Proton Decay and Flavor Violating Thresholds in SO(10) Models

Yukihiro Mimura (Texas A&M University)

Based on

Proton decay suppression in Collaboration with B. Dutta and R.N. Mohapatra

Phys. Rev. Lett. **94**, 091804 (2005); Phys. Rev. D**72**, 075009 (2005); arXiv:0712.1206 New!

Flavor part in Collaboration with B. Dutta

Phys. Rev. Lett. **97**, 241802 (2006); Phys. Rev. D**75**, 015006 (2007); arXiv:0708.3080

Talk at International Workshop on Grand Unified Theories (2007.12.18)

1. Motivation of GUT

- Unification of Strong & Electroweak interactions

  $\longrightarrow$ Gauge coupling unification (SUSY GUT)

- Unification of Quarks & Leptons

  $\longrightarrow$ Yukawa unification (bottom-tau)

- Proton decay $\longrightarrow$ Most important direct test

  Negative results for now.

- Charge quantization

Menu

1. Motivation of SO(10) GUT
2. A Minimal SO(10) Model with 126 Higgs
3. Proton Decay Suppression
   - Flavor Structure
   - Favorable GUT Thresholds
4. Conclusion

Motivation of SO(10) GUT

Unification of fermions in one irreducible representation

$$\psi(\mathbf{16}) = (u\ u\ u\ \nu\ \bar{u}\ \bar{u}\ \bar{u}\ \bar{\nu}\ d\ d\ d\ e\ \bar{d}\ \bar{d}\ \bar{d}\ \bar{e})$$

Full and sufficient unification of quarks and leptons

Predictive!

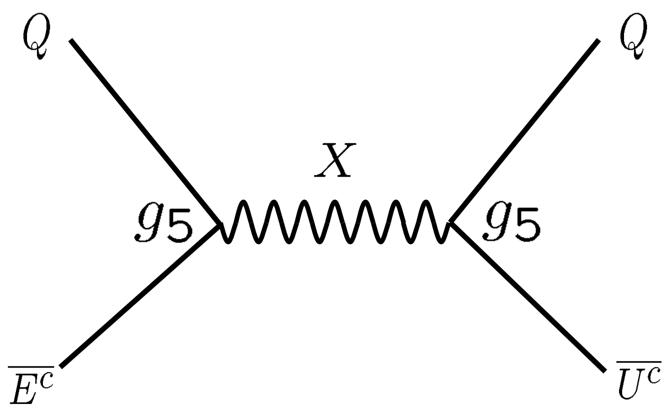
Variety of masses and mixings can be understood by less number of parameters.

cf SU(5) $\mathbf{10} : (u\ u\ u\ \bar{u}\ \bar{u}\ \bar{u}\ d\ d\ d\ \nu\ e),\quad \bar{\mathbf{5}} : (\bar{d}\ \bar{d}\ \bar{d}\ e\ \nu),\quad \mathbf{1} : (\bar{\nu})$

cf PS $(\mathbf{4}, \mathbf{2}, \mathbf{1}) : \begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix},\quad (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) : \begin{pmatrix} \bar{u} & \bar{u} & \bar{u} & \bar{\nu} \\ \bar{d} & \bar{d} & \bar{d} & \bar{e} \end{pmatrix}$

Proton Decay

- Dimension 6 operator



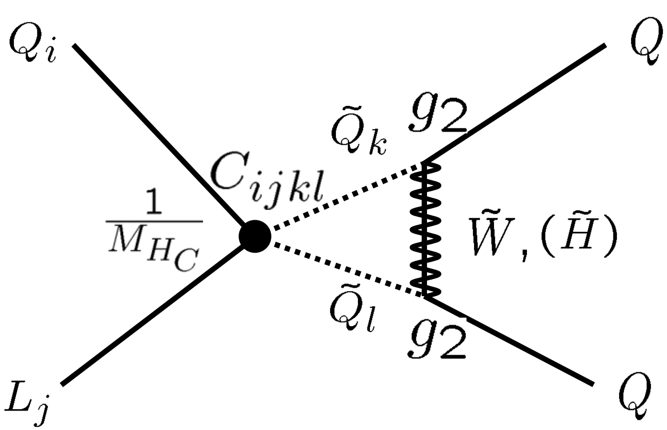
dominant mode : $p \to \pi^0 e^+$

$$A = \frac{g_5^2}{\boxed{M_X^2}}$$ (Up tp Hadron matrix element)

$$M_X \sim 2 \times 10^{16} \text{ GeV}$$

$$\tau_p \sim 10^{34} - 10^{36} \text{ years}$$

- Dimension 5 operator with gaugino or Higgsino dressing



dominant mode : $p \to K^+ \bar{\nu}$

$$A = \frac{\alpha_2}{4\pi \boxed{M_{H_C} m_{\text{SUSY}}}} C_{ijkl}$$

Severe constraint to models

Minimal SU(5) *v1.0*

(Hisano-Yanagida-Murayama)

$$W_Y = Y_u \mathbf{10}_i \mathbf{10}_j H_{\mathbf{5}} + Y_d \mathbf{10}_i \bar{\mathbf{5}}_j \overline{H}_{\bar{\mathbf{5}}} + Y_\nu \bar{\mathbf{5}}_i \mathbf{1}_j H_{\mathbf{5}} + M_R \mathbf{1}_i \mathbf{1}_j$$

$$Y_u = Y_u^{\top}, \qquad \underline{Y_d = Y_e^{\top}} \longleftarrow$$ Wrong prediction

$$W_H = M \Sigma_{\mathbf{24}}^2 + \lambda \Sigma_{\mathbf{24}}^3 + H_{\mathbf{5}}(\Sigma + M_H) H_{\bar{\mathbf{5}}}$$

Proton decay constraints kill the Minimal SUSY SU(5) *v1.0*.

(Goto-Nihei, Murayama-Pierce)

D5 Proton decay operators : $C_L^{ijkl} \simeq C_R^{ijkl} \simeq (Y_d)_{ij}(Y_u)_{kl}/M_{H_C}$

$$-W_5 = \frac{1}{2} C_L^{ijkl} q_k q_l q_i \ell_j + C_R^{ijkl} e_k^c u_l^c u_i^c d_j^c$$

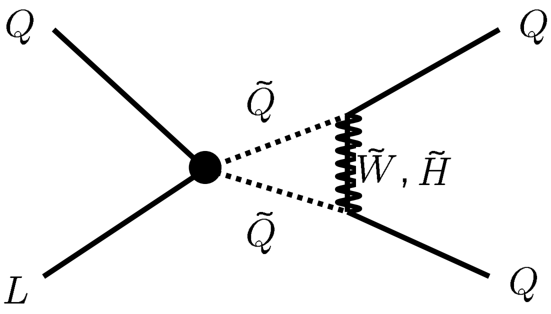
There is room for evading the constraints.

$\longrightarrow$ We will be back later.

Open Question

- Can D5 nucleon decay be suppressed
  and are the current constraints satisfied naturally?

$$\tau(p \to K\bar{\nu}) > 19 \times 10^{32} \text{ years}$$
$$\tau(n \to \pi\bar{\nu}) > 4.4 \times 10^{32} \text{ years}$$
$$\tau(n \to K\bar{\nu}) > 1.8 \times 10^{32} \text{ years}$$



In this talk, we will study the case where
the D5 operators do really exist,
and the D5 proton decay can be observed
in the next generation of water Cherenkov detector.

Dimension-5 Proton decay suppression

D5 Proton decay amplitude:

wino

$$A \simeq \frac{\alpha_2 \beta_p}{4\pi} \frac{1}{M_{H_C}} \frac{m_{\tilde{W}}}{m_{\tilde{q}}^2} C_{ijkl}$$

Lightest colored Higgs mass

squark

Function of h, f, h'

Possible solutions to suppress D5 proton decay

1. SUSY particles (squarks) are heavy.
   $\longrightarrow$ Look forward to LHC
2. Colored Higgs is heavy. (arXiv:0712.1206)
3. Typical flavor structure of Yukawa h, f, h'.
   $\longrightarrow$ Predictive to masses and mixings

(PRL**94** 091804; PRD**72** 075009)

This talk

A Minimal SO(10) Model

(Babu-Mohapatra,1992)

$$\mathbf{16} \times \mathbf{16} = \mathbf{10} + \mathbf{126} + \mathbf{120}$$

Higgs fields which couple to fermions $\psi(\mathbf{16})$:

$H(\mathbf{10})$, $\bar{\Delta}(\overline{\mathbf{126}})$ [and $D(\mathbf{120})$]

Renormalizable Yukawa terms:

$$\frac{1}{2}h_{ij}\psi_i\psi_j H + \frac{1}{2}f_{ij}\psi_i\psi_j\bar{\Delta} + \frac{1}{2}h'_{ij}\psi_i\psi_j D$$

h, f : symmetric

h' : anti-symmetric

GJ relation from CG coefficient $\frac{m_s}{m_b} \sim 3\frac{m_\mu}{m_\tau}$

$$\mathbf{126} = \mathbf{1} + \bar{\mathbf{5}} + \mathbf{10} + \overline{\mathbf{15}} + \mathbf{45} + \overline{\mathbf{50}}$$ under SU(5)

$$\mathbf{126} = (\mathbf{6}, \mathbf{1}, \mathbf{1}) + (\mathbf{10}, \mathbf{3}, \mathbf{1}) + (\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3}) + (\mathbf{15}, \mathbf{2}, \mathbf{2})$$

under SU(4)xSU(2)xSU(2)

☆ R-parity : automatic

$\longrightarrow$ Merit to use **126** Higgs (without **16** Higgs)

Babu-Mohapatra (1992)

…

Matsuda-Koide-Fukuyama-Nishiura (2001), Fukuyama-Okada (2002),
Bajc-Senjanovic-Vissani (2003), Goh-Mohapatra-Ng (2003),
Dutta-YM-Mohapatra (2004), Bertolini-Frigerio-Malinsky (2004),
Babu-Macesanu (2005),
Bajc-Melfo-Senjanovic-Vissani (2006), Bertolini-Scwetz-Malinsky (2006)

….

SO(10) breaking vacua down to SM

Minimal choice : $\mathbf{126}(\Delta) + \overline{\mathbf{126}}(\bar{\Delta}) + \mathbf{210}(\Phi)$

Vacua can be specified by a single parameter x.

(Aulakh-Bajc-Melfo-Senjanovic-Vissani)

$$W_H = \frac{1}{2} m_1 \Phi^2 + \frac{1}{6} \lambda_1 \Phi^3 + m_2 \Delta \bar{\Delta} + \lambda_2 \Phi \Delta \bar{\Delta}$$

$$\Phi_1 = -\frac{m_1}{\lambda_1} \frac{x(1-5x^2)}{(1-x)^2}, \quad \Phi_2 = -\frac{m_1}{\lambda_1} \frac{1-2x-x^2}{1-x}, \quad \Phi_3 = \frac{m_1}{\lambda_1} x$$

$$v_R \bar{v}_R = \frac{m_1^2}{\lambda_1 \lambda_2} \frac{x(1-3x)(1+x^2)}{(1-x)^2}, \qquad \frac{3-14x+15x^2-8x^3}{(1-x)^2} = \frac{m_2 \lambda_1}{m_1 \lambda_2}$$

$$\Phi_1 \equiv \langle \Phi_{1234} \rangle, \quad \Phi_2 \equiv \langle \Phi_{5678} \rangle = \langle \Phi_{7890} \rangle = \langle \Phi_{9056} \rangle,$$
$$\Phi_3 \equiv \langle \Phi_{1256} \rangle = \langle \Phi_{1278} \rangle = \langle \Phi_{1290} \rangle = \langle \Phi_{3456} \rangle = \langle \Phi_{3478} \rangle = \langle \Phi_{3490} \rangle,$$
$$v_R \equiv \langle \Delta_{24680} \rangle, \quad \bar{v}_R \equiv \langle \bar{\Delta}_{13579} \rangle$$

$$W_Y = \frac{1}{2}h_{ij}\psi_i\psi_j H + \frac{1}{2}f_{ij}\psi_i\psi_j\bar{\Delta} + \frac{1}{2}h'_{ij}\psi_i\psi_j D \qquad [H(\mathbf{10}),\ \bar{\Delta}(\overline{\mathbf{126}}),\ D(\mathbf{120})]$$

$$[\Delta(\mathbf{126}),\ \Phi(\mathbf{210})]$$

Higgs doublets

$$\varphi_d = (H_d^{10}, D_d^1, D_d^2, \bar{\Delta}_d, \Delta_d, \Phi_d) \qquad \varphi_u = (H_u^{10}, D_u^1, D_u^2, \Delta_u, \bar{\Delta}_u, \Phi_u)$$

$$\text{mass term : } (\varphi_d)_a (M_{\text{doub.}})_{ab} (\varphi_u)_b \qquad U M_{\text{doub.}} V^{\mathsf{T}} = M_{\text{doub.}}^{\text{diag}}$$

Light doublets

$$H_d = U_{1a}^*(\varphi_d)_a \qquad H_u = V_{1a}^*(\varphi_u)_a$$

U, V : unitary matrices

$$Y_u = \bar{h} + r_2\bar{f} + r_3\bar{h}'$$

$$Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e\bar{h}')$$

$$Y_\nu = \bar{h} - 3r_2\bar{f} + c_\nu\bar{h}'$$

$$\bar{h} = V_{11}h \qquad \bar{f} = U_{14}/(\sqrt{3}r_1)f$$

$$\bar{h}' = (U_{12} + U_{13}/\sqrt{3})/r_1 h'$$

$$r_1 = \frac{U_{11}}{V_{11}}$$

$$r_2 = r_1\frac{V_{15}}{U_{14}} \qquad r_3 = r_1\frac{V_{12} - V_{13}/\sqrt{3}}{U_{12} + U_{13}/\sqrt{3}}$$

$$c_e = \frac{U_{12} - \sqrt{3}U_{13}}{U_{12} + U_{13}/\sqrt{3}} \qquad c_\nu = r_1\frac{V_{12} + \sqrt{3}V_{13}}{U_{12} + U_{13}/\sqrt{3}}$$

Neutrino Mass

$$m_\nu^{\text{light}} = \underline{M_L - \underline{M_\nu^D M_R^{-1} (M_\nu^D)^\top}}$$

Type II Type I

$$M_L = 2\sqrt{2} f \langle \bar{\Delta}_L \rangle \qquad M_R = 2\sqrt{2} f \langle \bar{\Delta}_R \rangle$$

$$\bar{\Delta}_L \; : \; (\mathbf{1}, \mathbf{3}, 1) \qquad \bar{\Delta}_R \; : \; (\mathbf{1}, \mathbf{1}, 0)$$

$\mathrm{SU}(2)_L$ triplet

$$W_Y = \frac{1}{2} h_{ij} \psi_i \psi_j H + \frac{1}{2} f_{ij} \psi_i \psi_j \bar{\Delta} \; + \; \frac{1}{2} h'_{ij} \psi_i \psi_j D \qquad [H(\mathbf{10}),\; \bar{\Delta}(\overline{\mathbf{126}}),\; D(\mathbf{120})]$$

$$\psi\psi\bar{\Delta} \supset \ell\ell\bar{\Delta}_L + \bar{\nu}\bar{\nu}\bar{\Delta}_R$$

$$Y_u = \bar{h} + r_2 \bar{f} + r_3 \bar{h}'$$

$$Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e \bar{h}')$$

$$Y_\nu = \bar{h} - 3 r_2 \bar{f} + c_\nu \bar{h}'$$

$$\bar{h} = V_{11} h \qquad \bar{f} = U_{14}/(\sqrt{3} r_1) f$$

$$\bar{h}' = (U_{12} + U_{13}/\sqrt{3})/r_1 h'$$

$$r_1 = \frac{U_{11}}{V_{11}}$$

$$r_2 = r_1 \frac{V_{15}}{U_{14}} \qquad r_3 = r_1 \frac{V_{12} - V_{13}/\sqrt{3}}{U_{12} + U_{13}/\sqrt{3}}$$

$$c_e = \frac{U_{12} - \sqrt{3} U_{13}}{U_{12} + U_{13}/\sqrt{3}} \qquad c_\nu = r_1 \frac{V_{12} + \sqrt{3} V_{13}}{U_{12} + U_{13}/\sqrt{3}}$$

If Higgs potential is symmetric under the exchange $\Delta \leftrightarrow \bar{\Delta}$

$\longrightarrow$ $r_1 = 1 \quad (\tan\beta \sim 50)$.

