Corrections and renormalization in the Doublet-Triplet Higgs Model

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Outline I

Motivation

- The existence of massive neutrinos is a strong motivation for physics beyond the Standard Model (SM)
- Weinberg has talked about one dimension-five operator relevant for neutrino masses in the context of the SM: $(\kappa/\Lambda)\, l_L l_L HH$.
- At electroweak symmetry breaking, neutrinos acquire a Majorana mass: $m_\upsilon \sim \kappa \upsilon_0^2/\Lambda \lesssim 1eV$
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The Lagrangian Density of the HTM

The doublet and the triplet

- HTM is one of the extensions of the Standard Model in which neutrinos acquire a Majorana mass.
- The Higgs sector of the Lagrangian density of this model is extended by adding an $SU(2)_L$ Higgs triplet Δ in addition to the Higgs doublet.
- The Higgs doublet has the form:

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{1}$$

- While the Higgs triplet Δ is:

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \tag{2}$$

And the lagrangian density of this model is:

$$\mathcal{L}_{\mathtt{HTM}} = (D_\mu H)^\dagger (D^\mu H) + \mathtt{Tr}(D_\mu \Delta)^\dagger (D^\mu \Delta) + \mathcal{L}_Y - V(H, \Delta)$$

- $D_\mu H = \partial_\mu H + i g_1 T^a W_\mu^a H + i \frac{g_2}{2} B_\mu H$
- $D_\mu \Delta = \partial_\mu \Delta + i g_1 [T^a W_\mu^a, \Delta] + i g_2 \frac{Y}{2} B_\mu \Delta$
  - $T^a = \sigma^a / 2$
  - σ^a is the Pauli matrix
- $\mathcal{L}_Y = -Y_\upsilon l_L^T C i \sigma_2 \Delta l_L + h.c.$
  - Y_υ is a 3×3 symmetric and complex matrix.
  - $l_l^T = (\upsilon_L^T, e_L^T)$
  - C is conjugate charge operator.
  - σ_2 is the Pauli matrix.

The Scalar Potential

- The scalar interactions can be found in the potential term:

$$V(H,\Delta) = -m_H^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 + M_\Delta^2 Tr(\Delta^\dagger \Delta)$$
$$+ \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr\Delta^\dagger \Delta)^2 + \lambda_3 Tr(\Delta^\dagger \Delta)^2$$
$$+ \lambda_4 H^\dagger \Delta \Delta^\dagger H + (\mu H^T i\sigma_2 \Delta^\dagger H + h.c.)$$

- Imposing conditions of global minimum, one finds:

$$-m_H^2 + \frac{\lambda}{4}\upsilon_0^2 - \sqrt{2}\mu\upsilon_t + \frac{(\lambda_1+\lambda_4)}{2}\upsilon_t^2 = 0$$
$$-\frac{\mu\upsilon_0^2}{2} + \frac{M_\Delta^2 \upsilon_t}{\sqrt{2}} + \frac{\lambda_1+\lambda_4}{2\sqrt{2}}\upsilon_0^2\upsilon_t + \frac{\lambda_2+\lambda_3}{\sqrt{2}}\upsilon_t^3 = 0$$

Once the neutral component Δ gets the vev v, neutrinos acquire a a Majorana mass.

$$M_{\upsilon} = \sqrt{2} Y_{\upsilon} v_t, \texttt{then} Y_{\upsilon} = \frac{M_{\upsilon}}{\sqrt{2} v_t} \tag{3}$$

At electroweak symmetry breaking, seven physical massive bosons of Higgs show up in the spectrum

- $h^0 = \cos\alpha\phi^0 + \sin\alpha\Delta^0$
- $H^0 = -\sin\alpha\phi^0 + \cos\alpha\Delta^0$
- $A = -\sin\beta\xi^0 + \cos\beta\eta^0$
- $H^{\pm} = -\sin\beta'\phi^{\pm} + \cos\beta'\delta^{\pm}$
- $H^{\pm\pm} = \delta^{\pm\pm}$
- $G = \cos\beta\xi^0 + \sin\beta\eta^0$
- $G^{\pm} = \cos\beta'\phi^{\pm} + \sin\beta'\delta^{\pm}$

