

CP-violation and mixing in charmed mesons at Belle

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Rencontres du Vietnam
Beyond The Standard Model of Particle Physics



- Introduction
- Belle detector
- CPV mechanisms in D -mesons
- Direct CPV
 - $A_{CP}(D^0 \rightarrow K^+K^-, \pi^+\pi^-)$
 - $A_{CP}(D^\pm \rightarrow K_S^0\pi^\pm)$
- Mixing
 - x, y in $D^0 \rightarrow K_S^0\pi^+\pi^-$
 - y_{CP}, A_Γ in $D^0 \rightarrow K^+K^-, \pi^+\pi^-$
- Summary

Direct CPV ($\Delta C = 1$):

decay amplitudes for D^0 -, D^+ -decays and their flavor conjugated processes differ ($|A_f| \neq |\bar{A}_{\bar{f}}|$)

SM predictions:

$O(10^{-3})$ in singly Cabibbo suppressed decays (SCS)

smaller in Cabibbo favored and doubly Cabibbo suppressed (DCS)

(A. Petrov, arXiv:1101.3822v1; F. Buccella *et al.*, PLB 302, 319 (1993))

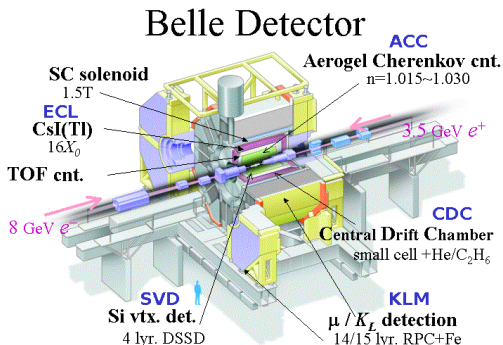
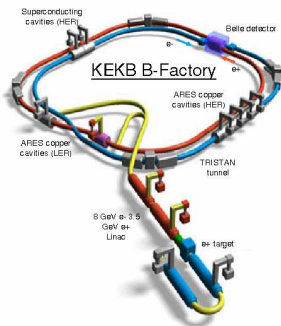
Indirect CPV:

- in mixing ($\Delta C = 2$): rates $D^0 \rightarrow \bar{D}^0$ and $\bar{D}^0 \rightarrow D^0$ differ
- in interference between mixing and direct decay amplitudes.

SM predictions:

$O(10^{-4} - 10^{-5})$ (I. Bigi *et al.*, JHEP 1106, 089 (2011))

Direct CPV depends on the final state,
indirect CPV is universal for all D -mixing processes.



The Belle detector (KEKB collider), KEK, Tsukuba, Japan:

- World luminosity record: $L \simeq 2.1 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$
- Data taking finished in 2010 \Rightarrow Upgrade to Belle II
- Total integrated luminosity: over 1 ab^{-1}
 - $711 \text{ fb}^{-1} \Upsilon(4S)$ ($772 \times 10^6 B\bar{B}$ -pairs)
 - $121 \text{ fb}^{-1} \Upsilon(5S)$
 - $\Upsilon(1S), \Upsilon(2S), \Upsilon(3S)$ scan

Time-integrated CP -asymmetry for final CP -eigenstate f :

$$a_{CP}^f = \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow f)}$$

Recent ΔA_{CP} measurements

$$\Delta A_{CP} = A_{CP}^{KK} - A_{CP}^{\pi\pi}$$

$(-0.82 \pm 0.21 \pm 0.11)\%$ (LHCb 0.62 fb^{-1} , PRL 108, 111602 (2012)) 3.5σ

$(-0.62 \pm 0.21 \pm 0.10)\%$ (CDF 9.7 fb^{-1} , arXiv:1207.2158v1) 2.7σ

SM or NP?

Theoretical predictions for SM contributions to this value can reach up to a few per mille. On the other hand, New Physics can also contribute to ΔA_{CP} .

