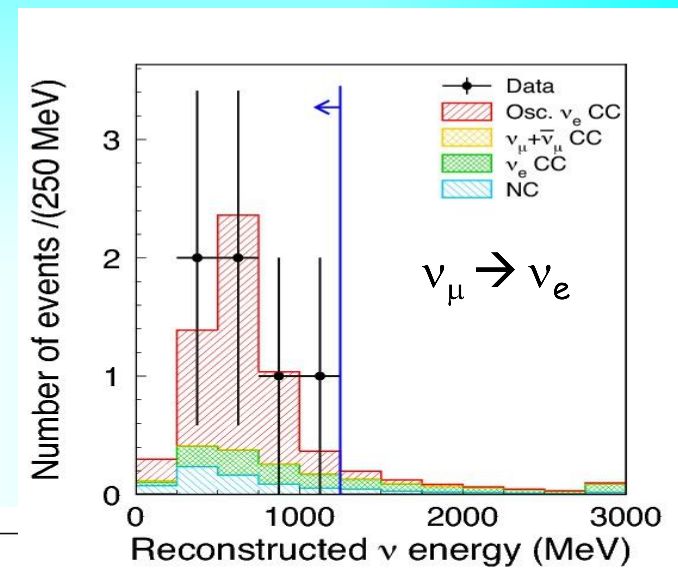
- Distance L and
- Inverse energy 1/E

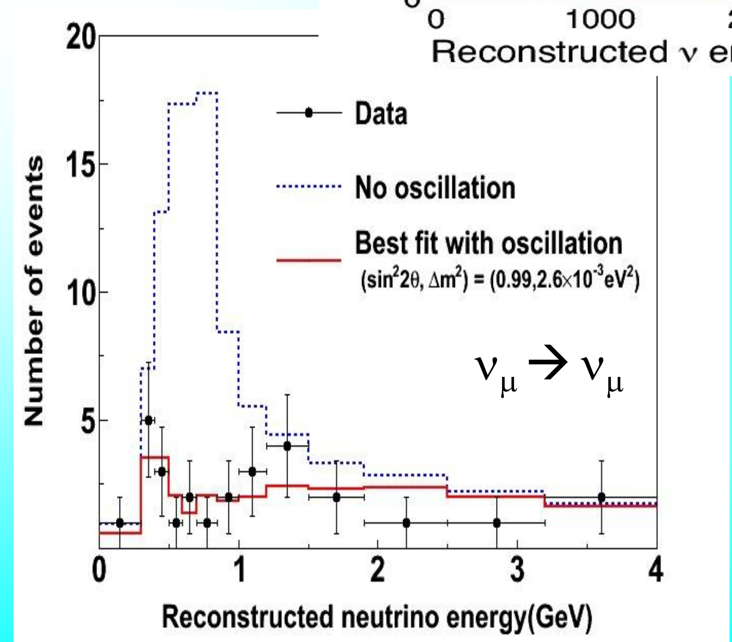
Observation of oscillations

KamLAND

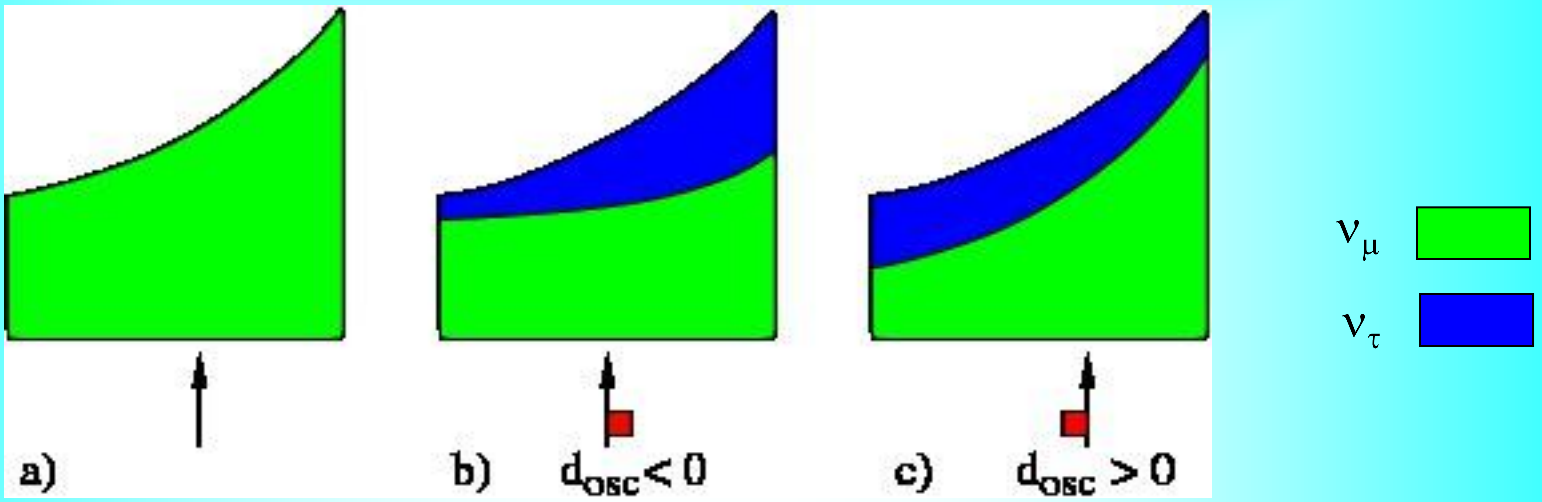


T2K





Oscillation distortion of shape factors



Distortion of the ν_μ - wave packet due to oscillations leads to shift of the center of mass of the packet

