B-$\bar{B}$ mixing and lepton flavor violation in SUSY Grand Unified Models

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Collaboration with Bhaskar Dutta

(based on PRL**97**,241802; PRD**75**,015006; arXiv:0708.3080)

1. Introduction & Basic Scenario
2. Possible discrepancies from SM
3. FCNCs in GUT models (SU(5), SO(10))
4. Relations in flavor violations
5. Conclusion

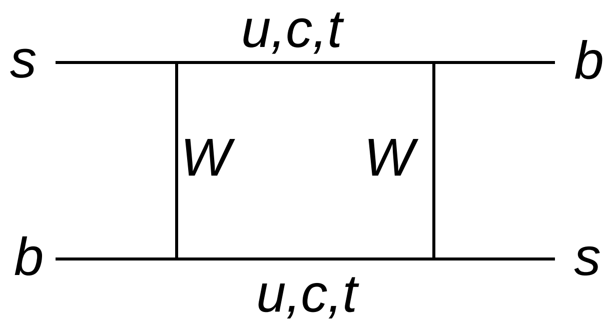
Talk at GUT07 Workshop, 12.17.2007

Introduction

Recent result: B_s-$\bar{B}_s$ mass difference (D0, CDF)

$$\Delta M_s = 17.77 \pm 0.10 \pm 0.07 \ \text{ps}^{-1}$$

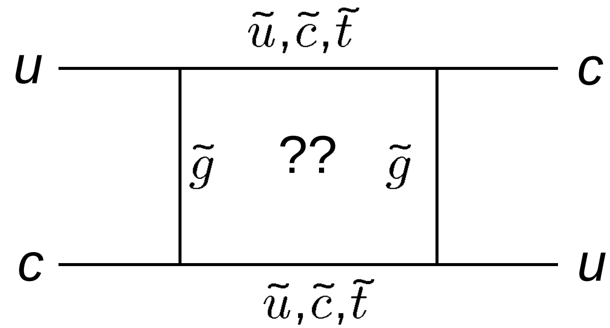
(CDF, hep-ex/0609040)



Recent result: D-$\bar{D}$ mixing (Babar, Bell, 2007)

$$x_D = 8.7^{+3.0}_{-3.4} \times 10^{-3}, \quad y_D = (6.6 \pm 2.1) \times 10^{-3} \quad \text{(HFAG)}$$

$$x_D \equiv \frac{\Delta M_D}{\Gamma_D}, \qquad y_D \equiv \frac{\Delta\Gamma_D}{2\Gamma_D},$$



Additional FCNC constraints

Basic Scenario

Too much FCNCs in general SUSY breaking masses.

Flavor universality of SUSY breaking is assumed.

Even if so, FCNCs are induced by RGEs.

In MSSM, the quark FCNCs are small due to tiny CKM mixings.

If there is a heavy particle, the loop corrections can induce sizable FCNCs.

(e.g. right-handed neutrino)

(Borzumati-Masiero)

Investigating accurate measurement of FCNCs in quarks and leptons is very important to find a footprint of the GUT models.

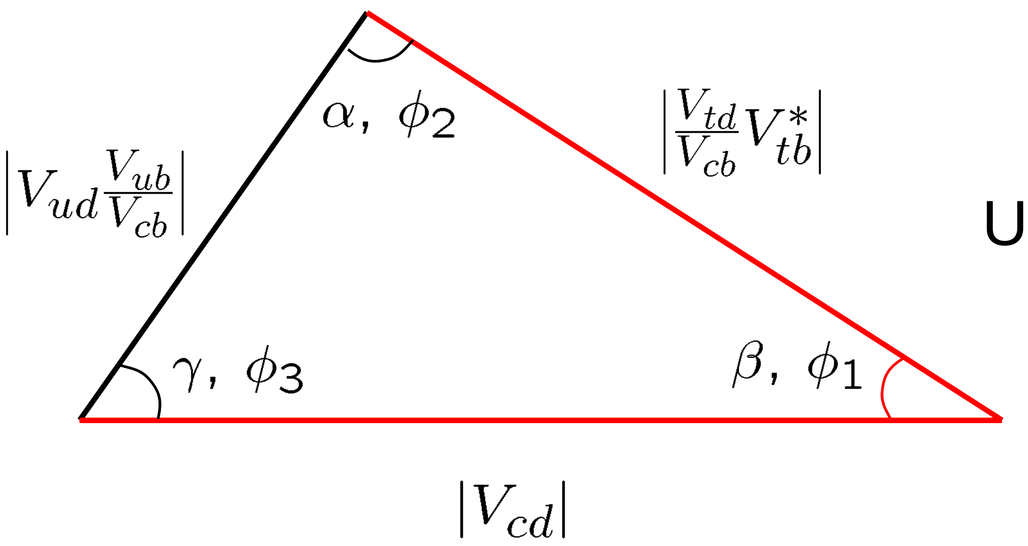
This talk: we study FCNCs from grand unified models SU(5), SO(10)

In this talk, we feature

1. $\sin 2\phi_1$-V_{ub} discrepancy in unitarity triangle

2. phase of B_s-$\bar{B}_s$ mixing

   CP violation of $B_s \to J/\psi\phi$ decay



Unitarity Triangle

$$\sin 2\phi_1 = 0.680 \pm 0.026$$ (World average, Belle and Babar)

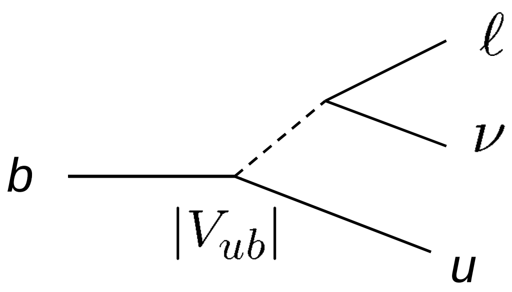
$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.206^{+0.008}_{-0.006} \qquad \frac{\Delta M_s}{\Delta M_d} = \xi^2 \frac{M_{B_s}}{M_{B_d}} \left|\frac{V_{td}}{V_{ts}}\right|^2 \qquad \left(\xi^2 = \frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2} = (1.21 \pm 0.06)^2\right)$$

$$|V_{cd}| = 0.2258, \quad |V_{cb}| = (41.6 \pm 0.6) \times 10^{-3}, \quad |V_{ts}| \simeq |V_{cb}|, \quad |V_{tb}| \simeq 1$$

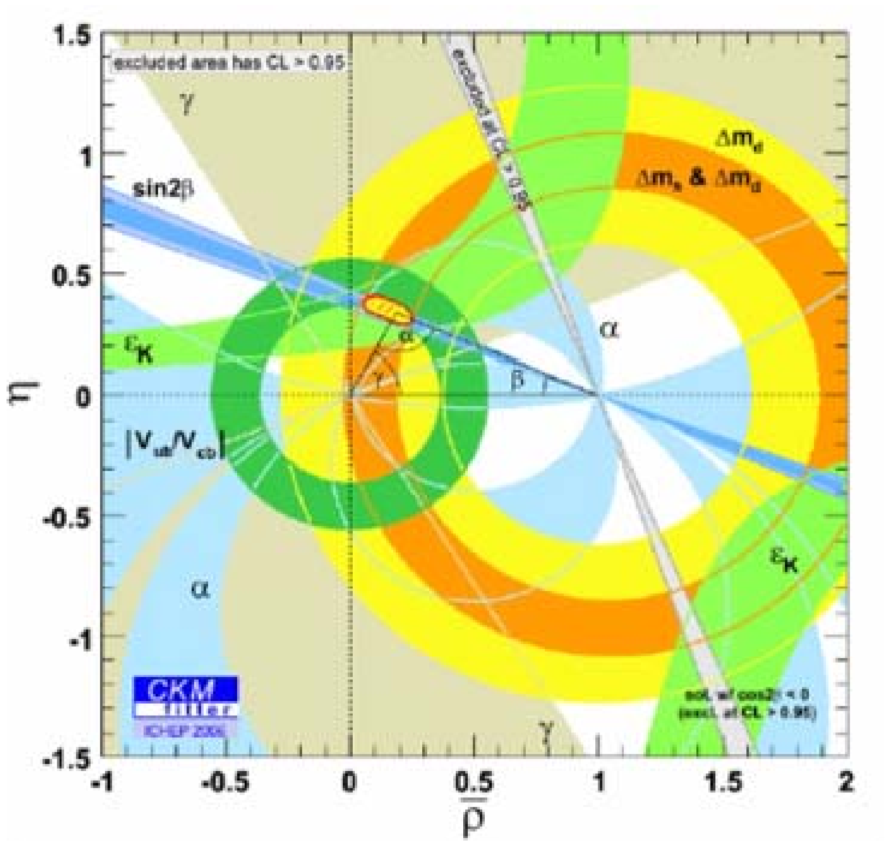
$$|V_{ub}| = (3.52 \pm 0.17) \times 10^{-3}$$ (Unitarity)

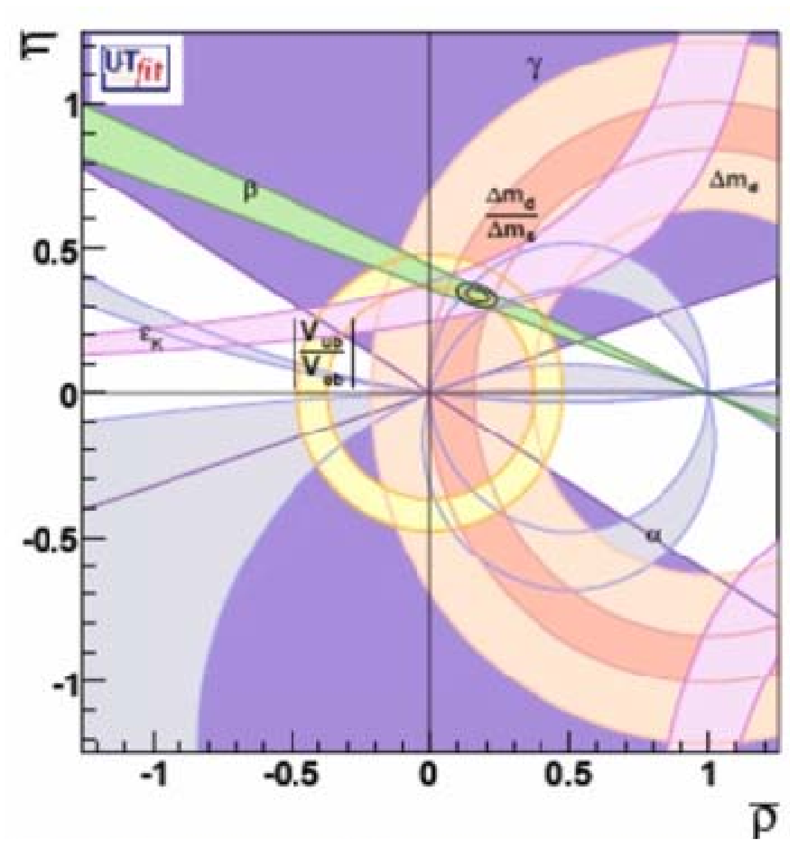
Experimental measurement of $|V_{ub}|$ (Tree-level dominant)

PDG average : $|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}$



(Recently, inclusive decay data become accurate.)





