

Differentiating Neutrino Models on the Basis of θ_{13} and Lepton Flavor Violation

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PRD 74, 113006 (2006), hep-ph/0608137

Paper in preparation

International Workshop on Grand Unified Theories

Ritsumeikan University, December 17, 2007

Present Oscillation Data and Unknowns

- Present data within 2σ accuracy

$$\Delta m_{21}^2 = (7.3 - 8.5) \times 10^{-5} \text{ eV}^2$$

$$\Delta m_{31}^2 = (2.2 - 3.0) \times 10^{-3} \text{ eV}^2$$

$$\sin^2 \theta_{12} = 0.26 - 0.36$$

$$\sin^2 \theta_{23} = 0.38 - 0.63$$

$$\sin^2 \theta_{13} \leq 0.025$$

- Data suggests the approximate tri-bimaximal mixing texture of Harrison, Perkins and Scott:

$$U_{PMNS} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \end{pmatrix}$$

with $\sin^2 \theta_{23} = 0.5$, $\sin^2 \theta_{12} = 0.33$ and $\sin^2 \theta_{13} = 0$.

- Present unknowns

Hierarchy and absolute mass scales

Whether neutrinos are Dirac or Majorana

CP-violating phases of mixing matrix

How close to zero is the reactor angle θ_{13} ?

How near maximal is the atmospheric mixing angle?

Is the approximate tri-bimaximal symmetry a softly-broken or accidental symmetry?

How large is charged lepton flavor violation?

- Scope of Survey

What do models say about θ_{13} , hierarchy, and lepton flavor violation?

Outline

- Present Oscillation Data and Unknowns
- Theoretical Framework and Models
- Survey of Model Predictions for θ_{13}
- Flavor Violation in Radiative Lepton Decays
- Examples of Predictive SUSY GUT Models
- Lepton Flavor Violation Predictions for Radiative Decays and $\mu - e$ Conversion
- Conclusions

Theoretical Framework and Models

- Neutrino oscillations require “massive” neutrinos

$$\Delta m_{21}^2 \simeq 7.9 \times 10^{-5}, \quad |\Delta m_{32}^2| \simeq 2.5 \times 10^{-3} \text{ eV}^2$$

$$\sum_i m_i \leq 0.17 \text{ eV} \rightarrow 2 \text{ eV} \text{ (WMAP, SDSS, Lyman alpha)}$$

- Possible extensions of the Standard Model
 - Introduce dim-5 effective non-renormalizable operators
 - Add RH neutrinos with Yukawa interactions
 - Add direct mass terms with RH Majorana couplings
 - Add Higgs triplet with LH Majorana couplings

- **General 6 x 6 neutrino mass matrix in flavor basis**

$\mathcal{B}(\nu_{\alpha L}, N_{\alpha L}^c)$ of the 6 LH fields:

$$\mathcal{M} = \begin{pmatrix} M_L & M_N^T \\ M_N & M_R \end{pmatrix}$$

where M_N is the Dirac neutrino mass matrix, M_L the LH and M_R the RH Majorana neutrino mass matrix.

- M_L only with Higgs triplets/or higher dimensional effective interactions and no RH neutrinos
- M_N only with Higgs doublets and Dirac Yukawa couplings
- Type I seesaw with $M_L = 0$, $M_N \ll M_R$:

$$M_\nu = -M_N^T M_R^{-1} M_N$$

- Type II seesaw with $M_L \neq 0$, $M_N \ll M_R$:

$$M_\nu = M_L - M_N^T M_R^{-1} M_N$$

- Neutrino masses and mixing matrix can then be determined:

- Effective light LH Majorana neutrino mass matrix is complex symmetric and diagonalized by the unitary transformation with real positive masses down diagonal:

$$M_{\nu}^{diag} = U_{\nu L}^T M_{\nu} U_{\nu L} = \text{diag}(m_1, m_2, m_3)$$

- Dirac charged lepton mass matrix is diagonalized by

$$M_{lept}^{diag} = U_{lept R}^{\dagger} M_{lept} U_{lept L} = \text{diag}(m_e, m_{\mu}, m_{\tau})$$

- Neutrino mixing matrix is then given by

$$V_{PMNS} \equiv U_{lept L}^{\dagger} U_{\nu L} = U_{PMNS} \Phi,$$

$$\Phi = \text{diag}(e^{i\chi_1}, e^{i\chi_2}, 1)$$

Models with Well-Defined Symmetry

- Examples with Lepton Flavor Symmetry

- $\mu - \tau$ Interchange Symmetry
- More restrictive S_3 or A_4 lepton flavor symmetry
- SO(3) or SU(3) Flavor Symmetries
- Texture Zeros

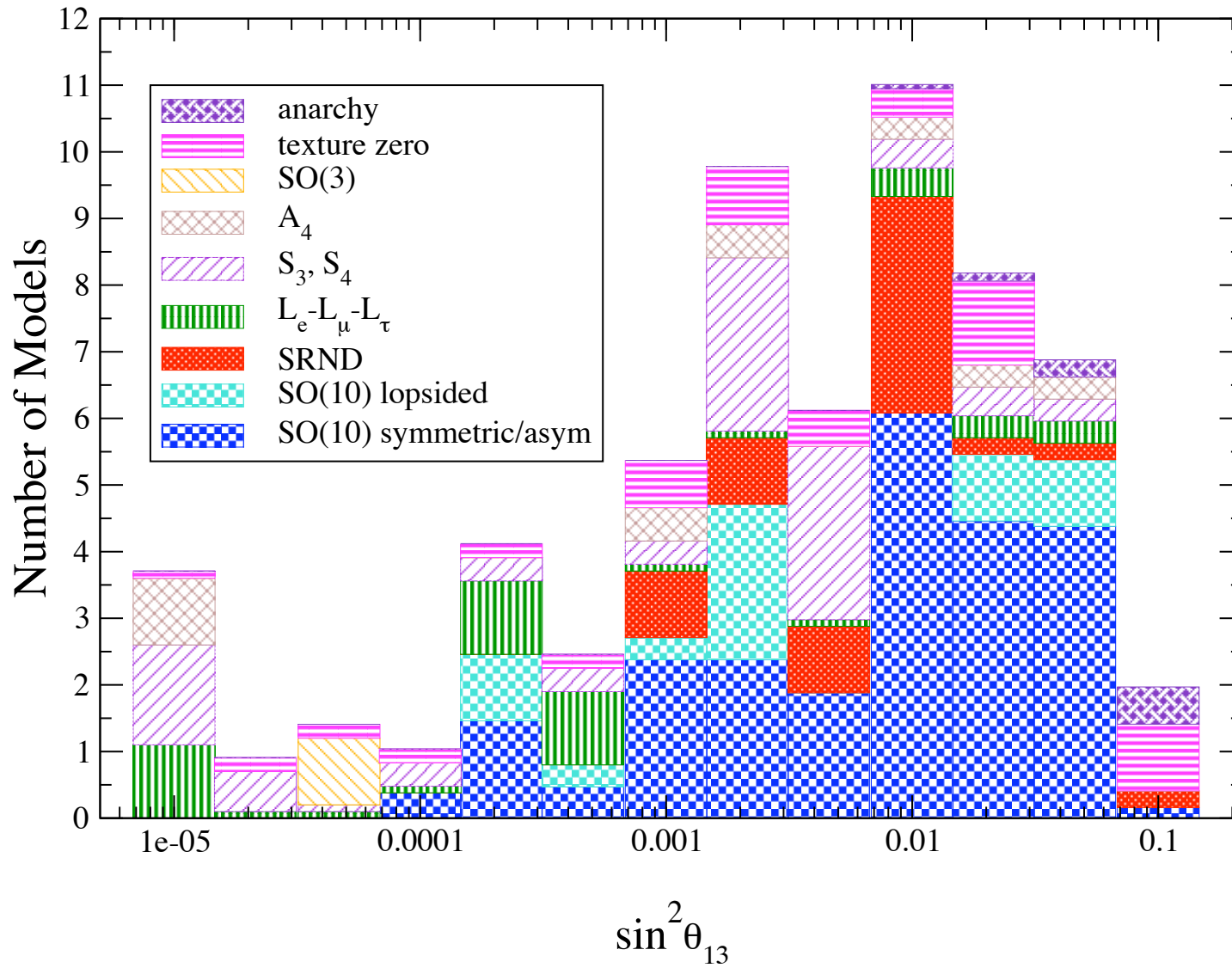
- Examples involving GUT Models

- “Minimal” SO(10) Models with Higgs in 10, 126, (120, 45, 54)
- “Lopsided” SO(10) Models with Higgs in 10, 16, 16bar, 45