At electroweak symmetry breaking, the gauge bosons acquire a mass:

$$M_W^2 = \frac{g_1^2(v_0^2 + 2v_t^2)}{4} \quad and \quad M_Z^2 = \frac{g^2(v_0^2 + 4v_t^2)}{4c_w^2} \tag{4}$$

The Scalar Potential

The mass of ten degrees of freedom

The ten degrees of freedom would have the following mass:

- $M^2_{H^{\pm\pm}} = \frac{\sqrt{2}\mu v_0^2 - \lambda_4 v_0^2 v_t - 2\lambda_3 v_t^3}{2v_t}$
- $M^2_{H^{\pm}} = \frac{v_0^2[2\sqrt{2}\mu - \lambda_4 v_t]}{4v_t}$
- $M^2_{A} = \frac{\mu(v_0^2 + 4v_t^2)}{\sqrt{2}v_t}$
- $M^2_{h^0} = \frac{1}{2}(A + C - \sqrt{(A-C)^2 + 4B^2})$
- $M^2_{H^0} = \frac{1}{2}(A + C + \sqrt{(A-C)^2 + 4B^2})$
- $M_{G^0} = 0, M_{G^{\pm}} = 0$

Where $A = \frac{\lambda}{2}v_0^2$, $B = v_t(-\sqrt{2}\mu + (\lambda_1 + \lambda_4)v_t)$ and

$$C = \frac{\sqrt{2}\mu v_0^2 + 4(\lambda_2 + \lambda_3)v_t^3}{2v_t}$$

The mixing angles are:

- $\tan\beta' = \frac{2\sqrt{2}\upsilon_t\upsilon_0}{\upsilon_0^2 - 2\upsilon_t}$
- $\tan 2\beta = \frac{4\upsilon_0\upsilon_t}{\upsilon_0^2 - 4\upsilon_t^2}$
- $\tan 2\alpha = \frac{4\upsilon_t\upsilon_0(-\sqrt{2}\mu + (\lambda_1+\lambda_4)\upsilon_t)}{\lambda\upsilon_0^2\upsilon_t - (\sqrt{2}\mu\upsilon_0^2 + 4(\lambda_2+\lambda_3)\upsilon_t^3)}$

β' and β are the mixing angles that generate the physical states of $H^{\pm}$ and A respectively.
α is the one that generates the physical state of h^0 and H^0

The parameters λ and $\lambda_{\{1,2,3,4\}}$ could be determined from the masses of the ten degrees of freedom and from the mixing angles as well:

- $\lambda_1 = -\frac{2}{\upsilon_0^2+4\upsilon_t^2} M_A^2 + \frac{4}{\upsilon_0^2+2\upsilon_t^2} M_{H^\pm}^2 + \frac{\sin 2\alpha}{2\upsilon_0\upsilon_t}\left[M_{H_1}^2 - M_{H_2}^2\right]$
- $\lambda_3 = \frac{1}{\upsilon_t^2}\left[\frac{-\upsilon_0^2}{\upsilon_0^2+4\upsilon_t^2} M_A^2 + \frac{2\upsilon_0^2}{\upsilon_0^2+2\upsilon_t^2} M_{H^\pm}^2 - M_{H^{\pm\pm}}^2\right]$
- $\lambda_4 = \frac{4}{\upsilon_0^2+4\upsilon_t^2} M_A^2 - \frac{4}{\upsilon_0^2+4\upsilon_t^2} M_{H^\pm}^2$
- $\lambda = \frac{2}{\upsilon_0^2}\left[s_\alpha^2 M_{h^0}^2 + c_\alpha^2 M_{H^0}^2\right]$ for $M_{H^0} > M_{h^0}$
- $\lambda_2 = \frac{1}{\upsilon_t^2}\left[\frac{c_\alpha^2 M_{h^0}^2 + s_\alpha^2 M_{H^0}^2}{2}\right]$ for $M_{H^0} > M_{h^0}$
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Radiative Corrections and Renormalization

Basics

- Four free parameters in gauge-Higgs sector (g, g', μ, λ), conventionally chosen:
  - $\alpha = 1/137.0359895(61)$
  - $G_F = 1.16637(1) \times 10^{-5} GeV^{-2}$
  - $M_Z = 91.1875 \pm 0.0021 GeV$
  - M_h
- Express everything else in terms of these parameters

