



**Proton Decay in  $SO(10)$   
with Stabilized Doublet–Triplet Splitting**

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**K.S. Babu, J.C. Pati, Z. Tavartkiladze (2007)**

## Outline

### ❖ Motivation/“Evidence” favoring SO(10) GUT

### ❖ Doublet-Triplet Splitting Issues:

- Dimopoulos-Wilczek Mechanism
- Stability of VEV structure
- Absence of pseudo-Goldstone bosons
- GUT threshold corrections and unification

### ❖ Proton decay

- d=5 and d=6 proton decay rates
- Correlation between the two

### ❖ Conclusions

## Motivations/Evidence favoring SO(10)

- Electric charge quantization
  - ◇  $Q_p = -Q_e$  to better than 1 part in  $10^{21}$
- Miraculous cancellation of anomalies
- Quantum numbers of quarks and leptons
- Existence of  $\nu_R$  and thus neutrino mass via seesaw
- Unification of gauge couplings with low energy SUSY
- $b - \tau$  unification
- Baryon asymmetry of the universe via leptogenesis

## Miraculous cancellation of anomalies

- $SU(3)_C^2 \times U(1)_Y: \frac{1}{2} \left[ 2 \times \left(\frac{1}{6}\right) + 1 \times \left(\frac{-2}{3}\right) + 1 \times \left(\frac{1}{3}\right) \right] = 0$

- $SU(2)_L^2 \times U(1)_Y: \frac{1}{2} \left[ 3 \times \left(\frac{1}{6}\right) + 1 \times \left(\frac{-1}{2}\right) \right] = 0$

- $(\text{gravity})^2 \times U(1)_Y:$

$$\left[ 3 \times 2 \times \left(\frac{1}{6}\right) + 3 \times \left(\frac{-2}{3}\right) + 3 \times \left(\frac{1}{3}\right) + 2 \times \left(\frac{-1}{2}\right) + 1 \times 1 \right] = 0$$

- $U(1)_Y^3:$

$$\left[ 3 \times 2 \times \left(\frac{1}{6}\right)^3 + 3 \times \left(\frac{-2}{3}\right)^3 + 3 \times \left(\frac{1}{3}\right)^3 + 2 \times \left(\frac{-1}{2}\right)^3 + 1 \times (1)^3 \right] = 0$$

Suggests  $SO(10)$  embedding

$SO(10)$  is automatically anomaly free

## Structure of matter multiplets

$$Q = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \sim (3, 2, \frac{1}{6})$$

$$u^c = (u_1^c \quad u_2^c \quad u_3^c) \sim (\bar{3}, 1, \frac{-2}{3})$$

$$d^c = (d_1^c \quad d_2^c \quad d_3^c) \sim (\bar{3}, 1, \frac{1}{3})$$

$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix} \sim (1, 2, \frac{-1}{2})$$

$$e^c \sim (1, 1, +1)$$

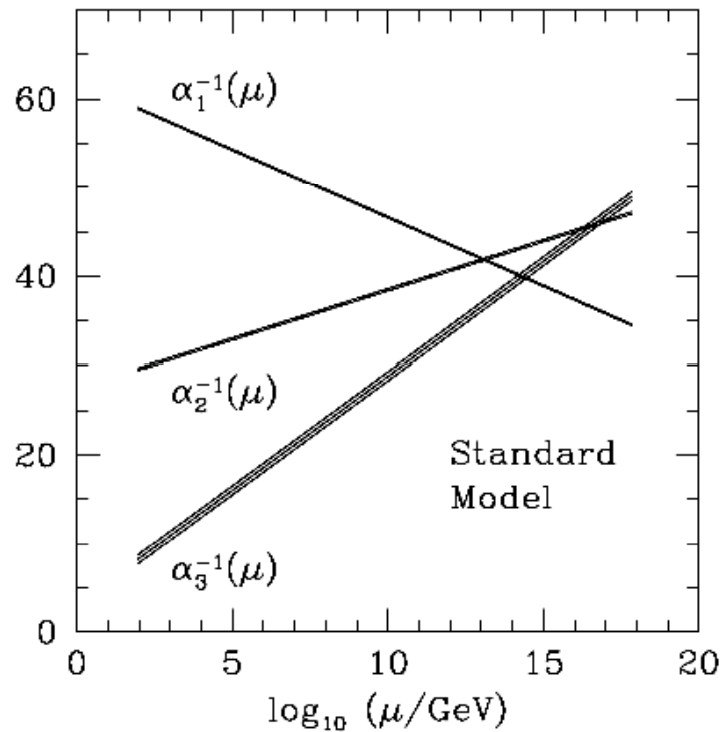
$$\nu^c \sim (1, 1, 0)$$

## Matter Unification in 16 of SO(10)



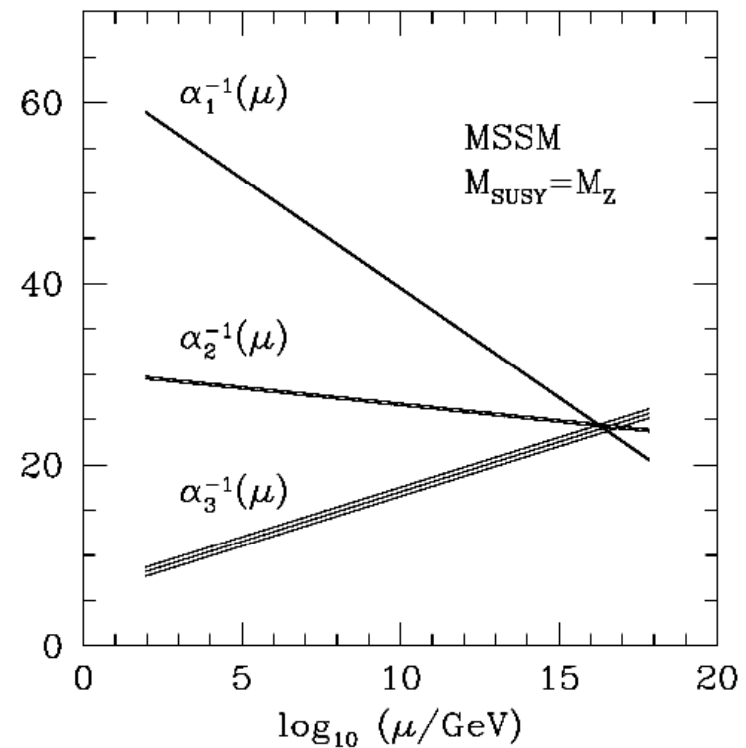
$u_1$	:		↑	↓	↑	↑	↓	>
$u_2$	:		↑	↓	↑	↓	↑	>
$u_3$	:		↑	↓	↓	↑	↑	>
$d_1$	:		↓	↑	↑	↑	↓	>
$d_2$	:		↓	↑	↑	↓	↑	>
$d_3$	:		↓	↑	↓	↑	↑	>
$u_1^c$	:		↓	↓	↑	↓	↓	>
$u_2^c$	:		↓	↓	↓	↑	↓	>
$u_3^c$	:		↓	↓	↓	↓	↑	>
$d_1^c$	:		↑	↑	↑	↓	↓	>
$d_2^c$	:		↑	↑	↓	↑	↓	>
$d_3^c$	:		↑	↑	↓	↓	↑	>
$\nu$	:		↑	↓	↓	↓	↓	>
$e$	:		↓	↑	↓	↓	↓	>
$e^c$	:		↓	↓	↑	↑	↑	>
$\nu^c$	:		↑	↑	↑	↑	↑	>

