Electroweak Corrections at the LHC

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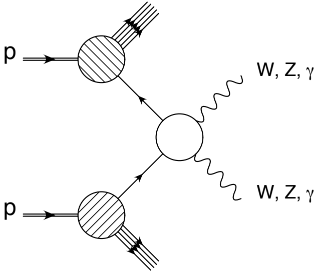
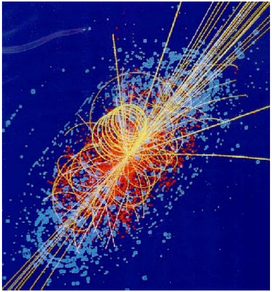
LHC: proton-proton collisions



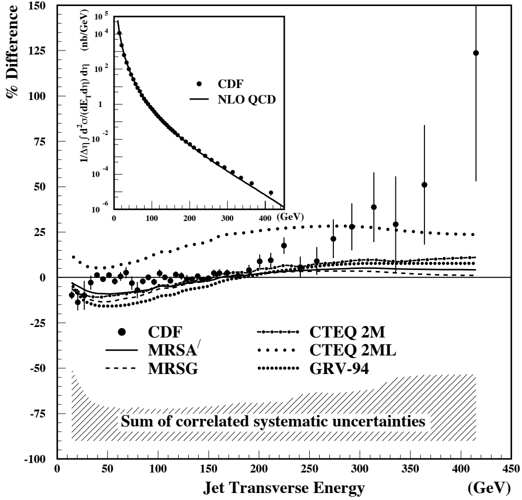
Theory



Experiment

Pitfalls

New physics?



LHC Processes

Typical LHC processes:

- jet production
- W, Z production (single or pair)
- leptons: 2, 3, 4, ...

with $E_{\text{cm}} = \sqrt{s} = Q$ up to a few TeV.

$$Q \sim 1\ \text{TeV}, \quad M_{W,Z} \sim 0.1\ \text{TeV}, \quad m_{\text{proton}} \sim 0.001\ \text{TeV}$$

Hierarchy of scales:

$$Q^2 \gg M_{W,Z}^2 \gg \Lambda_{\text{QCD}}^2$$

Note: M_Z effects at the LHC are the same size as m_b effects at LEP.

Worked out a general formalism for computing EW corrections at high energy.

Work done with Jui-Yu Chiu, Frank Golf, Andreas Fuhrer, Randall Kelley

Applicable to *all* hard scattering processes at high energy.

- A collinear function for each particle that depends on the energy

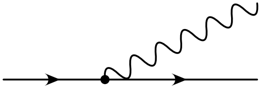
$$\mathcal{F}_i(E) : i = u_L, u_R, e_L, W, Z, \ldots$$

- A universal soft function that depends only the directions of the outgoing particles

$$\mathcal{S} \propto \sum_{\langle ij \rangle} T_i \cdot T_j \, \log \frac{n_i \cdot n_j}{2}$$

The corrections are large, e.g. 37% to *WW* production at 2 TeV.

Infrared Singularities in Radiation



The intermediate propagator is

$$\frac{1}{(p+k)^2 - m^2} = \frac{1}{2p\cdot k + k^2} = \frac{1}{2E_p\,\omega_k - 2\,|\mathbf{p}|\,|\mathbf{k}|\cos\theta + M^2}$$

For massless particles, $E_p = |\mathbf{p}|$ and $\omega_k = |\mathbf{k}|$

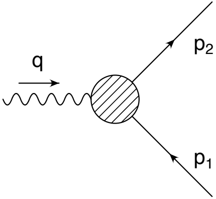
$$2E\omega\,(1-\cos\theta)$$

singularities as $\omega \to 0$ (soft) and $\theta \to 0$ (collinear).