$\longrightarrow$ top-bottom-tau Yukawa unification (approximately)

Even if $r_1 \neq 1$, we may have bottom-tau unification (approximately).

Top-bottom-tau unification:

Banks, Olechowski-Pokorski, Ananthanarayan-Lazarides-Shafi, Dimopoulos-Hall-Raby, Murayama-Olechowski-Pokorski Blazek-Dermisek-Raby, Baer-Ferrandis, Tobe-Wells, ….

Topics of Minimal SO(10)

- Maximal mixing for atmospheric neutrino can be realized in type II seesaw due to the bottom-tau mass convergence at GUT scale. (Bajc-Senjanovic-Vissani)

- Detailed numerical analyses (both type I & II, with & without 120) (Matsuda-Koide-Fukuyama-Nishiura, Fukuyama-Kikuchi-Okada, Goh-Mohapatra-Ng, Dutta-YM-Mohapatra, Bertolini-Frigerio-Malinsky, Yang-Wang, Babu-Macesanu, ….)

- Minimal model ($10+126+\overline{126}+210$ Higgses) is disfavored. (Bertolini-Schwetz-Malinsky, …)

- Higgs sector (Fukuyama-Ilakovac-Kikuchi-Meljanac-Okada, Bajc-Melfo-Senjanovic-Vissani, Aulakh-Girdhar)

Minimal model (10+126+$\overline{126}$+210 Higgses) is disfavored.
(Bertolini-Schwetz-Malinsky, …)

$$Y_u = \bar{h} + r_2 \bar{f}$$
$$Y_d = r_1(\bar{h} + \bar{f})$$
$$Y_e = r_1(\bar{h} - 3\bar{f})$$
$$Y_\nu = \bar{h} - 3 r_2 \bar{f}$$

1. r_2 is a function of vacua x.

$$r_2 = -\frac{3(x-1)(x+1)(2x-1)(x^3+5x-1)}{8x^6-27x^5+38x^4-70x^3+87x^2-31x+3}$$

2. Neutrino scale is a function of x.

$$v_R \bar{v}_R = \frac{m_1^2}{\lambda_1 \lambda_2} \frac{x(1-3x)(1+x^2)}{(1-x)^2}, \quad M_{(\mathbf{1},\mathbf{3},1)} = \frac{4x(4x^2-3x+1)m_2}{8x^3-15x^2+14x-3}$$

$\Downarrow$

Experimentally allowed solution : $x \sim \pm i$

But, gauge coupling unification spoils on the vacuum.

How to repair it in non-minimal SO(10) models

1. Add additional 10 or 120 Higgs.

$\longrightarrow$ r_2 is free.

2. Add additional 126+$\overline{126}$, or 54 Higgs for neutrino scale.

Type I seesaw Type II seesaw

The detail does not affect to the following discussion.

Alternative Higgs choices :

$$\mathbf{126}(\Delta) + \overline{\mathbf{126}}(\bar{\Delta}) \leftrightarrow \mathbf{16} + \overline{\mathbf{16}} \qquad \mathbf{210}(\Phi) \leftrightarrow \mathbf{45} + \mathbf{54}$$

$$\mathbf{126}(\Delta) + \overline{\mathbf{126}}(\bar{\Delta}) + \mathbf{210}(\Phi) \leftrightarrow \mathbf{144} + \overline{\mathbf{144}}$$

(Babu-Gogoladze-Nath-Syde)

Dimension-5 Proton decay suppression

D5 Proton decay amplitude:

wino

$$A \simeq \frac{\alpha_2 \beta_p}{4\pi} \frac{1}{M_{H_C}} \frac{m_{\tilde{W}}}{m_{\tilde{q}}^2} C_{ijkl}$$

Lightest colored Higgs mass

squark

Function of h, f, h'

Possible solutions to suppress D5 proton decay

1. SUSY particles (squarks) are heavy.
   $\longrightarrow$ Look forward to LHC
2. Colored Higgs is heavy. (arXiv:0712.1206)
3. Typical flavor structure of Yukawa h, f, h'.
   $\longrightarrow$ Predictive to masses and mixings

(PRL**94** 091804; PRD**72** 075009)

Q. What binds colored Higgs mass?
A. Gauge coupling unification condition.

$$\frac{1}{\alpha_3(m_Z)} = \frac{1}{\alpha_U(\Lambda)} + \frac{1}{2\pi}\left(3\ln\frac{m_Z}{\Lambda} - 4\ln\frac{m_{\mathrm{SUSY}}}{m_Z} + 4\ln\frac{M_X}{\Lambda} - \ln\frac{M_{H_C}}{\Lambda} - \sum_I \Delta b_3^I \ln\frac{M_I}{\Lambda}\right)$$

$$\frac{1}{\alpha_2(m_Z)} = \frac{1}{\alpha_U(\Lambda)} + \frac{1}{2\pi}\left(-\ln\frac{m_Z}{\Lambda} - \frac{25}{6}\ln\frac{m_{\mathrm{SUSY}}}{m_Z} + 6\ln\frac{M_X}{\Lambda} - \sum_I \Delta b_2^I \ln\frac{M_I}{\Lambda}\right)$$

$$\frac{1}{\alpha_1(m_Z)} = \frac{1}{\alpha_U(\Lambda)} + \frac{1}{2\pi}\left(-\frac{33}{5}\ln\frac{m_Z}{\Lambda} - \frac{5}{2}\ln\frac{m_{\mathrm{SUSY}}}{m_Z} + 10\ln\frac{M_X}{\Lambda} - \frac{2}{5}\ln\frac{M_{H_C}}{\Lambda} - \sum_I \Delta b_1^I \ln\frac{M_I}{\Lambda}\right)$$

I : fields except for X (heavy gauge boson) and H_C

(**3**,**2**,-5/6)

$$-\frac{2}{\alpha_3(m_Z)} + \frac{3}{\alpha_2(m_Z)} - \frac{1}{\alpha_1(m_Z)} = \frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_{H_C}}{m_Z} - 2\ln\frac{m_{\mathrm{SUSY}}}{m_Z} + \sum_I N_A^I \ln\frac{M_I}{\Lambda}\right)$$

$$-\frac{2}{\alpha_3(m_Z)} - \frac{3}{\alpha_2(m_Z)} + \frac{5}{\alpha_1(m_Z)} = \frac{1}{2\pi}\left(12\ln\frac{M_X^2\Lambda}{m_Z^3} + 8\ln\frac{m_{\mathrm{SUSY}}}{m_Z} + \sum_I N_B^I \ln\frac{M_I}{\Lambda}\right)$$

$$-\frac{2}{\alpha_3(m_Z)}+\frac{3}{\alpha_2(m_Z)}-\frac{1}{\alpha_1(m_Z)}=\frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_{H_C}}{m_Z}-2\ln\frac{m_{\mathrm{SUSY}}}{m_Z}+\sum_I N_A^I\ln\frac{M_I}{\Lambda}\right)$$

$$-\frac{2}{\alpha_3(m_Z)}-\frac{3}{\alpha_2(m_Z)}+\frac{5}{\alpha_1(m_Z)}=\frac{1}{2\pi}\left(12\ln\frac{M_X^2\Lambda}{m_Z^3}+8\ln\frac{m_{\mathrm{SUSY}}}{m_Z}+\sum_I N_B^I\ln\frac{M_I}{\Lambda}\right)$$

$$N_A^I=2\Delta b_3^I-3\Delta b_2^I+\Delta b_1^I,\qquad N_B^I=2\Delta b_3^I+3\Delta b_2^I-5\Delta b_1^I$$

Ex1. minimal SU(5) *v1.0*

$$M_{(\mathbf{8},\mathbf{1},0)}=M_{(\mathbf{1},\mathbf{3},0)}$$

$$-\frac{2}{\alpha_3(m_Z)}+\frac{3}{\alpha_2(m_Z)}-\frac{1}{\alpha_1(m_Z)}=\frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_{H_C}}{m_Z}-2\ln\frac{m_{\mathrm{SUSY}}}{m_Z}\right)$$

$$-\frac{2}{\alpha_3(m_Z)}-\frac{3}{\alpha_2(m_Z)}+\frac{5}{\alpha_1(m_Z)}=\frac{1}{2\pi}\left(12\ln\frac{M_X^2M_{(\mathbf{8},\mathbf{1},0)}}{m_Z^3}+8\ln\frac{m_{\mathrm{SUSY}}}{m_Z}\right)$$

Hisano-Yanagida-Murayama

$$3.5\times10^{14}\ \mathrm{GeV}<M_{H_C}<3.6\times10^{15}\ \mathrm{GeV}$$

$$(M_X^2M_{(\mathbf{8},\mathbf{1},0)})^{\frac{1}{3}}\simeq2\times10^{16}\mathrm{GeV}$$

(Murayama-Pierce)

Ex2. minimal SU(5) *v1.1* (Bajc-Fileviez Prez-Senjanovic)

$$W_H = M \,\mathrm{Tr}\, \Sigma_{\mathbf{24}}^2 + \lambda \,\mathrm{Tr}\, \Sigma_{\mathbf{24}}^3 + \underline{\kappa_1 \frac{1}{M_*} (\mathrm{Tr}\, \Sigma_{\mathbf{24}}^2)^2 + \kappa_2 \frac{1}{M_*} \mathrm{Tr}\, \Sigma_{\mathbf{24}}^4}$$

$$M_{(\mathbf{8},\mathbf{1},0)} \neq M_{(\mathbf{1},\mathbf{3},0)} \qquad M_{(\mathbf{1},\mathbf{3},0)} = 4 M_{(\mathbf{8},\mathbf{1},0)} \quad (\lambda \to 0)$$

$$-\frac{2}{\alpha_3(m_Z)} + \frac{3}{\alpha_2(m_Z)} - \frac{1}{\alpha_1(m_Z)} = \frac{1}{2\pi} \left(\frac{12}{5} \ln \frac{M_{H_C}}{m_Z} + 6 \ln \frac{M_{(\mathbf{8},\mathbf{1},0)}}{M_{(\mathbf{1},\mathbf{3},0)}} - 2 \ln \frac{m_{\mathrm{SUSY}}}{m_Z} \right)$$

$$-\frac{2}{\alpha_3(m_Z)} - \frac{3}{\alpha_2(m_Z)} + \frac{5}{\alpha_1(m_Z)} = \frac{1}{2\pi} \left(6 \ln \frac{M_X^4 M_{(\mathbf{8},\mathbf{1},0)} M_{(\mathbf{1},\mathbf{3},0)}}{m_Z^6} + 8 \ln \frac{m_{\mathrm{SUSY}}}{m_Z} \right)$$

If (**8**,**1**,0) is light, colored Higgs can be heavier.

e.g. Two **24** Higgs non-minimal model (Chkareuli-Gogoladze)

Ex3. SU(5) 75 Higgs

$$-\frac{2}{\alpha_3(m_Z)}+\frac{3}{\alpha_2(m_Z)}-\frac{1}{\alpha_1(m_Z)}=\frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_{H_C}}{m_Z}+6\ln\frac{M_{(8,1,0)}M^2_{(6,3,5/6)}M^2_{(3,1,5/3)}}{M^5_{(8,3,0)}}-2\ln\frac{m_{\mathrm{SUSY}}}{m_Z}\right)$$

$$-\frac{2}{\alpha_3(m_Z)}-\frac{3}{\alpha_2(m_Z)}+\frac{5}{\alpha_1(m_Z)}=\frac{1}{2\pi}\left(12\ln\frac{M_X^2 M^{\frac{1}{2}}_{(\mathbf{8},\mathbf{1},0)}M^{\frac{11}{2}}_{(8,3,0)}}{m_Z^3 M_{(6,3,5/6)}M^4_{(3,1,5/3)}}+8\ln\frac{m_{\mathrm{SUSY}}}{m_Z}\right)$$

$$W_H=m\mathbf{75}^2+\lambda\mathbf{75}^3$$

$$\Rightarrow\quad M_{(\mathbf{8},\mathbf{3},0)}:M_{(\mathbf{3},\mathbf{1},5/3)}:M_{(\mathbf{6},\mathbf{2},5/6)}:M_{(\mathbf{8},\mathbf{1},0)}=5:4:2:1$$

M_{H_C} bound becomes larger, but M_X becomes smaller.

e.g. (Modified) missing partner model (Hisano-Nomura-Yanagida)

$$-\frac{2}{\alpha_3(m_Z)}+\frac{3}{\alpha_2(m_Z)}-\frac{1}{\alpha_1(m_Z)}=\frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_{H_C}}{m_Z}-2\ln\frac{m_{\mathrm{SUSY}}}{m_Z}+\sum_I N_A^I\ln\frac{M_I}{\Lambda}\right)$$

$$-\frac{2}{\alpha_3(m_Z)}-\frac{3}{\alpha_2(m_Z)}+\frac{5}{\alpha_1(m_Z)}=\frac{1}{2\pi}\left(12\ln\frac{M_X^2\Lambda}{m_Z^3}+8\ln\frac{m_{\mathrm{SUSY}}}{m_Z}+\sum_I N_B^I\ln\frac{M_I}{\Lambda}\right)$$

$$N_A^I=2\Delta b_3^I-3\Delta b_2^I+\Delta b_1^I,\qquad N_B^I=2\Delta b_3^I+3\Delta b_2^I-5\Delta b_1^I$$

A condition to suppress D5 proton decay

The lightest colored Higgs mass is always comparable to (or smaller than) the heavy gauge boson mass.

Therefore, to increase the colored Higgs mass $> 10^{17}$ GeV, we need a light field whose N_A and N_B are both positive.

If such a field splits from the multiplet for a given vacuum, the lightest colored Higgs can be heavier.

Let us find fields whose N_A and N_B are both positive.

