Nucleon sigma term from lattice QCD

Tetsuya Onogi (YITP, Kyoto Univ.)
for JLQCD collaboration

International Workshop on Grand Unified Theories:
Current Status and Future Prospects

December 17, 2007, Ritsumeikan University

–

JLQCD Collaboration

KEK S. Hashimoto, T. Kaneko, H. Matsufuru, J. Noaki, E. Shintani, N. Yamada
RIKEN/Niels Bohr H. Fukaya
Tsukuba S. Aoki, T. Kanaya, N. Ishizuka, Y. Taniguchi, A. Ukawa, T. Yoshie
Hiroshima K.-I. Ishikawa, M. Okawa
YITP H. Ohki, T. Onogi

KEK BlueGene (10 racks, 57.3 TFlops)

Outline

1. Introduction
2. Basic Methods
3. Lattice calculation
4. Results
5. Comparisons with other results
6. Discussion and summary

1. Introduction

WMAP found existence of Cold Dark Matter

= Evidence of New Physics

Candidates: neutralino, gravitino, axion, ….

In SUSY GUT, neutralino is one of the promising candidates,

but it may not be unique.

Direct detection of the dark matter is important

for finding the property and constituents of DM.

Neutralino(LSP) as DM candidate

- Neutralino is a combination of gaugino and higgsino.

$$\chi = a_1 \tilde{B} + a_2 \tilde{W}_3 + a_3 \tilde{H}_1 + a_4 \tilde{H}_4$$

- Mass and couplings depend on the SUSY breaking parameters, which should be determined by LHC and ILC

- Then one can predict the direct DM detection rate, which can be measured independently by experiment.

A crucial test for identifying the DM constituent.

Detection rate R

$$R \ \propto \ \sigma_{\chi N} \times \frac{\rho}{M} \int dv v f(v)$$

DM-Nuclear
cross-section

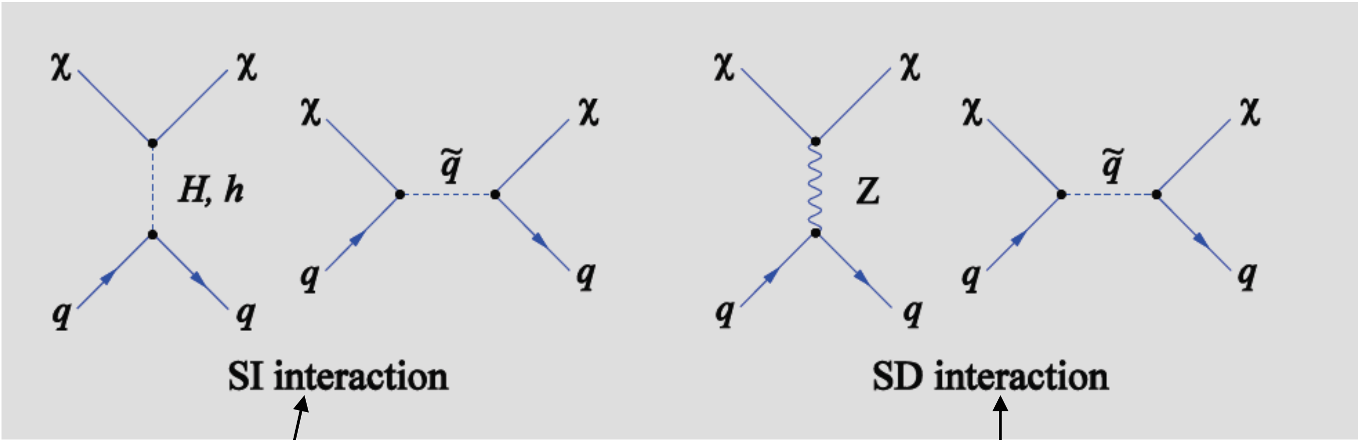
ρ : DM density at the earth
$f(v)$: velocity profile

Determined from SUSY model

Determined from N-body simulation

$\sigma_{\chi N}$ arises from the interaction with quark

- Spin-independent interaction : higgs exchange
- Spin-dependent interaction : Z and squark exchange



Coherent enhancement for large Atomic number

$\sigma_{\chi N}^{\mathrm{SI}} \propto A^2$

No Coherent enhancement

$\sigma_{\chi N}^{\mathrm{SD}} \propto \text{(total spin)}$

Spin independent interaction is much larger.

Why sigma term is important?

- A crucial parameter for the WIMP dark matter detection rate.

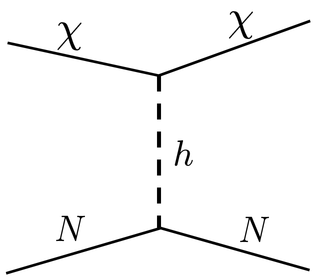
  The interaction with nucleon is mediated by the higgs boson exchange in the t-channel.

Higgs Yukawa coupling of the proton

$$y_{Hpp} = \frac{m_p}{250\text{MeV}}\left[\frac{2}{27} + \frac{25}{27} f_{T_s}\right] + \cdots$$

heavy quark loop

strange quark



K. Griest, Phys.Rev.Lett.62,666(1988)

Phys,Rev,D38, 2375(1988)

Baltz, Battaglia, Peskin, Wizanksy Phys. Rev. D74, 103521 (2006).

Sigma term is the parameter to convert the quark yukawa coupling to nucleon yukawa coupling.

Note: strange quark contribution is dominant.