\Rightarrow **Important to measure $A(KK)$ and $A(\pi\pi)$ with high precision!**

$$A_{CP}^{KK} = \frac{\Gamma(D^0 \rightarrow KK) - \Gamma(\bar{D}^0 \rightarrow KK)}{\Gamma(D^0 \rightarrow KK) + \Gamma(\bar{D}^0 \rightarrow KK)}, \text{ similarly for } A_{CP}^{\pi\pi}$$

$$N^{rec} = N_{prod D^*} \cdot \mathcal{B}(D^* \rightarrow D^0 \pi_S) \cdot \mathcal{B}(D^0 \rightarrow KK) \cdot \varepsilon_{KK} \cdot \varepsilon_{\pi_S}$$

For small asymmetries: $A^{rec} = A_{CP}^{KK} + A_{FB}^{D^*} + A_{CP}^{D^0 \pi_S} + A_{\varepsilon}^{KK} + A_{\varepsilon}^{\pi_S}$

- $A_{CP}^{D^0 \pi_S} = 0$ (strong decay)
- $A_{\varepsilon}^{KK} = 0$ (CP symmetric final state)
- $A_{\varepsilon}^{\pi_S}$: detector-induced asymmetry accounted for using control data samples ($D^0 \rightarrow K\pi$). **Important in other $A(D^0 \rightarrow f)$ measurements!**
- $A_{FB}^{D^*}$: due to interference of virtual γ and Z_0 in $e^+e^- \rightarrow c\bar{c}$ and is a function of $\cos(\theta^*)$, $A_{FB}^{D^*}(\cos(\theta^*)) = -A_{FB}^{D^*}(-\cos(\theta^*))$ ($\theta^* = D^*$ polar angle in CMS).

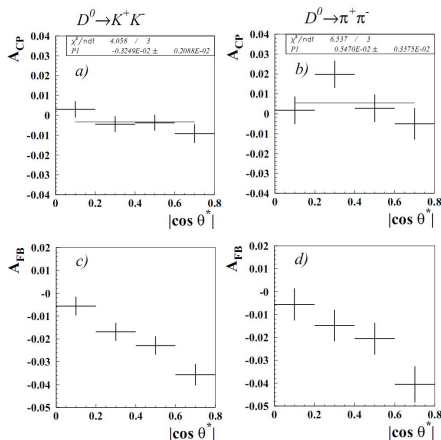
$$\Rightarrow A_{corr}^{rec} = A^{rec} - A_{\varepsilon}^{\pi_S},$$

$$A_{corr}^{rec}(\cos(\theta^*)) = A_{CP}^{KK} + A_{FB}^{D^*}(\cos(\theta^*))$$

$$A_{CP} = \frac{A_{corr}^{rec}(\cos(\theta^*)) + A_{corr}^{rec}(-\cos(\theta^*))}{2}$$

$$A_{FB} = \frac{A_{corr}^{rec}(\cos(\theta^*)) - A_{corr}^{rec}(-\cos(\theta^*))}{2}$$

- $D^{*+} \rightarrow D^0 \pi_S^+, D^{*-} \rightarrow \bar{D}^0 \pi_S^-$
- $p(D^*) > 2.5(3.1) \text{ GeV}/c$ for $\Upsilon(4S)/(5S)$ data
- $1.81 < M(D_{cand}^0) < 1.91 \text{ GeV}/c^2$
- $Q = M(\pi_S D_{cand}^0) - M(D_{cand}^0) - m(\pi_S) < 20 \text{ MeV}/c^2$
- $D^0 \rightarrow$ charged tracks, K/π separation: $\mathcal{L}(K) \leftarrow \text{CDC, ACC, TOF}$
- D^0 vertex fit
- D^* IP constrained fit


 976 fb^{-1} Belle data

	KK	$\pi\pi$
N_{ev}	282×10^3	123×10^3

Belle (976 fb⁻¹)

$$A_{CP}^{KK} = (-0.32 \pm 0.21 \pm 0.09)\%$$

$$A_{CP}^{\pi\pi} = (+0.55 \pm 0.36 \pm 0.09)\%$$

Experiment	$\Delta A_{CP}(\%)$	σ from zero	Reference
Belle, 540 fb ⁻¹	$-0.86 \pm 0.60 \pm 0.07$	1.4	PLB 670, 190 (2008)
LHCb, 0.62 fb ⁻¹	$-0.82 \pm 0.21 \pm 0.11$	3.5	PRL 108, 111602 (2012)
CDF, 9.7 fb ⁻¹	$-0.62 \pm 0.21 \pm 0.10$	2.7	arXiv:1207.2158v1
Belle, 976 fb ⁻¹	$-0.87 \pm 0.41 \pm 0.06$	2.1	

The results are in agreement with our previous measurement and with recent measurements from LHCb and CDF.

