Flavour structure from the seesaw

Michael A. Schmidt

University of Melbourne

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Alexei Smirnov Fest
What's nu? – Invisibles 12

M. Lindner, MS, A. Y. .Smirnov, JHEP **0507** (2005) 048
C. Hagedorn, MS, A. Y. .Smirnov, Phys. Rev. D **79**, 036002 (2009)
MS, A. Y. .Smirnov, Nucl.Phys. B857 (2012) 1-27

Outline

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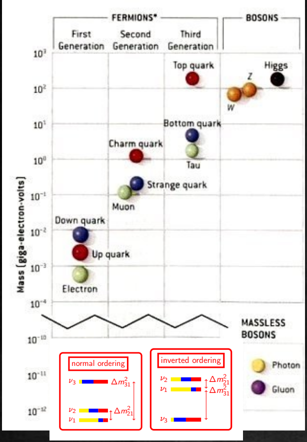
Fermion Masses



- Huge hierarchy of charged fermions:

$$m_t : m_c : m_u \sim 1 : 7 \cdot 10^{-3} : 10^{-5}$$

$$m_b : m_s : m_d \sim 1 : 2 \cdot 10^{-2} : 10^{-3}$$

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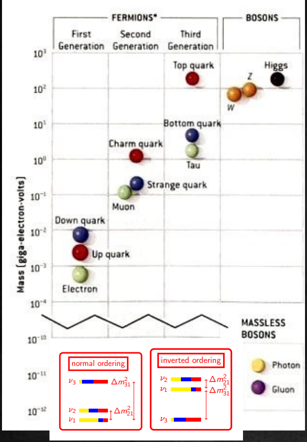

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- Small mixing angles in CKM matrix:

ϑ_{12}	ϑ_{23}	θ_{13}
13°	2.4°	0.23°

- Large mixing angles in PMNS matrix:

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34°	44°	9.3°	[Forero et. al (2012)]
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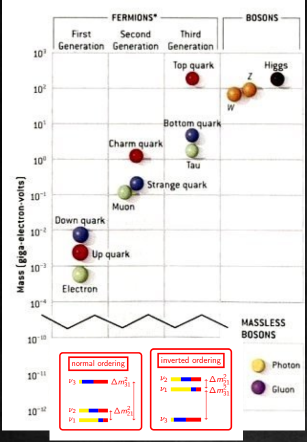

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- Explanation of different structures?
- Compatibility with GUTs?

Seesaw Mechanism

Standard Seesaw [Minkowski;Yanagida;Glashow;Gell-Mann,Ramond,Slansky;Mohapatra,Senjanovic]

- Introduction of right-handed (RH) neutrinos N

$$\mathcal{M} = \begin{pmatrix} 0 & m_D^T \\ \cdot & M_{NN} \end{pmatrix}$$

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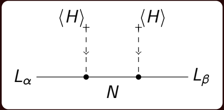

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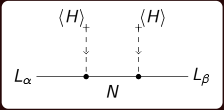

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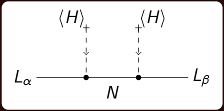

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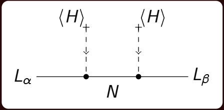

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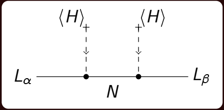

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- New scale below Λ_{GUT} needed

Solutions to Large Hierarchy from Seesaw

- Forget about SO(10), use e.g. SU(5)

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Alternative Seesaw

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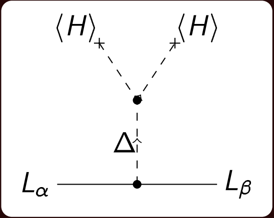
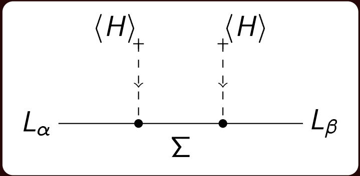


Type III (fermionic triplet) seesaw



Solutions to Large Hierarchy from Seesaw

- Forget about SO(10), use e.g. SU(5)
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- Cancel hierarchy through structure in RH Majorana mass matrix M_{NN}
- . . .

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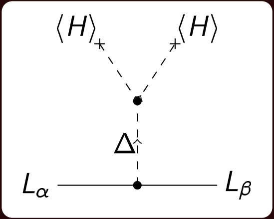

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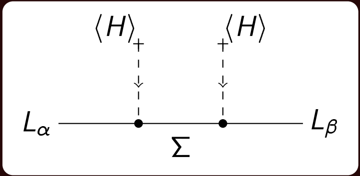

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Double Seesaw [Mohapatra,Valle;Barr]

Introduce additional singlets S

$$\mathcal{M} = \begin{pmatrix} 0 & m_D^T & 0 \\ . & 0 & M_{SN}^T \\ . & . & M_{SS} \end{pmatrix}$$

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- If M_{SS} singular, there are massless neutrinos.

Cancellation of Hierarchy

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- $F \propto 1$ (Dirac screening $\Rightarrow$ Dirac flavour structure is cancelled)

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How can this structure be obtained? – no SO(10)

Abelian Symmetry – L number

With the charges $L(\nu_L) = L(N) = 1$, $L(S) = 0$

$$M_{SS} SS$$

in basis $(\nu, N, S)^T$

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Minimal LR symmetry $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$

$$G(\mathcal{M}) = \begin{pmatrix} [3,1] & [2,2] & [2,1] \\ . & [1,3] & [1,2] \\ . & . & [1,1] \end{pmatrix}$$

Setup within SO(10)

Lagrangian

$$\alpha_{ij}\, \underline{\mathbf{16}}_i\, \underline{\mathbf{16}}_j\, H + \beta_{ij}\, S_i\, \underline{\mathbf{16}}_j\, \Delta + (M_{SS})_{ij}\, S_i S_j$$

	$\underline{\mathbf{16}}_i$	S_i	H	Δ	χ_i
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$\Rightarrow$ correlation between α and β needed.

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Cancellation Mechanism

- e.g. Frogatt Nielsen mechanism

$\rightarrow$ Hierarchy cancelled, anarchical spectrum

[Hall, Murayama, Weiner (1999); de Gouvêa, Murayama (2012)]

Realisation with Extended Gauge Symmetry I

$SO(10) \subset E_6$

$$\underline{\mathbf{27}} \rightarrow \underline{\mathbf{16}} \oplus \underline{\mathbf{10}} \oplus \underline{\mathbf{1}}$$

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In terms of $SU(3)_L \times SU(3)_R \times SU(3)_C$

- Leptons

$$L = \begin{pmatrix} L_1^{\dot{1}} & E^- & e^- \\ E^+ & L_2^{\dot{2}} & \nu \\ e^+ & \overline{\nu} & L_3^{\dot{3}} \end{pmatrix} \sim (\underline{\overline{\mathbf{3}}}, \underline{\mathbf{3}}, 1)$$

$$[Q_L \sim (\underline{\mathbf{3}}, 1, \underline{\overline{\mathbf{3}}}),\ Q_R \sim (1, \underline{\overline{\mathbf{3}}}, \underline{\mathbf{3}})]$$

Realisation with Extended Gauge Symmetry I

$SO(10) \subset E_6$

$$\underline{\mathbf{27}} \to \underline{\mathbf{16}} \oplus \underline{\mathbf{10}} \oplus \underline{\mathbf{1}}$$

$\Rightarrow$ Singlets are in the same representation

$$\underline{\mathbf{27}}_i\,\underline{\mathbf{27}}_j \left(Y^{\{ij\}}_{27}\,\underline{\mathbf{27}} + Y^{\{ij\}}_{351_S}\,\underline{\mathbf{351}}_S + Y^{[ij]}_{351_A}\,\underline{\mathbf{351}}_A \right)$$

In terms of $SU(3)_L \times SU(3)_R \times SU(3)_C$

- Leptons

$$L = \begin{pmatrix} L^{\dot 1}_1 & E^- & e^- \\ E^+ & L^{\dot 2}_2 & \nu \\ e^+ & \overline{\nu} & L^{\dot 3}_3 \end{pmatrix} \sim (\underline{\overline{3}}, \underline{3}, 1)$$

$[Q_L \sim (\underline{3}, 1, \underline{\overline{3}}),\ Q_R \sim (1, \underline{\overline{3}}, \underline{3})]$

