Scattering in Fixed Point Gravity

Gudrun Hiller, Daniel Litim, **Jan Schröder**

Department of Physics and Astronomy
Theoretical Particle Physics Group
University of Sussex

NExT PhD Workshop Sussex
20-22. August 2012

Ingredients

Aim: Examination of scalar scattering with Graviton exchange

Starting point: Linearized Einstein-Hilbert-Lagrangian in $d = 4 + n$ dimensions

- metric $g_{\mu\nu}$ is the carrier of the gravitational force
- n extra dimensions are large and compact
  $\rightarrow$ d-dimensional Planck Mass $M = \mathcal{O}(\mathrm{TeV})$
- extra dimensions generate Kaluza-Klein tower of Gravitons

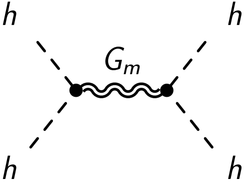
Fixed Point Gravity

But: KK-integral is not UV-safe
$\rightarrow$ use results from **F**ixed **P**oint **G**ravity (cf. Weinberg '79)
UV governed by interacting fixed point g^* of coupling g

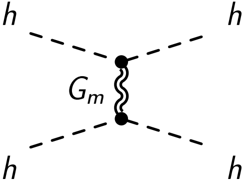
$$\mathbf{I^{(n)}(x)} \propto \int_0^\infty dm\, m^{n-1} P(x, m, \Lambda) \quad , P_{\mathrm{IR}}(x, m, \Lambda) = \frac{1}{x - m^2}$$

- KK-integral becomes UV-safe (no cutoff!)
- use different parametrization for IR-UV transition motivated by FPG [3]
- important parameter: crossover scale Λ, related to g^*

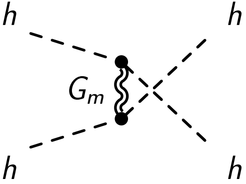
Feynman Diagrams for Higgs (Single Graviton)



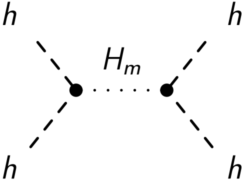
(a) s-channel: G



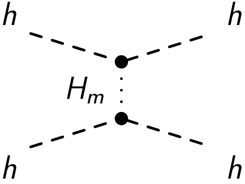
(b) t-channel: G



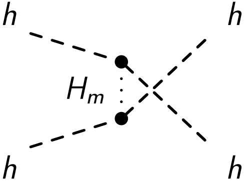
(c) u-channel: G



(d) s-channel: H



(e) t-channel: H



(f) u-channel: H

Higgs Scattering

$$M = I^{(n)}(s)\left(\frac{n\,s^2}{n+2} + 2t^2 + 2ts\right) + I^{(n)}(t)\left(\frac{n\,t^2}{n+2} + 2s^2 + 2ts\right)$$
$$+ I^{(n)}(-s-t)\left(s^2 + t^2 - \frac{2(s+t)^2}{n+2}\right)$$

remember: $I^{(n)}(x) \propto \int_0^\infty dm\, m^{n-1} P(x, m, \Lambda)$

- application of unitarity constraints to examine breakdown energy s resp. bounds on parameters

Unitarity Constraints I

partial wave amplitude for $J = 0$

$$a_0(s) = \frac{1}{16\,\pi\,s} \int_{-s}^{0} M(s,t)\,dt$$

unitarity of the S-matrix implies here:

$$\left|\mathrm{Re}\,(a_0)\right| < 1 \quad \vee \quad |a_0| < 2$$

s-channel unitarity leads to upper bound on Λ:

$$\Lambda_{\mathrm{crit}} = \sup\,\left\{\Lambda \mid \left|a_0^{(s)}(s,\Lambda)\right| < 2\right\}$$

Unitarity Constraints II

t/u channel unitarity is violated at s_{crit}:

$$\left|\text{Re}\left[a_0^{(\text{t,u})}(s_{\text{crit}})\right]\right| = 1$$

- s_{crit} depends on Λ and n
- t/u channel unitarity always violated for large s