Fields which mixed with would-be-Goldstone bosons:

	Δb_3	Δb_2	Δb_1	N_A	N_B
$(\mathbf{1},\mathbf{1},1)+c.c.$	0	0	$\frac{6}{5}$	$\frac{6}{5}$	-6
$(\mathbf{3},\mathbf{1},\frac{2}{3})+c.c.$	1	0	$\frac{8}{5}$	$\frac{18}{5}$	-6
$(\mathbf{3},\mathbf{2},-\frac{5}{6})+c.c.$	2	3	5	0	-12
$(\mathbf{3},\mathbf{2},\frac{1}{6})+c.c.$	2	3	$\frac{1}{5}$	$-\frac{24}{5}$	12

For heavy gauge bosons, multiply by (-2)

Doublet-triplet :

	Δb_3	Δb_2	Δb_1	N_A	N_B
$(\mathbf{1},\mathbf{2},\frac{1}{2})+c.c.$	0	1	$\frac{3}{5}$	$-\frac{12}{5}$	0
$(\mathbf{3},\mathbf{1},-\frac{1}{3})+c.c.$	1	0	$\frac{2}{5}$	$\frac{12}{5}$	0

$\mathbf{126} + \overline{\mathbf{126}}$:

	Δb_3	Δb_2	Δb_1	N_A	N_B	$SU(5)$
$(\mathbf{1}, \mathbf{1}, 2) + c.c.$	0	0	$\frac{24}{5}$	$\frac{24}{5}$	-24	$\mathbf{50}$
$(\mathbf{1}, \mathbf{3}, 1) + c.c.$	0	4	$\frac{18}{5}$	$-\frac{42}{5}$	-6	$\mathbf{15}$
$(\mathbf{3}, \mathbf{1}, -\frac{4}{3}) + c.c.$	1	0	$\frac{32}{5}$	$\frac{42}{5}$	-30	$\mathbf{45}$
$(\mathbf{3}, \mathbf{2}, \frac{7}{6}) + c.c.$	2	3	$\frac{49}{5}$	$\frac{24}{5}$	-36	$\mathbf{45}, \mathbf{50}$
$(\mathbf{3}, \mathbf{3}, -\frac{1}{3}) + c.c.$	3	12	$\frac{6}{5}$	$-\frac{144}{5}$	36	$\mathbf{45}$
$(\mathbf{6}, \mathbf{1}, -\frac{2}{3}) + c.c.$	5	0	$\frac{16}{5}$	$\frac{66}{5}$	-6	$\mathbf{15}$
$(\mathbf{6}, \mathbf{1}, \frac{1}{3}) + c.c.$	5	0	$\frac{4}{5}$	$\frac{54}{5}$	6	$\mathbf{45}$
$(\mathbf{6}, \mathbf{1}, \frac{4}{3}) + c.c.$	5	0	$\frac{64}{5}$	$\frac{114}{5}$	-54	$\mathbf{50}$
$(\mathbf{6}, \mathbf{3}, \frac{1}{3}) + c.c.$	15	24	$\frac{12}{5}$	$-\frac{198}{5}$	90	$\mathbf{50}$
$(\mathbf{8}, \mathbf{2}, \frac{1}{2}) + c.c.$	12	8	$\frac{24}{5}$	$\frac{24}{5}$	24	$\mathbf{45}, \mathbf{50}$

$\mathbf{210}$:

	Δb_3	Δb_2	Δb_1	N_A	N_B	$SU(5)$
$(\mathbf{1},\mathbf{2},\frac{3}{2})+c.c.$	0	1	$\frac{27}{5}$	$\frac{12}{5}$	-24	$\mathbf{40}$
$(\mathbf{1},\mathbf{3},0)$	0	2	0	-6	6	$\mathbf{24}$
$(\mathbf{3},\mathbf{1},\frac{5}{3})+c.c.$	1	0	10	12	-48	$\mathbf{75}$
$(\mathbf{3},\mathbf{3},\frac{2}{3})+c.c.$	3	12	$\frac{24}{5}$	$-\frac{126}{5}$	18	$\mathbf{40}$
$(\mathbf{6},\mathbf{2},-\frac{1}{6})+c.c.$	10	6	$\frac{2}{5}$	$\frac{12}{5}$	36	$\mathbf{40}$
$(\mathbf{6},\mathbf{2},\frac{5}{6})+c.c.$	10	6	10	12	-12	$\mathbf{75}$
$(\mathbf{8},\mathbf{1},0)$	3	0	0	6	6	$\mathbf{24},\mathbf{75}$
$(\mathbf{8},\mathbf{1},1)+c.c.$	6	0	$\frac{48}{5}$	$\frac{108}{5}$	-36	$\mathbf{40}$
$(\mathbf{8},\mathbf{3},0)$	9	16	0	-30	66	$\mathbf{75}$

$\mathbf{126} + \overline{\mathbf{126}} + \mathbf{210}$ thresholds

$$
\begin{aligned}
12.78 &= -\frac{2}{\alpha_3(m_Z)} + \frac{3}{\alpha_2(m_Z)} - \frac{1}{\alpha_1(m_Z)} \\
&= \frac{1}{2\pi}\Bigg(\frac{12}{5}\ln\frac{M_{H_C}}{m_Z} - 2\ln\frac{m_{\mathsf{SUSY}}}{m_Z} \\
&\quad + \frac{6}{5}\ln\frac{M^2_{G(3,2,1/6)}\,\det' M_{(1,1,1)}\,\det' M^3_{(3,1,2/3)}\,\det' M^2_{(3,1,1/3)}}{M^2_{G(1,1,1)} M^6_{G(3,1,2/3)}\,\det' M_{(3,2,1/6)}\,\det' M^2_{(1,2,1/2)}} \\
&\quad \times \frac{M^4_{(1,1,2)} M^7_{(3,1,4/3)}\,\det M^4_{(3,2,7/6)} M^{11}_{(6,1,2/3)} \boxed{M^9_{(6,1,1/3)}} M^{19}_{(6,1,4/3)} \boxed{\det M^4_{(8,2,1/2)}}}{M^7_{(1,3,1)} M^{21}_{(3,3,1/3)} M^{33}_{(6,3,1/3)}} \\
&\quad \times \frac{M^2_{(1,2,3/2)} M^{10}_{(3,1,5/3)} \boxed{M^2_{(6,2,1/6)}} M^{10}_{(6,2,5/6)} \boxed{\det M^5_{(8,1,0)}} M^{18}_{(8,1,1)}}{M^5_{(1,3,0)} M^{21}_{(3,3,5/3)} M^{25}_{(8,3,0)}}\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
189.4 &= -\frac{2}{\alpha_3(m_Z)} - \frac{3}{\alpha_2(m_Z)} + \frac{5}{\alpha_1(m_Z)} \\
&= \frac{1}{2\pi}\Bigg(8\ln\frac{m_{\mathsf{SUSY}}}{m_Z} \\
&\quad + 6\ln\frac{M^4_X M^2_{G(1,1,1)} M^2_{G(3,1,2/3)}\,\det' M^2_{(3,2,1/6)}}{m^6_Z M^4_{G(3,2,1/6)}\,\det' M_{(1,1,1)}\,\det' M_{(3,1,2/3)}\,\det' M^2_{(3,2,5/6)}} \\
&\quad \times \frac{M^6_{(3,3,1/3)} \boxed{M_{(6,1,1/3)}} M^{15}_{(6,3,1/3)} \boxed{\det M^4_{(8,2,1/2)}}}{M^4_{(1,1,2)} M_{(1,3,1)} M^5_{(3,1,4/3)}\,\det M^6_{(3,2,7/6)} M_{(6,1,2/3)} M^9_{(6,1,4/3)}} \\
&\quad \times \frac{M_{(1,3,0)} M^3_{(3,3,5/3)} \boxed{M^6_{(6,2,1/6)}} \boxed{\det M_{(8,1,0)}} M^{11}_{(8,3,0)}}{M^4_{(1,2,3/2)} M^8_{(3,1,5/3)} M^2_{(6,2,5/6)} M^6_{(8,1,1)}}\Bigg)
\end{aligned}
$$

Candidates to increase lightest colored Higgs mass :

	N_A	N_B	SO(10)	SU(5)
$(\mathbf{8},\mathbf{2},1/2)+c.c.$	$\frac{24}{5}$	24	$\mathbf{126}+\overline{\mathbf{126}},\ \mathbf{120}$	$\mathbf{45},\mathbf{50}$
$(\mathbf{6},\mathbf{1},1/3)+c.c.$	$\frac{54}{5}$	6	$\mathbf{126}+\overline{\mathbf{126}},\ \mathbf{120}$	$\mathbf{45}$
$(\mathbf{6},\mathbf{2},-1/6)+c.c.$	$\frac{12}{5}$	36	$\mathbf{210}$	$\mathbf{40}$
$(\mathbf{8},\mathbf{1},0)$	6	6	$\mathbf{210},\ \mathbf{45},\ \mathbf{54}$	$\mathbf{24},\mathbf{75}$

There are only four in the well motivated SO(10) reps.

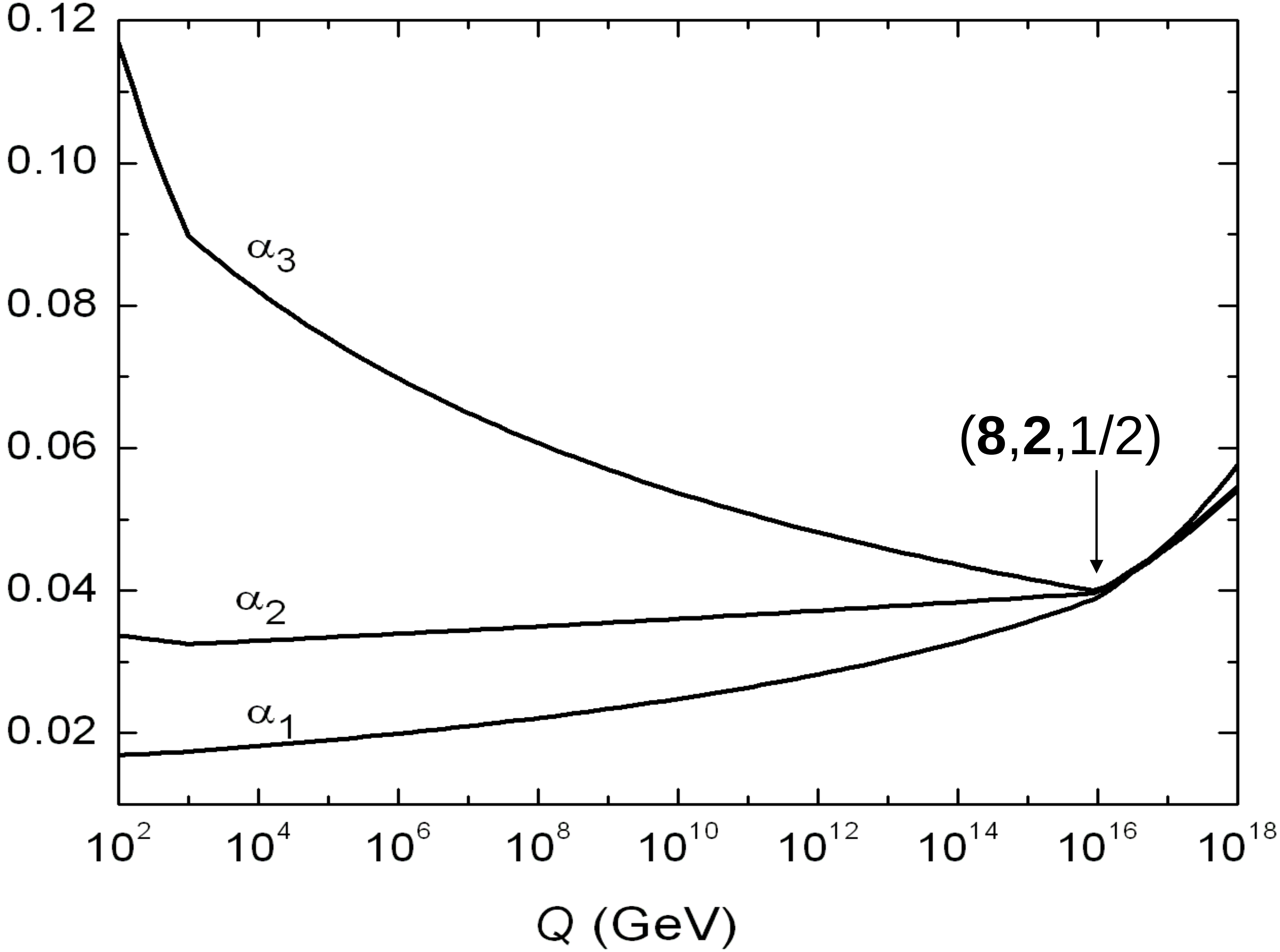
All four candidates are also included in $\mathbf{144}+\overline{\mathbf{144}}$.

Especially, above (**8,2**,1/2) threshold, $b_3=b_2=9, b_1=\frac{57}{5}$

(**6,2**,-1/6) threshold, $b_3=b_2=b_1=7$

If these fields are split from the multiplets,
colored Higgs can be easily made heavy.

MSSM+(**8**,**2**,1/2) threshold



Gauge symmetry does not recover, but couplings run almost unitedly.

Suggested possibility :

Approximated gauge coupling unification scale may be the scale of (**8,2**,1/2) or (**6,2**,-1/6) mass.

It gives a simple explanation why gauge couplings look unified though there is a little hierarchy to the gravity/string scale.

SO(10) symmetry is recovered at more than 10^{17} GeV (possibly the Planck scale)

Gauge couplings blow up when we meet the other decomposed fields from **126**+$\overline{\mathbf{126}}$ and **210**.
Maybe it connects to the strong coupling string theory.

SO(10) breaking vacua down to SM

Minimal choice : $\mathbf{126}(\Delta) + \overline{\mathbf{126}}(\bar{\Delta}) + \mathbf{210}(\Phi)$

Vacua can be specified by a single parameter x.