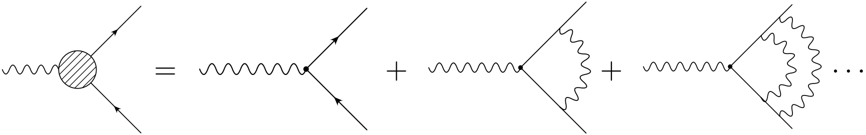
Sudakov Form Factor

- $(Q^2 \equiv -q^2 = -(p_2 - p_1)^2)$,

$$F_E(Q)\left[\bar{u}(p_2)\gamma^\mu u(p_1)\right] = \langle p_2|\, J^\mu_{EM}(q)\,|p_1\rangle =$$



- If coupling strength is small we calculate $F_E(Q^2)$ perturbatively in powers of $\alpha = \frac{e^2}{4\pi}$.



General perturbative structure of $F_E(Q)$

$$\mathsf{L} = \log \mathsf{Q}^2/\mathsf{M}^2$$

$$\begin{aligned} F_E(Q) = & \quad 1 & \text{LO} \\ & + \quad \alpha^1 \left(\mathsf{L}^2 + \mathsf{L}^1 + \mathsf{L}^0\right) & \text{NLO} \\ & + \quad \alpha^2 \left(\mathsf{L}^4 + \mathsf{L}^3 + \mathsf{L}^2 + \mathsf{L}^1 + \mathsf{L}^0\right) & \text{N}^2\text{LO} \\ & + \quad \alpha^3 \left(\mathsf{L}^6 + \mathsf{L}^5 + \mathsf{L}^4 + \mathsf{L}^3 + \mathsf{L}^2 + \mathsf{L}^1 + \mathsf{L}^0\right) & \text{N}^3\text{LO} \end{aligned}$$

The α^n term has powers of L up to L^{2n}.

$2n + 1$ terms at order n

Structure of Terms

$$\mathcal{A} = \begin{pmatrix} 1 & & & & \\ \alpha L^2 & \alpha L & \alpha & & \\ \alpha^2 L^4 & \alpha^2 L^3 & \alpha^2 L^2 & \alpha^2 L & \alpha^2 \\ \alpha^3 L^6 & & \ldots & & \\ \vdots & & & & \end{pmatrix}$$

In the leading-log regime $L \sim 1/\alpha$, the various terms are of order

$$\mathcal{A} = \begin{pmatrix} 1 & & & & \\ \frac{1}{\alpha} & 1 & \alpha & & \\ \frac{1}{\alpha^2} & \frac{1}{\alpha} & 1 & \alpha & \alpha^2 \\ \frac{1}{\alpha^3} & & \ldots & & \\ \vdots & & & & \end{pmatrix}.$$

Resummation: Exponentiated Form

Exponentiated form using EFT methods:

$$\log \mathcal{A} = \begin{pmatrix} \alpha L^2 & \alpha L & \alpha & & \\ \alpha^2 L^3 & \alpha^2 L^2 & \alpha^2 L & \alpha^2 & \\ \alpha^3 L^4 & \alpha^3 L^3 & \alpha^3 L^2 & \alpha^3 L & \alpha^3 \\ \alpha^4 L^5 & & \ldots & & \\ \vdots & & & & \end{pmatrix}$$

In the leading-log regime $\alpha L \sim 1$:

$$\log \mathcal{A} = \begin{pmatrix} \frac{1}{\alpha} & 1 & \alpha & & \\ \frac{1}{\alpha} & 1 & \alpha & \alpha^2 & \\ \frac{1}{\alpha} & 1 & \alpha & \alpha^2 & \alpha^3 \\ \frac{1}{\alpha} & & \ldots & & \\ \vdots & & & & \end{pmatrix}.$$

Resummation: Exponentiated Form

$$\begin{aligned}\log \mathcal{A} &= \mathsf{L}\, f_0(\alpha \mathsf{L}) + f_1(\alpha \mathsf{L}) + \alpha\, f_2(\alpha \mathsf{L}) + \ldots \\ &= \frac{1}{\alpha} f_0 + f_1 + \alpha f_2 + \ldots = \frac{1}{\alpha}\left[f_0 + \alpha f_1 + \alpha^2 f_2 + \ldots\right]\end{aligned}$$

so that f_1 and f_2 are corrections to $\log A$.