The oscillation phase which describes distortion

Key point

$$\phi_p = \frac{2\pi\ \sigma}{l_\nu\,(v - v_\pi)} = 2\,\pi\, l_p / l_\nu$$

equals the phase acquired over the region of formation of the wave packets

Probabilities

D. Henandez, A.S.

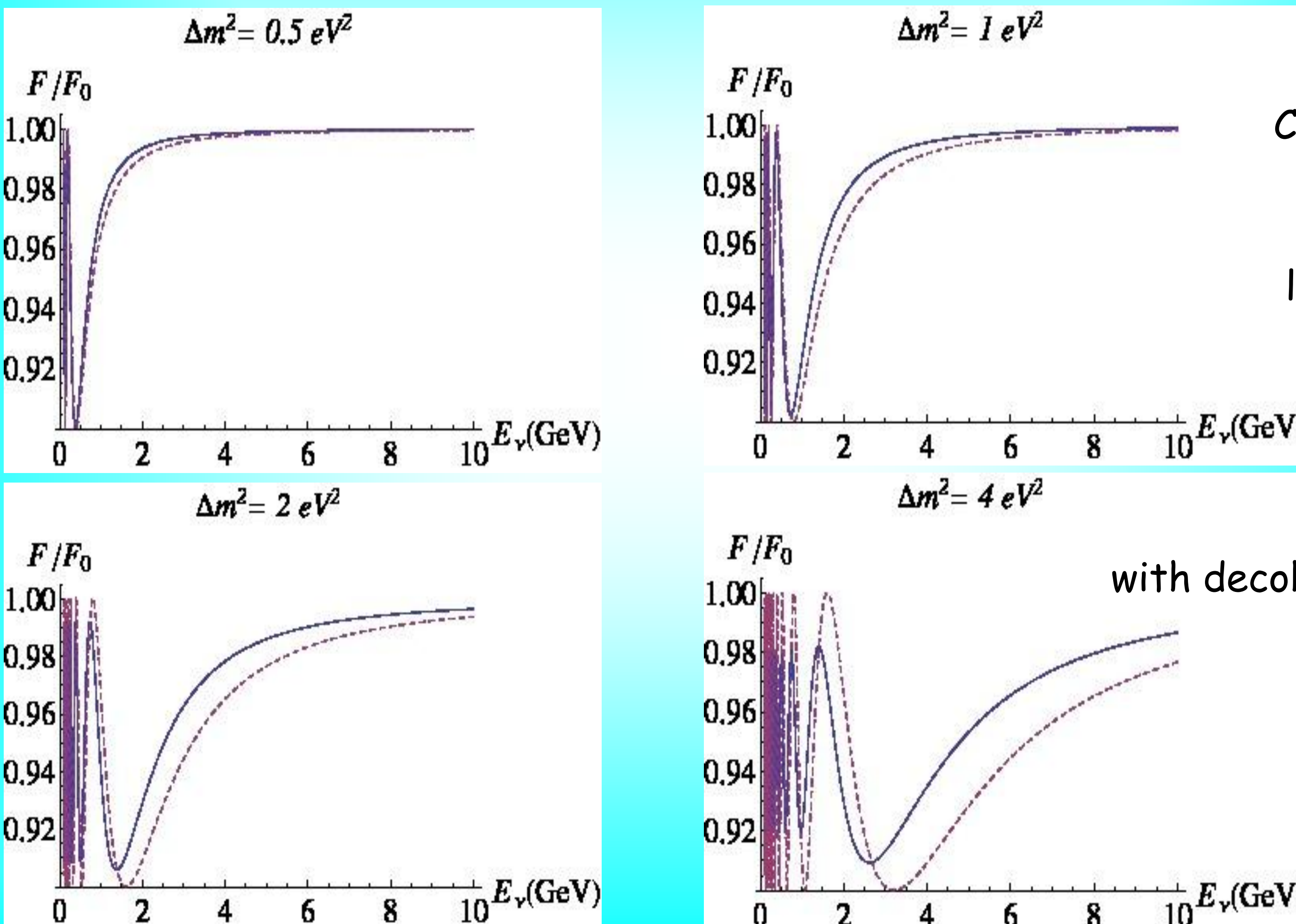
$P(\nu_\mu \rightarrow \nu_\mu)$

MINOS, ND

Coherence:

$\Delta E \sim \Gamma$

$l_{decay} \sim l_\nu$



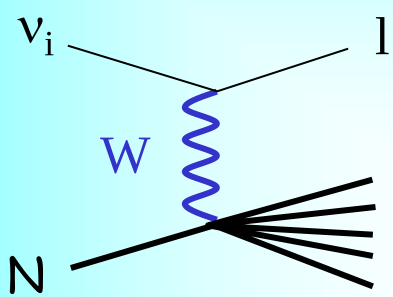
with decoherence

How to measure mixing?

Analogy with quark sector?

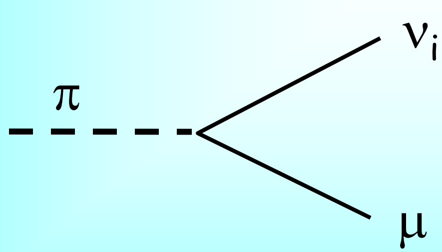
mass state

Scattering



$\sigma \sim |U_{il}|^2$

$\pi \rightarrow \mu\ \nu_i$



$\Gamma \sim |U_{i\mu}|^2$

In practice:

No beams of neutrino mass eigenstates

No neutrino mass spectrometer

The problem: neutrinos are in the coherent (flavor) states

Standard methods do not work

lead to new effects which allow to measure mixing parameters

To the theory of oscillations

Theory of oscillations:
Quantum mechanics at
macroscopic distances

Energy-momentum
conservation

Kinematic
entanglment

Coherence of production

Coherence in propagation

Academic interest?

