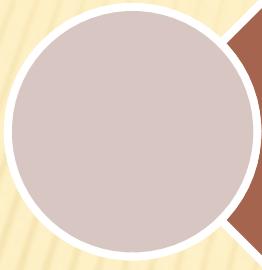


Darren Forde
(SLAC & UCLA)

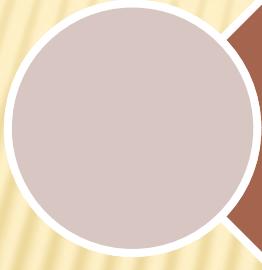
HIGHER ORDER QCD CALCULATIONS

OVERVIEW



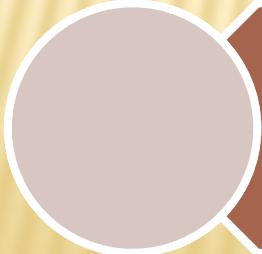
NLO amplitudes using Feynman diagram techniques

- The limitations.
- “State of the art” results.



New techniques required

- Unitarity approaches.
- Results.
- Progress towards automation.



Automated one-loop amplitude calculations.

- *BlackHat*

WHAT'S THE PROBLEM?

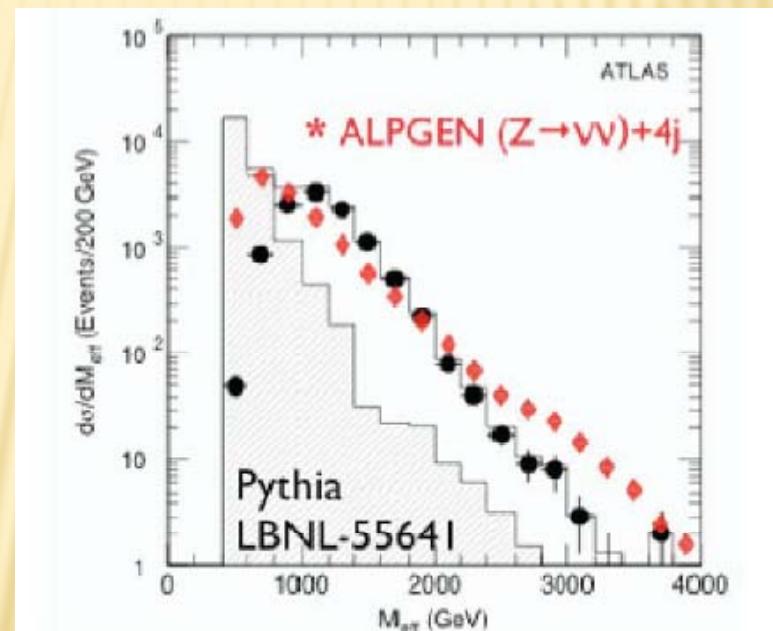
- ✖ QCD amplitudes are needed to understand the results from colliders– Tevatron and LHC (2008).



- ✖ High multiplicity events.

NLO CALCULATIONS

- ✖ Probe beyond the Standard Model,
 - + Signals in discovery channels can be close to backgrounds.



SUSY search:
Early ATLAS TDR
using PYTHIA.

- ✖ Maximise discovery potential
 - ⇒ Precise understanding of background processes.

WHAT DO WE NEED?

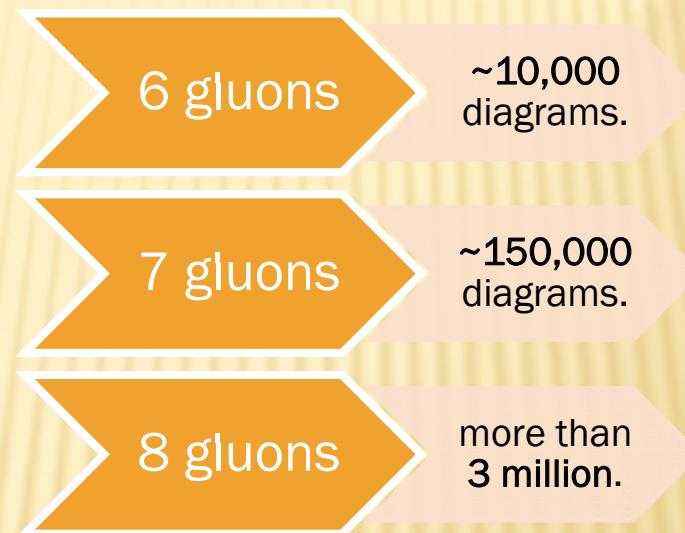
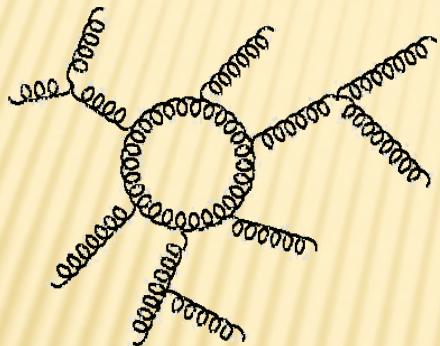
- ✖ One-loop high multiplicity processes.
“Famous” Les Houches list, (2005)

process ($V \in \{Z, W, \gamma\}$)	background to
1. $pp \rightarrow VV$ jet	$t\bar{t}H$, new physics
2. $pp \rightarrow H + 2$ jets	H production by vector boson fusion (VBF)
3. $pp \rightarrow t\bar{t} b\bar{b}$	$t\bar{t}H$
4. $pp \rightarrow t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \rightarrow VV b\bar{b}$	$VBF \rightarrow H \rightarrow VV, t\bar{t}H$, new physics
6. $pp \rightarrow VV + 2$ jets	$VBF \rightarrow H \rightarrow VV$
7. $pp \rightarrow V + 3$ jets	various new physics signatures
8. $pp \rightarrow VVV$	SUSY trilepton

Five, six or more legs in the loop.

WHAT'S THE HOLD UP?

- ✖ Calculating using Feynman diagrams is **Hard!**



- ✖ Difficulty increases greatly with the number of jets.
- ✖ Known results much **simpler** than would be expected!

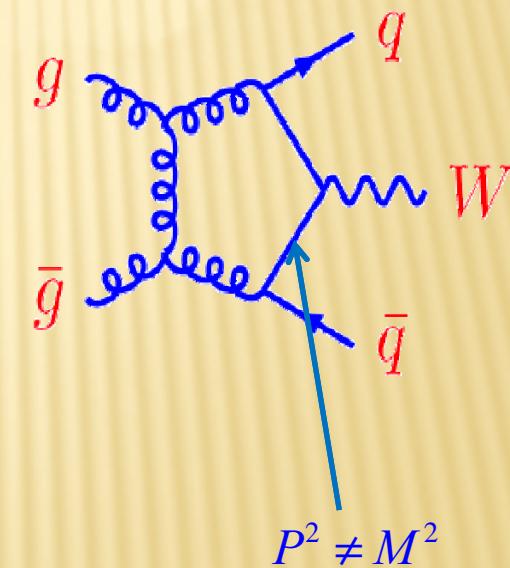
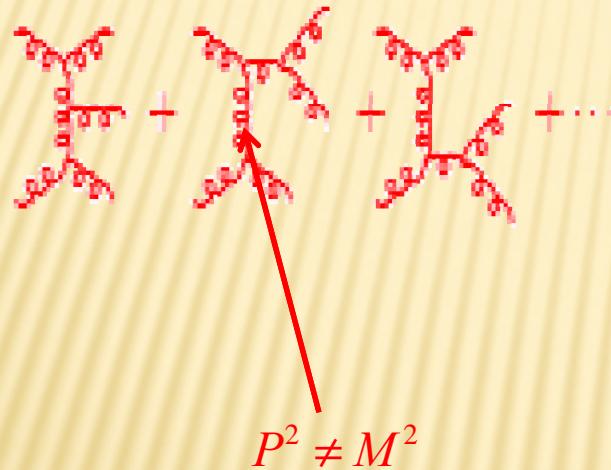
(Park, Taylor)

$$A_n = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle (n-1)n \rangle \langle n1 \rangle}$$

A Feynman diagram for a process involving n gluons. A central circle labeled A_n has four external gluon lines. The top-left line is labeled i^- , the top-right $+$, the bottom-left $+$, and the bottom-right j^- . Ellipses between the top and bottom lines indicate intermediate gluons.