# Evolution of gauge couplings



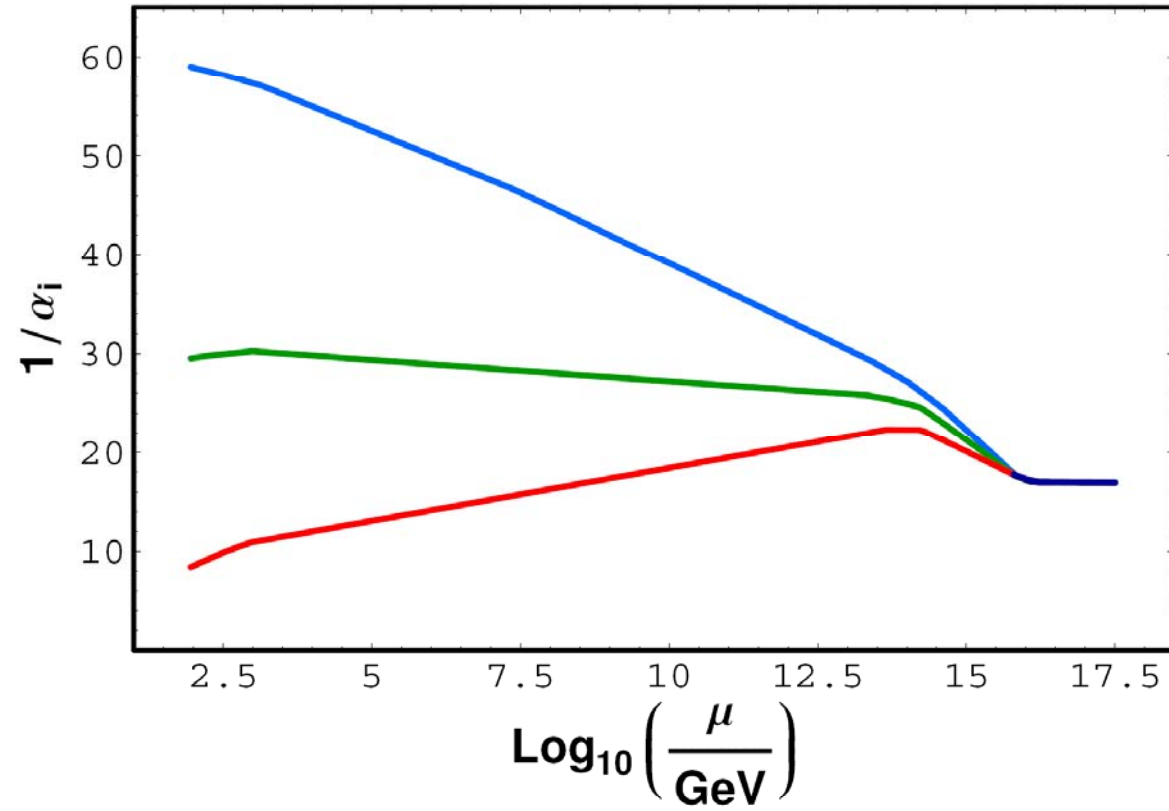
Standard Model

Unification far off



Supersymmetry

$\alpha_3(M_Z) \simeq 0.13$  predicted



Gauge coupling evolution in our SO(10) model



## GUT gauge groups

- $SU(5)$
- $SO(10)$
- $E_6$
- $E_8$
- ...

- $[SU(3)]^3$
- $[SU(5)]^2$
- $[SU(3)]^4$
- ...

# Minimal SUSY SU(5) GUT

**Matter multiplets:**  $\{10 + \bar{5} + 1\}$

$$10 : \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}$$

$$\bar{5} : (d_1^c, d_2^c, d_3^c, e, -\nu_e)$$

$$1 : \nu^c$$

**Higgs:**  $24_H, \{5_H, \bar{5}_H\} \Rightarrow$  Contain color triplets  $\{H_C, \bar{H}_C\}$

**Yukawa Couplings**  $Y_u^{ij} 10_i 10_j 5_H + Y_d^{ij} 10_i \bar{5}_j \bar{5}_H$

$$M_\ell = M_d^T \Rightarrow m_b = m_\tau, m_s = m_\mu, m_d = m_e$$

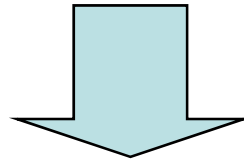
## MSSM Higgs doublets have color triplet partners in GUTs

$$H(1, 2, 1/2) \oplus H_c(3, 1, -1/3) = \mathbf{5} \text{ of } SU(5)$$

$$\bar{H}(1, 2, -1/2) \oplus \bar{H}_c(\bar{3}, 1, 1/3) = \bar{\mathbf{5}}$$

$H, \bar{H}$  must remain light

$H_c, \bar{H}_c$  must have GUT scale mass to prevent rapid proton decay



### Doublet-triplet splitting

Even if color triplets have GUT scale mass, d=5 proton decay is problematic

## Doublet-triplet splitting in SU(5)

$$W_{D-T} = \bar{5}_H (\lambda 24_H + M) 5_H$$

$$\langle 24_H \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{pmatrix} V$$

$$M_{H_c} = \lambda V + M \sim O(M_{GUT}); \quad M_H = -\frac{3}{2}\lambda V + M$$

FINE-TUNED TO  $O(M_W)$

### The GOOD

- (1) Predicts unification of couplings
- (2) Uses economic Higgs sector

### The BAD

- (1) Unnatural fine tuning
- (2) Large proton decay rate

# Realistic SO(10) model without fine-tuning

Quarks and leptons:  $\{16_i\}$

Babu, Pati, Wilczek (1998)

Barr, Raby (1997)

Babu, Pati, Tavartkiladze (2007)

## Economic Higgs system

$\{45_H + 16_H + \overline{16}_H\} \Rightarrow$  breaks symmetry to SM

$\{10_H + 10'_H\} \Rightarrow$  EW symmetry breaking

$\{16'_H + \overline{16}'_H\} \Rightarrow$  avoids pseudoGoldstones

$\{2 \text{ singlets : } S, Z\} \Rightarrow$  Fixes VEVs

Small Higgs representation  $\Rightarrow$  Small threshold effects for gauge couplings

A  $Z_2$ -assisted anomalous  $\mathcal{U}(1)$  symmetry guarantees stability

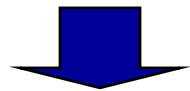
## Dimopoulos-Wilczek mechanism

$$W_{D-T} = \lambda(10_H 45_H 10'_H) + \dots$$

$$\langle 45_H \rangle = \begin{pmatrix} a & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes i\tau_2 \propto (B-L)$$

Dimopoulos, Wilczek (1981)  
Babu, Barr (1993)

- B-L VEV gives mass to triplets only
- If  $10_H$  only couples to fermions, no d=5 proton decay
- Doublets from  $10_H$  and  $10'_H$  light  
4 doublets, unification upset



Add mass term for  $10'_H$

$$W_{D-T} = \lambda(10_H 45_H 10'_H) + M 10'_H 10'_H$$

## Issues to be addressed with DW mechanism

- Can the VEV pattern for  $45_H$  be realized?
- Is the VEV structure stable?
- Are there flat directions?
- Are there pseudo-Goldstone bosons?
- Are threshold corrections small?
- Is  $d = 5$  proton decay consistent with data?

