SUSY_FLAVOR: a computational tool for FCNC and CP-violating processes in the MSSM

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based on A. Crivellin, P. Chankowski, A. Dedes, S. Jäger, JR and P. Tanedo,
Comput. Phys. Commun. 181:2180, 2010 and arXiv:1203.5023 [hep-ph].

- Sources of the CP and flavor violation in the MSSM
- SUSY_FLAVOR library
  - input parameters
  - hadronic variables
  - available physical observables
- Example of SUSY_FLAVOR application: leptonic B decays
- SUSY_FLAVOR limitations and plans for future development
- Summary

1. Sources of the CP and flavor violation in the MSSM

Flavor and CP violation in the SM:

- relatively simple - determined by the 3 angles and phase of the CKM matrix (also QCD strong phase?)
- neutral currents flavor conserving at the tree level.

Enough to generate very rich phenomenology!

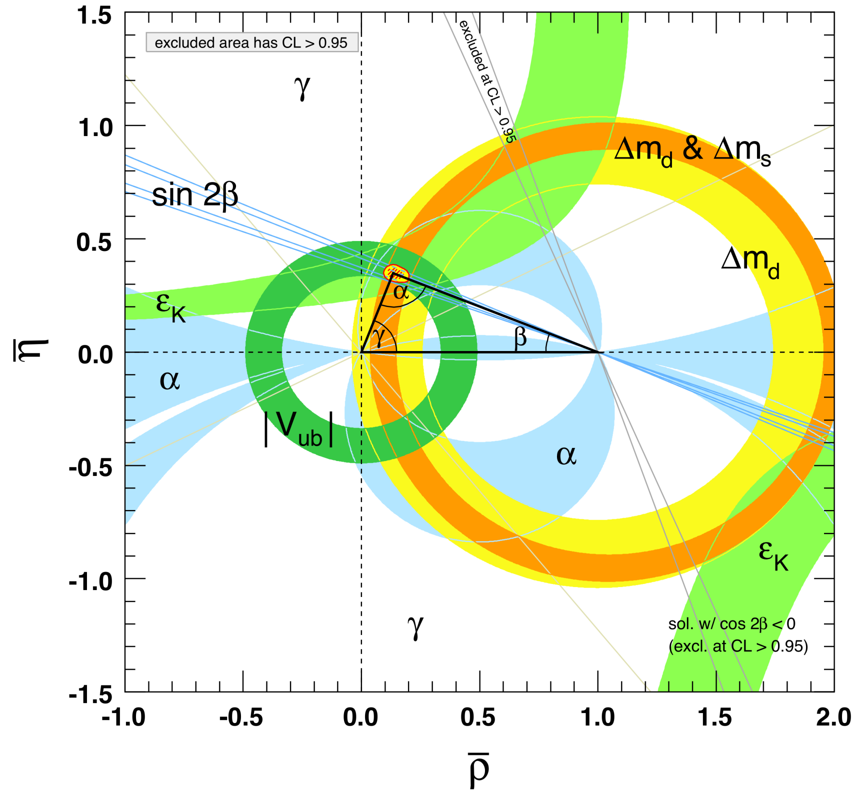
General MSSM much more complicated:

- SM flavor violating couplings replicated in new SUSY vertices
- numerous new sources of CP and flavor violation in the SUSY soft breaking sector.
- in general tree level FCNC present in supersymmetric vertices

General MSSM: 105 free parameters, most of them connected with flavor and CP violation (few hundreds if R-parity is not conserved and/or non-holomorphic soft terms present).

General MSSM: potentially very difficult technical problem - how to disentangle effects of interfering flavor and CP violation sources in each process?

Experiment: SM flavor violation seems to explain current measurements within experimental bounds! Standard test: "unitarity triangle" gives remarkable consistency (plot from PDG review)



Supersymmetric flavor violating terms should be small. How small?