Global fit :

Consistent with SM

But 1.5-2 sigma discrepancy in $\sin 2\phi_1$-V_{ub}

New physics is hiding there ? Or statistical phantom ?

2. phase of B_s-$\bar{B}_s$ mixing

CP violation in $B_s \to J/\psi\phi$ decay

$$M_{12} = \langle B_s|H|\bar{B}_s\rangle$$

$$\Delta M_s = 2|M_{12}| \qquad M_{12} = |M_{12}|e^{-2i\beta_s}$$

SM prediction : $2\beta_s = 0.03 - 0.04$ (rad)

DØ preliminary : $2\beta_s = -0.79^{+0.47}_{-0.39}$ (rad)

(hep-ex/0702030)

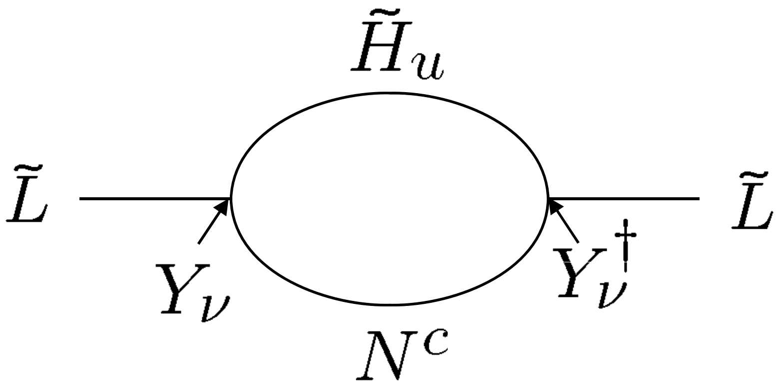
Waiting for more statistics.

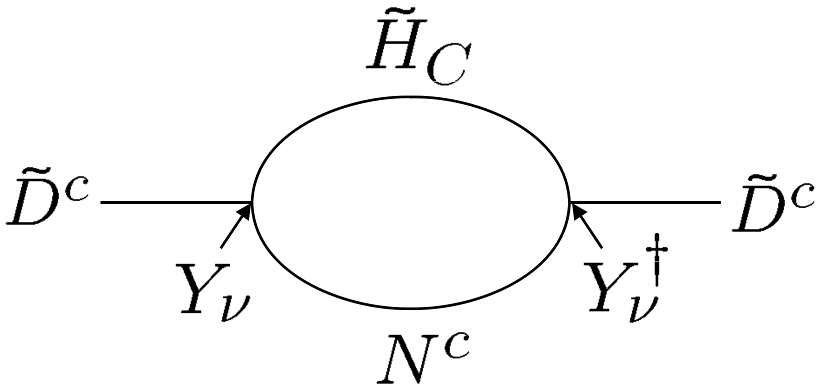
SU(5) GUT

Down quarks (D^c) and lepton doublet (L) are unified in $\bar{\mathbf{5}}$.

$Q, U^c, E^c\ :\ \mathbf{10}$ Right-handed neutrino : N^c

$$Y_u\,\mathbf{10}\cdot\mathbf{10}\,H_5 + Y_d\,\mathbf{10}\cdot\bar{\mathbf{5}}\,H_{\bar{5}} + Y_\nu\,\bar{\mathbf{5}}\,N^c\,H_5$$





Both RH down-squarks and sleptons can have sizable FCNC effects.

(Moroi, Akama-Kiyo-Komine-Moroi, Baek-Goto-Okada-Okumura, …)

SO(10) GUT

All Q, U^c, D^c, L, E^c, N^c are unified in $\mathbf{16}$.

$$h\,\mathbf{16}\cdot\mathbf{16}\,H_{10} + f\,\mathbf{16}\cdot\mathbf{16}\,H_{\overline{126}} + h'\mathbf{16}\cdot\mathbf{16}\,H_{120}$$

$$Y_u = h + r_2 f + r_3 h'$$
$$Y_d = r_1(h + f + h')$$
$$Y_e = r_1(h - 3f + c_e h')$$
$$Y_\nu = h - 3r_2 f + c_\nu h'$$

$$M_\nu^{\text{light}} = \underbrace{M_L}_{\text{Type II}} - \underbrace{Y_\nu M_R^{-1} Y_\nu^{\mathsf{T}} v_u^2}_{\text{Type I}}$$

$$M_L = f_L \langle \Delta_L^0 \rangle \qquad M_R = f_R \langle \Delta_R^0 \rangle$$

Naively, $V^e_{L,R} \sim \mathbf{1}$. $\quad (Y_\nu = V_L^e Y_\nu^{\text{diag}} V_R^{e\dagger})$

The right-handed neutrino loop effects are not very large.

However, $f\,\mathbf{16}\cdot\mathbf{16}\,H_{\overline{126}}$ coupling has large mixings.

The coupling includes the Majorana couplings : $f_L L L \Delta_L + f_R L^c L^c \Delta_R$

$$m_{16}^2 \;\simeq\; m_{\tilde{Q}}^2 \simeq m_{\tilde{U}^c}^2 \simeq m_{\tilde{D}^c}^2 \simeq m_{\tilde{L}}^2 \simeq m_{\tilde{E}^c}^2 \simeq m_{\tilde{N}^c}^2$$

$$m_{16}^2 \;\simeq\; m_0^2 \left(\mathbf{1} - \kappa\, U \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U^\dagger \right)$$

Threshold parameter : $\kappa \simeq \frac{(f_{33}^{\mathrm{diag}})^2}{8\pi^2} \left(3 + \frac{A_0^2}{m_0^2} \right) \ln \frac{M_*}{M_{\mathrm{GUT}}}$

M_*: String/Planck scale

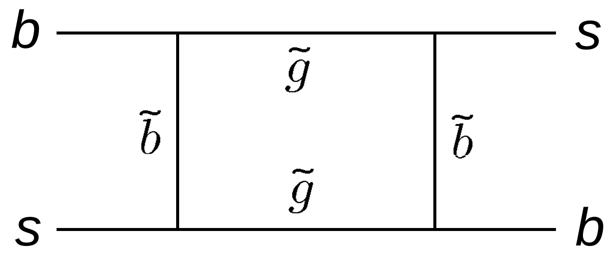
$$f = U f^{\mathrm{diag}} U^{\mathsf{T}} \qquad U \simeq U^*_{\mathrm{MNSP}} \qquad k_2 \simeq \frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}}$$

Both left- and right-squarks have sizable FCNC effects!

SUSY contributions in B-$\bar{B}$ mixings

$$M_{12} = \langle B|H|\bar{B}\rangle \qquad \Delta M = 2|M_{12}|$$

The gluino box diagram dominates.



Mass insertion approximation:

$$\frac{M_{12}^{\mathrm{SUSY}}}{M_{12}^{\mathrm{SM}}} \simeq a[(\delta^d_{LL})^2_{3i} + (\delta^d_{RR})^2_{3i}] - b(\delta^d_{LL})_{3i}(\delta^d_{RR})_{3i} + \cdots$$

$i = 1$ for B_d, $i = 2$ for B_s

$a \sim O(1)$, $b \sim O(100)$ for $m_{\mathrm{SUSY}} \sim 1$ TeV (Ball-Khalil-Kou)

$$\delta^d_{LL,RR} = (M^2_{\tilde{d}})_{LL,RR}/\tilde{m}^2 \qquad \tilde{m}: \text{ average squark mass}$$

$$(\tilde{d}_L, \tilde{d}_R)\begin{pmatrix} (M^2_{\tilde{d}})_{LL} & (M^2_{\tilde{d}})_{LR} \\ (M^2_{\tilde{d}})_{RL} & (M^2_{\tilde{d}})_{RR} \end{pmatrix}\begin{pmatrix} \tilde{d}^\dagger_L \\ \tilde{d}^\dagger_R \end{pmatrix}$$

$$(M^2_{\tilde{d}})_{LL} = m^2_{\tilde{Q}} + \cdots$$

$$(M^2_{\tilde{d}})_{RR} = (m^2_{\tilde{D}^c})^{\mathsf{T}} + \cdots$$

Both left- and right-squarks have sizable FCNC effects in SO(10).

$$\frac{M_{12}^{\mathrm{SUSY}}}{M_{12}^{\mathrm{SM}}} \simeq a[(\delta^d_{LL})^2_{3i} + (\delta^d_{RR})^2_{3i}] - b(\delta^d_{LL})_{3i}(\delta^d_{RR})_{3i} + \cdots$$

$i = 1$ for B_d, $i = 2$ for B_s

$a \sim O(1)$, $b \sim O(100)$ for $m_{\mathrm{SUSY}} \sim 1$ TeV

Flavor violating effects are larger in the box diagram in SO(10).

Cf. Only δ^d_{RR} is large in SU(5).

Remark :

Accurate measurement of mass difference is consistent with SM.