- Relevant Higgs multiplets

$$H \subset (\underline{\overline{\mathbf{3}}}, \underline{\mathbf{3}}, \underline{\mathbf{1}}) \subset \underline{\mathbf{27}}$$

$$H_S \subset (\underline{\overline{\mathbf{3}}}, \underline{\mathbf{3}}, \underline{\mathbf{1}}) + (\underline{\mathbf{6}}, \underline{\overline{\mathbf{6}}}, \underline{\mathbf{1}}) \subset \underline{\mathbf{351}}_S$$

$$H_A \subset (\underline{\overline{\mathbf{3}}}, \underline{\mathbf{3}}, \underline{\mathbf{1}}) + (\underline{\overline{\mathbf{3}}}, \underline{\overline{\mathbf{6}}}, \underline{\mathbf{1}}) + (\underline{\mathbf{6}}, \underline{\mathbf{3}}, \underline{\mathbf{1}}) \subset \underline{\mathbf{351}}_A$$

Realisation with Extended Gauge Symmetry II

Dirac Screening structure obtained from

$$\left\langle (H_A)^{\dot{1}}_{1} \right\rangle \simeq \mathcal{O}\left(\mathrm{SU}(2)_L \text{ breaking scale}\right)$$

$$\left\langle (H_A)^{\{33\}}_{1} \right\rangle \simeq \mathcal{O}\left(\mathrm{SU}(2)_R \text{ breaking scale}\right)$$

$$\left\langle (H_S)^{\{33\}}_{\{\dot{3}3\}} \right\rangle \simeq \left\langle (H_A)^{\dot{3}}_{3} \right\rangle \simeq \mathcal{O}\left(\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \text{ breaking scale}\right)$$

in basis $\left(\nu \sim L^{\dot{2}}_{3},\ \ N \sim L^{\dot{3}}_{2},\ \ S \sim L^{\dot{3}}_{3},\ \ S' \sim L^{\dot{1}}_{1},\ \ S'' \sim L^{\dot{2}}_{2}\right)$

$$\begin{pmatrix} 0 & -Y_{351_{A}} \left\langle (H_A)^{\dot{1}}_{1} \right\rangle & 0 & 0 & 0 \\ . & 0 & -Y_{351_{A}} \left\langle (H_A)^{\{33\}}_{1} \right\rangle & 0 & 0 \\ . & . & Y_{351_{S}} \left\langle (H_S)^{\{33\}}_{\{\dot{3}3\}} \right\rangle & Y_{351_{A}} \left\langle (H_A)^{\dot{1}}_{1} \right\rangle & 0 \\ . & . & . & 0 & Y_{351_{A}} \left\langle (H_A)^{\dot{3}}_{3} \right\rangle \\ . & . & . & . & 0 \end{pmatrix}$$

Realisation with Extended Gauge Symmetry II

Dirac Screening structure obtained from

$$\left\langle (H_A)_1^{\dot{1}} \right\rangle \simeq \mathcal{O}\left(\mathrm{SU}(2)_L \text{ breaking scale}\right)$$

$$\left\langle (H_A)_1^{\{33\}} \right\rangle \simeq \mathcal{O}\left(\mathrm{SU}(2)_R \text{ breaking scale}\right)$$

$$\left\langle (H_S)_{\{\dot{3}3\}}^{\{33\}} \right\rangle \simeq \left\langle (H_A)_3^{\dot{3}} \right\rangle \simeq \mathcal{O}\left(\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \text{ breaking scale}\right)$$

in basis $\left(\nu \sim L_3^{\dot{2}},\ \ N \sim L_2^{\dot{3}},\ \ S \sim L_3^{\dot{3}},\ \ S' \sim L_1^{\dot{1}},\ \ S'' \sim L_2^{\dot{2}}\right)$

$$\begin{pmatrix} 0 & -Y_{351_A}\left\langle (H_A)_1^{\dot{1}} \right\rangle & 0 & 0 & 0 \\ . & 0 & -Y_{351_A}\left\langle (H_A)_1^{\{33\}} \right\rangle & 0 & 0 \\ . & . & Y_{351_S}\left\langle (H_S)_{\{\dot{3}3\}}^{\{33\}} \right\rangle & Y_{351_A}\left\langle (H_A)_1^{\dot{1}} \right\rangle & 0 \\ . & . & . & 0 & Y_{351_A}\left\langle (H_A)_3^{\dot{3}} \right\rangle \\ . & . & . & . & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 0 & -Y_{351_A}\left\langle (H_A)^{\dot{1}}_{1}\right\rangle & 0 & 0 & 0 \\ . & 0 & -Y_{351_A}\left\langle (H_A)^{\{33\}}_{1}\right\rangle & 0 & 0 \\ . & . & Y_{351_S}\left\langle (H_S)^{\{33\}}_{\{\dot{3}\dot{3}\}}\right\rangle & Y_{351_A}\left\langle (H_A)^{\dot{1}}_{1}\right\rangle & 0 \\ . & . & . & 0 & Y_{351_A}\left\langle (H_A)^{\dot{3}}_{3}\right\rangle \\ . & . & . & . & 0 \end{pmatrix}$$

$\Rightarrow$ Construction of viable Higgs Potential important

Realisation with Flavour Symmetry within SO(10)

- Explain number of generations: $\underline{\mathbf{16}}_i \sim \underline{\mathbf{3}}$

Particle Content

	$\underline{\mathbf{16}}_i$	S_i	H	Δ	χ_i
SO(10)	$\underline{\mathbf{16}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{10}}$	$\overline{\underline{\mathbf{16}}}$	$\underline{\mathbf{1}}$

Realisation with Flavour Symmetry within SO(10)

- Explain number of generations: $\underline{\mathbf{16}}_i \sim \underline{\mathbf{3}}$
- Complex representation $\underline{\mathbf{3}}$, otherwise $\underline{\mathbf{3}} \times \underline{\mathbf{3}}$ contains singlet

$\Rightarrow$ A_4 not possible, but: T_7 [Luhn,Nasri,Ramond], $\Sigma(81)$ [Ma], ...

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	$\underline{\mathbf{16}}_i$	S_i	H	Δ	χ_i
SO(10)	$\underline{\mathbf{16}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{10}}$	$\overline{\underline{\mathbf{16}}}$	$\underline{\mathbf{1}}$
T_7	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{i}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{3}}^{\star}$

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- Explain difference in CKM and MNS matrix (in lowest order)

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SO(10)	$\underline{\mathbf{16}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{10}}$	$\overline{\underline{\mathbf{16}}}$	$\underline{\mathbf{1}}$
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$\Rightarrow$ A_4 not possible, but: T_7 [Luhn,Nasri,Ramond], $\Sigma(81)$ [Ma], ...

- Explain difference in CKM and MNS matrix (in lowest order)
- Flavons (gauge group singlets charged with respect to G_F) χ

$$W \supset \frac{\alpha_{ij}}{\Lambda} \underline{\mathbf{16}}_i\, \underline{\mathbf{16}}_j\, H\chi + \frac{\beta_{ij}}{\Lambda} S_i\, \underline{\mathbf{16}}_j\, \Delta\chi + (M_{SS})_{ij}\, S_i S_j\,,$$

Particle Content

	$\underline{\mathbf{16}}_i$	S_i	H	Δ	χ_i
SO(10)	$\underline{\mathbf{16}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{10}}$	$\overline{\underline{\mathbf{16}}}$	$\underline{\mathbf{1}}$
T_7	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}_\mathbf{i}$	$\underline{\mathbf{1}}_\mathbf{1}$	$\underline{\mathbf{1}}_\mathbf{1}$	$\underline{\mathbf{3}}^\star$

T_7: Realisation

Field	$\underline{\mathbf{16}}_i$	S_i	H	Δ	χ_i
$SO(10)$	$\underline{\mathbf{16}}$	$\underline{\mathbf{1}}$	$\underline{\mathbf{10}}$	$\overline{\underline{\mathbf{16}}}$	$\underline{\mathbf{1}}$
T_7	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}_{\mathbf{i}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{3}}^{\star}$

Superpotential

$$
\begin{aligned}
W \supset\ & \alpha\, H\,(\underline{\mathbf{16}}_3\,\underline{\mathbf{16}}_3\,\chi_1 + \underline{\mathbf{16}}_1\,\underline{\mathbf{16}}_1\,\chi_2 + \underline{\mathbf{16}}_2\,\underline{\mathbf{16}}_2\,\chi_3)/\Lambda \\
& + \beta_1\,\Delta\,S_1\,(\underline{\mathbf{16}}_1\,\chi_1 + \underline{\mathbf{16}}_2\,\chi_2 + \underline{\mathbf{16}}_3\,\chi_3)/\Lambda \\
& + \beta_2\,\Delta\,S_2\,(\underline{\mathbf{16}}_1\,\chi_1 + \omega\,\underline{\mathbf{16}}_2\,\chi_2 + \omega^2\,\underline{\mathbf{16}}_3\,\chi_3)/\Lambda \\
& + \beta_3\,\Delta\,S_3\,(\underline{\mathbf{16}}_1\,\chi_1 + \omega^2\,\underline{\mathbf{16}}_2\,\chi_2 + \omega\,\underline{\mathbf{16}}_3\,\chi_3)/\Lambda \\
& + A\,S_1\,S_1 + B\,(S_2\,S_3 + S_3\,S_2) + \text{h.c.}
\end{aligned}
$$

with $\omega = e^{2\pi\,\mathrm{i}\,/3}$

T_7: Lowest Order

$$m_D = \frac{\alpha \langle H \rangle}{\Lambda} \begin{pmatrix} \langle \chi_2 \rangle & 0 & 0 \\ 0 & \langle \chi_3 \rangle & 0 \\ 0 & 0 & \langle \chi_1 \rangle \end{pmatrix}$$

