Three family GUT-like models from heterotic string

Based on arXiv:0707.3355 [hep-th]

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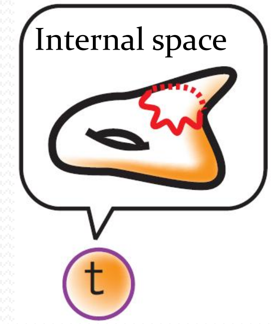
Introduction

- How can we realize the Standard Model as a low energy effective theory of string theory?
- As a perturbative approach, we assume the compactification: 4D + 6D compact space

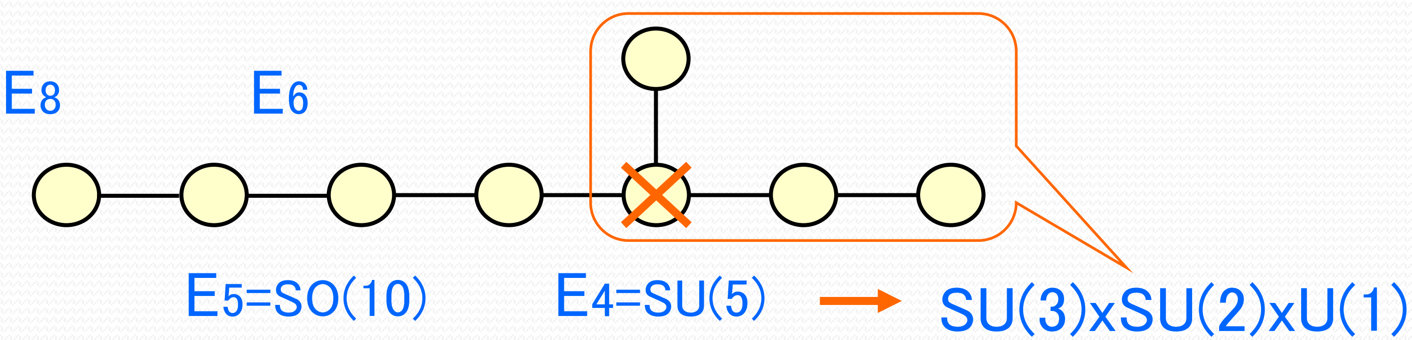


- Several approaches are proposed so far:

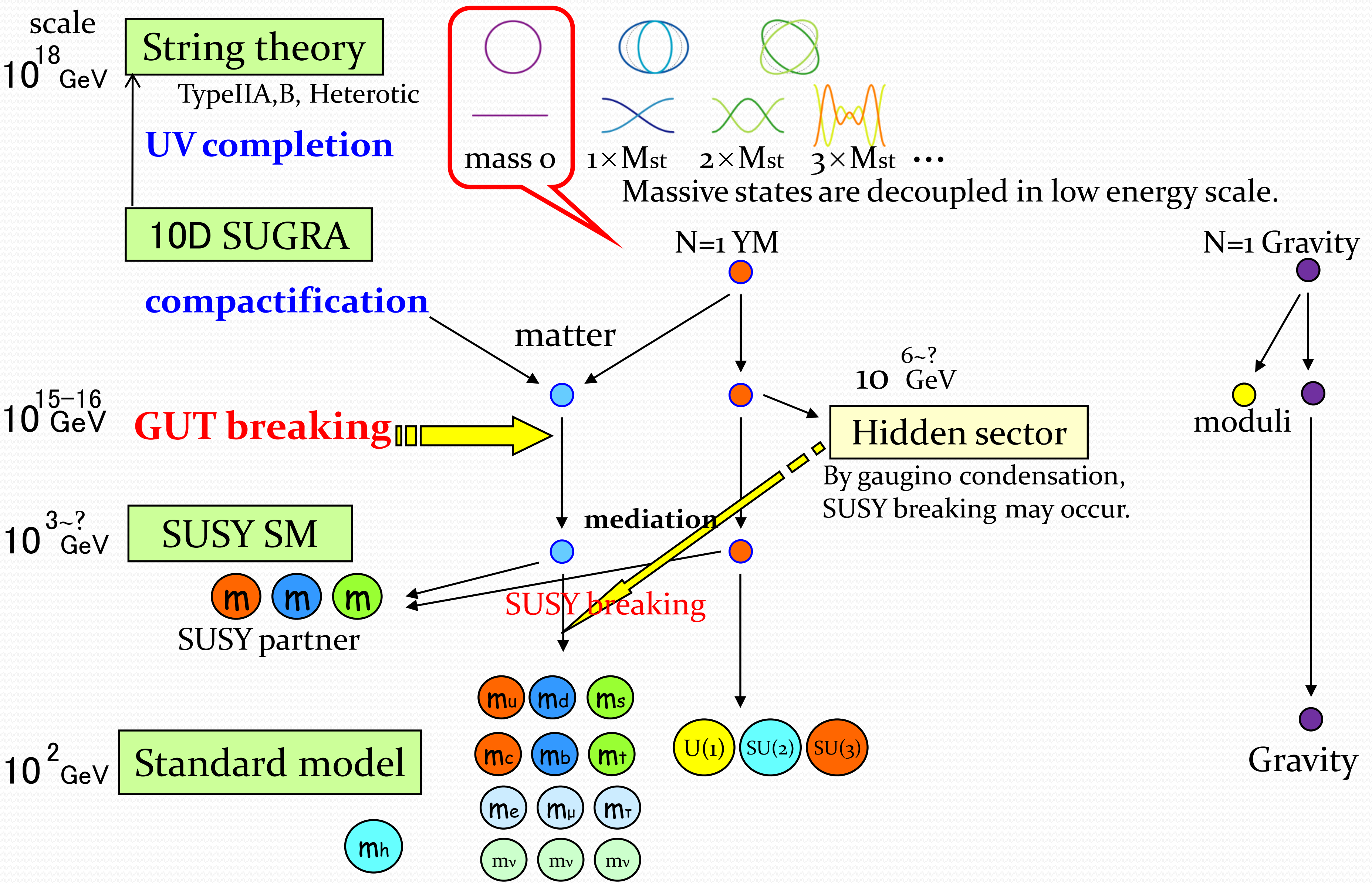
  Heterotic string → Calabi-Yau, orbifold…

  Type II string → Intersecting D-brane, flux,…

- GUT and SM gauge groups are naturally included in E_8.



A standard-like story to the Standard Model



Why orbifold?

- N=1 supersymmetry:

Torus compactification gives N=4 model.

N=2 and 4 is not chiral.

Requirement of naturalness (for higgs mass), but is not necessary.

Stability of compact space (no tadpole, no tachyon).

- Appearance of "matter":

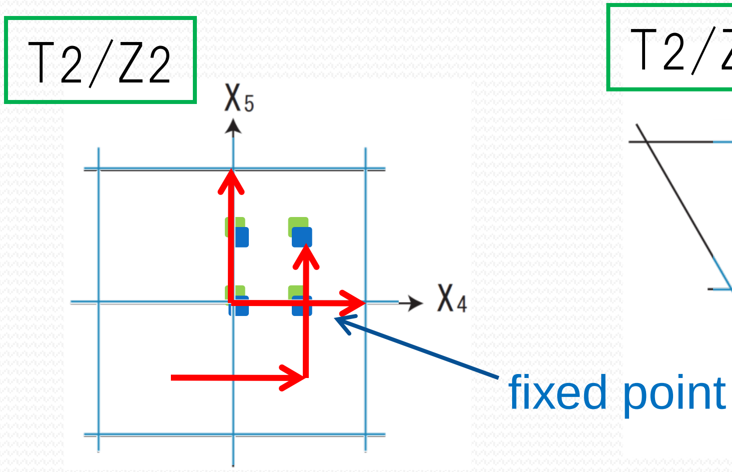
Geometry of compact space generate localized string states, which are not states obtained from dimensional reduction.

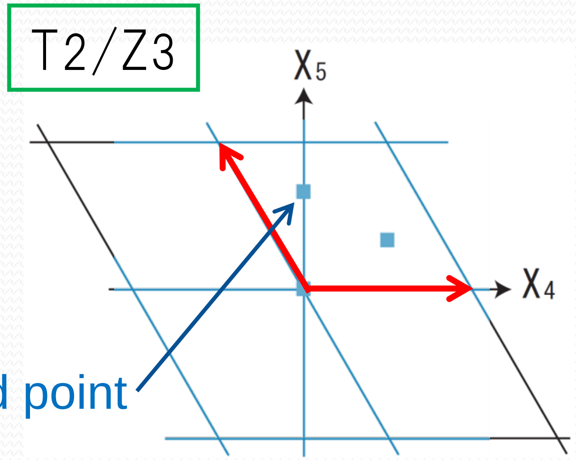
- I think orbifold provides good playground for model construction.