Source of syst.	$A_{CP}^{KK}(\%)$	$A_{CP}^{\pi\pi}(\%)$	$\Delta A_{CP}(\%)$
Signal counting	0.055	0.023	0.037
Slow π eff. corr.	0.065	0.067	0.014
A_{CP} extraction	0.006	0.050	0.051
Quadrature sum	0.085	0.087	0.064

$$A_{CP}(D \rightarrow K_S^0 \pi) = \frac{\Gamma(D^+ \rightarrow K_S^0 \pi^+) - \Gamma(D^- \rightarrow K_S^0 \pi^-)}{\Gamma(D^+ \rightarrow K_S^0 \pi^+) + \Gamma(D^- \rightarrow K_S^0 \pi^-)} = A_{CP}^{D^+} + A_{CP}^{\bar{K}_0}$$

- $A_{CP}^{D^+}$: CPV from D^+ decay
- $A_{CP}^{\bar{K}_0}$: CPV in K^0 system, $(-0.332 \pm 0.006)\%$ (J. Phys. G **37** 075021 (2010)).
If measured $A_{CP}(D \rightarrow K_S^0 \pi)$ is significantly inconsistent with $A_{CP}^{\bar{K}_0}$, this could be an indication of NP.

$$A_{rec}(D^+ \rightarrow K_S^0 \pi^+) = A_{CP}(D \rightarrow K_S^0 \pi) + A_{FB}^{D^+} + A_\epsilon^{\pi^+} + A_D$$

- $A_{FB}^{D^+}$: D^+ production asymmetry, odd function of $\cos(\theta_{D^+}^{CMS})$
- $A_\epsilon^{\pi^+}$: π^\pm detection asymmetry, depends on $p_{T\pi^+}^{lab}, \cos(\theta_{\pi^+}^{lab})$, corrected for using control data samples ($D^+ \rightarrow K^- \pi^+ \pi^+, D^0 \rightarrow K^- \pi^+ \pi^0$)
- A_D : asymmetry due to difference in interaction of K^0 and \bar{K}^0 with detector material, depends on $p_{K_S^0}^{lab}$. Calculated according to PRD **84**, 111501 (2011).

$$\Rightarrow A_{rec}^{corr} = A_{CP}^{D^+ \rightarrow K_S^0 \pi^+} + A_{FB}^{D^+}$$

$$A_{CP}^{D^+ \rightarrow K_S^0 \pi^+} = \frac{A_{rec}^{corr}(\cos\theta_D) + A_{rec}^{corr}(-\cos\theta_D)}{2}$$

$$A_{FB}^{D^+} = \frac{A_{rec}^{corr}(\cos\theta_D) - A_{rec}^{corr}(-\cos\theta_D)}{2}$$

$$N_{\text{sign}}(D^+ + D^-) = (1738 \pm 2) \times 10^3$$

$$A_{CP}(D^+ \rightarrow K_S^0 \pi^+) = (-0.363 \pm 0.094 \pm 0.067)\%$$

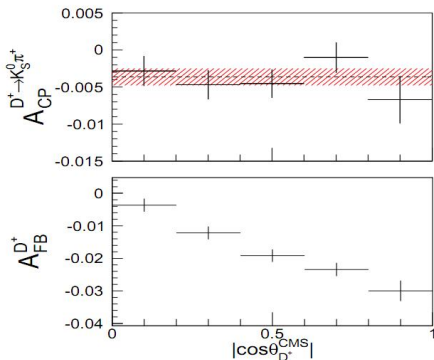
consistent with $A_{CP}^{K_S^0}$,

PRL 109, 021601 (2012)

$$A_{CP}^{D^+} = (-0.018 \pm 0.094 \pm 0.068)\%$$

consistent with zero.

Most precise measurement of $A_{CP}(D^+ \rightarrow K_S^0 \pi^+)$ to date.