(Aulakh-Bajc-Melfo-Senjanovic-Vissani)

$$W_H = \frac{1}{2} m_1 \Phi^2 + \frac{1}{6}\lambda_1 \Phi^3 + m_2 \Delta\bar{\Delta} + \lambda_2 \Phi\Delta\bar{\Delta}$$

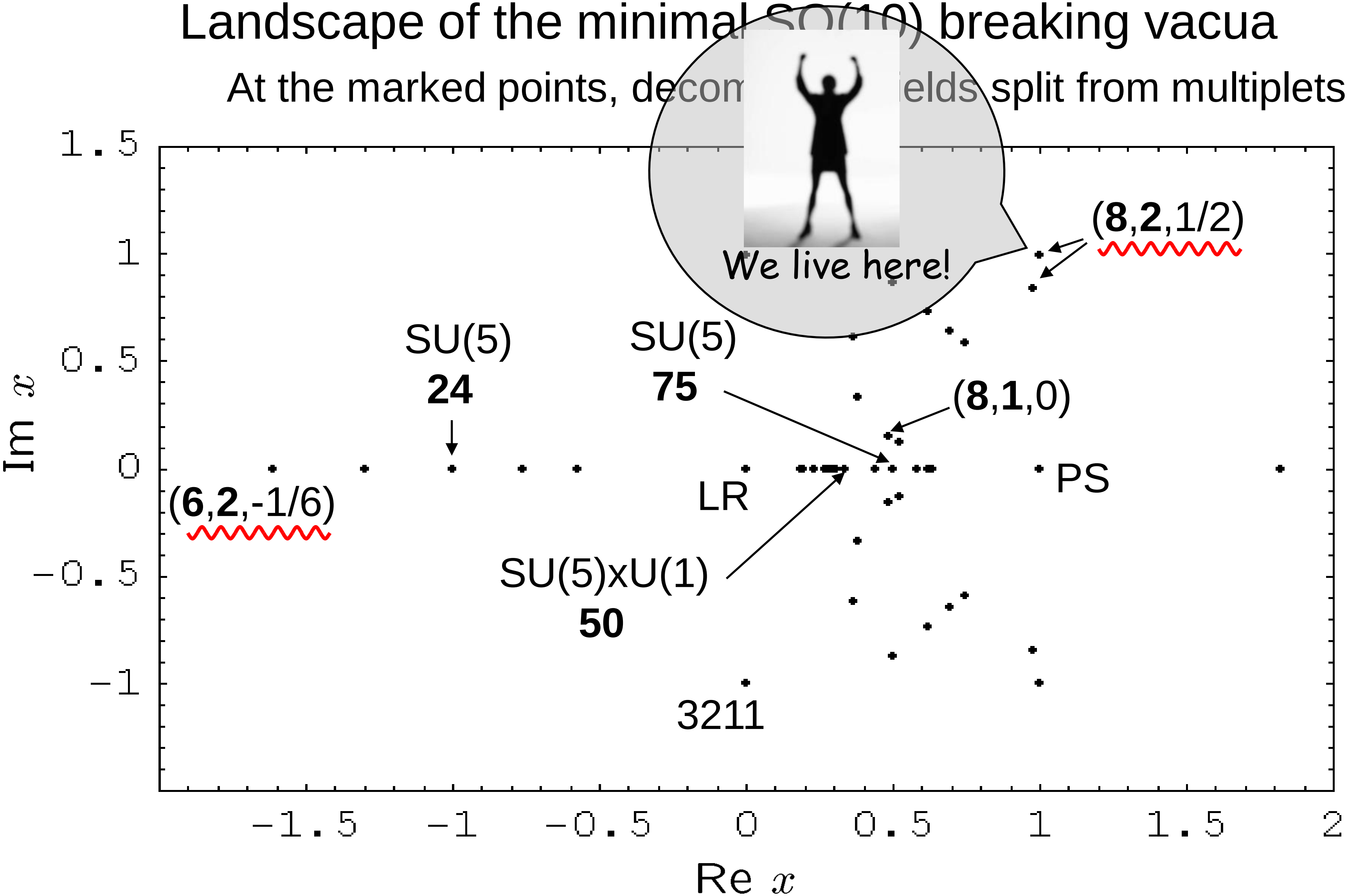
$$\Phi_1 = -\frac{m_1}{\lambda_1}\frac{x(1-5x^2)}{(1-x)^2}, \quad \Phi_2 = -\frac{m_1}{\lambda_1}\frac{1-2x-x^2}{1-x}, \quad \Phi_3 = \frac{m_1}{\lambda_1}x$$

$$v_R\bar{v}_R = \frac{m_1^2}{\lambda_1\lambda_2}\frac{x(1-3x)(1+x^2)}{(1-x)^2}, \qquad \frac{3-14x+15x^2-8x^3}{(1-x)^2} = \frac{m_2\lambda_1}{m_1\lambda_2}$$

$$\begin{aligned}
&\Phi_1 \equiv \langle\Phi_{1234}\rangle, \quad \Phi_2 \equiv \langle\Phi_{5678}\rangle = \langle\Phi_{7890}\rangle = \langle\Phi_{9056}\rangle, \\
&\Phi_3 \equiv \langle\Phi_{1256}\rangle = \langle\Phi_{1278}\rangle = \langle\Phi_{1290}\rangle = \langle\Phi_{3456}\rangle = \langle\Phi_{3478}\rangle = \langle\Phi_{3490}\rangle, \\
&v_R \equiv \langle\Delta_{24680}\rangle, \quad \bar{v}_R \equiv \langle\bar{\Delta}_{13579}\rangle
\end{aligned}$$

Landscape of the minimal SO(10) breaking vacua

At the marked points, decom[...]ields split from multiplets.



Higgs spectrum : Fukuyama-Ilakovac-Kikuchi-Meljanac-Okada
Bajc-Melfo-Senjanovic-Vissani, Aulakh-Girdhar

	N_A	N_B	SO(10)	SU(5)
$(\mathbf{8},\mathbf{2},1/2)+c.c.$	$\frac{24}{5}$	24	$\mathbf{126}+\overline{\mathbf{126}},\ \mathbf{120}$	$\mathbf{45},\mathbf{50}$
$(\mathbf{6},\mathbf{1},1/3)+c.c.$	$\frac{54}{5}$	6	$\mathbf{126}+\overline{\mathbf{126}},\ \mathbf{120}$	$\mathbf{45}$
$(\mathbf{6},\mathbf{2},-1/6)+c.c.$	$\frac{12}{5}$	36	$\mathbf{210}$	$\mathbf{40}$
$(\mathbf{8},\mathbf{1},0)$	6	6	$\mathbf{210},\ \mathbf{45},\ \mathbf{54}$	$\mathbf{24},\mathbf{75}$

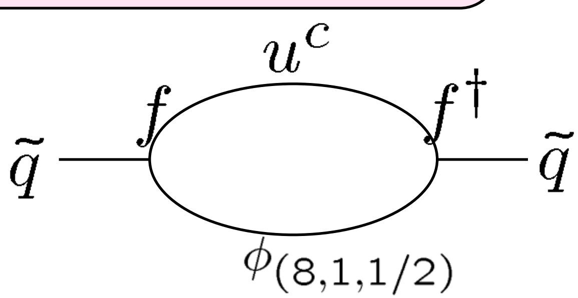
(**8**,**2**,1/2) and (**6**,**1**,1/3) can couple to fermions.

$$q u^c \phi_{(\mathbf{8},\mathbf{2},1/2)} + q d^c \phi_{(\mathbf{8},\mathbf{2},-1/2)} + q q \phi_{(\bar{\mathbf{6}},\mathbf{1},-1/3)} + u^c d^c \phi_{(\mathbf{6},\mathbf{1},1/3)}.$$

Through the threshold, flavor violation for both left- and right-handed squark mass matrices can be generated.

Impact on the meson mixings

(Yesterday’s my talk)



$$qu^c\phi_{(\mathbf{8},\mathbf{2},1/2)} + qd^c\phi_{(\mathbf{8},\mathbf{2},-1/2)} + qq\phi_{(\bar{\mathbf{6}},\mathbf{1},-1/3)} + u^c d^c\phi_{(\mathbf{6},\mathbf{1},1/3)}.$$

Bottom Yukawa is modified through the threshold.
If the couplings are order 1, it decreases the bottom Yukawa coupling at low energy.

Good direction for bottom-tau unification!

Bottom-tau can be unified for any $\tan\beta$.

Cf. For MSSM running, bottom-tau unification can happen only when $\tan\beta \sim 2$ or ~ 50.

	N_A	N_B	SO(10)	SU(5)
$(\mathbf{8},\mathbf{2},1/2)+c.c.$	$\frac{24}{5}$	24	$\mathbf{126}+\overline{\mathbf{126}},\ \mathbf{120}$	$\mathbf{45},\mathbf{50}$
$(\mathbf{6},\mathbf{1},1/3)+c.c.$	$\frac{54}{5}$	6	$\mathbf{126}+\overline{\mathbf{126}},\ \mathbf{120}$	$\mathbf{45}$
$(\mathbf{6},\mathbf{2},-1/6)+c.c.$	$\frac{12}{5}$	36	$\mathbf{210}$	$\mathbf{40}$
$(\mathbf{8},\mathbf{1},0)$	6	6	$\mathbf{210},\ \mathbf{45},\ \mathbf{54}$	$\mathbf{24},\mathbf{75}$

None of the four candidate to increase colored Higgs mass couples to leptons.

Large lepton flavor violation is not expected for the direction to increase the colored Higgs mass.

$$\tau \to \mu\gamma,\ \tau \to e\gamma,\ \mu \to e\gamma,$$

Note : Large LFVs are not generated from right-handed neutrino loops neither since Dirac neutrino coupling does not have large mixing in the naive SO(10) fit.

$\mathbf{126} + \overline{\mathbf{126}}$:

	Δb_3	Δb_2	Δb_1	N_A	N_B	$SU(5)$
$(\mathbf{1},\mathbf{1},2)+c.c.$	0	0	$\frac{24}{5}$	$\frac{24}{5}$	-24	$\mathbf{50}$
$(\mathbf{1},\mathbf{3},1)+c.c.$	0	4	$\frac{18}{5}$	$-\frac{42}{5}$	-6	$\mathbf{15}$
$(\mathbf{3},\mathbf{1},-\frac{4}{3})+c.c.$	1	0	$\frac{32}{5}$	$\frac{42}{5}$	-30	$\mathbf{45}$
$(\mathbf{3},\mathbf{2},\frac{7}{6})+c.c.$	2	3	$\frac{49}{5}$	$\frac{24}{5}$	-36	$\mathbf{45},\mathbf{50}$
$(\mathbf{3},\mathbf{3},-\frac{1}{3})+c.c.$	3	12	$\frac{6}{5}$	$-\frac{144}{5}$	36	$\mathbf{45}$
$(\mathbf{6},\mathbf{1},-\frac{2}{3})+c.c.$	5	0	$\frac{16}{5}$	$\frac{66}{5}$	-6	$\mathbf{15}$
$(\mathbf{6},\mathbf{1},\frac{1}{3})+c.c.$	5	0	$\frac{4}{5}$	$\frac{54}{5}$	6	$\mathbf{45}$
$(\mathbf{6},\mathbf{1},\frac{4}{3})+c.c.$	5	0	$\frac{64}{5}$	$\frac{114}{5}$	-54	$\mathbf{50}$
$(\mathbf{6},\mathbf{3},\frac{1}{3})+c.c.$	15	24	$\frac{12}{5}$	$-\frac{198}{5}$	90	$\mathbf{50}$
$(\mathbf{8},\mathbf{2},\frac{1}{2})+c.c.$	12	8	$\frac{24}{5}$	$\frac{24}{5}$	24	$\mathbf{45},\mathbf{50}$

Type II seesaw may decrease the unification scale if SU(5) **15** multiplet splits.

If the candidates are lighter enough, there is a solution to increase Higgsino mass.

There may be a hint to distinguish type I and II.

Note : Minimal SU(5) *v2.0* with **45** Higgs
(which reproduces GJ relation)
can have the same threshold effects.

	N_A	N_B	SO(10)	SU(5)
$(\mathbf{8}, \mathbf{2}, 1/2) + c.c.$	$\frac{24}{5}$	24	$\mathbf{126} + \overline{\mathbf{126}},\ \mathbf{120}$	$\mathbf{45}, \mathbf{50}$
$(\mathbf{6}, \mathbf{1}, 1/3) + c.c.$	$\frac{54}{5}$	6	$\mathbf{126} + \overline{\mathbf{126}},\ \mathbf{120}$	$\mathbf{45}$
$(\mathbf{6}, \mathbf{2}, -1/6) + c.c.$	$\frac{12}{5}$	36	$\mathbf{210}$	$\mathbf{40}$
$(\mathbf{8}, \mathbf{1}, 0)$	6	6	$\mathbf{210},\ \mathbf{45},\ \mathbf{54}$	$\mathbf{24}, \mathbf{75}$

To explain GJ relation from renormalizable coupling,
(**8,2,**1/2) is incorporated in the model.

LHC/ILC results

$\longrightarrow$ More accurate information of gauge and Yukawa couplings above TeV scale

$\longrightarrow$ It gives important probe of the GUT thresholds.

Flavor violations are also important to probe the thresholds.

Keep watching:

$\sin 2\phi_1$-V_{ub} discrepancy in unitarity triangle

phase of B_s-$\bar{B}_s$ mixing

(CP violation of $B_s \to J/\psi\phi$ decay)

$\tau \to \mu\gamma$, $\tau \to e\gamma$, $\mu \to e\gamma$, μ-e conversion ...

Dimension-5 Proton decay suppression

D5 Proton decay amplitude:

wino

$$A \simeq \frac{\alpha_2 \beta_p}{4\pi} \frac{1}{M_{H_C}} \frac{m_{\tilde{W}}}{m_{\tilde{q}}^2} C_{ijkl}$$

Lightest colored Higgs mass

squark

Function of h, f, h'

Possible solutions to suppress D5 proton decay

1. SUSY particles (squarks) are heavy.
   $\longrightarrow$ Look forward to LHC
2. Colored Higgs is heavy. (arXiv:0712.1206)
3. Typical flavor structure of Yukawa h, f, h'.
   $\longrightarrow$ Predictive to masses and mixings

(PRL**94** 091804; PRD**72** 075009)

Dimension 5 operators : $-W_5 = \frac{1}{2} C_L^{ijkl} \underbrace{q_k q_l}_{\text{symmetric}} q_i \ell_j + C_R^{ijkl} e_k^c u_l^c u_i^c d_j^c$

Higgs triplets $(\bar{\mathbf{3}}, \mathbf{1}, 1/3) + c.c.$

$$\varphi_{\bar{T}} = (H_{\bar{T}}, D_{\bar{T}}, D'_{\bar{T}}, \bar{\Delta}_{\bar{T}}, \Delta_{\bar{T}}, \Delta'_{\bar{T}}, \Phi_{\bar{T}})$$

$$\varphi_T = (H_T, D_T, D'_T, \Delta_T, \bar{\Delta}_T, \bar{\Delta}'_T, \Phi_T)$$

mass term : $(\varphi_{\bar{T}})_a (M_T)_{ab} (\varphi_T)_b$

$$X M_T Y^{\mathsf{T}} = M_T^{\text{diag}}$$

H_T and $\bar{\Delta}_T$ have opposite D-parity.

$$C_L^{ijkl} = \sum_a \frac{1}{M_{T_a}} (X_{a1} h + X_{a4} f + \sqrt{2} X_{a3} h')_{ij} (Y_{a1} h + Y_{a5} f)_{kl}$$

Opposite signature

no h'

$$C_R^{ijkl} = \sum_a \frac{1}{M_{T_a}} (X_{a1} h - X_{a4} f + \sqrt{2} X_{a2} h')_{ij} (Y_{a1} h - (Y_{a5} - \sqrt{2} Y_{a6}) f + \sqrt{2} (Y_{a3} - Y_{a2}))_{kl}$$

$$\left(\begin{array}{l} C_L^{ijkl} = c h_{ij} h_{kl} + x_1 f_{ij} f_{kl} + x_2 f_{ij} f_{kl} + x_3 f_{ij} h_{kl} + x_4 h'_{ij} h_{kl} + x_5 h'_{ij} f_{kl} \\ C_R^{ijkl} = c h_{ij} h_{kl} + y_1 f_{ij} f_{kl} + y_2 f_{ij} f_{kl} + y_3 f_{ij} h_{kl} + y_4 h'_{ij} h_{kl} + y_5 h'_{ij} f_{kl} + \cdots \end{array} \right)$$

$\underline{x_3 = -y_3}$ (Dutta-YM-Mohapatra)

Proton decay suppression

SU(5) limit : $C_L^{ijkl} \simeq C_R^{ijkl} \simeq (Y_d)_{ij}(Y_u)_{kl}/M_{T_1}$

In SO(10), we may have cancellation among different Higgs couplings,

$$C_L^{ijkl} = ch_{ij}h_{kl} + x_1 f_{ij}f_{kl} + x_2 f_{ij}f_{kl} + x_3 f_{ij}h_{kl} + x_4 h'_{ij}h_{kl} + x_5 h'_{ij}f_{kl}$$
$$C_R^{ijkl} = ch_{ij}h_{kl} + y_1 f_{ij}f_{kl} + y_2 f_{ij}f_{kl} + y_3 f_{ij}h_{kl} + y_4 h'_{ij}h_{kl} + y_5 h'_{ij}f_{kl} + \cdots$$

But,

$$C_L^{ijkl} = \sum_a \frac{1}{M_{T_a}}(X_{a1}h + X_{a4}f + \sqrt{2}X_{a3}h')_{ij}(Y_{a1}h + Y_{a5}f)_{kl}$$

Opposite signature

no h'

$$C_R^{ijkl} = \sum_a \frac{1}{M_{T_a}}(X_{a1}h - X_{a4}f + \sqrt{2}X_{a2}h')_{ij}(Y_{a1}h - (Y_{a5} - \sqrt{2}Y_{a6})f + \sqrt{2}(Y_{a3} - Y_{a2}))_{kl}$$

$$Y_u = \bar{h} + r_2\bar{f} + r_3\bar{h}'$$
$$Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

$$m_u/m_t \ll m_d/m_b,\ m_c/m_t \ll m_s/m_b$$

$$m_u/m_t \ll m_d/m_b,\ m_c/m_t \ll m_s/m_b$$

Up-type quark masses are hierarchical rather than down-type ones.

$$Y_u = \bar{h} + r_2\bar{f} + r_3\bar{h}'$$
$$Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

Typical solutions:

Sol. 1: $r_2, r_3 \sim O(1)$

roughly, $r_1\bar{h} \sim Y_d$

Sol. 2: $r_2, r_3 \ll O(1)$

roughly, $\bar{h} \sim Y_u$

In Sol. 1, cancellation among $\bar{h}$, $r_2\bar{f}$, and $r_3\bar{h}'$ is needed.