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$$M_{SN} = \frac{\langle \Delta \rangle_N}{\Lambda} \begin{pmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \beta_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} \langle \chi_1 \rangle & 0 & 0 \\ 0 & \langle \chi_2 \rangle & 0 \\ 0 & 0 & \langle \chi_3 \rangle \end{pmatrix}$$

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$$M_{SS} = \begin{pmatrix} A & 0 & 0 \\ \cdot & 0 & B \\ \cdot & \cdot & 0 \end{pmatrix}$$

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$$M_{SS} = \begin{pmatrix} A & 0 & 0 \\ \cdot & 0 & B \\ \cdot & \cdot & 0 \end{pmatrix}$$

$$m_\nu \approx \left(\frac{\alpha \langle H \rangle}{\langle \Delta \rangle_N} \right)^2 D_\chi \begin{pmatrix} \tilde{A} + 2\tilde{B} & \tilde{A} - \tilde{B} & \tilde{A} - \tilde{B} \\ \cdot & \tilde{A} + 2\tilde{B} & \tilde{A} - \tilde{B} \\ \cdot & \cdot & \tilde{A} + 2\tilde{B} \end{pmatrix} D_\chi$$

$$\tilde{A} \equiv \frac{A}{9\beta_1^2}, \quad \tilde{B} \equiv \frac{B}{9\beta_2\beta_3}, \quad D_\chi \equiv \mathrm{diag}\left(\frac{\langle \chi_2 \rangle}{\langle \chi_1 \rangle}, \frac{\langle \chi_3 \rangle}{\langle \chi_2 \rangle}, \frac{\langle \chi_1 \rangle}{\langle \chi_3 \rangle} \right)$$

T_7: Lowest Order

$$m_D = \frac{\alpha \langle H\rangle \langle \chi_1\rangle}{\Lambda} \begin{pmatrix} \epsilon^2 & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{SN} = \frac{\langle \Delta\rangle_N \langle \chi_1\rangle}{\Lambda} \begin{pmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \beta_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \epsilon^2 & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$

$$M_{SS} = \begin{pmatrix} A & 0 & 0 \\ \cdot & 0 & B \\ \cdot & \cdot & 0 \end{pmatrix}$$

$$m_\nu \approx \left(\frac{\alpha \langle H\rangle}{\langle \Delta\rangle_N \epsilon}\right)^2 \begin{pmatrix} (\tilde{A}+2\tilde{B})\epsilon^6 & (\tilde{A}-\tilde{B})\epsilon^3 & (\tilde{A}-\tilde{B})\epsilon^3 \\ \cdot & \tilde{A}+2\tilde{B} & \tilde{A}-\tilde{B} \\ \cdot & \cdot & \tilde{A}+2\tilde{B} \end{pmatrix}$$

$\tilde{A} \equiv \frac{A}{9\beta_1^2}$, $\tilde{B} \equiv \frac{B}{9\beta_2\beta_3}$, $D_\chi \equiv \mathrm{diag}\left(\frac{\langle\chi_2\rangle}{\langle\chi_1\rangle}, \frac{\langle\chi_3\rangle}{\langle\chi_2\rangle}, \frac{\langle\chi_1\rangle}{\langle\chi_3\rangle}\right) = \mathrm{diag}\left(\epsilon, \epsilon^{-2}, \epsilon^{-2}\right)$

T_7: Solar Mixing Angle

$$m_\nu^{LS} \approx -\left[m_D^T\, M_{SN}^{-1}\, m_{S\nu} + (\dots)^T\right]$$

$m_{S\nu}$ originates from $S_i\, \underline{\mathbf{16}}_j\, \Delta\, \chi \Rightarrow m_\nu^{LS}$ diagonal

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$$m_{S\nu} = \frac{\langle \Delta' \rangle_\nu}{\Lambda} \begin{pmatrix} \beta'_1 & 0 & 0 \\ 0 & \beta'_2 & 0 \\ 0 & 0 & \beta'_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} \langle\chi_1\rangle & 0 & 0 \\ 0 & \langle\chi_2\rangle & 0 \\ 0 & 0 & \langle\chi_3\rangle \end{pmatrix}$$

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Leading order

$$m_\nu \approx \left(\frac{\alpha\,\langle H\rangle}{\langle\Delta\rangle_N\,\epsilon}\right)^2 \begin{pmatrix} -2\,X\sum_{i=1}^{3}\tilde\beta_i\,\epsilon^3 & -X\sum_{i=1}^{3}\tilde\beta_i\,\omega^{1-i} & -X\sum_{i=1}^{3}\tilde\beta_i\,\omega^{i-1} \\ . & \tilde A + 2\tilde B & \tilde A - \tilde B \\ . & . & \tilde A + 2\tilde B \end{pmatrix}$$

$$X = \frac{\langle\Delta\rangle_N\,\langle\Delta'\rangle_\nu\,\langle\chi_1\rangle\,\epsilon}{3\,\alpha\,\langle H\rangle\,\Lambda} \Rightarrow \langle\Delta\rangle_N\,\epsilon \sim \tilde A, \tilde B\,, \quad \tilde\beta_i = \beta_i'/\beta_i$$

T_7: Phenomenology

Dominant 2-3 block in neutrino mass matrix preserved, θ_{12}, θ_{13} can be fitted:

$$\tan\theta_{12} \approx \frac{X\,|\tilde{\beta}_2 - \tilde{\beta}_3|}{\sqrt{6}\,|\tilde{B}|}\,, \quad \sin\theta_{13} \approx \frac{X\,|2\tilde{\beta}_1 - \tilde{\beta}_2 - \tilde{\beta}_3|}{\sqrt{2}\,|2\tilde{A} + \tilde{B}|}\,, \quad \theta_{23} \approx \frac{\pi}{4}$$

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m_1 and m_2 especially changed:

$$m_1 \approx \left(\frac{\alpha\,\langle H\rangle}{\langle\Delta\rangle_N\,\epsilon}\right)^2 \left|3\,|\tilde{B}|\tan^2\theta_{12} - |2\tilde{A} + \tilde{B}|\sin^2\theta_{13}\right|$$

$$m_2 \approx \left(\frac{\alpha\,\langle H\rangle}{\langle\Delta\rangle_N\,\epsilon}\right)^2 3\,|\tilde{B}|\,\left|1 - \tan^2\theta_{12}\right|$$

$$m_3 \approx \left(\frac{\alpha\,\langle H\rangle}{\langle\Delta\rangle_N\,\epsilon}\right)^2 |2\tilde{A} + \tilde{B}|\,\left|1 + \sin^2\theta_{13}\right|$$

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$$m_2 \approx \left(\frac{\alpha\,\langle H\rangle}{\langle\Delta\rangle_N\,\epsilon}\right)^2 3\,|\tilde{B}|\left|1 - \tan^2\theta_{12}\right|$$

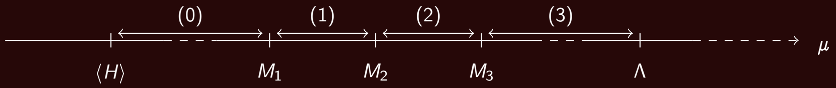
$$m_3 \approx \left(\frac{\alpha\,\langle H\rangle}{\langle\Delta\rangle_N\,\epsilon}\right)^2 |2\tilde{A} + \tilde{B}|\left|1 + \sin^2\theta_{13}\right|$$

$$\Delta m^2_{21} \approx \left(\frac{\alpha\,\langle H\rangle}{\langle\Delta\rangle_N\,\epsilon}\right)^4 9\,|\tilde{B}|^2(1 - 2\tan^2\theta_{12})$$

$$\Delta m^2_{32} \approx \left(\frac{\alpha\,\langle H\rangle}{\langle\Delta\rangle_N\,\epsilon}\right)^4 \left(|2\tilde{A} + \tilde{B}|^2 - 9|\tilde{B}|^2\left(1 - \tan^2\theta_{12}\right)^2\right)$$

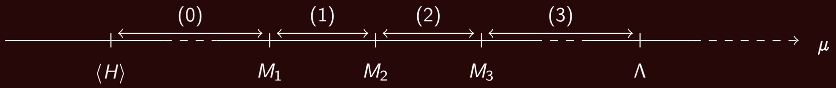
Outline

Stability with respect to RG



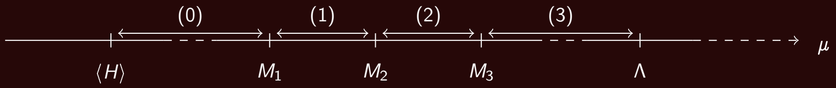
- $m_\nu = m_D^T M_{SN}^{-1} M_{SS} {M_{SN}^{-1}}^T m_D$ stable with respect to RG?