Experiment	$A_{CP}(D^+ \rightarrow K_S^0 \pi^+)$, %	Reference
FOCUS	$-1.6 \pm 1.5 \pm 0.9$	PRL 88, 041602 (2002)
CLEO	$-1.3 \pm 0.7 \pm 0.3$	PR D 81, 052013 (2010)
BaBar	$-0.44 \pm 0.13 \pm 0.10$	PR D 83, 071103 (2011)
New world ave.	-0.41 ± 0.09	

Hamiltonian eigenstates of $D^0-\bar{D}^0$ system:

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle, \quad p^2 + q^2 = 1$$

(can be CP -eigenstate if $\frac{q}{p} = 1$, i.e. no CPV)

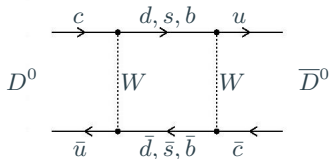
Time evolution:

$$|D_{1,2}(t)\rangle = e_{1,2}(t)|D_{1,2}(0)\rangle,$$

$$e_{1,2}(t) = e^{-i(m_{1,2} - \frac{i}{2}\Gamma_{1,2})t}$$

$$\begin{aligned} \Rightarrow \frac{dN(D^0 \rightarrow f)}{dt} &= |A(D^0(t) \rightarrow f)|^2 = \\ &= e^{-\Gamma t} |A_f - \frac{q}{p} \frac{y+ix}{2} \Gamma t \bar{A}_f|^2 \end{aligned}$$

$$x = \frac{m_1 - m_2}{\Gamma}, \quad y = \frac{\Gamma_1 - \Gamma_2}{2\Gamma}, \quad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$$



SM predictions for x, y :

quark box diagrams $\sim O(10^{-7} - 10^{-6})$, long distance effects $\sim O(10^{-3} - 10^{-2})$.

New Physics can provide additional box diagrams.

3-body charge self conjugated modes: x, y can be extracted using t -dependent Dalitz analysis. $D^0 \rightarrow K_S^0 \pi^+ \pi^-$: $A_f = A(m_-^2, m_+^2)$,
 $m_-^2 = m^2(K_S^0 \pi^-)$, $m_+^2 = m^2(K_S^0 \pi^+)$

Assuming no CPV ($\bar{A}(m_-^2, m_+^2) = A(m_+^2, m_-^2)$, $q/p = 1$), decay rate(time):

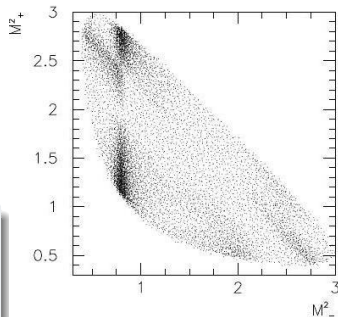
$$|M(m_+^2, m_-^2, t)|^2 =$$

$$(|A_1|^2 e^{-y\Gamma t} + |A_2|^2 e^{y\Gamma t} +$$

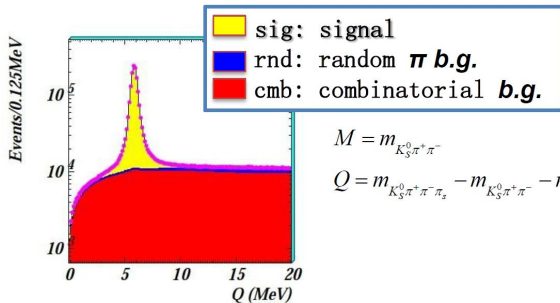
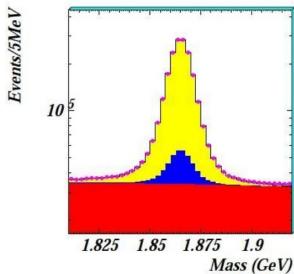
$$2\text{Re}(A_1 A_2^*) \cos(x\Gamma t) + 2\text{Im}(A_1 A_2^*) \sin(x\Gamma t)) e^{-\Gamma t}$$

$$A_{1,2} = \frac{1}{\sqrt{2}}(A_f + \bar{A}_f)$$

Similarly for $|\bar{M}(m_+^2, m_-^2, t)|^2 \dots$



t -integrated Dalitz plot (DP)
 $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ distribution.