We now have a complete model where all issues are successfully addressed

# Complete SO(10) Model

## $Z_2$ -assisted anomalous $\mathcal{U}(1)$

	$A(45)$	$H(10)$	$H'(10)$	$C(16)$	$\bar{C}(\bar{16})$	$Z$	$S$	$C'(16)$	$\bar{C}'(\bar{16})$	$16_i$
$\mathcal{U}(1)$	0	1	-1	$\frac{k+4}{2k}$	$-\frac{1}{2}$	$\frac{2}{k}$	$\frac{2}{k}$	$\frac{k-4}{2k}$	$-\frac{k+8}{2k}$	$-\frac{1}{2} + a_i$
$Z_2$	-	+	-	+	+	-	+	+	+	$P_i$

Superpotential:

$$W(A) = M_A \text{tr} A^2 + \frac{\lambda_A}{M_*} (\text{tr} A^2)^2 + \frac{\lambda'_A}{M_*} \text{tr} A^4$$

$$W(A, C, C') = C \left( \frac{a_1}{M_*} Z A + \frac{b_1}{M_*} C \bar{C} + c_1 S \right) \bar{C}' + C' \left( \frac{a_2}{M_*} Z A + \frac{b_2}{M_*} C \bar{C} + c_2 S \right) \bar{C}$$

$$W(DT) = \lambda_1 H A H' + \lambda_{H'} \frac{S^k}{M_*^{k-1}} (H')^2 + \lambda_2 H \bar{C} \bar{C}$$



## Fixing the VEVs

$$\langle A \rangle = i\sigma_2 \otimes \text{Diag}(a, a, a, 0, 0)$$

$$\langle C \rangle = \langle \bar{C} \rangle = c, \quad \langle C' \rangle = \langle \bar{C}' \rangle = 0$$

$$\langle S \rangle = s, \quad \langle Z \rangle = z$$

$$F_A = 0 \Rightarrow a^2 = \frac{M_A M_*}{2(6\lambda_A + \lambda'_A)}$$

$$\mathcal{U}(1) \text{ Dterm} \Rightarrow c^2 + z^2 + s^2 = \frac{k}{2}\xi \quad \left(\xi = \frac{g_A^2 M_P^2}{192\pi^2} \text{Tr}Q\right)$$

$$F_{C'} = 0 \Rightarrow -\frac{3a_1}{M_*}za + \frac{b_1}{M_*}c^2 + c_1s = 0$$

$$F_{\bar{C}'} = 0 \Rightarrow \frac{3a_2}{M_*}za + \frac{b_2}{M_*}c^2 + c_2s = 0 .$$

$$F_S = 0 \Rightarrow \langle C' \rangle = 0$$

$$F_Z = 0 \Rightarrow \langle \bar{C}' \rangle = 0$$

$$\Rightarrow s = \frac{c^2}{M_*} \frac{b_1 a_2 - b_2 a_1}{a_1 c_2 - a_2 c_1}, \quad z = \frac{c^2}{3a} \frac{b_1 c_2 - b_2 c_1}{a_1 c_2 - a_2 c_1}$$

All the VEVs are fixed, in desired directions

## Doublet-Triplet Mass Matrix

$$M_{D,T} = \begin{matrix} \bar{5}_H \\ \bar{5}_{H'} \\ \bar{5}_C \\ \bar{5}_{C'} \end{matrix} \begin{pmatrix} 5_H & 5_{H'} & 5_{\bar{C}} & 5_{\bar{C}'} \\ 0 & \eta_{D,T} a & \lambda_2 c & 0 \\ -\eta_{D,T} a & M_{H'} & 0 & 0 \\ 0 & 0 & 0 & \kappa_{D,T} Y_1 \\ 0 & 0 & \kappa_{D,T} Y_2 & M' \eta \end{pmatrix}$$

with  $\eta_D = 0$ ,  $\eta_T = \lambda_1$ ,  $\kappa_D = 3$ ,  $\kappa_T = 2$

$M_{H'} = \lambda_{H'} s^k / M_*^{k-1}$ ,  $Y_{1,2} = 2a_{1,2} z a / (M_*) \sim M_{\text{GUT}}^2 / M_*$

$M' \subset Z^2 S C' \bar{C}' / M^2$ ,  $\eta = 1$  or  $0$

All color triplets heavy, one pair of Higgs doublets light

$H_u$  is in  $5_H$ ,  $H_d$  in  $\bar{5}_H + 16'_H \Rightarrow \tan \beta \neq \frac{m_t}{m_b}$

## Stability of doublet mass

$$M_{D,T} = \begin{matrix} & 5_H & 5_{H'} & 5_{\bar{C}} & 5_{\bar{C}'} \\ \begin{matrix} \bar{5}_H \\ \bar{5}_{H'} \\ \bar{5}_C \\ \bar{5}_{C'} \end{matrix} & \begin{pmatrix} 0 & \eta_{D,T} a & \lambda_2 c & 0 \\ -\eta_{D,T} a & M_{H'} & 0 & 0 \\ 0 & 0 & 0 & \kappa_{D,T} Y_1 \\ 0 & 0 & \kappa_{D,T} Y_2 & M' \eta \end{pmatrix} \end{matrix}$$

Zeros in first column protected by  $\mathcal{U}(1) \times Z_2$  symmetry

Mass for doublet needs negative charge VEV

All GUT scale VEVs have positive  $\mathcal{U}(1)$  charge

Superpotential must be holomorphic

$A^n (C\bar{C})^m$  type terms would destabilize  $(B - L)$  VEV

Such terms forbidden by  $\mathcal{U}(1)$  symmetry

$$W_\mu = \frac{1}{\tilde{M}} H C' C' S \Rightarrow \mu \sim 10^2 \text{ GeV}$$

This shows all order stability of doublet mass

## Spectrum of doublets and triplets

$$\frac{M_{D_1} M_{D_2} M_{D_3}}{M_{T_1} M_{T_2} M_{T_3} M_{T_4}} = \frac{9}{4 M_{\text{eff}} \cos \gamma}$$

with  $\tan \gamma = \frac{\lambda_2 c}{3 Y_2}$ ,  $\frac{1}{M_{\text{eff}}} = (M_T^{-1})_{11} = \frac{M_{H'}}{\lambda_1^2 a^2}$

$M_{\text{eff}}$  controls  $d = 5$  proton decay:  $\mathcal{A}(d = 5) \propto 1/M_{\text{eff}}$

$M_{\text{eff}} \sim 10^{19}$  realized naturally, as  $M_{H'} \ll M_{\text{GUT}}$