ISSUES WITH FEYNMAN DIAGRAMS

- ✗ Feynman diagrams are gauge dependent objects.
- ✗ Built up from off-shell quantities.



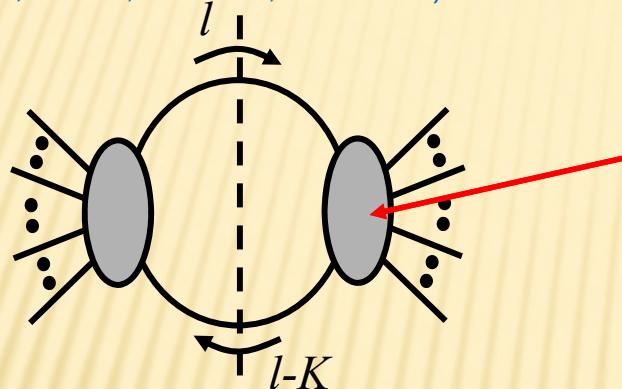
- ✗ Want an approach that utilises more symmetries.
- ✗ Work with on-shell quantities, i.e. amplitudes rather than Feynman diagrams.

“STATE OF THE ART” FEYNMAN DIAGRAMS

- ✖ Using analytic and numerical techniques
 - + QCD corrections to vector boson pair production (W^+W^- , $W^\pm Z$ & ZZ) via vector boson fusion (VBF). (Jager, Oleari, Zeppenfeld)+(Bozzi)
 - + QCD and EW corrections to Higgs production via VBF. (Ciccolini, Denner, Dittmaier)
 - + $pp \rightarrow WW + j + X$. (Campbell, Ellis, Zanderighi). (Dittmaier, Kallweit, Uwer)
 - + $pp \rightarrow \text{Higgs} + 2 \text{ jets}$. (Campbell, Ellis, Zanderighi), (Ciccolini, Denner, Dittmaier).
 - + $pp \rightarrow ZZZ$, $pp \rightarrow t\bar{t}H$. (Lazopoulos, Petriello, Melnikov)
 - + $gg \rightarrow gggg$ amplitude, 1 point in 9 seconds. (Ellis, Giele, Zanderighi)
 - + 6 photons (Nagy, Soper), (using other approaches (Ossola, Papadopoulos, Pittau), (Binoth, Heinrich, Gehrmann, Mastrolia))

UNITARITY CUTTING TECHNIQUES

- Basic idea, glue together **tree** amplitudes to form loops.
(Bern,Dixon,Dunbar,Kosower)



On-shell tree amplitudes
Use compact tree amplitudes
⇒ Compact loop result

- Cut propagators replaced with delta function,

$$\frac{1}{(l-K_i)^2 + i\epsilon} \rightarrow (2\pi) \delta((l-K_i)^2)$$

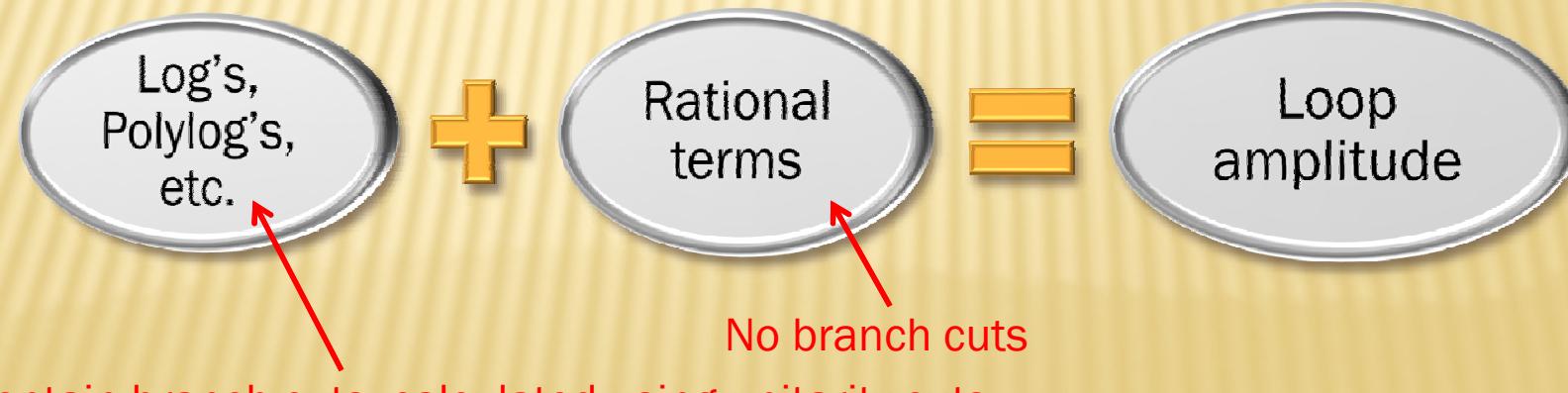
- Reconstruct both real and imaginary pieces of the amplitude due to the underlying Feynman diagram representation.

ONE-LOOP INTEGRAL BASIS

- A one-loop amplitude decomposes into Rational terms

$$R_n + r_\Gamma \frac{(\mu^2)^\varepsilon}{(4\pi)^{2-\varepsilon}} \left(\sum_i b_i \text{ (1-loop scalar integral)} + \sum_{ij} c_{ij} \text{ (triangle integral)} + \sum_{ijk} d_{ijk} \text{ (square integral)} \right)$$

- The analytic structure of a one-loop amplitude



RATIONAL TERMS

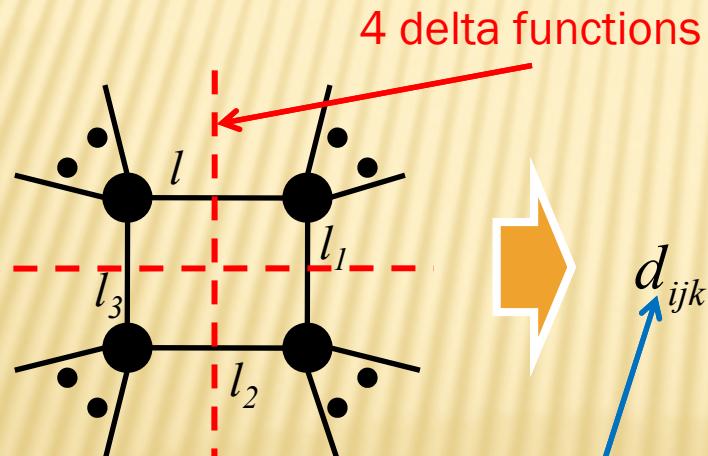
- ✖ Unitarity tells us about objects with branch cuts.
- ✖ Cannot compute the rational terms when cut legs are in $D=4$ dimensions. ([Bern, Dixon, Dunbar, Kosower](#))
 - + Only “Cut-constructible” terms computed from unitarity.
 - + Rational terms need a separate technique.
- ✖ Rational terms multiplied by $(-s)^{-\varepsilon}$ when cut legs are in $D=4-2\varepsilon$
 - + Rational terms now contain branch cuts from $\log(-s)$.
 - + Requires trees in $D=4-2\varepsilon$, more complicated than in $D=4$.