$$M = m_{K_S^0 \pi^+ \pi^-}$$

$$Q = m_{K_S^0 \pi^+ \pi^- \pi_s} - m_{K_S^0 \pi^+ \pi^-} - m_{\pi}$$

- $D^{*+} \rightarrow D^0 \pi^+$, $D^{*-} \rightarrow \bar{D}^0 \pi^-$,
 $D^0(\bar{D}^0) \rightarrow K_S^0 \pi^+ \pi^-$
- K_S^0 : $|M_{\pi^+ \pi^-} - M_{K_S}| < 10 \text{ MeV}/c^2$
- D^0 proper decay time measurement:
 $t = \frac{l_{dec}}{c\beta\gamma}$, $\beta\gamma = \frac{p_{D^0}}{M_{D^0}}$
- $\sigma_t < 1000 \text{ fs}$

$\Upsilon(4S)$, $\Upsilon(5S)$ full data set (920 fb^{-1}):

Signal	1.23×10^6
Rand. π_s bckgr.	0.01×10^6
Comb. bckgr.	0.05×10^6

(Signal purity = 95.6%)

$$2\ln\mathcal{L} =$$

$$2 \sum_{i=1}^n \ln(f_{sig} p_{sig}(m_{-i}^2, m_{+i}^2, t_i) + f_{rnd} p_{rnd}(m_{-i}^2, m_{+i}^2, t_i) + f_{cmb} p_{cmb}(m_{-i}^2, m_{+i}^2, t_i))$$

$$p_{sig}(m_{-i}^2, m_{+i}^2, t_i) = \frac{\int_0^{\infty} dt' R_{sig}(t_i - t') |M(m_{-i}^2, m_{+i}^2, t')|^2 \varepsilon(m_{-i}^2, m_{+i}^2)}{\int_0^{\infty} dt \int_D dm_-^2 dm_+^2 |M(m_-^2, m_+^2, t)|^2 \varepsilon(m_-^2, m_+^2)}, \quad f_{cm} = (1 - f_{si} - f_{rn})$$

- f_x determined by 2D $M - Q$ fit,
- DP efficiency functions $\varepsilon(m_-^2, m_+^2)$ estimated from MC sim., parameterized by 3rd order polynomial,
- background shape estimated from $M - Q$ sidebands,
- signal DP distribution convolved with resolution function R_{sig}

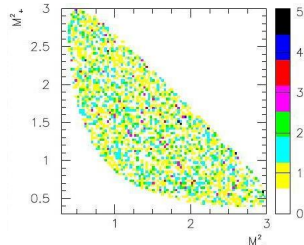
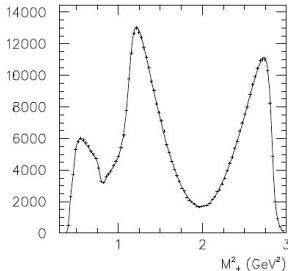
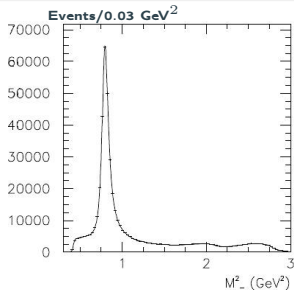
Dalitz model (best fit description)

$$A(m_-^2, m_+^2) = B_{r \neq S} + K_{\pi\pi, S} + L_{K\pi, S}$$

- P-, D-wave:
Breit-Wigner, 12 resonances
- $\pi^+\pi^-$ S-wave: **K-matrix model**
- $K\pi$ S-wave: **LASS model**

Resonances: $K^*(892)$, $K_0^*(1430)^-$, $K_2^*(1430)^-$, $K^*(1410)^-$, $K^*(1680)^-$, $K^*(892)^+$, $K_0^*(1430)^+$, $K_2^*(1430)^+$, $K^*(1410)^+$, $K^*(1680)^+$, $\rho(770)$ (fixed), $\omega(782)$, $f_2(1270)$, $\rho(1450)$, $K_S\pi$ and $\pi\pi$ S-waves.

Fit results (preliminary):



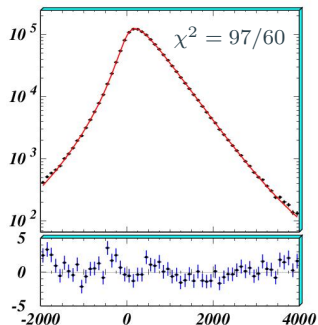
$\chi^2 = 1.246$ for $N_{df} = 3653 - 49$

	fit results			95% C.L. int. (stat. only)
$x(\%)$	0.56 ± 0.19	+0.03 -0.09	+0.06 -0.09	(0.09, 1.03)
$y(\%)$	0.30 ± 0.15	+0.04 -0.05	+0.03 -0.06	(-0.08, 0.68)

Uncertainties: stat., experimental and model syst.