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{\pi\alpha}{2\left(1-\frac{M_W^2}{M_Z^2}\right)M_W^2}$$

- $=>$ Predicts M_W

- Mixing angle is predicted quantity
- On-shell definition $\cos{\theta_W}^2 = M_W^2/M_Z^2$
- Predict M_W
  - $M_W^2 = \pi\sqrt{2}\frac{\alpha}{G_F}\left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}}\right)^{-1}$
- Plug in numbers:
  - Predicted $M_W = 80.939$ GeV
  - Experimental $M_W = 80.399 \pm 0.023$GeV

Inedequacy of Tree Level Calculations

Need to calculate beyond tree level

Modifications of tree level relations

Radiative Corrections and Renormalization

- The expression of the Fermi coupling constant at one loop

$$G_F = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin{\theta_W}^2}\frac{1}{1-\Delta r}$$

- Contributions to Δr as a physical quantity which incorporates 1-loop corrections

$$\Delta r = \frac{\Sigma^{WW}(0)-\delta M_W^2}{M_W^2} + \Pi^{\gamma}(0) - \frac{\delta s_\theta^2}{s_\theta^2} + 2\frac{c_\theta}{s_\theta}\frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \delta_{VB}$$

  - $\delta_{VB} = -\frac{\alpha}{4\pi s_\theta^2}\left[6 + \frac{10-10s_\theta^2 3(R/c_\theta^2)(1-2s_\theta^2)}{2(1-R)}\ln R\right]$ summarizes the vertex corrections
  - $R = M_W^2/M_Z^2$

- The Amplitudes calculated at one-loop level are UV-Divergent.
- We need to perform renormalization in the on-shell scheme.
- It is convenient to treat the s_θ^2 as an additional independent input parameter and treat its counter term δs_θ^2 by an appropriate renormalization condition.

Radiative Corrections and Renormalization

Renormalization of the vector boson self-energy

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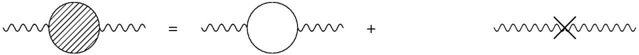
The renormalized self energies can be written as follows:



$$\hat{\Sigma}^{\gamma\gamma}(k^2) = \Sigma^{\gamma\gamma}(k^2) + \delta Z_2^{\gamma}\, k^2$$

$$\hat{\Sigma}^{ZZ}(k^2) = \Sigma^{ZZ}(k^2) + \delta Z_2^{Z}(k^2 - M_Z^2) - \delta M_Z^2$$

$$\hat{\Sigma}^{WW}(k^2) = \Sigma^{WW}(k^2) + \delta Z_2^{W}(k^2 - M_W^2) - \delta M_W^2$$

$$\hat{\Sigma}^{\gamma Z}(k^2) = \Sigma^{\gamma Z}(k^2) + (\delta Z_1^{\gamma Z} - \delta Z_2^{\gamma Z})M_Z^2 - \delta Z_2^{\gamma Z}\, k^2$$

with

$$\delta Z_i^{\gamma Z} = \frac{c_\theta s_\theta}{c_\theta^2 - s_\theta^2}(\delta Z_i^{Z} - \delta Z_i^{\gamma}) \quad .$$

The on-shell renormalization conditions determine the counter terms as follows:

$$
\begin{aligned}
\delta M_W^2 &= \mathrm{Re}\Sigma^{WW}(M_W^2) \\
\delta M_Z^2 &= \mathrm{Re}\left[\Sigma^{ZZ}(M_Z^2) - \frac{(\hat{\Sigma}^{\gamma Z}(M_Z^2))^2}{M_Z^2 + \hat{\Sigma}^{\gamma\gamma}(M_Z^2)}\right] \\
\delta Z_1^\gamma &= -\Pi^\gamma(0) - \frac{s_\theta}{c_\theta}\frac{\Sigma^{\gamma Z}(0)}{M_Z^2} \quad \text{with} \quad \Pi^\gamma(0) = \frac{\partial \Sigma^{\gamma\gamma}}{\partial k^2}(0) \\
\delta Z_2^\gamma &= -\Pi^\gamma(0) \\
\delta Z_1^Z &= -\Pi^\gamma(0) - \frac{3c_\theta^2 - 2s_\theta^2}{s_\theta c_\theta}\frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{c_\theta^2 - s_\theta^2}{c_\theta^2}\frac{\delta s_\theta^2}{s_\theta^2} \\
\delta Z_2^Z &= -\Pi^\gamma(0) - 2\frac{c_\theta^2 - s_\theta^2}{s_\theta c_\theta}\frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{c_\theta^2 - s_\theta^2}{c_\theta^2}\frac{\delta s_\theta^2}{s_\theta^2} \\
\delta Z_1^W &= -\Pi^\gamma(0) - \frac{3 - 2s_\theta^2}{s_\theta c_\theta}\frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{\delta s_\theta^2}{s_\theta^2} \\
\delta Z_2^W &= -\Pi^\gamma(0) - 2\frac{c_\theta}{s_\theta}\frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{\delta s_\theta^2}{s_\theta^2} \quad .
\end{aligned}
$$