Torus and symmetry

- The rotations of an orbifold, i.e. θ and ϕ, should be given by the symmetry of the torus.

Examples in 2D :



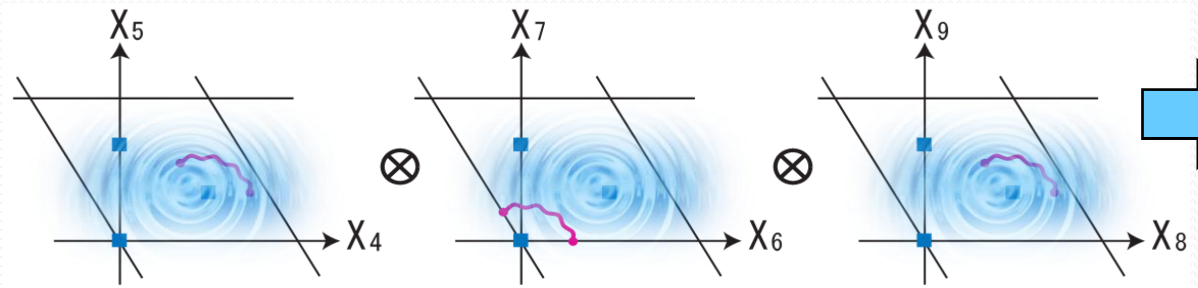


- We generalize these concepts to six dimensional torus.

String on orbifolds:

Geometry and structure → spectrum

- An example of Z_3 orbifold on factorizable torus: $T^2 \otimes T^2 \otimes T^2$

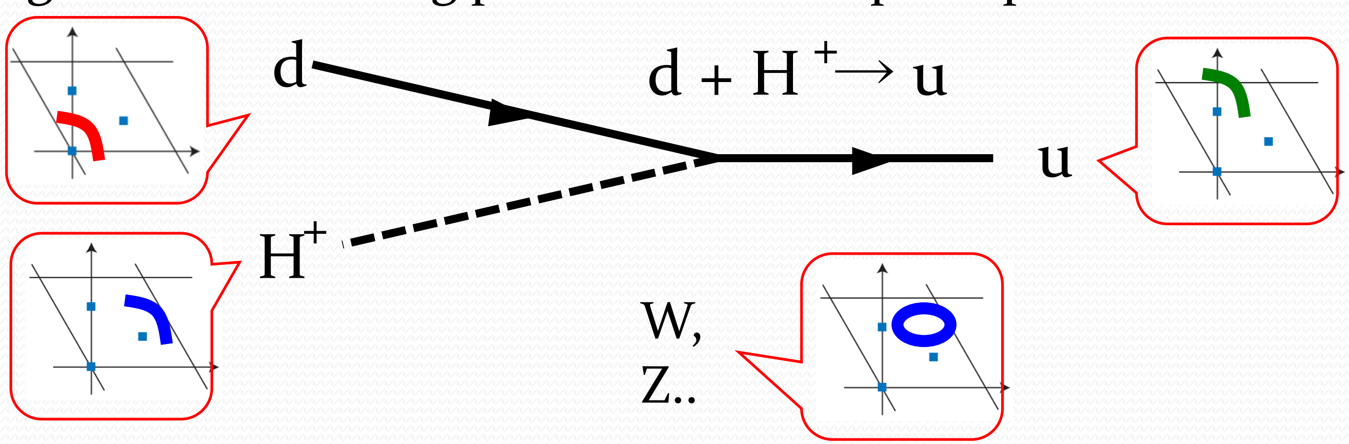


3×3×3=27
twisted sectors
+
9 untwisted sectors

36 generations of matter

Boundary cond. $X(2\pi)=\theta X(0)+v$, θ : rotation, v: shift ($v \in \Lambda$)

- In this description, an interaction of particles could be interpreted as a change of the localizing points in the compact space.