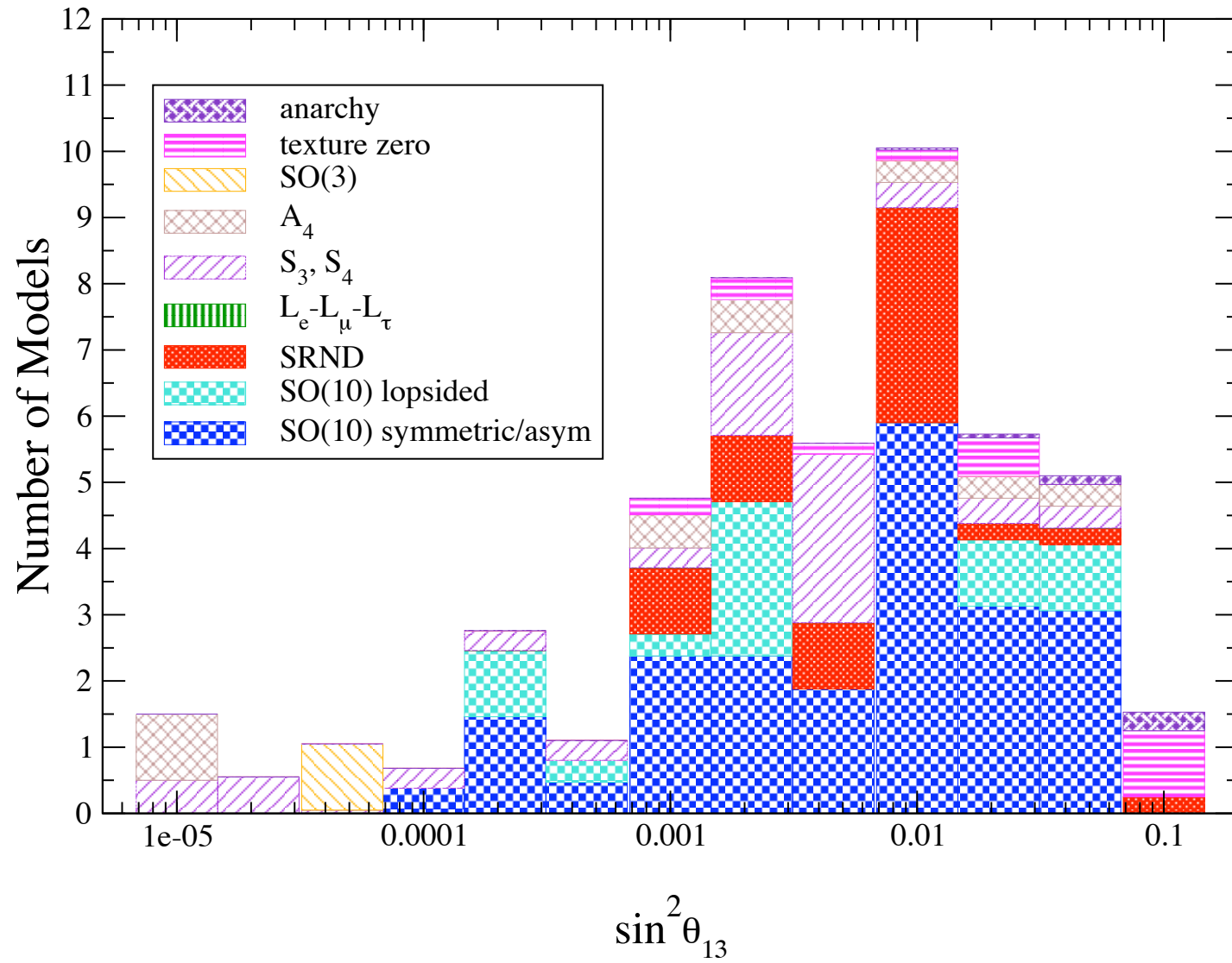
Survey of Predictions for θ_{13} and Hierarchy

- Survey made of 63 models in literature which give the LMA solution for the solar neutrino oscillations and firm and reasonably restrictive predictions for the reactor neutrino angle. (Cutoff date: May 2006)
- Most of models predict $10^{-4} < \sin^2 \theta_{13} < 0.04$
- Normal hierarchy is preferred 3 : 1
- Planned reactor experiments will reach $\sin^2 2\theta_{13} \sim 0.01$, so half of models will be eliminated if no $\bar{\nu}_e$ disappearance.
- Meanwhile MEG will probe $\mu \rightarrow e\gamma$ for LFV, so this may serve as even more immediate selector of models.

