String Compactification and Unification of Forces

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Based on papers with B. Kyae, J.-H. Kim, I-W Kim
JHEP 0706.034 [hep-ph/0702278] (R-parity,etc.)
PLB 647, 275; PLB 651, 407;
PLB 656, 207 [0707.3292](GMSB) ;
arXiv: 0712.1596 (KK mass)

1. Introduction

2. R-parity

3. SU(5)' hidden sector from $Z_{12\text{-}I}$ orbifold compactification

A. FCNC, B. FCNC conditions,

C. Z(12-I) models without exotics

4. Threshold correction toward unification

5. Conclusion

1. Introduction

- With LHC in sight, TeV scale SUSY will be in check. Superstring relies on this SUSY phenomenology.
- The $E_8 x E_8$ heterotic string gives a good gauge groups and string phenomenology is most successful in compactifications of the heterotic string.
- TeV SUSY is based on supergravity Lagrangian in the last 24 years. [Cremmer et al]
- Gravitino phenomenology: reheating temp.<10^{9-7} GeV [EKN, KKM]

  Attempts exist to detect it at LHC via neutralino decay to gravitino [Buchmuller et al.]
- R-parity for proton longevity: most string models are ruled out. The $u^c d^c d^c$ coupling must be forbidden.

- The μ-problem, or more generally the MSSM problem [KN, Giudice-Masiero]. But why only one pair?
- Little hierarchy problem: at present fine-tuning of order 1% needed(10-100 TeV SUSY particle masses). Negative stop mass considered to raise it to 5-10% fine-tuning [Dermisik-H.D. Kim]
  ----we hope that it will be understood in the end.
- Strong CP problem: string axions
  [Witten, CK, I-W Kim & K, Conlon]
- SUSY flavor problem: GMSB, AMSB
  [Dine-Fischler-Raby, Dine-Nelson, Seiberg et al, ISS]
- KKLT led to the consideration of mirage mediation
  [Choi-Nilles, others]
- Exotics or no exotics?

Most string models accompany exotics. Chiral exotics are dangerous phenomenologically. Some build models with chiral exotics, which is outrageously wrong as a phenomenological model.

So, all exotics must be made vectorlike.

This is a nontrivial condition.

Until recently, we did not find exotics-free models. But it seems that there are exotics-free models. The weak mixing angle here is not 3/8. Except this, the condition on singlet VEVs is not so strong as in models with exotics.

This talk is a top-down approach.

Specific examples if needed will be in Z_{12-I} orbifold models.

2. R-parity

R-parity (or matter parity) in the MSSM is basically put in by hand: q and l are odd, H are even

SO(10) GUT advocates that it has a natural R-parity

16 odd, 10 even

But it is nothing but the disparity between spinor-vector difference of S & V:

SSV coupling allowed, but $\boxed{\text{SSS}}$ coupling not allowed

$u^c d^c d^c$

In heterotic string compactification, we note the E_8 adjoint has

S=(++-+-+++), etc +=1/2, -=-1/2

V=(1 -1 0 0 0 0 0 0), etc

Therefore, in heterotic string compactification the strategy is to put matter representations in S type and Higgs reps. in V type of the original E_8.

As a GUT, E_6 is not good in this sense because matter 16 and Higgs 10 are put in the same 27.

27 = 16+10+1

U(1) charge Γ: by even Γ VEV, Γ=(2 2 2 0 0 0 0 0) → P

Then, Γ =odd integer for S

Γ =even integer for V. Then P is good.

$$S = (++--++++), etc.$$
$$V = (10-100000\ ), etc.$$

126 belongs to V [Mohapatra]

There are 4 possibilities of U(1)s:

B-L ~ (2 2 2 0 0 0 0), **X** ~ (2 2 2 2 2 0 0 0)

$\mathbf{Q_1}$ ~ (0 0 0 0 0 2 0 0), $\mathbf{Q_2}$ ~ (0 0 0 0 0 0 2 0)

3. SU(5)' hidden sector from Z_{12-I} orbifold compactification:

no exotics, R-parity, and MSSM

- The orbifold compactification is well known by now. [book: K.-S. Choi and K]
- The $E_8 x E_8$ heterotic string gives a good gauge groups and string phenomenology is most successful here.
- Any orbifold has a same order of complexity. Even though Z_3 looks the simplest, 27 fixed points makes it very complicated. Z_{12-I} looks complicated, but it is simple in Wilson lines and makes it intuitively superior to others.

	Lattice	Effective Order	Condition
$\mathbf{Z}_3$	$SU(3)^3$	$3a_1=0, 3a_3=0, 3a_5=0$	$a_1=a_2, a_3=a_4, a_5=a_6$
$\mathbf{Z}_4$	$SU(4)^2$	$2a_1=0, 2a_4=0$	$a_1=a_2=a_3, a_4=a_5=a_6$
	$SU(4)\times SO(5)\times SU(2)$	$2a_1=0, 2a_5=0, 2a_6=0$	$a_1=a_2=a_3, a_4=0$
	$SO(5)^2\times SU(2)^2$	$2a_2=0, 2a_4=0, 2a_5=0, 2a_6=0$	$a_1=a_3=0$
$\mathbf{Z}_6$-I	$SU(3)\times SU(3)^2$	$3a_1=0$ ←	$a_1=a_2, a_3=a_4=a_5=a_6=0$
$\mathbf{Z}_6$-II	$SU(2)\times SU(6)$	$2a_1=0$ ←	$a_2=a_3=a_4=a_5=a_6=0$
	$SU(3)\times SO(8)$	$3a_1=0, 2a_5=0$	$a_1=a_2, a_3=a_4=0, a_5=a_6$
	$SU(2)^2\times SU(3)^2$	$3a_1=0, 2a_3=0, 2a_4=0$	$a_1=a_2, a_5=a_6=0$
$\mathbf{Z}_7$	$SU(7)$	$7a_1=0$ ←	$a_1=a_2=a_3=a_4=a_5=a_6$
$\mathbf{Z}_8$-I	$SO(8)\times SO(5)$	$2a_1=0, 2a_6=0$	$a_1=a_2=a_3=a_4, a_5=0$
$\mathbf{Z}_8$-II	$SO(10)\times SU(2)$	$2a_4=0, 2a_6=0$	$a_1=a_2=a_3=0, a_4=a_5$
	$SU(2)^2\times SO(8)$	$2a_1=0, 2a_5=0, 2a_6=0$	$a_1=a_2=a_3=a_4$
$\mathbf{Z}_{12}$-I	E_6	no restriction	$a_1=a_2=a_3=a_4=a_5=a_6=0$
	$SU(3)\times SO(8)$	$\boxed{3a_1=0}$	$\boxed{a_1=a_2}, a_3=a_4=a_5=a_6=0$
$\mathbf{Z}_{12}$-II	$SU(2)^2\times SO(8)$	$2a_1=0, 2a_2=0$	$a_3=a_4=a_5=a_6=0$

Some simple cases of Wilson lines

A. FCNC

- SUSY flavor problem led to the GMSB.
- GMSB : SO(10), 16+10,   SU(5), 10+5* [Veneziano, ADS, PT]
- Unstable vacuum: SU(5), 6(5+5*), 7(5+5*), etc. [ISS]

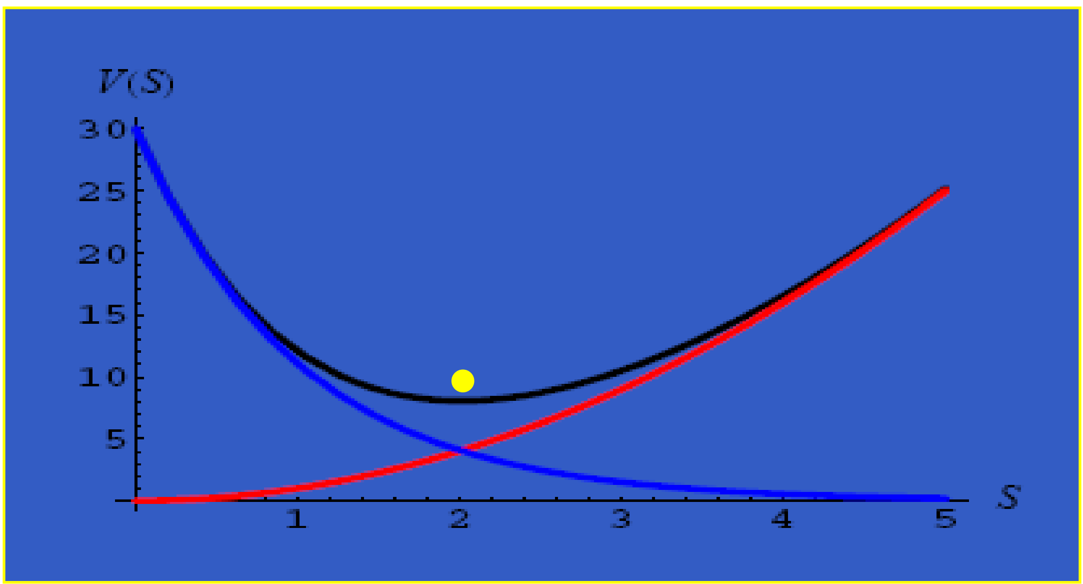
The messenger sector {f, …} [e.g. Murayama-Nomura]

$$Unstable: W_{tree} = m\bar{Q}Q + \frac{\lambda}{M_{Pl}} Q\bar{Q}f\bar{f} + Mf\bar{f}, \qquad Q = h \quad quark$$

$$Stable: L = \int d^2\theta \left[\frac{1}{M^2} \bar{f}fW'^{\alpha} W'_{\alpha} + M_f \bar{f}f \right] + h.c.$$

In string compactification, there are many heavy charged fields which can act as the messenger sector.