E6 torus

- E6 root lattice:

$$\begin{aligned}
\alpha_1 &= (1,0,0,0,0,0)\,, \\
\alpha_2 &= \left(-\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}, 0,0,0,0\right), \\
\alpha_3 &= \left(0, -\tfrac{1}{\sqrt{3}}, 0, -\tfrac{1}{\sqrt{3}}, 0, -\tfrac{1}{\sqrt{3}}\right), \\
\alpha_4 &= \left(0,0,-\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}, 0,0\right), \\
\alpha_5 &= (0,0,1,0,0,0)\,, \\
\alpha_6 &= \left(0,0,0,0,-\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right).
\end{aligned}$$

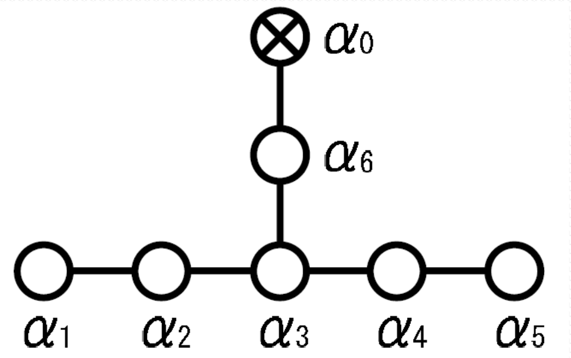


Figure 1: Extended E_6 diagram

$$\begin{aligned}
\alpha_0 &\equiv -\alpha_1 - 2\alpha_2 - 3\alpha_3 - 2\alpha_4 - \alpha_5 - 2\alpha_6 \\
&= (0,0,0,0,1,0)\,,
\end{aligned}$$

- Orbifold action of $Z_3 \times Z_3$:

rotation

$$\begin{aligned}
\theta &\equiv r_{\alpha_2} r_{\alpha_1} r_{\alpha_5} r_{\alpha_4}, \quad \Rightarrow \left(\tfrac{1}{3}, -\tfrac{1}{3}, 0\right), \\
\phi &\equiv r_{\alpha_4} r_{\alpha_5} r_{\alpha_0} r_{\alpha_6}. \quad \left(0, \tfrac{1}{3}, -\tfrac{1}{3}\right).
\end{aligned}$$

- 3 fixed tori appear in θ-twisted sector !

$$\theta\text{-sector} \Rightarrow \begin{array}{l} (0,0,0,0,x,y), \\ (0, \tfrac{1}{\sqrt{3}}, 0,0,x,y), \\ (0,0,0,\tfrac{1}{\sqrt{3}}, x, y), \end{array}$$

This is similar for ϕ, $\theta\,\phi$-sectors.

- $\theta\,\phi_2$-sector includes 27 fixed points. ⇒ $\chi = \frac{1}{N} \sum_{[g,h]=0} \chi_{g,h} = 72$

A model with the standard embedding

- An explicit model on $Z_3 \times Z_3$ orbifold on E_6 lattice.

To satisfy the modular invariance we must embed the twist in 6D space to $E_8 \times E_8$ gauge space.

$$v = \left(\tfrac{1}{3}, -\tfrac{1}{3}, 0\right) \;\rightarrow\; V = \left(\tfrac{1}{3}, -\tfrac{1}{3}, 0, 0, 0, 0, 0, 0\right)(0, 0, 0, 0, 0, 0, 0, 0)\,,$$
$$w = \left(0, \tfrac{1}{3}, -\tfrac{1}{3}\right) \;\rightarrow\; W = \left(0, \tfrac{1}{3}, -\tfrac{1}{3}, 0, 0, 0, 0, 0\right)(0, 0, 0, 0, 0, 0, 0, 0)\,.$$

- Gauge group: $E_6 \times U(1)^2 \times E_8'$
- The massless spectrum:

Untwisted sector:	$3 \times \mathbf{27}$,	θ-sector:	$6 \times \mathbf{27}$,	ϕ-sector:	$6 \times \mathbf{27}$,
		$\theta\phi$-sector:	$6 \times \mathbf{27}$,	$\theta\phi^2$-sector:	$15 \times \mathbf{27}$,

+ singlets + N=1 Gravity + N=1 YM

→ 36 generations

half of χ

An SU(5) model

- Quite simple assumptions:

  $Z_3 \times Z_3$ orbifold on E6 root lattice

  + gauge embeddings:

$$V = \left(0,0,0,0,0,\frac{1}{3},\frac{1}{3},\frac{2}{3}\right)\left(0,0,0,0,0,0,\frac{1}{3},\frac{1}{3}\right),$$

$$W = \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{6},\frac{5}{6},\frac{5}{6},-\frac{5}{6},\frac{5}{6}\right)\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},0,0,0,0,0\right).$$

- Gauge group:

visible

$SU(5)\times SU(2)_L \times SU(2)_R \times U(1)^2$

hidden

$\times [SU(6)\times SU(3)\times U(1)]'$

	untwisted sector		twisted sector			
		representation		representation		representation
visible sector	U_2	$(\mathbf{5},\mathbf{1},\mathbf{1})_{0,-6,0}$	$\bar{A}$	$3(\mathbf{10},\mathbf{1},\mathbf{1})_{-2,0,-2}$	A	$3(\mathbf{1},\mathbf{1},\mathbf{2})_{2,3,-4}$
	U_3	$(\bar{\mathbf{5}},\mathbf{1},\mathbf{1})_{6,0,0}$	A	$3(\bar{\mathbf{5}},\mathbf{1},\mathbf{1})_{-4,0,2}$	A	$3(\mathbf{1},\mathbf{1},\mathbf{2})_{2,3,2}$
	U_2	$(\mathbf{10},\mathbf{1},\mathbf{2})_{0,3,0}$	A	$3(\mathbf{1},\mathbf{2},\mathbf{1})_{5,0,2}$	$\bar{A}$	$3(\mathbf{1},\mathbf{1},\mathbf{2})_{-2,-3,-2}$
	U_3	$(\overline{\mathbf{10}},\mathbf{2},\mathbf{1})_{-3,0,0}$				
	U_1	$(\bar{\mathbf{5}},\mathbf{1},\mathbf{2})_{-6,-3,0}$				
	U_1	$(\mathbf{5},\mathbf{2},\mathbf{1})_{3,6,0}$				
	U_1	$(\mathbf{1},\mathbf{2},\mathbf{2})_{3,-3,0}$				
	U_2	$(\mathbf{1},\mathbf{2},\mathbf{1})_{-9,-6,0}$				
	U_3	$(\mathbf{1},\mathbf{1},\mathbf{2})_{6,9,0}$				
messenger sector	U_1	$\mathbf{1}_{0,0,-6}$	B	$3(\mathbf{1},\mathbf{2},\mathbf{1})(\mathbf{1},\mathbf{3})'_{[illegible],4,0}$	C	$3(\mathbf{1},\mathbf{2},\mathbf{1})(\mathbf{6},\mathbf{1})'_{-3,-2,-1}$
			$\bar{B}$	$3(\mathbf{1},\mathbf{1},\mathbf{2})(\mathbf{1},\bar{\mathbf{3}})_{-4,-1,0}$	A	$3\times\mathbf{1}_{-4,-6,2}$
hidden sector	U_1	$(\mathbf{20},\mathbf{1})'_{0,0,3}$	D	$15(\bar{\mathbf{6}},\mathbf{1})'_{-2,2,-1}$	$\bar{C}$	$3(\mathbf{1},\bar{\mathbf{3}})'_{-6,-4,-2}$
	U_3	$(\mathbf{15},\mathbf{3})'_{0,0,0}$	D	$6(\mathbf{1},\bar{\mathbf{3}})'_{-2,2,2}$		
	U_2	$(\mathbf{6},\bar{\mathbf{3}})'_{0,0,-3}$				

Higgs

3-family matter

Messenger

- No adjoint higgs! $\leftarrow$ Not realistic.
- Assume the vector-like 6's of SU(6)' have large mass (~M_{GUT}), the coupling of SU(6)' become strong at ~ 10^9 GeV.