- Charged lepton sector - very strong constraints (neutrino sector?).
- Kaon sector ($1 \leftrightarrow 2$ transitions) - relative off-diagonal soft terms ("mass insertions") of the order of 10^{-3} or less.
- B meson sector ($1, 2 \leftrightarrow 3$ transitions) - constraints more relaxed.

More experiments and data coming, tools needed for efficient analysis!

Calculations of each process usually straightforward (at least at 1-loop SUSY level) but tedious, often higher order effects have to be included.

Numerous analyses by many authors, not easy to combine due to different conventions, model assumptions etc.

Solution for non-flavor experts, experimentalists: ready publicly available "libraries" calculating flavor and CP transitons.

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2. SUSY_FLAVOR library

Few available public "flavor codes" for FCNC calculations mostly restricted to "Minimal Flavor Violation" (MFV) scenario, where CKM elements remain the only source of flavour violation also in MSSM. Examples:

- CPsuperH (Comput. Phys. Commun. 180 (2009) 312)
- SuperIso (arXiv:0808.3144)
- SusyBSG (arXiv:0712.3265) Br($B \to s\gamma$) only, but two loop SUSY corrections

Some flavor observables (MFV version) calculated also by: MasterCode, SPHeno, Micromegas.

Exception: SUSY_FLAVOR v2.0 (Comput. Phys. Commun. **181** (2010) 2180, version 2.0: arXiv:1203.5023). designed to work with general R-parity conserving MSSM, all possible flavor violating terms in the sfermion sector allowed!

SUSY_FLAVOR assumptions:

- Calculations done in the general R-parity conserving MSSM (including even so called "non-holomorphic" trilinear A terms).
- Input parameters set at the SUSY scale in the SLHA2 convention
- SUSY_FLAVOR version 2: chirally enhanced higher order corrections summed to all orders, also for non-minimal flavor violation
- QCD corrections and hadronic matrix elements treated mostly on the basis calculations done in the frame of SM; hadronic related quantities documented and user accessible, for future updates.
- Program written in FORTRAN 77; SUSY_FLAVOR v2.0 contains approximately 15000 lines of code and ~600 subroutines.

Output: currently 30 physical observables in lepton and quark sectors, new ones are being added. Based on series of papers on MSSM flavor physics - 15 publications of 12 authors from 1996 till 2012.

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Calculations in SUSY_FLAVOR pass through the following steps:

1. Parameter Initialization (default: SLHA2 conventions)
2. Calculation of the tree level physical masses and mixing matrices of all MSSM particles
3. Resummation of leading chirally enhanced corrections, calculation of bare and renormalized Yukawa couplings and CKM elements
4. Calculation of Wilson coefficients at the SUSY/EW scale (2-, 3- and 4- point Green functions expanded in the basis of appropriate effective operators. All calculations done in mass eigenstate basis, "mass insertion approximation" not used.
5. Implementation of QCD running from SUSY/EW to low energy scale, evaluation of hadronic matrix elements.
6. Evaluation and printing of physical observables.

The program runs fairly quickly ($\lesssim$ 1 sec for 1 MSSM parameter point on standard PC computer).

SUSY_FLAVOR **input parameters.**

- Standard SM parameters, i.e. $\alpha_{em}, \alpha_s(M_Z), M_Z$, physical charged lepton masses, running quark masses at given scale, CKM parameters (no neutrino masses and U_{PMNS} yet).
- CP-odd Higgs mass M_A, $\tan\beta$, μ parameter and gaugino masses M_1, M_2, M_3 (μ, M_1, M_2 complex in general).
- General form of sfermion soft terms:

$$\begin{aligned} &- (M_Q^2)^{IJ} Q_{Li}^{I*} Q_{Li}^J - (M_D^2)^{IJ} D_R^{I*} D_R^J - (M_U^2)^{IJ} U_R^{I*} U_R^J \\ &- (M_L^2)^{IJ} L_{Li}^{I*} L_{Li}^J - (M_E^2)^{IJ} E_R^{I*} E_R^J \\ &+ \epsilon_{ij} (A_d^{IJ} H_i^1 Q_{Lj}^I D_R^J + A_u^{IJ} H_i^2 Q_{Lj}^I U_R^J + A_l^{IJ} H_i^1 L_{Lj}^I E_R^J + \text{H.c.}) \end{aligned}$$