UNITARITY IN $D=4-2\epsilon$

- ✖ Relate cut momentum in $D=4-2\epsilon$ to cut momentum in $D=4$ and an extra integral over μ a vector in -2ϵ . (Anastasiou, Britto, Kunszt, Mastrolia)
 - + Internal massive legs computed with a similar approach. (Britto, Feng)
- ✖ Coefficients found by rewriting a two-particle cut integrand and identifying bubbles, triangles and boxes term separately. (Britto, Cachazo, Feng) (Britto, Feng)
 - + Used in $D=4$ to compute the remaining unknown 6 gluon cut pieces $A_6(-+-+-+)$, $A_6(--++-+)$ (Britto, Feng, Mastrolia)
 - + Recently used to produce a general condensed formula for bubble and triangle coefficients. (Britto, Feng)
- ✖ 6 photon amplitude. (Binoth, Heinrich, Gehrmann, Mastrolia), (Nagy, Soper), (Ossola, Papadopoulos, Pittau)

GENERALIZED UNITARITY

- ✖ Cut more than two legs in the loop.
 - + Gives the leading singularity, again directly related to the full amplitude. (Bern, Dixon, Kosower) (Britto, Cachazo, Feng)
- ✖ Quadruple cuts freeze the integral \Rightarrow boxes (Britto, Cachazo, Feng)



Multiplies a scalar Box Integral

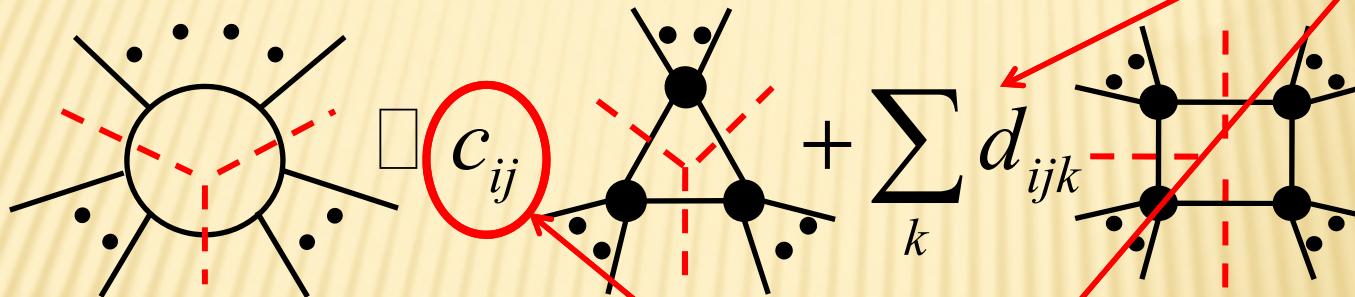
In 4 dimensions 4 integrals
 $\Rightarrow l^2 = 0, l_1^2 = 0, l_2^2 = 0, l_3^2 = 0 \Rightarrow l = l_{ijk}$

$$d_{ijk} = \frac{1}{2} \sum_{a=1}^2 A_1(l_{ijk;a}) A_2(l_{ijk;a}) A_3(l_{ijk;a}) A_4(l_{ijk;a})$$

Rational coefficient

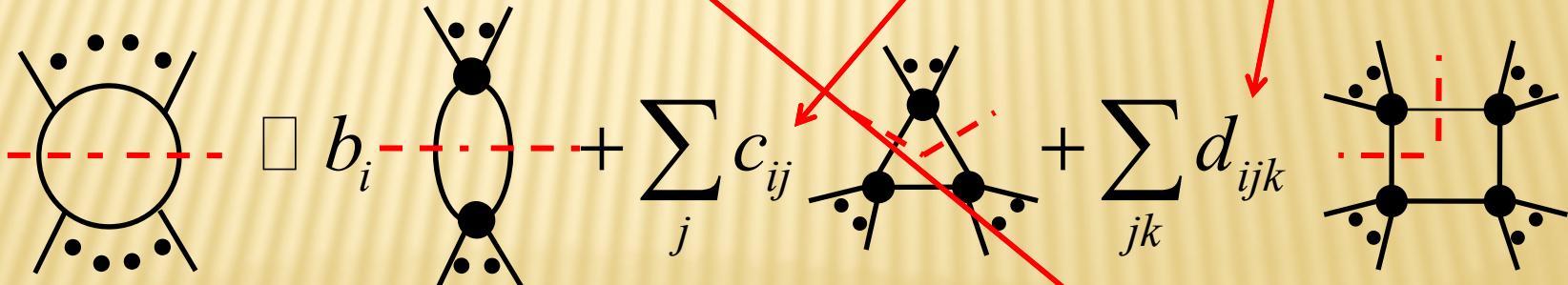
TWO-PARTICLE AND TRIPLE CUTS

- What about bubble and triangle terms?
- Triple cut \Rightarrow Scalar triangle coefficients?



Additional coefficients

- Two-particle cut \Rightarrow Scalar bubble coefficients?



Isolates a single triangle

- Disentangle these coefficients.

DISENTANGLING COEFFICIENTS

- ✖ Approaches,
 - + Unitarity technique, (Bern, Dixon, Dunbar, Kosower)
 - + MHV vertex techniques, (Bedford, Brandhuber, Spence, Traviglini), (Quigley, Rozali)
 - + Unitarity cuts & integration of spinors, (Britto, Cachazo, Feng) + (Mastrolia) + (Anastasiou, Kunszt)
 - + Recursion relations, (Bern, Bjerrum-Bohr, Dunbar, Ita)
 - + Solving for coefficients, (Ossola, Papadopoulos, Pittau), (Ellis, Giele, Kunszt)
 - + Large loop momentum behaviour. (DF), (Kilgore)
- ✖ Finding all coefficients \Rightarrow “cut-constructible” part of amplitude.
 - + All one-loop scalar integrals are known. (Ellis, Zanderighi)

THE OPP METHOD

- Write one-loop integral as (Ossola, Papadopoulos, Pittau)

Unconstrained parameters in triangles & bubbles

$$\frac{d_0 + d_0(n \cdot l_{ijk})}{D_0 D_1 D_2 D_3} + \frac{c_0 + t^{-3} c_{-3} + \dots - t c_3 + b_{00} + t^{-2} b_{0-2} + \dots + y t^2 b_{12}}{D_0 D_1 D_2}$$

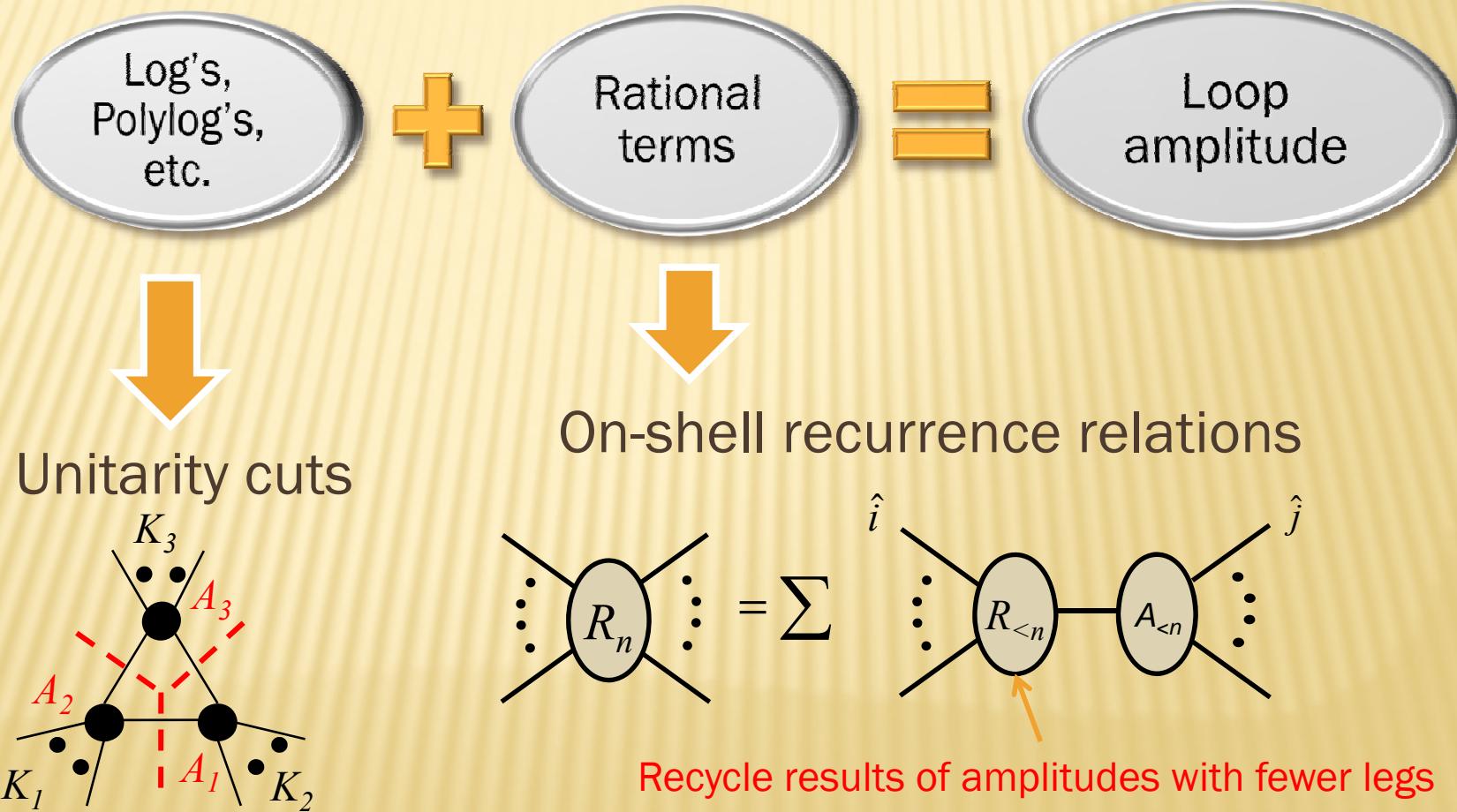
- Apply cuts for boxes then triangles and then bubbles
 - Solve for the coefficients of these sets of equations hierarchically.
 - Want the scalar integral coefficients d_0 , c_0 , and b_{00} .
- OPP have implemented this in a program called **CutTools**
 - Used to compute the complete 6 photon amplitude. (Nagy, Soper), (Binoth, Heinrich, Gehrmann, Mastrolia), (for scalar QED (Bernicot, Guillet))
- Technique separately implemented by Ellis, Giele, Kunszt to compute the gluon $A_6(--++++)$ amplitude, (10,000 points in 107s, 900 times faster than Feynman diagram approach).