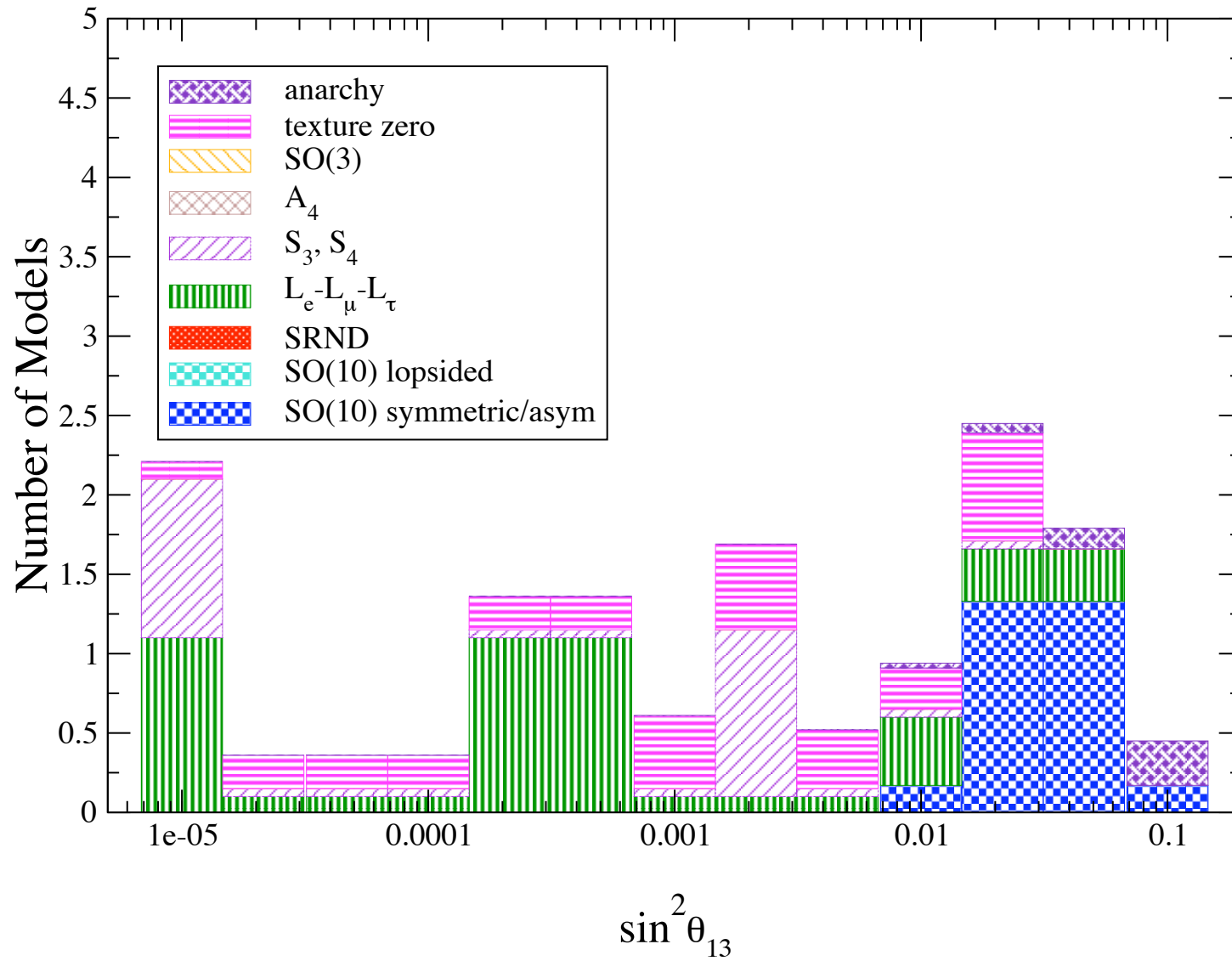
Predictions of All 63 Models



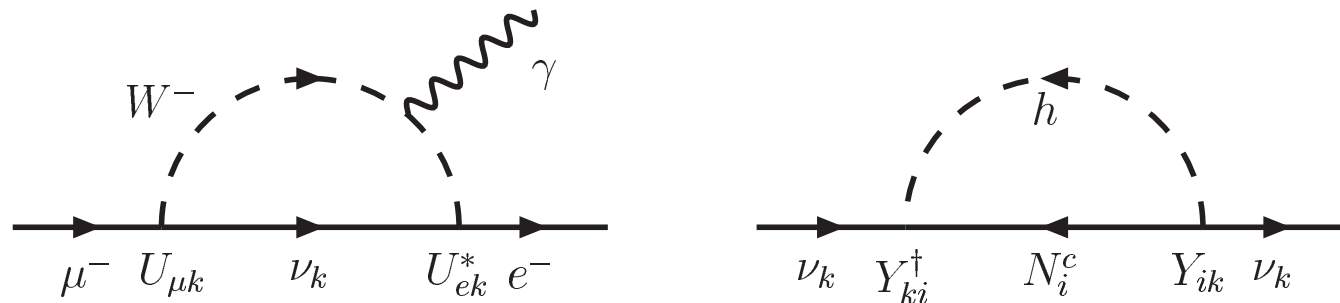
Models with Normal Hierarchy



Models with Inverted Hierarchy



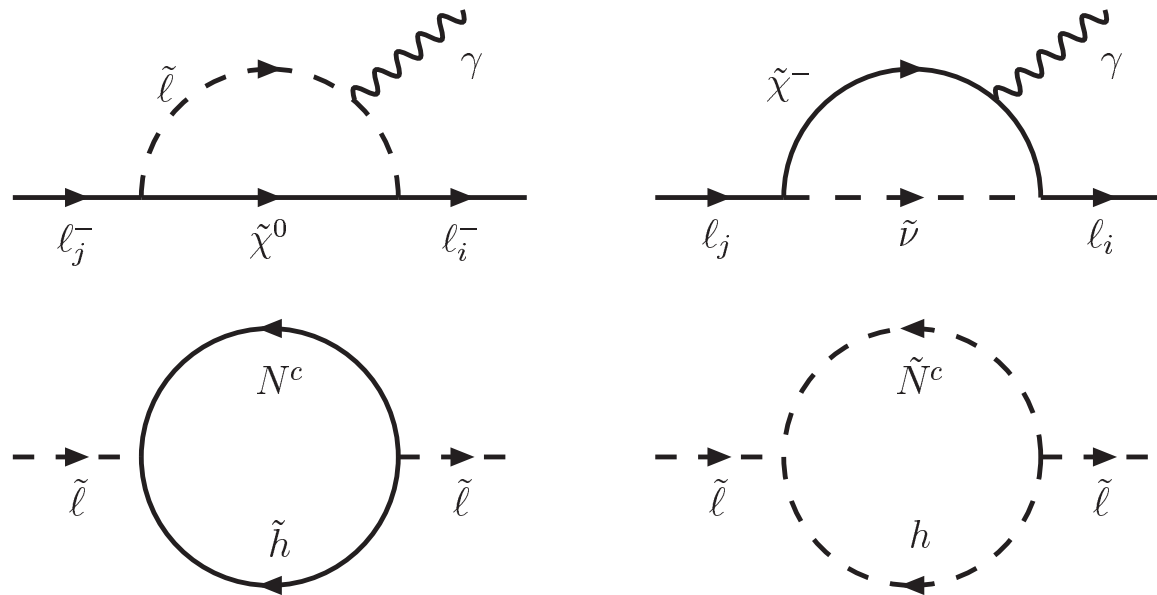
Lepton Flavor Violation in Radiative Decays



- In SM with 3 massive N^c 's, individual L_e , L_μ , L_τ are not conserved. LFV arises in 1-loop where the neutrino insertion involves lepton flavor-changing Yukawa couplings.

$$\begin{aligned}
 BR_{21} &\equiv \Gamma(\mu \rightarrow e\gamma)/\Gamma(\mu \rightarrow \nu_\mu e \bar{\nu}_e) \\
 &= \frac{3\alpha}{32\pi} \left| \sum U_{\mu k}^* \frac{m_k^2}{M_W^2} U_{ke} \right|^2 \sim \frac{3\alpha}{128\pi} \left(\frac{\Delta m_{21}^2}{M_W^2} \right)^2 \sin^2 2\theta_{12} \\
 &\sim 10^{-54}
 \end{aligned}$$

- In SUSY GUT models slepton - neutralino and sneutrino - chargino loops contribute to radiative lepton decays.



- In the CMSSM version with universal soft masses and trilinear couplings, LFV arises from evolution of Yukawa couplings and soft parameters.
- With more comparable heavy masses in the loops, the LFV branching ratios can be much larger.

- In the LLA, largest contribution comes from the LL slepton mass matrix yielding

$$\text{BR}(\ell_j \rightarrow \ell_i \gamma) = \frac{\alpha^3}{G_F^2 m_S^8} |(m_{\tilde{\ell}}^2)_{ji}|^2 \tan^2 \beta$$

where

$$(m_{\tilde{\ell}}^2)_{ji} = -\frac{1}{8\pi^2} m_0^2 (3 + A_0^2/m_0^2) Y_{jk}^\dagger \log \frac{M_G}{M_k} Y_{ki}$$

- Full evolution effects are extremely well approximated by

$$m_S^8 \simeq 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2 \quad \text{Petcov et al.}$$

- **MEG experiment only has a chance of seeing a positive signal from a SUSY GUT model. All other models considered here will give negative results.**

Generic Approach to LFV in CMSSM Seesaw

- With M_E and M_R diagonal, seesaw formula can be inverted to yield the Yukawa Dirac neutrino coupling matrix (Casas-Ibarra)

$$Y_\nu = \frac{1}{v \sin \beta} D_N(\sqrt{M_i}) R D_\nu(\sqrt{m_j}) U_{PMNS}^\dagger$$

in terms of a complex orthogonal R matrix which allows for various unknown RH mixings.

- By using soft SUSY benchmarks, adopting various heavy RH masses and various R parametric angles and phases, one can “predict” the radiative LFV branching ratios.
- One finds results strongly depend on $\tan \beta$, M_3 , θ_{13} , and the R parameters: Arganda, Herrero, et al.
 - BR's increase by powers of 10 as θ_{13} varies from $0^\circ \rightarrow 10^\circ$.
 - BR21($\mu \rightarrow e \gamma$) restricts $M_3 \leq 10^{14} - 10^{15}$ GeV.