  $M_Q^2, M_U^2, M_D^2, M_L^2, M_E^2$ are hermitian and A_d, A_u, A_l general complex 3×3 matrices.
- 'Non-holomorphic' trilinear terms A'_d, A'_u, A'_l, again general complex:

$$\epsilon_{ij} (A'^{IJ}_d H_i^{2\star} Q_{Lj}^I D_R^J + A'^{IJ}_u H_i^{1\star} Q_{Lj}^I U_R^J + (A'^{IJ}_l H_i^{2\star} L_{Lj}^I E_R^J + \text{H.c.})$$

Parton-level form factors calculated by SUSY_FLAVOR v2.0

Available set of quark and lepton 2-, 3- and 4-point Green functions:

Box	Penguin	Self energy
$dddd$	$Z\bar{d}d$, $\gamma\bar{d}d$, $g\bar{d}d$	d-quark
$uuuu$	$H_i^0\bar{d}d$, $A_i^0\bar{d}d$	u-quark
$ddll$	$H_i^0\bar{u}u$, $A_i^0\bar{u}u$	charged lepton l
$dd\nu\nu$	$\gamma\bar{l}l$	

where e.g. $dddd$ denote all flavor combinations $d^I d^J d^K d^L$ etc.

More in preparation : $llll$ box (e.g. $\mu \to eee$ decay) etc.

Combinations of Wilson coefficients calculated already by SUSY_FLAVOR allow to calculate many more processes then currently implemented - e.g. $B \to Xll$, various asymmetries, hadronic decays etc. Lot of room for future development.

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Resummation of leading chirally enhanced effects.

Example: correction to bottom quark Yukawa-mass relation

$$Y_b = \frac{m_b\sqrt{2}}{v_d(1-\epsilon_b \tan\beta)}$$

ϵ_b term radiatively generated, of the order of typical EW correction, $\sim$few %. But $\epsilon_b \tan\beta \sim \mathcal{O}(1)$ possible for $\tan\beta \sim 50$!

General MSSM - difficult problem, interplay of CKM and sfermion flavour violation sources (Crivellin, Hofer, JR 2011). Features implemented in SUSY_FLAVOR:

- Finite renormalization of quark and lepton masses and CKM matrix.
- Analytical expressions use in the "decoupling limit" $v \ll M_{SUSY}$ (usually good approximation); iterative calculations used in general case.
- Formulae for the effective resummed gluino, chargino, neutralino and Higgs vertices given in terms of physical masses, renormalized Yukawa couplings and CKM matrix.

Hadronic parameters in SUSY_FLAVOR.

Imported from SM analyses - SUSY corrections hopefully small.

Whenever possible, FORTRAN "common blocks" (global variables) store user accessible hadronic and QCD quantities, can be modified when new results become available. Example - neutral $\bar{K}K$ mixing:

```
common/meson_data/dmk,amk,epsk,fk,dmd,amd,fd,amb(2),dmb(2),gam_b(2),fb(2)
```

M_K^{exp}	amk $= 0.497672$
ΔM_K^{exp}	dmk $= 3.49 \cdot 10^{-15}$
ε_K^{exp}	epsk $= 2.26 \cdot 10^{-3}$
f_K	fk $= 0.1598$

```
common/bx_4q/bk(5),bd(5),bb(2,5),amu_k,amu_d,amu_b
```