$\tau = 410.3 \pm 0.45$ fs, consistent with

$\tau_{PDG} = 410.1 \pm 1.5$ fs



$\chi^2 = 97/60$

	$x(\%)$			$y(\%)$		
Belle 540 fb⁻¹ PRL 99, 131803 (2007)	0.80 ± 0.29	+0.09 -0.07	+0.10 -0.14	0.33 ± 0.24	+0.08 -0.12	+0.06 -0.08
BaBar 469 fb⁻¹ PRL 105, 081803 (2010)	$0.16 \pm 0.23 \pm 0.12 \pm 0.08$			$0.57 \pm 0.20 \pm 0.13 \pm 0.07$		
Belle 920 fb⁻¹	0.56 ± 0.19	+0.03 -0.09	+0.06 -0.09	0.30 ± 0.15	+0.04 -0.05	+0.03 -0.06

Most precise determination of x, y to date!

Mixing parameter y_{CP}

$$y_{CP} = \frac{\tau_{K\pi}}{\tau_{CP}} - 1, CP : KK, \pi\pi$$

- CP conservation: $y_{CP} = y$
- CP violation: $y_{CP} = y \cos\phi - \frac{1}{2} A_M x \sin\phi, \left|\frac{q}{p}\right|^2 = 1 + A_M, \phi = \arg\left(\frac{q}{p}\right)$

CP asymmetry in lifetimes

$$A_\Gamma = \frac{\tau(\bar{D}^0 \rightarrow CP) - \tau(D^0 \rightarrow CP)}{\tau(\bar{D}^0 \rightarrow CP) + \tau(D^0 \rightarrow CP)}$$

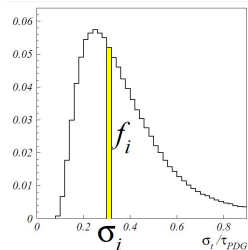
- CP conservation: $A_\Gamma = 0$
- CP violation: $A_\Gamma = \frac{1}{2} A_M y \cos\phi - x \sin\phi$

Belle 2007 measurement (540 fb^{-1}):

- $y_{CP} = (1.31 \pm 0.32 \pm 0.25)\%$
 - $A_\Gamma = (0.01 \pm 0.30 \pm 0.15)\%$
- \Rightarrow first evidence for mixing (3.2σ)

$$f(t) = \frac{N}{\tau} \int e^{-t'/\tau} R(t-t') dt' + B(t)$$

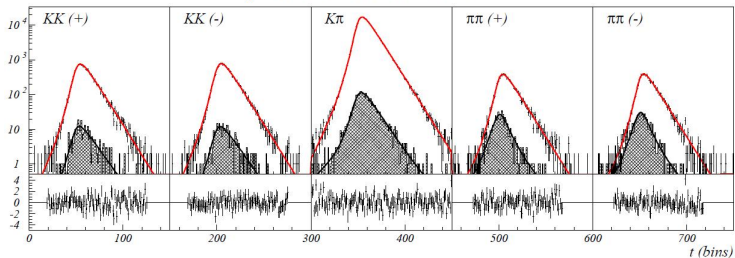
- N : normalization (free fit parameter)
- τ : lifetime (free fit parameter)
- $R(t-t')$: signal resolution function
- $B(t)$: signal-window background estimated from $M-Q$ sidebands
- Large disagreement between data and MC: up to 5% of lifetime \Rightarrow Measurement must be performed in bins of $\cos(\theta^*)$ to reduce systematics + add cut $|\cos(\theta^*)| < 0.9$ (1% signal lost). θ^* – polar angle of D^0 in CMS.



Resolution function constructed for each $\cos(\theta^*)$ bin by fitting $(t - t_{gen})/\sigma_t$.