Examples of Predictive SUSY GUT Models

- LFV has been studied in a number of papers in rather generic GUT models. Here we wish to differentiate between specific GUT models and draw some conclusions.
- **SO(10) Models with indicated Flavor Symmetry and Higgs IRs**
 - (1) **AB (Albright-Barr):** $U(1) \times Z_2 \times Z_2$ with 10, 16, $\overline{16}$, 45
 - (2) **CM (Chen - Mahanthappa):** $SU(2) \times (Z_2)^3$ with 10, $\overline{126}$
 - (3) **CY (Cai - Yu):** S_4 with 10, $\overline{126}$
 - (4) **DR (Dermisek - Raby):** D_3 with 10, 45
 - (5) **GK (Grimus - Kuhblok):** Z_2 with 10, 120, $\overline{126}$

Models	$SO(10)$ IRs	Flavor Symmetry	M_R 's	$\tan \beta$	$\sin^2 \theta_{13}$	Interesting Features
A - B	10, 16, $\overline{16}$, 45	$U(1) \times Z_2 \times Z_2$	2.4×10^{14} 4.5×10^8 4.5×10^8	5	0.0020 (2.6°)	Large M_R hierarchy with lightest two nearly degenerate leads to resonant leptogenesis.
C - M	10, $\overline{126}$	$SU(2) \times (Z_2)^3$	7.0×10^{12} 4.5×10^9 1.1×10^7	10	0.013 (6.5°)	Large M_R hierarchy with heaviest more than 3 orders of magnitude below GUT scale; large $\sin^2 \theta_{13}$.
C - Y	10, $\overline{126}$	S_4	2.6×10^{12} 2.6×10^{12} 2.6×10^{12}	10	0.0029 (3.1°)	Degenerate M_R spectrum 4 orders of magnitude below GUT scale.
D - R	10, 45	D_3	5.5×10^{13} 9.3×10^{11} 1.1×10^{10}	50	0.0024 (2.8°)	Mild M_R hierarchy almost 3 orders of magnitude below GUT scale.
G - K	10, 120, $\overline{126}$	Z_2	2.0×10^{15} 4.1×10^{14} 6.7×10^{12}	10	0.00059 (1.4°)	Mild M_R hierarchy just 1 order of magnitude below GUT scale; rather small $\sin^2 \theta_{13}$.

Radiative Lepton Flavor Violation Predictions

- In CMSSM with universal soft parameters m_0 , $M_{1/2}$, A_0 , for given $\tan\beta$ and $\text{sgn}(\mu)$, a variety of plots are possible.

1) BR vs. $M_{1/2}$ for fixed $A_0 = 0$ and different choices of m_0 .

2) Ratio of the branching ratios, BR32/BR21 on log - log plot:

$$\log BR32 = \log BR21 + \log \left| \frac{(Y_\nu^\dagger LY_\nu)_{32}}{(Y_\nu^\dagger LY_\nu)_{21}} \right|^2$$

with unit slope and intercept the second term on right.

Length of straight line depends on the soft parameters.

3) A_0/m_0 vs. $M_{1/2}$ scatterplot with a color scheme to indicate branching ratio ranges.

- **Soft Parameter constraints imposed**

For $\tan \beta = 5, 10$:

$$m_0 : 50 \rightarrow 400 \text{ GeV}$$

$$M_{1/2} : 200 \rightarrow 1000 \text{ GeV}$$

$$A_0 : -4000 \rightarrow 4000 \text{ GeV}$$

For $\tan \beta = 50$:

$$m_0 : 500 \rightarrow 4000 \text{ GeV}$$

$$M_{1/2} : 200 \rightarrow 1500 \text{ GeV}$$

$$A_0 : -50 \rightarrow 50 \text{ TeV}$$

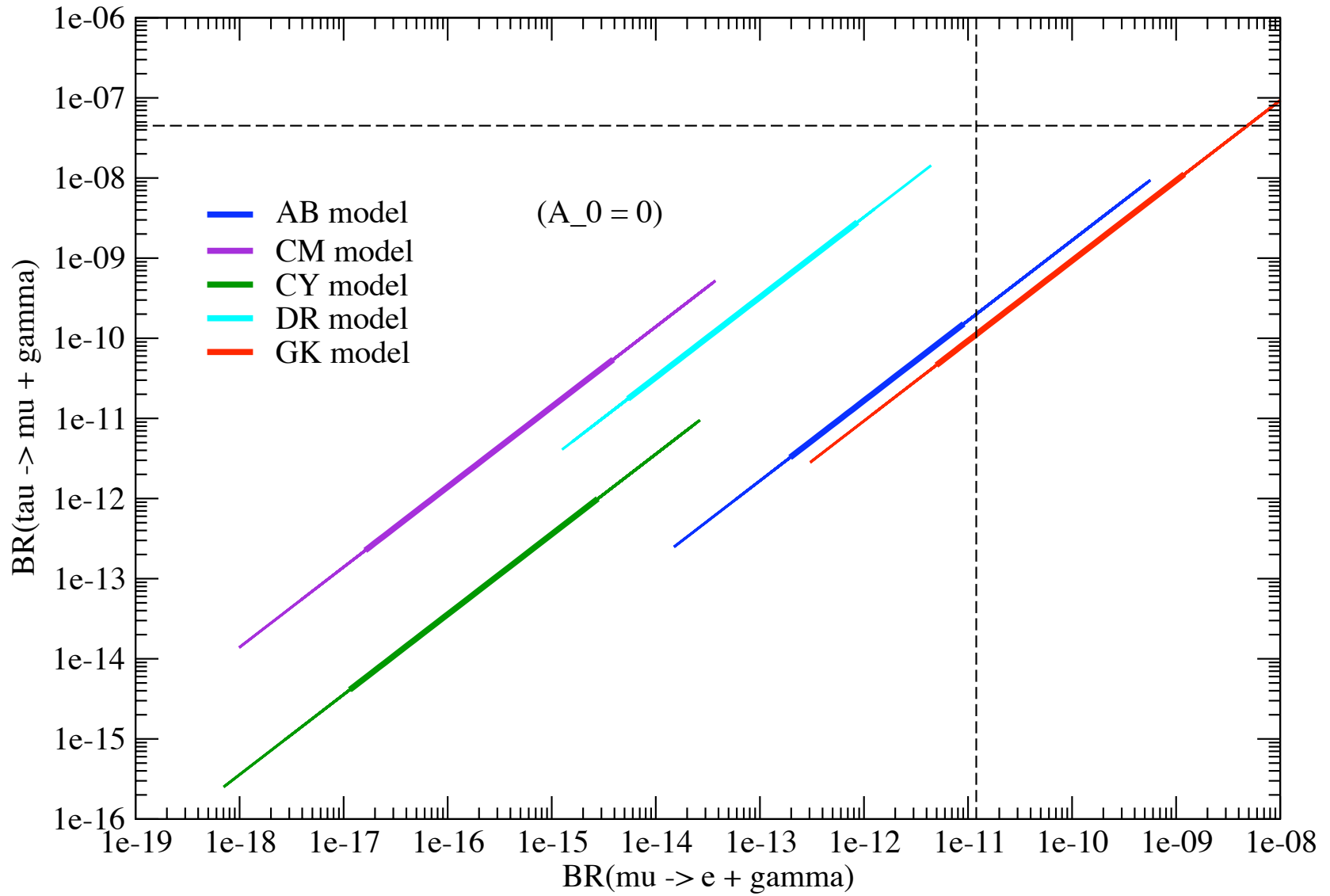
- **WMAP DM constraints in coannihilation regions**