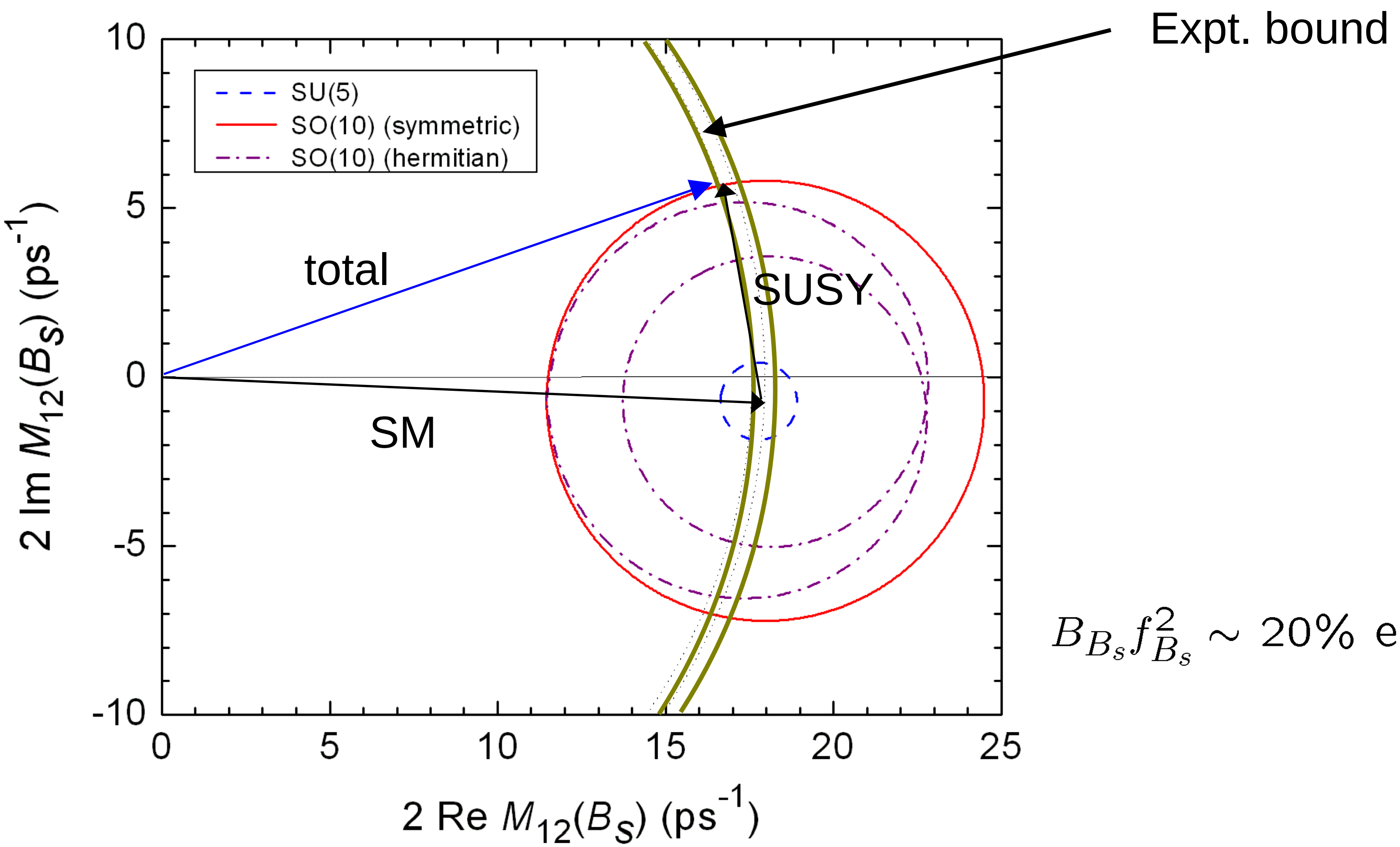
However, experimental result of $\Delta M_s = 2|M_{12}(B_s)|$
does not constrain size of SUSY contribution $|M_{12}^{\mathsf{SUSY}}|$ much.

$$M_{12} = M_{12}^{\mathsf{SM}} + M_{12}^{\mathsf{SUSY}}$$

arg M_{12}^{SUSY} is a free parameter in the model:

due to free phase in Yukawa couplings

There is room for sizable SUSY contribution. $\Longrightarrow$ Next page



$B_{B_s} f_{B_s}^2 \sim 20\%$ error

One can always find a solution as long as $|M_{12}^{\mathrm{SUSY}}| < 2|M_{12}^{\mathrm{SM}}|$.

Accurate measurement of not only the mass difference but also the B_s-$\bar{B}_s$ phase is very important.

$$\frac{M_{12}^{\mathsf{SUSY}}}{M_{12}^{\mathsf{SM}}} \propto (\delta_{LL}^{d})_{ji}(\delta_{RR}^{d})_{ji}$$

$$ji = 12 \;:\; K\text{-}\bar{K}$$
$$ji = 13 \;:\; B_d\text{-}\bar{B}_d$$
$$ji = 23 \;:\; B_s\text{-}\bar{B}_s$$

$$(M_{\tilde{d}}^2)_{LL} = m_{\tilde{Q}}^2 + \cdots, \qquad (M_{\tilde{d}}^2)_{RR} = (m_{\tilde{D}^c}^2)^{\mathsf{T}} + \cdots$$

$$m_{\tilde{Q}}^2 \simeq m_{\tilde{U}^c}^2 \simeq m_{\tilde{D}^c}^2 \simeq \; m_0^2 \left(\mathbf{1} \underbrace{\boxed{- \kappa\, U \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U^\dagger}}_{\delta_{ji}} \right)$$

Parameterization of phase:

$$U = P U_q \qquad P = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix}$$

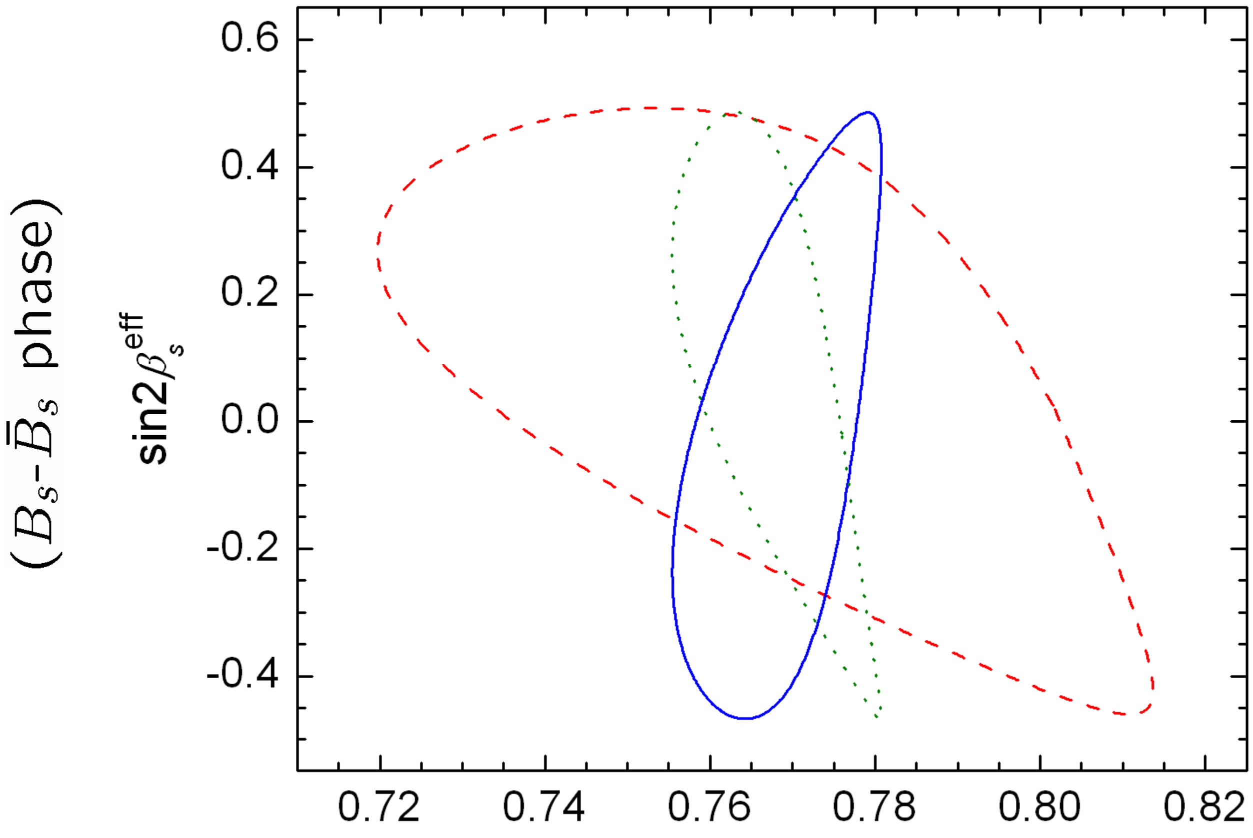
Phases in *P* are cancelled in the SUSY contribution.

$$Y_u = V_{\mathsf{CKM}}^{\mathsf{T}} Y_u^{\mathsf{diag}} P_u V_{\mathsf{CKM}} \qquad Y_d = Y_d^{\mathsf{diag}} P_d$$

P_d provides the phase of SUSY contribution of K-$\bar{K}$, B_d-$\bar{B}_d$, B_s-$\bar{B}_s$