$$\mathcal{A} = \exp\left[\frac{1}{\alpha} f_0 + f_1 + \alpha f_2 + \ldots\right] = e^{\frac{1}{\alpha} f_0} \times e^{f_1} \times e^{\alpha f_2} \times \ldots$$

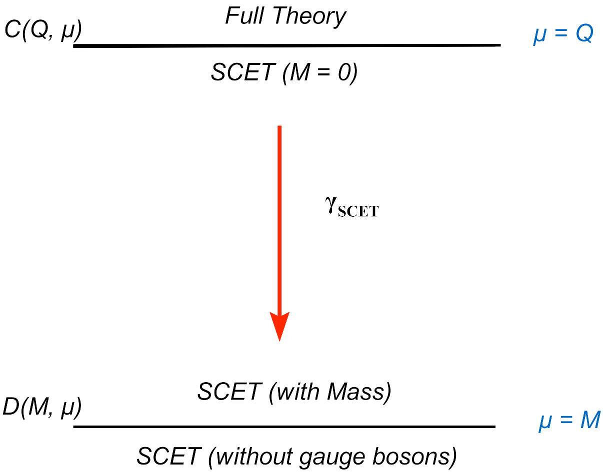
Must include the LL and NLL series. The NLL series is *not* a correction.

LL: one-loop cusp

NLL: two-loop cusp, one-loop non-cusp, one-loop D_1

NNLL: three-loop cusp, two-loop non-cusp and D_1, one-loop C and D_0

Outline of Calculation



SCET Formula

$$\begin{aligned}\log F_E(Q^2) &= C(\alpha(Q)) \\ &+ \int_Q^M \frac{\mathrm{d}\mu}{\mu}\left[A(\alpha(\mu))\log\frac{\mu^2}{Q^2} + B(\alpha(\mu))\right] \\ &+ D_0(\alpha(M)) + D_1(\alpha(M))\log\frac{Q^2}{M^2}\end{aligned}$$

- C: matching at Q
- $A \log \mu^2/Q^2 + B$: SCET anomalous dimension
- $D_0 + D_1 \log Q^2/M^2$: matching at M
- There is a log Q in the matching at M

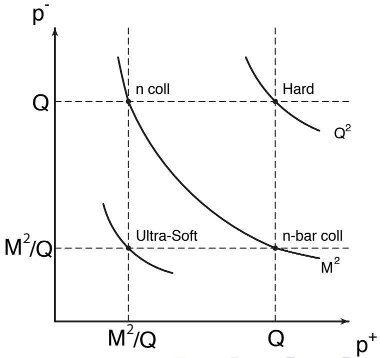
SCET degrees of freedom (modes)

$p^+ = E - p_z, \qquad p^- = E + p_z$

For a particle in the $+z$ direction:

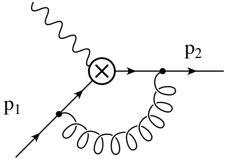
$$p^- \sim 2E, \qquad p^+ \sim \frac{M^2}{2E}$$

- Light Cone Coordinates:
- Hard Modes: $p^2 \sim Q^2$
  integrated out
- Collinear modes: $p^2 \sim M^2$
- Ultra-Soft modes: $p^2 \sim M^4/Q^2$
  do not contribute

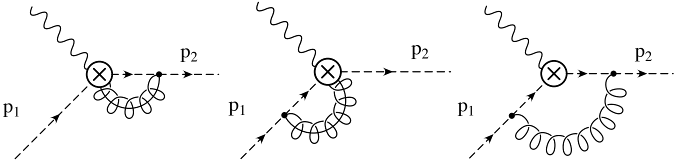


High scale matching: $\mu \sim Q$

- full theory:



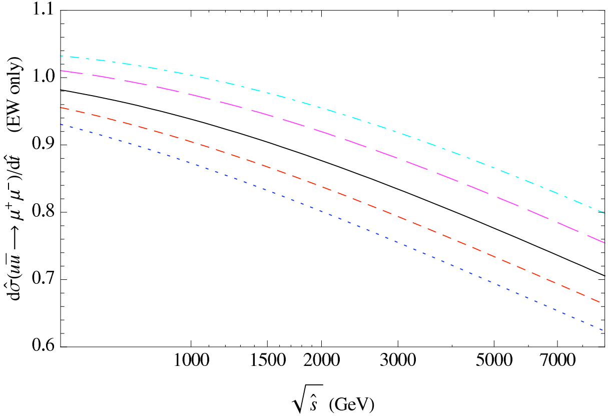
- EFT:



Scattering

The intermediate gauge boson is off-shell by Q^2, and can be shrunk to a point.

Study 4-particle operators in the effective theory.



Electroweak corrections to $u\bar{u} \to \mu^+\mu^-$ as a function of $\sqrt{\hat{s}}$ in GeV

$\hat{t} = -0.2\hat{s}$, (dotted blue)

$\hat{t} = -0.35\hat{s}$ (dashed red)

$\hat{t} = -0.5\hat{s}$ (solid black)

$\hat{t} = -0.65\hat{s}$ (long-dashed magenta)

$\hat{t} = -0.8\hat{s}$ (dot-dashed cyan)

t quark forward backward asymmetry

$q\bar{q} \to t\bar{t}$

Forward: t in direction of q

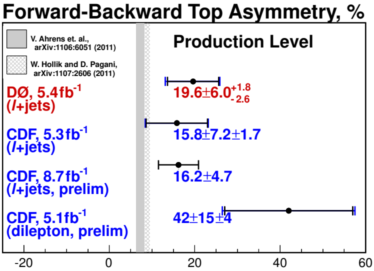
CDF collaboration arXiv:1101.0034

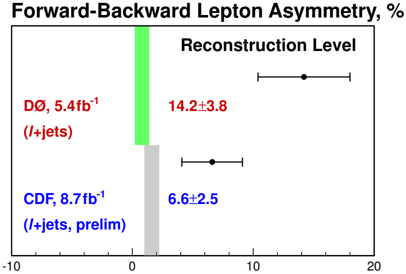
$$A_{FB} = 0.475 \pm 0.114$$

Theory:

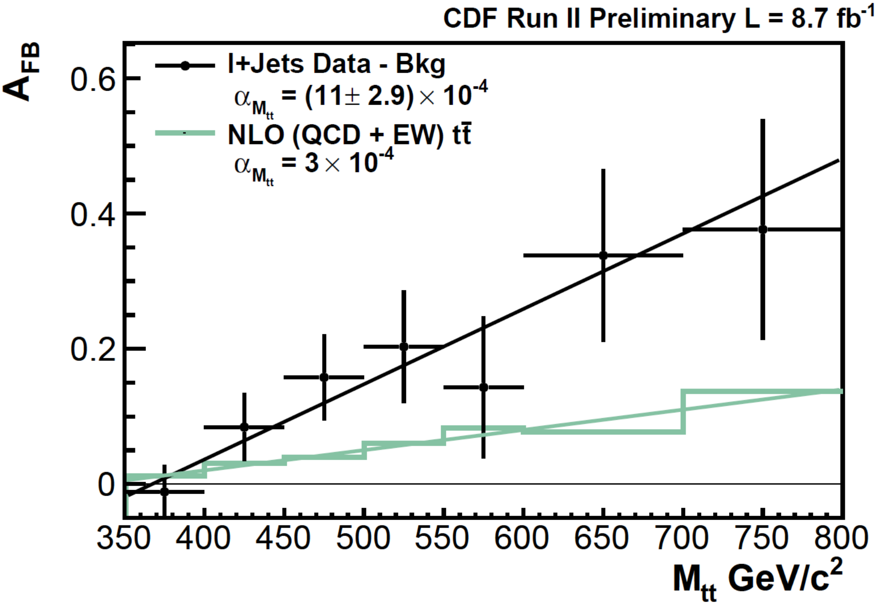
$$A_{FB} = 0.088 \pm 0.013$$

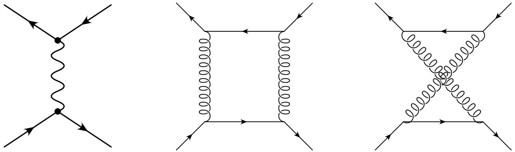
Large discrepancy





- For more information: arXiv:1107.4995


CDF Run II Preliminary L = 8.7 fb^{-1}
A_{FB}
0.6
0.4
0.2
0
l+Jets Data - Bkg
$\alpha_{M_{tt}} = (11 \pm 2.9) \times 10^{-4}$
NLO (QCD + EW) $t\bar{t}$
$\alpha_{M_{tt}} = 3 \times 10^{-4}$
350 400 450 500 550 600 650 700 750 800
M_{tt} GeV/c^2



$$\sigma_{\text{tot}} = \alpha_s^2 + \alpha_s^3 + \ldots$$
$$\sigma_{\text{FB}} = 0 \cdot \alpha_s^2 + \alpha_s^3 + \ldots$$

Tree-level EW gauge boson exchange can produce FB asymmetry because of V-A coupling.

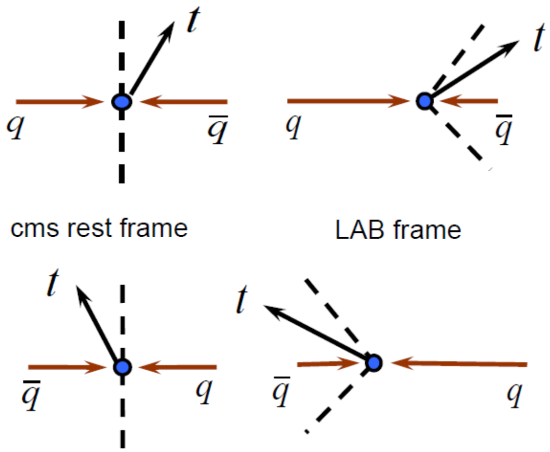
$$A = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

Many models proposed to explain the FB asymmetry. Usually a new production mechanism that is constrained by the total dijet rate.

$$A = \frac{(\sigma_F - \sigma_B)_{SM} + (\sigma_F - \sigma_B)_{NP}}{(\sigma_F + \sigma_B)_{SM} + (\sigma_F + \sigma_B)_{NP}}$$

$$\sigma_{\text{tot}} = (\sigma_F + \sigma_B)_{SM} + (\sigma_F + \sigma_B)_{NP}$$

Tevatron: $p\bar{p}$ LHC: pp



$$A_C = \frac{N(\Delta|y| > 0) - N(\Delta|y| < 0)}{N(\Delta|y| > 0) + N(\Delta|y| < 0)}, \quad \Delta|y| = |y|_t - |y|_{\bar{t}}$$

A. Manohar and M. Trott arXiv:1201.3926

Computed the resummed EW corrections

$$\mathcal{R}_{\rm FB}(t) = \frac{\sigma_{\rm FB}^{\rm QCD+EWS}(t\bar{t})}{\sigma_{\rm FB}^{\rm QCD}(t\bar{t})}, \quad \mathcal{R}_t = \frac{\sigma_{t\bar{t}}^{\rm QCD+EW}}{\sigma_{t\bar{t}}^{\rm QCD}}.$$