For the SUSY flavor problem, we need a dynamical symmetry breaking (DSB) scale less than $10^{11\text{-}12}$ GeV. Then the gravity mediation is subdominant, and we expect the SUSY flavor problem is remedied. Before 2006/02, DSB is the obtained by the fight between the runaway dynamical solution and the steep strong solution [Abel],



Nelson-Seiberg argued the need for R-symmetry to break SUSYdynamically at the ground state.

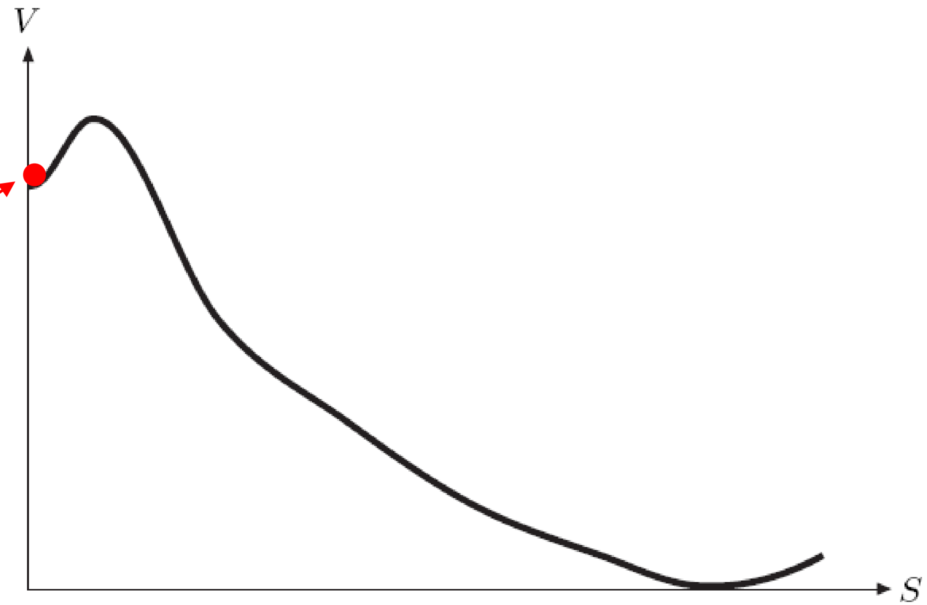
ISS looked for a sufficiently long lived unstable vaccum, where the need of R-symmetry is discarded.

$$W_{ISS} = \bar{b}_i S^{ij} b_j - \frac{\det S^{ij}}{\Lambda^{N_f - 3}} - m_i \Lambda S^{ii}$$

SU(5) with 6 flavors [MN]

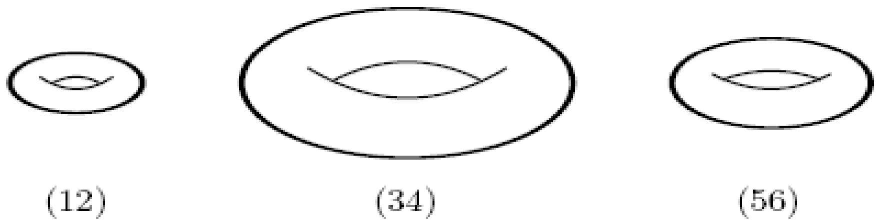
The R-symmetry breaking is introduced by the tree level W_{tree}, including the messengers f. S develops an F-term and SUSY breaking is mediated by f sector (generating F term by W_{tree}) to the observable sector.

Since 2006, the unstable minimum near the origin of the field space has been considered. It lives sufficiently long so that we can consider it as an acceptable vacuum. [ISS]



Acceptable vacuum if lifetime>> the age of the universe

To fulfil the condition for the DSB to occur at a relatively low energy scale, we introduce different radii for three tori. It is reminiscent of Horava-Witten's introduction of a distance between two branes.



Also extra particles in the desert may be used to fit the data.

B. SUSY FCNC Conditions

SUSY flavor constraint

The stringent SUSY flavor bound [Masiero-Silvestrini] generally falls in the region,

$$\delta \approx O(10^{-2}, \quad 10^{-3})$$

Different flavors can have different s-fermion couplings, in particular in the Kaehler potential. So, the SUSY flavor problem is generic.

The SUSY desert hypothesis fixes α_{GUT} around 1/25 at (2-3)x10^{16} GeV, but this can be changed by populating the desert with incomplete multiplets. We may allow α_{GUT} between 1/20 – 1/30.

For the SUSY flavor problem, the gravity mediation must be sub-dominant the GMSB, thus we may requires the SUSY breaking scale in the GMSB scenario below $10^{11\text{-}12}$ GeV.

The GMSB scenario needs two ingredients

- SUSY breaking sector: a confining gauge group, e.g. SU(5)', with quark Q: scale Λ_h
- Messeger scale M_X

So, we consider the following scale

$$\frac{\Lambda_h^3}{M_P^2} \le 10^{-3}\,TeV \Rightarrow \Lambda_h \le 2\times 10^{12}\,GeV$$

$$\frac{(\xi\Lambda_h)^2}{M_X} \approx 10^3\,GeV$$

The one-loop coupling running is

$$\frac{1}{\alpha_{GUT}^{h}} = \frac{1}{\alpha_{j}^{h}(\mu)} + \frac{-b_{j}^{h}}{2\pi} \ln\left|\frac{M_{GUT}^{h}}{\mu}\right|$$

$$A = 1/\alpha_{GUT}, \qquad A' = 1/\alpha_{GUT}^{h}$$

$$\boxed{A' - 1 = \frac{-b_{j}^{h}}{2\pi} \ln\left|\frac{M_{GUT}^{h}}{\Lambda_{h}}\right| \qquad (A')}$$

For example, if $-b_j^h$ is given, we can relate A' and Λ_h,

SU(4) no matter (any such model?), 12→A'=27.4
SU(5) 7 flavors, 8→A'=18.6

The confining scale is estimated as

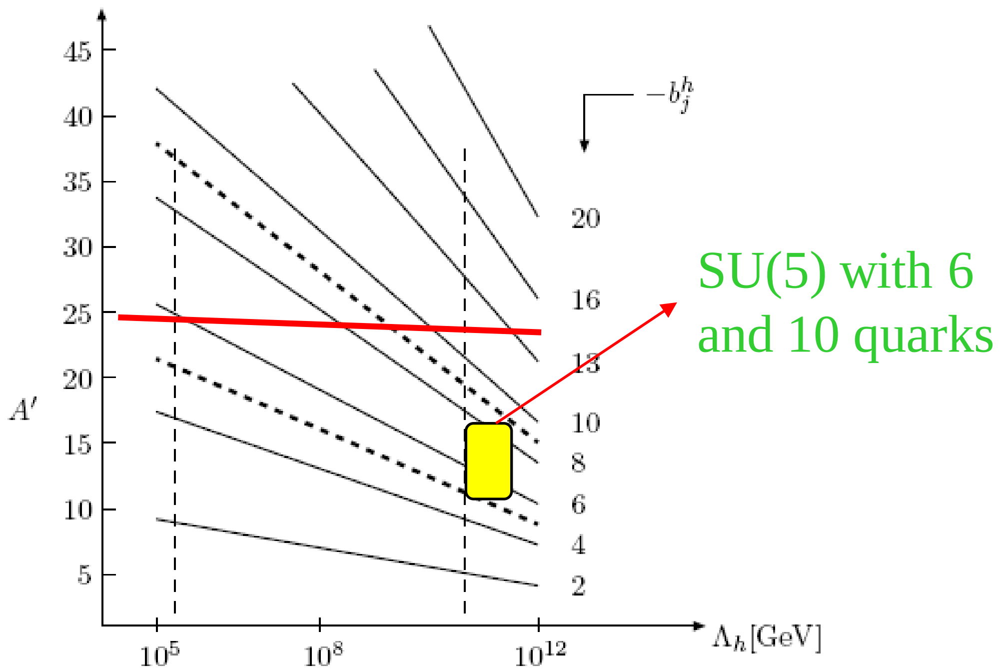


FIG. 2: Constraints on A'. The confining scale is defined as the scale μ where $\alpha_j^h(\mu) = 1$. Using $\xi = 0.1, M_X = 2 \times 10^{16}$ GeV in the upper bound region and $\xi = 0.1, M_X = \frac{1}{2} \times 10^6$ GeV in the lower bound region, we obtain the region bounded by dashed vertical lines. Thick dash curves are for $-b_j^h = 5$ and 9.