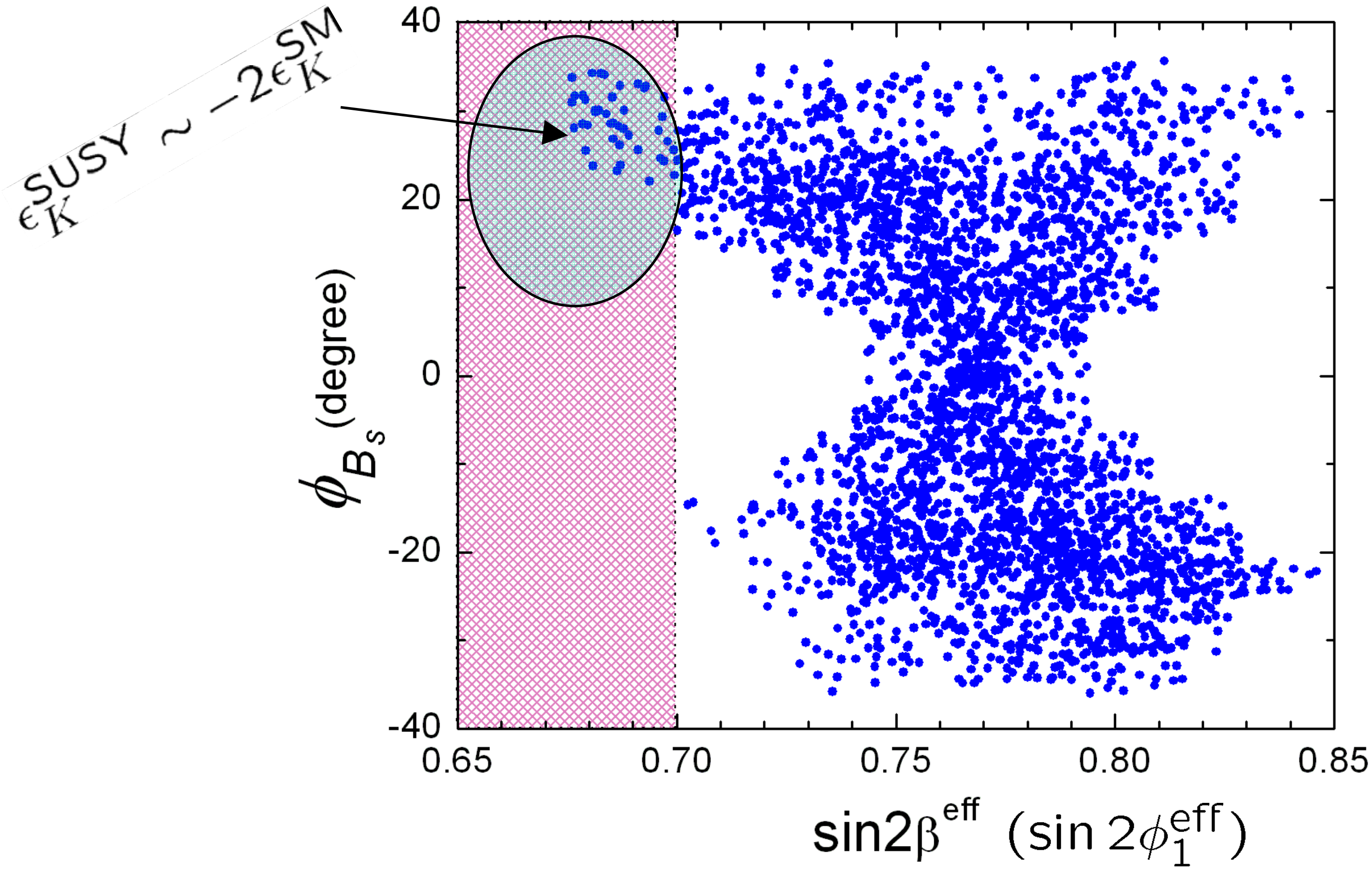
There are only two phase freedom (approximately) for SUSY contributions to K-$\bar{K}$, B_d-$\bar{B}_d$, B_s-$\bar{B}_s$ mixing amplitudes.

The SUSY modifications of the phases are related.



SUSY K-$\bar{K}$ phase is fixed as $0, \pi/2, \pi$.

ϕ_{B_s} vs $\sin 2\beta$ with ϵ_K constraint



$\beta_s^{SM} \simeq 1^\circ$ $\qquad$ $\beta_s^{eff} = \beta_s^{SM} - \phi_{B_s} = (-23 \pm 16)^\circ$

DØ preliminary

$$m^2_{\tilde{Q}} \simeq m^2_{\tilde{U}^c} \simeq m^2_{\tilde{D}^c} \simeq \; m^2_0 \left(\mathbf{1} - \kappa\, U \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U^\dagger \right)$$

$$U = P U_q \qquad U_q = (\theta^q_{12}, \theta^q_{13}, \theta^q_{23}, \delta^q)$$

$$K\text{-}\bar{K}\ : \qquad |\delta^d_{12}| \simeq \kappa \left| \tfrac{1}{2} k_2 \sin 2\theta^q_{12} \cos\theta^q_{23} + e^{i\delta^q} \sin\theta^q_{13} \sin\theta^q_{23} \right|$$

$$B_d\text{-}\bar{B}_d\ : \qquad |\delta^d_{13}| \simeq \kappa \left| \tfrac{1}{2} k_2 \sin 2\theta^q_{12} \sin\theta^q_{23} - e^{i\delta^q} \sin\theta^q_{13} \cos\theta^q_{23} \right|$$

$$B_s\text{-}\bar{B}_s\ : \qquad |\delta^d_{23}| \simeq \tfrac{1}{2} \kappa \sin 2\theta^q_{23}$$

SUSY contribution of K-$\bar{K}$ mixing can be cancelled when $\delta^q \simeq \pi$.

D-$\bar{D}$ mixing amplitude can be calculated in a similar formula, but it is calculated in the up-type quark Yukawa diagonal basis.

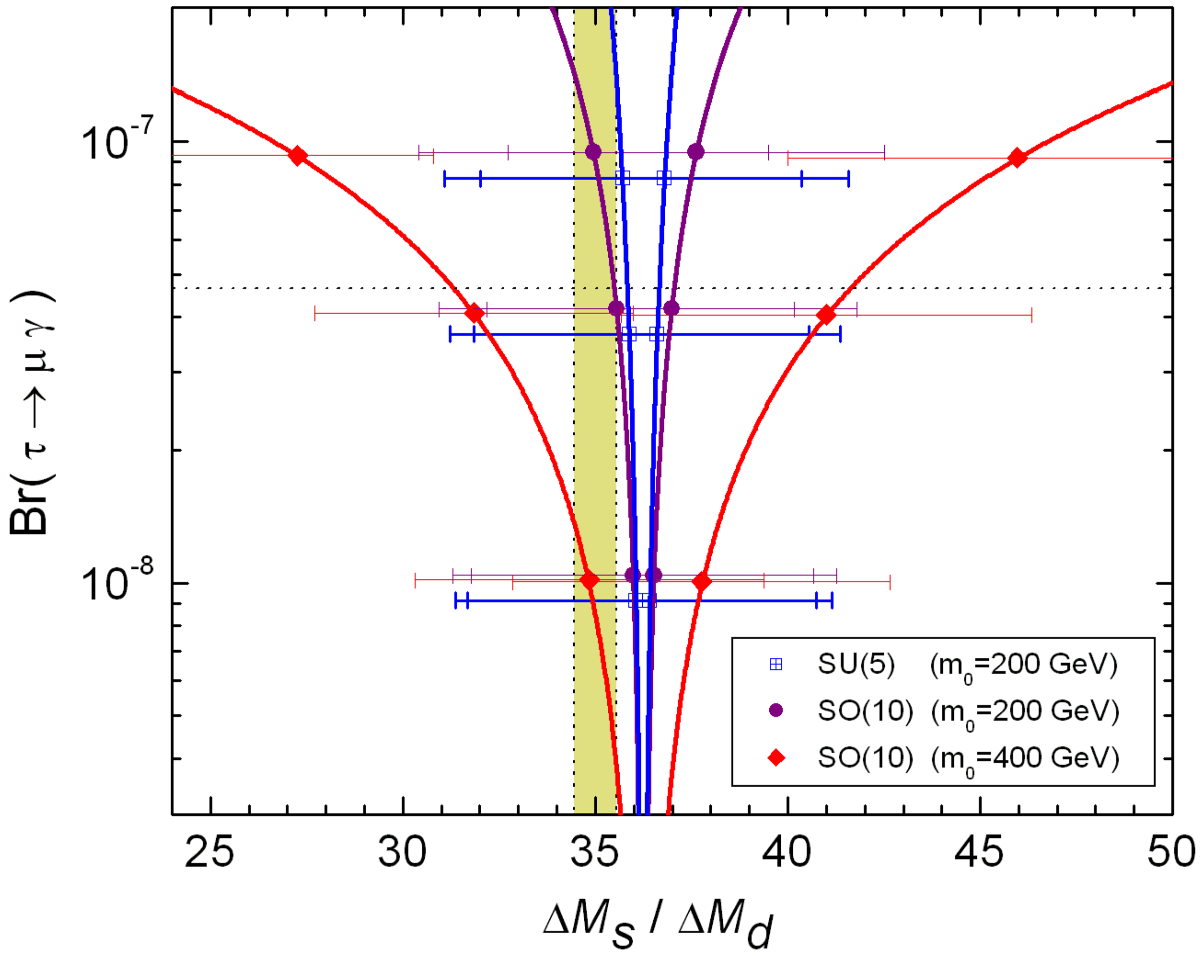
$$\delta^u_{12} \simeq [V^*_{\rm CKM}(\delta^d)V^{\rm T}_{\rm CKM}]_{12} \simeq \delta^d_{12} + V_{us}\kappa \sin^2\theta^q_{23}$$

K-$\bar{K}$ and D-$\bar{D}$ cannot be cancelled simultaneously if $\kappa \sin^2\theta^q_{23}$ is sizable.