Stability with respect to RG



- $m_\nu = m_D^T M_{SN}^{-1} M_{SS} M_{SN}^{-1\,T} m_D$ stable with respect to RG?
- Two contributions to m_ν from
  - Active N: $Y_\nu^T M^{-1} Y_\nu$
  - Effective D5 operator: κ

Stability with respect to RG



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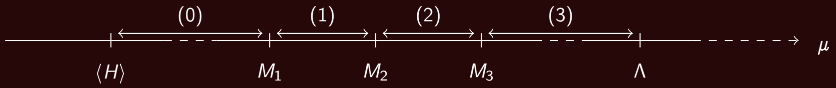

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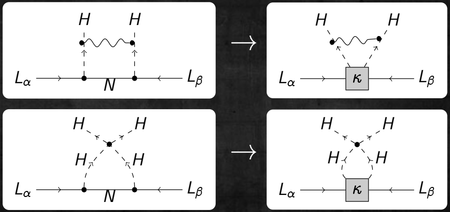
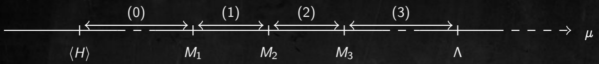
- SM: additional vertex corrections



[Antusch, Kersten, Lindner, Ratz (2002)]

Running between Mass Thresholds



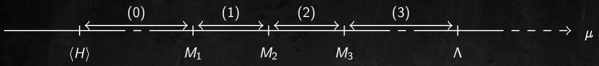
RG transformation

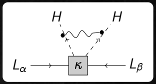
$$\overset{(n)}{Y_\nu} \xrightarrow{\text{RG}} \overset{(n)}{Z_N}{}^{T}\, \overset{(n)}{Y_\nu}\, \overset{(n)}{Z}_{\text{ext}}$$

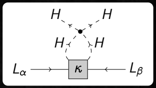
$$\overset{(n)}{M_{NN}} \xrightarrow{\text{RG}} \overset{(n)}{Z_N}{}^{T}\, \overset{(n)}{M_{NN}}\, \overset{(n)}{Z_N}$$

$$\overset{(n)}{\kappa} \xrightarrow{\text{RG}} \overset{(n)}{Z}_{\text{ext}}{}^{T}\, \overset{(n)}{\kappa}\, \overset{(n)}{Z}_{\text{ext}}\, \overset{(n)}{Z_\kappa}$$

Running between Mass Thresholds







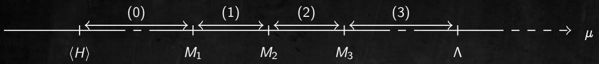
RG transformation

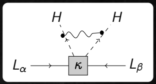
$$\overset{(n)}{Y_\nu} \xrightarrow{\text{RG}} \overset{(n)}{Z_N}{}^T \, \overset{(n)}{Y_\nu} \, \overset{(n)}{Z_{\text{ext}}}$$

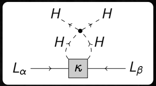
$$\overset{(n)}{M_{NN}} \xrightarrow{\text{RG}} \overset{(n)}{Z_N}{}^T \, \overset{(n)}{M_{NN}} \, \overset{(n)}{Z_N}$$

$$\overset{(n)}{\kappa} \xrightarrow{\text{RG}} \overset{(n)}{Z_{\text{ext}}}{}^T \, \overset{(n)}{\kappa} \, \overset{(n)}{Z_{\text{ext}}} \, \overset{(n)}{Z_\kappa}$$

Running between Mass Thresholds







$$Z_\kappa \equiv \mathrm{diag}\left(\overset{(0)}{Z_\kappa}, \overset{(0-1)}{Z_\kappa}, \overset{(0-2)}{Z_\kappa}\right)$$

$$\overset{(0-n)}{Z_\kappa} = 1 + \frac{1}{16\pi^2}\left(\lambda + \frac{9}{10}g_1^2 + \frac{3}{2}g_2^2\right)\ln\frac{\langle\phi\rangle}{M_{n+1}}$$

RG transformation

$$\overset{(n)}{Y_\nu} \xrightarrow{\mathrm{RG}} \overset{(n)}{Z_N}{}^T \, \overset{(n)}{Y_\nu} \, \overset{(n)}{Z_{\mathrm{ext}}}$$

$$\overset{(n)}{M_{NN}} \xrightarrow{\mathrm{RG}} \overset{(n)}{Z_N}{}^T \, \overset{(n)}{M_{NN}} \, \overset{(n)}{Z_N}$$

$$\overset{(n)}{\kappa} \xrightarrow{\mathrm{RG}} \overset{(n)}{Z_{\mathrm{ext}}}{}^T \, \overset{(n)}{\kappa} \, \overset{(n)}{Z_{\mathrm{ext}}} \, \overset{(n)}{Z_\kappa}$$

$$M_{NN} = -M_{SN}^T M_{SS}^{-1} M_{SN}$$

$$V_N^T M_{NN} V_N = D_N \equiv (M_i)$$

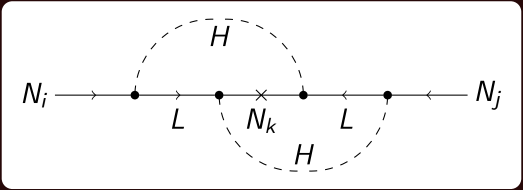
Leading Order Correction to Neutrino Mass

Rescaling of RH neutrino masses M_i:

$$m_\nu \approx -\langle H\rangle^2 Z_{\mathrm{ext}}^T \left[Y_\nu^T V_N Z_\kappa D_N^{-1} V_N^T Y_\nu\right] Z_{\mathrm{ext}}$$

Two Loop Contribution to RH Neutrino Masses

Renormalization of RH neutrinos



$$\Delta M_{ij} = \frac{2}{(16\pi^2)^2} \sum_k (Y_\nu^\dagger Y_\nu)_{ik} \left(Y_\nu^\dagger Y_\nu\right)_{jk} M_k \left(\frac{1}{\epsilon} + \frac{1}{2} + \ln \frac{\mu^2}{M_k^2}\right)$$

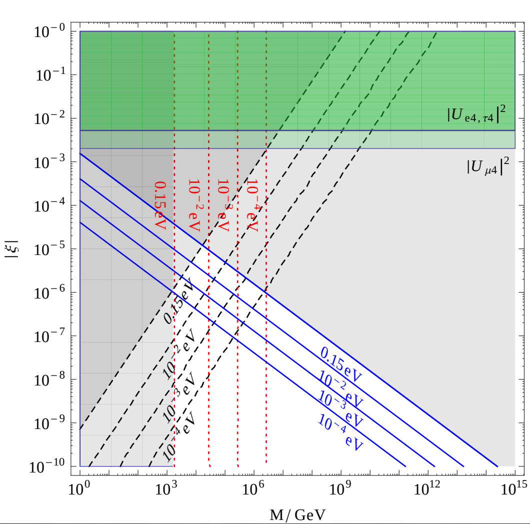
The finite Higgs mass has been neglected. There are $\mathcal{O}\left(\mu_H^2/M_k^2\right)$ corrections.

