Symmetries in multi-Higgs-doublet models

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My typical conversation at *Rencontres de Vietnam* starts with:

— What experiment are you from?

— I am a theorist.

— Oh, I see...

What this talk is about

I am not going to:

- promote some very specific models beyond SM,
- or give detailed predictions for the LHC or astroparticle observables.

I will show some general results about symmetries in the scalar sector with several Higgs doublets.

My motivaton is very pragmatic: many people study numerous variants of this model based on various symmetry groups. But they do it by trial and error. We found a way to approach this problem systematically.

Our goal is to learn what's possible symmetry-wise in models with N Higgs doublets (NHDM).

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Multi-Higgs-doublet models

NHDM are among the most actively studied models of EWSB beyond SM:

- Conceptually simple: "Higgs generations".
- 2HDM is used in MSSM and is interesting on its own.
- Many specific variants of NHDM for $N \geq 3$ were studied (only individual groups are mentioned):

  Weinberg, PRL37, 657 (1976); Adler, PRD59, 015012 (1999); Ma, Rajasekaran, PRD64, 113012 (2001); Ferreira, Silva, PRD78, 116007 (2008); Lavoura, Kuhbock, EPJC55, 303 (2008); Morisi, Peinado, PRD 80, 113011 (2009); Ivanov, JHEP 1007, 020 (2010); de Adelhart Toorop et al, JHEP 1103, 035 (2011); Machado, Montero, Pleitez, PLB 697, 318 (2011); de Medeiros Varzielas, Emmanuel-Costa, PRD84, 117901 (2011); Olaussen, Osland, Solberg, JHEP 1107, 020 (2011); Aranda, Bonilla, Diaz-Cruz, 1204.5558...
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- The focus has been on the fermion mass matrices. However, there is rich physics in the scalar sector, with astroparticle consequences, which is still poorly known.

Scalar sector in 2HDM

Two electroweak doublets, ϕ_1 and ϕ_2, can interact via:

$$\begin{aligned} V &= -\frac{1}{2}\left[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2\,*}(\phi_2^\dagger\phi_1)\right] \\ &\quad + \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &\quad + \left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2) + \lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + \mathrm{h.c.} \end{aligned}$$

V contains 14 free parameters: 4 free parameters m_{ab}^2 and 10 λ's.

Scalar sector in NHDM

We introduce ϕ_a, $a = 1, \dots, N$, and construct the general scalar potential from $(\phi_a^\dagger \phi_b)$'s:

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{abcd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)\,,$$

with N^2 independent components in Y and $N^2(N^2+1)/2$ independent components in Z (e.g. 54 free parameters for $N = 3$).

The main problem: the general potential cannot be minimized explicitly $\to$ other methods are needed to learn at least something about the general NHDM. Many people focus on simple particular variants. But understanding the general case with N doublets is also needed; it is an activity complementary to detailed phenomenological calculations.

One important issue: possible symmetries in the NHDM potential.

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Realizable symmetry groups

Definition:
we call a symmetry group G realizable if there exists a G-symmetric potential which is not symmetric under a larger symmetry group containing G.

Example:
$(\phi_1^\dagger \phi_1)$ is symmetric under cyclic group $\mathbb{Z}_p$, for any integer p, generated by phase rotations by $2\pi/p$. But these $\mathbb{Z}_p$ symmetries are trivial consequences of $(\phi_1^\dagger \phi_1)$ being $U(1)$-invariant under arbitrary phase rotation.
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Symmetries in 2HDM

Some questions concerning symmetries of the Higgs potential:

- Which groups can be realized as symmetry groups of the NHDM Higgs potential?
- Do these groups spontaneously break after EWSB?
- How to find the symmetry group of a given potential?

In 2HDM, all these questions have been answered. The full list of groups realizable as symmetry groups is

$$\mathbb{Z}_2,\ (\mathbb{Z}_2)^2,\ (\mathbb{Z}_2)^3,\ U(1),\ U(1)\times\mathbb{Z}_2,\ SU(2)\,.$$

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Going beyond two doublets

For any $N > 2$ these questions remain unanswered.

Although many people used various (finite) symmetry groups to construct examples of NHDM, the general classification of possible symmetry groups has not been found.

We solved this problem for 3HDM. I will show two results:

- classification of abelian symmetry groups for any N,
- the full list of finite groups realizable in the 3HDM potential.

In both cases I will only focus on groups of unitary transformations (i.e. Higgs-basis transformations).

Antiunitary (i.e. generalized-CP transformations) can be included as well.

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Finding abelian symmetry groups

Outline of the strategy.

- the reparametrization group is not $SU(N)$ but $PSU(N) \simeq SU(N)/\mathbb{Z}_N$;
- we find all maximal abelian subgroups of $PSU(N)$;
- we identify all realizable subgroups in these maximal groups.

"*maximal abelian*" = "*not contained in a larger abelian*".

Maximal abelian subgroups

All maximal abelian groups in $SU(N)$ are maximal tori $T_0 = [U(1)]^{N-1}$.
All of them are conjugate to the group of pure phase rotations (= Cartan subalgebra of $su(N)$).

In $PSU(N)$, there are two types of maximal abelian groups :

- maximal tori $T = [U(1)]^{N-1}$, which are images of T_0 under $SU(N) \to PSU(N)$.
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3HDM example

- maximal torus $T = U(1)_1 \times U(1)_2$, where

$$U(1)_1 = \alpha_1(-1, 1, 0)\,, \quad U(1)_2 = \alpha_2\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)\,,$$

  with $\alpha_i \in [0, 2\pi)$.
- one additional group $\mathbb{Z}_3 \times \mathbb{Z}_3$ which is the image of the extraspecial 3-group generated by

$$a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$

  with $\omega = \exp(2\pi i/3)$. It can be checked that $[a, b] := a^{-1}b^{-1}ab \in Z(SU(N))$, so its image is trivial.