RATIONAL TERMS

- ✖ Rational terms are harder to get.
- ✖ Rational terms in OPP generated by working in $D=4-2\varepsilon$,
 - + Introduces powers of μ^2 in the numerators \Rightarrow relate these to the rational terms.
- ✖ Related to similar approaches using the UV behaviour of the amplitude.
 - + Rational terms of the gluonic $A_6(-+-+-+)$, $A_6(--++-+)$ and other 6-point amplitudes. ([Xiao, Yang, Zhu](#))
 - + 6 photon rational terms vanish. ([Binoth, Heinrich](#)) + ([Gehrmann, Mastrolia](#))

THE UNITARITY BOOTSTRAP

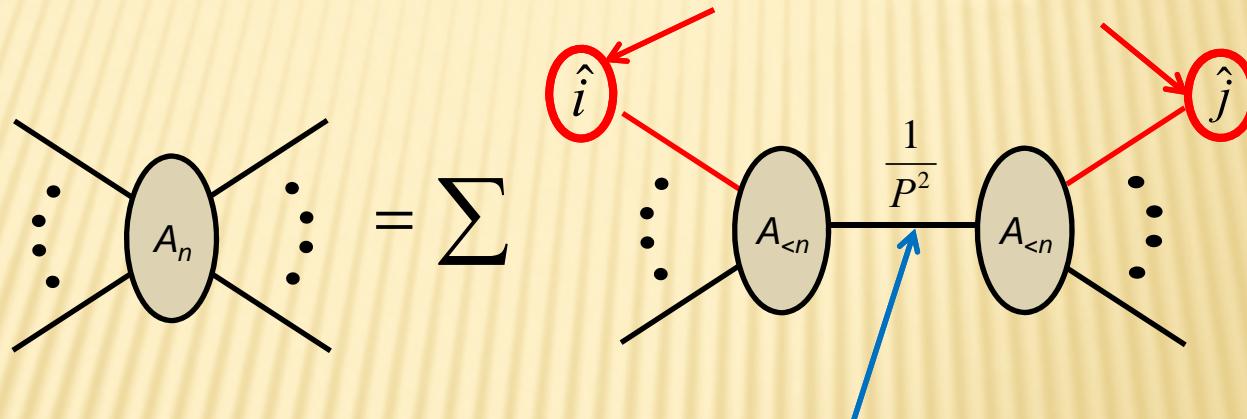
- Use the most **efficient** approach for each piece,



ON-SHELL RECURSION RELATIONS

- Recursion using **on-shell** amplitudes with fewer legs,

Two reference legs “shifted” by a complex parameter z

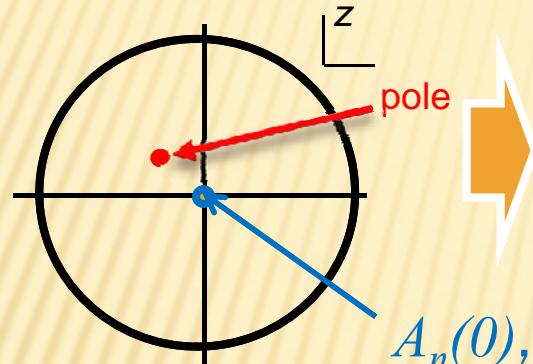


Intermediate momentum leg is on-shell.

- Shift keeps both legs **on-shell** and **conserves momentum**. Requires **complex** momentum.
- Final result independent of choice of shift.
- Complete amplitude at tree level. (Britto, Cachazo, Feng) + (Witten)

A SIMPLE IDEA

- Function of a **complex** variable containing only simple poles



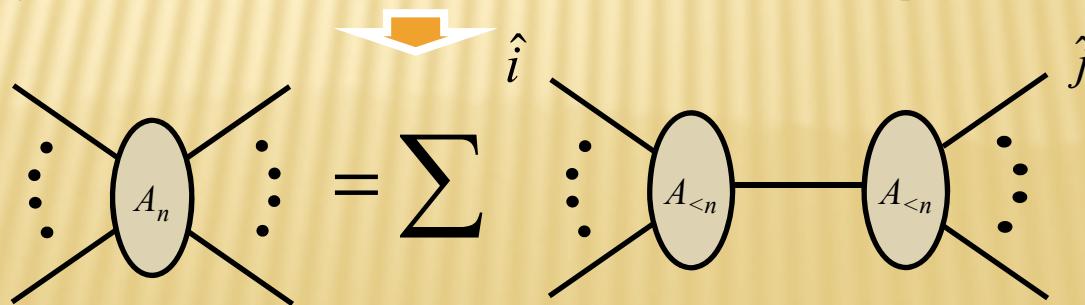
$$\frac{1}{2i\pi} \oint_C dz \frac{A_n(z)}{z} = 0$$

$$A_n(0) = - \sum_{\text{poles}} \text{Res}_z \frac{A_n(z)}{z}$$

$A_n(0)$, the amplitude with real momentum

- Position of poles from **complex** factorisation of the amplitude.

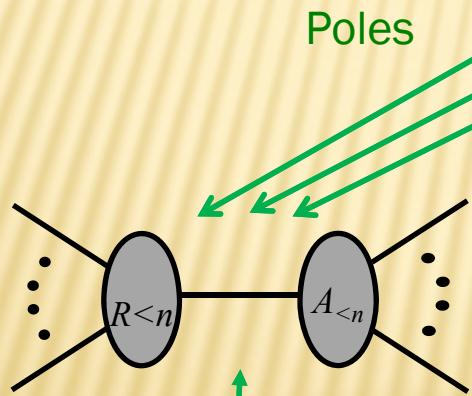
$$A_n \xrightarrow{P^2 \rightarrow 0} \sum_{i \in L, j \in R} A_L(..., i, ..., P) \frac{1}{P^2} A_R(..., j, ..., P) \rightarrow - \sum_{\text{poles}} \text{Res} \frac{A_n(z)}{z}$$



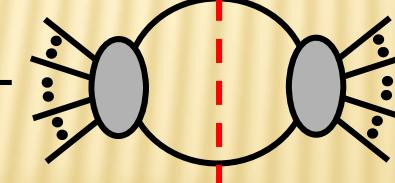
BRANCH CUTS

- ✖ What about loops?
- ✖ Rational part of one-loop amplitude.
(Bern, Dixon, Kosower) + (Berger, DF)