Manipulating with
oscillation setup

Search for new realizations
of the oscillation setup
Phenomenological consequences

Collective
effects

Interplay of oscillations
with other non-standard
effects

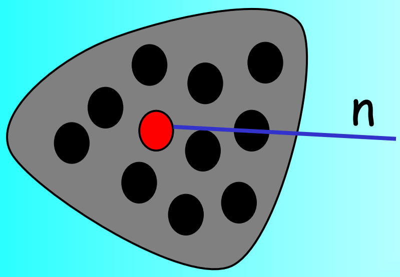
Identify origins of neutrino mass

E-p conservation

Space-time localization → energy-momentum uncertainty, DE , D p

Generic feature of oscillation set-up

Nuclei decay



Energy-momentum is conserved in whole the system which includes also particles whose interactions localize neutrino parents (walls, etc).

E-p are not conserved in the sub-system: particles which Directly involved in the process of neutrino production

E-p can be conserved for individual components of the wave packets. Uncertainty is restored when the probability is convoluted with wave packets

Entanglement

Exact energy-momentum conservation: kinematic entanglement

$\pi \rightarrow \nu_\mu + \mu$

$\pi \rightarrow \nu_i + \mu$ $\quad i = 1,2$

Final state: $|f\rangle = \Sigma_i |\mu_i\rangle |\nu_i\rangle$

μ_i is the state which (kinematically) corresponds mass eigenstate ν_i

Evolved: $|f\rangle = \Sigma |\mu_i\rangle |\nu_i\rangle \, e^{i\phi_i(\nu)} \, e^{i\phi_i(\mu)}$

Does recoil (muon) phase contribute to the oscillation phase?

For muon: $p^2 = E^2 - m^2$ ➡ $2p\,\Delta p = 2E\Delta E$ ➡ $\Delta p = E/p\,\Delta E = 1/v_g\,\Delta E$

$v_g = p/E$

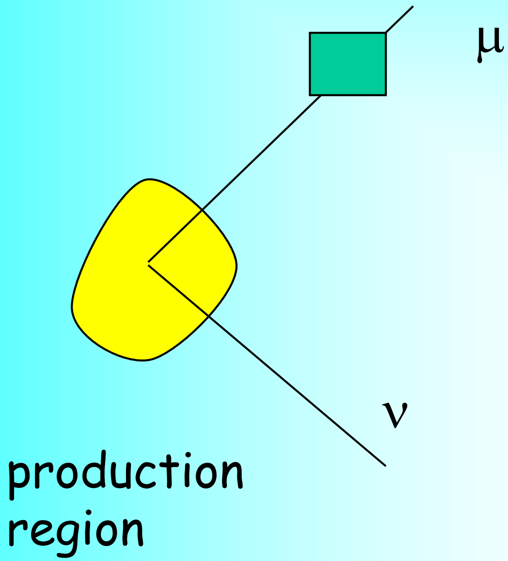
$\phi = \Delta E t - \Delta p x = \Delta E(t - x/v_g) \sim \Delta E\,\sigma_x$

$x = v_g t$

no mass difference between two muon components

Related to uncertainty of the phase due to localization

As it should be...



Flavor of neutrino state is fixed at the production

This does not depend on where and how muon is detected and on what is the phase difference between the muon components

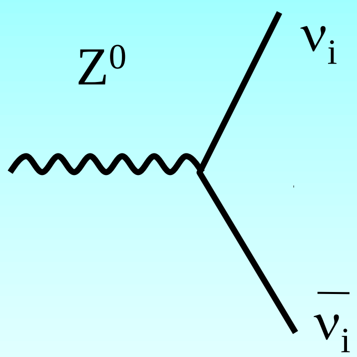
Neutrino phase difference is determined by neutrino parameters

Can NC interactions prepare mixed state?

Entanglement & EPR

Z f nd

What is the neutrino state produced in the Z-decay in the presence of mixing?