$\bar{D}$-$\bar{D}$ data constrain $\kappa \sin^2\theta^q_{23}$. (arXiv:0708.3080)

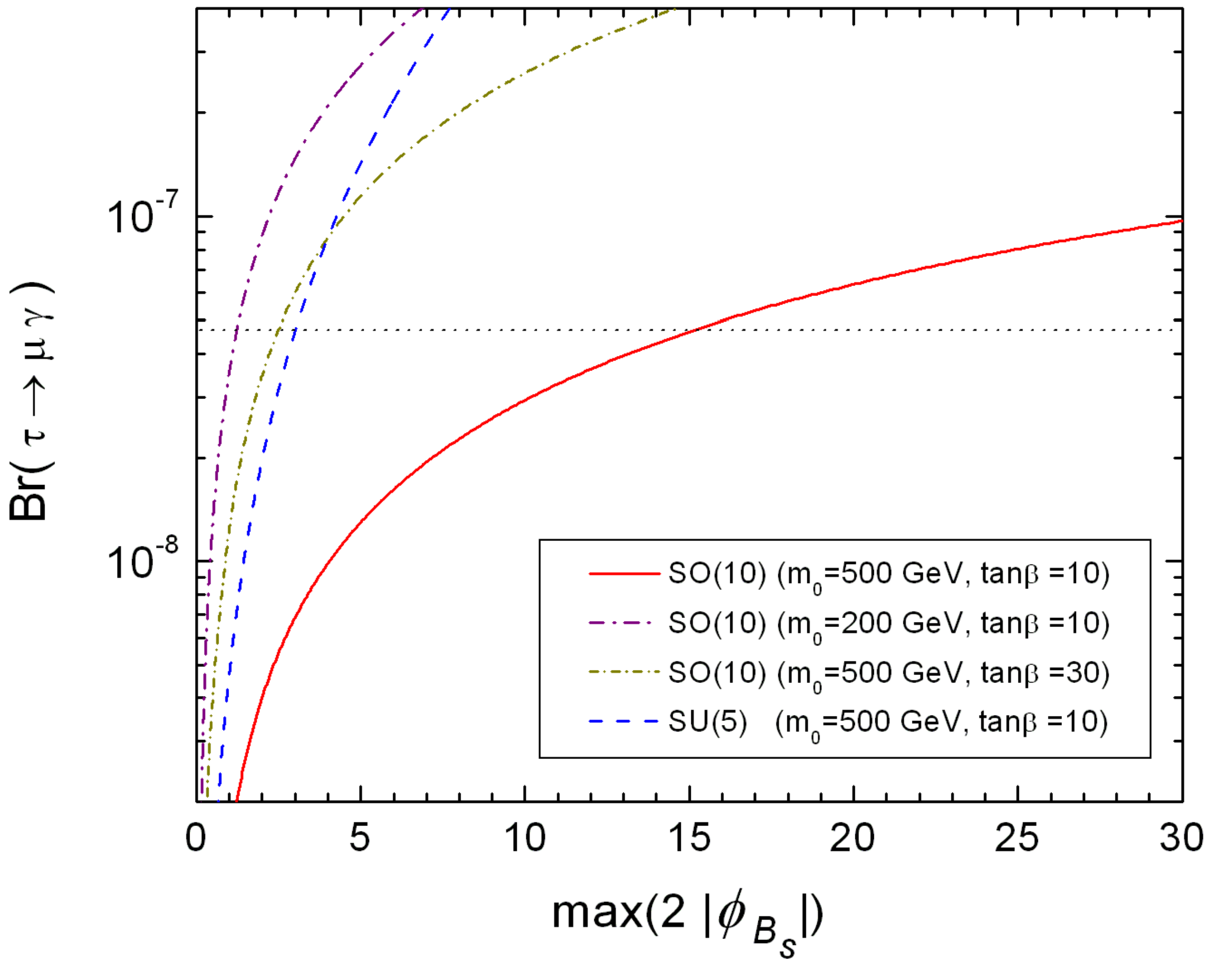
In the scenario where $\sin 2\phi_1$ and V_{ub} discrepancy is solved by the SUSY contribution, SUSY contribution may also affect to the recently measured D-$\bar{D}$ mixing.

B_s-$\bar{B}_s$ mixing and $\tau \to \mu\gamma$



$m_{1/2} = 300$ GeV, $\tan\beta = 10$

$\mathrm{Br}(\tau \to \mu\gamma)$ constrains the CP phase of B_s decay



$\mathrm{BR} < 4.5 \times 10^{-8}$

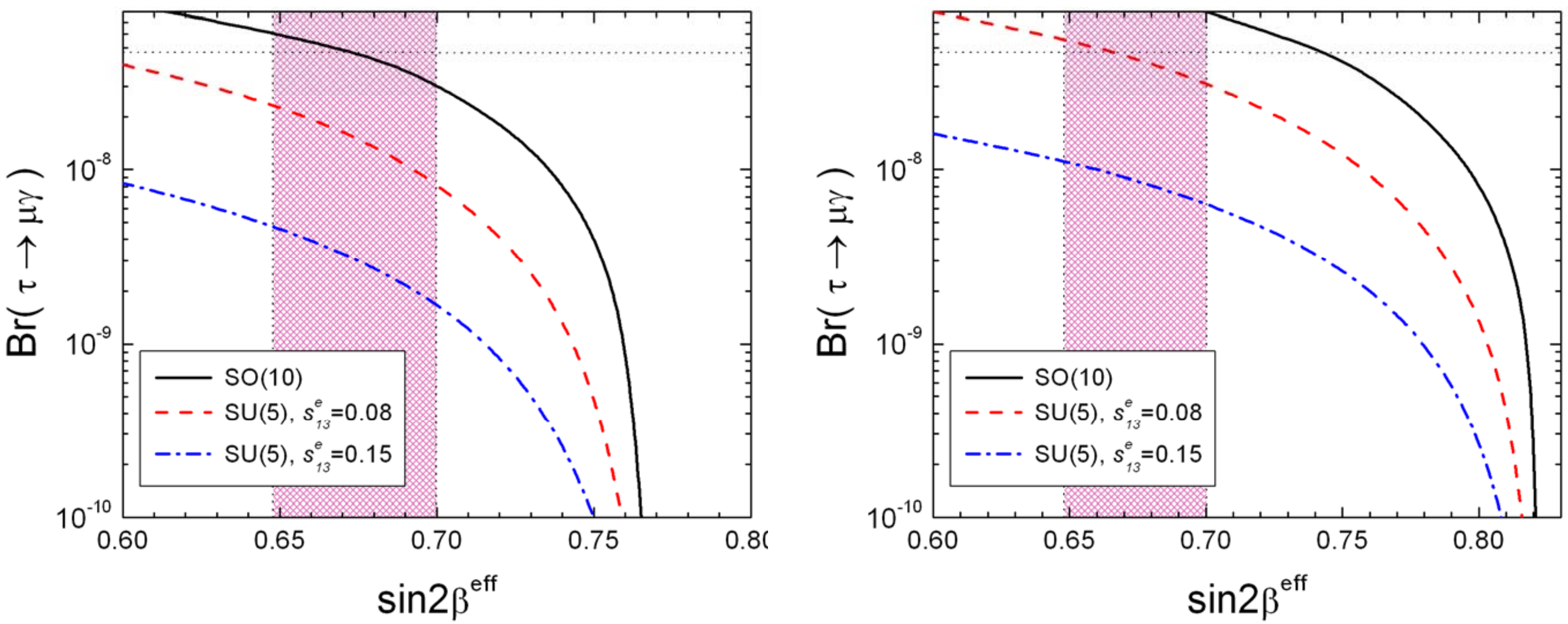
(Belle)

$$C_{B_s} e^{2i\phi_{B_s}} = \frac{M_{12}^{\text{full}}}{M_{12}^{\text{SM}}}$$

$$\beta_s^{\text{SM}} \simeq 1^\circ, \qquad \beta_s^{\text{eff}} = \beta_s^{\text{SM}} - \phi_{B_s} = (-23 \pm 16)^\circ$$

DØ preliminary

$\mathrm{Br}(\tau \to \mu\gamma)$ vs $\sin 2\phi_1$



$|V_{ub}| = 0.0041$ $\qquad$ $|V_{ub}| = 0.0045$

$$m_0 = 1.2 \text{ TeV},\ m_{1/2} = 300 \text{ GeV},\ \tan\beta = 10,\ A_0 = 0$$

We have assumed

$$m_{16}^2 \simeq m_{\tilde{Q}}^2 \simeq m_{\tilde{U}^c}^2 \simeq m_{\tilde{D}^c}^2 \simeq m_{\tilde{L}}^2 \simeq m_{\tilde{E}^c}^2 \simeq m_{\tilde{N}^c}^2$$

$$m_{16}^2 \simeq m_0^2 \left(\mathbf{1} - \kappa\, U \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U^\dagger \right)$$

But, κ may depend on the fermion species when (some of) decomposed fields from 126 and 120 Higgses split from the others.

Variety of induced FCNCs depends on the light decomposed field.

Accurate measurement of quark-lepton FCNCs deviation from SM

Information of GUT breaking vacua

quark-quark-Higgs coupling

$(qq)_s$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	colored Higgs	
$(qq)_a$	$(\bar{\mathbf{3}}, \mathbf{3}, \frac{1}{3})$	cause proton decay	
$(qq)_s$	$(\mathbf{6}, \mathbf{3}, \frac{1}{3})$	light in flipped-$SU(5)$	
$(qq)_a$	$(\mathbf{6}, \mathbf{1}, \frac{1}{3})$	favorable to suppress p decay	(*)
qu^c, qd^c	$(\mathbf{1}, \mathbf{2}, \pm\frac{1}{2})$		
qu^c, qd^c	$(\mathbf{8}, \mathbf{2}, \pm\frac{1}{2})$	favorable to suppress p decay	(*)
$u^c u^c$	$(\mathbf{3}, \mathbf{1}, -\frac{4}{3})$	cause proton decay	
$u^c d^c$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	colored Higgs	
$d^c d^c$	$(\mathbf{3}, \mathbf{1}, \frac{2}{3})$	PS higgsino	
$u^c u^c$	$(\bar{\mathbf{6}}, \mathbf{1}, -\frac{4}{3})$		
$u^c d^c$	$(\bar{\mathbf{6}}, \mathbf{1}, -\frac{1}{3})$	favorable to suppress p decay	(*)
$d^c d^c$	$(\bar{\mathbf{6}}, \mathbf{1}, \frac{2}{3})$	light in flipped-$SU(5)$	

