Quark and Lepton Masses from Top Loops

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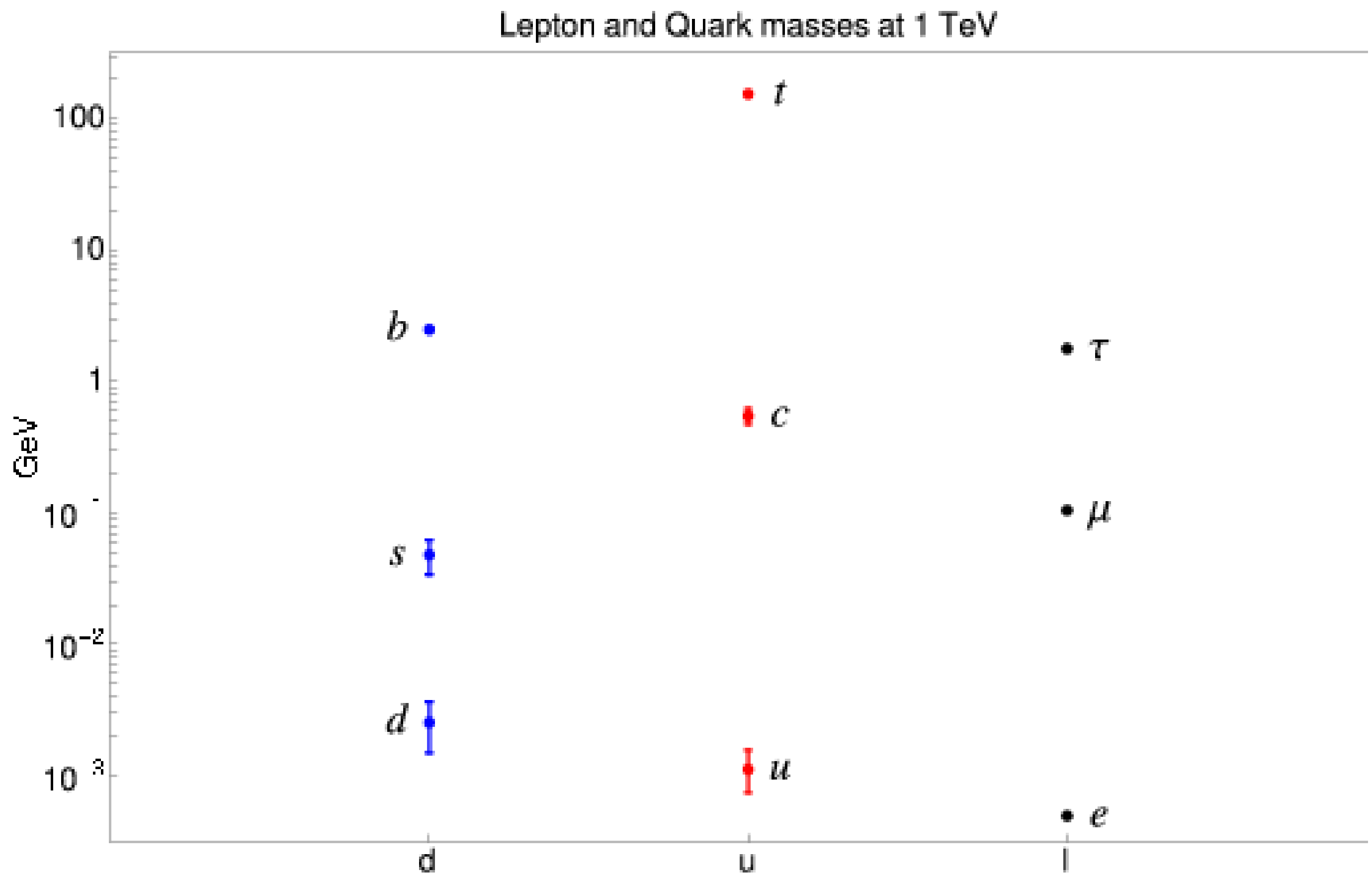
Work with Paddy Fox

Quark and lepton masses at the 1 TeV scale:



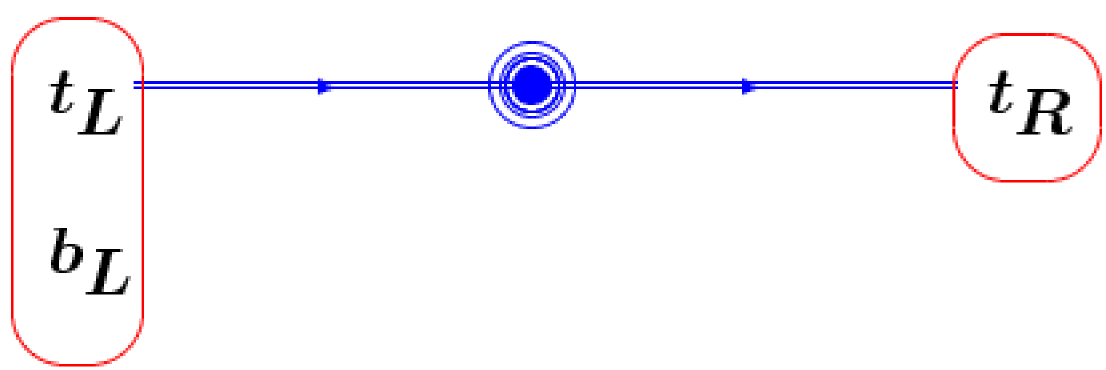
How is electroweak symmetry breaking communicated to the fermions?

All Standard Model fermions are chiral.

The two top quarks:

- "left-handed" top *(feels the weak interaction)*
- "right-handed" top *(no interaction with $W^{\pm}$)*

Top mass: t_L turns into t_R and vice-versa

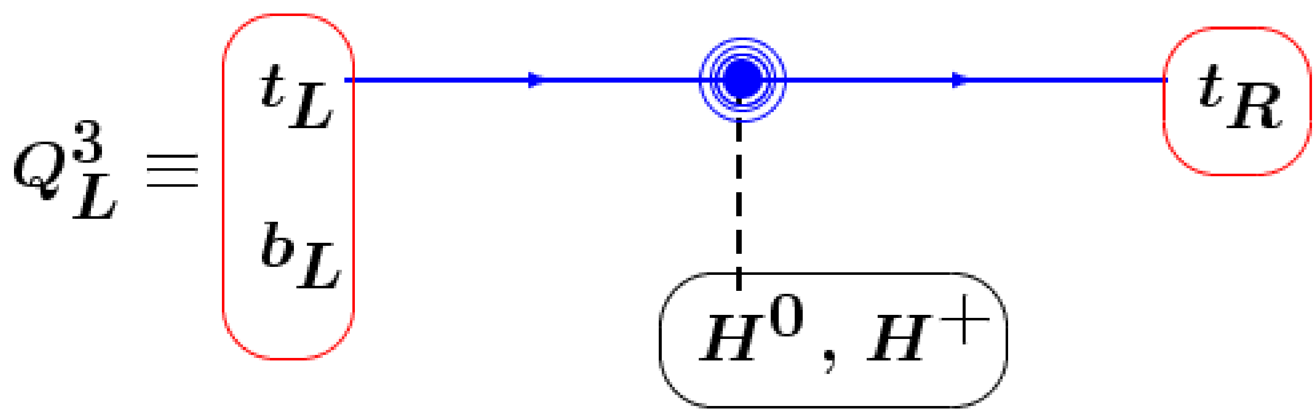


All Standard Model fermions are chiral.

Top quark gets a mass from its interaction with the vacuum:

$$y_t\, \bar{t}_R \langle H^0 \rangle t_L \;, \qquad \langle H^0 \rangle \approx 174 \text{ GeV}$$

Measured top mass $\Rightarrow$ top Yukawa coupling is $y_t \approx 1$.