But such cancellation hardly happen in both C_L and C_R

$$C_L^{ijkl} = \sum_a \frac{1}{M_{T_a}}(X_{a1}h + X_{a4}f + \sqrt{2}X_{a3}h')_{ij}(Y_{a1}h + Y_{a5}f)_{kl}$$

Opposite signature

no h'

$$C_R^{ijkl} = \sum_a \frac{1}{M_{T_a}}(X_{a1}h - X_{a4}f + \sqrt{2}X_{a2}h')_{ij}(Y_{a1}h - (Y_{a5} - \sqrt{2}Y_{a6})f + \sqrt{2}(Y_{a3} - Y_{a2}))_{kl}$$

Proton decay amplitude $A = \frac{\alpha_2 \beta_p}{4\pi M_T m_{\rm SUSY}} \tilde{A}$

$\tau(p \to K\bar{\nu}) > 19 \times 10^{32}$ years
$\tau(n \to \pi\bar{\nu}) > 4.4 \times 10^{32}$ years
$\tau(n \to K\bar{\nu}) > 1.8 \times 10^{32}$ years

$$\tilde{A} = c\tilde{A}_{hh} + x_1 \tilde{A}_{ff} + x_2 \tilde{A}_{hf} + x_3 \tilde{A}_{fh} + x_4 \tilde{A}_{h'h} + x_5 \tilde{A}_{h'f}$$

To satisfy current bounds,

$$\tilde{A}(p \to K\bar{\nu}) < 10^{-8}, \quad \tilde{A}(n \to \pi\bar{\nu}) < 2 \cdot 10^{-8}, \quad \tilde{A}(n \to K\bar{\nu}) < 5 \cdot 10^{-8}$$

$$\left[M_T = 2 \cdot 10^{16}\ \text{GeV},\ M_{\tilde{q}} = 1\ \text{TeV},\ M_{\tilde{W}} = 250\ \text{GeV}\right]$$

When $r_2,\ r_3 \sim O(1)$, $\quad \tilde{A}_{hh} \sim 10^{-4} - 10^{-5}$

(roughly, $\bar{h} \sim Y_d$)

Cancellation is unnatural.

When $r_2,\ r_3 \ll 1$ and $\bar{h} \sim Y_u$, $\quad \tilde{A}_{hh} \sim 10^{-7}$

One still needs to care about cancellation in each decay mode, $p \to K\bar{\nu}_{e,\mu,\tau}$

A special choice is more preferable.

$$\bar{h} \simeq \text{diag}(\ll \lambda_u, \ll \lambda_c,\ \lambda_t)$$

$$(\tilde{A}_{hh} \ll 10^{-8})$$

$$Y_u = \bar{h} + r_2 \bar{f} + r_3 \bar{h}'$$
$$Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

A special choice of texture (in $\bar{h}$-diagonal basis)

$$\bar{h} \simeq \mathrm{diag}(\ll \lambda_u, \ll \lambda_c,\ \lambda_t) \qquad \lambda \sim 0.2$$

$$\bar{f} \simeq \begin{pmatrix} \sim 0 & \sim 0 & \lambda^3 \\ \sim 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}, \qquad \bar{h}' \simeq i \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix}$$

Relations are clear under this texture!

e.g. Charm Yukawa: $\lambda_c \simeq r_2 m_s/m_b$ ⇨ $|r_2| \simeq 0.1 - 0.15$

$m_d m_s m_b \simeq c_e^2 m_e m_\mu m_\tau$ ⇨ $|c_e| \sim 1.$

$$Y_u = \bar{h} + r_2 \bar{f} + r_3 \bar{h}' \qquad Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e \bar{h}') \qquad m_\nu^{\text{light}} \propto \bar{f} \quad \text{(type II)}$$

$$\bar{h} \simeq \mathrm{diag}(0,0,1), \quad \bar{f} \simeq \begin{pmatrix} \sim 0 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}, \quad \bar{h}' \simeq i \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix}$$

Rough structures of the Yukawa matrices

$$Y_u \simeq \begin{pmatrix} \sim 0 & \sim 0 & r_2\lambda^3 \\ \sim 0 & r_2\lambda^2 & r_2\lambda^2 \\ r_2\lambda^3 & r_2\lambda^2 & 1 \end{pmatrix}, \quad Y_d \simeq r_1 \begin{pmatrix} \sim 0 & i\lambda^3 & \sim \lambda^3 \\ -i\lambda^3 & \lambda^2 & \sim \lambda^2 \\ \sim \lambda^3 & \sim \lambda^2 & 1 \end{pmatrix}$$

$$Y_e \simeq r_1 \begin{pmatrix} \sim 0 & ic_e\lambda^3 & \sim \lambda^3 \\ -ic_e\lambda^3 & -3\lambda^2 & \sim \lambda^2 \\ \sim \lambda^3 & \sim \lambda^2 & 1 \end{pmatrix}, \quad m_\nu^{\text{light}} \propto \begin{pmatrix} \sim 0 & \sim 0 & \lambda^3 \\ \sim 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}$$

$$Y_u = \bar{h} + r_2 \bar{f} + r_3 \bar{h}' \qquad Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e \bar{h}') \qquad m_\nu^{\text{light}} \propto \bar{f}$$ (type II)

Simple explanation of successful fermion mass and mixings

Fermion mass matrices are rank 1 (h) + corrections (f, h')

Let us consider a limit where 1st generation is massless.

$h' = 0$, f is also rank 1.

f-diagonal basis. $\bar{f}_{33}$ small $\longrightarrow$ CKM small.

$$\bar{h} = \bar{h}_{33} \begin{pmatrix} c^2 & bc & ac \\ bc & b^2 & ab \\ ac & ab & a^2 \end{pmatrix}, \quad \bar{f} = \bar{f}_{33} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In the limit $\lambda_1, \lambda_2 \to 0$,

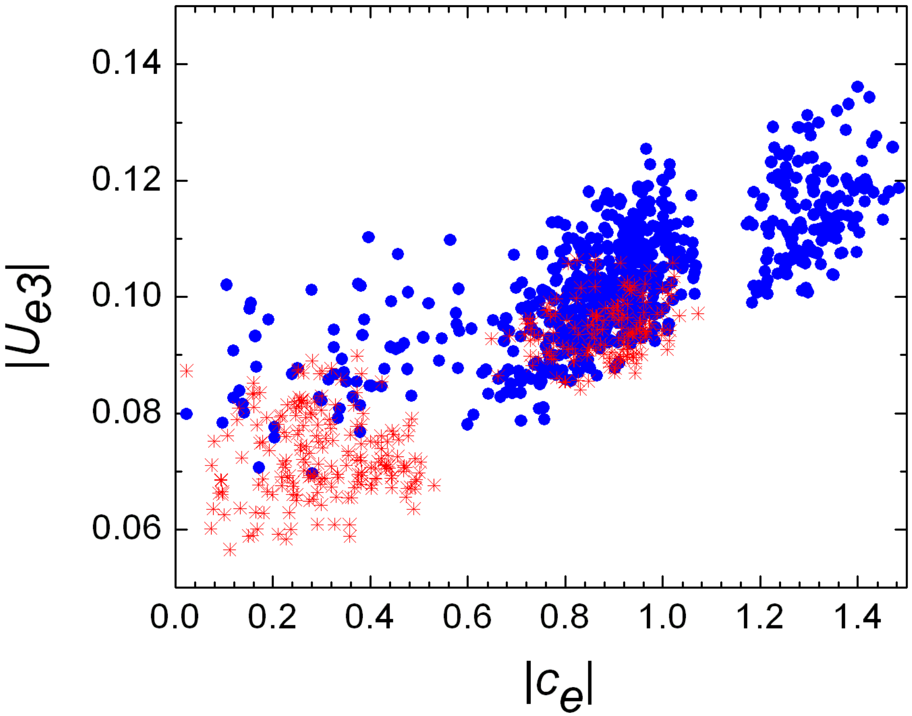
$$\tan^2\theta_{\text{atm}} = \frac{b^2+c^2}{a^2}, \qquad \tan^2\theta_{\text{sol}} = \frac{c}{b}, \qquad \sin\theta_{13} = 0.$$

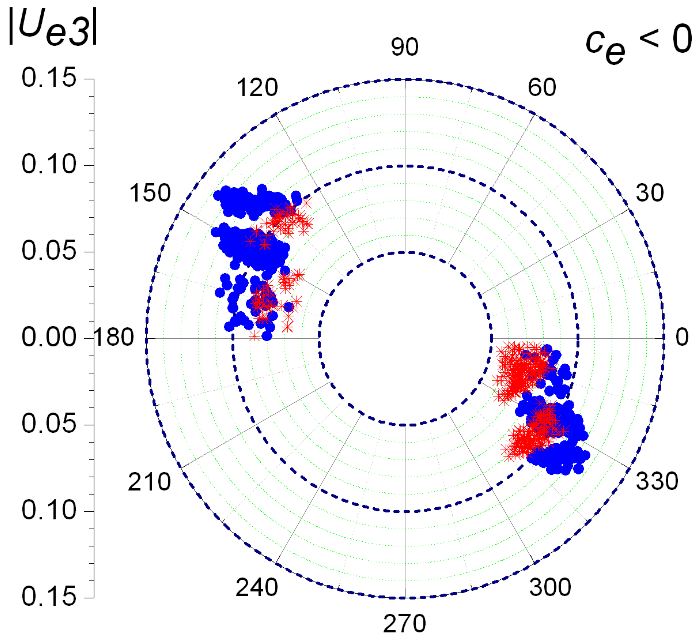
2 large, 1 small neutrino mixings are automatic.

$$Y_u = \bar{h} + r_2\bar{f} + r_3\bar{h}' \qquad Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e\bar{h}') \qquad m_\nu^{\text{light}} \propto \bar{f} \text{ (type II)}$$

$$|U_{e3}| = \sin\theta_{13} \text{ is corrected by } \sim \sqrt{\frac{m_e}{m_\mu}}, \qquad \sim \sin\theta_{\text{sol}} \sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m_{\text{atm}}}}$$





$$\delta_{\text{MNSP}} \simeq 130^\circ - 150^\circ,\ 310^\circ - 330^\circ$$

(Hermiticity of fermion mass matrix is assumed.)

PRD**72**, 075009 (2005)

Measurable and testable in near-future experiments.

Fortunately, many of the SO(10) models predicts measurable 13 neutrino mixing.

It will be measured near future.

HyperKamiokande will be designed (hopefully).

Nucleon decay experiments can disentangle the models.

Summary

- We study the proton decay suppression by
(1) increasing the colored Higgs mass
(2) selecting suitable flavor structure
in SO(10) models.

- There are four candidates in the SO(10) multiplets to increase the colored Higgs mass.

- We present a possibility for the interpretation of coupling unification scale [(**8**,**2**,1/2) and (**6**,**2**,1/6)].

- Proton decay constraints may imply quark flavor violation rather than leptonic one.

● We study a suitable flavor structure to suppress proton decay naturally.
The fermion masses and mixings are predictive.

● Measurement of 13 neutrino mixing and a phase, accurate analyses of quark and lepton flavor violating processes, squark-slepton masses (LHC/ILC) will lead us a scheme of vast scope to investigate GUT scale physics.

Back up slides

Type I seesaw

$$Y_\nu \bar{f}^{-1} Y_\nu^\top$$

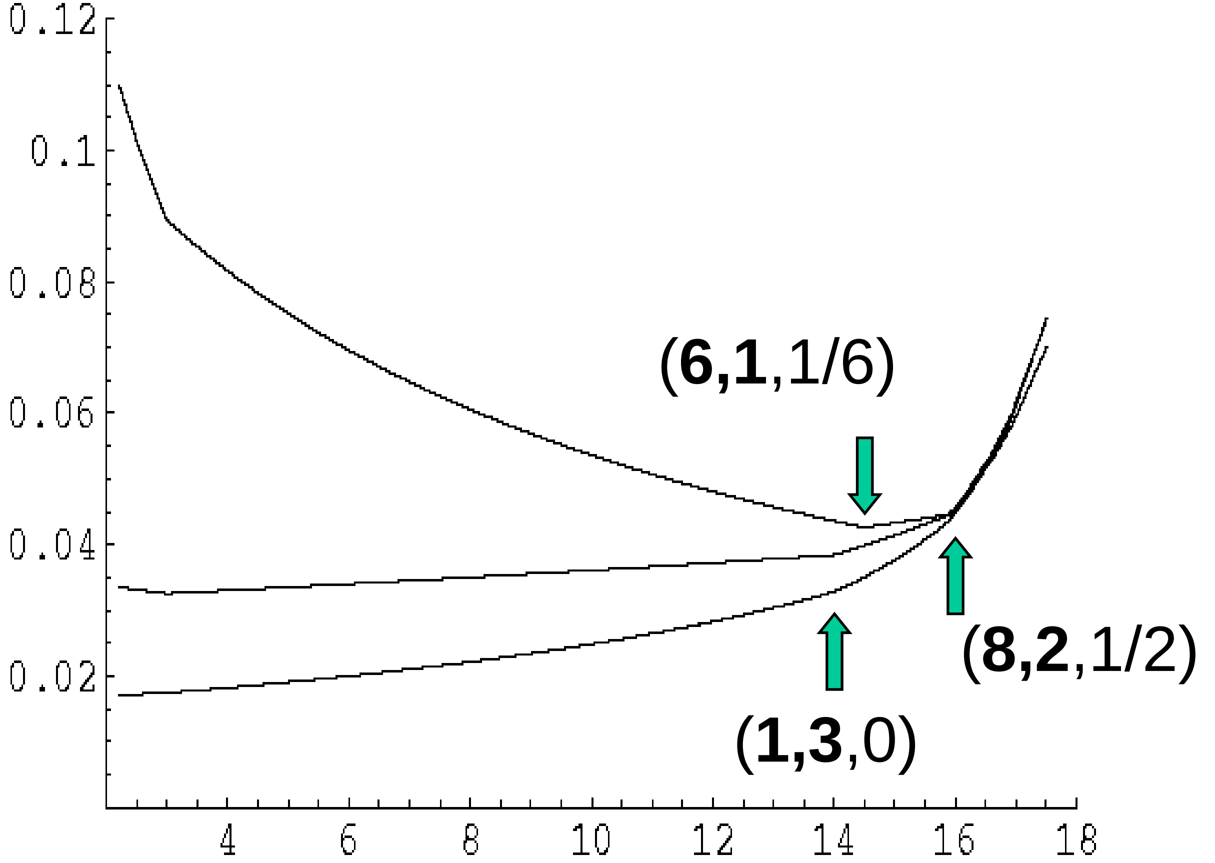
$$= (\bar{h} - 3r_2\bar{f} + c_\nu \bar{h}')\bar{f}^{-1}(\bar{h} - 3r_2\bar{f} - c_\nu \bar{h}')$$

$$= \underline{(\bar{h} + c_\nu \bar{h}')\bar{f}^{-1}(\bar{h} - c_\nu \bar{h}') - 6r_2\bar{h}} + 9r_2^2\bar{f}$$

Cancellation is needed.

$$\bar{h} \simeq \mathrm{diag}(0,0,1), \qquad \bar{f} \simeq \begin{pmatrix} \sim 0 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}, \qquad \bar{h}' \simeq i \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix}$$

Compatible to $\bar{f}_{33}^{-1}$ suppression


0.12
0.1
0.08
0.06
0.04
0.02
4
6
8
10
12
14
16
18
(**6,1**,1/6)
(**8,2**,1/2)
(**1,3**,0)

16 Higgs Scenario

Matter : $\psi_{16} + \psi_{10}$ Higgs : $H_{10} + H_{16}$

Yukawa couplings : $\psi_{16}\psi_{16}H_{10} + \psi_{16}\psi_{10}H_{16}$

$$\psi_{16} \supset 10 + 1, \quad \psi_{10} \supset \bar{5} \text{ under SU(5)}$$

Fermions are not completely unified.
Quantitative predictivity is less.