Nucleon sigma term for strange quark is most important.

What is sigma term ?

- scalar form factor of the nucleon at zero recoil

$$\sigma_{\pi N} \equiv m_{ud}\left(\langle N|\bar{u}u+\bar{d}d|N\rangle - V\langle 0|\bar{u}u+\bar{d}d|0\rangle\right),$$
$$\sigma_{KN} \equiv m_s\left(\langle N|\bar{s}s|N\rangle - V\langle 0|\bar{s}s|0\rangle\right),$$

where $\langle N(p)|N(p')\rangle = (2\pi)^3\delta^3(p-p'),\ \ m_{ud} \equiv \frac{m_u+m_d}{2}$

V : spatial volume

- related quantities

$$y \equiv \frac{2\left(\langle N|\bar{s}s|N\rangle - V\langle 0|\bar{s}s|0\rangle\right)}{\langle N|\bar{u}u+\bar{d}d|N\rangle - V\langle 0|\bar{u}u+\bar{d}d|0\rangle}$$
$$f_{T_s} \equiv \frac{m_s\left(\langle N|\bar{s}s|N\rangle - V\langle 0|\bar{s}s|0\rangle\right)}{m_N}$$

We will denote $\langle N|\bar{q}q|N\rangle - V\langle 0|\bar{q}q|0\rangle$ as $\langle N|\bar{q}q|N\rangle$ for simplicity.

How well is sigma term known?

Theory	Group		method	$\sigma_{\pi N}$ [MeV]	y
ChPT	Gasser et al.(91)	1-loop	spectrum	44(9)	~ 0.2
	Borasoy-Meissner(96)	1-loop	spectrum	48(10)	0.2(2)
	Borasoy-Meissner(97)	2-loop	spectrum	36(7)	0.21(20)
Lattice	Kuramashi et al(95)	$n_f = 0$ Wilson	3pt/2pt	40-60	0.66(15)
	Dong et al(96)	$n_f = 0$ Wilson	3pt/2pt	50(3)	0.36(3)
	SEASAM(99)	$n_f = 2$ Wilson	3pt/2pt	18(5)	0.59(13)
	UKQCD(02)	$n_f = 2$ Clover	spectrum(unsubtracted)		0.53(12)
			spectrum(subtracted)		-0.30(34)
ChPT+Lattice	Procura et al.(04)	NNLO, $n_f = 2$ Clover	spectrum	49(3)	

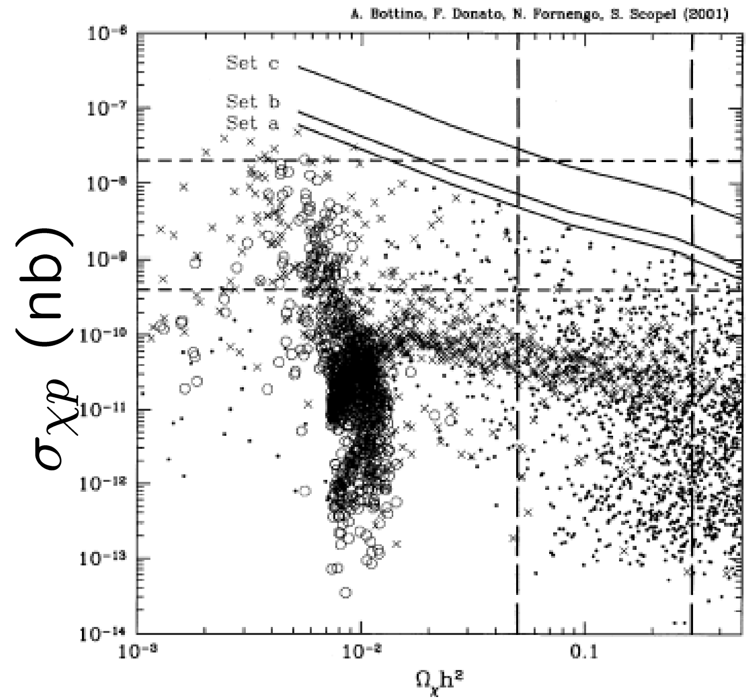
$\sigma_{\pi N}$=30-50 MeV from ChPT
y=0-0.2 from ChPT, y=0-0.5 from lattice QCD

The strange quark content has 100% uncertainty.

Experimental status

SUSY model prediction of DM cross-section $\sigma_{\chi p}$ choosing $y = 0.2$

Bottino et al. Astroparticle Phys.18(2002)205



Present senstivity

New experiments XMASS, SuperCDM
can improve the sensitivity
by 2-3 orders of magnitude.

Determination of y is getting
more and more important.

Our goal

Determine the nucleon sigma term in unquenced QCD
using the dynamical quark (overlap fermion),
which has an exact chiral symmetry on the lattice.