(*) arXiv:0712.1206 (Dutta-YM-Mohapatra) lepton-lepton-Higgs coupling

$(\ell\ell)_s$	$(\mathbf{1},\mathbf{3},-1)$	important in type II seesaw
$(\ell\ell)_a$	$(\mathbf{1},\mathbf{1},-1)$	$SU(2)_R$ higgsino
$\ell\nu^c, \ell e^c$	$(\mathbf{1},\mathbf{2},\pm\frac{1}{2})$	important in type I seesaw
$\nu^c\nu^c$	$(\mathbf{1},\mathbf{1},0)$	
$\nu^c e^c$	$(\mathbf{1},\mathbf{1},1)$	$SU(2)_R$ higgsino
$e^c e^c$	$(\mathbf{1},\mathbf{1},2)$	light in PS & LR vacua

quark-lepton-Higgs coupling

$q\ell$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})$	colored Higgs
$q\ell$	$(\mathbf{3},\mathbf{3},-\frac{1}{3})$	cause proton decay
$q\nu^c$	$(\mathbf{3},\mathbf{2},\frac{1}{6})$	flipped-$SU(5)$ higgsino
qe^c	$(\mathbf{3},\mathbf{2},\frac{7}{6})$	
ℓu^c	$(\bar{\mathbf{3}},\mathbf{2},-\frac{7}{6})$	
ℓd^c	$(\bar{\mathbf{3}},\mathbf{2},-\frac{1}{6})$	flipped-$SU(5)$ higgsino
$u^c\nu^c$	$(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$	PS higgsino
$u^c e^c$	$(\bar{\mathbf{3}},\mathbf{1},\frac{1}{3})$	colored Higgs
$d^c\nu^c$	$(\bar{\mathbf{3}},\mathbf{1},\frac{1}{3})$	important in lopsided structure
$d^c e^c$	$(\bar{\mathbf{3}},\mathbf{1},\frac{4}{3})$	cause proton decay

If a field from **120** Higgs is light, the FCNC inducing fermion coupling is antisymmetric.

$$h'h'^{\dagger} = \begin{pmatrix} 0 & c & b \\ -c & 0 & a \\ -b & -a & 0 \end{pmatrix} \begin{pmatrix} 0 & -c^* & -b^* \\ c^* & 0 & -a^* \\ b^* & a^* & 0 \end{pmatrix}$$

$$= (a^2 + b^2 + c^2) I - \begin{pmatrix} a^2 & a^* b & a^* c \\ a b^* & b^2 & b^* c \\ a c^* & b c^* & c^2 \end{pmatrix}$$

$$b \sim c \sim \lambda a$$

$$\lambda \sim 0.2$$

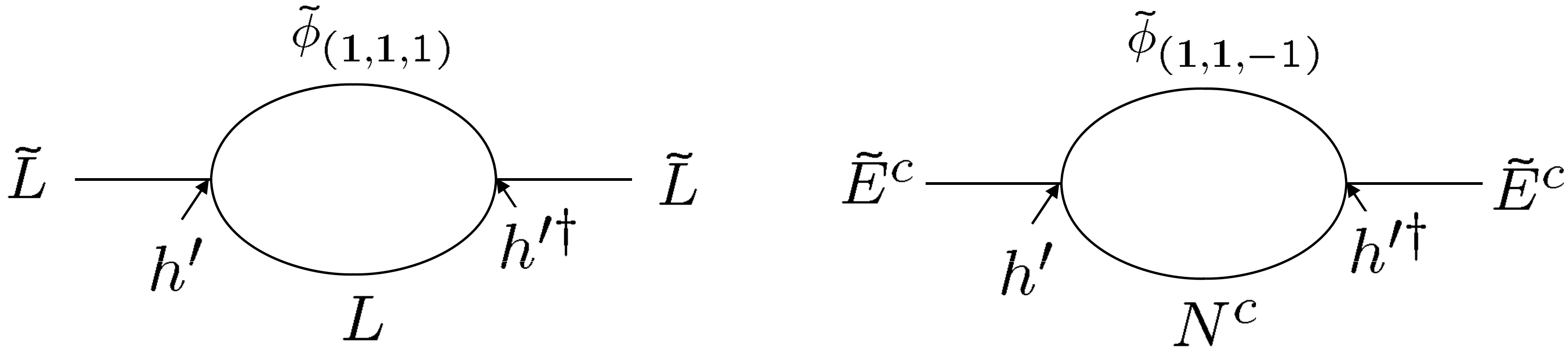
Hierarchy is inverted.

E.g. If (8,2,1/2) splits from **120** or **126**, flavor violations are induced for both left- and right-handed squarks.

$$q u^c \phi_{(\mathbf{8},\mathbf{2},1/2)} + q d^c \phi_{(\mathbf{8},\mathbf{2},-1/2)}$$

If it comes from **120**, contribution to B_s-$\bar{B}_s$ is small, but V_{ub}-sin $2\phi_1$ discrepancy can be solved.

E.g. If $\mathrm{SU}(2)_R$ remains below SO(10) breaking scale, and right-handed chargino $(\mathbf{1},\mathbf{1},\pm 1)$ comes from **120** mainly, $BR(\tau \to e\gamma)$ is enhanced rather than $BR(\tau \to \mu\gamma)$.



Cf. In usual scenario, $BR(\tau \to e\gamma)$ is much suppressed since 13 neutrino mixing is small.

Detail of induced FCNC is related to the SO(10) breaking pattern.

Summary

- We study the flavor violation in the context of SUSY GUTs.
- SO(10) model has impact on the modification of meson mixing amplitudes since both left- and right-handed squarks can have sizable flavor violation.
- The future observation of FCNCs for both quarks and leptons as well as sfermion mass (maybe for upcoming decade) can probe GUT scale physics.

Back up slides

$\mu \to e\gamma$ bound : $\quad \mathrm{BR} < 1.1 \times 10^{-11}$

$$m^2_{\tilde{D}^c} = m^2_{\tilde{L}} \simeq m^2_0 \left(\mathbf{1} - \kappa\, U^*_{\mathrm{MNSP}} \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U^{\mathrm{T}}_{\mathrm{MNSP}} \right)$$

$\delta^{\tilde{\ell}}_{12}$: small $\Rightarrow$ $\underline{\sin\theta_{13} \sim k_2 \sin 2\theta_{12} \text{ and } \delta_{\mathrm{MNSP}} \sim \pi}$

SU(5), type I

$$k_2 \simeq \sqrt{\frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}} \frac{M_2}{M_3}}$$

$$\sin\theta_{13} \lesssim 0.18$$

$\sin 2\beta$ contribution : large

SO(10), type II

$$k_2 \simeq \frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}}$$

$\Rightarrow \sin\theta_{13} \sim 0.02$

$\sin 2\beta$ contribution : small

$\tau \to \mu\gamma$ bound : $\quad$ $\mathrm{BR} < 4.5 \times 10^{-8}$

$m_{1/2} \sim 300$ GeV $\quad$ ($m_{\tilde{g}} \sim 1$ TeV)

SUSY contribution of B-$\bar{B}$ box diagram becomes maximal

at $m_0 \sim 1$ TeV.

Squark masses (diagonal elements)
do not depend much if $m_0 < 1$ TeV

Off-diagonal elements becomes large for larger m_0

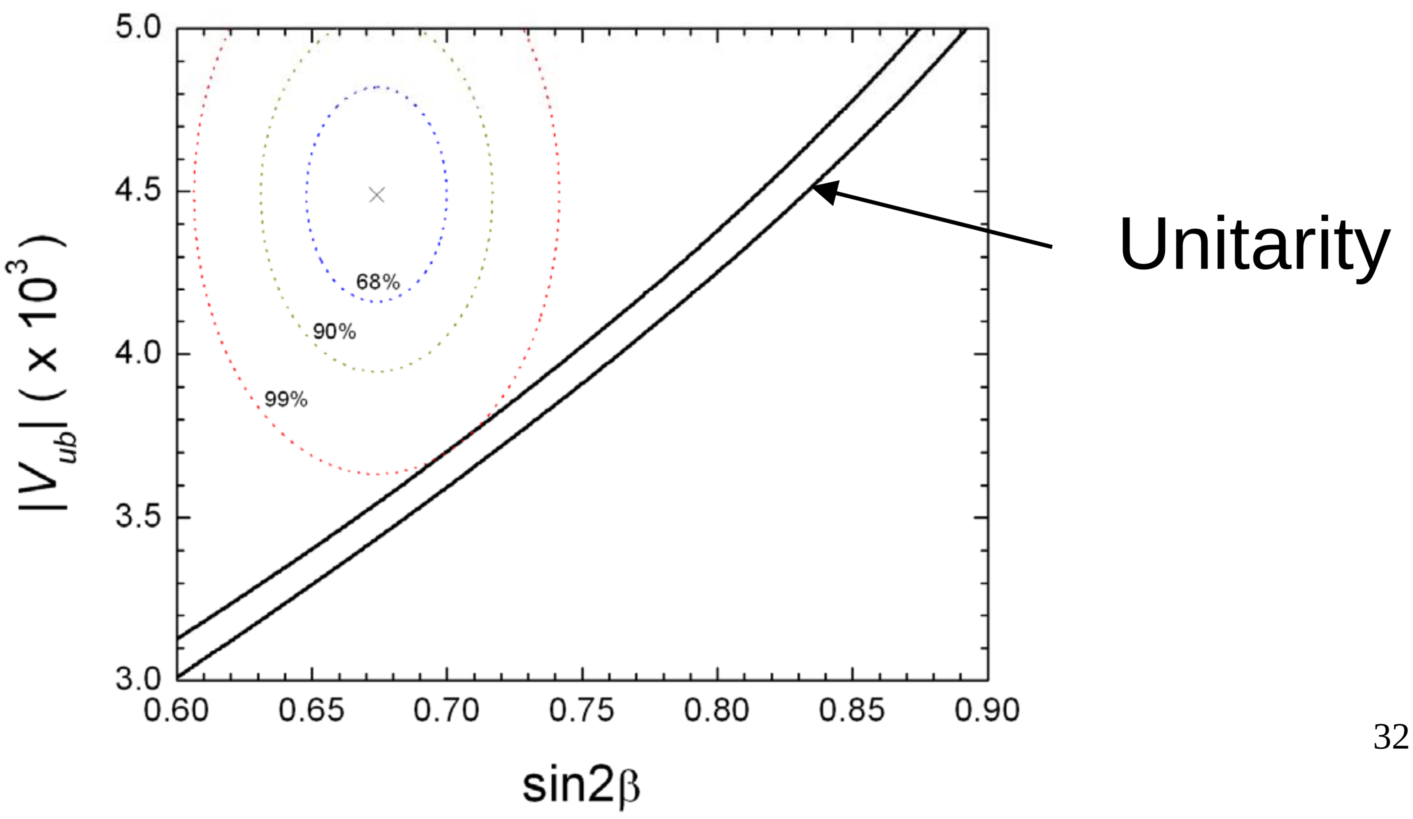
Sleptons become heavy for larger m_0

LFVs are suppressed at $m_0 \sim 1$ TeV.