$B_1^{\mathrm{VLL}}(\mu_K)$	bk(1) $= 0.61$
$B_1^{\mathrm{SLL}}(\mu_K)$	bk(2) $= 0.76$
$B_2^{\mathrm{SLL}}(\mu_K)$	bk(3) $= 0.51$
$B_1^{\mathrm{LR}}(\mu_K)$	bk(4) $= 0.96$
$B_2^{\mathrm{LR}}(\mu_K)$	bk(5) $= 1.30$
Renormalization scale μ_K	amu_k $= 2$

```
common/sm_4q/eta_cc,eta_ct,eta_tt,eta_b,bk_sm,bd_sm,bb_sm(2)
```

B_{SM}^{VLL}	bk_sm $= 0.724$
η_{cc}	eta_cc $= 1.44$
η_{ct}	eta_ct $= 0.47$
η_{tt}	eta_tt $= 0.57$

All hadronic variables documented in the manual (arXiv or SUSY_FLAVOR web page) and papers cited inside.

SUSY_FLAVOR output: list of observables calculated in v2.0 and their currently measured values or 95% C.L bounds.

Observable	Experiment
$\Delta F = 1$	
$\text{Br}(\mu \to e\gamma)$	$< 2.8 \times 10^{-11}$
$\text{Br}(\tau \to e\gamma)$	$< 3.3 \times 10^{-8}$
$\text{Br}(\tau \to \mu\gamma)$	$< 4.4 \times 10^{-8}$
$\text{Br}(K_L \to \pi^0\nu\nu)$	$< 6.7 \times 10^{-8}$
$\text{Br}(K^+ \to \pi^+\nu\nu)$	$17.3^{+11.5}_{-10.5} \times 10^{-11}$
$\text{Br}(B_d \to ee)$	$< 1.13 \times 10^{-7}$
$\text{Br}(B_d \to \mu\mu)$	$< 0.8 \times 10^{-9}$
$\text{Br}(B_d \to \tau\tau)$	$< 4.1 \times 10^{-3}$
$\text{Br}(B_s \to ee)$	$< 7.0 \times 10^{-5}$
$\text{Br}(B_s \to \mu\mu)$	$< 4.2 \times 10^{-9}$
$\text{Br}(B_s \to \tau\tau)$	$--$
$\text{Br}(B_s \to \mu e)$	$< 2.0 \times 10^{-7}$
$\text{Br}(B_s \to \tau e)$	$< 2.8 \times 10^{-5}$
$\text{Br}(B_s \to \mu\tau)$	$< 2.2 \times 10^{-5}$
$\text{Br}(B^+ \to \tau^+\nu)$	$(1.65 \pm 0.34) \times 10^{-4}$
$\frac{\text{Br}(B_d \to D\tau\nu)}{\text{Br}(B_d \to Dl\nu)}$	$(0.407 \pm 0.12 \pm 0.049)$
$\text{Br}(B \to X_s\gamma)$	$(3.52 \pm 0.25) \times 10^{-4}$

Observable	Experiment
$\Delta F = 0$	
$\frac{1}{2}(g-2)_e$	$(1159652188.4 \pm 4.3) \times 10^{-12}$
$\frac{1}{2}(g-2)_\mu$	$(11659208.7 \pm 8.7) \times 10^{-10}$
$\frac{1}{2}(g-2)_\tau$	$< 1.1 \times 10^{-3}$
$\|d_e\|$(ecm)	$< 1.6 \times 10^{-27}$
$\|d_\mu\|$(ecm)	$< 2.8 \times 10^{-19}$
$\|d_\tau\|$(ecm)	$< 1.1 \times 10^{-17}$
$\|d_n\|$(ecm)	$< 2.9 \times 10^{-26}$
$\Delta F = 2$	
$\|\epsilon_K\|$	$(2.229 \pm 0.010) \times 10^{-3}$
ΔM_K	$(5.292 \pm 0.009) \times 10^{-3}$ ps^{-1}
ΔM_D	$(2.37^{+0.66}_{-0.71}) \times 10^{-2}$ ps^{-1}
ΔM_{B_d}	(0.507 ± 0.005) ps^{-1}
ΔM_{B_s}	(17.77 ± 0.12) ps^{-1}

3. Example of SUSY_FLAVOR application: leptonic B decays

LHC has first chance to measure leptonic $B \to l^+ l^-$ decay, very rare in the SM (prediction first calculated by Buchalla and Buras).