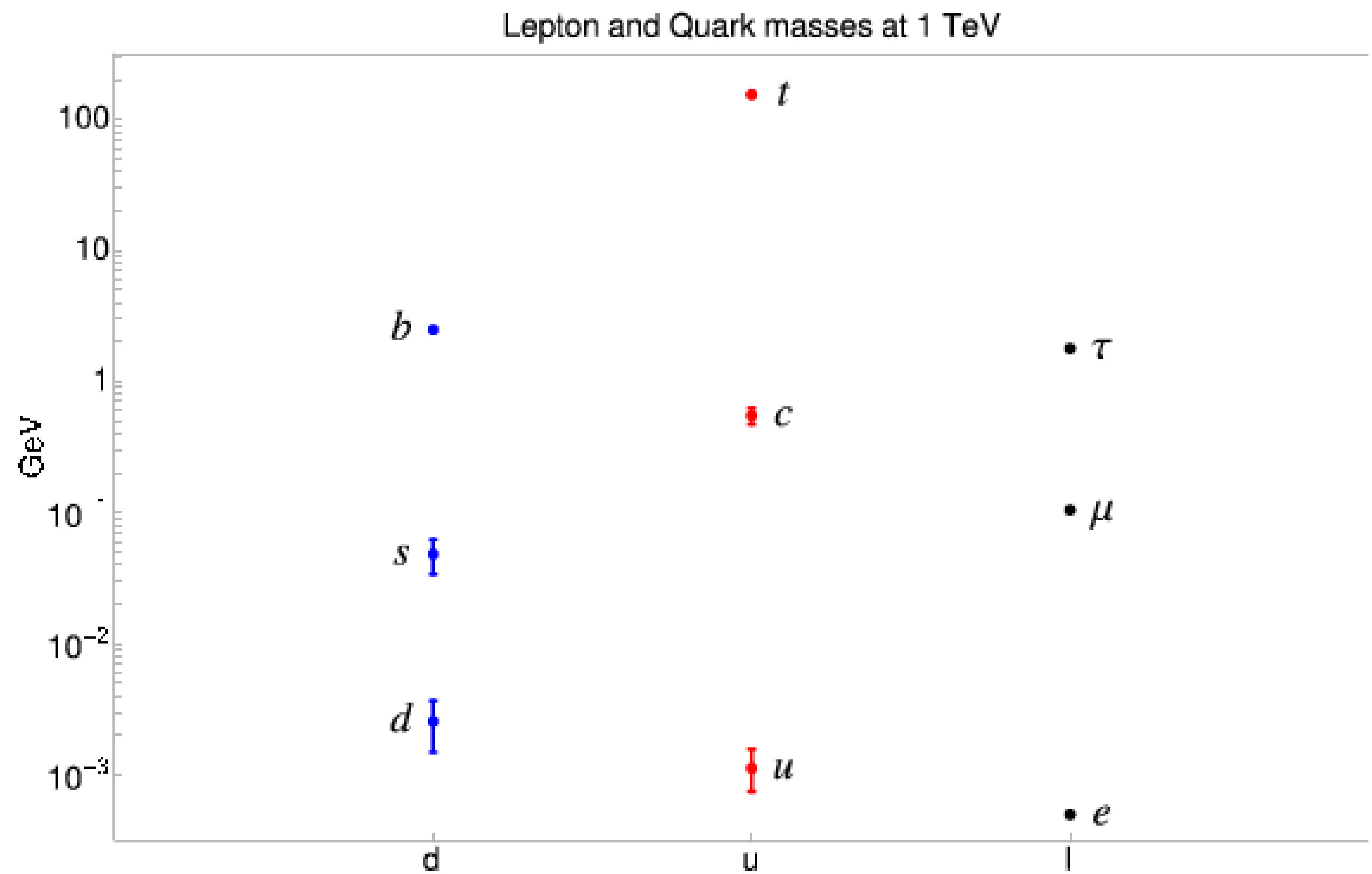
Many attempts at explaining the hierarchy of Yukawa couplings:

- **discrete symmetries** $\longrightarrow (\langle\phi\rangle/M)^n$ **suppressions.**
- **GUT relations.**
- **wave function overlaps in extra dimensions.**
- ...
- **loop suppressions:**

  Georgi, Glashow, 1972 – **attempts to calculate the electron mass as a one loop contribution involving the muon mass.**

  Many papers in the 1980's (*e.g., Balakrishna, Kagan, Mohapatra, 1988*)

  Most ambitious schemes: *3rd generation masses at tree level,*
2nd generation masses at one loop,
1st generation masses at two loops.



Let us assume that only the top quark gets its mass at tree level, $y_t\,\bar{t}_R Q^3_L\,H$, and introduce some interactions that communicate EWSB to the other quarks and leptons.