C. A $Z_{12\text{-I}}$ Model without Exotics

- 3 chiral families: this restricts very much the possibility of good representations (10+5-bar, or many 5 & 5-bar flavors are desirable, since 3 families already have many chiral fields)
- Vectorlike exotics, or no exotics: This is another strong restriction.

Why $Z_{12\text{-I}}$?

Probably, most restrictive in Yukawa couplings. Wilson line is simple Z_3, not like $Z_{12\text{-II}}$. In this sense, it is a very simple model.

Restrictiveness because of 12 used:

Approximate R-parity in flipped SU(5) [IWK-K-Kyae, ph/0612365]

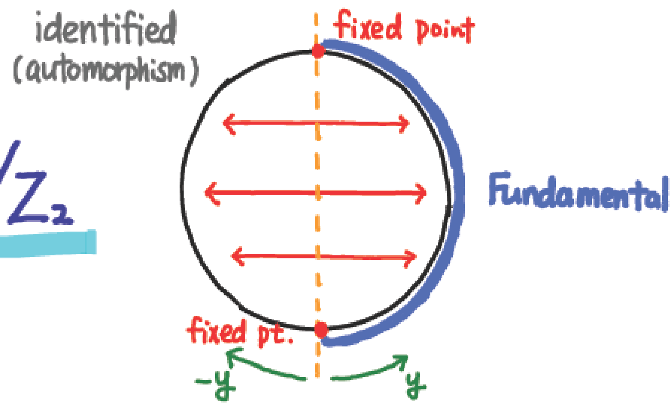
Effective R-parity in SM [K-JHKim-Kyae,ph/0702278, JHEP 0706, 034 (2007)]

Orbifold

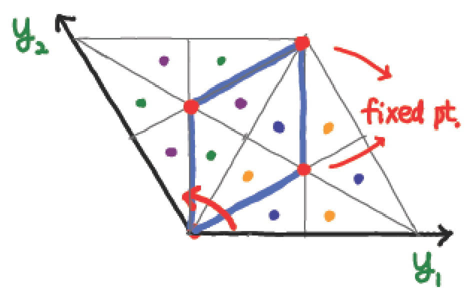
manifold / discrete sym.

ex)

S^1/Z_2



T^2/Z_3



g: $S^1/Z_2 \quad y \xrightarrow[\text{map}]{} -y$

$T^2/Z_3 \quad z \xrightarrow[\text{map}]{} e^{2\pi i/3} z$

$$S^1/Z_2 \begin{cases} V_y(y) = -V_y(-y) \\ V_\mu(y) = +V_\mu(-y) \end{cases}$$

$$T^2/Z_3 \begin{cases} V_z(z) = e^{2\pi i/3}\, V_z(e^{2\pi i/3} z) \\ V_\mu(z) = 1 \;\; V_z(e^{2\pi i/3} z) \end{cases}$$

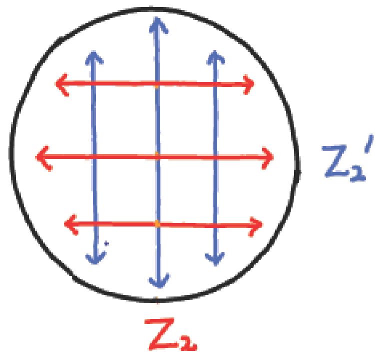
5D SUSY GUT

Kawamura (2000), Hall & Nomura (2001

$S^1/Z_2 \times Z_2'$

$\downarrow$ $Z_2 \times$ transl.

$g: (\theta, v)$



N=2 SUSY $\longrightarrow$ N=1 SUSY

SU(5) $\longrightarrow$ SM

$$24_{SU(5)} = \left(\begin{array}{c:c} SU(3)^{++}\ U(1)_Y & (3,2)^{+-}_{-5/6} \\ \hdashline (\bar{3},2)^{+-}_{5/6} & SU(2)^{++}\ U(1)_Y \end{array}\right)$$

Gauge

$\lambda_1^{SM\,++}$, $A_\mu^{SM\,++}$ — $N=2$ SUSY — $\lambda_2^{SM\,--}$, $A_y^{SM}, \phi^{SM\,--}$

$\lambda_1^{SU(5)/SM\,+-}$, $A_\mu^{SU(5)/SM\,+-}$ — $N=2$ SUSY — $\lambda_2^{SU(5)/SM\,-+}$, $A_y^{SU(5)/SM}, \phi^{SU(5)/SM\,-+}$

Hyper

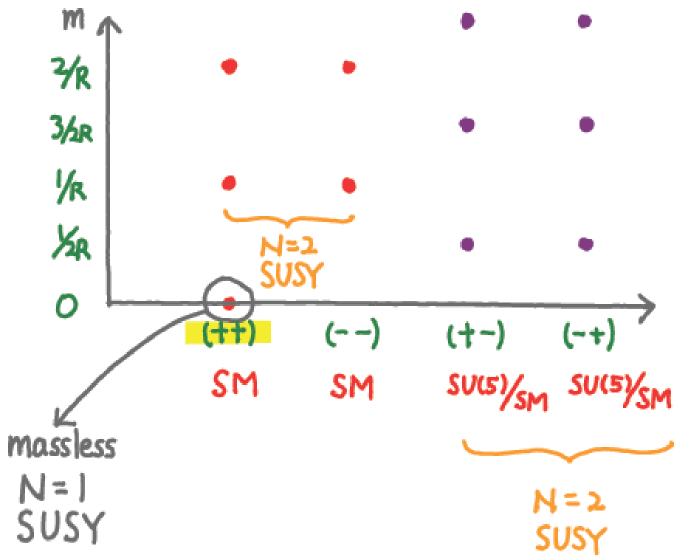
$$5 = \left(\begin{array}{c} 3^{+-} \\ \hdashline H_u^{++} \end{array}\right), \quad \bar{5} = \left(\begin{array}{c} \bar{3}^{+-} \\ \hdashline H_d^{++} \end{array}\right)$$

$\updownarrow N=2$ $\qquad$ $\updownarrow N=2$

$$5^c = \left(\begin{array}{c} 3^{c\,-+} \\ \hdashline H_u^{c\,--} \end{array}\right), \quad \bar{5}^c = \left(\begin{array}{c} \bar{3}^{c\,-+} \\ \hdashline H_d^{c\,--} \end{array}\right)$$

KK Tower

Field theory

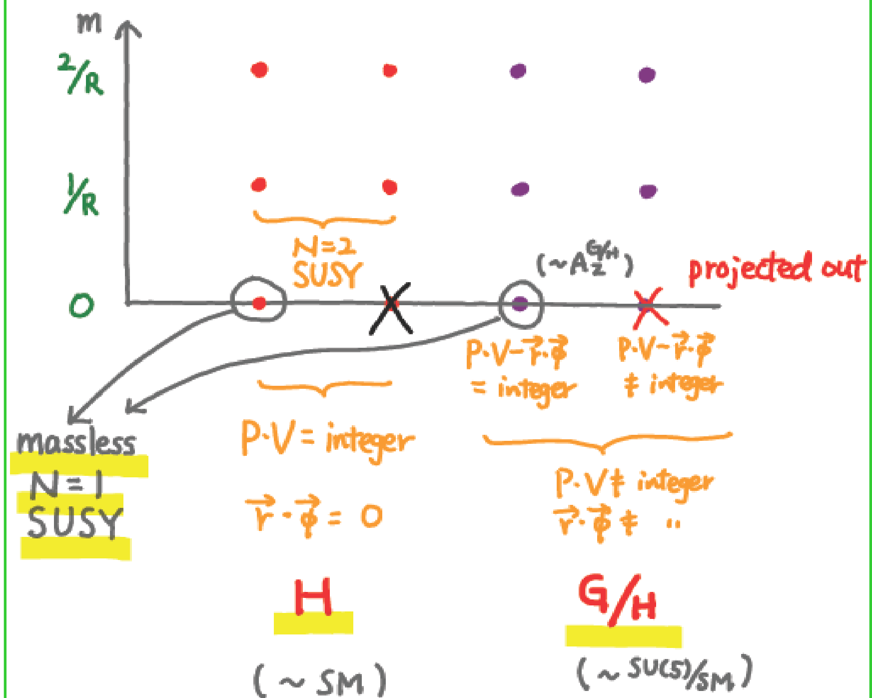


Only (++) modes contain massless states.

String

$G \longrightarrow H$ gauge sym. broken

$N=2$ SUSY $\longrightarrow$ $N=1$ SUSY



Field theory

String

Threshold Corr. by KK modes

N=1, SM

$$\frac{1}{g_i^2(\mu)} \approx \frac{1}{g_\Lambda^2} + b_i^{\circ} \ln\frac{\Lambda}{\mu} - (\overbrace{b_i^{++} + b_i^{--}}^{SM}) \ln\frac{\Lambda}{M_R}$$

$$+ \underbrace{(\underbrace{b_i^{++} + b_i^{--}}_{SM} + \underbrace{b_i^{+-} + b_i^{-+}}_{SU(5)/SM})}_{N=2,\, SU(5)} \left[\frac{\Lambda}{M_c} - 1\right]$$

linearly div.

not reliable

Still $\frac{1}{g_i^2(\mu)} - \frac{1}{g_j^2(\mu)} \sim \mathrm{Log}\frac{\Lambda}{\mu}$: reliable

Modula Inv.