Assuming $y_3 \gg y_2 \gg y_1$ and not all $U_{R,i3}$ vanish:

$$\Delta M_i \sim \frac{y_3^4}{(8\pi^2)^2} \sum_k \left[U_{R,k3}^* U_{R,i3}\right]^2 M_k \ln\left(\Lambda^2/M_k^2\right)$$

[Aparici, Herrero-Garcia, Rius, Santamaria (2011); MS, Smirnov (2011)]

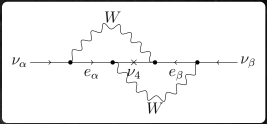
Fourth Generation



$$\begin{pmatrix} 0 & 0 & f_L & m \\ \dots & 0 & m_{E4} & f_R^T \\ \dots & \dots & M_4 & 0 \\ \dots & \dots & \dots & M \end{pmatrix}$$

$$m_{\alpha\beta}^{\text{tree}} \simeq m_{E4} \sum_k \frac{m_{\alpha k}}{M_k} \times$$

$$\times (U_L)_{\beta 4} (U_R)_{k4} + (\alpha \leftrightarrow \beta)$$



[Petcov, Toshev (1984); Babu,Ma (1988)]

$$m_D = m_e = 511\,\text{keV}, \; \xi = (U_L)_{\alpha 4} (U_R)_{i4}$$

Outline

1. Introduction
2. Implementations of Double Seesaw Structure
3. Stability with respect to Quantum Corrections
4. Conclusions

Summary & Conclusions

- Double Seesaw structure can accomodate different hierarchies in charged and neutral fermion masses
- A complete cancellation of the Dirac structure can be obtained
- Standard (Fermionic singlet) seesaw within SO(10) $\times$ G_f possible

Summary & Conclusions

- Double Seesaw structure can accomodate different hierarchies in charged and neutral fermion masses
- A complete cancellation of the Dirac structure can be obtained
- Standard (Fermionic singlet) seesaw within SO(10) $\times$ G_f possible
- Study of RG stability of double seesaw structure
- Within MSSM, double seesaw structure is stable
- Threshold corrections in non-SUSY dominantly lead to a rescaling of RH neutrino masses

$\Rightarrow$ Structure of formula in cancellation mechanism modified

Thank you, Alexei!

For the collaboration and
for everything, what I learned from you!

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for everything, what I learned from you!

All the best for the next 60 years!

T_7: Group Theory

- $T_7 \cong Z_7 \rtimes Z_3 \subset \mathrm{SU}(3)$, also called Frobenius group
- Smallest group with complex $\underline{\mathbf{3}}$: order 21
- Irreducible representations: $\underline{\mathbf{1}}_{\mathbf{1}}$, $\underline{\mathbf{1}}_{\mathbf{2}}$, $\underline{\mathbf{1}}_{\mathbf{3}} \cong \underline{\mathbf{1}}_{\mathbf{2}}^*$ and $\underline{\mathbf{3}}$, $\underline{\mathbf{3}}^\star$
- $\underline{\mathbf{1}}_{\mathbf{i}}$ like in Z_3: $\underline{\mathbf{1}}_{\mathbf{1}}$ and $\underline{\mathbf{1}}_{\mathbf{2}} \otimes \underline{\mathbf{1}}_{\mathbf{3}}$ are invariant
- Generators of $\underline{\mathbf{3}}$:

$$A = \begin{pmatrix} e^{2\pi\,\mathrm{i}\,/7} & 0 & 0 \\ 0 & e^{4\pi\,\mathrm{i}\,/7} & 0 \\ 0 & 0 & e^{8\pi\,\mathrm{i}\,/7} \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

- $\{\underline{\mathbf{3}} \otimes \underline{\mathbf{3}}\} = \underline{\mathbf{3}} \oplus \underline{\mathbf{3}}^*$:
  $(a_1,\ a_2,\ a_3)^T \sim \underline{\mathbf{3}}$: $(a_3\,a_3, a_1\,a_1, a_2\,a_2)^T \sim \underline{\mathbf{3}}$,
  $(a_{\{2}\,a_{3\}}, a_{\{3}\,a_{1\}}, a_{\{1}\,a_{2\}})^T \sim \underline{\mathbf{3}}^*$
- $\underline{\mathbf{3}} \otimes \underline{\mathbf{1}}_{\mathbf{i}} = \underline{\mathbf{3}}$:
  $(a_1,\ a_2,\ a_3)^T \sim \underline{\mathbf{3}}$, $c \sim \underline{\mathbf{1}}_{\mathbf{i}}$: $(a_1\,c, \omega^{i-1}\,a_2\,c, \omega^{1-i}\,a_3\,c)$ with $\omega = e^{\mathrm{i}\,2\pi/3}$

Higher-Dimensional Operators and Mass Scales

- Higher-dimensional operators up to $\frac{m_u}{\alpha} \sim \epsilon^4 \eta$, $\eta = \frac{\langle \chi_1 \rangle}{\Lambda} \sim 0.48$

$$\chi_1^n \xrightarrow{A} e^{-\frac{2\pi i}{7} n} \chi_1^n \sim \mathcal{O}(1)$$

$$\chi_1^{n-1}\chi_3 \xrightarrow{A} e^{-\frac{2\pi i}{7}(n+3)} \chi_1^{n-1}\chi_3 \sim \mathcal{O}\left(\epsilon^2\right)$$

$\Rightarrow$ Introduction of Z_7

- Problem: $M_{SS} \sim \mathcal{O}\left(\epsilon^4 M_{Pl}\right)$, but contributions like

$$M_{SS} \sim SS \langle \chi \rangle^n / \Lambda^{n-1}$$

- Solution: forbid tree-level and generate M_{SS} at higher order
- Observe: only one covariant

$$SS\chi^3/\Lambda^2 \sim (a\, S_1 S_1 + b\,(S_2 S_3 + S_3 S_2))\,\chi_1\chi_2\chi_3/\Lambda^2$$

Field	$\underline{\mathbf{16}}_i$	S_i	H	Δ	χ_i
T_7	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}_i$	$\underline{\mathbf{1}}_1$	$\underline{\mathbf{1}}_1$	$\underline{\mathbf{3}}^*$
Z_7	3	2	0	1	1

T_7: Higher-Dimensional Operators

Consider operators up to order $\frac{m_u}{\alpha} \sim \epsilon^2 \eta = \eta^{17}$, $\eta = \frac{\langle \chi_1 \rangle}{\Lambda} \sim 0.48$

Additional Z_7 to forbid operators

Field	$\underline{\mathbf{16}}_i$	S_i	H	Δ	χ_i
T_7	$\underline{\mathbf{3}}$	$\underline{\mathbf{1}}_i$	$\underline{\mathbf{1}}_1$	$\underline{\mathbf{1}}_1$	$\underline{\mathbf{3}}^*$
Z_7	3	0	0	3	1

→ All higher-dimensional operators suppressed by η^7 compared to LO.
→ Vanishing entries are filled.

Structure of covariants periodic in 7 due to subgroup Z_7 of T_7

Structure	Transformation Properties under Generator A	Order in ϵ
χ_1^n	$e^{-\frac{2\pi i}{7} n} \chi_1^n$	$\mathcal{O}(1)$
$\chi_1^{n-1} \chi_2$	$e^{-\frac{2\pi i}{7}(n+1)} \chi_1^{n-1} \chi_2$	$\mathcal{O}(\epsilon^2)$
$\chi_1^{n-1} \chi_3$	$e^{-\frac{2\pi i}{7}(n+3)} \chi_1^{n-1} \chi_3$	$\mathcal{O}(\epsilon)$
$\chi_1^{n-2} \chi_3^2$	$e^{-\frac{2\pi i}{7}(n+6)} \chi_1^{n-2} \chi_3^2$	$\mathcal{O}(\epsilon^2)$

T_7: Cabibbo Angle and Charged Lepton Masses

Cabibbo Angle

By introduction of $\underline{\mathbf{16}}_H$, $\underline{\mathbf{16}}'_H \sim (\underline{1}_{T_7}, 6_{Z_7})$

$$\frac{1}{M}\left(\underline{\mathbf{16}}_i\, \underline{\mathbf{16}}_j\, \underline{\mathbf{16}}_H\, \underline{\mathbf{16}}'_H\right)\left(\frac{\chi}{\Lambda}\right)^3$$

contributes to down type and charged lepton mass matrix

$$m_{down} \approx \langle \underline{\mathbf{16}}_H \rangle_\nu \left(\frac{\langle \underline{\mathbf{16}}'_H \rangle_N}{M}\right) \begin{pmatrix} \mathcal{O}(\epsilon^4\eta^3) & \mathcal{O}(\eta^3) & \mathcal{O}(\epsilon^6\eta^3) \\ . & \mathcal{O}(\epsilon^4\eta^3) & \mathcal{O}(\epsilon^2\eta^3) \\ . & . & 0 \end{pmatrix}$$

Charged Lepton Mass Matrix

The introduction of $\underline{\mathbf{45}}_H \sim (\underline{1}_{T_7}, 4_{Z_7})$

$$\frac{1}{M'}\left(\underline{\mathbf{16}}_i\, \underline{\mathbf{16}}_j\, H\, \underline{\mathbf{45}}_H\right)\left(\frac{\chi}{\Lambda}\right)^4$$

can generate needed Georgi-Jarlskog factor.