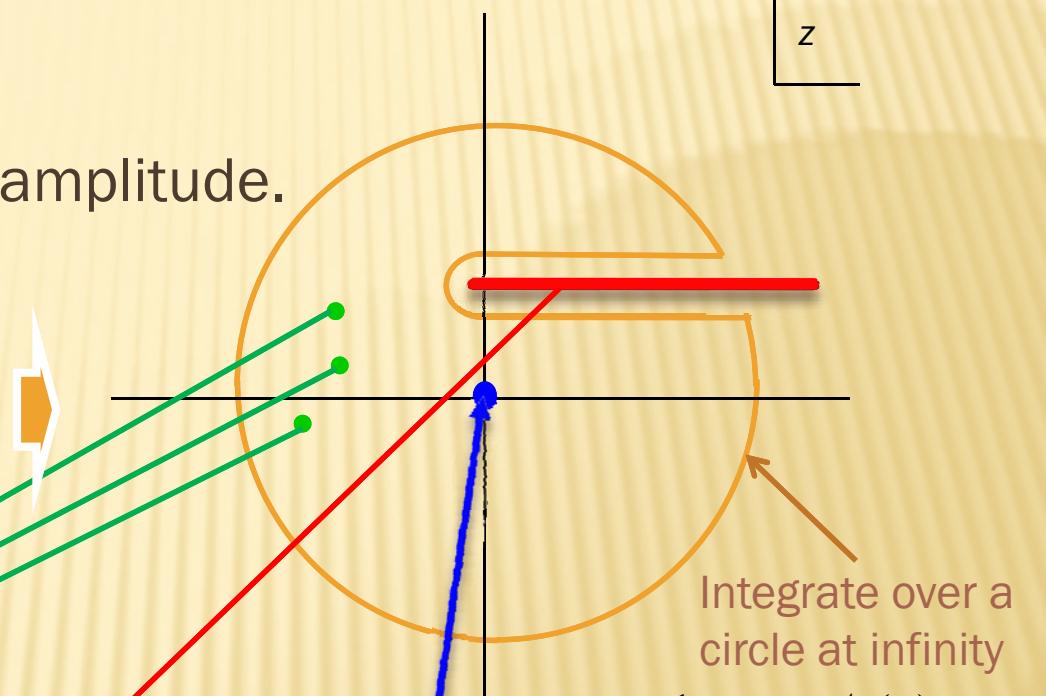
$$A_n^1(\dots, k_i^{h_i}(z), \dots, k_j^{h_j}(z), \dots)$$



On-shell
recurrence relation



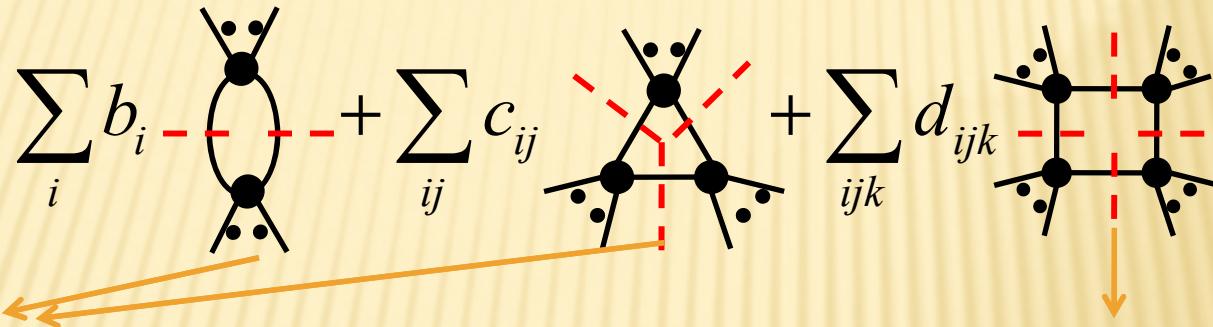
Unitarity techniques



$$\frac{1}{2i\pi} \oint_C dz \frac{A_n(z)}{z} = 0$$

BUBBLES & TRIANGLES

- What about bubble and triangle contributions?



Taking the loop momentum

Large allows us to suppress box

coefficients and get c_0 . Similarly for b_{00} . (DF) (extension to massive loops (Kilgore))

Quadruple cuts, give box coefficients

- For unitarity techniques growth in no. of terms proportional to the number of coefficients.
- Generally applicable, including “wish list” processes.

“ON-SHELL BOOTSTRAP APPROACH”

❖ Has been used to calculate

+ 2 minus all-multiplicity amplitudes (DF, Kosower) (Berger, Bern, Dixon, DF, Kosower)

We can then write the result for the unrenormalized amplitude $A_{n,s}^{\text{loop}} = V^s A_n^{\text{tree}} + iF^s$ in the following form,

$$V_n^s A_n^{\text{tree}} + iF_n^s = c_1 [\hat{C}_n + \hat{R}_n] + \frac{1}{2} A_n^{\text{tree chiral}} + \frac{2}{9} A_n^{\text{tree}}$$

where \hat{C}_n are the cut-containing contributions computed in ref. [1], completed so as to remove $s_1 \rightarrow s_2$ spurious singularities,

$$\begin{aligned} \hat{C}_n = & -\frac{1}{3s_{12}} A^{\text{tree}}(1^-, 2^-, 3^+, \dots, n^+) \\ & \times \sum_{j=4}^{n-1} \frac{L_2((-s_{2-(m-1)})) / (-s_{2-(m-1)})}{s_{2-m}^2} \text{tr}_+ [\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_m \mathbf{k}_{m-1}] \text{tr}_+ [\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_{m-1} \mathbf{k}_m] \text{tr}_+ [\mathbf{k}_1 \mathbf{k}_2 (\mathbf{k}_m \mathbf{k}_{m-1} - \mathbf{k}_{m-1} \mathbf{k}_m)] \end{aligned} \quad (2)$$

The computations then yield,

$$\begin{aligned} \hat{R}_n(1^-, 2^-, 3^+, \dots, n^+) = & \frac{1}{3} A^{\text{tree}}(1^-, 2^-, 3^+, \dots, n^+) \\ & \times \sum_{l=1}^{n-1} \left(\sum_{i_2=i_1+1}^{n-1} \left[C_1(n; i_1, i_2) (T_1(n; i_1, i_2, i_2) + T_1(n; i_1, i_2, i_2+1)) \right. \right. \\ & \quad \left. \left. + C_2(n; i_1, i_2) (T_2(n; i_1, i_2) + T_{28}(n; i_1, i_2)) \right. \right. \\ & \quad \left. \left. + C_3(n; i_1, i_2) (T_{30}(n; i_1, i_2) + T_{36}(n; i_1, i_2) + T_{36}(n; i_1, i_2)) \right] \right) + T_4(n; i_1) \end{aligned} \quad (3)$$

In this equation,

$$\begin{aligned} C_1(n; i_1, i_2) = & \frac{\langle (i_1+1) | (i_1+2) \rangle}{\langle 1^- | \mathcal{K}_{(i_2+1) \dots n} \mathcal{K}_{(i_1+2) \dots i_2} | (i_1+1)^+ \rangle} \langle 1^- | \mathcal{K}_{(i_2+1) \dots i_2} \mathcal{K}_{(i_1+2) \dots i_2} | (i_1+2)^+ \rangle, \\ C_2(n; i_1, i_2) = & \frac{s_{(i_1+2) \dots i_2}}{s_{(i_1+2) \dots i_2} \langle 1^- | \mathcal{K}_{2-(i_1+1)} \mathcal{K}_{(i_1+2) \dots i_2} | (i_2+1)^+ \rangle} C_1(n; i_1, i_2), \\ C_3(n; i_1, i_2) = & s_{(i_1+2) \dots i_2}^2 C_2(n; i_1, i_2). \end{aligned} \quad (4)$$

The terms T_i are given by,

$$\begin{aligned} T_1(n; i_1, i_2, j) = & \frac{\langle i_2^- | \mathcal{K}_{(i_2+1) \dots n} \mathcal{K}_{(i_1+2) \dots i_2} | j^+ \rangle \langle 1^- | \mathcal{K}_{2-(i_1+1)} \mathcal{K}_{(i_1+2) \dots i_2} | (j-1)^+ \rangle - \langle i_2^- | \mathcal{K}_{(i_1+2) \dots (i_2-1)} | j^+ \rangle^2}{2 \langle 1^- | \mathcal{K}_{2-(i_1+1)} \mathcal{K}_{(i_1+2) \dots i_2} | j^+ \rangle^2}; \end{aligned} \quad (5)$$