## Spectrum heavy states from 45

$$M_{e_A^c} = M_3 = \frac{1}{2} M_8 \equiv \frac{1}{2} M_\Sigma, \quad \text{with} \quad M_\Sigma = \frac{2\lambda'_A}{6\lambda_A + \lambda'_A} M_A.$$

$$M_3 : M(1, 3, 0), \quad M_8 : M(8, 1, 0), \quad M_{e^c} : M(1, 1, 1)$$

Remaining states are in  $10 + \overline{10}$  of  $SU(5)$

## Spectrum from 10 + 10-bar

$$M(\Psi^{10}) = \begin{pmatrix} \Psi_A^{10} \\ \Psi_C^{10} \\ \Psi_{C'}^{10} \end{pmatrix} \begin{pmatrix} \bar{\Psi}_A^{10} & \bar{\Psi}_C^{10} & \bar{\Psi}_{C'}^{10} \\ M_\Psi & 0 & X_1 \\ 0 & 0 & \kappa_\Psi Y_1 \\ X_2 & \kappa_\Psi Y_2 & M'\eta \end{pmatrix},$$

with  $\Psi = (u^c, q, e^c)$ ,  $\kappa_\Psi = (2, 1, 0)$ ,  $M_\Psi = (0, 0, M_\Sigma/2)$

$$X_{1,2} = 4a_{1,2}zc/M_*$$

$$u_1^c u_2^c = Y_1 Y_2 (4 + \tilde{p}^2), \quad Q_1 Q_2 = Y_1 Y_2 (1 + \tilde{p}^2), \quad \mathcal{E}_1^c \mathcal{E}_2^c = Y_1 Y_2 \tilde{p}^2$$

$$\text{with } \tilde{p}^2 = \frac{|X_1|^2}{|Y_1|^2} = \frac{|X_2|^2}{|Y_2|^2}, \quad \hat{p}^2 = \tilde{p}^2 \left| 1 - \frac{M_\Sigma M'}{2X_1 X_2} \eta \right|$$

## Gauge boson spectrum

$$M^2(X, Y) = g^2 a^2 \equiv M_X^2, \quad M^2(X', Y') = M_X^2(1 + p^2)$$

$$M^2(V_{uc}, \bar{u}c) = M_X^2(4 + p^2), \quad M^2(V_{ec}, \bar{e}c) = M_X^2 p^2, \quad \text{with } p^2 = \frac{4c^2}{a^2}$$

Note:  $p = \tilde{p}$  in the model

⇒ Apparent  $N = 4$  SUSY in  $\{10 + \overline{10}\}$  spectrum

⇒ Threshold corrections from  $\{10 + \overline{10}\}$  cancels

⇒ Model almost as predictive as minimal SUSY  $SU(5)$

# Threshold corrections

$$\alpha^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\Lambda}{M_Z} + \Delta_{i,w}^{(2)} + \Delta_i^{\text{GUT}},$$

## Weak scale threshold

**Case 1 :**  $\tan \beta = 3$ ,  $m_0 \simeq 300$  GeV,  $m_{1/2} \simeq 352.4$  GeV

$$\Rightarrow \Delta_{i,w}^{(2)} \simeq (0.2602, 0.349, 1.207)$$

**Case 2 :**  $\tan \beta = 3$ ,  $m_0 \simeq 930$  GeV,  $m_{1/2} \simeq 146.8$  GeV

$$\Rightarrow \Delta_{i,w}^{(2)} \simeq (0.4021, 0.2264, 0.9483);$$

**Case 3 :**  $\tan \beta = 3$ ,  $m_0 \simeq 1.97$  TeV,  $m_{1/2} \simeq 146.8$  GeV,

$$\Rightarrow \Delta_{i,w}^{(2)} \simeq (0.6431, 0.4739, 1.209).$$

## GUT scale threshold

Hisano, Murayama, Yanagida (1993)  
Hisano, Moroi, Tobe, Yanagida (1995)

$$\begin{aligned} \ln \frac{M_{\text{eff}} \cos \gamma}{M_Z} &= \frac{5\pi}{6} \left( 3(\alpha_2^{-1} + \Delta_{2,w}^{(2)} - \frac{1}{6\pi}) - 2(\alpha_3^{-1} + \Delta_{3,w}^{(2)} - \frac{1}{4\pi}) - (\alpha_1^{-1} + \Delta_{1,w}^{(2)}) \right) \\ &\quad - \ln \frac{4\kappa^{5/2}}{9} + \ln \frac{(4+p^2)^{3/2}(1+\tilde{p}^2)^2}{(4+\tilde{p}^2)^{3/2}(1+p^2)^2} + \ln \frac{p}{\tilde{p}} \\ \ln \frac{(M_X^2 M_\Sigma)^{1/3}}{M_Z} &= \frac{\pi}{18} \left( 5(\alpha_1^{-1} + \Delta_{1,w}^{(2)}) - 3(\alpha_2^{-1} + \Delta_{2,w}^{(2)} - \frac{1}{6\pi}) - 2(\alpha_3^{-1} + \Delta_{3,w}^{(2)} - \frac{1}{4\pi}) \right) \\ &\quad + \frac{1}{6} \ln \kappa - \frac{1}{6} \ln \frac{(4+p^2)(1+\tilde{p}^2)^2}{(4+\tilde{p}^2)(1+p^2)^2} - \frac{1}{3} \ln \frac{p}{\tilde{p}}, \end{aligned}$$