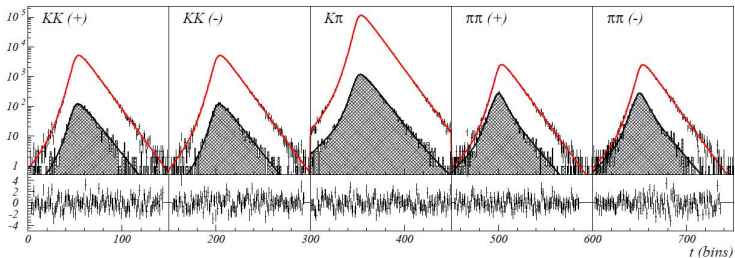
Lifetime fit performed simultaneously for all decay modes:
 $D^0 \rightarrow KK$, $\bar{D}^0 \rightarrow KK$, $D^0 \rightarrow K^- \pi^+$, $D^0 \rightarrow \pi\pi$, $\bar{D}^0 \rightarrow \pi\pi$:

$$\chi^2/n\text{df} = 545.0/542 \quad (\text{CL} = 45.6\%)$$

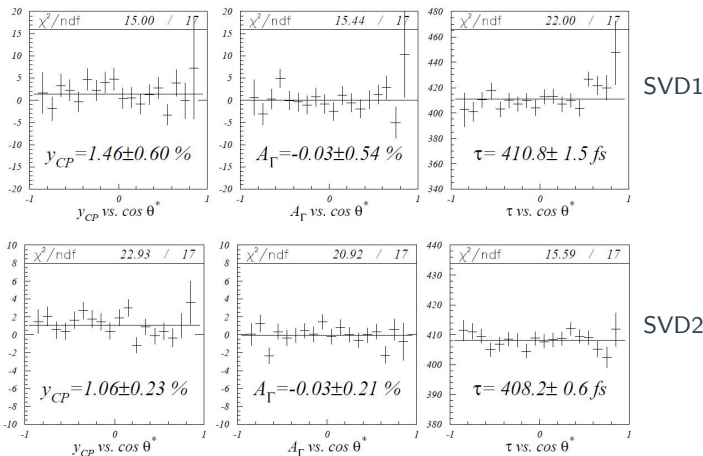


SVD1

$$\chi^2/n\text{df} = 792.9/684 \quad (\text{CL} = 0.2\%)$$



SVD2



$y_{CP}(\%)$	$A_{\Gamma}(\%)$
1.11 ± 0.22	-0.03 ± 0.20

$\tau_{K\pi} = 408.56 \pm 0.54 \text{ fs}$
 (consistent with $\tau_{PDG} = 410.1 \pm 1.5 \text{ fs}$)

source	Δy_{CP} (%)	ΔA_{Γ} (%)
acceptance	0.050	0.044
SVD misalignments	0.060	0.041
mass window position	0.007	0.009
background	0.059	0.050
resolution function	0.030	0.002
binning	0.021	0.010
sum in quadrature	0.11	0.08

$$y_{CP} = (1.11 \pm 0.22 \pm 0.11)\%$$

$$A_{\Gamma} = (-0.03 \pm 0.20 \pm 0.08)\%$$

- y_{CP} : 4.5σ away from zero, stat. + syst. error combined (4.5σ , stat. error only)
- A_{Γ} : consistent with zero

Big data sample and good detector performance allow us to measure A_{CP} and mixing in different decay modes using different techniques.

- $A_{CP}^{KK} = (-0.32 \pm 0.21 \pm 0.09)\%$
 $A_{CP}^{\pi\pi} = (+0.55 \pm 0.36 \pm 0.09)\%$
 $\Delta A_{CP} = (-0.87 \pm 0.41 \pm 0.06)\%$ (agreement with LHCb and CDF)

- $A_{CP}(D^+ \rightarrow K_S^0 \pi^+) = (-0.363 \pm 0.094 \pm 0.067)\%$ is consistent with $A_{CP}^{K_S^0}$
 $A_{CP}^{D^+} = (-0.018 \pm 0.094 \pm 0.068)\%$ consistent with zero.

Most precise measurement to date.

- Mixing in $D^0 \rightarrow K_S^0 \pi^+ \pi^-$:

$$x = \begin{pmatrix} 0.56 \pm 0.19 & +0.03 & +0.06 \\ & -0.09 & -0.09 \end{pmatrix} \% \quad y = \begin{pmatrix} 0.30 \pm 0.15 & +0.04 & +0.03 \\ & -0.05 & -0.06 \end{pmatrix} \%$$

Most precise measurement to date.