Unitarity : $|V_{ub}| = (3.49 \pm 0.17) \times 10^{-3}$

Inclusive decays :

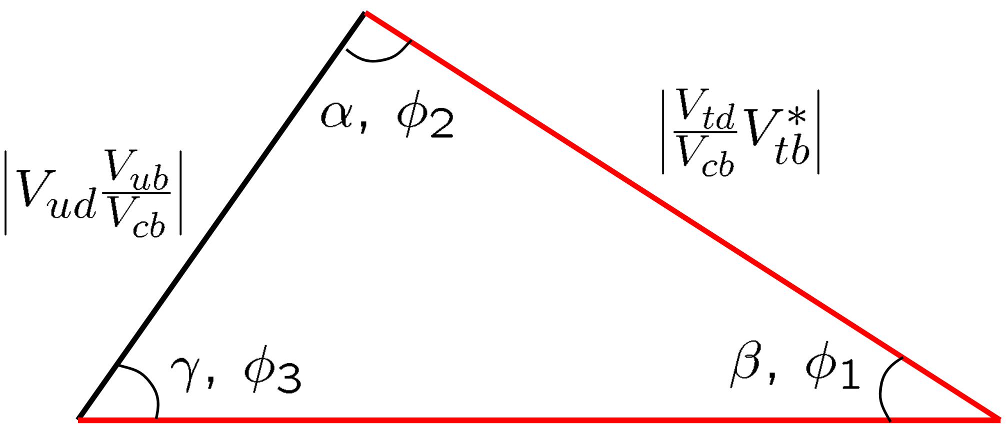
$$|V_{ub}| = (4.49 \pm 0.19 \pm 0.27) \times 10^{-3}$$



$$\frac{\Delta M_s}{\Delta M_d} = \xi^2 \frac{M_{B_s}}{M_{B_d}} \left|\frac{V_{td}}{V_{ts}}\right|^2 \quad \left(\xi^2 = \frac{B_{B_s} f_{B_s}^2}{B_{B_d} f_{B_d}^2} = (1.21 \pm 0.06)^2\right)$$

Lattice calculation

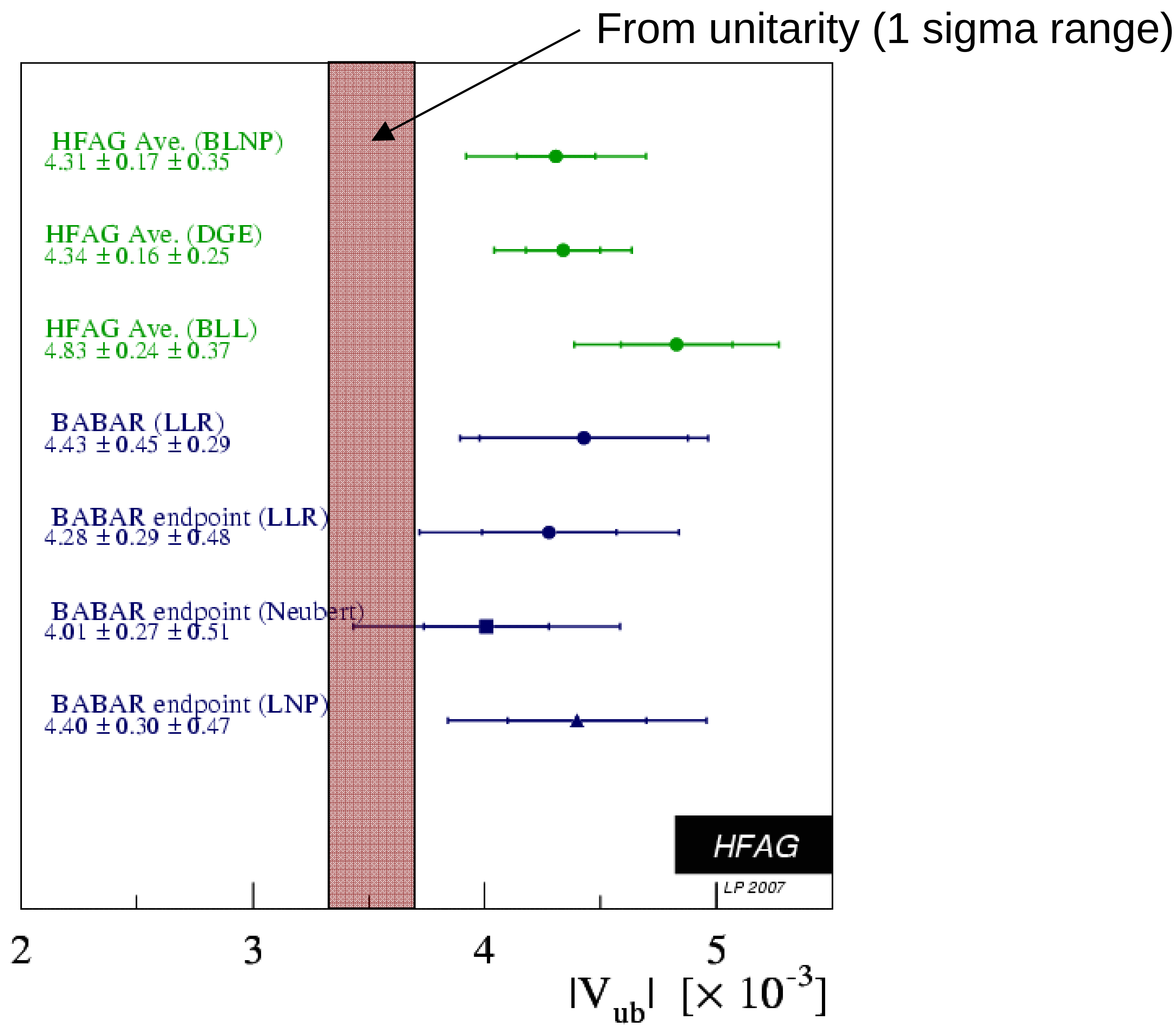
$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.206^{+0.008}_{-0.006}$$



$|V_{ts}| \simeq |V_{cb}|$

$|V_{tb}| \simeq 1$

$|V_{cd}| = 0.2258$


From unitarity (1 sigma range)
HFAG Ave. (BLNP)
4.31 ± 0.17 ± 0.35
HFAG Ave. (DGE)
4.34 ± 0.16 ± 0.25
HFAG Ave. (BLL)
4.83 ± 0.24 ± 0.37
BABAR (LLR)
4.43 ± 0.45 ± 0.29
BABAR endpoint (LLR)
4.28 ± 0.29 ± 0.48
BABAR endpoint (Neubert)
4.01 ± 0.27 ± 0.51
BABAR endpoint (LNP)
4.40 ± 0.30 ± 0.47
HFAG
LP 2007
2
3
4
5
$|V_{ub}|$ [$\times 10^{-3}$]

Small FCNC

Flavor universality at boundary condition.
FCNCs are induced by radiative corrections (RGEs).

$$
\begin{aligned}
16\pi^2 \frac{dm^2_{\tilde{Q}}}{d\ln\mu} &= Y_u Y_u^\dagger m^2_{\tilde{Q}} + m^2_{\tilde{Q}} Y_u Y_u^\dagger + Y_d Y_d^\dagger m^2_{\tilde{Q}} + m^2_{\tilde{Q}} Y_d Y_d^\dagger \\
&+ 2(Y_u m^2_{\tilde{U}^c} Y_u^\dagger + Y_d m^2_{\tilde{D}^c} Y_d^\dagger + Y_u Y_u^\dagger m^2_{H_u} + Y_d Y_d^\dagger m^2_{H_d} + A_u A_u^\dagger + A_d A_d^\dagger) \\
&- 4(\frac{1}{30} g_1^2 M_1^2 + \frac{2}{3} g_2^2 M_2^2 + \frac{8}{3} g_3^2 M_3^2)
\end{aligned}
$$

Origin : Yukawa couplings and A-terms (scalar trilinear couplings)

When Yukawa matrices $(Y_u,\ Y_d)$
and A-term coupling matrices $(A_u,\ A_d)$ are not simultaneously diagonalized, off-diagonal elements of $m^2_{\tilde{Q}}$ are generated.

MSSM (mSUGRA boundary condition)

all the scalar mass : m_0 $\quad A_{ij} = A_0 Y_{ij}$

FCNC : very small (CKM mixings are small)

Introduce the neutrino couplings:

Sizable leptonic FCNC is expected due to large neutrino mixings. (Borzumati-Masiero, '86)

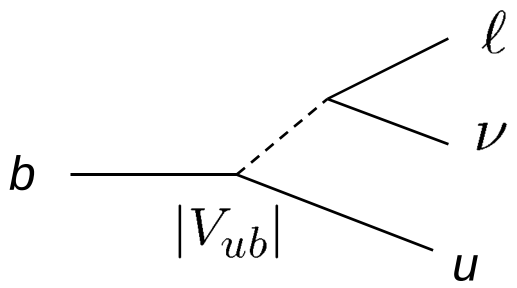
Seesaw neutrino mass:

$$M_\nu^{\rm light} = \underbrace{M_L}_{\text{Type II}} - \underbrace{Y_\nu M_R^{-1} Y_\nu^{\rm T} v_u^2}_{\text{Type I}}$$

$$M_L = f_L \langle \Delta_L^0 \rangle \qquad M_R = f_R \langle \Delta_R^0 \rangle$$

$$f_L L L \Delta_L + f_R L^c L^c \Delta_R$$

Experimental measurement of $|V_{ub}|$ (Tree-level dominant)