T_7: Flavon Potential

T_7

- $W = \kappa\,\chi_1\,\chi_2\,\chi_3$
- F-terms: $F_{\chi_1} = \dfrac{\partial W}{\partial \chi_1} = \kappa\,\chi_2\,\chi_3$ and cyclic $\Rightarrow \langle\chi_{2,3}\rangle = 0$, $\langle\chi_1\rangle \neq 0$

$T_7 \times Z_7$

- Renormalizable part forbidden
- Introduce $U(1)_R$: superpotential: +2, matter: +1, Higgs/flavons: 0 and driving field $\phi \sim (\underline{\mathbf{3}}^*, 5)_{+2} \Rightarrow$ superpotential linear in ϕ
- $W = \kappa\,\phi\,\chi^2 = \kappa\,\phi_1\,\chi_2\,\chi_3 +$ cyclic
- $F_{\phi_1} = \kappa\,\chi_2\,\chi_3$ and cyclic $\Rightarrow \langle\chi_{2,3}\rangle = 0$, $\langle\chi_1\rangle \neq 0$

- Leading order can be obtained,
- Further investigation needed to generate viable flavon potential

$\Sigma(81)$: Group Theory

- $\Sigma(81) \subset \mathrm{U}(3)$: order 81
- Irreducible representations: $\underline{\mathbf{1}}_{\mathbf{i}}$, $i = 1, \ldots, 9$ and $\underline{\mathbf{3}}_{\mathbf{i}}$, $i = 1, \ldots, 8$

Rep.		$\underline{\mathbf{1}}_4$	$\underline{\mathbf{1}}_5$	$\underline{\mathbf{1}}_6$	$\underline{\mathbf{3}}_3$	$\underline{\mathbf{3}}_5$	$\underline{\mathbf{3}}_7$
Rep.*		$\underline{\mathbf{1}}_7$	$\underline{\mathbf{1}}_8$	$\underline{\mathbf{1}}_9$	$\underline{\mathbf{3}}_4$	$\underline{\mathbf{3}}_6$	$\underline{\mathbf{3}}_8$

- Kronecker products:
  - $\underline{\mathbf{1}}_{\mathbf{i}} \otimes \underline{\mathbf{1}}_{\mathbf{j}} = \underline{\mathbf{1}}_{\mathbf{i+j \bmod 3}}\,, \qquad i, j = 1, 2, 3$
  - $\underline{\mathbf{3}}_{\mathbf{i}} \otimes \underline{\mathbf{1}}_{\mathbf{j}} = \underline{\mathbf{3}}_{\mathbf{i}}\,, \quad i = 1, 2\,; j = 1, 2, 3$:
    $(a_1,\ a_2,\ a_3)^T \sim \underline{\mathbf{3}}_{\mathbf{i}}$, $c \sim \underline{\mathbf{1}}_{\mathbf{j}}$: $(a_1\, c, \omega^{i-1}\, a_2\, c, \omega^{1-i}\, a_3\, c)$
  - $\{\underline{\mathbf{3}}_1 \otimes \underline{\mathbf{3}}_1\} = \underline{\mathbf{3}}_2 \oplus \underline{\mathbf{3}}_4$:
    $(a_1,\ a_2,\ a_3)^T \sim \underline{\mathbf{3}}_1$: $(a_1\, a_1, a_2\, a_2, a_3\, a_3)^T \sim \underline{\mathbf{3}}_2$ with $\omega = e^{\mathrm{i}\,2\pi/3}$
  - $\underline{\mathbf{3}}_1 \otimes \underline{\mathbf{3}}_2 = \underline{\mathbf{1}}_1 \oplus \underline{\mathbf{1}}_2 \oplus \underline{\mathbf{1}}_3 \oplus \underline{\mathbf{3}}_7 \oplus \underline{\mathbf{3}}_8$:
    $(a_1, a_2, a_3)^T \sim \underline{\mathbf{3}}_1$, $(b_1, b_2, b_3)^T \sim \underline{\mathbf{3}}_2$: $(a_1\, b_1, a_2\, b_2, a_3\, b_3)^T \sim \underline{\mathbf{1}}_1$

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Rep.	$\underline{\mathbf{1}}_1$	$\underline{\mathbf{1}}_2$	$\underline{\mathbf{1}}_4$	$\underline{\mathbf{1}}_5$	$\underline{\mathbf{1}}_6$	$\underline{\mathbf{3}}_1$	$\underline{\mathbf{3}}_3$	$\underline{\mathbf{3}}_5$	$\underline{\mathbf{3}}_7$
Rep.*	$\underline{\mathbf{1}}_1$	$\underline{\mathbf{1}}_3$	$\underline{\mathbf{1}}_7$	$\underline{\mathbf{1}}_8$	$\underline{\mathbf{1}}_9$	$\underline{\mathbf{3}}_2$	$\underline{\mathbf{3}}_4$	$\underline{\mathbf{3}}_6$	$\underline{\mathbf{3}}_8$

- Kronecker products:
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  - $\underline{\mathbf{3}}_{\mathbf{i}} \otimes \underline{\mathbf{1}}_{\mathbf{j}} = \underline{\mathbf{3}}_{\mathbf{i}}\,, \quad i = 1, 2\,; j = 1, 2, 3$:
    $(a_1,\ a_2,\ a_3)^T \sim \underline{\mathbf{3}}_{\mathbf{i}},\ c \sim \underline{\mathbf{1}}_{\mathbf{j}}$: $(a_1\, c, \omega^{i-1}\, a_2\, c, \omega^{1-i}\, a_3\, c)$
  - $\{\underline{\mathbf{3}}_1 \otimes \underline{\mathbf{3}}_1\} = \underline{\mathbf{3}}_2 \oplus \underline{\mathbf{3}}_4$:
    $(a_1,\ a_2,\ a_3)^T \sim \underline{\mathbf{3}}_1$: $(a_1\, a_1, a_2\, a_2, a_3\, a_3)^T \sim \underline{\mathbf{3}}_2$ with $\omega = e^{\mathrm{i}\, 2\pi/3}$
  - $\underline{\mathbf{3}}_1 \otimes \underline{\mathbf{3}}_2 = \underline{\mathbf{1}}_1 \oplus \underline{\mathbf{1}}_2 \oplus \underline{\mathbf{1}}_3 \oplus \underline{\mathbf{3}}_7 \oplus \underline{\mathbf{3}}_8$:
    $(a_1, a_2, a_3)^T \sim \underline{\mathbf{3}}_1,\ (b_1, b_2, b_3)^T \sim \underline{\mathbf{3}}_2$: $(a_1\, b_1, a_2\, b_2, a_3\, b_3)^T \sim \underline{\mathbf{1}}_1$

$\Sigma(81)$: Group Theory

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Rep.	$\underline{\mathbf{1}}_1$	$\underline{\mathbf{1}}_2$	$\underline{\mathbf{1}}_4$	$\underline{\mathbf{1}}_5$	$\underline{\mathbf{1}}_6$	$\underline{\mathbf{3}}_1$	$\underline{\mathbf{3}}_3$	$\underline{\mathbf{3}}_5$	$\underline{\mathbf{3}}_7$
Rep.*	$\underline{\mathbf{1}}_1$	$\underline{\mathbf{1}}_3$	$\underline{\mathbf{1}}_7$	$\underline{\mathbf{1}}_8$	$\underline{\mathbf{1}}_9$	$\underline{\mathbf{3}}_2$	$\underline{\mathbf{3}}_4$	$\underline{\mathbf{3}}_6$	$\underline{\mathbf{3}}_8$

- Kronecker products:
  - $\underline{\mathbf{1}}_\mathbf{i} \otimes \underline{\mathbf{1}}_\mathbf{j} = \underline{\mathbf{1}}_{\mathbf{i+j \bmod 3}}\,, \qquad i, j = 1, 2, 3$
  - $\underline{\mathbf{3}}_\mathbf{i} \otimes \underline{\mathbf{1}}_\mathbf{j} = \underline{\mathbf{3}}_\mathbf{i}\,, \quad i = 1, 2\,; j = 1, 2, 3$:
    $(a_1,\, a_2,\, a_3)^T \sim \underline{\mathbf{3}}_\mathbf{i},\ c \sim \underline{\mathbf{1}}_\mathbf{j}$: $(a_1\, c, \omega^{i-1}\, a_2\, c, \omega^{1-i}\, a_3\, c)$
  - $\{\underline{\mathbf{3}}_1 \otimes \underline{\mathbf{3}}_1\} = \underline{\mathbf{3}}_2 \oplus \underline{\mathbf{3}}_4$:
    $(a_1,\, a_2,\, a_3)^T \sim \underline{\mathbf{3}}_1$: $(a_1\, a_1, a_2\, a_2, a_3\, a_3)^T \sim \underline{\mathbf{3}}_2$ with $\omega = e^{\mathrm{i}\, 2\pi/3}$
  - $\underline{\mathbf{3}}_1 \otimes \underline{\mathbf{3}}_2 = \underline{\mathbf{1}}_1 \oplus \underline{\mathbf{1}}_2 \oplus \underline{\mathbf{1}}_3 \oplus \underline{\mathbf{3}}_7 \oplus \underline{\mathbf{3}}_8$:
    $(a_1, a_2, a_3)^T \sim \underline{\mathbf{3}}_1$, $(b_1, b_2, b_3)^T \sim \underline{\mathbf{3}}_2$: $(a_1\, b_1, a_2\, b_2, a_3\, b_3)^T \sim \underline{\mathbf{1}}_1$