$\tilde{r}$: scalar field transforming as (3,2,+7/6) under

$$SU(3)_c \times SU(2)_W \times U(1)_Y$$

Most general renormalizable interactions with SM fermions

$$\lambda_{ij}\,\tilde{r}\,\overline{u}^i_R L^j_L + \lambda'_{ij}\,\tilde{r}\,\overline{Q}^i_L e^j_R \qquad (\tilde{r} \text{ is a leptoquark})$$

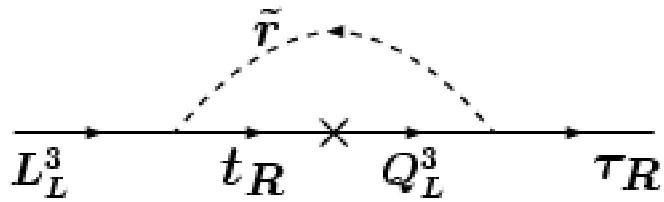
break explicitly the chiral symmetries of the

quarks $U(2)_Q \times U(2)_u \times U(1)_t \times U(3)_d \rightarrow U(1)_u \times U(3)_d$

and leptons $U(3)_L \times U(3)_e \rightarrow U(1)_L$

$\Rightarrow$ all up-type quarks and electrically-charged leptons get masses at some loop level.

The one-loop diagram responsible for the tau mass.



Only one linear combination of L_L^j couples to t_R
$\Rightarrow m_\mu = m_e = 0$ at 1-loop

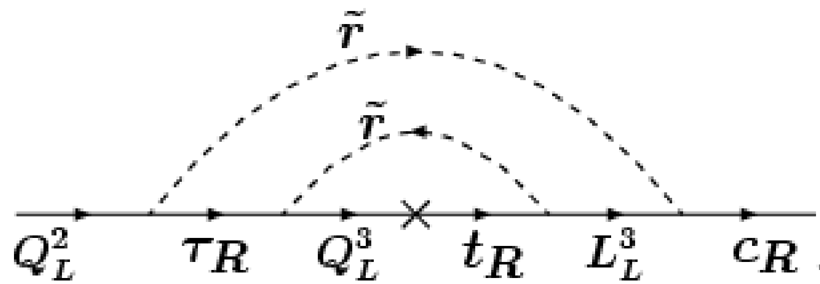
Only one linear combination of e_R^j couples to Q_L^3

$$m_\tau \simeq \lambda_{33}\lambda'_{33}\, m_t \frac{N_c}{16\pi^2} \ln\left(\frac{\Lambda^2}{M_{\tilde{r}}^2}\right)$$

Some new physics cuts off the loop integral at a scale Λ:
a superpartner of $\tilde{r}$, or some dynamics if $\tilde{r}$ is a composite particle.

m_τ *depends only on the ratio* $\Lambda/M_{\tilde{r}}$
(only a lower limit on $M_{\tilde{r}}$ *may be derived from phenomenology).*

Charm mass induced by a two-loop "rainbow" diagram:



Only one linear combination of Q_L^1 and Q_L^2 couples to $\tau_R \Rightarrow m_u = 0$ at 1-loop

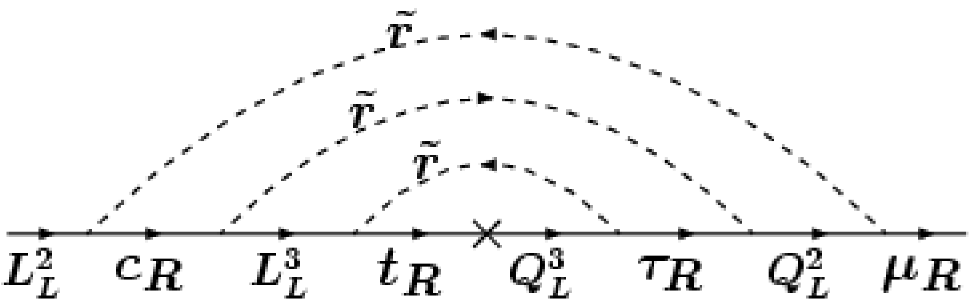
Only one linear combination of u_R^1 and u_R^2 couples to L_L^3