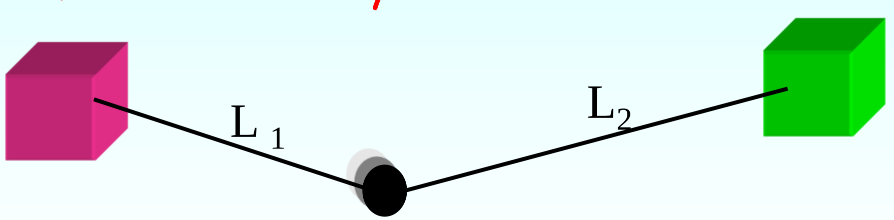


$$|f\rangle = \frac{1}{\sqrt{3}} \left[|\bar{\nu}_1 \nu_1\rangle + |\bar{\nu}_2 \nu_2\rangle + |\bar{\nu}_3 \nu_3\rangle \right]$$

$$|\langle f|H|Z\rangle|^2 = 3\,|\langle \bar{\nu}_1 \nu_1|H|Z\rangle|^2$$

Do neutrinos from Z^0- decay oscillate?

Two detectors experiment: detection of both neutrinos



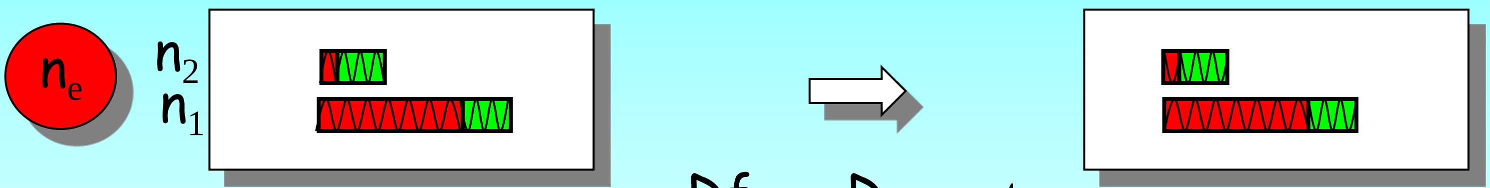
If the flavor of one of the neutrino is fixed, another neutrino oscillates

$$P = \sin^2 2\theta \; \sin^2 [\, \pi (L_1 + L_2)/ l_\nu]$$

Vacuum Oscillations

- Flavors of mass eigenstates do not change
- Admixtures of mass eigenstates do not change: no n_1 <-> n_2 transitions

} Determined by q



Df = 0

Df = $Dv_{phase}\, t$

- Due to difference of masses n_1 and n_2 have different phase velocities

oscillations:

effects of the phase difference increase which changes the interference pattern

Oscillation probability

$$P(n_m) = \frac{A_p}{2}\left(1 - \cos\frac{2px}{l_n}\right) = \sin^2 2q \sin^2 \frac{px}{l_n}$$

Features of neutrino oscillations in vacuum:

Oscillations -- effect of the phase difference increase between mass eigenstates

Admixtures of the mass eigenstates n_i in a given neutrino state do not change during propagation

Flavors (flavor composition) of the eigenstates are fixed by the vacuum mixing angle

A Yu Smirnov

At one glance

Sources:

Sun

Atmosphere

The Earth
Geo-nu

SN1987A

Universe (indirectly)

Accelerators

Reactors

Rad. Sources

All well established/confirmed results are described by

Standard Model + **Mass and Mixing**

of three neutrinos with rather peculiar pattern

and nothing more?

Introduction of neutrino mass and mixing may have negligible impact on the rest of SM

mass is generated by $\frac{1}{\Lambda} L\, L\, H\, H$

other effects ~ Λ^{-2}

Neutrinos in SM

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$I_W = 1/2$

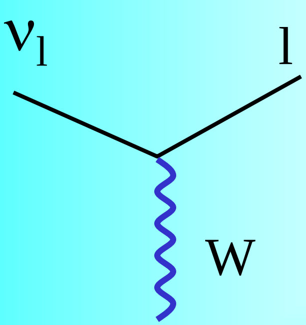
$I_{3W} = 1/2$

Chiral components

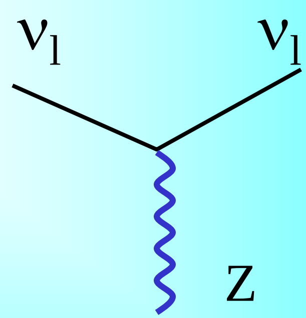
$\nu_L = \frac{1}{2}(1 - \gamma_5)\,\nu$

$\nu_R = \frac{1}{2}(1 + \gamma_5)\,\nu$?