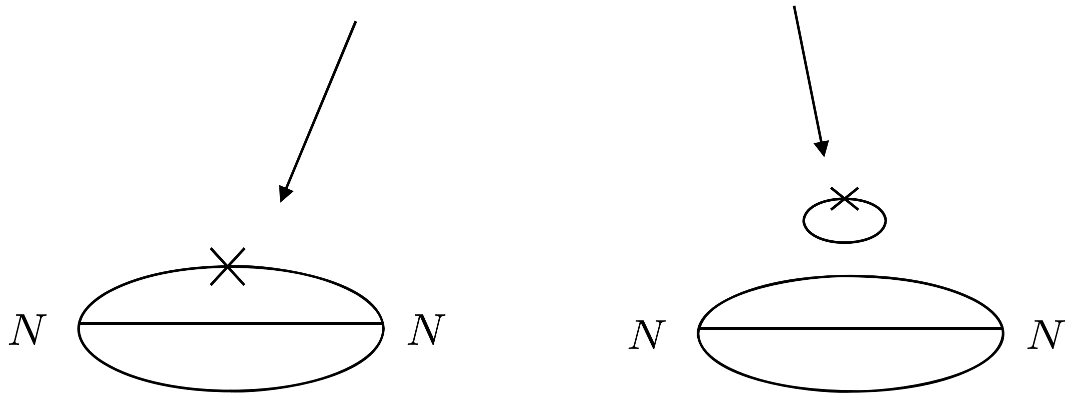
- The advantage of the exact chiral symmetry
  - No power divergence → subtraction of the vacuum condensate is numerical much more stable
  - No unwanted operator mixing
- In this study, we work in nf=2 unquenched QCD
  nf=2+1 QCD will be studied very soon
- We exploit mass spectrum method (explained later)

Basic Methods

Feynman - Hellman theorem

$$\frac{dm_N}{dm_q} = \langle N|\bar{q}q|N\rangle$$

Moreover, partial derivatives with respect to the valence and sea quark masses give contributions from ‘connected’ and ‘disconnected’ diagrams.



$$\frac{\partial m_N(m_{\text{val}}, m_{\text{sea}})}{\partial m_{\text{val}}} = \langle N|\bar{q}q|N\rangle_{\text{conn.}}$$

$$\frac{\partial m_N(m_{\text{val}}, m_{\text{sea}})}{\partial m_{\text{sea}}} = \langle N|\bar{q}q|N\rangle_{\text{disc.}}$$

3. Lattice calculation

Many unquenched simulations are performed or starting now.
In addition to rooted staggered by MILC collab.,
Wilson-type fermions and Ginsparg-Wilson fermions are in progress.
Important for cross-check and theoretically clean

Group	Action	n_f	a (fm)	m_π (MeV)
MILC	Staggered	2+1	0.09, 0.12	$\geq$300
Del Debbio et al.	Wilson, O(a)-imp Wilson	2	0.052-0.075	$\geq$ 300
PACS-CS	O(a)-imp Wilson	2+1	0.07, 0.10, 0.12	$\geq$210
ETMC	twisted Wilson	2	0.075, 0.096	$\geq$270
JLQCD	Overlap	2 (2+1)	0.11	$\geq$300
RBC UKQCD	Domain wall	2+1	0.09-0.13	$\geq$ 310

Ginsparg-Wilson fermion

- Ginsparg-Wilson relation   Ginsparg and Wilson, Phys.Rev.D 25(1982) 2649.

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D$$

Exact chiral symmetry on the lattice   (index theorem)

Hasenfratz, Laliena and Niedermayer, Phys.Lett. B427(1998) 125

Luscher, Phys.Lett.B428(1998)342.

$$\psi \rightarrow \psi + i\gamma_5(1 - aD)\psi = \psi + i\widehat{\gamma_5}\psi$$
$$\bar{\psi} \rightarrow \bar{\psi} + i\bar{\psi}\gamma_5$$

- Overlap fermion (explicit construction by Neuberger)

$$D = \frac{1}{a}\left[1 + \gamma_5 sign(H_W)\right], \qquad H_W \equiv \gamma_5(D_W + M_0)$$

D_W : Wilson Dirac op., M_0 : negative mass

Numerical simulation

Dynamical simulation with Nf=2 overlap fermion

- 16^3 x 32, a=0.12 fm, L=1.9 fm
- quark mass 6 values in the range of ms/6-ms
- fixed topology
- At Q=0 accumulated 10,000 trajectories

A good test for real calculation with Nf=2+1.

Nf=2+1(u,d,s) simulation will finish within 1-2 months.

Parameters for our study

We have 6 and 9 quark masses
for the sea and valence quarks, respectively.

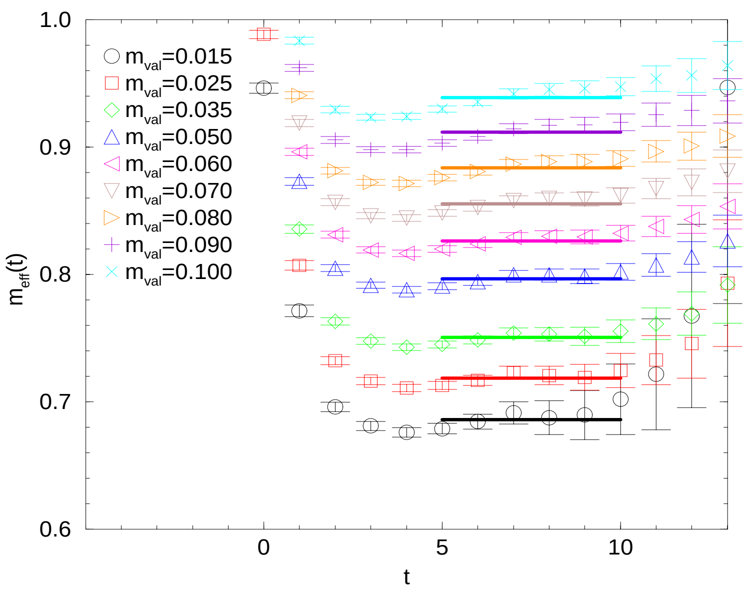
am_{sea}	m_π [GeV]	confs
0.015	0.3063(20)	500
0.025	0.3905(14)	500
0.035	0.4635(14)	500
0.050	0.5549(14)	500
0.070	0.6608(11)	500
0.100	0.7993(15)	500

$$L \sim 1.9 \text{ fm}$$
$$Q = 0$$
$$a^{-1} = 1.69 \text{ GeV}$$
$$am_{\text{val}} = 0.015, 0.025, 0.035, 0.050, 0.060, 0.070, 0.080, 0.090, 0.100$$