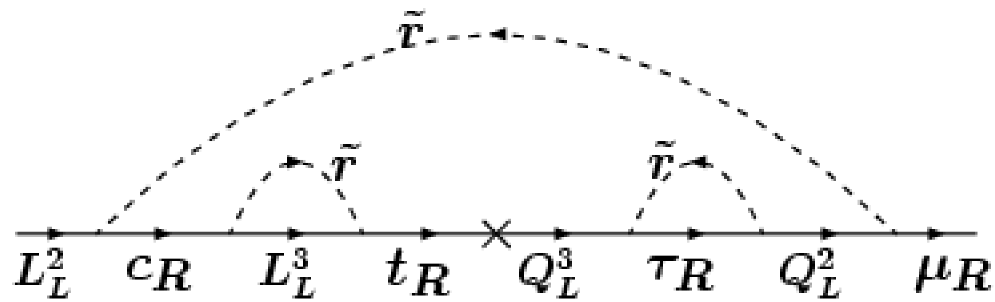
$$m_c \simeq \lambda'_{23}\lambda_{23}\, m_\tau \frac{1}{16\pi^2} \ln\left(\frac{\Lambda^2}{M_{\tilde{r}}^2}\right)$$

Assuming that there are no other contributions to m_c,

the m_c/m_τ *ratio at 1 TeV requires* $\lambda_{23}\lambda'_{23} \approx (3.3)^2$ *for* $\Lambda \approx 10 M_{\tilde{r}}$.

Muon mass induced by 3-loop "rainbow" and "bug" diagrams:



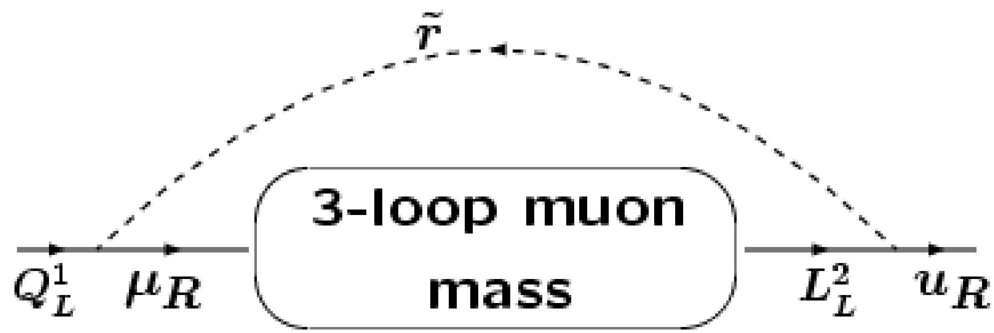


$$m_\mu \simeq \lambda'_{22}\lambda_{22}\, m_c\, (1+x)\, \frac{N_c}{16\pi^2} \ln\left(\frac{\Lambda^2}{M_{\tilde{\tau}}^2}\right)$$