$\cancel{\text{gauge sym.}}$ $\quad$ $\cancel{\text{SUSY}}$

$$N \times (V^2 - \phi^2) = \text{integer}$$

Gauge sym. must be broken.

Renormalized gauge coupling

$$\frac{16\pi^2}{g_H^2(\mu)} \approx \frac{16\pi^2}{g_*^2} + b_H^{\circ} \ln\frac{M_*^2}{\mu^2} - \frac{1}{4} \underset{\Downarrow\ (\text{Not } b_H^{N=2})}{b_G^{N=2}} \ln\frac{M_*^2}{M_R^2}$$

$$+ \frac{1}{4} b_G^{N=2} \left[\frac{2\pi}{\sqrt{3}} \frac{M_*^2}{M_c^2} - 2.19\right]$$

Wilson line shifts up (KK) masses.

(Wilson line breaking in string theory ~ orbifold breaking in field theory)

To study $N=2$ SUSY KK masses,
$\exists$ modulus (~ radius)

$\longrightarrow$ Non-prime orbifold

higher twist sectors include
sub-lattice inv. under g.

To discuss KK masses, GSO projection,
Wilson line eff., threshold corr.,
need to Partition Function.

$Z_{12\text{-}I}$ has the twist

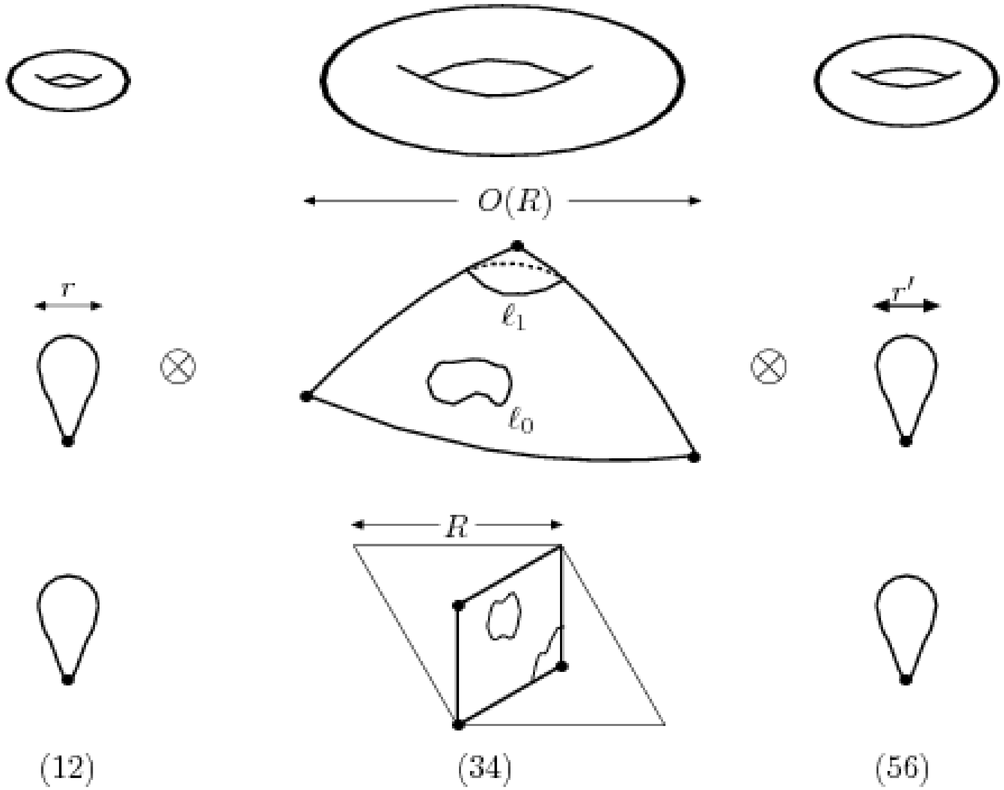
$$\phi = \left(\frac{5}{12}, \frac{4}{12}, \frac{1}{12}\right)$$

$\uparrow$

1/3

Z_3 has the twist

$$\phi = \left(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$$



$O(R)$
r
r'
ℓ_1
ℓ_0
R
(12)
(34)
(56)

- Wilson lines distinguish 3 fixed points. Only one (34)-torus and 3 fixed points for Z_{12-I} and 3x3x3=27 fixed points in Z_3. In the end, in terms of Wilson lines it is not as complicated as Z_3. The geometric discussion is simpler since we pay attention to the (34)-torus only. Much of breaking E_8 is directly done by *V* only, which is the reason *a* can be simple.

The modular invariance conditions to be satisfied are: (20)

$12(V^2-\phi^2)$=even integer, 8

$12a_3^2$=even integer, 4

$12V\cdot a_3$=integer, 0 in our case. $a_3=a_4$, others $a_1=\cdots=0$

Masslessness conditions are for k=0(U), 1, 2, ⋯ ,12

$$L-mover: \frac{(P+kV)^2}{2}+\sum_i N_i^L\tilde{\phi}_i-\tilde{c}_k=0$$

$$R-mover: \frac{(\vec{r}+k\vec{\phi})^2}{2}+\sum_i N_i^R\tilde{\phi}_i-c_k=0$$

$$(P+kV)\cdot a_3=0,\pm1,\pm2,\cdots,\quad k=0,3,6,9$$

Generalized GSO projection

$$P_k(f)=\frac{1}{12\cdot 3}\sum_{l=0}^{N-1}\tilde{\chi}(\theta_k,\theta_l)e^{2\pi i\Theta_f}$$

$$\Theta_f=\sum_i(N_i^L-N_i^R)\hat{\phi}_i-\frac{k}{2}(V_f^2-\phi^2)+(P+kV_f)\cdot V_f-(\vec{r}+k\vec{\phi})\cdot\vec{\phi}$$

$$V_f=V+m_f a_3$$

$$V=\frac{1}{12}(2,2,2,4,4,1,3,6)(3,3,3,3,3,1,1,1)'$$

$$a_3=\frac{1}{3}(0,0,0,0,0,0,0,0)(0,0,0,0,0,2,-1,-1)'$$

$$SU(4)\times SU(2)_W\times SU(2)_V\times SU(2)_n\times U(1)_a\times U(1)_b$$
$$\times SU(5)'\times SU(3)'\times U(1)'^2$$

$$SU(4):\begin{cases}\alpha_1= & (0\ 1\ -1\ 0\ 0\ ;\ 0\ 0\ 0)\\ \alpha_2= & (\frac12\ \frac{-1}{2}\ \frac12\ \frac12\ \frac12\ ;\ \frac{-1}{2}\ \frac{-1}{2}\ \frac{-1}{2})\\ \alpha_3= & (\frac12\ \frac{-1}{2}\ \frac{-1}{2}\ \frac{-1}{2}\ \frac{-1}{2}\ ;\ \frac12\ \frac12\ \frac12)\end{cases}$$

$$SU(2)_W:\alpha_W=\ (0\ 0\ 0\ 1\ -1;\ 0\ 0\ 0)$$

$$SU(2)_V:\alpha_V=\ (\frac12\ \frac12\ \frac12\ \frac12\ \frac12\ ;\ \frac12\ \frac12\ \frac12)$$

$$SU(2)_n:\alpha_n=\ (\frac12\ \frac12\ \frac12\ \frac{-1}{2}\ \frac{-1}{2}\ ;\ \frac{-1}{2}\ \frac{-1}{2}\ \frac12)\,.$$

Pati-Salam Classification
We use 4* (4) instead of 4 (4*)

SU(2)$_V$ is like SU(2)$_R$* in the Pati-Salam model. Rather than obtaining the SM directly, we go through the intermediate SU(4). This helps in making the classification job simple. The electroweak hypercharge contains the E$_8$' part,

$$Y = \tau_3 + Y_4 + Y'$$
$$Y_4 = diag.\left(\tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, -\tfrac{1}{2}\right) \quad for \quad 3$$
$$Y_4 = diag.\left(-\tfrac{1}{6}, -\tfrac{1}{6}, -\tfrac{1}{6}, \tfrac{1}{2}\right) \quad for \quad \overline{3}$$
$$\tau_3 \quad is \quad SU(2)_V \quad generator$$

$$Y' = (0^8)\left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, 0, 0, 0\right)'$$