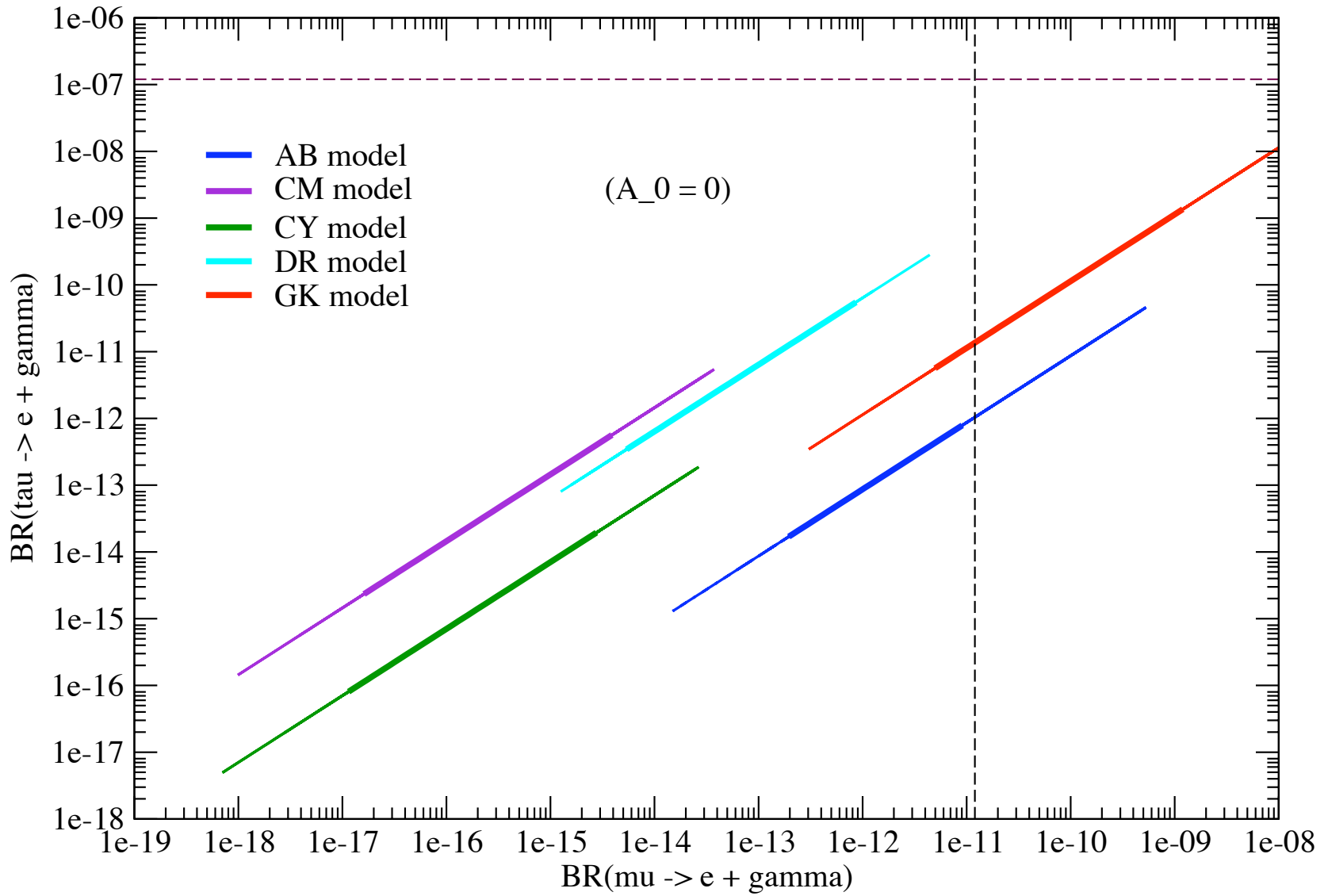
$$m_0 = c_0 + c_1 M_{1/2} + c_2 M_{1/2}^2$$

$$c_i = c_i(A_0, \tan \beta) \quad \text{Stark, Hafliger, Biland, Paus}$$

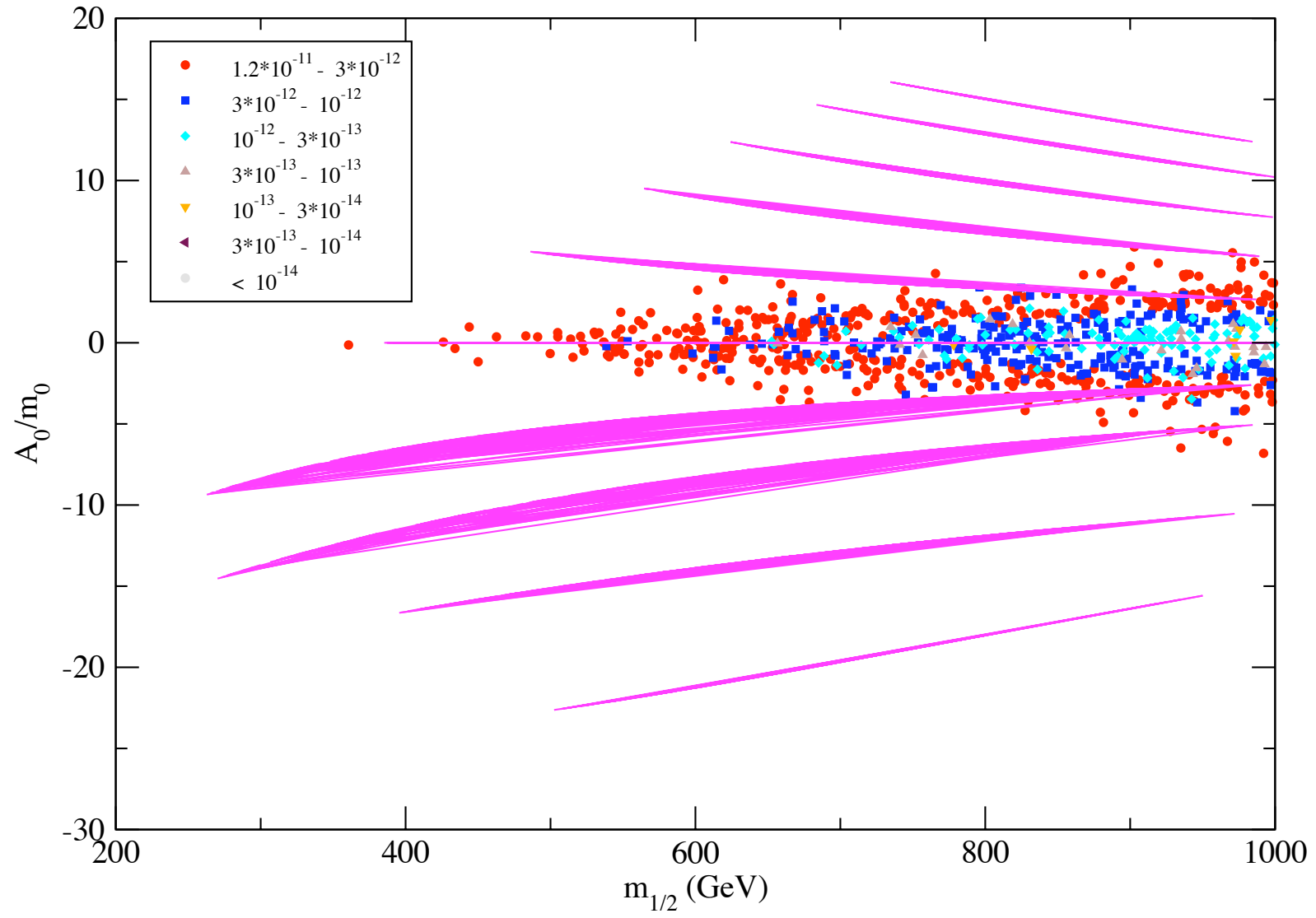
If $M_{1/2}$ is too small, $m_h > 114 \text{ GeV}$ is violated.

If $M_{1/2}$ is too large, $\tilde{\chi}^0$ relic density is too large.