m_μ/m_c **ratio requires** $\lambda_{22}\lambda'_{22}(1+x) \approx (1.5)^2$

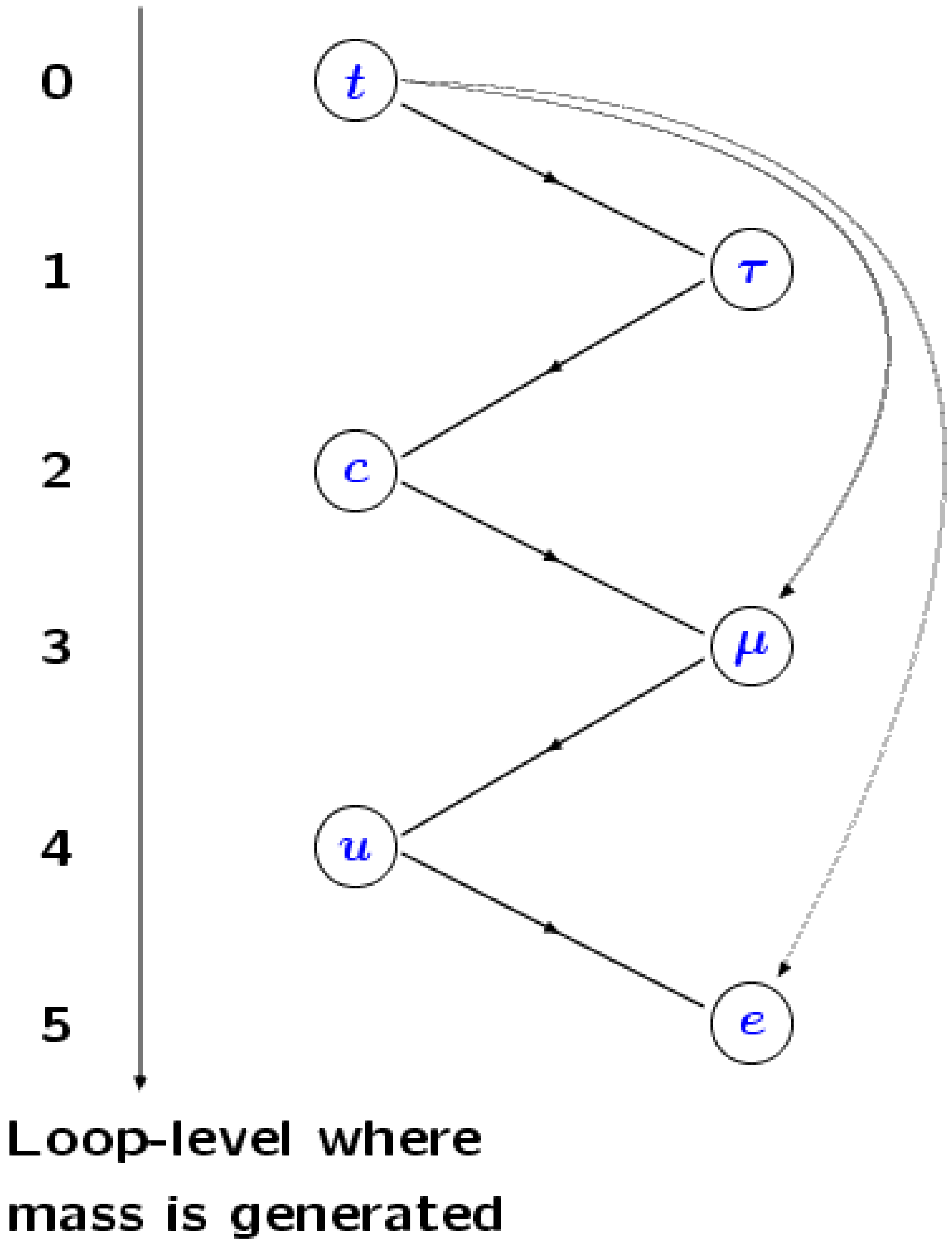
At 3-loops the electron is still massless!