Matter in (SU(4), $SU(2)_W$,$SU(2)_V)_{Y'}$ notation

$$
\begin{aligned}
&U_1: \ \underline{(\overline{\mathbf{4}},\mathbf{2},\mathbf{1})_0}, \ 2(\mathbf{6},\mathbf{1},\mathbf{1})_0 \\
&U_2: \ 2(\mathbf{4},\mathbf{1},\mathbf{2})_0, \ (\mathbf{6},\mathbf{1},\mathbf{1})_0 \\
&U_3: \ (\mathbf{4},\mathbf{1},\mathbf{2})_0, \ 2(\mathbf{1},\mathbf{2},\mathbf{2})_0, \ (\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2};\mathbf{1},\mathbf{1})_0 \\
&T_{1_0}: \ (\overline{\mathbf{4}},\mathbf{1},\mathbf{1})_{1/2}, \ (\mathbf{1},\mathbf{2},\mathbf{1})_{1/2}, \ (\mathbf{1},\mathbf{1},\mathbf{2})_{1/2} \\
&T_{1_+}: \ (\mathbf{1},\mathbf{2},\mathbf{1})_{-1/2}, \ (\mathbf{1},\mathbf{1},\mathbf{2})_{-1/2} \\
&T_{1_-}: \ (\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{1};\mathbf{5}';\mathbf{1})_{-1/10} \\
&T_{2_0}: \ (\mathbf{6},\mathbf{1},\mathbf{1})_0, \ \mathbf{2}^n_0, \ \mathbf{1}_0 \\
&T_{2_+}: \ \mathbf{5}'_{2/5}, \ \overline{\mathbf{3}}'_0, \\
&T_{2_-}: \ (\mathbf{1},\mathbf{2},\mathbf{2})_0, \ \mathbf{3}'_0, \ \mathbf{2}^n_0, \ 2\cdot\mathbf{1}_0 \\
&T_3: \ (\overline{\mathbf{4}},\mathbf{1},\mathbf{1})_{1/2}, \ (\mathbf{4},\mathbf{1},\mathbf{1})_{-1/2}, \ (\mathbf{4},\mathbf{1},\mathbf{1})_{1/2}, \ 2(\overline{\mathbf{4}},\mathbf{1},\mathbf{1})_{-1/2}, \ 3(\mathbf{1},\mathbf{2},\mathbf{1})_{1/2}, \\
&\qquad 2(\mathbf{1},\mathbf{2},\mathbf{1})_{-1/2}, \ 2(\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{2};\mathbf{1};\mathbf{1})_{1/2}, \ (\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{2};\mathbf{1};\mathbf{1})_{-1/2}, \\
&\qquad (\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1};\mathbf{5}';\mathbf{1})_{-1/10}, \ 2\cdot(\mathbf{1},\mathbf{2},\mathbf{1};\mathbf{1};\overline{\mathbf{5}}';\mathbf{1})_{1/10} \\
&T_{4_0}: \ 2(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2};\mathbf{1};\overline{\mathbf{3}}')_0, \ 2\cdot\overline{\mathbf{3}}'_0 \\
&T_{4_+}: \ 2\underline{(\overline{\mathbf{4}},\mathbf{2},\mathbf{1})_0}, \ 2(\mathbf{4},\mathbf{1},\mathbf{2})_0, \ 2(\mathbf{6},\mathbf{1},\mathbf{1})_0, \ 7\cdot\mathbf{2}^n_0, \ 9\cdot\mathbf{1}_0 \\
&T_{4_-}: \ 2(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{2};\mathbf{1};\mathbf{3}')_0, \ 2\cdot\mathbf{3}'_0 \\
&T_{7_+}: \ (\overline{\mathbf{4}},\mathbf{1},\mathbf{1})_{1/2}, \ (\mathbf{1},\mathbf{1},\mathbf{2})_{1/2} \\
&T_{7_-}: \ (\overline{\mathbf{4}},\mathbf{1},\mathbf{1})_{-1/2}, \ (\mathbf{1},\mathbf{1},\mathbf{2};\mathbf{2};\mathbf{1};\mathbf{1})_{-1/2}, \ (\mathbf{1},\mathbf{1},\mathbf{2})_{-1/2} \\
&T_6: \ 6\cdot\overline{\mathbf{5}}'_{-2/5}, \ 5\cdot\mathbf{5}'_{2/5},
\end{aligned}
$$

This model does not have exotics.

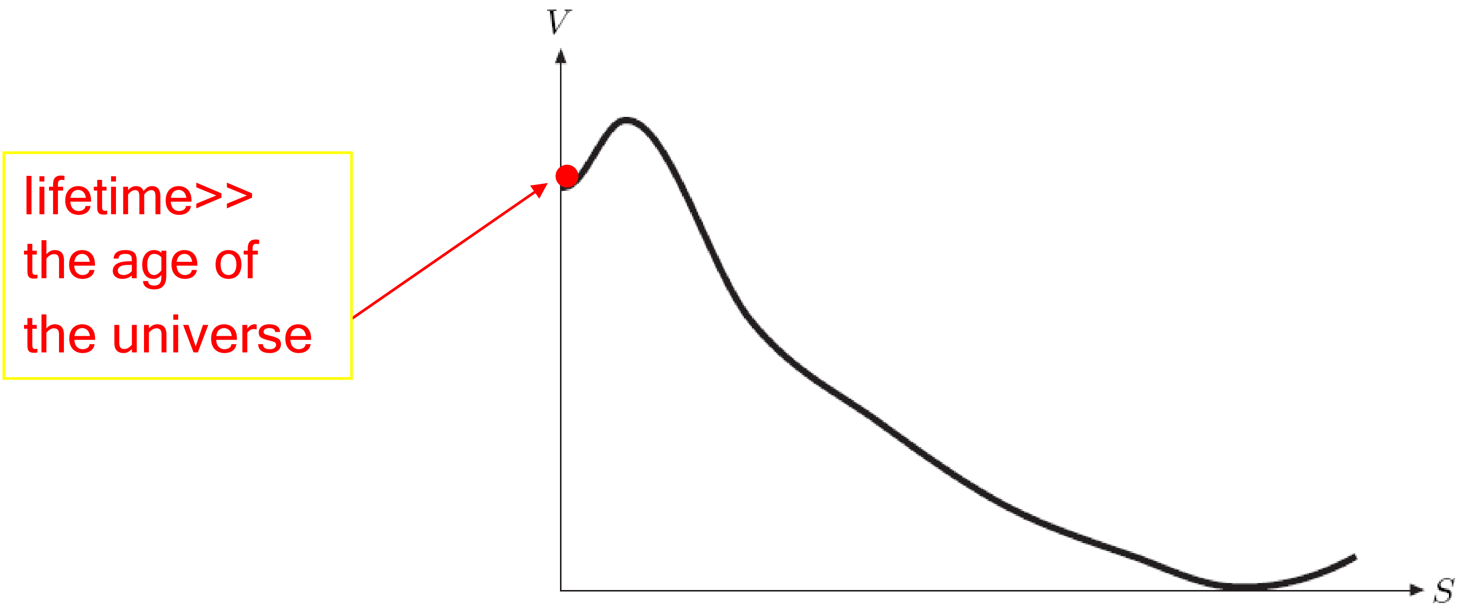
The set with SU(5)' singlet fields is anomaly free in the SM gauge group.

SU(5)' Hidden-sector

But nontrivial under SM group

Hidden sector SU(5)' quarks: N_f=10

$P+n[V\pm a]$	χ	No.$\times$(Repts.)$_{Y,Q_1,Q_2}$
$(\underline{\tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{12}\ \tfrac{1}{4}\ \tfrac{1}{2}})(\underline{\tfrac{3}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4})'_{T1_-}$	L	$(\mathbf{1},\mathbf{1},\mathbf{2};1;\mathbf{5}',1)^L_{-1/10,-1/6,-4/3}$
$(\tfrac{-1}{6}\ \tfrac{-1}{6}\ \tfrac{-1}{6}\ \ \tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{-1}{3}\ 0\ \tfrac{1}{2})(\underline{1\ 0\ 0\ 0\ 0}\ 0\ 0\ 0)'_{T2_+}$	L	$(\mathbf{1},\mathbf{1},\mathbf{1};1;\mathbf{5}',1)^L_{2/5,-1/3,-8/3}$
$(0\ 0\ 0\ \underline{\tfrac{1}{2}\ \tfrac{-1}{2}}\ \tfrac{-1}{4}\ \tfrac{1}{4}\ 0)(\underline{\tfrac{3}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4})'_{T3}$	L	$(\mathbf{1},\boxed{\mathbf{2}},\mathbf{1};1;\mathbf{5}',1)^L_{-1/10,-1/2,0}$
$(0\ 0\ 0\ \underline{\tfrac{1}{2}\ \tfrac{-1}{2}}\ \tfrac{1}{4}\ \tfrac{-1}{4}\ 0)(\underline{\tfrac{-3}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4})'_{T9}$	L	$2(\mathbf{1},\boxed{\mathbf{2}},\mathbf{1};1;\overline{\mathbf{5}}',1)^L_{1/10,1/2,0}$
$(0\ 0\ 0\ 0\ 0\ \tfrac{-1}{2}\ \tfrac{1}{2}\ 0)(\underline{-1\ 0\ 0\ 0\ 0}\ 0\ 0\ 0)'_{T6}$	L	$4(\mathbf{1},\mathbf{1},\mathbf{1};1;\overline{\mathbf{5}}',1)^L_{-2/5,-1,0}$
$(0\ 0\ 0\ 0\ 0\ \tfrac{-1}{2}\ \tfrac{1}{2}\ 0)(\underline{1\ 0\ 0\ 0\ 0}\ 0\ 0\ 0)'_{T6}$	L	$2(\mathbf{1},\mathbf{1},\mathbf{1};1;\mathbf{5}',1)^L_{2/5,-1,0}$
$(0\ 0\ 0\ 0\ 0\ \tfrac{1}{2}\ \tfrac{-1}{2}\ 0)(\underline{-1\ 0\ 0\ 0\ 0}\ 0\ 0\ 0)'_{T6}$	L	$2(\mathbf{1},\mathbf{1},\mathbf{1};1;\overline{\mathbf{5}}',1)^L_{-2/5,1,0}$
$(0\ 0\ 0\ 0\ 0\ \tfrac{1}{2}\ \tfrac{-1}{2}\ 0)(\underline{1\ 0\ 0\ 0\ 0}\ 0\ 0\ 0)'_{T6}$	L	$3(\mathbf{1},\mathbf{1},\mathbf{1};1;\mathbf{5}',1)^L_{2/5,1,0}$