An SO(10) model

- Similarly

with the gauge embeddings:

$$V = \left(\frac{1}{3},\frac{1}{3},\frac{2}{3},0,0,0,0,0\right)\left(\frac{1}{3},\frac{1}{3},0,0,0,0,0,0\right),$$

$$W = \left(-\frac{2}{3},0,0,0,0,0,0,0\right)\left(\frac{1}{3},0,\frac{1}{3},\frac{1}{3},\frac{1}{3},0,0,0\right).$$

- Gauge group:

visible

$SO(10)\times SU(2)\times U(1)^2$

hidden

$\times\,[SU(7)'\times U(1)'^2]$

- No adjoint Higgs.

	untwisted sector		twisted sector			
		representation		representation		representation
visible sector	U_3	$(\mathbf{16},\mathbf{1})_{3,6,0,0}$	A	$3(\mathbf{16},\mathbf{1})_{-1,-2,4,2}$	$\bar{B}$	$3(\mathbf{10},\mathbf{1})_{-2,0,-2,-8}$
	U_2	$(\overline{\mathbf{16}},\mathbf{1})_{-3,6,0,0}$	$\bar{A}$	$3(\mathbf{10},\mathbf{1})_{-2,-4,-4,-2}$	B	$3(\mathbf{1},\mathbf{2})_{2,-6,2,8}$
	U_1	$(\mathbf{10},\mathbf{2})_{0,6,0,0}$	$\bar{A}$	$3(\mathbf{1},\mathbf{2})_{-2,2,-4,-2}$	$\bar{C}$	$3(\mathbf{1},\mathbf{2})_{2,2,6,-4}$
	U_2	$(\mathbf{1},\mathbf{2})_{6,6,0,0}$				
	U_3	$(\mathbf{1},\mathbf{2})_{-6,6,0,0}$				
messenger sector	U_1	$\mathbf{1}_{0,-12,0,0}$	C	$3(\mathbf{1},\mathbf{2})(\mathbf{7})'_{-2,-2,0,-2}$	D	$27\times\mathbf{1}_{0,4,8,18}$
	U_2	$\mathbf{1}_{0,0,1\ldots}$ [illegible]	C	$3\times\mathbf{1}_{4,4,6,10}$	D	$15\times\mathbf{1}_{0,-8,2,-6}$
			A	$3\times\mathbf{1}_{-4,4,4,2}$	B	$3\times\mathbf{1}_{-4,0,2,8}$
			A	$3\times\mathbf{1}_{-4,4,-8,-4}$		
hidden sector	U_3	$(\mathbf{35})'_{0,0,0,-6}$	B	$3(\bar{\mathbf{7}})'_{-4,0,2,-4}$	D	$6(\mathbf{7})'_{0,4,-4,0}$
	U_1	$(\overline{\mathbf{21}})'_{0,0,6,0}$	$\bar{C}$	$3(\bar{\mathbf{7}})'_{-4,-4,0,2}$	D	$6(\bar{\mathbf{7}})'_{0,4,2,6}$
	U_2	$(\mathbf{7})'_{0,0,-6,-12}$				
	U_3	$(\mathbf{7})'_{0,0,0,12}$				
	U_2	$(\bar{\mathbf{7}})'_{0,0,-6,6}$				

3-family matter

Higgs

Messenger

The allowed 3-point functions

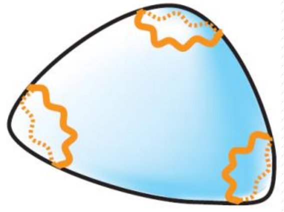
- There are mainly three requirements for the allowed couplings.

1. Gauge invariance (charge conservation).

2. H-momentum (10D angular momentum) conservation.

$$U_1U_2U_3,\quad U_3A\bar{A},\quad U_1B\bar{B},\quad U_2C\bar{C},\quad DDD,$$
$$\bar{A}BD,\quad ACD,\quad \bar{B}\bar{C}D,\quad AB\bar{C},\quad \bar{A}\bar{B}C,$$

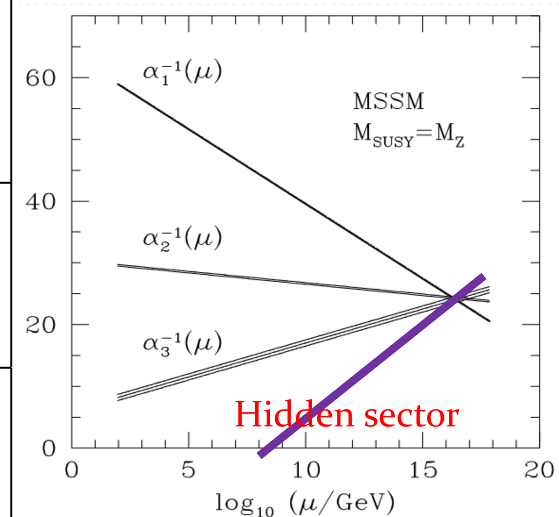
3. Geometric restrictions (instanton effect).