Current experimental status for BR($B_s \to \mu^+ \mu^-$):

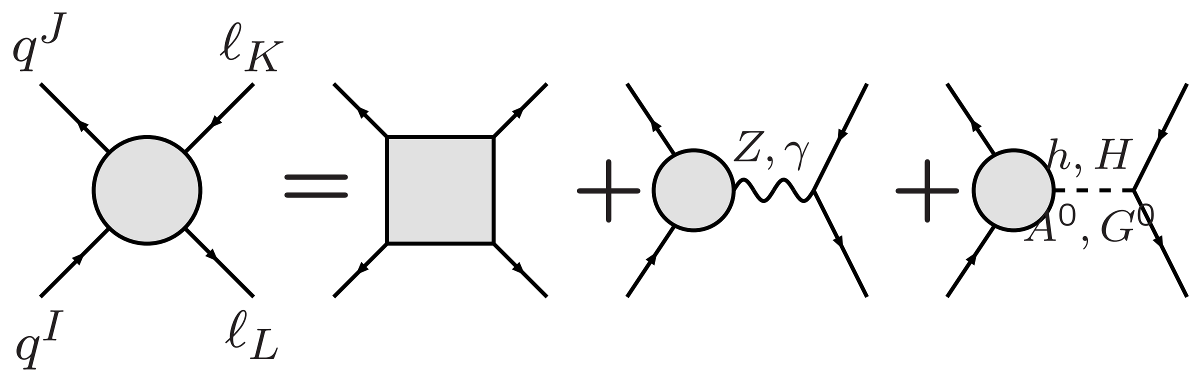
Expt.	Bound (95% CL)	SM Prediction
CMS	$< 7.7 \times 10^{-9}$	
LHCb	$< 4.2 \times 10^{-9}$	$(3.2 \pm 0.2) \times 10^{-9}$

ATLAS and CMS will be able to reconstruct the SM-like $B_s^0 \to \mu^+ \mu^-$ signal with significance of 3σ after $\approx$ 30 fb^{-1}. LHCb already probing $B_s^0 \to \mu^+ \mu^-$ down to the SM prediction, discovery soon?

In MSSM very strong constraints on parameters! For large values of $\tan\beta$ Higgs penguin contribution dominates. In MFV scenario:

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) \approx 5 \cdot 10^{-7} \left(\frac{\tan\beta}{50}\right)^6 \left(\frac{300\ \mathrm{GeV}}{M_A}\right)^4 ,$$

Low(er) $\tan\beta$ regime - gauge penguins and box diagrams with contributions from squark flavor violating terms have to be included.