Very similar to minimal SUSY  $SU(5)$  threshold

A single new parameter beyond  $SU(5)$

$\Rightarrow$  better  $\alpha_3(M_Z)$

$M_{HC}$  of  $SU(5)$  replaced by  $M_{\text{eff}} \cos \gamma$

$\Rightarrow d = 5$  proton decay under control



## Correlation between d=5 and d=6 proton decay

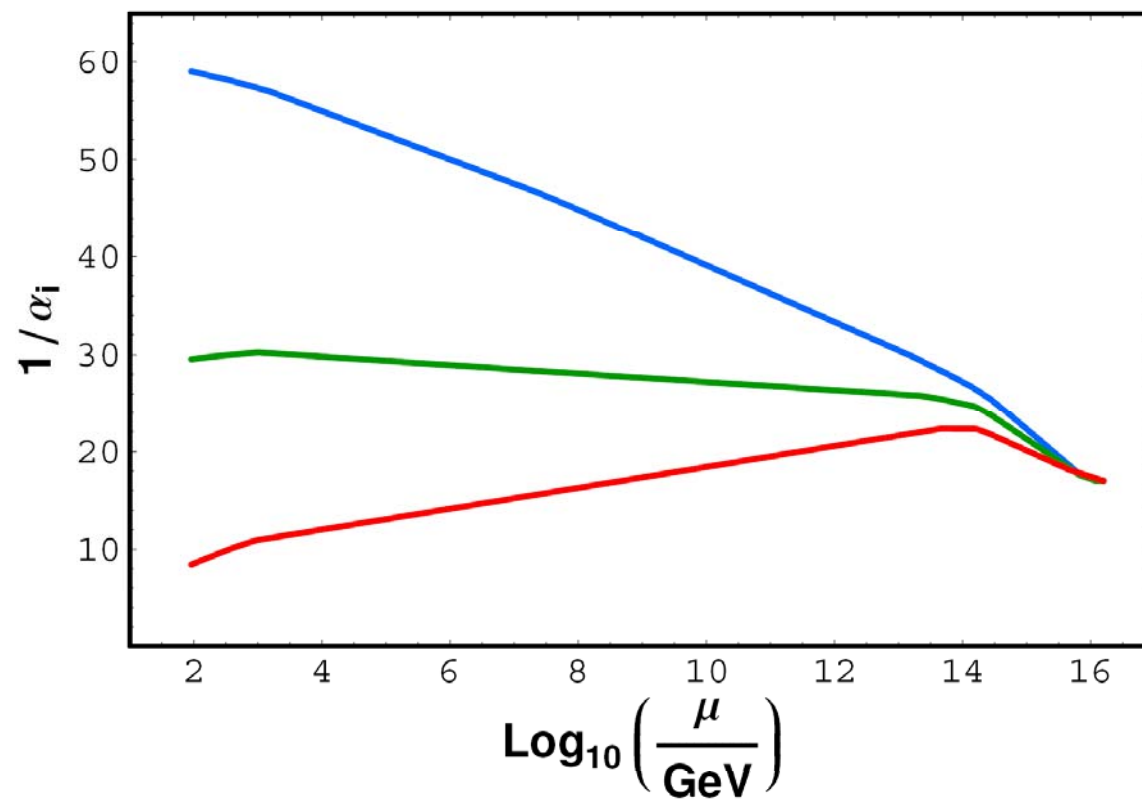
$$M_{\text{eff}} \simeq 10^{18} \text{GeV} \cdot \left( \frac{10^{16} \text{GeV}}{M_X} \right)^3 \left( \frac{3}{\tan \beta} \right) \left( \frac{1/25}{r} \right) \frac{\exp[2\pi(\Delta_{2,w}^{(2)} - \Delta_{3,w}^{(2)} - \delta\alpha_3^{-1})]}{1.42 \cdot 10^{-2}}.$$

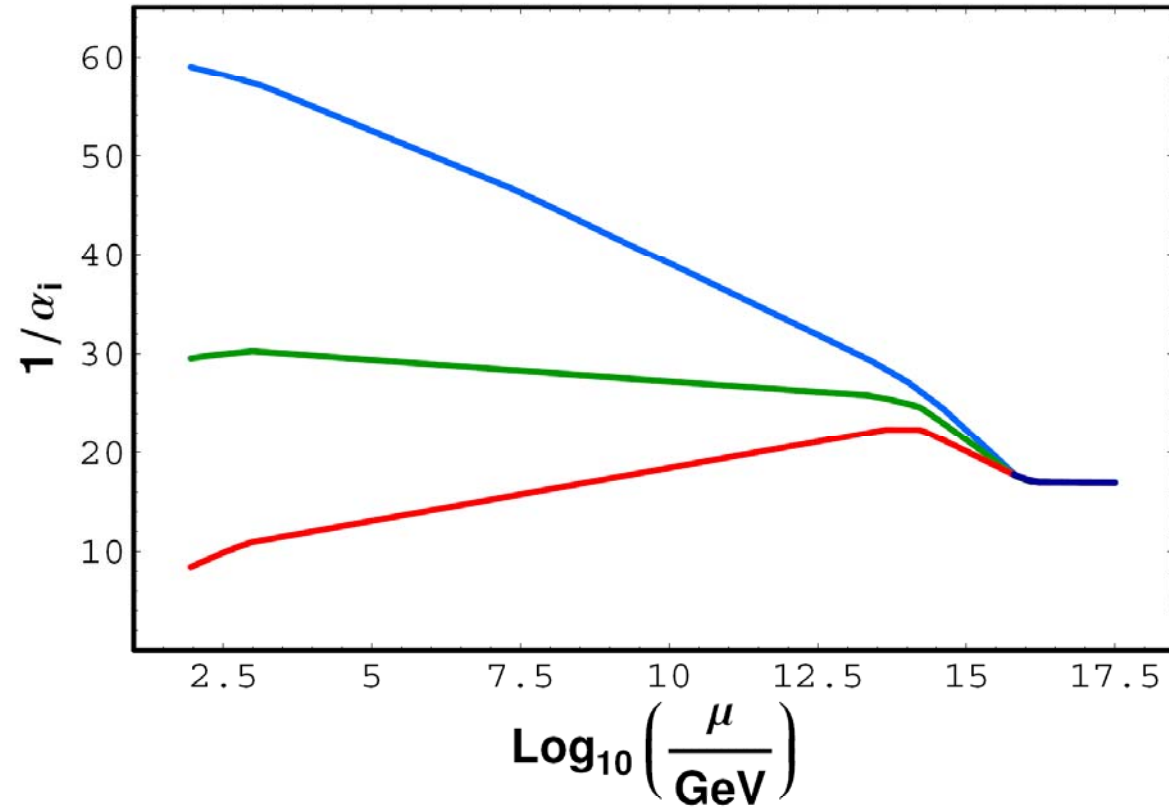
$$r \equiv M_\Sigma / M_X \ll 1, \quad r = (1/25 - 1/200)$$

$$\alpha_3^{-1}(M_X) = \alpha_3^{-1}(M_Z) + \Delta_{3,w}^{(2)} - \frac{1}{4\pi} + \frac{3}{2\pi} \ln \frac{M_X}{M_Z}$$