4. Results

- Nucleon masses from 2-pt functions

$$m_N^{\text{eff}}(t) \equiv -\ln\left(\frac{C_2(t+1)}{C_2(t)}\right)$$



Effective mass plot for amq=0.035

Solid lines are the mass from the fit

Nice plateau for t >4

We fit the 2-pt function with a single expoential function

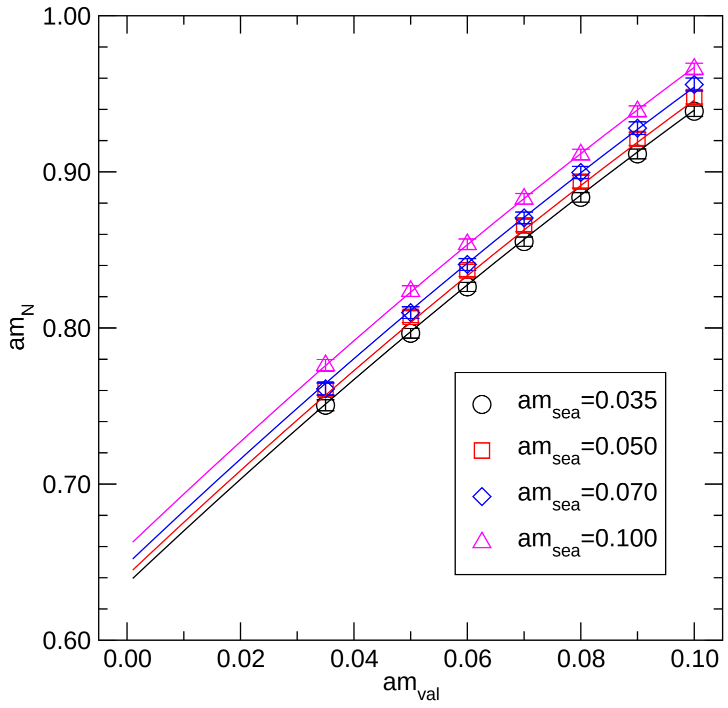
$$C_2^{\text{fit}}(t) \equiv Z_2 \exp(-m_N t)$$

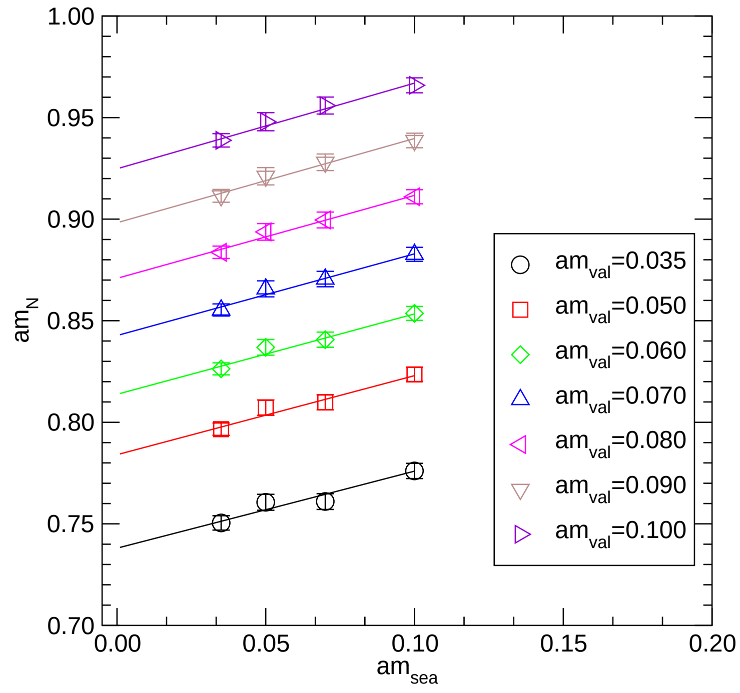
with fitting range t=5-10

Sea and valence quark mass dependences

The valence quark mass dependence is very clear,

while the sea quark mass dependence is small.





Fit of the quark mass dependence

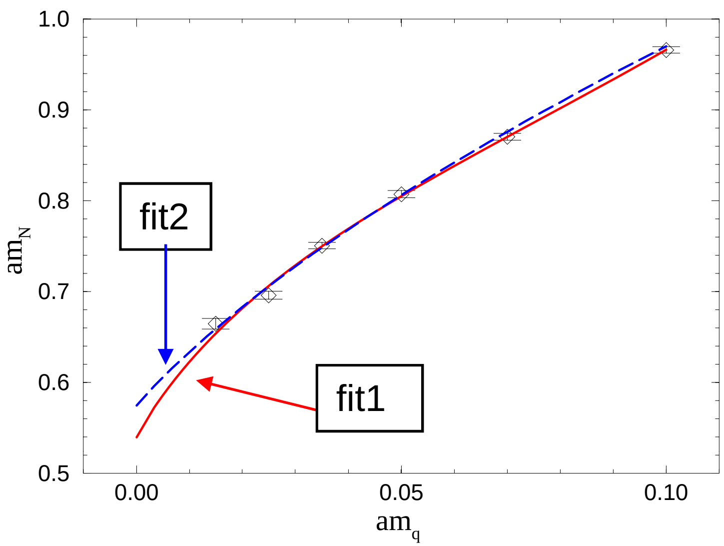
1. Fit of diagonal (unitary) points with Chiral Perturbation Theory (ChPT)