A little phenomenology of the SO(10) model

- Assuming that all the pairs of 7 and $\overline{7}$ generate mass terms and decouple, the hidden sector scale is $\Lambda_{hidden} \sim 1.3 \times 10^8 \mathrm{GeV}$.
- Because $\overline{C}$ (1,2) couples only to C (1,2)(7) and U2($\overline{7}$), the tree level superpotential includes W= W_{ADS} + $\overline{C}$ C U2 . ⟶ ~~SUSY~~ ?

	untwisted sector		twisted sector			
		representation		representation		representation
visible sector	U_3	$(\mathbf{16},\mathbf{1})_{3,6,0,0}$	A	$3(\mathbf{16},\mathbf{1})_{-1,-2,4,2}$	$\bar{B}$	$3(\mathbf{10},\mathbf{1})_{-2,0,-2,-8}$
	U_2	$(\overline{\mathbf{16}},\mathbf{1})_{-3,6,0,0}$	$\bar{A}$	$3(\mathbf{10},\mathbf{1})_{-2,-4,-4,-2}$	B	$3(\mathbf{1},\mathbf{2})_{2,-6,2,8}$
	U_1	$(\mathbf{10},\mathbf{2})_{0,6,0,0}$	$\bar{A}$	$3(\mathbf{1},\mathbf{2})_{-2,2,-4,-2}$	$\bar{C}$	$3(\mathbf{1},\mathbf{2})_{2,2,6,-4}$
	U_2	$(\mathbf{1},\mathbf{2})_{6,6,0,0}$				
	U_3	$(\mathbf{1},\mathbf{2})_{-6,6,0,0}$				
messenger sector	U_1	$\mathbf{1}_{0,-12,0,0}$	C	$3(\mathbf{1},\mathbf{2})(\mathbf{7})'_{2,-2,0,-2}$	D	$27 \times \mathbf{1}_{0,4,8,18}$
	U_2	$\mathbf{1}_{0,0,12,6}$	C	$3 \times \mathbf{1}_{4,4,6,10}$	D	$15 \times \mathbf{1}_{0,-8,2,-6}$
			A	$3 \times \mathbf{1}_{-4,4,4,2}$	B	$3 \times \mathbf{1}_{-4,0,2,8}$
			A	$3 \times \mathbf{1}_{-4,4,-8,-4}$		
hidden sector	U_3	$(\mathbf{35})'_{0,0,0,-6}$	B	$3(\bar{\mathbf{7}})'_{-4,0,2,-4}$	D	$6(\mathbf{7})'_{0,4,-4,0}$
	U_1	$(\mathbf{21})'_{0,0,6,0}$	$\bar{C}$	$3(\bar{\mathbf{7}})'_{-4,-4,0,2}$	D	$6(\bar{\mathbf{7}})'_{0,4,2,6}$
	U_2	$(\mathbf{7})'_{0,0,-6,-12}$				
	U_3	$(\mathbf{7})'_{0,0,0,12}$				
	U_2	$(\bar{\mathbf{7}})'_{0,0,-6,6}$				



Summary

- The origin of three generation matter are explained by the three fixed tori in a twisted sector in the models.
- Due to small number of fixed tori, the spectra are very simple.
- In $Z_3 \times Z_3$ orbifold, three point (Yukawa) interactions with flavor mixing terms, which are suppressed by e^{-S}, are generated by instanton effect.
- They include strongly coupled sector in the low energy, and may cause spontaneous SUSY breaking.
- Nevertheless they are toy models without adjoint higgs.
- By inclusion of Wilson lines, we can obtain directly $SU(3) \times SU(2) \times U(1)$ gauge group. → More realistic models?

- String theory indicates various symmetries or textures in the low energy spectra. Model constructions including many phenomenological features (realization of the spectrum, SUSYV, its mediation, moduli stabilization, etc) will be more exciting in the future.