(Note that $T_1(n; i_1, n-1, n) = 0$.)

$$T_{28}(n; i_1, i_2) = \sum_{l=i_2+1}^{n-1} \langle 1^- | \mathcal{K}_{(i_2+1) \dots n} \mathcal{K}_{(i_1+2) \dots i_2} | l^+ \rangle f_1(n; l, i_1, i_2); \quad (6)$$

$$\begin{aligned} T_{36}(n; i_1, i_2) = & - \sum_{l=i_2+1}^{i_2-1} \sum_{p=l+1}^{i_2} \frac{f_2(n; l, p; i_1, i_2)}{\langle 1^- | \mathcal{K}_{(i_2+1) \dots n} \mathcal{K}_{(i_1+2) \dots i_2} \mathcal{K}_{(i_1+2) \dots (l-1)} \mathcal{K}_{l-(p-1)} | p^+ \rangle} \\ & \times \frac{\langle (l-1) | \mathcal{K}_{(i_2+1) \dots n} \mathcal{K}_{(i_1+2) \dots i_2} \mathcal{K}_{(i_1+2) \dots (l-1)} \mathcal{K}_{(l-1)+} | (l-1)^+ \rangle^3}{s_{(i_1+2) \dots i_2} \langle 1^- | \mathcal{K}_{(i_2+1) \dots n} \mathcal{K}_{(i_1+2) \dots i_2} \mathcal{K}_{(i_1+2) \dots (l-1)} | (l-1)^+ \rangle}, \end{aligned} \quad (7)$$

$$\begin{aligned} T_{36}(n; i_1, i_2) = & \sum_{l=i_2+1}^{n-1} \frac{\langle 1 | \langle (l-1) | \langle 1^- | \mathcal{K}_{(i_2+1) \dots n} \mathcal{K}_{(i_1+2) \dots i_2} | l^+ \rangle | \rangle}{\langle l | (l+1) \rangle} \langle 1^- | \mathcal{K}_{(i_2+1) \dots n} \mathcal{K}_{(i_1+2) \dots i_2} | l^+ \rangle; \end{aligned} \quad (8)$$

$$T_{10}(n; i_1, i_2) = \frac{\langle 1^- | \mathcal{K}_{2-(i_2)} | (i_2+1)^- \rangle \langle 1^- | \mathcal{K}_{2-(i_1+1)} \mathcal{K}_{2-(i_2)} | l^+ \rangle \langle 1 | (i_2+1)^2}{\langle 1^- | \mathcal{K}_{2-(i_1+1)} \mathcal{K}_{(i_1+2) \dots i_2} | (i_2+1)^+ \rangle^2}; \quad (9)$$

$$\begin{aligned} T_{3c}(n; i_1, i_2) = & \sum_{l=i_2+1}^{n-1} \sum_{p=l+1}^{n-1} \frac{\langle 1^- | \mathcal{K}_{l-p} \mathcal{K}_{(p+1) \dots n} | l^+ \rangle^3}{\langle 1^- | \mathcal{K}_{(p+1) \dots n} \mathcal{K}_{l-(p-1)} | p^+ \rangle^3} \\ & \times \frac{\langle (p+1) | \langle 1^- | \mathcal{K}_{2-(l-1)} | \mathcal{F}(l, p) |^2 \mathcal{K}_{(p+1) \dots n} | 1^+ \rangle | f_3(n; l, p; i_1, i_2)}{s_{l-p} \langle 1^- | \mathcal{K}_{2-(l-1)} \mathcal{K}_{l-(p-1)} | p^+ \rangle \langle 1^- | \mathcal{K}_{2-(l-1)} \mathcal{K}_{l-p} | (p+1)^+ \rangle}, \end{aligned} \quad (10)$$

$$T_4(n; i_1) = \frac{[(i_1+2)(i_1+3)] \langle (i_1+3) |}{2 \langle 1^- | \mathcal{K}_{2-(i_1+1)} | (i_1+2) \rangle}. \quad (11)$$

The f_i appearing in the above equations are given by,

$$\begin{aligned} f_1(n; l, i_1, i_2) = & \begin{cases} -s_{(i_1+2) \dots i_2}^2 \langle 1^- | \mathcal{K}_{(i_1+2) \dots i_2} \mathcal{K}_{2-(i_1+1)} | 1^+ \rangle \\ \times \langle 1^- | \mathcal{K}_{2-(i_2)} \mathcal{K}_{(i_1+2) \dots (i_2-1)} | i_2^+ \rangle \langle i_2^+ | \mathcal{K}_{2-(i_2-1)} | 1^+ \rangle, & l = i_2 \\ \langle 1^- | \mathcal{K}_{2-(i_2)} \mathcal{K}_{(i_1+2) \dots i_2} | (l+1)^+ \rangle \langle l+1 | \mathcal{K}_{2-(i_2-1)} | i_2^+ \rangle, & (i_1+2) \leq l < i_2 \end{cases} \end{aligned} \quad (12)$$

$$\begin{aligned} f_2(n; l, p, i_1, i_2) = & \begin{cases} \langle i_2^- | \mathcal{K}_{(i_2+2) \dots n} \mathcal{K}_{2-(i_1+1)} | 1^+ \rangle & p = i_2 \\ \frac{s_{(i_1+2) \dots i_2}}{s_{(i_1+2) \dots i_2} \langle 1^- | \mathcal{K}_{(i_2+1) \dots n} \mathcal{K}_{(i_1+2) \dots (l-1)} \mathcal{K}_{2-(i_2+1)} | 1^+ \rangle} \langle p | (p+1) | & l+1 \leq p < i_2 \\ \langle 1^- | \mathcal{K}_{(i_2+1) \dots n} \mathcal{K}_{(i_1+2) \dots i_2} \mathcal{K}_{(i_1+2) \dots (l-1)} \mathcal{K}_{l-(p-1)} | (p+1)^+ \rangle & \end{cases} \end{aligned} \quad (13)$$

$$\begin{aligned} f_3(n; l, p, i_1, i_2) = & \begin{cases} \frac{\langle l^- | \mathcal{K}_{2-(i_1+1)} \mathcal{K}_{(i_1+2) \dots i_2} | (l+1)^+ \rangle}{\langle 1^- | \mathcal{K}_{(i_1+1) \dots n} \mathcal{K}_{(i_1+2) \dots i_2} \mathcal{K}_{(i_1+2) \dots (l-1)} | (l-1)^+ \rangle}, & l = i_2+1 \\ \frac{\langle (l-1) | l |}{\langle 1^- | \mathcal{K}_{(i_1+1) \dots n} \mathcal{K}_{l-(p-1)} | (l-1)^+ \rangle}, & l > i_2+1 \end{cases} \end{aligned} \quad (14)$$

and [2],

$$f(l, p) = \sum_{i=l}^{p-1} \sum_{m=i+1}^p \mathbf{k}_i \mathbf{k}_m. \quad (15)$$

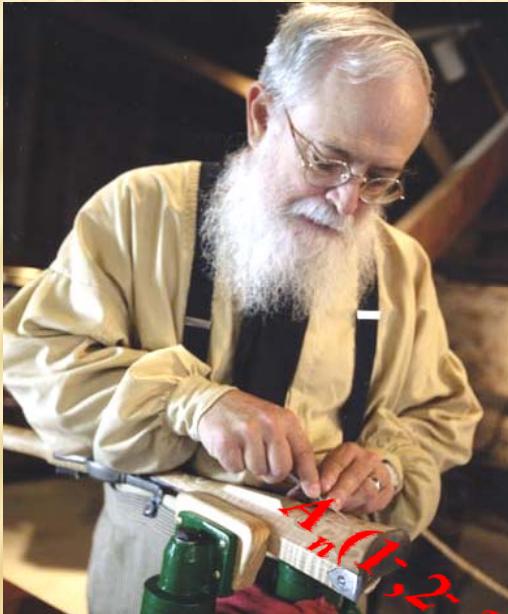
+ 3-minus split helicity amplitude. (Berger, Bern, Dixon, DF, Kosower)