$$m_N = B_0 + B_1 m_q + B_2 m_q^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3$$

g_A: axial coupling of nucleon
Phenomenological value: $g_A = 1.267$

fit 1: gA fixed (gA=1.267), fit 2: gA free

Both fits uses 6 points $m_q = m_{\text{val}} = m_{\text{sea}} = 0.015, 0.025, 0.035, 0.050, 0.070, 0.100$



	B_0	B_1	B_2	g_A
fit1	0.5398(77)	12.05(29)	59.8(2.4)	1.267(fixed)
fit2	0.575(26)	7.2(3.4)	13(33)	0.73(52)

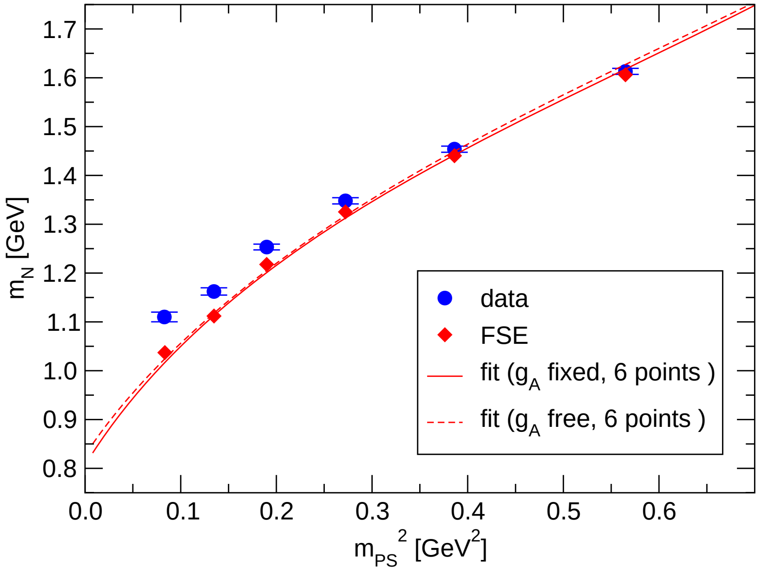
ChPT fit with fixed gA or free gA give consistent results.

- # Finite size correction

Box size of L=1.9 fm is rather small for baryon with light quarks.

We can estimate the Finite Size Effect(FSE) using ChPT.

After correcting the lattice data including FSE, we can redo the ChPT fit.



	B_0	B_1	B_2	g_A
Without FSE correction				
g_A(fixed)	0.5398(76)	12.05(29)	59.8(2.4)	1.267(fixed)
g_A(free)	0.575(26)	7.2(3.4)	13(33)	0.73(52)
With FSE correction				
g_A(fixed)	0.4855(73)	13.20(28)	53.3(2.3)	1.267(fixed)
g_A(free)	0.498(25)	11.5(3.3)	36(32)	1.10(33)

- Result of sigma term from ChPT fit

	$\sigma_{\pi N}$ [MeV]	
	FSE uncorrected	FSE corrected
g_A fixed	$48(2)_{\rm stat}$	$54(1)_{\rm stat}$
g_A free	$33(11)_{\rm stat}$	$49(10)_{\rm stat}$

1. Although there are finite size effects (FSE), sigma term gets only about 12% change.
2. ChPT fit (gA fixed) without considering FSE gives reasonable result.
3. We take ChPT fit (gA fixed, FSE uncorrected) as our best fit, assuming possible 12% finite size error.

$$\sigma_{\pi N} = 48(2)_{\rm stat}(^{+0}_{-15})_{\rm extrap.}(^{+6}_{-0})_{\rm FSE}\ [\mathrm{MeV}]$$