Radiative Corrections and Renormalization

On-shell counter terms

- In the SM, δM_Z^2 is given by $\delta M_Z^2 = \text{Re}\left[\Sigma^{ZZ}(M_Z^2)\right]$ instead of $\delta M_Z^2 = \text{Re}\left[\Sigma^{ZZ}(M_Z^2) - \frac{(\hat{\Sigma}^{\gamma Z}(M_Z^2))^2}{M_Z^2 + \hat{\Sigma}^{\gamma\gamma}(M_Z^2)}\right]$
- The angle counter term δs_θ^2 is related to the fundamental renormalization constant by:

$$\frac{\delta s_\theta^2}{s_\theta^2} = \frac{c_\theta}{s_\theta}\,(3\,\delta Z_2^{\gamma Z} - 2\,\delta Z_1^{\gamma Z}) \quad . \tag{5}$$

- Note that $c_W = \frac{M_W}{M_Z}$ and $c_\theta = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}$
- For the SM case, we have:

$$\frac{\delta s_\theta^2}{s_\theta^2} = \frac{\delta s_W^2}{c_W^2} = \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} = \frac{c_W}{s_W}\,(3\,\delta Z_2^{\gamma Z} - 2\,\delta Z_1^{\gamma Z}) \tag{6}$$

Radiative Corrections and Renormalization

Renormalization Condition

- Thus an extra renormalization condition is required to fix δs_θ^2
- A natural condition would be to define the mixing angle for the leptons in terms of the dressed coupling constants:

Renormalization Condition

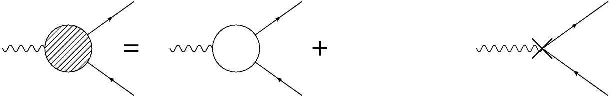
$$\frac{\mathrm{Re}(g_V^e)}{\mathrm{Re}(g_A^e)} = 1 - 4s_\theta^2$$

- The couplings constants g_V^e and g_A^e at one loop appear through the counter term of the weak vertex *Zee*

Radiative Corrections and Renormalization

The renormalized expression of $Zf\bar{f}$

The renormalized gauge boson fermion vertices $\hat{\Gamma}$ would be:



$$\hat{\Gamma}_\mu^{Zff} = \Gamma_\mu^{Zff} + i\frac{e}{2s_\theta c_\theta}\,\gamma_\mu\, C_V^{Zff} - i\frac{e}{2s_\theta c_\theta}\,\gamma_\mu\gamma_5\, C_A^{Zff}$$

$$\hat{\Gamma}_\mu^{W\tilde{f}f} = \Gamma_\mu^{W\tilde{f}f} + i\frac{e}{2\sqrt{2}s_\theta}\,\gamma_\mu(1-\gamma_5)\, C_L^{W\tilde{f}f}$$

$$
\begin{aligned}
\text{with} \quad C_V^{Zff} &= v_f(\delta Z_1^Z - \delta Z_2^Z) + 2 s_\theta c_\theta Q_f (\delta Z_1^{\gamma Z} - \delta Z_2^{\gamma Z}) \\
&+ (v_f \delta Z_V^f + a_f \delta Z_A^f) \\
C_A^{Zff} &= a_f(\delta Z_1^Z - \delta Z_2^Z) + (v_f \delta Z_A^f + a_f \delta Z_V^f) \\
C_L^{W\tilde{f}f} &= \delta Z_1^W - \delta Z_2^W + \delta Z_L \\
\text{and} \quad v_f &= I_3^f - 2 s_\theta^2 Q_f \quad , \quad a_f = I_3^f \quad .
\end{aligned} \tag{7}
$$