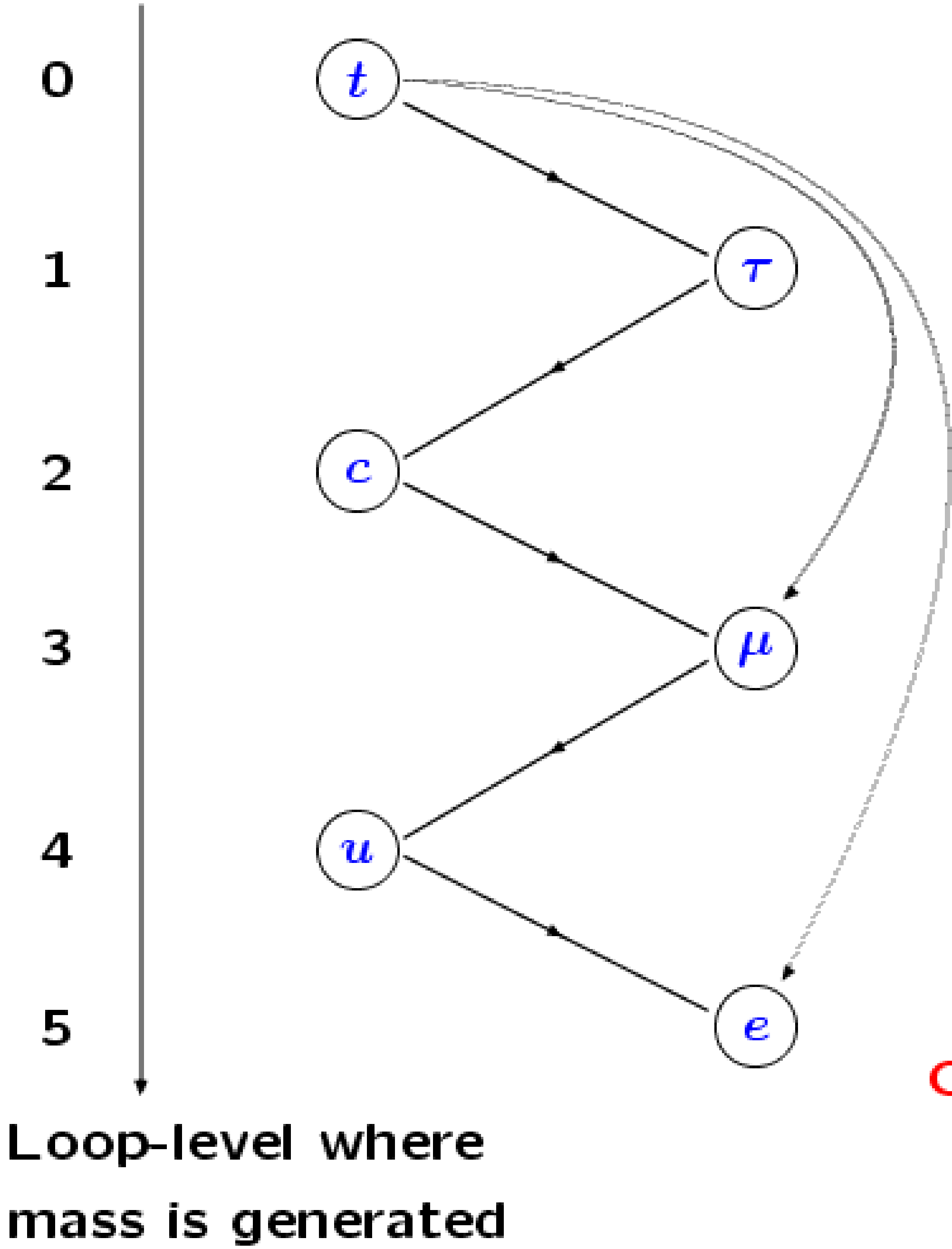
Up-quark mass induced at 4-loops:

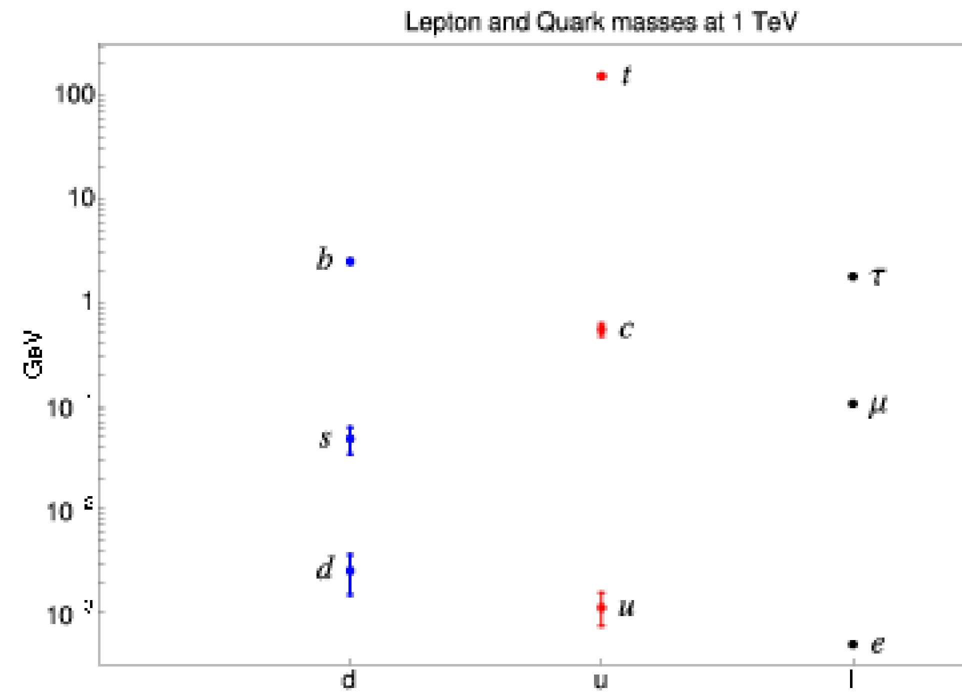


$$m_u \simeq \lambda'_{12}\lambda_{12}\, m_\mu \frac{1}{16\pi^2} \ln\left(\frac{\Lambda^2}{M_{\tilde{r}}^2}\right)$$