- Mixing in $D^0 \rightarrow KK, \pi\pi$:

$$y_{CP} = (1.11 \pm 0.22 \pm 0.11)\% \rightarrow \text{confirmed evidence for mixing in } D \text{ decays}$$

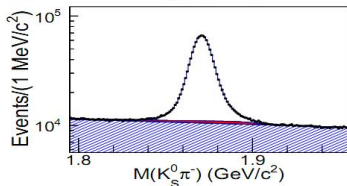
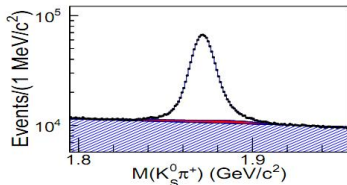
$$A_{\Gamma} = (-0.03 \pm 0.20 \pm 0.08) \rightarrow \text{consistent with zero}$$

We hope for more experimental opportunities with the upgraded Belle II detector.

Thank you.

- $D^+ \rightarrow K_S^0 \pi^+$, $K_S^0 \rightarrow \pi^+ \pi^-$
- $p(D^+) > 2.5(3.0) \text{ GeV}/c$ for $\Upsilon(4S/5S)$ data
- $1.795 < M(K_S^0 \pi^+) < 1.955 \text{ GeV}/c^2$
- $1.855 < M(\pi^+ \pi^-) < 1.885 \text{ GeV}/c^2$
- K_S^0 vertex fit
- D^+ IP constrained fit

$$N_{\text{sign}}(D^+ + D^-) = (1738 \pm 2) \times 10^3$$



Resonance	Amplitude	Phase (deg)		
$K^*(892)^-$	1.593 ± 0.005	131.6 ± 0.2		
$K_0^*(1430)^-$	2.071 ± 0.025	-196.1 ± 2.2		
$K_2^*(1430)^-$	1.136 ± 0.011	-41.2 ± 0.6		
$K^*(1410)^-$	0.503 ± 0.022	83.5 ± 2.3		
$K^*(1680)^-$	1.676 ± 0.157	-85.5 ± 3.5		
$K^*(892)^+$	0.140 ± 0.002	-41.5 ± 0.7		
$K_0^*(1430)^+$	0.174 ± 0.011	-104.8 ± 3.8		
$K_2^*(1430)^+$	0.082 ± 0.009	-34.8 ± 6.8		
$K^*(1410)^+$	0.252 ± 0.017	-141.5 ± 4.0		
$K^*(1680)^+$	1.409 ± 0.117	89.9 ± 5.0		
$\rho(770)$	1 (fixed)	0 (fixed)	$K\pi$ S-wave parameters	
$\omega(782)$	0.0369 ± 0.0005	115.2 ± 0.7	$M(MeV/c^2)$	1462.5 ± 2.4
$f_2(1270)$	1.284 ± 0.019	-31.4 ± 0.9	$\Gamma(MeV/c^2)$	266.1 ± 4.6
$\rho(1450)$	0.556 ± 0.052	84.8 ± 5.0	F	0.4270 ± 0.0309
$\pi\pi$ S-wave			$\phi_F(rad)$	0.261 ± 0.022
β_1	4.26 ± 0.03	163.0 ± 0.5	R	1(fixed)
β_2	10.82 ± 0.04	15.8 ± 0.4	$\phi_R(rad)$	2.528 ± 0.056
β_3	38.2 ± 0.5	1.8 ± 1.0	$a(GeV/c^{-1})$	0.177 ± 0.006
β_4	14.4 ± 0.2	-10.1 ± 0.8	$r(GeV/c^{-1})$	-19.9 ± 0.8
f_{11}^{prod}	12.60 ± 0.07	-161.7 ± 0.5	$K^*(892)$ parameters	
f_{12}^{prod}	14.0 ± 0.2	-175.9 ± 0.7	$M_{K^*(892)} MeV/c^2$	893.69 ± 0.05
f_{13}^{prod}	9.6 ± 0.5	-120.9 ± 2.5	$\Gamma_{K^*(892)} MeV/c^2$	47.63 ± 0.10