- Inclusive decays $b \to u\ell\nu$

Weak quark decay + QCD corrections

$\to$ OPE in α_s and $1/m_b$

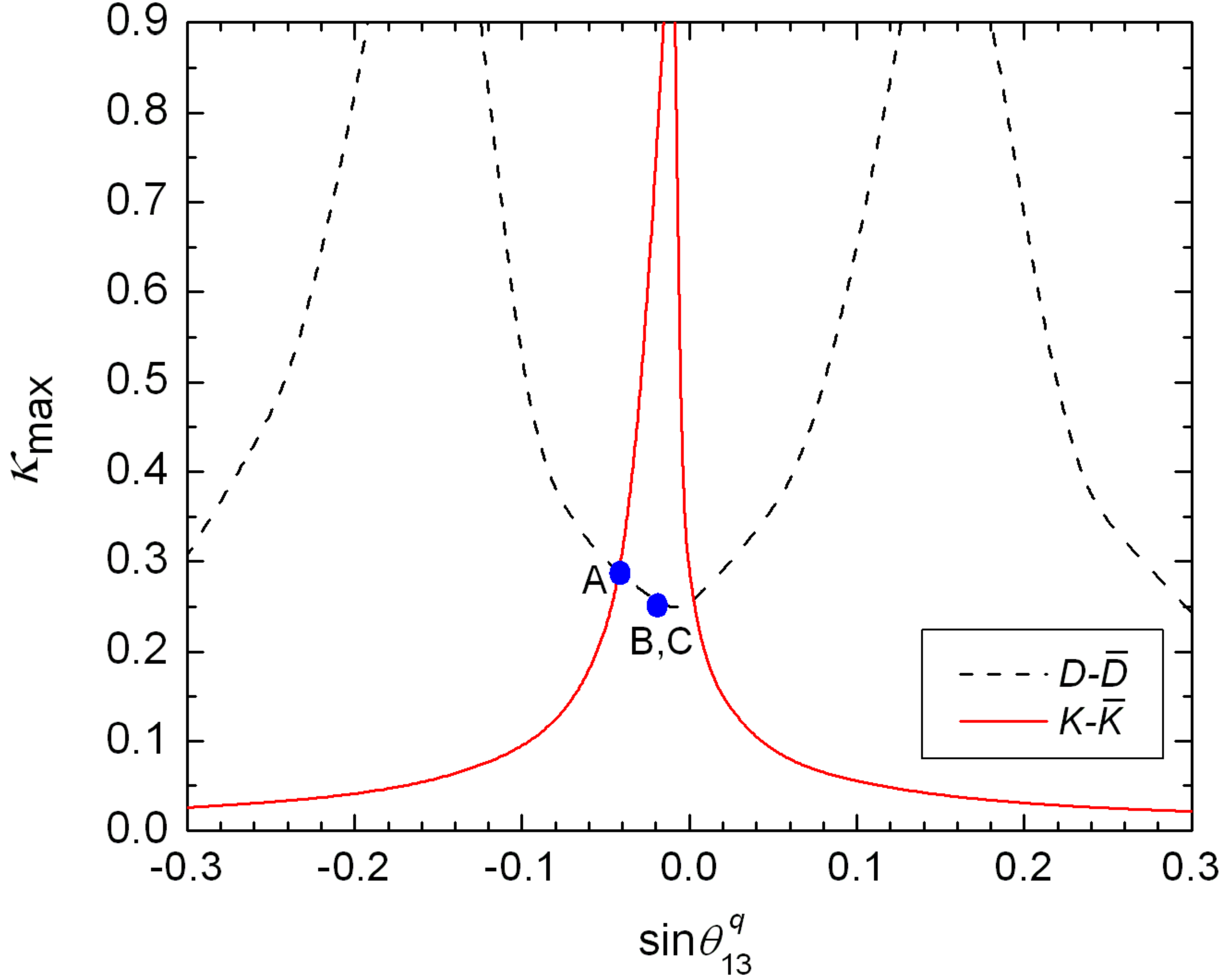
$$|V_{ub}| = (4.49 \pm 0.19 \pm 0.27) \times 10^{-3}$$

- Exclusive decays $B \to X_u \ell\nu$

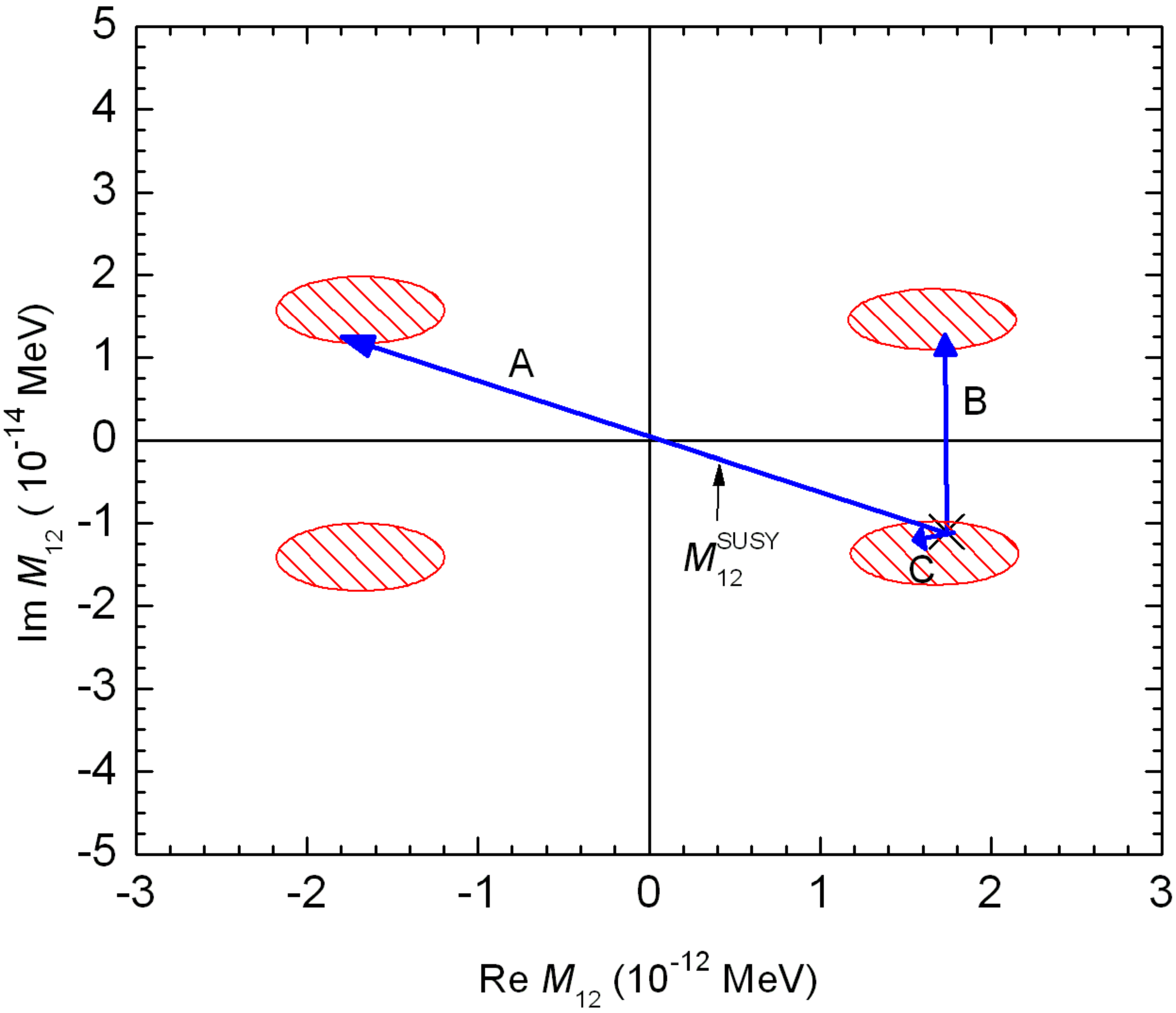
Form factors : need lattice QCD (large error)

$$|V_{ub}| = (3.84^{+0.67}_{-0.49}) \times 10^{-3}$$

PDG average : $|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}$



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
κ_{max}
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
$\sin\theta_{13}^{q}$
A
B,C
D-$\bar{D}$
K-$\bar{K}$



A
B
C
M_{12}^{SUSY}
Im M_{12} (10^{-14} MeV)
Re M_{12} (10^{-12} MeV)

$$m^2_{\bar{5}} = m^2_{\tilde{D}^c} = m^2_{\tilde{L}} \simeq m_0^2 \left(\mathbf{1} - \kappa\, V_L^e \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} V_L^{e\dagger} \right)$$

$$M_R = \mathrm{diag}(M_1, M_2, M_3) \qquad Y_\nu = V_L^e Y_\nu^{\mathrm{diag}} V_R^{e\dagger}$$

When $V_R^e \simeq \mathbf{1}$,

$$V_L^e \simeq U^*_{\mathrm{MNSP}} \qquad k_2 \simeq \sqrt{\frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}}\frac{M_2}{M_3}}$$

Yukawa couplings

$$Y_u = \underline{V_{qeL}} V_{\mathrm{CKM}} Y_u^{\mathrm{diag}} P_u \underline{V_{uR}}$$

$$Y_d = \underline{V_{qeL}} Y_d^{\mathrm{diag}} P_d \underline{V^\dagger_{qeR}}$$

$$Y_e = Y_e^{\mathrm{diag}} P_e$$

$P_{u,d,e}$:

Diagonal phase matrices

Minimality:

$V_{qeL}, V_{qeR}, V_{uR} \sim \mathbf{1}$

If triplet part dominates the seesaw formula :

$$
\begin{aligned}
Y_u &= h + r_2 f + r_3 h' \\
Y_d &= r_1(h + f + h') \\
Y_e &= r_1(h - 3f + c_e h') \\
Y_\nu &= h - 3r_2 f + c_\nu h'
\end{aligned}
\qquad
h \sim \begin{pmatrix} \cdot & & \\ & \cdot & \\ & & 1 \end{pmatrix}, \quad
f \sim \begin{pmatrix} & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} \lambda^2
$$

$$
\begin{aligned}
m_{16}^2 &= m_{\tilde{Q}}^2 = m_{\tilde{U}^c}^2 = m_{\tilde{D}^c}^2 = m_{\tilde{L}}^2 = m_{\tilde{E}^c}^2 = m_{\tilde{N}^c}^2 \\
&\simeq m_0^2 \left(\mathbf{1} - \kappa_{16}\, U \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U^\dagger \right)
\end{aligned}
$$

$$
f = U f^{\mathrm{diag}} U^\top \qquad U \simeq U^*_{\mathrm{MNSP}} \qquad k_2 \simeq \frac{\Delta m^2_{\mathrm{sol}}}{\Delta m^2_{\mathrm{atm}}}
$$

Both left- and right-squarks have sizable FCNC effects!