Results: Critical s

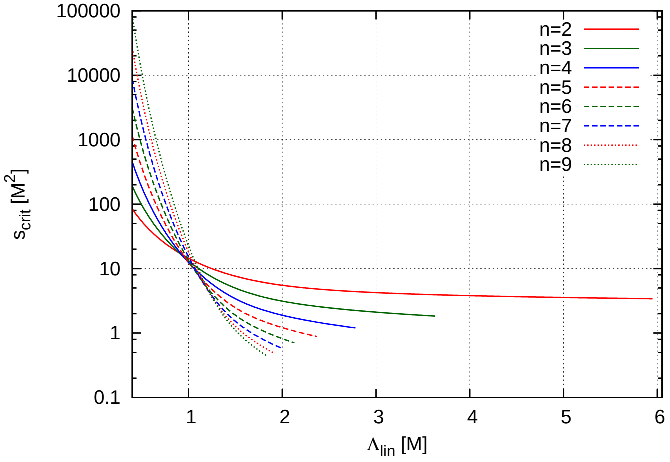


Figure: t/u channel: "Minus", lin, re

Eikonal Approximation

Multiple Graviton Exchange

- for spin 2 exchanges, the eikonal approximation is the nlo contribution in the kinematic regime of low angle scattering $t \ll s$
- eikonal approximation is the summation of an infinite class of ladder diagrams
- **restriction**: only Graviton exchanges, only t-channel diagrams, only leading kinematic contribution in s/t

Eikonal Amplitude

$$A_{\rm Eik} = -4\,\pi\,i\,s \int_0^\infty db\, J_0(q\,b)\,\left(\exp[i\,\chi(b)] - 1\right)$$

$$\chi(b) = \frac{1}{2\,s} \int_0^\infty \frac{dq}{2\,\pi}\, J_0(b\,q)\, A_{\rm Born}(q^2)$$

- q: absolute exchanged momentum $t = -q^2$
- b: impact parameter, [length], Fourier conjugate variable to q

Aims

- calculate χ and A_{Eik} and capture the essential behaviour in arbitrary extra dimensions
- compare results to other scenarios for Eikonal Graviton Exchange, namely Dimensional Regularisation (Giudice, Rattazzi, Wells), Finite Brane Width Scenario (Sjödahl, Gustafson), Effective Field Theory (Litim, Old - work in progress)
- examine unitarity behaviour

Summary

- KK-integration can be made UV-safe using results from **Fixed Point Gravity**
- unitarity breakdown as studied in Higgs scattering via single-graviton exchange typically occurs beyond the fundamental Planck scale: $\sqrt{s_{\mathrm{crit}}} > M$
- crossover scale Λ has upper bound: $\Lambda_{\mathrm{max}} = \mathcal{O}(M)$
- Eikonal calculation will give further insight into the FPG phenomenology of scalar scattering

References

[1] M. Niedermaier, *The Asymptotic Safety Scenario in Quantum Gravity - An Introduction*, arXiv:gr-qc/0610018v2, 19 Jul 2007

[2] J. Brinkmann, *Unitarity of Higgs Scattering in Extra Dimensional Models within Fixed Point Gravity*, Diploma Thesis, Dec 2009

[3] E. Gerwick, D. Litim and T. Plehn, *Asymptotic safety and Kaluza-Klein gravitons at the LHC*, arXiv:1101.5548 [hep-ph].

Parametrizations of the KK Integral

renormalization approach:

$$\int_0^\infty dm\, m^{n-1} \frac{1}{x - m^2}\, Z_{\mathrm{i}}^{-1}(\mu)$$

anomalous dimension approach:

$$\int_0^\infty dm\, m^{n-1} \frac{\Lambda^{2\,\Delta_{\mathrm{i}}}}{(x - m^2)^{1+\Delta_{\mathrm{i}}}}$$

minus approach:

$$\int_0^\infty dm\, m^{n-1} \frac{\Lambda^{2\,\Delta_{\mathrm{i}}}}{(x - m^2)(|x| + m^2)^{\Delta_{\mathrm{i}}}}$$

Approximations for Z and η

quench:

$$Z(\mu) = \begin{cases} 1 & \mu < \Lambda \\ \frac{\mu^{2+n}}{\Lambda^{2+n}} & \mu \geq \Lambda \end{cases}, \quad \eta(\mu) = \begin{cases} 0 & \mu < \Lambda \\ -2-n & \mu \geq \Lambda \end{cases}$$

linear:

$$Z(\mu) = 1 + \frac{\mu^{2+n}}{\Lambda^{2+n}}, \quad \eta(\mu) = -(2+n)(1 - Z^{-1}(\mu))$$

quadratic:

$$Z(\mu) = \sqrt{1 + \left(\frac{\mu^{2+n}}{2\,\Lambda^{2+n}}\right)^2} + \frac{\mu^{2+n}}{2\,\Lambda^{2+n}}, \quad \eta(\mu) = -(2+n)\frac{Z^2(\mu) - 1}{Z^2(\mu) + 1}$$

Λ Matching

different Λ_{param} are matched through IR condition:

$$\lim_{x \to 0} \int_0^\infty dm\, m^{n-1} P_{\text{i}}(x, m, \Lambda_{\text{i}}) = \lim_{x \to 0} \int_0^\infty dm\, m^{n-1} P_{\text{k}}(x, m, \Lambda_{\text{k}})$$