126 Higgs Scenario

R-parity : automatic

$\Rightarrow$ No dimension 4 baryon-number violating operator
Lightest SUSY Particle is stable. (Dark Matter candidate)

Explanation of factor 3 in GJ relation due to CG coefficient

Relation of bottom-tau mass conversion and large atm. mixing

(Bajc-Senjanovic-Vissani)

$$\left\{\begin{array}{l} M_d = r_1(\bar{h} + \bar{f})\, v_d \\ M_e = r_1(\bar{h} - 3\bar{f})\, v_d \\ m_\nu^{\mathrm{II}} \propto \bar{f} \end{array}\right. \quad \Rightarrow \quad \boxed{M_d - M_e \propto m_\nu^{\mathrm{II}}}$$

$$\left.\begin{array}{l} M_d \simeq \begin{pmatrix} \cdot & \cdot & O(\lambda^3) \\ \cdot & O(\lambda^2) & O(\lambda^2) \\ O(\lambda^3) & O(\lambda^2) & 1 \end{pmatrix} m_b \\ \\ M_e \simeq \begin{pmatrix} \cdot & \cdot & O(\lambda^3) \\ \cdot & O(\lambda^2) & O(\lambda^2) \\ O(\lambda^3) & O(\lambda^2) & 1 \end{pmatrix} m_\tau \end{array}\right\} \quad m_\nu^{\mathrm{II}} \propto \begin{pmatrix} \cdot & \cdot & \lambda \\ \cdot & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

$$\underline{m_b - m_\tau \sim O(\lambda^2) m_b}$$

Our Model

- We assume that Yukawa matrices are Hermitian.

$$(\bar{h},\ \bar{f}\ :\ \text{real},\quad \bar{h}'\ :\ \text{pure imaginary})$$

$$\bar{h} = V_{11} h, \quad \bar{f} = U_{14}/(\sqrt{3} r_1) f$$
$$\bar{h}' = (U_{12} + U_{13}/\sqrt{3})/r_1 h'$$

$$\mathrm{SO}(10) \times \mathcal{C}$$

Spontaneous CP violation by VEV of **45** Higgs.

VEVs of **45** Higgs are pure imaginary.

120 Higgs mixing is pure imaginary.

Merits of the assumption:

1. Number of parameters is less. ⟹ Predictive
2. No cancellation between $\bar{f}$ and $\bar{h}'$.

   ⟹ The predictions are stable.
3. A solution of SUSY CP and strong CP problems

$$\bar{\theta} = \theta + \arg(\det M_u M_d)$$

Note:

When $r_3 \to 0$ and $U \simeq V$ ($\tan\beta \simeq 50$),

then $c_e \simeq -1$, $c_\nu \simeq 2$.

$$U M_{\text{doub.}} V^{\top} = M_{\text{doub.}}^{\text{diag}}$$

$$r_1 = \frac{U_{11}}{V_{11}} \qquad r_3 = r_1 \frac{V_{12} - V_{13}/\sqrt{3}}{U_{12} + U_{13}/\sqrt{3}}$$

$$c_e = \frac{U_{12} - \sqrt{3}U_{13}}{U_{12} + U_{13}/\sqrt{3}} \qquad c_\nu = r_1 \frac{V_{12} + \sqrt{3}V_{13}}{U_{12} + U_{13}/\sqrt{3}}$$

Note:

When $\bar{f}_{11}, \bar{f}_{12} \to 0$, $\quad \det M_e \simeq |c_e|^2 \det M_d$

$|c_e| \simeq 1$ satisfies Georgi-Jarskog relation.

$$Y_d = r_1(\bar{h} + \bar{f} + \bar{h}') \qquad Y_e = r_1(\bar{h} - 3\bar{f} + c_e \bar{h}')$$

$\bar{h},\ \bar{f}$: real symmetric,
$\bar{h}'$: pure imaginary antisymmetric } 12 parameters

Higgs parameters : $r_1,\ r_2,\ r_3,\ c_e,\ c_\nu$ } 5 parameters

Total 17 parameters in the Yukawa matrices

4 parameters are insensitive to fit masses and mixings

$(\bar{h}_{11},\ \bar{h}_{22},\ r_3,\ c_\nu)$

$m_u,\ m_c,\ m_t$

$m_e,\ m_\mu,\ m_\tau$

$V_{us},\ V_{cb},\ V_{ub},\ \delta_{\mathrm{CKM}}$

$\theta_{\mathrm{atm}},\ \theta_{\mathrm{sol}},\ \Delta m^2_{\mathrm{sol}}/\Delta m^2_{\mathrm{atm}}$

Inputs

$m_d,\ m_s,\ m_b$

$|U_{e3}|,\ \delta_{\mathrm{MNSP}}$

Outputs

Note on the model without **120** Higgs

$\bar{h},\ \bar{f}$: complex symmetric } 9+(6) parameters

Higgs parameters : $r_1,\ r_2$ } 2+(1) parameters

Total 18 parameters in the Yukawa matrices

$$Y_u = \bar{h} + r_2 \bar{f}$$
$$Y_d = r_1(\bar{h} + \bar{f})$$
$$Y_e = r_1(\bar{h} - 3\bar{f})$$
$$Y_\nu = \bar{h} - 3 r_2 \bar{f}$$

Naively, $m_e \simeq 3 m_d$

⇓

We need cancellation in [1-2] block.

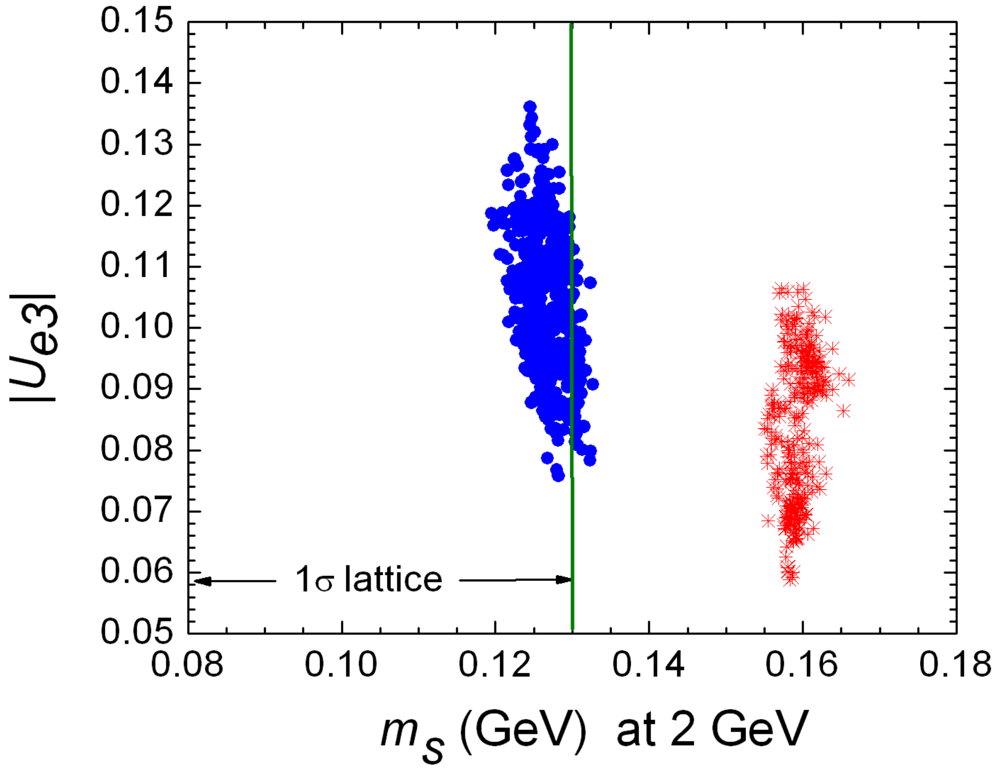
$h_{11},\ h_{22}$ are needed.

⇓

Every property is just fit.

Predictions

1. Strange quark mass (roughly, $m_s \sim \frac{1}{3} m_\mu (1 \pm O(\lambda^2))$)



- $m_s \simeq 120 - 130$ MeV
- $m_s \simeq 155 - 165$ MeV

Prediction: $m_s/m_d \simeq 17 - 18,\ 19 - 20.5$

Non-lattice value
$m_s/m_d = 18.9 \pm 0.8$
(Leutwyler)

$m_b \sim m_\tau (1 \pm O(\lambda^2))$ ⇒ large $\tan\beta$ ($\tan\beta \sim 50$)

Predictions

$$Y_d = r_1(\bar{h} + \bar{f} + \bar{h}')$$

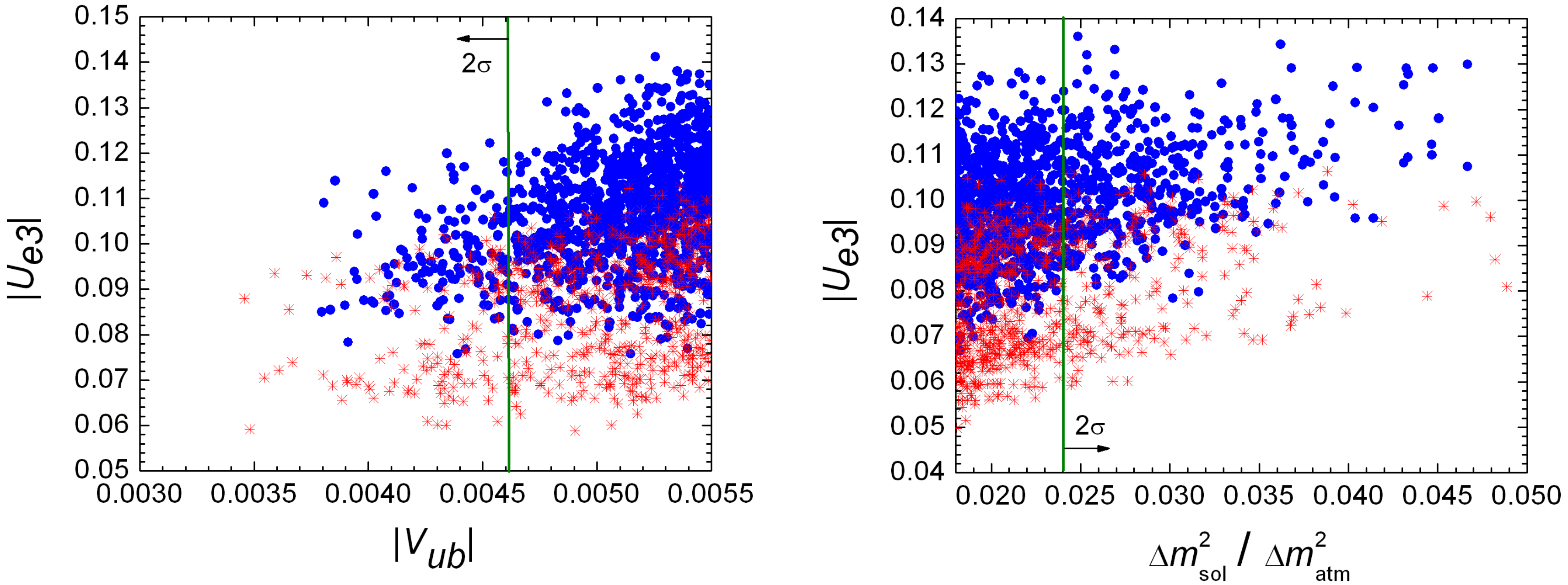
$$m_\nu^{\text{light}} \simeq \begin{pmatrix} \sim 0 & \sim 0 & \lambda^3 \\ \sim 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}$$

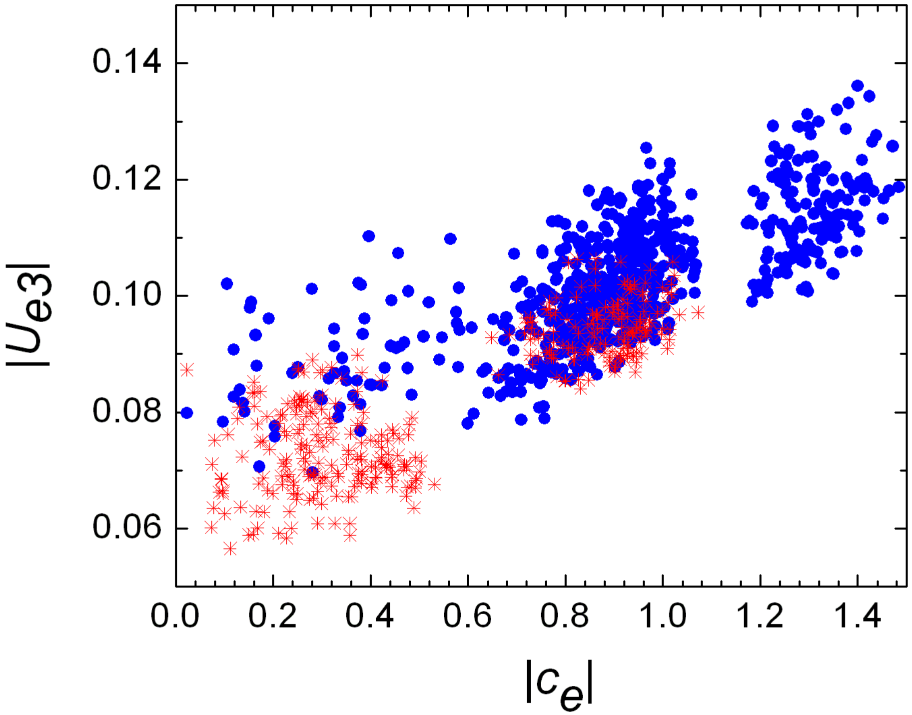
2. $|U_{e3}|$

$f_{12} < f_{13}$

Approximated relations :

$$|U_{e3}| \approx \frac{1}{\sqrt{2}} \left| \frac{V_{ub}}{V_{cb}} \right|, \qquad |U_{e3}|^2 \approx \frac{\tan^2 \theta_{\text{sol}}}{1 - \tan^4 \theta_{\text{sol}}} \Delta m^2_{\text{sol}} / \Delta m^2_{\text{atm}}$$





Prediction: $|U_{e3}| \simeq 0.08 - 0.12 \quad (|c_e| \simeq 1)$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e \bar{h}')$$

Note:

When $r_3 \to 0$ and $U \simeq V$ $(\tan\beta \simeq 50)$,

then $c_e \simeq -1$, $c_\nu \simeq 2$.