$\nu_e \quad \nu_\mu \quad \nu_\tau$ are neutrino flavor states form doublets (charged currents) with definite charged leptons,



$l = e,\ \mu,\ \tau$



Neutral current interaction

$$\frac{g}{2\sqrt{2}}\,\bar{l}\,\gamma^\mu (1 - \gamma_5)\,\nu_l\, W^+_\mu \; + \text{h.c.}$$

$$\frac{g}{4}\,\bar{\nu}_l\,\gamma^\mu (1 - \gamma_5)\nu_l\, Z_\mu$$

Conservation of lepton numbers L_e, L_μ, L_τ

Two effects

Solar neutrinos

KamLAND

Atmospheric neutrinos

Double Chooz

Daya Bay

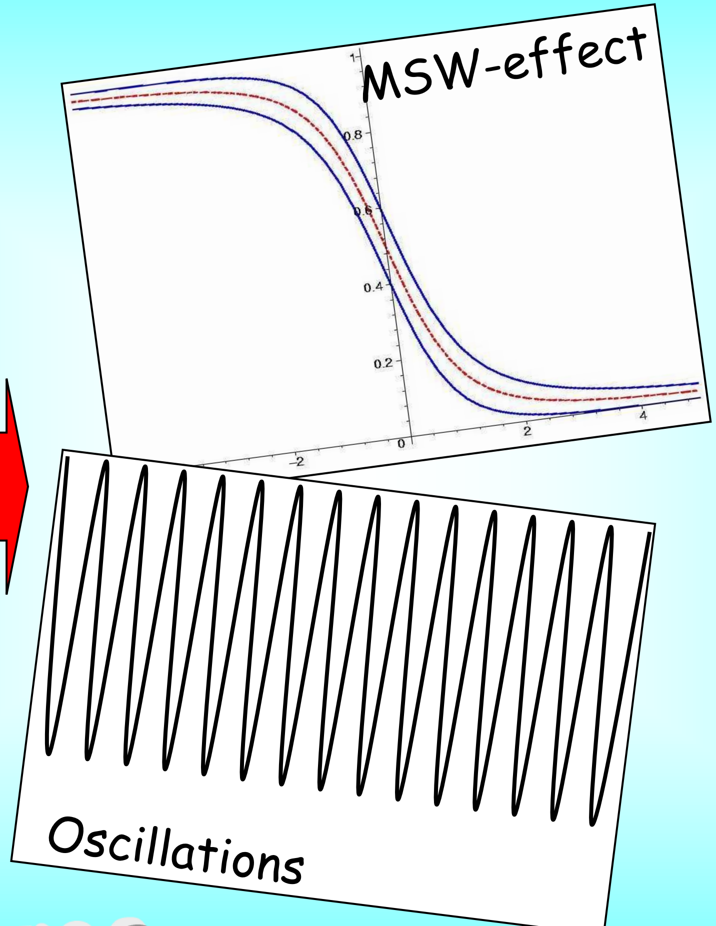
DeepCore

K2K

Antares

T2K

RENO



Δm^2

θ

Can be resonantly enhanced in matter

Mixing and mass matrices

Origin of mixing: off diagonal mass matrices

$M_l \neq M_\nu$

Diagonalization: → Mixing matrix → Mass spectrum

$M_l = U_{lL} m_l^{diag} U_{lR}^+$ $\quad M_\nu = U_{\nu L} m_\nu^{diag} U_{\nu L}^T$ (Majorana neutrinos)