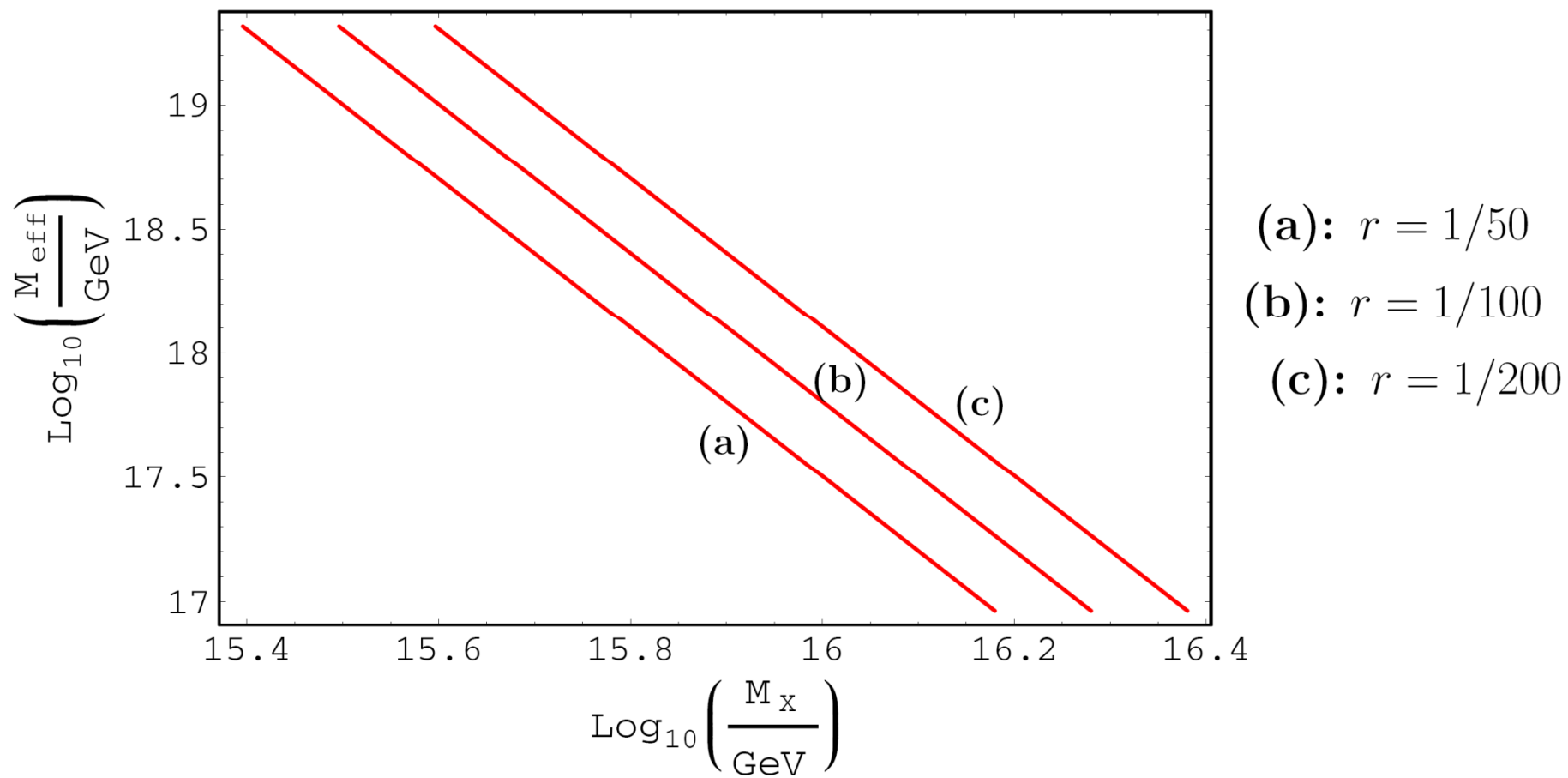
$$-\frac{1}{2\pi} \ln \frac{M_X^4}{M_{T_1} M_{T_2} M_{T_3} M_{T_4}} + \frac{3}{2\pi} \ln r - \frac{1}{2\pi} \ln \frac{M_X^2}{U_1^c U_2^c} - \frac{1}{\pi} \ln \frac{M_X^2}{Q_1 Q_2}.$$