So, the θ^0 component VEVs of 5-5* condensate mesons is almost zero and $SU(2)_W$ is not broken at the SUSY breaking scale by θ^0 component VEVs. But θ^2 components are large and carry $SU(2)_W$ quantum numbers. So, our model, even though very attractive, is breaking SM at the SUSY breaking scale (by meson F-term and baryon VEVs) and not working as a realistic model. [Planck-07, Warsaw]

Another Z_{12-I} Model

This model is very interesting in

- 3 families
- No exotics
- One pair of Higgs doublets
- GMSB at a stable vacuum
- But $\sin^2\Theta_W \neq 3/8$

The shift vector and Wilson line is taken as

$V = (1/12)(6\ 6\ 6\ 2\ 2\ 2\ 3\ 3)(3\ 3\ 3\ 3\ 3\ 1\ 1\ 1)'$

$a_3 = (1/12)(1\ 1\ 2\ 0\ 0\ 0\ 0\ 0)(0\ 0\ 0\ 0\ 0\ 1\ 1\ {-2})'$

Gauge group is

$SU(3)_c \times SU(3)_W \times SU(5)' \times SU(3)' \times U(1)s$

Lee-Weinberg electroweak model and no exotics

$$\Gamma = \frac{1}{3}Q_2 + Q_3 + W_8, \quad Lee-Weinberg \quad SU(3)_W$$

Note that $U(1)_\Gamma$ charges of SM fermions are odd and Higgs doublets are even. Extra vectorlike doublets are given superheavy masses. By breaking by VEVs of even Γ singlets, we break $U(1)_\Gamma$ to a discrete matter parity P. R-parity is achieved here. [JEK, plb 656, 207 (2007) [arXiv:0707.3292]

$P+[kV+ka]$	No.$\times$(Repts.)$_{Y\,[Q_1,Q_2,Q_3,Q_4,Q_5]}$	Γ	Label
$(\underline{\tfrac{-1}{3}\ \tfrac{-1}{3}\ \tfrac{-2}{3}}\ \underline{\tfrac{2}{3}\ \tfrac{-1}{3}}\ \tfrac{-1}{3}\ 0\ 0)(0^8)'_{T_{4_-}}$	$3\cdot(\mathbf{3},\mathbf{2})^L_{1/6\ [0,0,0;0,0]}$	1	$q_1,\ q_2,\ q_3$
$(\underline{\tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{5}{6}}\ \tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{1}{2}\ \tfrac{1}{2})(0^8)'_{T_{4_-}}$	$2\cdot(\bar{\mathbf{3}},\mathbf{1})^L_{-2/3\ [-3,3,2;0,0]}$	3	$u^c,\ c^c$
$(\underline{\tfrac{-1}{3}\ \tfrac{-1}{3}\ \tfrac{-2}{3}}\ \tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{-1}{4}\ \tfrac{-1}{4})(\tfrac{1}{4}^5\ \tfrac{1}{12}\ \tfrac{1}{12}\ \tfrac{1}{12})'_{T_{7_+}}$	$(\bar{\mathbf{3}},\mathbf{1})^L_{-2/3\ [0,6,-1;5,1]}$	1	t^c
$(\underline{\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}}\ \tfrac{-1}{6}\ \tfrac{-1}{6}\ \tfrac{-1}{6}\ 0\ 0)(0^5\ \tfrac{-1}{3}\ \tfrac{-1}{3}\ \tfrac{-1}{3})'_{T_{2_0}}$	$(\bar{\mathbf{3}},\mathbf{1})^L_{1/3\ [3,-3,0;0,-4]}$	-1	d^c
$(\underline{\tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{5}{6}}\ \tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{-1}{2}\ \tfrac{-1}{2})(0^8)'_{T_{4_-}}$	$2\cdot(\bar{\mathbf{3}},\mathbf{1})^L_{1/3\ [-3,3,-2;0,0]}$	1	$s^c,\ b^c$
$(\tfrac{-1}{3}\ \tfrac{-1}{3}\ \tfrac{1}{3}\ \underline{\tfrac{2}{3}\ \tfrac{-1}{3}}\ \tfrac{2}{3}\ 0\ 0)(0^8)'_{T_{4_-}}$	$(\mathbf{1},\mathbf{2})^L_{-1/2\ [-6,6,0;0,0]}$	1	l_1,l_2,l_3
$(0\ 0\ 0\ \underline{\tfrac{2}{3}\ \tfrac{-1}{3}}\ \tfrac{2}{3}\ \tfrac{-1}{4}\ \tfrac{-1}{4})(\tfrac{1}{4}^5\ \tfrac{1}{12}\ \tfrac{1}{12}\ \tfrac{1}{12})'_{T_{1_0}}$	$(\mathbf{1},\mathbf{2})^L_{1/2\ [0,6,-1;5,1]}$	0	H_u
$(\tfrac{-1}{3}\ \tfrac{-1}{3}\ \tfrac{1}{3}\ \underline{\tfrac{1}{3}\ \tfrac{-2}{3}}\ \tfrac{1}{3}\ \tfrac{-1}{4}\ \tfrac{-1}{4})(\tfrac{1}{4}^5\ \tfrac{1}{12}\ \tfrac{1}{12}\ \tfrac{1}{12})'_{T_{7_+}}$	$(\mathbf{1},\mathbf{2})^L_{-1/2\ [-6,0,-1;5,1]}$	-2	H_d

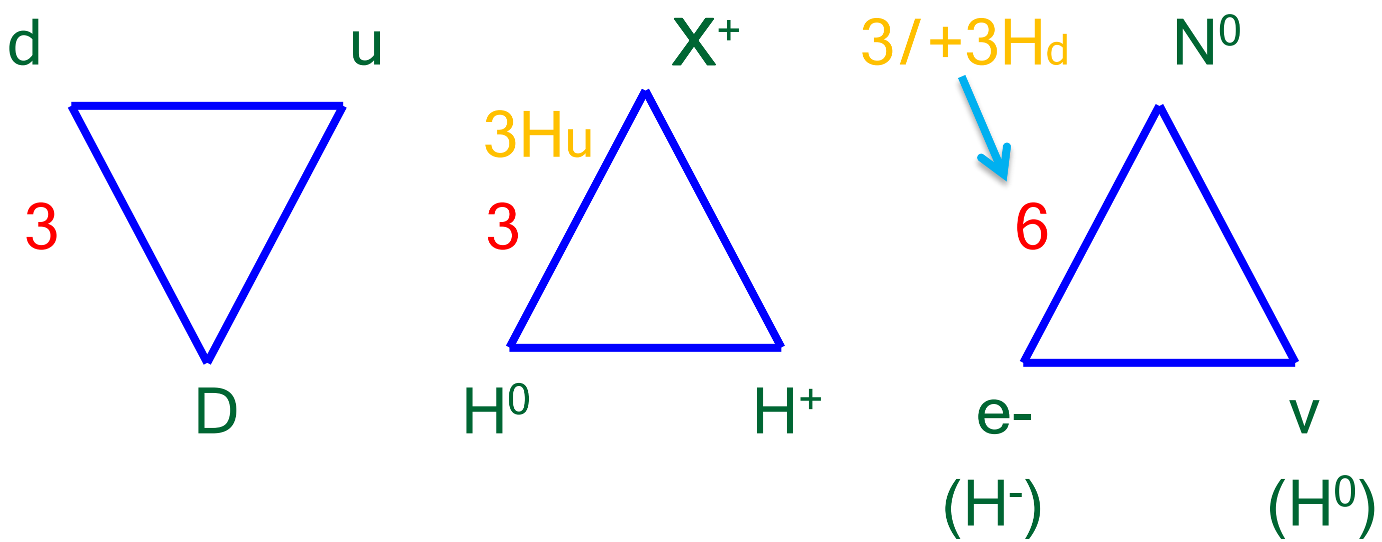
The SM spectrum

Three quark families appear as

$3\,(3_c, 3_W)$

At low energy, we must have nine 3^*_W

to cancel $SU(3)_W$ anomaly.