$\Sigma(81)$: Realisation

Particle Content

Field	$\underline{16}_i$	S_i	H	Δ	χ_i
$SO(10)$	$\underline{16}$	$\underline{1}$	$\underline{10}$	$\overline{\underline{16}}$	$\underline{1}$
$\Sigma(81)$	$\underline{3}_1$	$\underline{1}_i$	$\underline{1}_1$	$\underline{1}_1$	$\underline{3}_2 \cong \underline{3}_1^\star$

Lagrangian

$$
\begin{aligned}
\mathscr{L} \supset\ & \alpha\, H\, (\underline{16}_3\, \underline{16}_3\, \chi_1^* + \underline{16}_1\, \underline{16}_1\, \chi_2^* + \underline{16}_2\, \underline{16}_2\, \chi_3^*)/\Lambda \\
& + \beta_1\, \Delta\, S_1\, (\underline{16}_1\, \chi_1 + \underline{16}_2\, \chi_2 + \underline{16}_3\, \chi_3)/\Lambda \\
& + \beta_2\, \Delta\, S_2\, (\underline{16}_1\, \chi_1 + \omega\, \underline{16}_2\, \chi_2 + \omega^2\, \underline{16}_3\, \chi_3)/\Lambda \\
& + \beta_3\, \Delta\, S_3\, (\underline{16}_1\, \chi_1 + \omega^2\, \underline{16}_2\, \chi_2 + \omega\, \underline{16}_3\, \chi_3)/\Lambda \\
& + A\, S_1\, S_1 + B\, (S_2\, S_3 + S_3\, S_2) + \text{h.c.}
\end{aligned}
$$

$\Sigma(81)$: Lowest Order

$$m_D = \frac{\alpha \langle H \rangle}{\Lambda} \begin{pmatrix} \langle \chi_1 \rangle^* & 0 & 0 \\ 0 & \langle \chi_2 \rangle^* & 0 \\ 0 & 0 & \langle \chi_3 \rangle^* \end{pmatrix}$$

$$M_{SN} = \frac{\langle \Delta \rangle_N}{\Lambda} \begin{pmatrix} \beta_1 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & \beta_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} \langle \chi_1 \rangle & 0 & 0 \\ 0 & \langle \chi_2 \rangle & 0 \\ 0 & 0 & \langle \chi_3 \rangle \end{pmatrix}$$

$$M_{SS} = \begin{pmatrix} A & 0 & 0 \\ \cdot & 0 & B \\ \cdot & \cdot & 0 \end{pmatrix}$$

$$m_\nu \approx \left(\frac{\alpha \langle H \rangle}{\langle \Delta \rangle_N} \right)^2 \begin{pmatrix} \tilde{A} + 2\tilde{B} & \tilde{A} - \tilde{B} & \tilde{A} - \tilde{B} \\ \cdot & \tilde{A} + 2\tilde{B} & \tilde{A} - \tilde{B} \\ \cdot & \cdot & \tilde{A} + 2\tilde{B} \end{pmatrix}$$

$$\tilde{A} \equiv \frac{A}{9\beta_1^2}, \quad \tilde{B} \equiv \frac{B}{9\beta_2\beta_3}$$

$\Sigma(81)$: Phenomenology

Charged Fermions

- Quark mass hierarchy $\langle\chi_1\rangle^* : \langle\chi_2\rangle^* : \langle\chi_3\rangle^* = \epsilon^2 : \epsilon : 1$, $\epsilon \sim 3 \cdot 10^{-3}$
- Zero mixing in quark sector
- $m_t \Rightarrow \langle\chi_3\rangle^* \sim \Lambda \Rightarrow$ higher-dimensional operators are relevant

Neutrinos

- Dirac mass hierarchy exactly drops out

$$m_\nu \approx \left(\frac{\alpha \langle H\rangle}{\langle\Delta\rangle_N}\right)^2 \begin{pmatrix} \left(\tilde{A} + 2\tilde{B}\right) & \left(\tilde{A} - \tilde{B}\right) & \left(\tilde{A} - \tilde{B}\right) \\ \cdot & \tilde{A} + 2\tilde{B} & \tilde{A} - \tilde{B} \\ \cdot & \cdot & \tilde{A} + 2\tilde{B} \end{pmatrix}$$

- Neutrino mass matrix diagonalized by tri-bimaximal mixing matrix
- $m_2 = 3\left|\frac{\alpha \langle H\rangle}{\langle\Delta\rangle_N}\right|^2 \left|\tilde{A}\right|$, $m_{1,3} = 3\left|\frac{\alpha \langle H\rangle}{\langle\Delta\rangle_N}\right|^2 \left|\tilde{B}\right|$

$\Sigma(81)$: Higher-Dimensional Operators (General)

Order in ϵ	Structure	Representation
$\mathcal{O}(1)$	$\chi_3^m\,(\chi_3^\star)^{n-m}$ $(m=0,\ldots,n)$	$\underline{\mathbf{1}}_{\mathbf{1,2,3}}$ for $(2\,m-n) \mod 3 = 0$ $3^{\rm rd}$ comp. of $\underline{\mathbf{3}}_{\mathbf{1}}$ for $(2\,m-n) \mod 3 = 1$ $3^{\rm rd}$ comp. of $\underline{\mathbf{3}}_{\mathbf{2}}$ for $(2\,m-n) \mod 3 = 2$
$\mathcal{O}(\epsilon)$	...	...

- Structure of operators calculable to arbitrary order
- All higher-dimensional operators suppressed compared to LO.
- Vanishing entries are filled.
- Additional Z_n symmetry can be introduced to suppress higher-dimensional operators, e.g. Z_3

$\Rightarrow$ Numerical Example to show the possibility to fit the data

$\Sigma(81)$: Higher-Dimensional Operators (Numerical)

$$m_D = \begin{pmatrix} 1.1589 \cdot 10^{-6} & 0 & 8.6454 \cdot 10^{-7} \\ . & 1.0051 \cdot 10^{-3} & 3.4268 \cdot 10^{-4} \\ . & . & 0.63863 \end{pmatrix} \langle H \rangle \ ,$$

$$M_{SN} = \begin{pmatrix} 7.4031 \cdot 10^{-6} & 4.6288 \cdot 10^{-6} & 3.2038 \cdot 10^{-6} \\ 3.0486 \cdot 10^{-3} & 1.9009 \cdot 10^{-3}\,\omega & 1.4336 \cdot 10^{-3}\,\omega^2 \\ 1.2503 & 0.91423\,\omega^2 & 0.71852\,\omega \end{pmatrix} \langle \Delta \rangle_N \ ,$$

$$M_{SS} = \begin{pmatrix} 1 & 1.7689 \cdot 10^{-2}\,\omega^2 & 3.8688 \cdot 10^{-2}\,\omega \\ . & 1.1516 \cdot 10^{-2}\,\omega & -0.7475 \\ . & . & 2.3890 \cdot 10^{-2}\,\omega^2 \end{pmatrix} M_{Pl}$$

$$m_\nu \approx \begin{pmatrix} 1.1809 \cdot e^{\mathrm{i}\,0.019} & 1.7675 \cdot e^{\mathrm{i}\,3.12} & 1.5297 \cdot e^{-\mathrm{i}\,3.08} \\ . & 2.5403 \cdot e^{-\mathrm{i}\,0.031} & 3.4549 \cdot e^{\mathrm{i}\,3.11} \\ . & . & 1.8254 \end{pmatrix} \cdot 10^{-2}\,\mathrm{eV}$$

$$\Delta m^2_{21} = 7.9\,10^{-5}\,\mathrm{eV}^2 \ , \quad \Delta m^2_{32} = 2.5\,10^{-3}\,\mathrm{eV}^2 \ , \quad \theta_{12} = 33.0^\circ \ ,$$
$$\theta_{13} = 4.5^\circ \ , \quad \theta_{23} = 49.5^\circ \ , \quad \delta = 137^\circ \ , \quad \varphi_1 = 313^\circ \ , \quad \varphi_2 = 162^\circ$$

$\Sigma(81)$: Flavon Potential

In polar coordinates $\chi_i = X_i e^{\mathrm{i}\,\xi_i}$

$$V_\chi(X_j,\,\xi_j) = M^2 \sum_i X_i^2 + \lambda_1 \sum_i X_i^4 + \lambda_2 \sum_{i\neq k} X_i^2 X_k^2 + 2\kappa \sum_i X_i^3 \cos\left(\alpha + 3\xi_i\right)$$