Radiative Corrections and Renormalization

Self-energy of fermions

- The self-energy of fermions can be written in the form:

$$\Sigma^f(k) = \not{k}\Sigma_V^f(k^2) + \not{k}\gamma_5\Sigma_A^f(k^2) + m_f\Sigma_S^f(k^2)$$

- The doublet field renormalization constant has the form:

$$\delta Z_L = -\Sigma_L(m_f^2) - m^2[\Sigma_L'(m_f^2) + \Sigma_R'(m_f^2) + 2\ \Sigma_S'(m_f^2)]$$

- The vectorial axial renormalization constans have the form:

$$\begin{aligned} \delta Z_V &= -\Sigma_V(m_f^2) - 2m_f^2[\Sigma_V'(m_f^2) + \Sigma_S'(m_f^2)] \\ \delta Z_A &= \Sigma_A(m_f^2) \end{aligned} \tag{8}$$

with the scalar functions $\Sigma_{V,A}$ are related to $\Sigma_{L,R}$ via :

$$\Sigma_L = \Sigma_V - \Sigma_A, \quad \Sigma_R = \Sigma_V + \Sigma_A \tag{9}$$

Radiative Corrections and Renormalization

Effective couplings g_V and g_A

- The effective vector and axial vector couplings of the fermions to the Z Boson are determined at one loop:

$$\begin{aligned} g_V^f &= \left(\rho \frac{1-\Delta\tilde{r}}{1+\hat{\Pi}^Z(M_Z^2)}\right)^{\frac{1}{2}} \cdot \left[v_f + 2 s_\theta c_\theta Q_f \hat{\Pi}^{\gamma Z}(M_Z^2)\right. \\ &+ \left.\hat{\Lambda}_V^{Zff}(M_Z^2)\right] \\ g_A^f &= \left(\rho \frac{1-\Delta\tilde{r}}{1+\hat{\Pi}^Z(M_Z^2)}\right)^{\frac{1}{2}} \cdot \left[a_f + \hat{\Lambda}_A^{Zff}(M_Z^2)\right] \end{aligned} \tag{10}$$

- where $\hat{\Lambda}_{A,V}^{Zff}$ is the correction term of the renormalized vector or axial vector upon decomposing the vertex Zff

$$\hat{\Gamma}_\mu^{Zff} = i\frac{e}{2 s_\theta c_\theta}\left[\gamma_\mu(v_f - a_f\gamma_5) + \gamma_\mu \hat{\Lambda}_V^{Zff} + \gamma_\mu\gamma_5\hat{\Lambda}_A^{Zff}\right] \tag{11}$$

Radiative Corrections and Renormalization

Renormalization Condition

- And the $\hat{\Pi}^Z(M_Z^2)$ is the correction to the Z propagator:

$$\hat{\Pi}^Z(M_Z^2) = \text{Re}\, \frac{\partial \hat{\Sigma}^Z(k^2)}{\partial k^2}\bigg|_{k^2=M_Z^2} \tag{12}$$

- and the γZ mixing:

$$\hat{\Pi}^{\gamma Z}(M_Z^2) = \frac{\hat{\Sigma}^{\gamma Z}(M_Z^2)}{M_Z^2 + \hat{\Sigma}^{\gamma\gamma}(M_Z^2)} \tag{13}$$

- We can finally solve the Renormalization Condition to get:

$$\text{Re}\left\{ -\frac{\hat{\Pi}^{\gamma Z}(M_Z^2)}{v_f} + \frac{1}{2 s_\theta c_\theta}\left(\frac{\hat{\Lambda}_V^{Zff}(M_Z^2)}{v_f} - \frac{\hat{\Lambda}_A^{Zff}(M_Z^2)}{a_f} \right) \right\} = 0 \tag{14}$$

- This above equation can be solved yielding to the counter term:

$$\frac{\delta s_\theta^2}{s_\theta^2} = \text{Re}\left\{ \frac{c_\theta}{s_\theta}\left[\frac{v_f^2 - a_f^2}{2 s_\theta c_\theta a_f} \Sigma_A^f(m_f^2) + \frac{\Sigma^{\gamma Z}(M_Z^2)}{M_Z^2} - \frac{v_f}{2 s_\theta c_\theta}\left(\frac{\Lambda_V^{Zff}(M_Z^2)}{v_f} - \frac{\Lambda_A^{Zff}(M_Z^2)}{a_f} \right) \right] \right\} \tag{15}$$