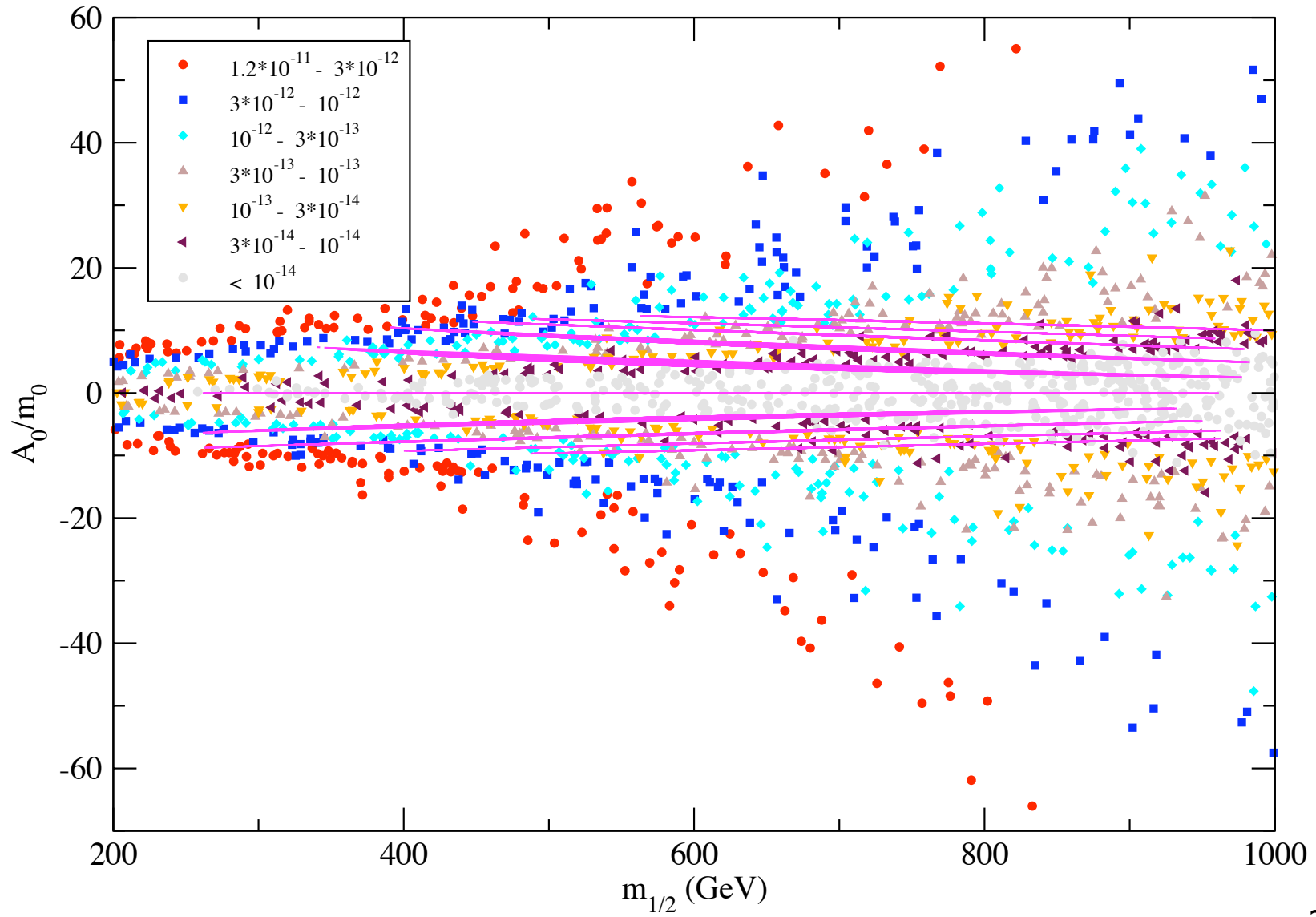




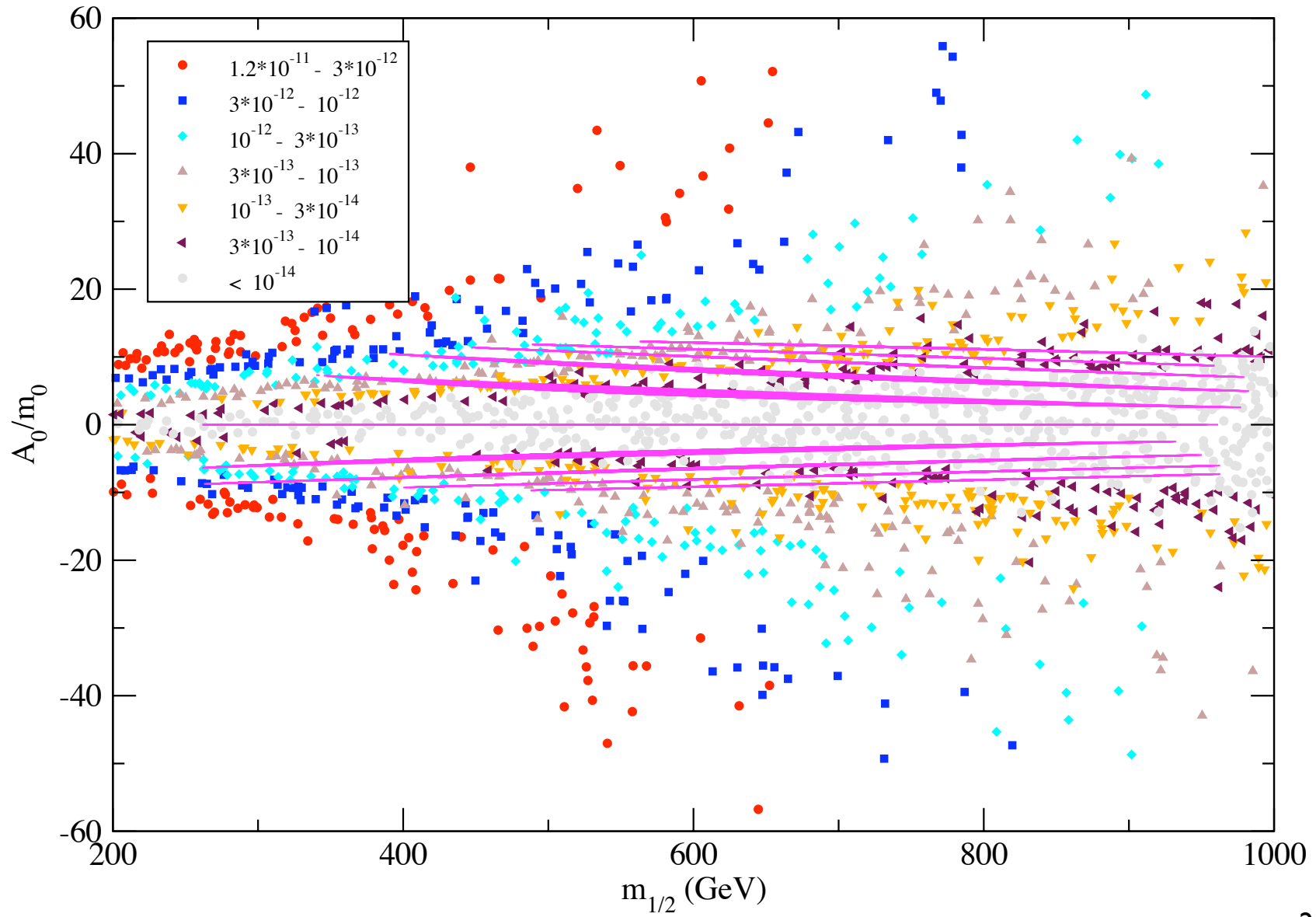
BR21AB4



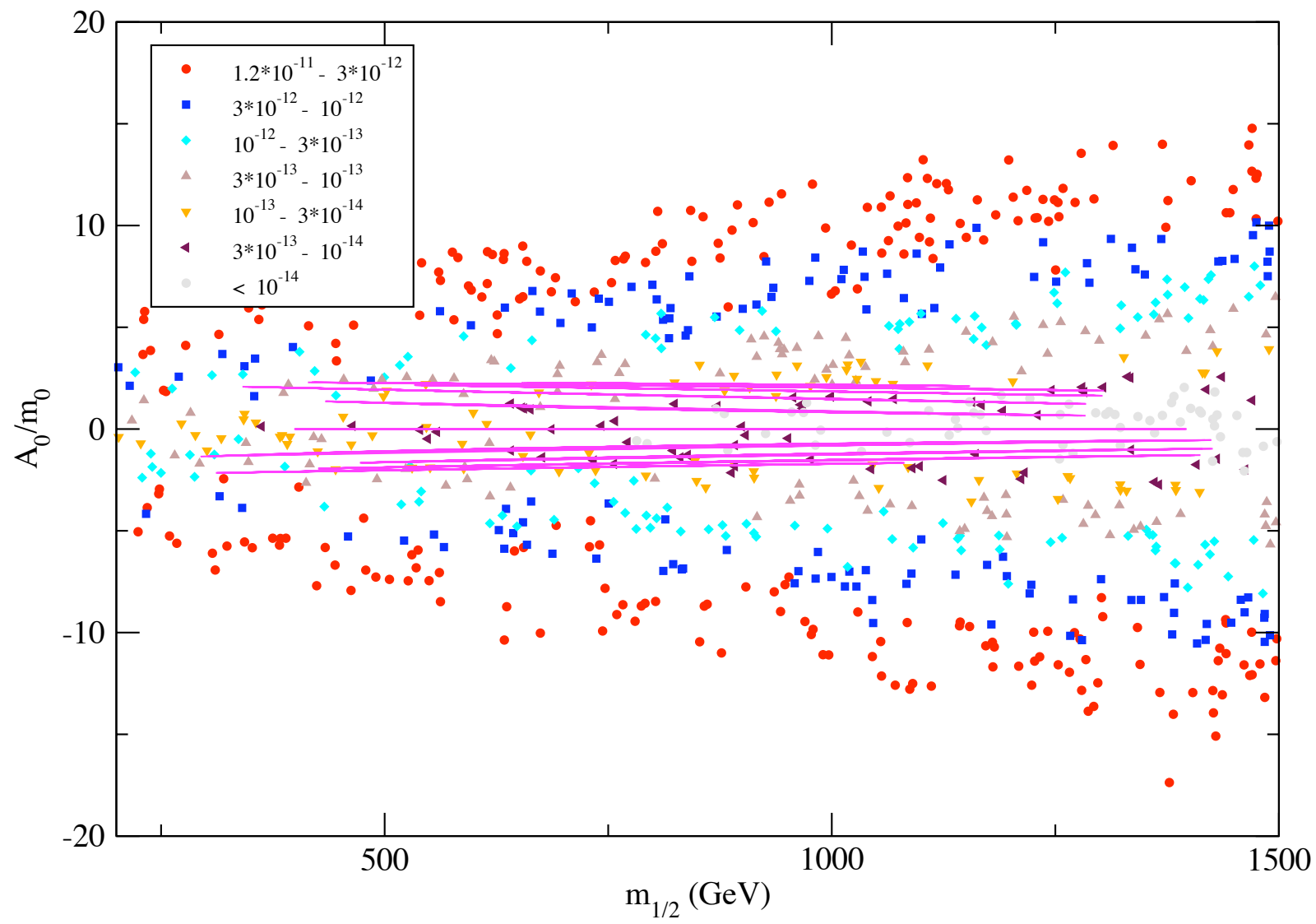
BR21CM4



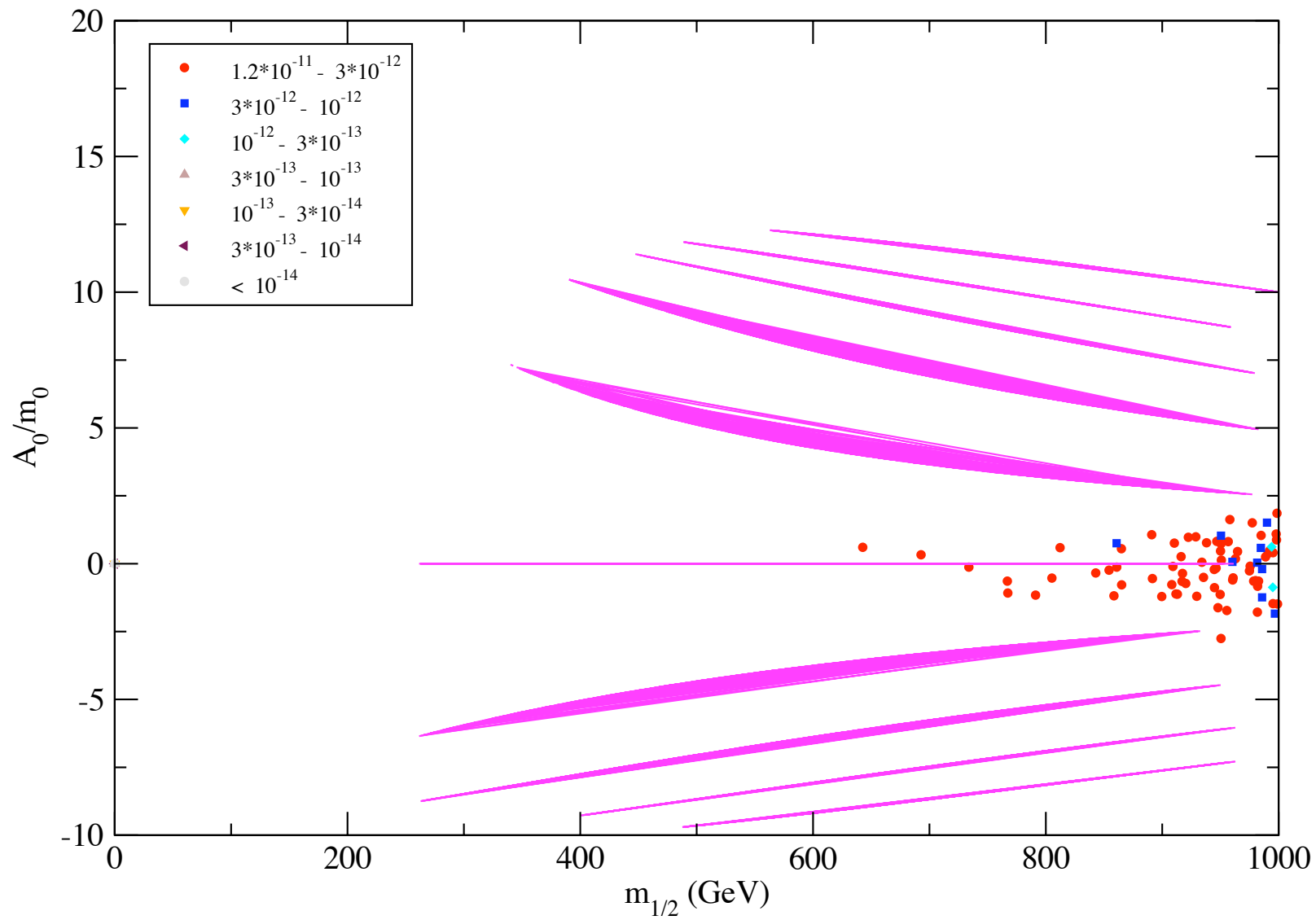
BR21CY4



BR21DR4

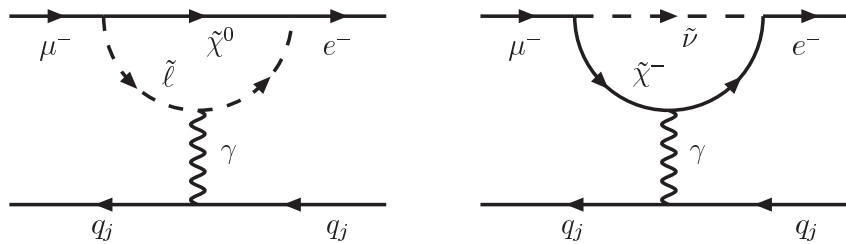


BR21GK4



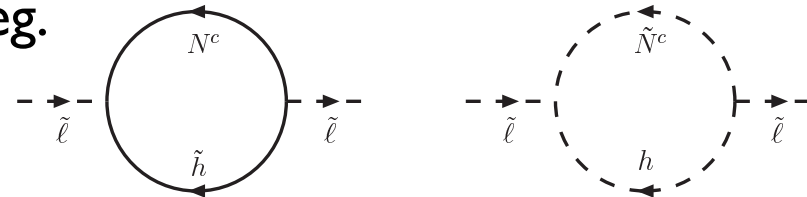
Lepton Flavor Violation in $\mu - e$ Conversion

- Here one-loop diagrams involving gamma, Z, and Higgs penguins all contribute, but in the CMSSM the gamma penguin dominates:

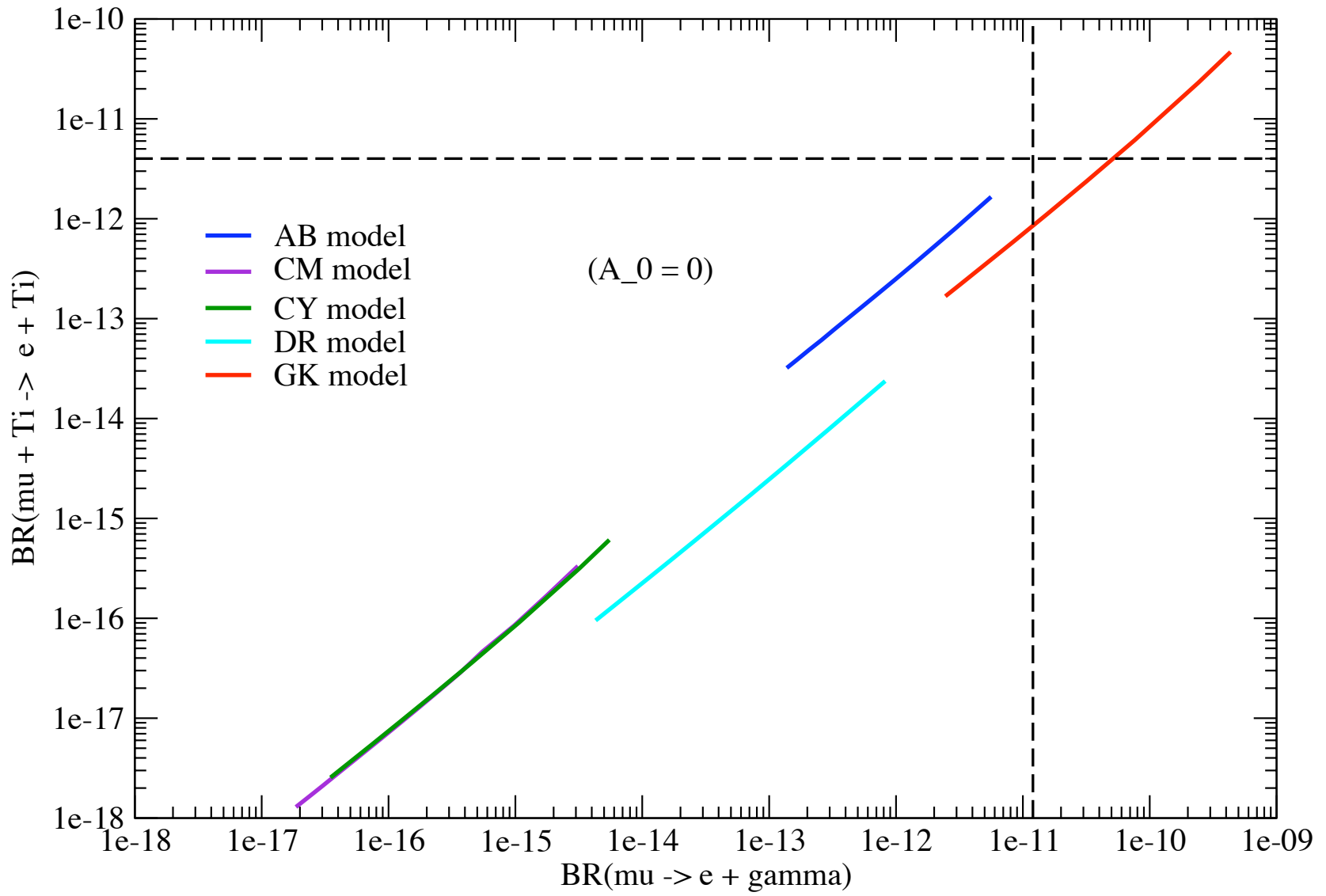


Hisano, Moroi, Tobe,
Yamaguchi
Arganda, Herrero et al.

where the effects of the virtual N^c and \tilde{N}^c with their Yukawa couplings appear in $\tilde{\ell}$ loops, eg.



- The $\mu - e$ conversion rate (relative to the capture rate) on Ti vs. $BR21(\mu \rightarrow e\gamma)$ is plotted for the 5 GUT models, where the tighter WMAP DM constraints have been imposed.



Models	$SO(10)$ IRs	$\sin^2 \theta_{13}$	M_R 's	$\tan \beta$	$ A_0/m_0 _{\max}$	BR21($\mu \rightarrow e\gamma$) $< 1.2 \times 10^{-11}$ $\rightarrow \times 10^{-13}$	BR32($\tau \rightarrow \mu\gamma$) $< 4.5 \times 10^{-8}$ $\rightarrow \times 10^{-9}$	BR($\mu + Ti \rightarrow e + Ti$) $< 4 \times 10^{-12}$ $\rightarrow \times 10^{-18}$
A - B	10, 16, $\overline{16}$, 45	0.0020 (2.6°)	2.4×10^{14} 4.5×10^8 4.5×10^8	5	5	$(0.2 - 9) \times 10^{-12}$	$(0.03 - 1) \times 10^{-10}$	$(0.03 - 2) \times 10^{-12}$
C - M	10, $\overline{126}$	0.013 (6.5°)	7.0×10^{12} 4.5×10^9 1.1×10^7	10	12	$(0.02 - 4) \times 10^{-15}$	$(0.02 - 5) \times 10^{-11}$	$(0.01 - 3) \times 10^{-16}$
C - Y	10, $\overline{126}$	0.0029 (3.1°)	2.6×10^{12} 2.6×10^{12} 2.6×10^{12}	10	12	$(0.02 - 5) \times 10^{-15}$	$(0.04 - 9) \times 10^{-13}$	$(0.03 - 6) \times 10^{-16}$
D - R	10, 45	0.0024 (2.8°)	5.5×10^{13} 9.3×10^{11} 1.1×10^{10}	50	2.5	$(0.05 - 8) \times 10^{-13}$	$(0.02 - 3) \times 10^{-9}$	$(0.01 - 2) \times 10^{-14}$
G - K	10, 120, $\overline{126}$	0.00059 (1.4°)	2.0×10^{15} 4.1×10^{14} 6.7×10^{12}	10	2	$(0.4 - 80) \times 10^{-11}$	$(0.004 - 1) \times 10^{-8}$	$(0.02 - 5) \times 10^{-11}$

Conclusions

Tried to differentiate models based on neutrino mass hierarchy, $\sin^2 \theta_{13}$, and charged lepton flavor violation predictions.

- Study initially based on 60+ models in literature (< 6/06)
 - Normal hierarchy preferred 3 : 1
 - Double CHOOZ and Daya Bay reactors will be able to eliminate roughly half of the 63 neutrino models surveyed, if their sensitivity reaches $\sin^2 2\theta_{13} \simeq 0.01$ as planned.
 - Of the order of 5 models have similar values for $\sin^2 \theta_{13}$ in the interval 0.001 - 0.08.
 - If the MEG experiment sees positive signals for $\mu \rightarrow e\gamma$, all non-SUSY models or non-NP models will be ruled out.

- Study narrowed to 5 predictive SO(10) SUSY GUT models
 - All 5 models have type I seesaws implying normal hierarchy.
 - $\sin^2 2\theta_{13}$ predictions:
CM (~ 0.05); AB, CY, DR (~ 0.01); GK (~ 0.001)
 - Previous studies of generic SO(10) models have concluded that the LFV branching ratios depend critically on θ_{13} and M_{R3} . Here we find that M_{R3} appears to be more important.
 - Branching ratio plots given for $A_0 = 0$ represent lower limits with higher predictions obtained for $|A_0/m_0| > 0$.
 - If the MEG experiment can reach an upper bound of $\text{BR}(\mu \rightarrow e\gamma) < 10^{-13}$, it will rule out the GK and AB models.
 - If $\mu - e$ conversion can be performed and reach a branching ratio limit of 10^{-18} as expected, it can potentially rule out all 5 models considered.