$$\alpha_G = (1/19 - 1/22)$$

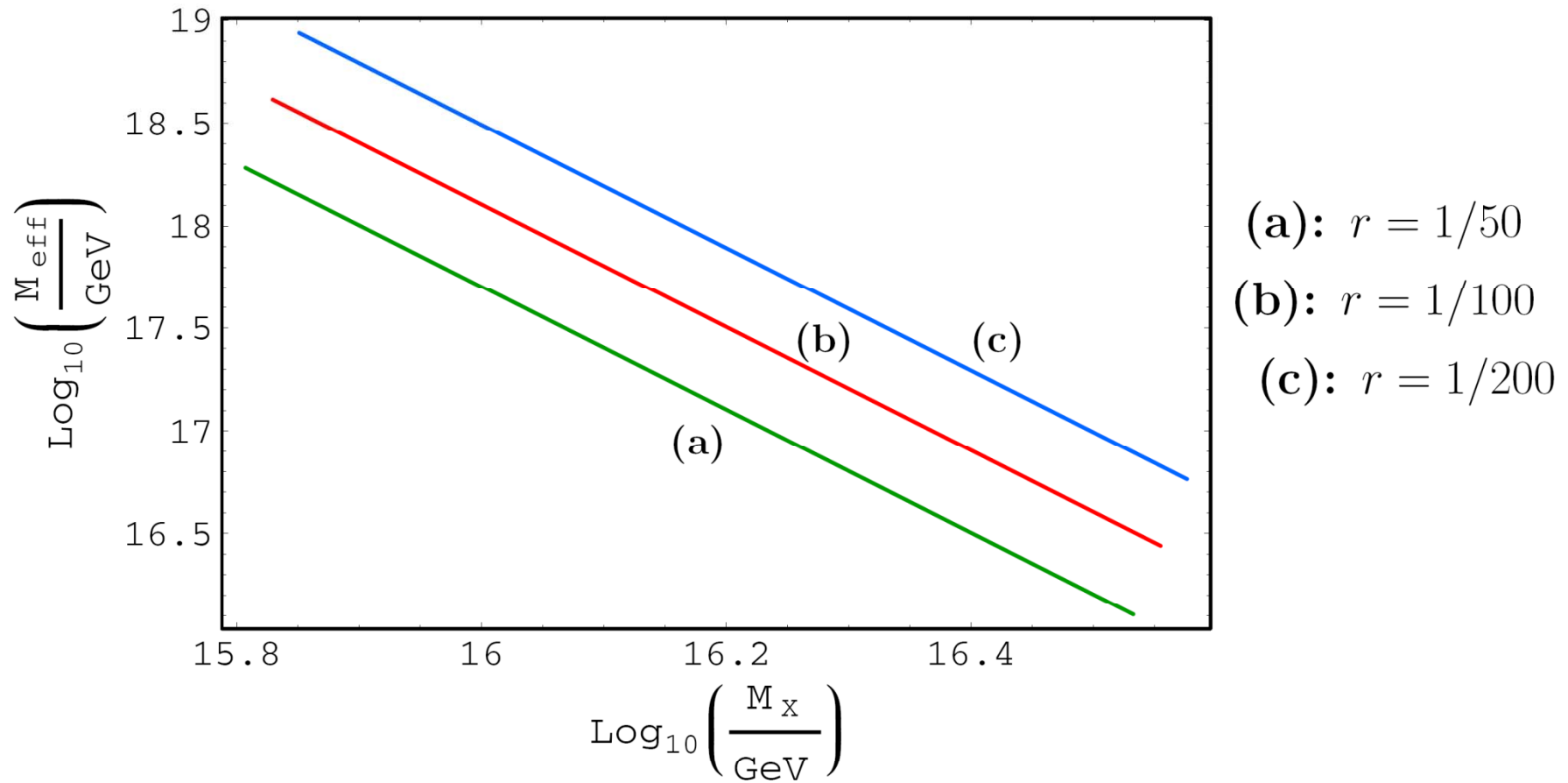




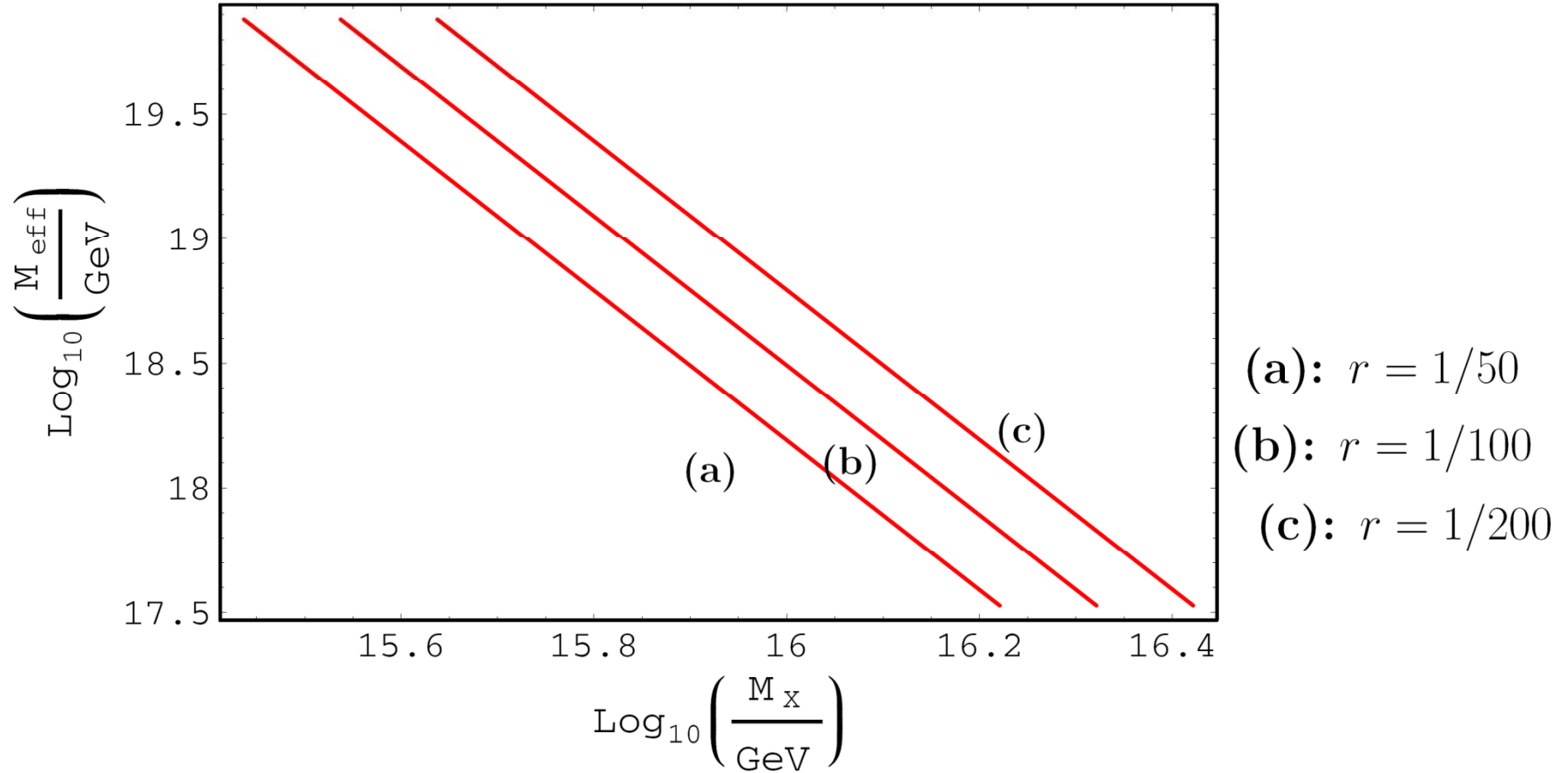
Gauge coupling evolution in our SO(10) model



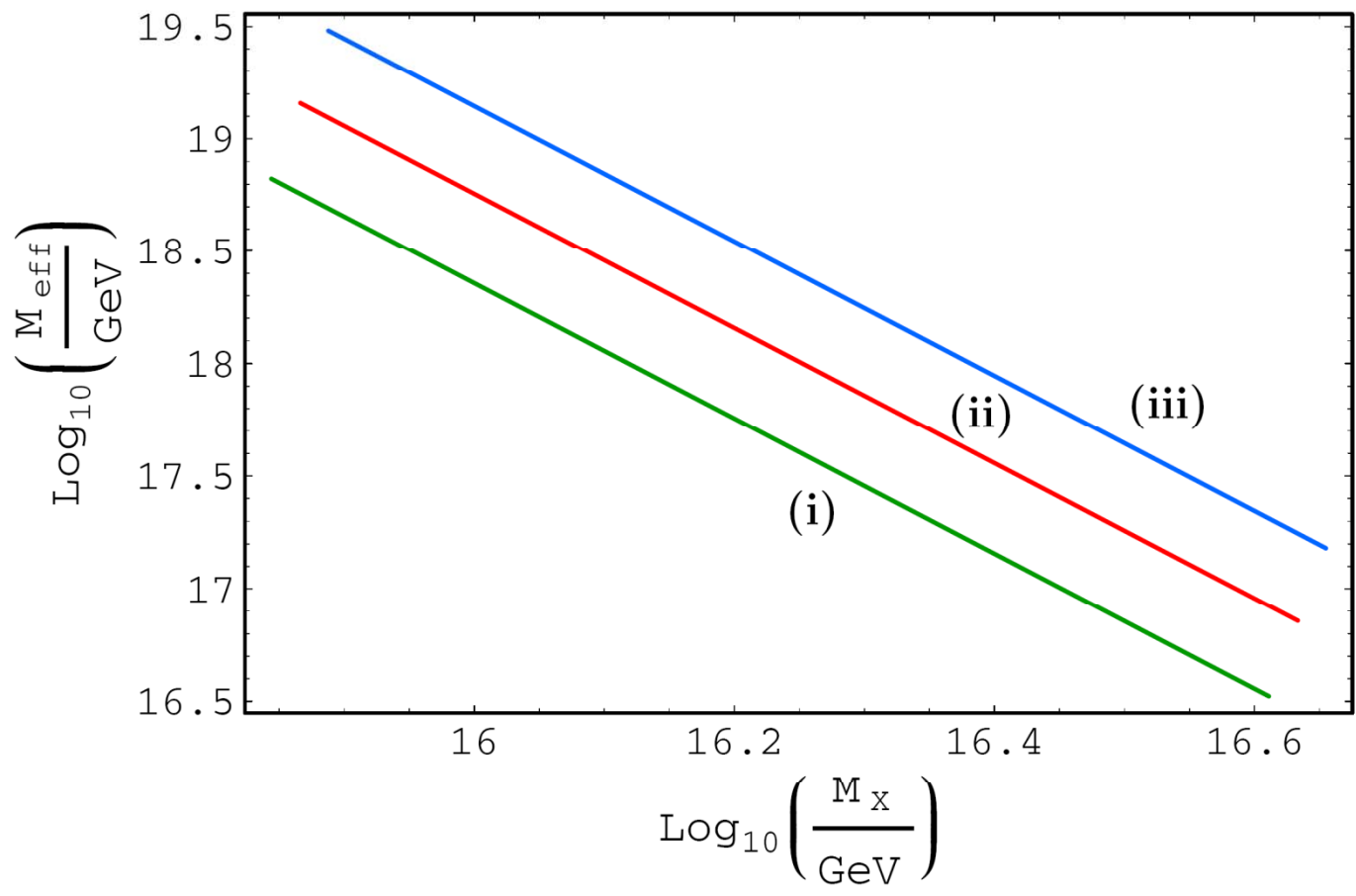
Light SUSY spectrum,  $\alpha_3(M_Z) \simeq 0.1176$ ,  $\tan \beta = 3$ .