Radiative Corrections and Renormalization

Feynman diagrams of the self energy of W boson

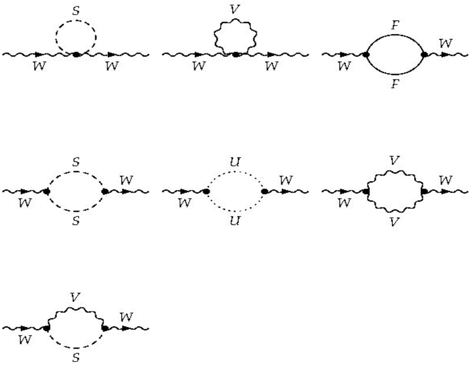


Figure: The contributing particles in the self energy of the W boson

Radiative Corrections and Renormalization

Feynman diagrams of the self energy of Z boson

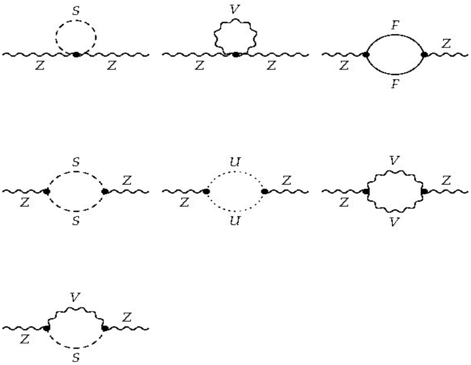


Figure: The contributing particles in the self energy of the Z boson

Radiative Corrections and Renormalization

Feynman diagrams of the self energy of γ boson

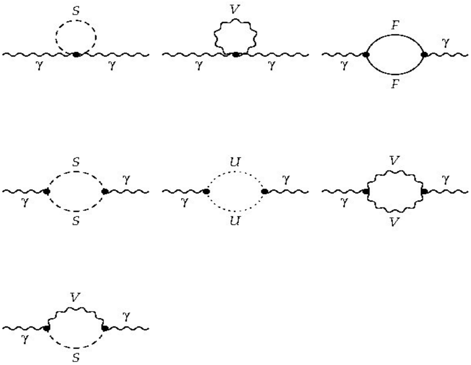


Figure: The contributing particles in the self energy of the γ boson

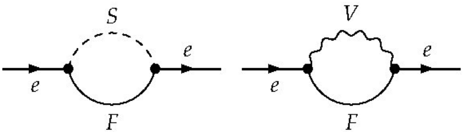


Figure: The contributing particles in the self energy of the electrons, $V \neq \gamma$

Radiative Corrections and Renormalization

Feynman diagrams of the *Zee* vertex

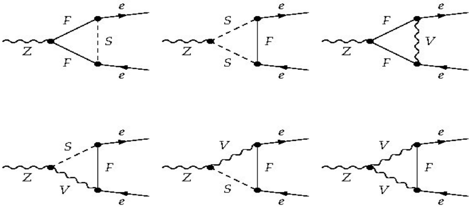


Figure: The contributing particles in the *Zee* vertex, $V \neq \gamma$

The ρ parameter

- The ρ parameter is the ratio between the charged current to the neutral current i.e. $\rho = \frac{M_W^2}{c_w^2 M_Z^2}$
- The ρ parameter in the SM at tree level equals to 1.
- it deviates from unity upon extending the Higgs sector by an additional scalar triplet with one extra vaccum expectation value $v_t > 0$
  - For the seesaw type II at tree level, it is given by $\rho = \frac{v_d^2+2v_t^2}{v_d^2+4v_t^2} \neq 1$
- At higher orders, the general expression of the deviation of ρ is given by: $\delta\rho = \frac{\hat{\Sigma}^{ZZ}(0)}{M_Z^2} - \frac{\hat{\Sigma}^{WW}(0)}{M_W^2}$
- Once we include the one loop corrections to rho, we hope to have the dependance of δ_ρ over other parameters like charged Higgs masses and other higges.