Remain three pairs of $3^*_W(H^+)$ and $3^*_W(H^-)$ after 3^*_W(lepton)

Both H_u and H_d appear from 3*. It is in contrast to the other cases such as in SU(5) or SO(10). The H_u and H_d coupling must come from

$$3^*_W\ 3^*_W\ 3^*_W \text{ coupling.}$$

Thus, there appears the Levi-Civita symbol and two epsilons are appearing, in $SU(3)_W$ space, a, b, c and in flavor space, I, J, ….

Therefore, in the flavor space the H_u-H_d mass matrix is antisymmetric and hence its determinant is zero.

One pair of Higgs doublets is massless:

MSSM problem resolved.

It is interesting to compare

Introduction of color:

56 of old SU(6) in 1960s = completely symm.

But spin-half quarks are fermions leads to

antisymmetric index= SU(3) color [Han-Nambu]

Introduction of flavor in the Higgs sector:

Lee-Weinberg SU(3)-weak gives

3*-3*-3* SU(3)-weak singlet = antisymmetric gives

antisymmetric bosonic flavor symmetry (SUSY)!

$P+n[V\pm a]$	Γ	$\mathrm{No.}\times(\mathrm{Repts.})_{Y[Q_1,Q_2,Q_3,Q_4,Q_5]}$
$(\tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{-1}{6}\ \tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{1}{4}\ \tfrac{1}{4})(\underline{\tfrac{-3}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4})'_{T1_-}$	2	$(1;\overline{5}',1)^L_0\ [3,3,1;1,-1]$
$(\tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{-1}{6}\ \tfrac{-1}{6}\ \tfrac{-1}{6}\ \tfrac{-1}{6}\ 0\ 0)(\underline{\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{-1}{2}\ \tfrac{-1}{2}\ \tfrac{-1}{2}}\ \tfrac{-1}{6}\ \tfrac{-1}{6}\ \tfrac{-1}{6})'_{T2_+}$	-1	$\boxed{\star\ (1;10',1)^L_0\ [3,-3,0;-2,-2]}$
$(0^6\ \underline{\tfrac{1}{4}\ \tfrac{-3}{4}})(\underline{\tfrac{3}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4})'_{T3}$	-1	$(2_n;5',1)^L_0\ [0,0,-1;-1,3]$
$(0^6\ \underline{\tfrac{3}{4}\ \tfrac{-1}{4}})(\underline{\tfrac{-3}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4})'_{T9}$	1	$(2_n;\overline{5}',1)^L_0\ [0,0,1;1,-3]$
$(0^3\ \tfrac{-1}{3}\ \tfrac{-1}{3}\ \tfrac{-1}{3}\ \tfrac{1}{4}\ \tfrac{1}{4})(\underline{\tfrac{-3}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}}\ \tfrac{1}{12}\ \tfrac{1}{12}\ \tfrac{1}{12})'_{T7_0}$	-1	$\boxed{\star\ (1;\overline{5}',1)^L_0\ [0,-6,1;1,1]}$
$(\tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{-1}{6}\ \tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{1}{6}\ \tfrac{-1}{4}\ \tfrac{-1}{4})(\underline{\tfrac{3}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}\ \tfrac{-1}{4}}\ \tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4})'_{T7_-}$	0	$(1;5',1)^L_0\ [3,3,-1;-1,3]$
$(0^6\ \tfrac{-1}{2}\ \tfrac{-1}{2})(\underline{-1\ 0\ 0\ 0\ 0}\ 0\ 0\ 0)'_{T6}$	-2	$3\cdot(1;\overline{5}',1)^L_0\ [0,0,-2;-4,0]$
$(0^6\ \tfrac{-1}{2}\ \tfrac{-1}{2})(\underline{1\ 0\ 0\ 0\ 0}\ 0\ 0\ 0)'_{T6}$	-2	$2\cdot(1;5',1)^L_1\ [0,0,-2;4,0]$
$(0^6\ \tfrac{1}{2}\ \tfrac{1}{2})(\underline{-1\ 0\ 0\ 0\ 0}\ 0\ 0\ 0)'_{T6}$	2	$2\cdot(1;\overline{5}',1)^L_{-1}\ [0,0,2;-4,0]$
$(0^6\ \tfrac{1}{2}\ \tfrac{1}{2})(\underline{1\ 0\ 0\ 0\ 0}\ 0\ 0\ 0)'_{T6}$	2	$3\cdot(1;5,1)^L_0\ [0,0,2;4,0]$

The SU(5)' spectrum

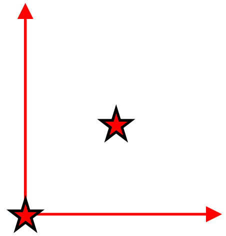
The chiral representation 10+5* remain. It is known that SUSY is broken dynamically [Veneziano,….].

Murayama-Nomura, PRD:

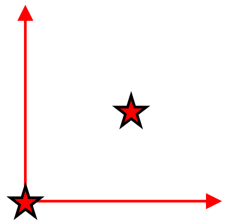
$$L = \int d^2\theta \left[\frac{1}{M^2} \bar{f} f W'^{\alpha} W'_{\alpha} + M_f \bar{f} f \right] + h.c.$$

M is the parameter, presumably above 10^{12} GeV. D-type quarks can be colored messengers.

$$T_3\,\mathit{sector} \quad : \qquad 3\phi = \left(\frac{1}{4}, 0, \frac{1}{4}\right)$$

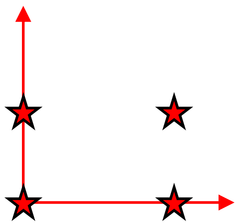


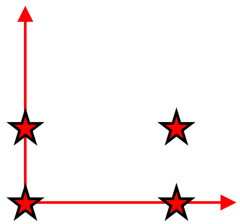
(12) torus



(56) torus

$$T_6\,\mathit{sector} \quad : \qquad 6\phi = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$$





For example, in T_6 we may consider S_4 symmetry. The Yukawa couplings must respect this kind of discrete symmetry. Can be used to obtain nonabelian discrete symmetries.

6. Threshold correction

In string compactification, the threshold correction comes from non-prime orbifolds. A general form was discussed before. The non-prime orbifolds have a substructure where a large radius R can be introduced. The simplest case is for Z_3 substructure. Namely, $Z_{6\text{-I}}$ or $Z_{12\text{-I}}$, and $Z_{12\text{-I}}$ has phenomenologically interesting models.

As an example, we use

$$\phi = \left(\frac{5}{12}\ \frac{4}{12}\ \frac{1}{12}\right)$$

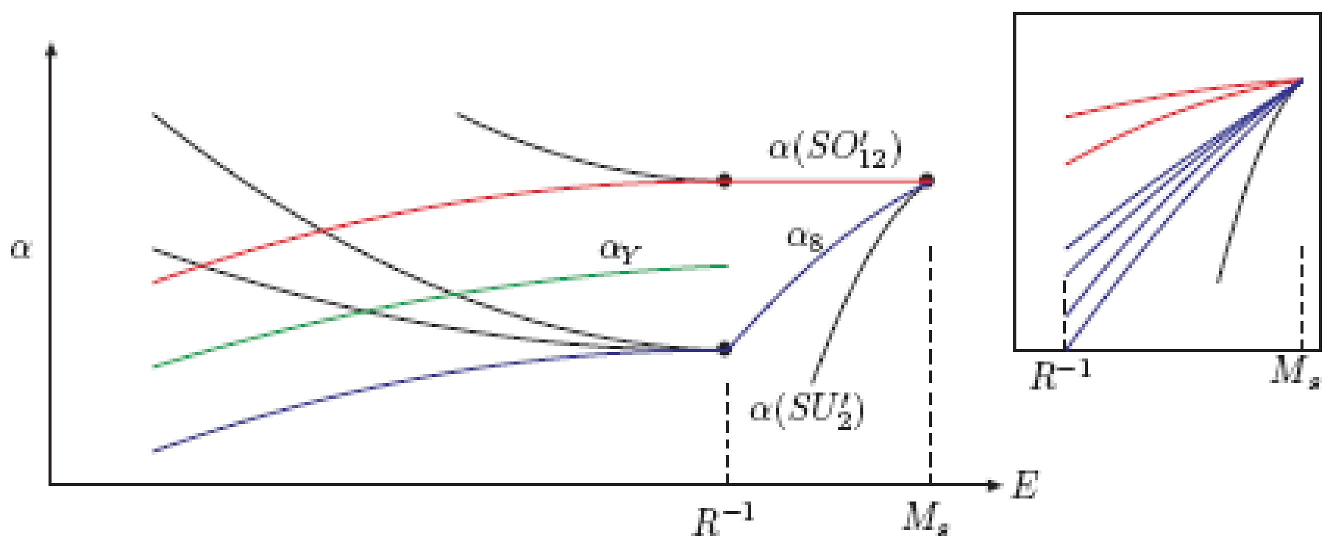
$$V = \left(\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ \frac{1}{4}\ ;\frac{5}{12}\ \frac{5}{12}\ \frac{1}{12}\right)\left(\frac{1}{4}\ \frac{3}{4}\ 0\ ;0^5\right)'$$

$$a_3 = \left(\frac{2}{3}\ \frac{2}{3}\ \frac{2}{3}\ \frac{-2}{3}\ \frac{-2}{3}\ ;\frac{2}{3}\ 0\ \frac{2}{3}\right)\left(0\ \frac{2}{3}\ \frac{2}{3}\ ;0^5\right)'.$$

J.-H. Kim, JEK, B. Kyae, JHEP 0706, 034 (2007): 0702.278

The 4D gauge group is

$$SU(3)\times SU(2)\times U(1)_Y\times U(1)^4$$
$$\times SO(10)'\times U(1)'^3$$



We use partition function approach. Dixon-Kaplunovsky-Louis developed the threshold correction with the shift vector V. We generalized it to include the Wilson lines. The simplest nontrivial example for using this is for the case of $Z_{12\text{-}I}$.