Minimisation:

$$\frac{\partial V_\chi}{\partial X_1} = 2X_1\left(M^2 + 2\lambda_1 X_1^2 + \lambda_2 X_2^2 + \lambda_2 X_3^2 + 3\kappa X_1 \cos\left(\alpha + 3\xi_1\right)\right) \stackrel{!}{=} 0$$

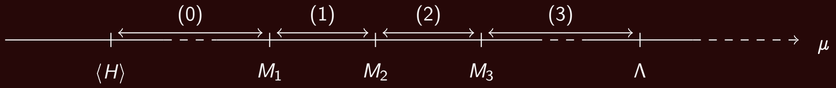
$$\frac{\partial V_\chi}{\partial \xi_1} = -6\kappa X_1^3 \sin\left(\alpha + 3\xi_1\right) \stackrel{!}{=} 0 \qquad \text{and cyclic}$$

Minimum

$$\langle X_1\rangle = \langle X_2\rangle = 0\,, \quad \langle X_3\rangle = \frac{3\kappa + \sqrt{9\kappa^2 - 8M^2\lambda_1}}{4\lambda_1}\,, \quad \langle \xi_3\rangle = -\frac{\alpha \pm \pi}{3}$$

possible in a certain region of parameter space $(M^2,\,\lambda_1,\,\lambda_2,\,\kappa,\,\alpha)$.

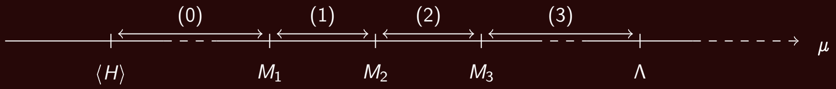
RG Evolution EFT by EFT



Neutrino mass operator

$$\overset{(3)}{O}_M(\Lambda) = \begin{pmatrix} 0 & \overset{(3)}{Y}{}_\nu^T H \\ \cdot & \overset{(3)}{M}_{NN} \end{pmatrix}$$

RG Evolution EFT by EFT



Neutrino mass operator

$$\overset{(3)}{O}_M(\Lambda) = \begin{pmatrix} 0 & \overset{(3)}{Y_\nu^T} H \\ \cdot & \overset{(3)}{M}_{NN} \end{pmatrix}$$

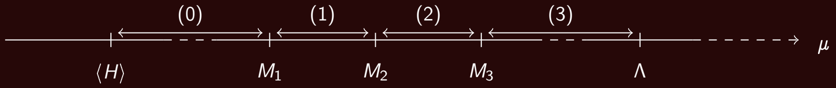
RG transformation

$$Y_\nu \xrightarrow{\text{RG}} Z_N^T \, Y_\nu \, Z_{\text{ext}}$$

$$M_{NN} \xrightarrow{\text{RG}} Z_N^T \, M_{NN} \, Z_N$$

$$\kappa \xrightarrow{\text{RG}} Z_{\text{ext}}^T \, \kappa \, Z_{\text{ext}} \, Z_\kappa$$

RG Evolution EFT by EFT



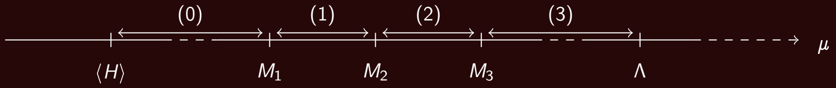
Neutrino mass operator

$$\overset{(3)}{O}_M(\Lambda) = \begin{pmatrix} 0 & \overset{(3)}{Y}{}_\nu^T \, H \\ \cdot & \overset{(3)}{M}_{NN} \end{pmatrix}$$

RG Evolution

$$\overset{(3)}{O}_M(M_3) = \begin{pmatrix} 0 & \overset{(3)}{Z}{}_{\text{ext}}^T \, \overset{(3)}{Y}{}_\nu^T \, H \, \overset{(3)}{Z}_N \\ \cdot & \overset{(3)}{Z}{}_N^T \, \overset{(3)}{M}_{NN} \, \overset{(3)}{Z}_N \end{pmatrix}$$

RG Evolution EFT by EFT



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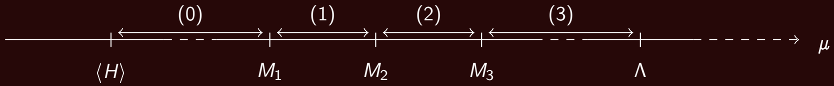
Diagonalization

$$\overset{(3)}{U}{}_N^T \, \overset{(3)}{Z}{}_N^T \, \overset{(3)}{M}_{NN} \, \overset{(3)}{Z}_N \, \overset{(3)}{U}_N = \begin{pmatrix} \overset{(2)}{M}_{NN} & 0 \\ 0 & M_3 \end{pmatrix}$$

$$\overset{(3)}{Z}{}_{\text{ext}}^T \, \overset{(3)}{Y}{}_\nu^T \, \overset{(3)}{Z}_N \, \overset{(3)}{U}_N \equiv \begin{pmatrix} \overset{(2)}{Y}{}_\nu^T, & y_3^T \end{pmatrix}$$

RG Evolution EFT by EFT



Neutrino mass operator

$$\overset{(3)}{O}_M(\Lambda) = \begin{pmatrix} 0 & \overset{(3)}{Y}{}_\nu^T\, H \\ \cdot & \overset{(3)}{M}_{NN} \end{pmatrix}$$

RG Evolution

$$\overset{(3)}{O}_M(M_3) = \begin{pmatrix} 0 & \overset{(3)}{Z}{}_{\text{ext}}^T\, \overset{(3)}{Y}{}_\nu^T\, H\, \overset{(3)}{Z}_N \\ \cdot & \overset{(3)}{Z}{}_N^T\, \overset{(3)}{M}_{NN}\, \overset{(3)}{Z}_N \end{pmatrix}$$

Diagonalization

$$\overset{(3)}{U}{}_N^T\, \overset{(3)}{Z}{}_N^T\, \overset{(3)}{M}_{NN}\, \overset{(3)}{Z}_N\, \overset{(3)}{U}_N = \begin{pmatrix} \overset{(2)}{M}_{NN} & 0 \\ 0 & M_3 \end{pmatrix}$$

$$\overset{(3)}{Z}{}_{\text{ext}}^T\, \overset{(3)}{Y}{}_\nu^T\, \overset{(3)}{Z}_N\, \overset{(3)}{U}_N \equiv \begin{pmatrix} \overset{(2)}{Y}{}_\nu^T, & y_3^T \end{pmatrix}$$

Integrate Out

$$\overset{(2)}{O}_M(M_3) = \begin{pmatrix} -y_3^T\, M_3^{-1}\, y_3\, H^2 & \overset{(2)}{Y}{}_\nu^T\, H \\ \cdot & \overset{(2)}{M}_{NN} \end{pmatrix}$$

RG corrections to Neutrino Mass – Technical details

$$m_\nu = -\frac{1}{2} Z_{\text{ext}}^T \left[\overset{(3)}{m}{}_D^T \left(X_N \overset{(3)}{M}{}_{NN}^{-1} + \overset{(3)}{M}{}_{NN}^{-1} X_N^T \right) \overset{(3)}{m}_D \right] Z_{\text{ext}}$$

$$X_N \equiv \overset{(3)}{Z}_N \overset{(3)}{U}_N \overset{(2)}{Z'_N} \overset{(2)}{U'_N} Z_\kappa \overset{(2)}{U}{}_N^{\prime\dagger} \overset{(2)}{Z}{}_N^{\prime -1} \overset{(3)}{U}{}_N^{\dagger} \overset{(3)}{Z}{}_N^{-1}$$

$$\overset{(2)}{Z'_N} \equiv \begin{pmatrix} \overset{(2)}{Z}_N & 0 \\ 0 & 1 \end{pmatrix}, \quad \overset{(2)}{U'_N} \equiv \begin{pmatrix} \overset{(2)}{U}_N & 0 \\ 0 & 1 \end{pmatrix}, \quad Z_\kappa \equiv \text{diag}\left(\overset{(0)}{Z}_\kappa, \overset{(0-1)}{Z_\kappa}, \overset{(0-2)}{Z_\kappa} \right)$$

Approximation

$$V_N^T \overset{(3)}{M}_{NN} V_N = D_N \equiv \text{diag}(M_1, M_2, M_3) \Rightarrow X_N \approx V_N Z_\kappa V_N^\dagger$$

$$m_\nu \approx -\langle H \rangle^2 Z_{\text{ext}}^T \left[\overset{(3)}{Y}{}_\nu^T V_N Z_\kappa D_N^{-1} V_N^T \overset{(3)}{Y}_\nu \right] Z_{\text{ext}}$$

$\Rightarrow$ Dominantly rescaling of right-handed neutrino masses