ANALYTIC ONE-LOOP RESULTS

- ✖ Important contributions to the **complete** analytic form for the 6 gluon amplitude, (Numerical result ([Ellis, Giele, Zanderighi](#)))
 - + Cut completion terms ([Bern,Dixon,Kosower](#)) , ([Britto,Feng,Mastrolia](#)) , ([Bern,Bjerrum-Bohr,Dunbar,Ita](#)), ([Berger,Bern,Dixon,DF,Kosower](#)), ([Bedford,Brandhuber,Spence,Travaglini](#))
 - + Rational terms ([Xiao,Yang,Zhu](#)) ,([Berger,Bern,Dixon,DF,Kosower](#))
- ✖ Higgs boson with gluons or a pair of quarks,
 $A_n(\varphi, -, \dots, -) \cdot A_n(\varphi, \pm, +, \dots, +)$ and $A_n(\varphi, -, -, +, , \dots, +)$. ([Badger, Glover](#)) ([Berger, Del Duca, Dixon](#)) ([Badger, Glover, Risager](#))
- ✖ Analytic results for mainly massless amplitudes.
Results including massive particles (e.g. top) desirable.
- ✖ Computed using systematic methods \Rightarrow **Automatable**.

AUTOMATION

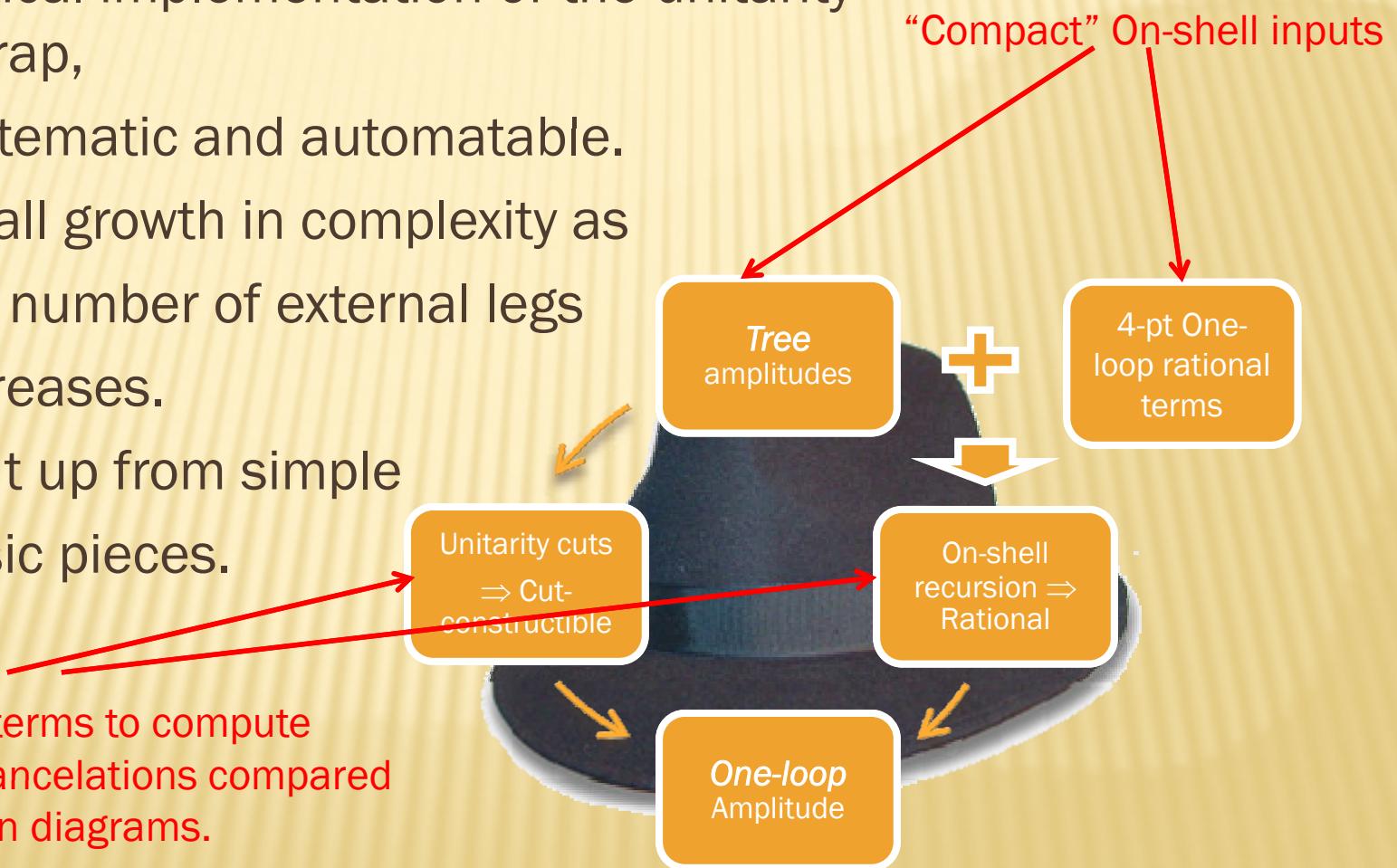
- ✖ Large number of processes to calculate (for the LHC),
 - + Automatic procedure highly desirable.
- ✖ We want to go from



- ✖ Implement new methods numerically.

BLACKHAT

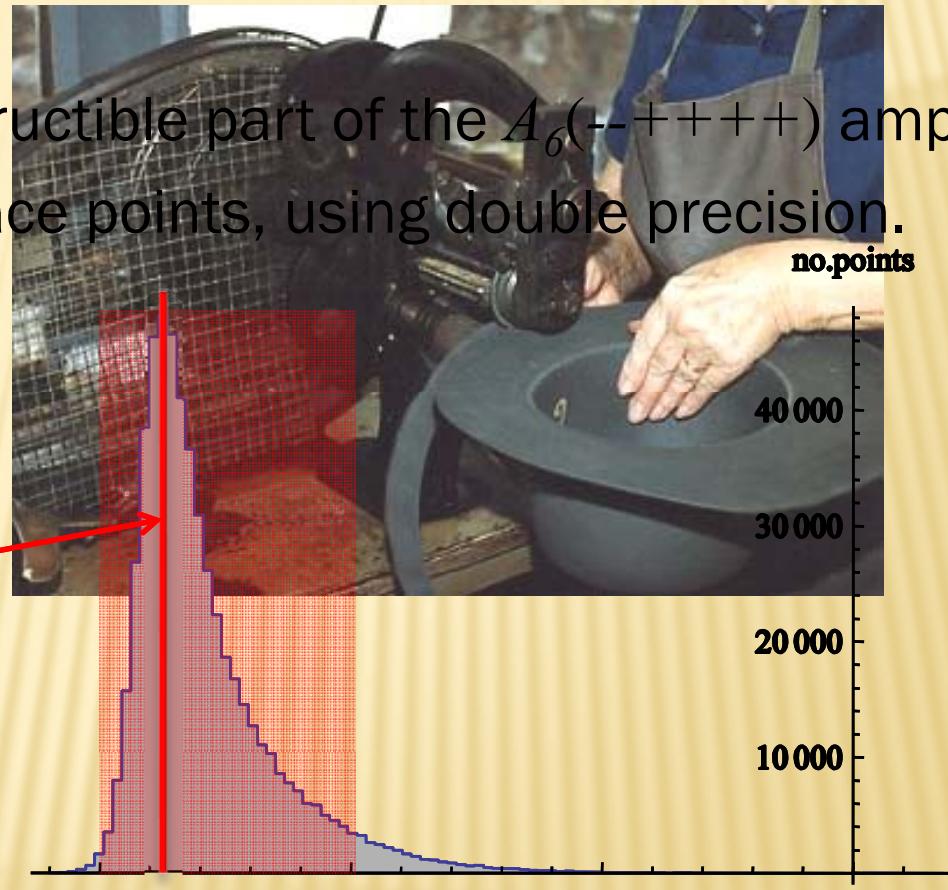
- ✖ Numerical implementation of the unitarity bootstrap,
 - + Systematic and automatable.
 - + Small growth in complexity as the number of external legs increases.
 - + Built up from simple basic pieces.