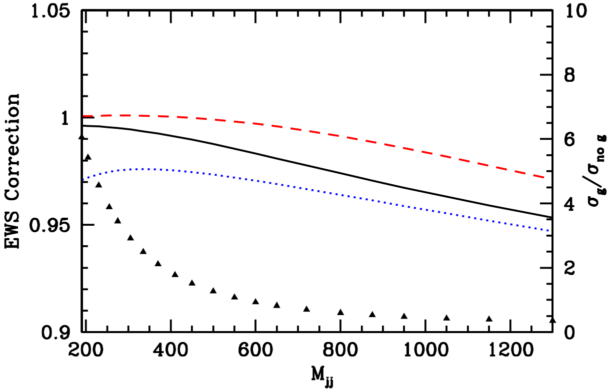
Bin [GeV]	$A_{\rm FB}^{t\bar{t}}(\%)$		$\mathcal{R}_{\rm FB}(t)$	$\mathcal{R}_t$
$[2\,m_{t\bar{t}}, 1960]$	7.7	7.5	1.02	0.98
$[2\,m_{t\bar{t}}, 450]$	5.6	5.4	1.02	0.98
$[450, 900]$	11	12	1.02	0.97

Bin[GeV]	$A_{\rm FB}^{b\bar{b}}$ (%)		$\mathcal{R}_{\rm FB}(b)$	$\mathcal{R}_b$	$A_{\rm FB}^{c\bar{c}}$ (%)		$\mathcal{R}_{\rm FB}(c)$	$\mathcal{R}_c$
$[50, 1960]$	0.4	0.4	1.06	0.99	0.3	0.3	0.99	0.99
$[50, 350]$	0.4	0.4	1.06	0.99	0.3	0.3	0.98	0.99
$[350, 650]$	8.1	7.8	1.00	1.00	6.7	6.6	1.04	1.00
$[650, 950]$	20	17	0.97	0.98	18	16	1.06	0.99

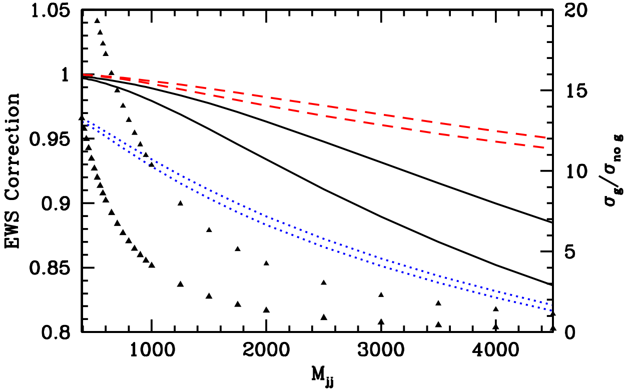
LHC

7 and 14 TeV

Bin [GeV]	$\mathcal{R}_t$		$\mathcal{R}_b$		$\mathcal{R}_c$	
$[50, 3000]$	–	–	0.99	0.99	0.99	0.99
$[350, 3000]$	0.97	0.97	–	–	–	–
$[50, 250]$	–	–	0.99	0.99	0.99	0.99
$[250, 500]$	–	–	1.00	1.00	1.00	1.00
$[350, 500]$	0.98	0.98	–	–	–	–
$[500, 750]$	0.97	0.97	0.99	0.99	0.99	0.99
$[750, 1000]$	0.95	0.95	0.98	0.98	0.98	0.98
$[1000, 1500]$	0.94	0.94	0.97	0.97	0.96	0.96
$[1500, 2000]$	0.92	0.92	0.95	0.95	0.95	0.95
$[2000, 2500]$	0.90	0.91	0.93	0.94	0.93	0.93
$[2500, 3000]$	0.88	0.89	0.92	0.93	0.92	0.92
$[3000, 3500]$	0.87	0.88	0.90	0.91	0.91	0.91



EWS correction (left axis) to the Tevatron dijet spectrum as a function of dijet invariant mass (solid black). Also shown are the corrections to dijet processes involving external gluons (red dashed), and no external gluons (blue dotted). The black triangles are the ratio of cross sections (right axis) with and without external gluons.



LHC at $\sqrt{s} = 7$ TeV (lower curves) and 14 TeV (upper curves).

Large change in the dijet rate.

Summary

- EW corrections are now largest source of uncertainties in current calculations.
- Have a formalism that can compute them all in the regime where they are important
- Working the C. Bauer to include them in the GENEVA code
- Should be able to do it in a way that others can use it
- A universal formula in the regime where they are important (high energy)