Note:

When $\bar{f}_{11}, \bar{f}_{12} \to 0$,

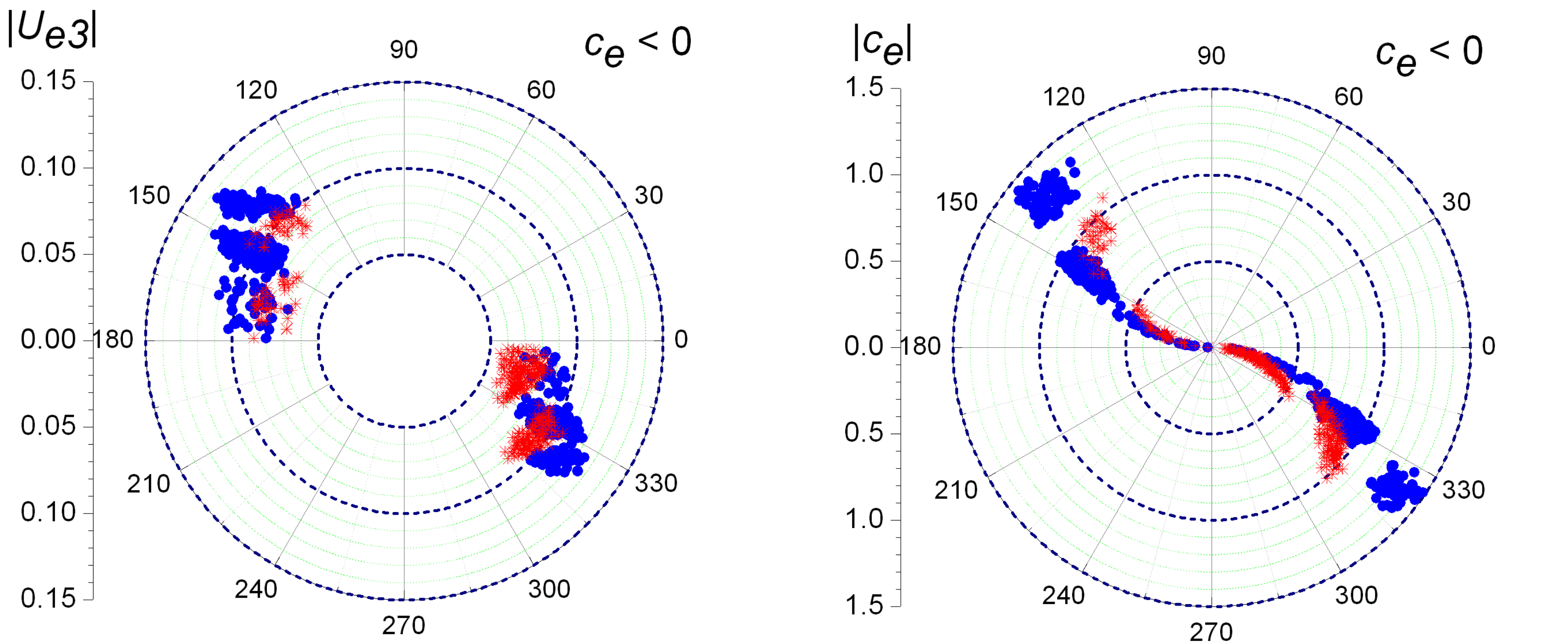
$\det M_e \simeq |c_e|^2 \det M_d$

Predictions

3. MNSP phase

Approximated relation (type II dominance) :

$$\sin\delta_{\rm MNSP} \sim \frac{1}{\sqrt{2}}\frac{\sin\theta^e_{12}}{\sin\theta^\nu_{13}}\sin\left(\tan^{-1}\frac{c_e\bar{h}'_{12}}{3\bar{f}_{12}}\right)$$



Prediction : $\delta_{\rm MNSP} \simeq 130^{\rm o} - 150^{\rm o},\ 310^{\rm o} - 330^{\rm o} \qquad (c_e \simeq -1)$

$[\sin\delta_{\rm MNSP} \sim \pm(0.5 - 0.7)]$

$$m_\nu^{\text{light}} = \underbrace{M_L}_{\text{Type II}} - \underbrace{M_\nu^D M_R^{-1} (M_\nu^D)^\mathsf{T}}_{\text{Type I}}$$

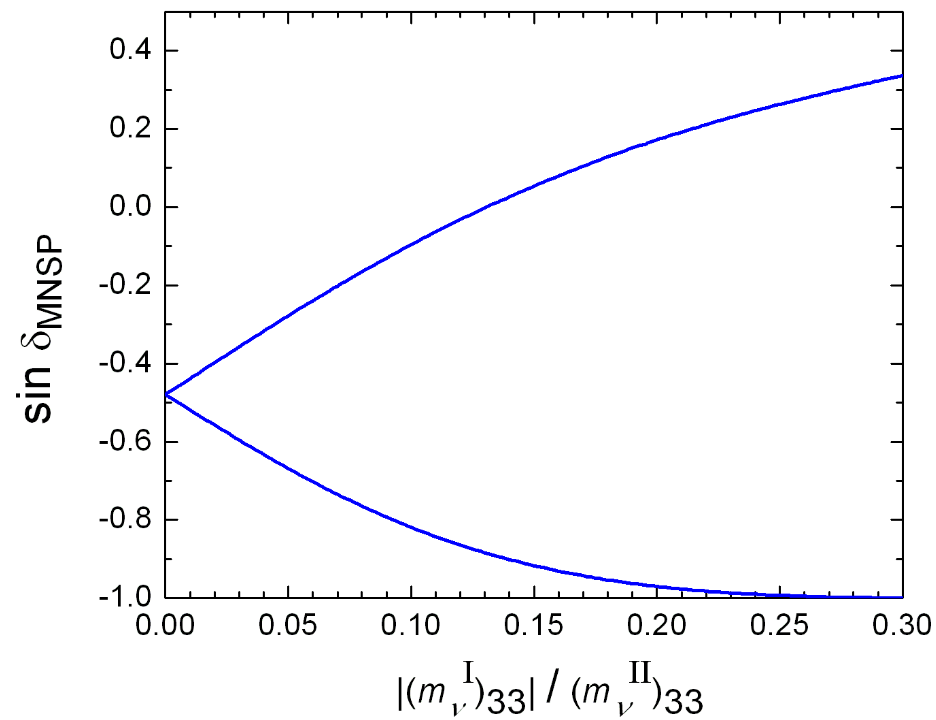
$$M_L = 2\sqrt{2} f \langle \bar{\Delta}_L \rangle \qquad M_R = 2\sqrt{2} f \langle \bar{\Delta}_R \rangle$$

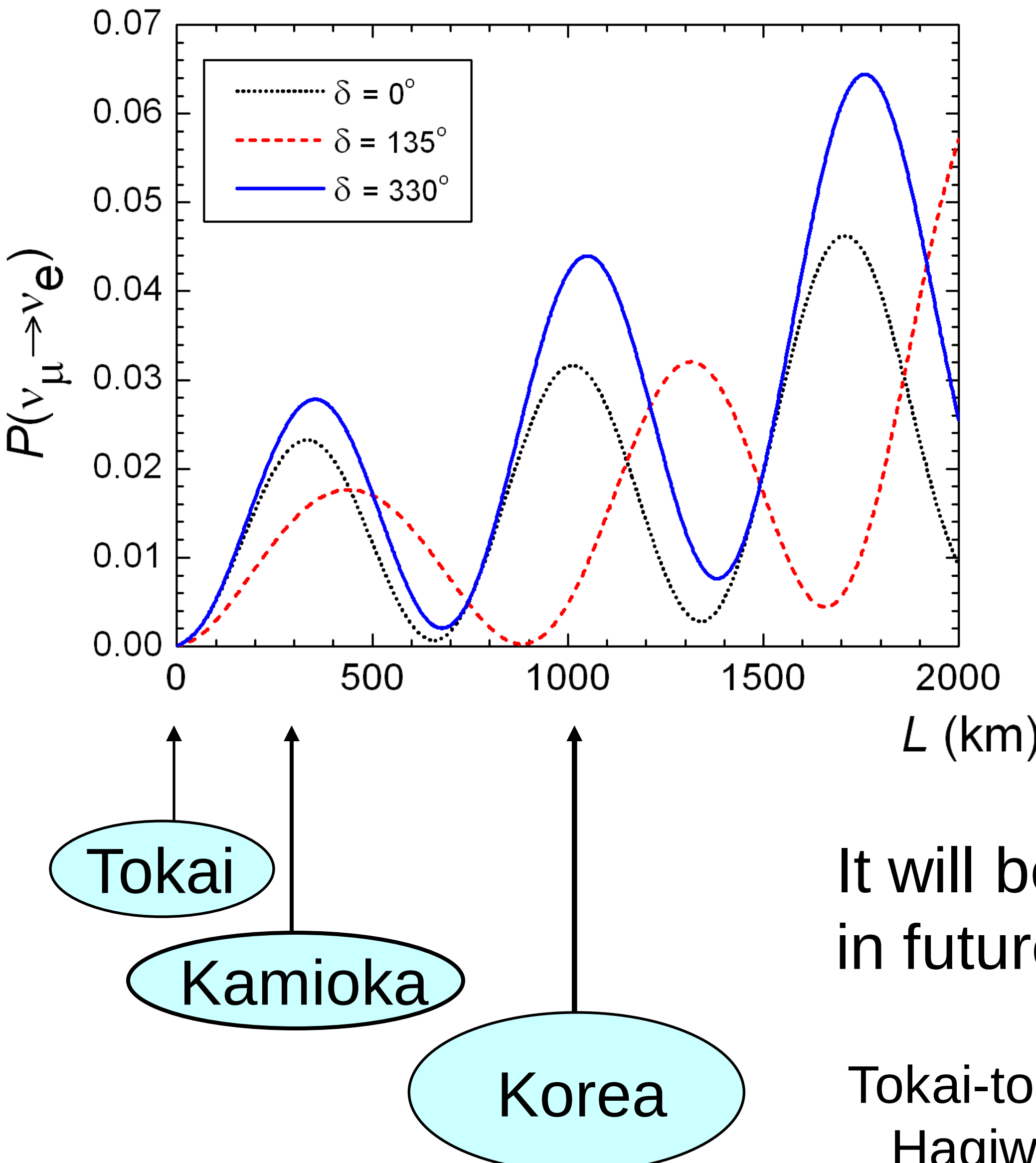
When $\langle \Delta_R \rangle / U_{14} \sim 2 \times 10^{16}$ GeV, type II contribution is dominant.

When $\langle \Delta_R \rangle / U_{14} \sim 2 \times 10^{15}$ GeV, $|(m_\nu^I)_{33}| / (m_\nu^{II})_{33} \sim 0.2$

$|U_{e3}|$: unchanged

MNSP phase : modified





$\Delta m^2_{13} = 2.5 \times 10^{-3}\ \mathrm{eV}^2,$

$\Delta m^2_{12} = 8 \times 10^{-5}\ \mathrm{eV}^2,$

$|U_{e3}| = 0.1$

$\tan^2\theta_{12} = 0.4$

$\sin^2 2\theta_{23} = 1$

$E_\nu = 0.7\ \mathrm{GeV}$

$\rho = 3\ \mathrm{g/cm^3}$

It will be measured
in future experiments.

Tokai-to-Korea proposal:
Hagiwara-Okamura-Senda
Ishitsuka-Kajita-Minakata-Nunokawa

Conclusion

1. A Minimal SO(10) Model with suppressed proton decay is presented.

2. Predictions :

$$|U_{e3}| \simeq 0.08 - 0.12$$

$$\delta_{\mathrm{MNSP}} \simeq 130^{\circ} - 150^{\circ},\ \ 310^{\circ} - 330^{\circ}$$

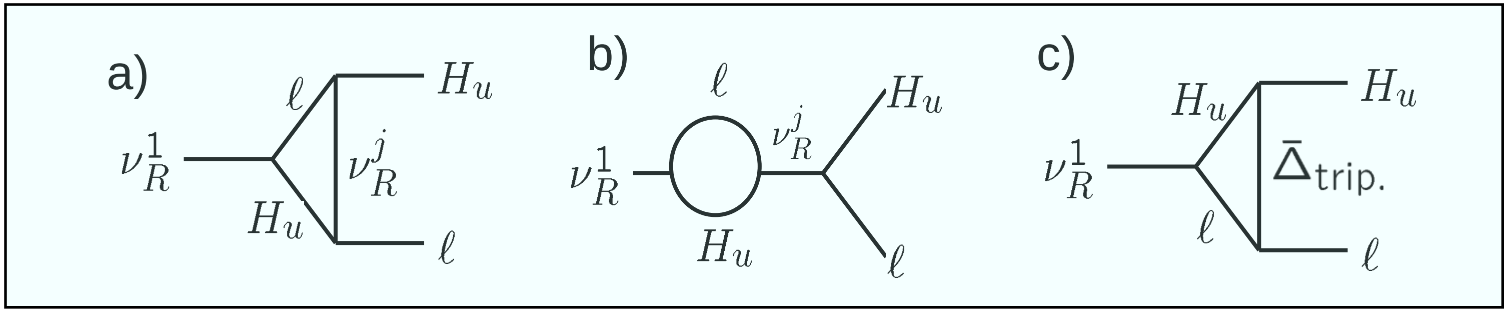
(type II dominant case)

3. It will be verified in the future experiments.

T2K, KASKA, NOvA, double-CHOOZ, ….