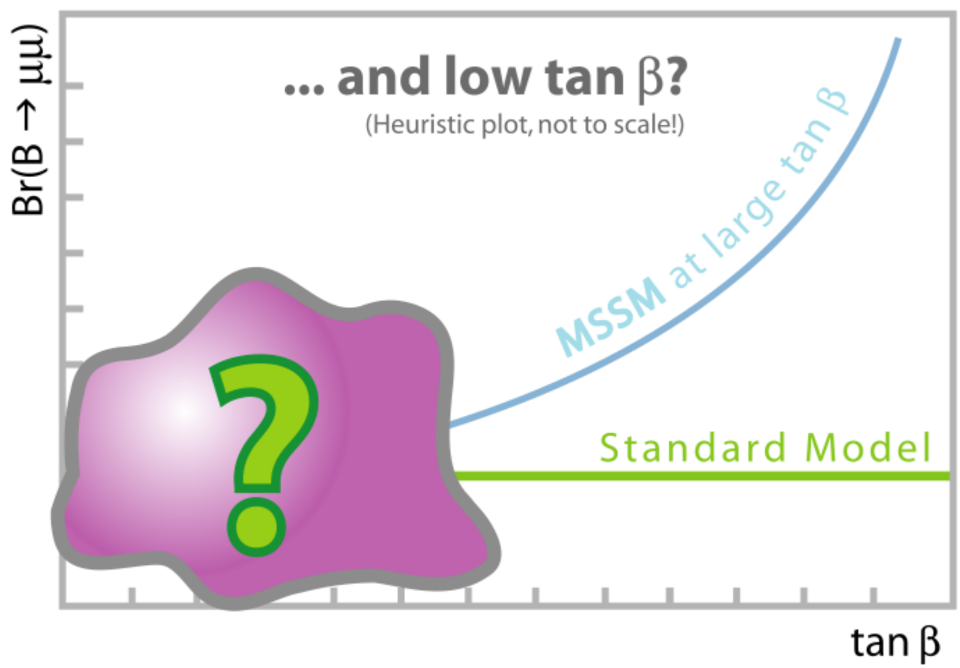
General formulae rather complicated - details in Dedes, JR, Tanedo, arXiv: 0812.4320 [hep-ph].

Simplified expression ($m_\mu/M_{B_q} \to 0$):

$$\mathcal{B}(B_q^0 \to \mu^- \mu^+) \approx \frac{\tau_{B_q} M_{B_q}}{8\pi} \left(|F_S|^2 + |F_P + 2\, m_\mu\, F_A|^2 \right)$$

where formfactors F_S, F_P, F_A are given by SUSY loops.

MFV models - branching ratio enhanced comparing to SM. What in general MSSM?



Two possible scenarios:

1. $\tan\beta \gtrsim 10$, Higgs penguin domination $|F_S| \approx |F_P| \gg 2m_\ell |F_A|$ due to $\tan^2\beta$ enhancement. Thoroughly investigated in the literature, mostly in the MFV limit. Full branching ratio enhanced.

2. $\tan\beta \lesssim 10$, comparable box and Z-penguin contributions, Higgs penguins small. Either an enhancement or a suppression of the branching ratios is possible depending on the choice of MSSM parameters.

Suppression below the SM prediction also possible! Requires a cancellation between various terms. Important from the point of view of Tevatron and LHC searches!

SUSY_FLAVOR a great tool for numerical scan searching for cancellation regions, using multiprocess analysis.

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Numerical setup.

Multi-dimensional scan over the following MSSM parameters:

Parameter	Symbol	Min	Max	Step
Ratio of Higgs vevs	$\tan\beta$	2	30	varied
CKM phase	γ	0	π	$\pi/25$
CP-odd Higgs mass	M_A	100	500	200
SUSY Higgs mixing	μ	-450	450	300
$SU(2)$ gaugino mass	M_2	100	500	200
Gluino mass	M_3	$3M_2$	$3M_2$	0
SUSY scale (1st & 2nd squark generation)	M_{SUSY}	500	1000	500
Slepton Masses	$M_{\tilde{\ell}}$	$M_{\mathrm{SUSY}}/3$	$M_{\mathrm{SUSY}}/3$	0
Left top squark mass	$M_{\tilde{Q}_L}$	200	500	300
Right bottom squark mass	$M_{\tilde{b}_R}$	200	500	300
Right top squark mass	$M_{\tilde{t}_R}$	150	300	150
Mass insertion	δ^{13}_{dLL}, δ^{23}_{dLL}	-1	1	1/10
Mass insertion	δ^{13}_{dLR}, δ^{23}_{dLR}	-0.1	0.1	1/100

δ^{IJ}_{dLL}, δ^{IJ}_{dLR}, μ and M_2 parameters chosen to be real, the trilinear soft couplings set to $A_t = A_b = M_{\tilde{Q}_L}$ and $A_{\tilde{\tau}} = M_{\tilde{\ell}}$.