3HDM example

The realizable groups are

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad U(1), \quad U(1) \times \mathbb{Z}_2, \quad U(1) \times U(1)\,.$$

The additional group $\mathbb{Z}_3 \times \mathbb{Z}_3$ is not realizable: a $\mathbb{Z}_3 \times \mathbb{Z}_3$-symmetric potential is automatically symmetric under a larger group, at least $(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_2$.

3HDM example

Examples of the potential. First, we start with a T-symmetric potential

$$V_0 = -\sum_a m_a^2(\phi_a^\dagger\phi_a) + \sum_{a,b}\lambda_{ab}(\phi_a^\dagger\phi_a)(\phi_b^\dagger\phi_b) + \sum_{a\neq b}\lambda'_{ab}(\phi_a^\dagger\phi_b)(\phi_b^\dagger\phi_a)\,,$$

and then add additional terms which select out a desired symmetry group.

- a $\mathbb{Z}_3$-symmetric potential contains

$$\lambda_{1232}(\phi_1^\dagger\phi_2)(\phi_3^\dagger\phi_2) + \lambda_{2313}(\phi_2^\dagger\phi_3)(\phi_1^\dagger\phi_3) + \lambda_{3121}(\phi_3^\dagger\phi_1)(\phi_2^\dagger\phi_1) + h.c.$$

- a $\mathbb{Z}_4$-symmetric potential contains

$$\lambda_{1323}(\phi_1^\dagger\phi_3)(\phi_2^\dagger\phi_3) + \lambda_{1212}(\phi_1^\dagger\phi_2)^2 + h.c.$$

Results for general N

Some results on realizable finite abelian groups A for any N:

- $|A| \leq 2^{N-1}$.
- any cyclic group $\mathbb{Z}_p$, $p \leq 2^{N-1}$, is realizable.
- If $N-1 = \sum_{i=1}^{k} n_i$ is a partitioning of $N-1$ into a sum of non-negative integers n_i, then, the finite group

  $$A = \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2} \times \cdots \times \mathbb{Z}_{p_k}$$

  is realizable for any $0 < p_i < 2^{n_i}$.

For $N = 3, 4, 5$: any finite abelian group with order $\leq 2^{N-1}$ is realizable; starting from $N = 6$ we cannot yet claim this property.

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Finding finite symmetry groups for 3HDM

The strategy:

- Take the list of abelian finite symmetry groups A we recently found for 3HDM.
- Group-theoretic part: prove that any finite symmetry group G must be of the form $A \rtimes K$, where $K \subset Aut(A)$.
- Calculational part: check all possible A's and K's and see whether the potential supports this group.

Structure of finite groups in 3HDM

We start from the list of finite abelian groups:

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad \mathbb{Z}_3 \times \mathbb{Z}_3 \,.$$

This list is complete: imposing any other finite abelian symmetry group on the 3HDM scalar potential unavoidably leads to continuous symmetry group.

Symmetry group G must contain only these abelian subgroups $\Rightarrow$ $|G| = 2^p 3^q$ $\Rightarrow$ by Burnside's theorem, G is solvable $\Rightarrow$ it contains a normal abelian subgroup A.

We proved that this A is maximal $\Rightarrow$ $G/A \subseteq Aut(A)$, and G can then be constructed as an extension of A by $K \subset Aut(A)$.

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Constructing G by extensions

Example: $A = \mathbb{Z}_4$. Then $Aut(\mathbb{Z}_4) = \mathbb{Z}_2$, so G is extension of $\mathbb{Z}_4$ by $\mathbb{Z}_2$.

There are several possibilities:

- extensions which lead to larger abelian groups ($\mathbb{Z}_8$, $\mathbb{Z}_4 \times \mathbb{Z}_2$) are immediately excluded;
- dihedral group D_8, the symmetry group of the square.

$$D_8 = \langle a, b \rangle \text{ with conditions } a^4 = 1,\ b^2 = 1,\ ab = ba^3 .$$

$a, b \in SU(3)$ satisfying these properties can be found and are essentially unique. This symmetry group is realizable.

- quaternion group Q_8:

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We find that imposing Q_8 on the potential leads to a continuous symmetry $\to$ Q_8 is not realizable in 3HDM.

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Results:

$$\mathbb{Z}_2\,, \quad \mathbb{Z}_3\,, \quad \mathbb{Z}_4\,, \quad \mathbb{Z}_2\times\mathbb{Z}_2\,, \quad D_6\,, \quad D_8\,,$$
$$T\simeq A_4\,, \quad O\simeq S_4\,, \quad (\mathbb{Z}_3\times\mathbb{Z}_3)\rtimes\mathbb{Z}_2\,, \quad (\mathbb{Z}_3\times\mathbb{Z}_3)\rtimes\mathbb{Z}_4\,.$$

This list is complete: trying to impose any other finite Higgs-family symmetry group on the 3HDM potential will lead to a potential symmetric under a continuous group.

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Possible uses

- Patterns of spontaneous breaking of each of these symmetry groups can be studied → first systematic analysis of scalar sector phenomenologies possible with three doublets [*work in progress*]..
- Examples of scalar dark matter models based on group $\mathbb{Z}_p$ rather than $\mathbb{Z}_2$ with desired microscopic dynamics can be easily constructed [*Ivanov, Keus, PRD...*].
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