Preliminary

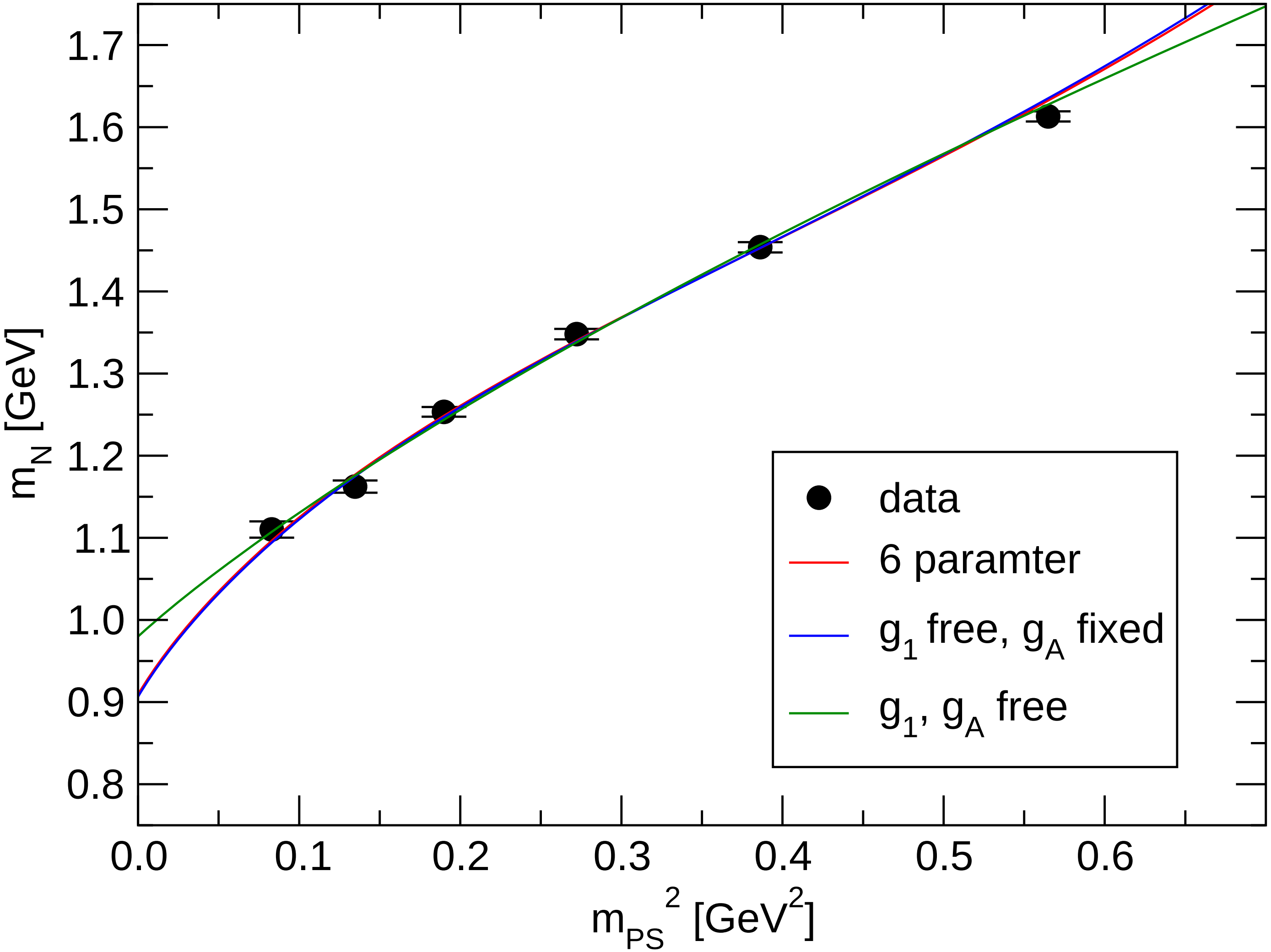
2. Global fit using partially quenched chiral perturbation (PQChPT)

$$m_N = B_{00} + B_{10}m_{val} + B_{01}m_{sea} + B_{11}m_{sea}m_{val} + B_{20}m_{val}^2 + B_{02}m_{sea}^2$$
$$-\frac{1}{16\pi f_\pi^2}\{\frac{g_A^2}{12}\left[-7(m_\pi^{vv})^3 + 16(m_\pi^{vs})^3 + 9m_\pi^{vv}(m_\pi^{ss})^2)\right]$$
$$\frac{g_1^2}{12}\left[-19(m_\pi^{vv})^3 + 10(m_\pi^{vs})^3 + 9m_\pi^{vv}(m_\pi^{ss})^2)\right]$$
$$\frac{g_1 g_A}{3}\left[-13(m_\pi^{vv})^3 + 4(m_\pi^{vs})^3 + 9m_\pi^{vv}(m_\pi^{ss})^2)\right]\}$$

$(m_\pi^{vv})^2 = Am_{val}$, $(m_\pi^{vs})^2 = \frac{A}{2}(m_{val} + m_{sea})$, $(m_\pi^{ss})^2 = Am_{sea}, A$: constant

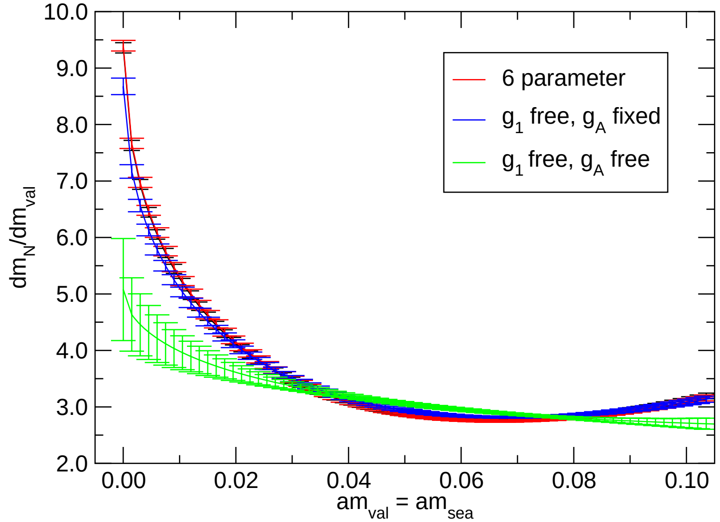
g_A, g_1: axial couplings of nucleon
Phenomenological values:
$g_A = 1.267$, $g_1 = -(0.4 - 0.6)$