$m_\nu^{diag} = (m_1, m_2, m_3)$

CC in terms of mass eigenstates: $\bar{l} \gamma^\mu (1 - \gamma_5) U_{PMNS} \nu_{mass}$

→ $U_{PMNS} = U_{lL}^+ U_{\nu L}$

Flavor basis: $M_l = m_l^{diag}$ $\quad U_{PMNS} = U_{\nu L}$

Parameterization

$U_{PMNS} = U_{23}\ I_\delta\ U_{13}\ I_{-\delta}\ U_{12}$

$I_\delta = \mathrm{diag}\,(1,\ 1,\ e^{i\delta})$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ s_{12}c_{23} + c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_{12} = \cos\theta_{12}$, etc.

δ is the Dirac CP violating phase

θ_{12} is the ``solar" mixing angle

θ_{23} is the ``atmospheric" mixing angle

θ_{13} is the mixing angle determined by T2K, Daya Bay, DC, RENO …

Challenging the theory of oscillations

Why the standard oscillation formula is so robust?
Corrections required ?

New oscillation setups
Higher precision …

LBL experiments:
- Long life-time parents: pions, muons, nuclei;
- large decay pipes, straight lines of storage rings

Neutrino anomalies as consequences of modified description of oscillations

Nature of neutrino mass differs from nature of electron or top quark…

Neutrinos in SM

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

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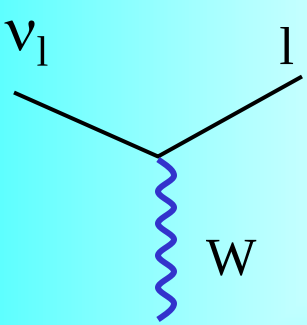

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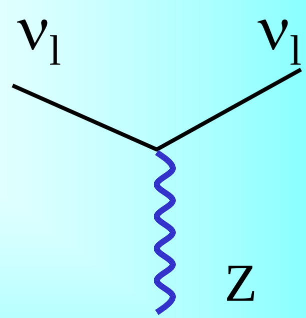

Neutral current interaction

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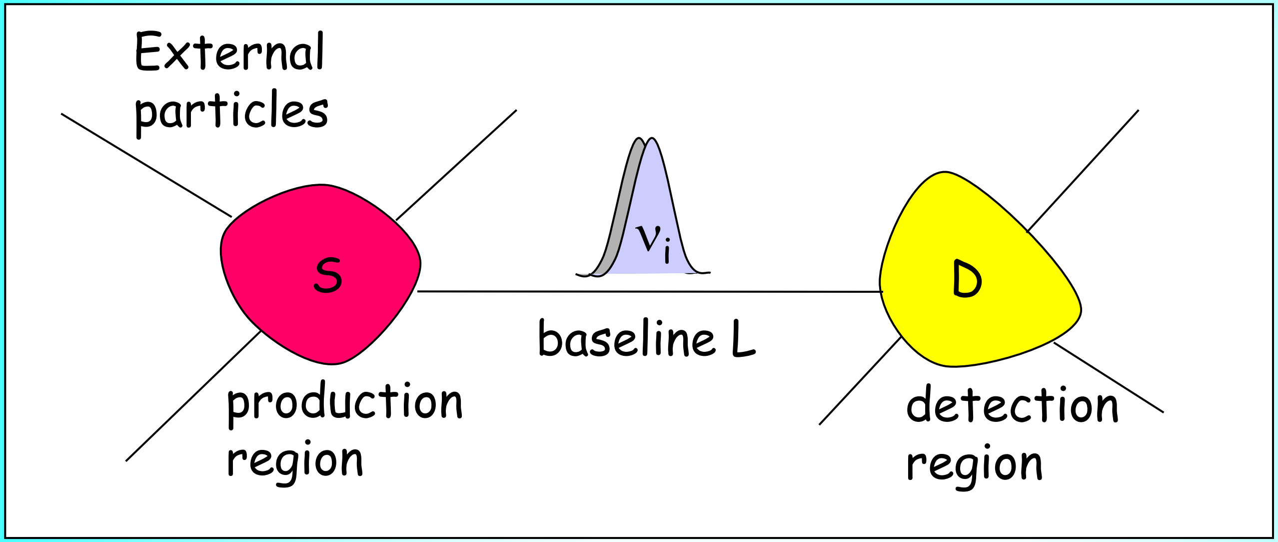
$$\frac{g}{4}\,\bar{\nu}_l\,\gamma^\mu(1 - \gamma_5)\nu_l\, Z_\mu$$