Correct m_u/m_μ ratio requires $\lambda_{12}\lambda'_{12} \approx (0.6)^2$


0
1
2
3
4
5
t
τ
c
μ
u
e
Loop-level where
mass is generated





Only input: $\lambda_{ij}\,\tilde{r}\,\overline{u}^i_R L^j_L + \lambda'_{ij}\,\tilde{r}\,\overline{Q}^i_L e^j_R$ and m_t.

Down-type quark masses

$\tilde{d}$: **scalar field with charges** $(3, 1, -1/3)$ **under** $SU(3) \times SU(2) \times U(1)$.

Renormalizable couplings to quarks: $\kappa_{ij}\, \tilde{d}\, \overline{Q}^{ci}_L Q^j_L + \kappa'_{ij}\, \tilde{d}\, \overline{d}^{ci}_R u^j_R$

m_b **induced by a 1-loop rainbow diagram.**

m_s **induced by 3-loop rainbow and bug diagrams.**

m_d **induced by a mixed 4-loop diagram involving two** $\tilde{d}$ **lines and two** $\tilde{r}$ **lines.**

Similar 4-loop mixed diagram (with three $\tilde{r}$ *and one* $\tilde{d}$*) contributes to the electron mass.*

Phenomenological constraints

Tree level flavor-changing processes induced by $\tilde{r}$ exchange:

$\mu \to e$ conversion

$K^+ \to \pi^0 \mu^+ e^-$, ...

$\tau^+ \to K^0 e^+$, ...

$\pi^+ \to e^+ \nu$ versus $\pi^+ \to \mu^+ \nu$

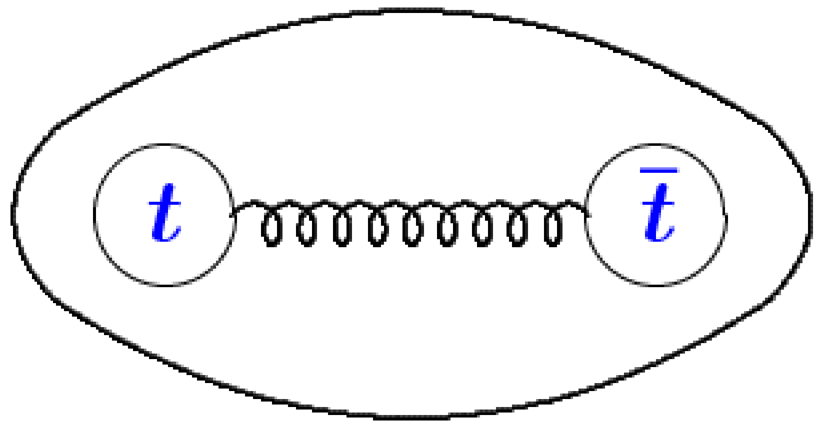
...

$\Rightarrow \quad M_{\tilde{r}} > O(5 - 50)$ TeV.

Top condensation $\Rightarrow$ Higgs boson is a $\bar{t}t$ bound state!

(Nambu; Miransky, Tanabashi, Yamawaki; Bardeen, Hill, Lindner, ...)

Binding may be due to some strongly-interacting heavy gauge bosons



New heavy quarks (vectorlike) could bring scale of Higgs compositeness down to a few TeV.

Explicit models: top seesaw, QCD in extra dimensions, ...

Conclusions

- **Quark and lepton masses follow intriguing patterns.**

- **If EWSB is communicated from the top quark to the other fermions by couplings to some new scalar fields, then realistic mass spectra are induced by multi-loop contributions.**

- **Searches for new flavor-changing processes may unravel the origin of quark and lepton masses.**

Work done with Paddy Fox at Fermilab.