Most relevant parameters chosen for scan, other do not lead to significant variations of the $\mathcal{B}(B^0_s \to \mu^+\mu^-)$.

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Many variables - constraints from other processes necessary to get meaningful results.

We use constraints on observables calculated by `SUSY_FLAVOR` and bounds on direct SUSY searches (additionally LEP data for the Higgs mass bound, $m_h \geq 92.8 - 114$ GeV depending on the value of $\sin^2(\alpha - \beta)$).

Quantity	Current Measurement	Experimental Error
$m_{\chi_1^0}$	> 46 GeV	
$m_{\chi_1^\pm}$	> 94 GeV	
$m_{\tilde{b}}$	> 89 GeV	
$m_{\tilde{t}}$	> 95.7 GeV	
m_h	> 92.8 GeV	
$\|\epsilon_K\|$	$2.232 \cdot 10^{-3}$	$0.007 \cdot 10^{-3}$
$\|\Delta M_K\|$	$3.483 \cdot 10^{-15}$	$0.006 \cdot 10^{-15}$
$\|\Delta M_D\|$	$< 0.46 \cdot 10^{-13}$	
ΔM_{B_d}	$3.337 \cdot 10^{-13}$ GeV	$0.033 \cdot 10^{-13}$ GeV
ΔM_{B_s}	$116.96 \cdot 10^{-13}$ GeV	$0.79 \cdot 10^{-13}$ GeV
$\mathrm{Br}(B \to X_s \gamma)$	$3.34 \cdot 10^{-4}$	$0.38 \cdot 10^{-4}$
$\mathrm{Br}(K_L \to \pi^0 \nu \bar{\nu})$	$< 1.5 \cdot 10^{-10}$	
$\mathrm{Br}(K^+ \to \pi^+ \nu \bar{\nu})$	$1.5 \cdot 10^{-10}$	$1.3 \cdot 10^{-10}$
Electron EDM	$< 0.07 \cdot 10^{-26}$	
Neutron EDM	$< 0.63 \cdot 10^{-25}$	

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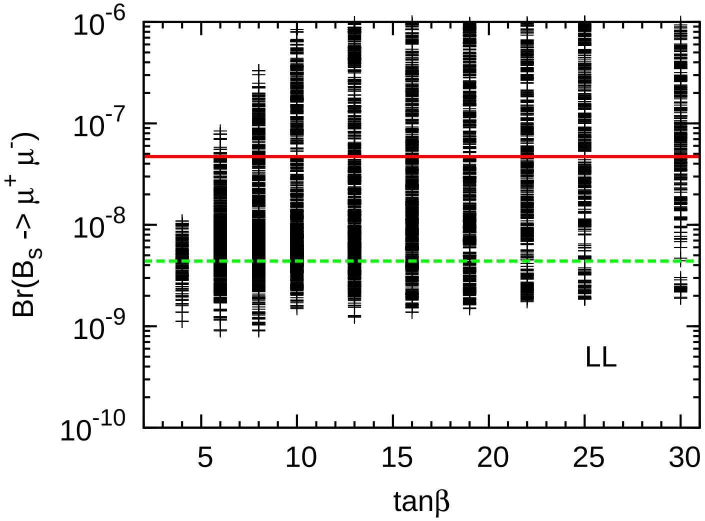
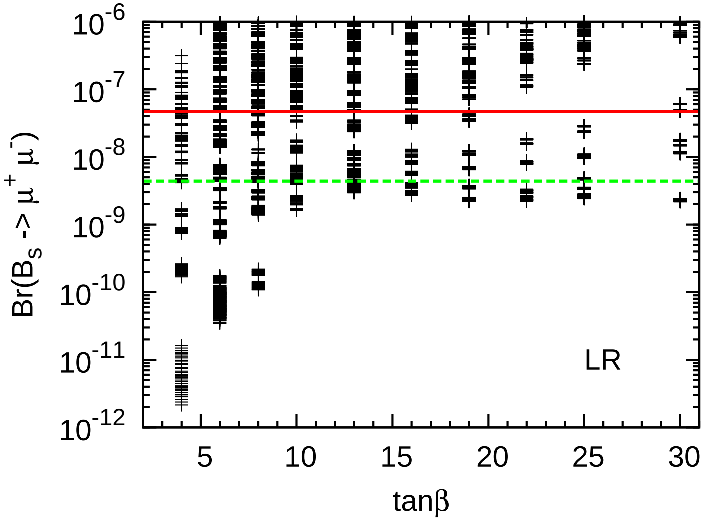
Scan results, $\mathcal{B}(B_s \to \mu^+\mu^-)$ versus $\tan\beta$ (green line: SM prediction):