Conservation of lepton numbers L_e, L_μ, L_τ

Where to truncate?

How external particles
Should be described?

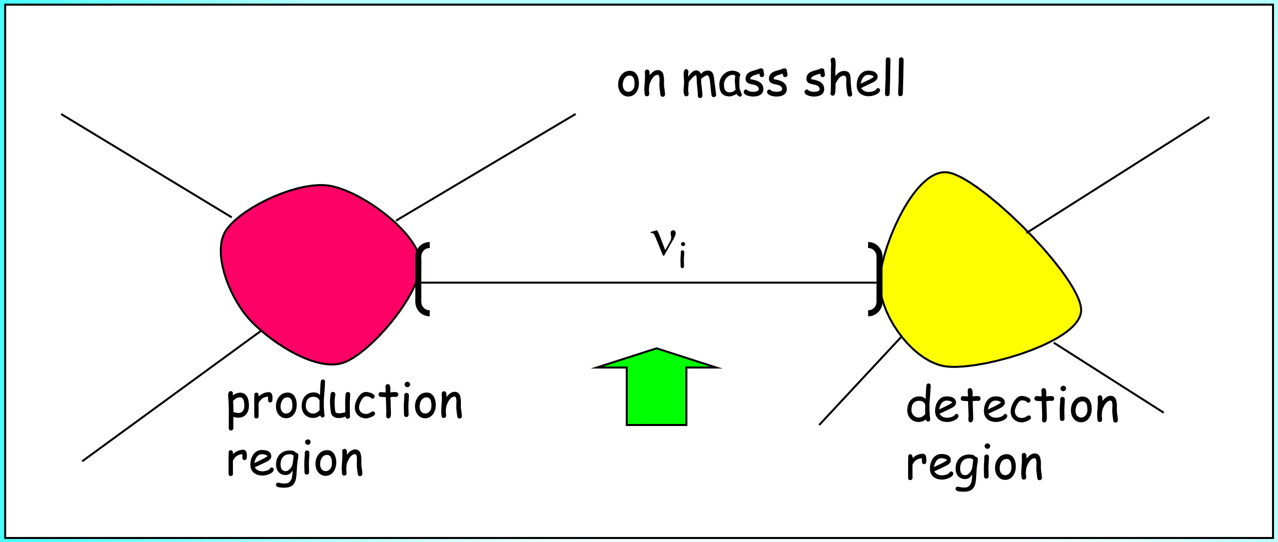


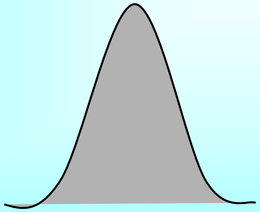
detection/production areas
are determined by localization
of particles involved in neutrino
production and detection
not source/detector volume
(still to integrate over)

→ wave packets for
external particles

→ Describe by plane waves
But introduce finite integration

Wave packets





Neutrinos real particles described by the wave packets
Which encode information about production and detection
as well on oscillations in the production region

Flavors and mixing

Flavor neutrino states:

ν_e ν_μ ν_τ

e μ τ

- correspond to certain charged leptons
- interact in pairs

flavor is characteristic of interactions

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$\pi \rightarrow \mu + \nu_\mu$$

Mass eigenstates

ν_1 ν_2 ν_3

m_1 m_2 m_3

Mixing

Flavor states $\neq$ Mass eigenstates

Entanglement & EPR