Light SUSY spectrum,  $\alpha_3(M_Z) \simeq 0.1176$ ,  $\tan \beta = 3$ .



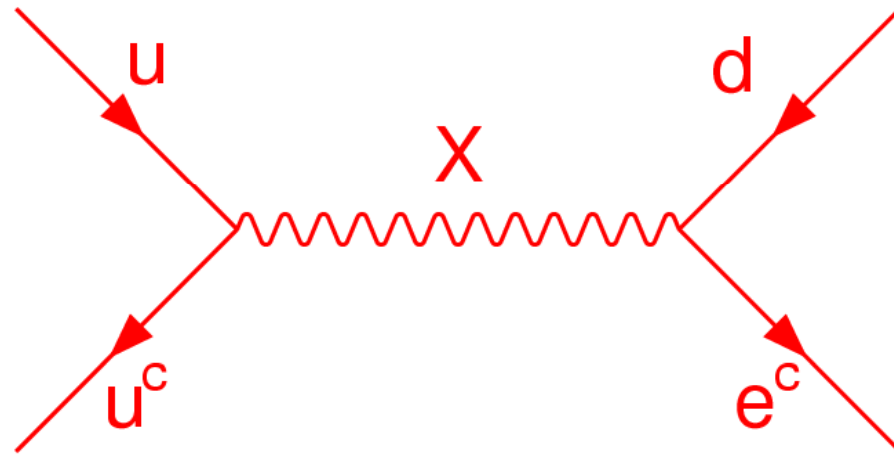
Medium-heavy SUSY spectrum,  $\alpha_3(M_Z) \simeq 0.1176$ ,  $\tan \beta = 3$ .



- (i):  $\alpha_3 = 0.1156$
- (ii):  $\alpha_3 = 0.1176$
- (iii):  $\alpha_3 = 0.1196$

Medium-heavy SUSY spectrum,  $r = 1/200$ ,  $\tan \beta = 3$ .

## Gauge boson induced d=6 proton decay



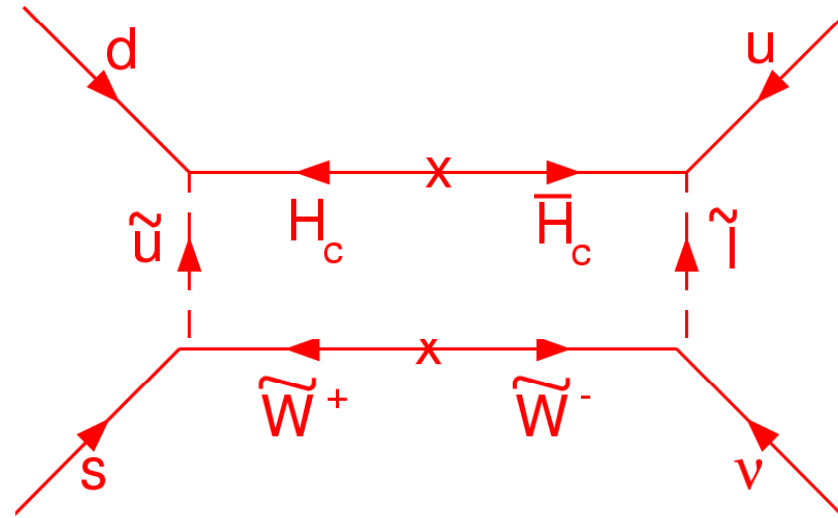
$$p \rightarrow e^+ \pi^0, \tau_p^{-1} \approx \left[ \frac{g^2}{M_X^2} \right]^2 m_p^5 \approx [10^{36 \pm 1} \text{yr}]^{-1}$$



# Higgsino mediated d=5 proton decay

Sakai, Yanagida (1982)

Weinberg (1982)



$$p \rightarrow \bar{\nu} K^+$$

$$\tau_p^{-1} \approx \left[ \frac{f^2}{M_{H_c} M_{SUSY}} \right]^2 \left( \frac{\alpha}{4\pi} \right)^2 m_p^5 \approx [10^{28} - 10^{32} \text{ yr}]^{-1}$$

## Proton Decay in the present SO(10) model

$$\tau_p^{d=5}(p \rightarrow \bar{\nu}K^+) \simeq 2.3 \cdot 10^{33} \text{ yrs} \times \left( \frac{0.01 \text{ GeV}^3}{\tilde{\beta}} \right)^2 \left( \frac{1.74}{A_S^t} \right)^2 \left( \frac{1.34}{A_L} \right)^2 \times \\ \times \left( \frac{M_{\text{eff}}}{1.51 \cdot 10^{19} \text{ GeV}} \right)^2 \left( \frac{m_{\tilde{q}}}{1.8 \text{ TeV}} \right)^4 \left( \frac{125 \text{ GeV}}{M_{\tilde{W}}} \right)^2 .$$

Analysis similar to BPW (1998)

$$\Rightarrow M_{\text{eff}} > 1.2 \times 10^{19} \text{ GeV}$$

## d = 6 Proton decay

$$\Gamma(p \rightarrow e^+ \pi^0) \simeq \frac{m_p}{64\pi f_\pi^2} (1+D+F)^2 \bar{\alpha}^2 \left( \frac{g_5^2 A_R}{M_X^2} \right)^2 \left( 5 + \frac{2}{1+p^2} + \frac{1}{(1+p^2)^2} \right),$$

$$\Gamma(p \rightarrow \bar{\nu} \pi^+) \simeq \frac{m_p}{32\pi f_\pi^2} (1+D+F)^2 \bar{\alpha}^2 \left( \frac{g_5^2 A_R}{M_X^2} \right)^2 \left( 1 + \frac{1}{1+p^2} \right)^2.$$

For  $p \lesssim 1/3$ ,  $\Gamma(p \rightarrow e^+ \pi^0) \simeq \Gamma(p \rightarrow \bar{\nu} \pi^+)$ ,

$p \sim 1$ ,  $\Gamma(p \rightarrow e^+ \pi^0) \sim 1.4 \cdot \Gamma(p \rightarrow \bar{\nu} \pi^+)$ .

$p \gg 1$ ,  $\Gamma(p \rightarrow e^+ \pi^0) \simeq 2.5 \Gamma(p \rightarrow \bar{\nu} \pi^+)$ .

$$\tau_p(p \rightarrow e^+ \pi^0) = (0.8 - 7.6) \cdot 10^{34} \text{ years},$$

$$\tau_p(p \rightarrow \bar{\nu} \pi^+) = (0.83 - 18) \cdot 10^{34} \text{ years}.$$

# Summary and Conclusions

- New  $SO(10)$  model with natural doublet–triplet mass splitting has been proposed
- Mass hierarchy stable to all orders
- No pseudo-Goldstone bosons
- GUT scale threshold effects surprisingly small
- Better values of  $\alpha_3(M_Z)$  predicted
- $d = 5$  proton decay adequately but not fully suppressed
- $d = 5$  and  $d = 6$  proton decay rates correlated
- Both  $p \rightarrow \bar{\nu}K^+$  and  $p \rightarrow e^+\pi^0$  should be within reach of HyperKamiokande and DUSEL experiments