Left panel: $\delta^{23}_{d\,LL}$ varied, $\delta^{23}_{d\,LR} = 0$, right panel: $\delta^{23}_{d\,LL} = 0$, $\delta^{23}_{d\,LR}$ varied.

Values both above below the SM predictions possible, for all $\tan\beta$ range!

Varying $\delta^{23}_{d\,LL}$: minimal $\mathcal{B}(B^0_s \to \mu^+\mu^-)_{min} \approx 10^{-9}$.
Varying $\delta^{23}_{d\,LR}$: minimal $\mathcal{B}(B^0_s \to \mu^+\mu^-)_{min} \approx 10^{-12}$, 3 orders below the SM (strong cancellation around $\delta^{23}_{d\,LR} \approx -0.01$ and $\tan\beta \lesssim 10$) - effectively unobservable at the LHC!

SUSY hard to discover, but also very hard to kill by any experiment...

4. Limitations and plans for future development

SUSY_FLAVOR limitations

Resummation of chiraly enhanced corrections may became numerically unstable and unreliable in some extreme cases of very large $\tan\beta$ or trilinear A terms - control output for size of corrections to Yukawa coupling and CKM elements always printed, for user judgment.

Some new results in QCD calculations could be accommodated only changing code structure, not just updating variables - new forms of contributions (e.g. some QCD corrections to $b \to s\gamma$ decay).

QCD improvements currently not a priority in SUSY_FLAVOR development - typically few % refinements, flavor violating soft terms can generate effects several orders of magnitude larger than SM predictions.

Plans for future development:

- add observables for new lepton flavor violating processes like $\ell^J \to \ell^K \ell^L \ell^M$, neutrino-related observables, explicit dependence on U_{PMNS} matrix
- add more observables in the B-meson system, e.g. the CP asymmetries in $B\bar{B}$ meson mixing and in $B \to X_s \gamma$ decay, observables associated with $B \to K l^+ l^-$ decay etc. - all SUSY loop formfactors already available, phenomenological formulae to be added.
- include quantities related to FCNCs in the top sector, like $t \to cX$ with $X = \gamma, Z, g, H$, in order to probe the flavor violation in up-squark mass matrices that are (almost) unconstrained to this moment.
- ...

5. Conclusions

- I presented SUSY_FLAVOR, a numerical library for calculating FCNC and CP violating processes in the general R-parity conserving MSSM
- SUSY_FLAVOR v2 calculates 30 interesting FCNC and CP-violating processes.
- Interfaced to SLHA2 for comparisons with other codes
- Powerful tool for multi-process flavor analyses, as shown for example of $B_s^0 \to \mu^+\mu^-$ decay.
- Project under development, new features and processes will be added.
- Hopefully useful both for theorists and experimentalists! Code and documentation can be downloaded from:

http://www.fuw.edu.pl/susy_flavor