- We obtained the R dependence [(34)-torus]

$$\frac{4\pi}{\alpha_{H_i}(\mu)} = \frac{4\pi}{\alpha_*} + b^0_{H_i}\log\frac{M_*^2}{\mu^2} - \frac{b_H}{\mu^2}\left[\log\frac{R^2}{\alpha'} + 1.89\right] + \frac{(b_H + b_{G/H})}{4}\left[\frac{2\pi R^2}{\sqrt{3}\alpha'} - 0.30\right]$$

reliable value for coupling constant at scale μ

But extradimensional field theory cannot calculate constant and R2 term. On the other hand, we have a definite prediction for these terms.

We can obtain 6D field theory by compactifying 4 internal spaces. This is another check of our partition function approach. Between R and string scale, the contribution to beta function coefficient is given by bH :
the corresponding group may not be the SM group.

$$\Delta_i = \frac{|G'|}{|G|} b_i^{N=2} \int_\Gamma \frac{d^2\tau}{\tau_2} (\hat{Z}_{torus}(\tau,\bar{\tau}) - 1)$$

Integration in the modular space along the above formula a la Dixon-Kaplunovsky-Louis gives the compactification size dependence.

We use the modular parameter with the metric,

$$\begin{cases} \vec{e}_1 = (\sqrt{2}, 0) \\ \vec{e}_2 = (-\sqrt{1/2}, \sqrt{3/2}) \end{cases}; \qquad g_{ab} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\begin{cases} \vec{e}^{*1} = (1/\sqrt{2}, \sqrt{1/6}) \\ \vec{e}^{*2} = (0,, \sqrt{2/3}) \end{cases}; \qquad g^{ab} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

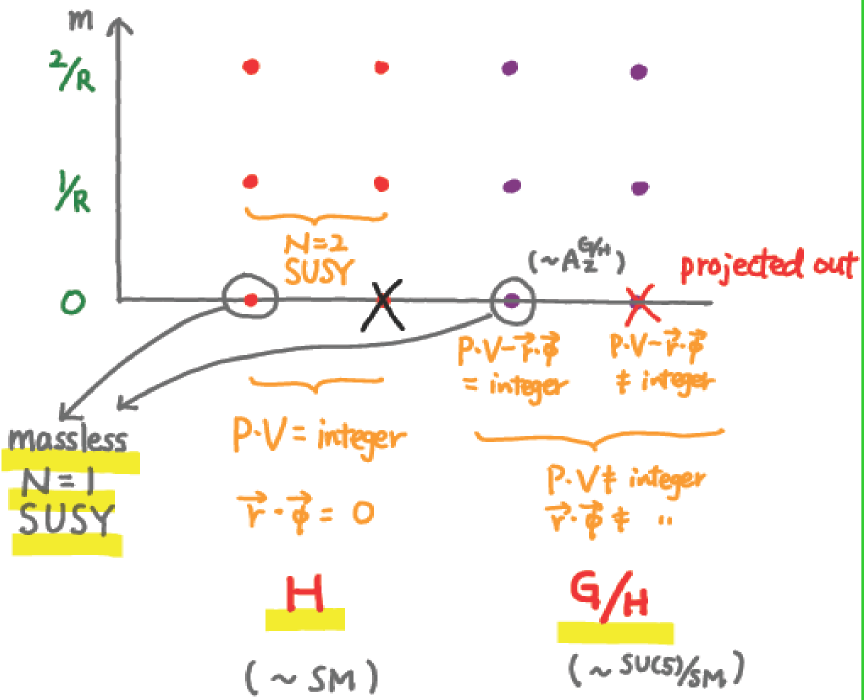
and obtain the R dependence,

$$\frac{4\pi}{\alpha_{H_i}(\mu)} = \frac{4\pi}{\alpha_*} + b_{H_i}^0 \log\frac{M_*^2}{\mu^2} - \frac{b_H}{\mu^2}\left[\log\frac{R^2}{\alpha'} + 1.89\right] + \frac{(b_H + b_{G/H})}{4}\left[\frac{2\pi R^2}{\sqrt{3}\alpha'} - 0.30\right]$$

It is a reliable calculation, not like the expressions written in extra dimensional field theory [Dienes-Dudas-Ghergetta]. R-squared and constant terms are reliable, and predicts how gauge couplings behave above the so-called GUT scale.

$G \longrightarrow H$ gauge sym. broken

$N=2$ SUSY $\longrightarrow$ $N=1$ SUSY



Modula Inv.

gauge ~~sym.~~ ↖ ↗ ~~SUSY~~

$$N \times (V^2 - \phi^2) = \text{integer}$$

Gauge sym. must be broken.

Renormalized gauge coupling

$$\frac{16\pi^2}{g_H^2(\mu)} \approx \frac{16\pi^2}{g_*^2} + b_H^0 \ln\frac{M_*^2}{\mu^2} - \frac{1}{4} b_G^{N=2} \ln\frac{M_*^2}{M_R^2}$$

⇓ (Not $b_H^{N=2}$)

$$+ \frac{1}{4} b_G^{N=2}\left[\frac{2\pi}{\sqrt{3}}\frac{M_*^2}{M_c^2} - 2.19\right]$$

$$N=2: \quad b_{H+G/H} = 24, \quad 0, \quad 48$$

SU(8) SO(12)' SU(2)'

$$N=2: \quad b_{H} = 13, \quad 11, \quad 89/5, \quad -2, \quad 5$$

SU(3) SU(4) U(1)e SO(10)' U(1)c'

$$N=1: \quad b_{H\,0} = -3, \quad 1, \quad 33/5$$

SU(3) SU(4) U(1)y

Actually, we need singlet Higgs VEVs to give large masses for exotic particles. Since SU(4) gives complcated form for its U(1) subgroup, we break SU(4) by VEVs of singlets. So, we consider only SU(2) of broken SU(4) and conside the N=2 bi in terms of another parameter hi,

$$b_i = h_i \left(\log \frac{M_s^2}{M_R^2} + 1.89 \right)$$

The hypercharge definition must be made judiciously to avoid chiral exotics or even to remove all exotics.

$$\text{Model E}: \quad Y = \left(\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{-1}{2}\,\frac{-1}{2}\,;0\,0\,0\right)\left(0\,0\,0;\,0\,0\,0\,0\,0\right)', \qquad \sin^2\theta_W=3/8$$

$$\text{Model S}: \quad \tilde{Y} = \left(\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3}\,\frac{-1}{2}\,\frac{-1}{2}\,;0\,0\,0\right)\left(0\,0\,1;\,0\,0\,0\,0\,0\right)', \qquad \sin^2\theta_W=3/14$$

$$Model\,E: \quad 34.78 \le (7h_3 - 12h_2 + 5h_1) \le 98.02$$

$$\sin^2\theta_W(M_Z) = 0.22306 \pm 0.00033, \qquad \alpha_3 = 0.1216 \pm 0.0017$$

$$Model\,S: \quad \frac{M_R}{M_Z} \approx 1.70\times 10^{15}, \qquad \frac{M_s}{M_R} \approx 3.68$$

6. Conclusion

We saw interesting explanations of SUGRA problems by orbifold compactification. Have pursued the GMSB possibility in orbifold compactification with desirable MSSM spectrum. Also, 6D SUSY GUT realized with KK mass dependent threshold corrections. These corrections are reliable unlike in extra-dimensional field theory. In some models,

- 3 families with no exotics. The GMSB with SU(5)' 10+5* at a stable vacuum. The MSSM spectrum: just one pair of Higgs doublets. R-parity embedding successful.

- Gauge coupling unification: nonprime orbifold with KK mass with R dep.

$$\frac{4\pi}{\alpha_{H_i}(\mu)} = \frac{4\pi}{\alpha_*} + b^0_{H_i}\log\frac{M_*^2}{\mu^2} - \frac{b_H}{\mu^2}\left[\log\frac{R^2}{\alpha'} + 1.89\right] + \frac{(b_H + b_{G/H})}{4}\left[\frac{2\pi R^2}{\sqrt{3}\alpha'} - 0.30\right]$$

- The orbifold compactification of $E_8 x E_8$ heterotic string gives enough good phenomenologies, not competed in other superstrings. Yet, to resolve the moduli stabil.