	B_{00}	B_{01}	B_{10}	B_{11}	B_{20}	B_{02}	g_1	g_A
g_A, g_1 fixed	0.519(8)	2.9(3)	9.81(7)	4.4(7)	3.8(26)	47.0(3)	-0.4(fixed)	1.267(fixed)
g_A free, g_1 fixed	0.517(8)	3.7(4)	9.0(1)	18(2)	4.5(26)	33(2)	-0.38(1)	1.267(fixed)
g_A, g_1 free	0.55(9)	2.6(7)	6.6(11)	8.2(6)	-0.6(39)	18(7)	-0.29(4)	0.94(20)

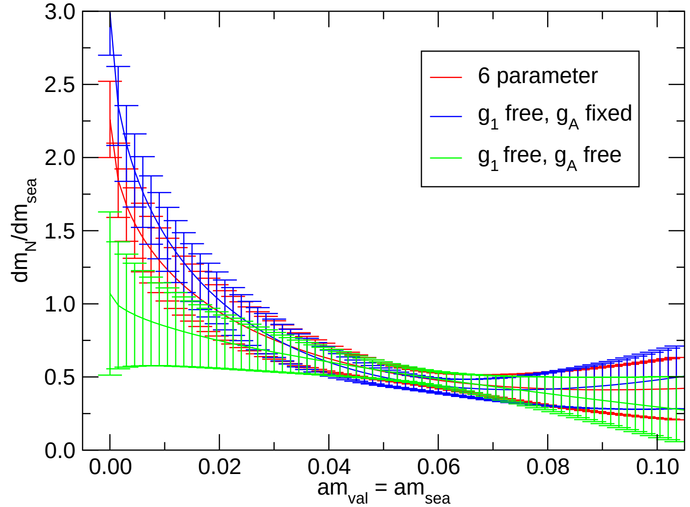


1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
m_N [GeV]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
m_PS^2 [GeV^2]
data
6 paramter
g_1 free, g_A fixed
g_1, g_A free

Connected and disconnected contributions for $m_{\text{val}} = m_{\text{sea}}$



Connected



Disconnected

fit	m_q	$\partial m_N/\partial m_{\text{val}}$	$\partial m_N/\partial m_{\text{sea}}$
g_A, g_1 fixed	0.0033	7.10(9)	1.65(25)
g_A fixed, g_1 free	0.0033	6.55(12)	2.06(26)
g_A, g_1 free	0.0033	4.50(33)	0.95(38)
g_A, g_1 fixed	0.080	-	0.42(11)
g_A fixed, g_1 free	0.080	-	0.42(11)
g_A, g_1 free	0.080	-	0.39(11)
g_A, g_1 fixed	0.090	-	0.41(16)
g_A fixed, g_1 free	0.090	-	0.44(16)
g_A, g_1 free	0.090	-	0.34(16)

Connected and disconnected contributions for $m_{\rm val} = m_{\rm sea}$

Taking PQChPT fit with fixed g1,gA, the disconnected contribution turns out as

$$2\langle N|\bar{q}q|N\rangle_{\rm disc.} = 1.65(25) \quad \text{for } m_q = m_{ud} = 0.0033$$
$$= 0.41(16) \quad \text{for } m_q = m_s = 0.090$$

It is not possible to extract strange quark content $\langle N|\bar{s}s|N\rangle$ from 2-flavor QCD.
For final result, we should wait for 2+1-flavor QCD result (coming soon).

In the present study,
We take $m_q = 0.09$ result as our best estimate
and use $m_q = 0.0033$ result to estimate the chiral extrapolation error.

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = 0.05(2)_{\rm stat.}(^{+14}_{-0})_{\rm extrap.}$$ Preliminary

Comparison with other results

$\sigma_{\pi N}$

- ChPT results and previous results are consistent.
- Our results with ChPT is consistent
- Previous lattice calculation sea/valence. Is larger than 1
- Our lattice calculation sea/valence n is about 0.1

y

- ChPT predicts $y = 0 - 0.4$
- Previous lattice results $y \sim 0.5$ due to large sea quark contribution
- Our results gives $y = 0.04 - 0.2$

Why is y so different ?

- ChPT

  Large uncertainty from Low Energy Constants.

- Previous lattice caculation

  Why disconnected contribution is so large?

  mixing with wrong chirality?

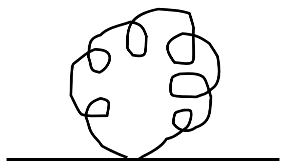
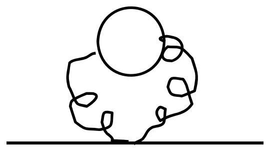
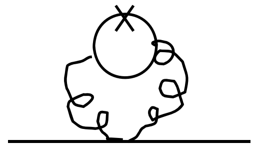
Operator mixing due to Wilson fermion artifact

If there is an additive mass shift there can be operator mixing which should be subtracted (but not subtracted except for UKQCD) for disconnected diagram.