PRELIMINARY RESULTS

- ✗ Work in progress.
- ✗ Example: Cut-constructible part of the A_6 (--++++) amplitude
- ✗ 100,000 phase space points, using double precision.

Usually get ~10-15 significant figures of accuracy



- ✗ Compute 100,000 points in ~28 minutes.

CONCLUSION

High multiplicity amplitudes needed

- Feynman diagram approaches limited in practical terms to ~five legs.

Unitarity Techniques

- Compact results.
- Minimal growth as no. of legs increases.
- Fast numerically stable results.
- Multiple approaches

Automatic computation of one-loop amplitudes.

- Needed to compute processes for the LHC.
- *BlackHat*

“One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane.” In J. Schwinger, “Particles, Sources and Fields”, Vol. I.

MORE DETAILS

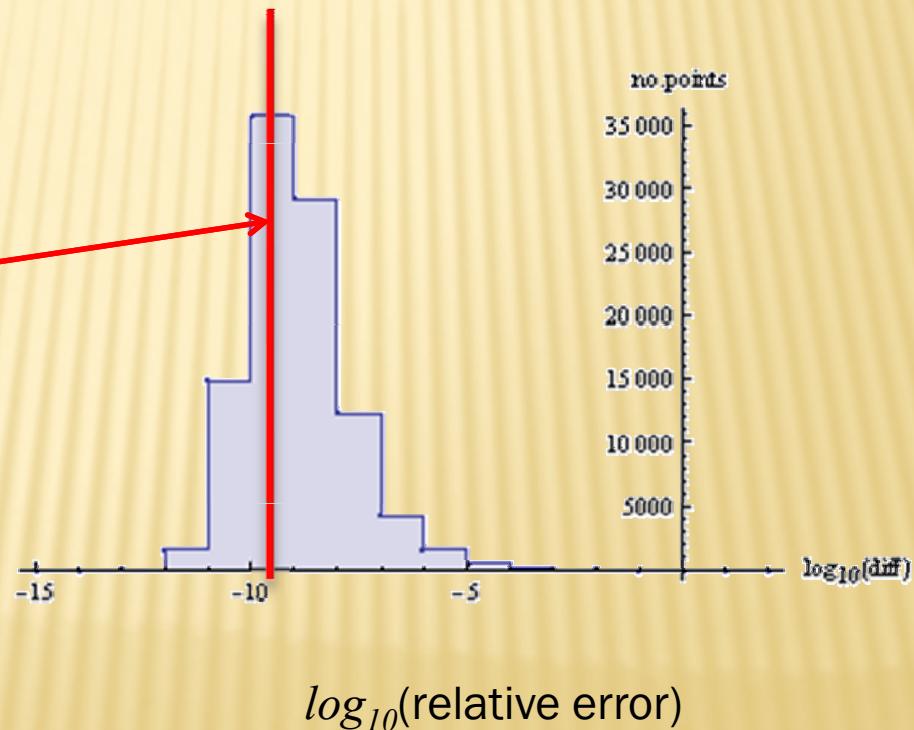
✖ Six gluon pieces

Amplitude	$N=4$	$N=I$	Scalar	Rational
(--++++)	BDDK'94	BDDK'94	BDDK'94	BDK'05, KF'05
(-+-+++)	BDDK'94	BDDK'94	BBST'04	BBDFK'06, XYZ'06
(-++-++)	BDDK'94	BDDK'94	BBST'04	BBDFK'06, XYZ'06
(---+++)	BDDK'94	BBDD'04	BBDI'05, BFM'06	BBDFK'06
(--+-++)	BDDK'94	BBCF'05, BBDP'05	BFM'06	XYZ'06
(-+-+-+)	BDDK'94	BBCF'05, BBDP'05	BFM'06	XYZ'06

RATIONAL TERMS

- ✖ Rational terms, cut pieces feed into these. Accuracy measurement for 100,000 phase space points we have

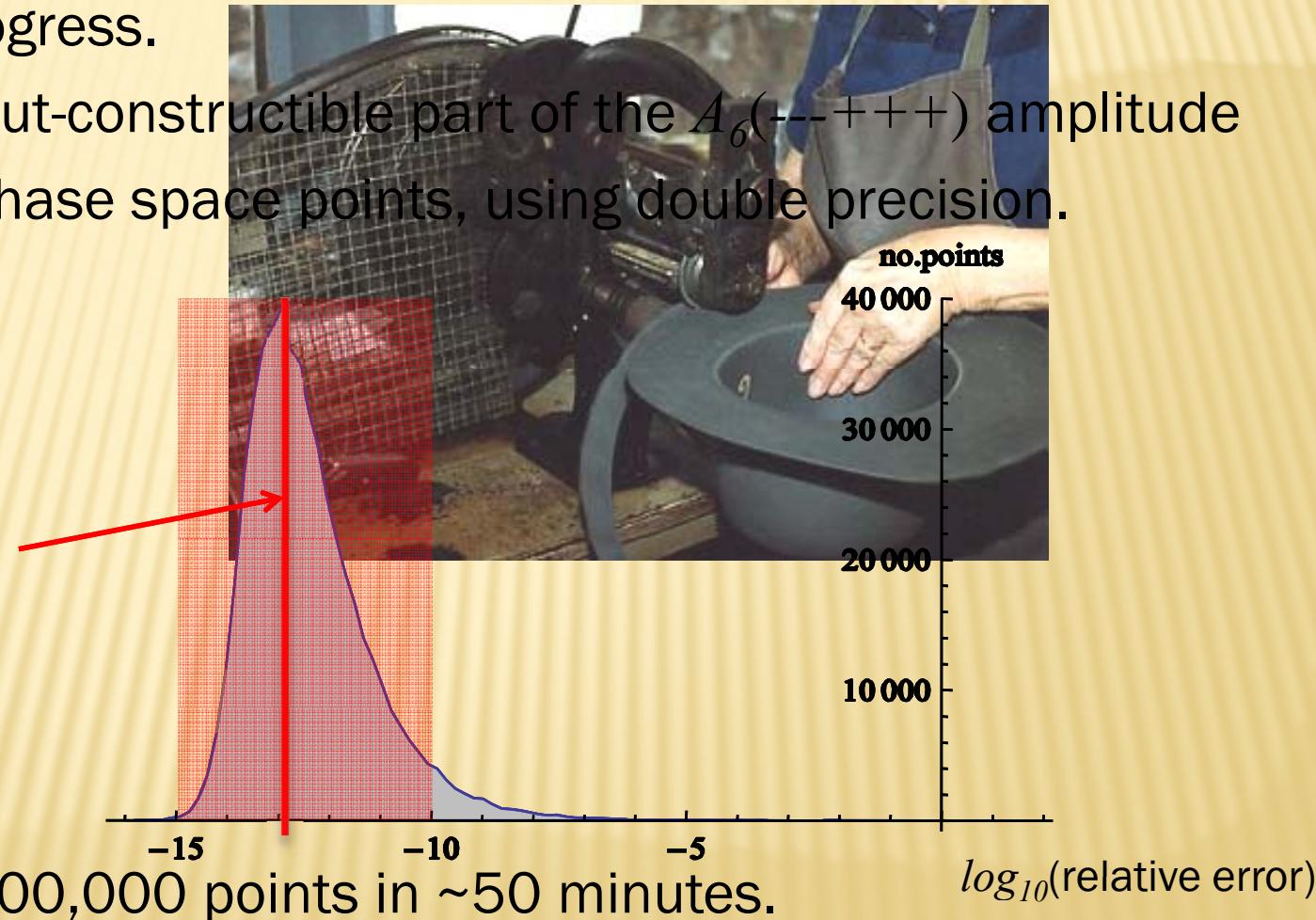
Usually get 8-10 significant figures of accuracy



PRELIMINARY RESULTS

- Work in progress.
- Example: Cut-constructible part of the $A_6(\text{---}+++)$ amplitude
- 100,000 phase space points, using double precision.

Usually get ~10-15 significant figures of accuracy



- Compute 100,000 points in ~50 minutes.