Lepton asymmetry via right-handed neutrino decay

$$\epsilon_1 = \sum_i \frac{\Gamma(N_1 \to \ell_i H_u^*) - \Gamma(N_1 \to \bar{\ell}_i H_u)}{\Gamma(N_1 \to \ell_i H_u^*) + \Gamma(N_1 \to \bar{\ell}_i H_u)}$$



Prediction: $|\epsilon_1| = (1-4) \times 10^{-4} (c_\nu/2)$ diagram a) & b)

$$|\epsilon_1^\Delta| = (2.5\text{–}6) \times 10^{-4} (c_\nu/2) \left(\frac{M_{R_1}}{10^{13}\text{GeV}}\right) \left(\frac{m_{\nu_3}^{\rm II}}{0.05\text{eV}}\right) G(y)$$

$$G(y) = y \ln \frac{y+1}{y} \qquad y = M^2_{\Delta_{\rm trip}} / M^2_{R_1}$$

When $M_{\Delta_{\rm trip}}/M_{R_1} \sim 0.1$, ϵ_1^Δ is negligible compared to ϵ_1.

$$Y_\nu = \bar{h} - 3 r_2 \bar{f} + c_\nu \bar{h}'$$

In thermal leptogenesis scenario

using WMAP data : $\eta_B = (6.3 \pm 0.3) \times 10^{-10}$

Lightest right-handed Majorana mass :

$$M_{R_1} \simeq (0.4 - 1)(2/c_\nu) \times 10^{13}\ \text{GeV}$$

VEV of $\bar{\Delta}_R$:

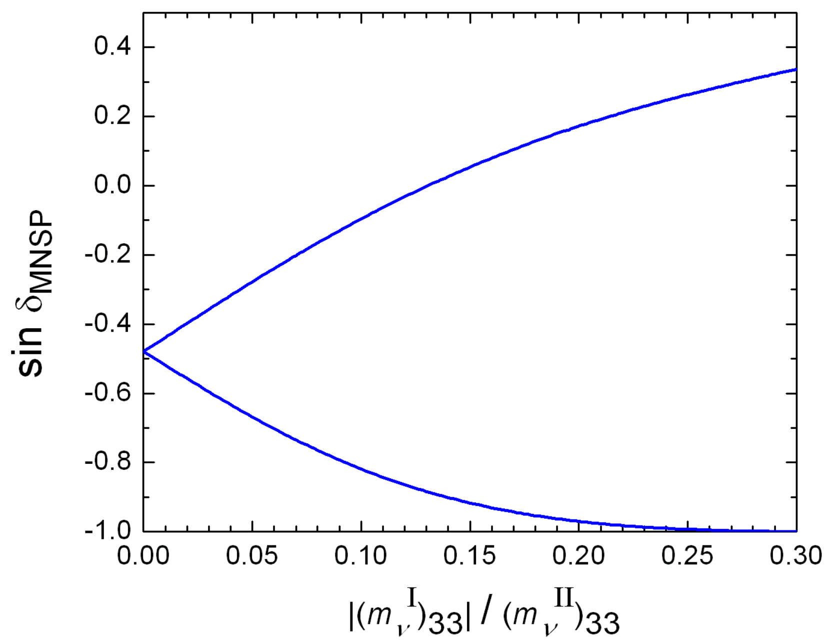
$$v_R \simeq U_{14}(2/c_\nu)(1 - 2.5) \times 10^{15}\ \text{GeV}$$

Under this choice,

$$|(m_\nu^I)_{33}|/(m_\nu^{II})_{33} \sim 0.2$$

$|U_{e3}|$: unchanged

MNSP phase : modified



Two possibilities:

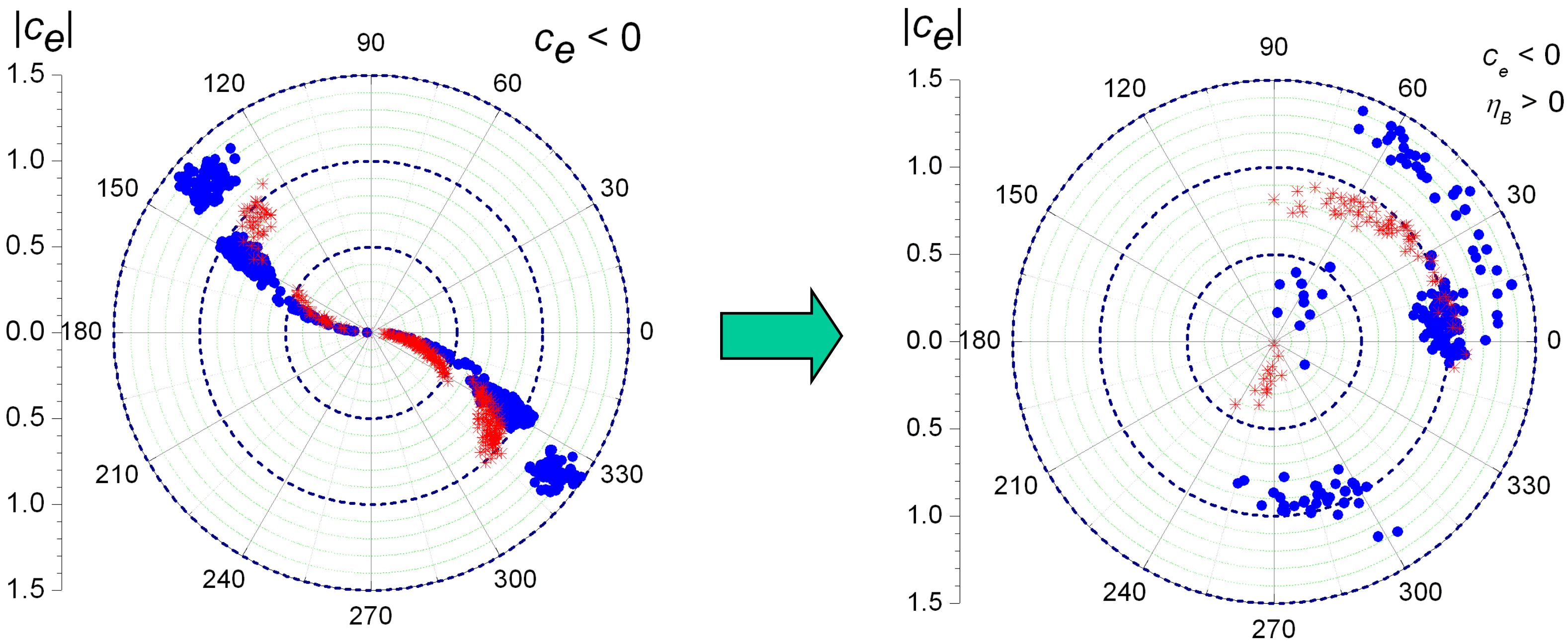
1. $v_R/U_{14} > 2 \times 10^{16}$ GeV

Type II seesaw dominant : MNSP phase predicted.

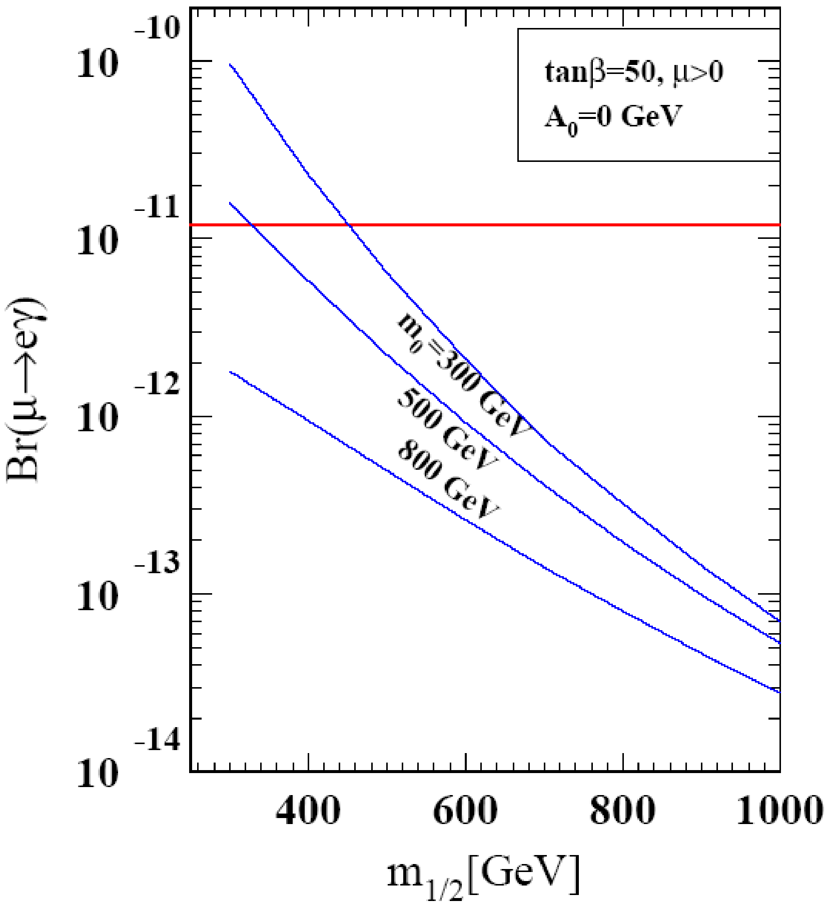
2. $v_R/U_{14} \sim (1 - 2.5) \times 10^{15}$ GeV

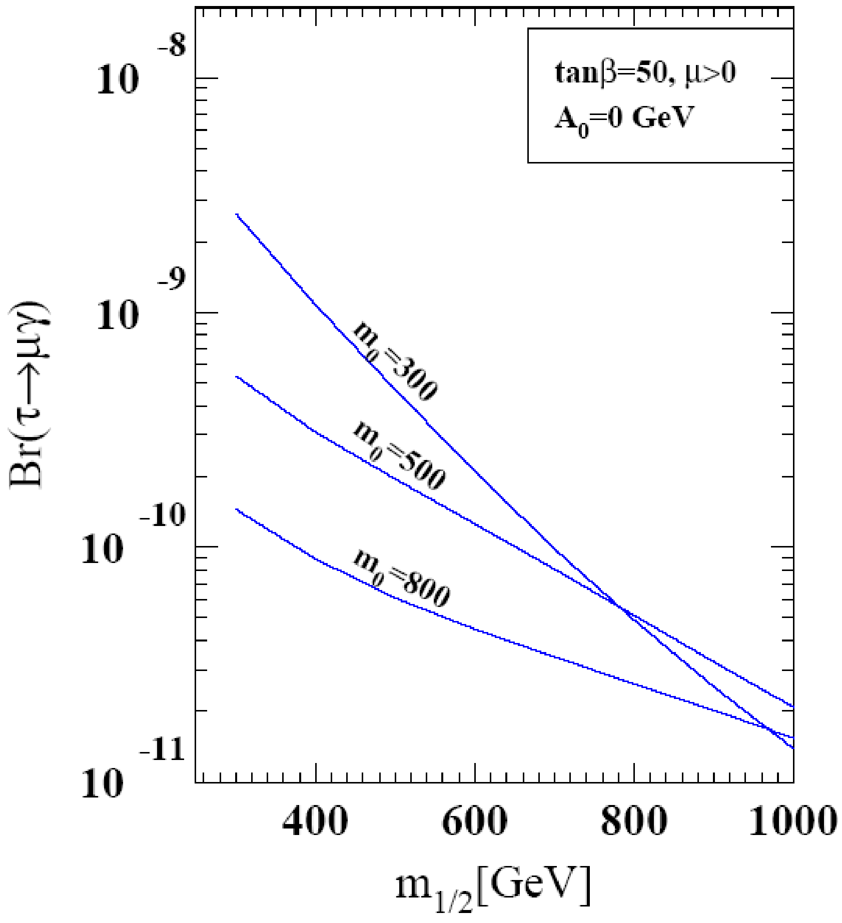
Type I seesaw contributes : MNSP phase modified.

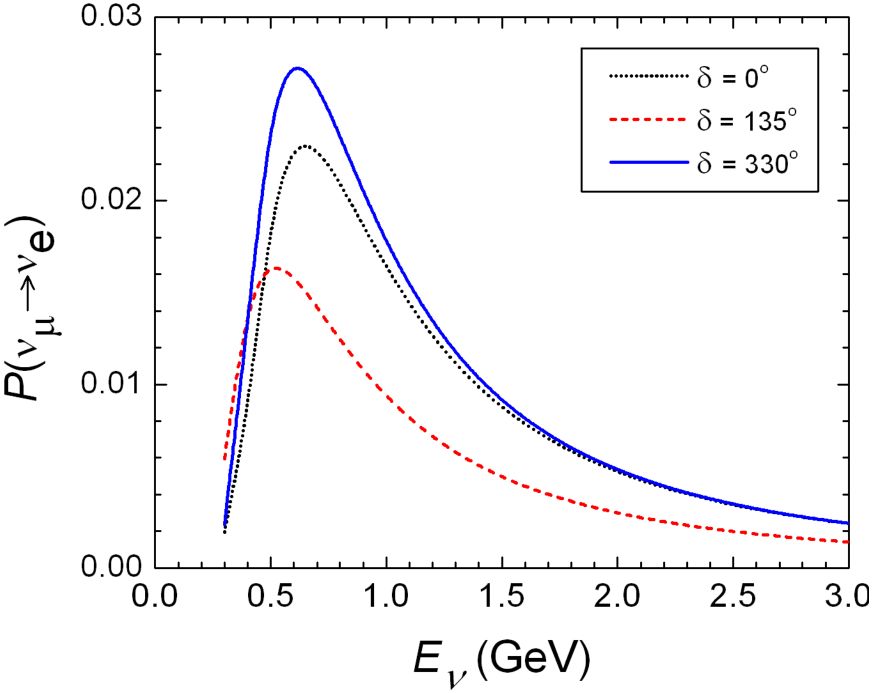
To satisfy WMAP data, SU(2) triplet contribution is not needed.

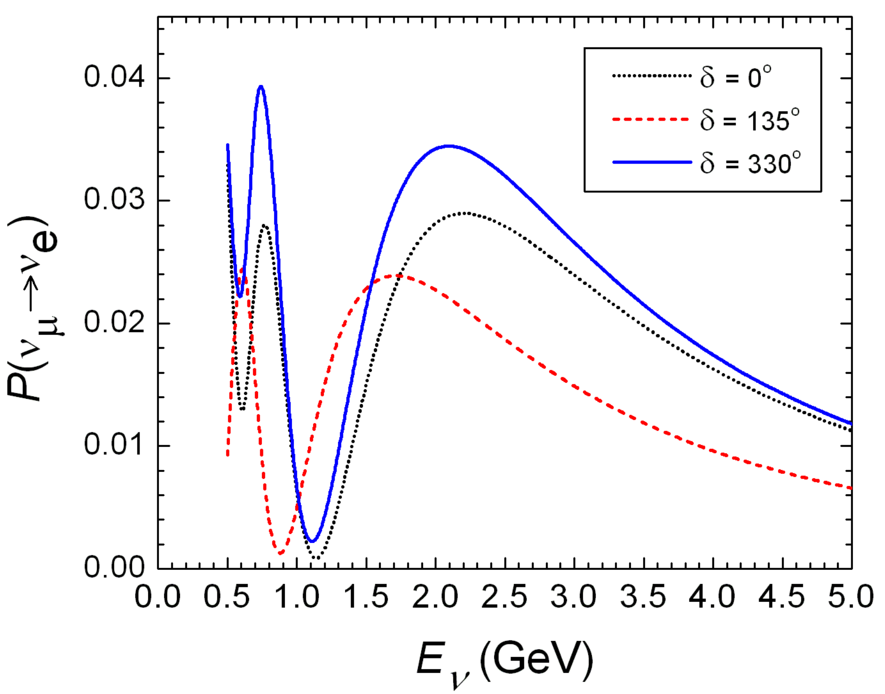


Lepton flavor violation









at Kamioka

$L = 295$ km

at Korea

$L = 1100$ km