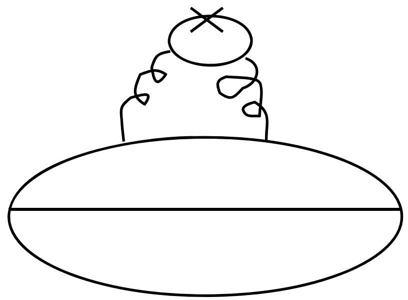
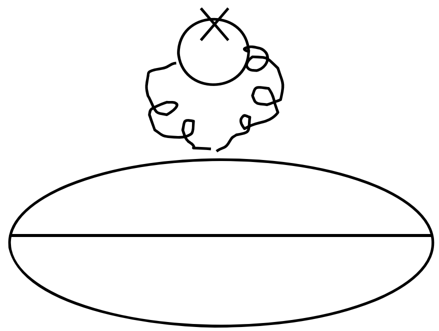
$$
\begin{aligned}
&\left(\bar{\psi}_{\text{sea}}\psi_{\text{sea}}\right)^{\text{lat}} \\
= \;& C_0 I + Z_S\left[\left(\bar{\psi}_{\text{sea}}\psi_{\text{sea}}\right)^{\bar{M}S} + \frac{\partial \Delta m_q}{\partial m_{\text{sea}}}\left(\bar{\psi}_{\text{val}}\psi_{\text{val}}\right)^{\bar{M}S}\right] + O(a)
\end{aligned}
$$

Subtracting this mixing effect by using the sea quark mass dependence of the quark mass shift, the disconnected contribution becomes tiny (consistent with zero).

Additive mass shift and sigma term

$\Delta m_q =$  $+$  $\Longrightarrow \frac{d\Delta m_q}{dm_{\mathrm{sea}}} =$ 

Wilson fermion has an additive mass shift through tadpole diagram which mimics mass operator insertion to the valence quark due to the lack of chiral symmetry

$\langle N|\,(\bar{q}q)^{\mathrm{lat}}\,|N\rangle_{\mathrm{disc.}} =$  $+$ 

Normal disconnected contribution

(long distance effect)

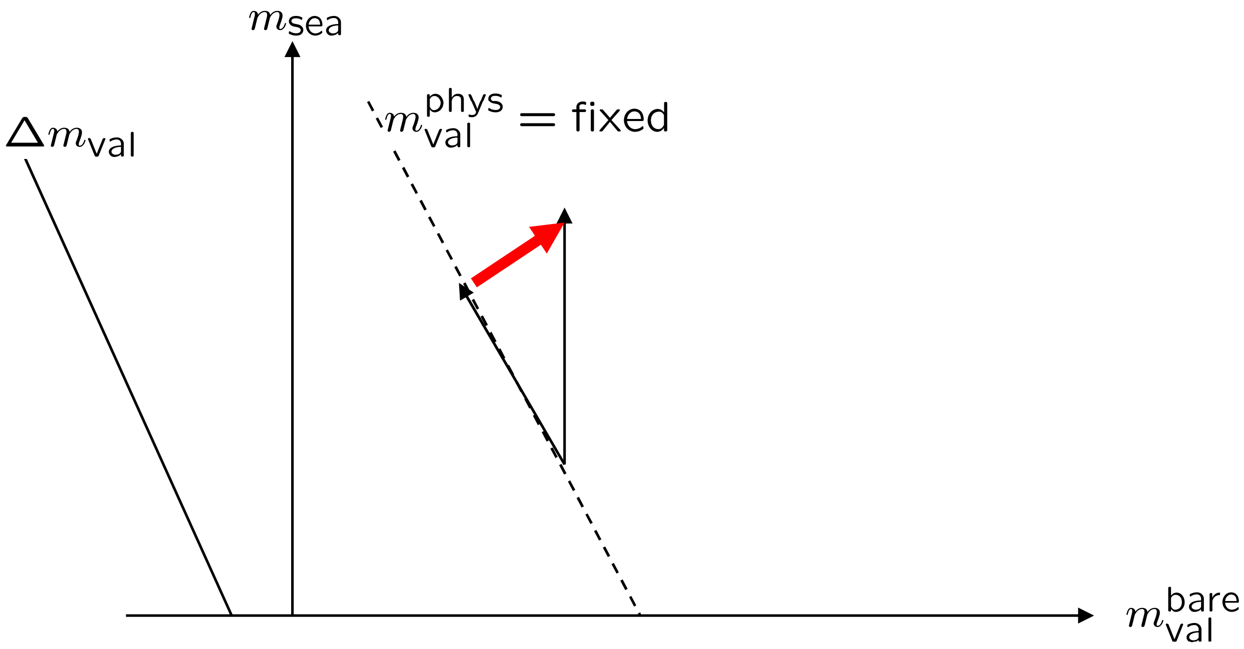
Operator mixing induced connected contribution

(short distance effect)

Additive mass shift and sigma term

Same phenomena can be understood in the mass spectrum approach

Fixing the bare valence quark mass and changing the sea quark mass effectively induces the change of the 'physical' valence quark mass due to the additive mass shift.



Discussion and summary

- We studied the nucleon mass spectrum for nf=2 unquenched QCD using exactly chirally symmetric dynamical fermion.
- It is expected that our calculation is free from dangerous lattice artifacts (power divergence, operator mixing)
- Our result is consistent with ChPT prediction.
- We found disconnected (strange quark content) part is tiny.
- We pointed out that the descrepancies from previous lattice calculation can come from artifact in Wilson fermion.

σ_{KN} (polynomial)

$$2\langle N|\bar{s}s|N\rangle_{\text{disc.}} = \frac{\partial m_N}{\partial m_{\text{sea}}}\Big|_{m_{\text{val}}=m_{ud}, m_{\text{sea}}=m_s} = B_{01} + B_{11} m_{ud}$$

$$2\langle N|\bar{s}s|N\rangle_{\text{disc.}} = 0.36(9)_{\text{stat}}$$

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = 0.045(11)$$

Preliminary