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How to Evade a NO-GO Theorem in Flavor Symmetries

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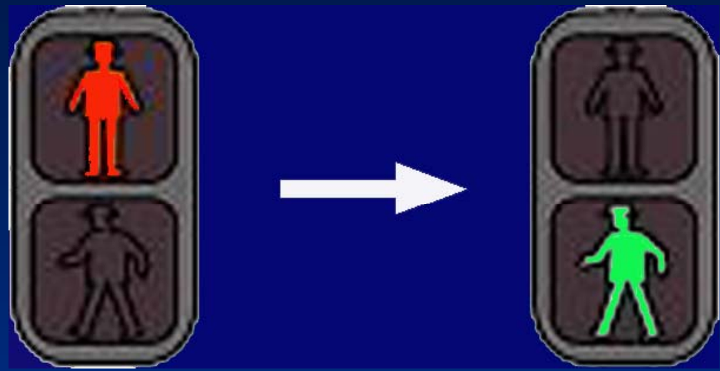
How to 

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5 Summary

1 Introduction

When we see history of physics, we will find that "symmetries" always play a key role in the new physics. In investigating the origin of flavors, too, we may expect that an approach based on symmetries will be a powerful instrument for the investigation.

Especially, how to treat the flavor symmetry is a big concern in grand unification model-building.

However, when we want to introduce a symmetry (discrete one, U(1), and any others) into our mass matrix model, we always encounter an obstacle,

a No-Go theorem in flavor symmetries:

We cannot bring any flavor symmetry into a mass matrix model within the framework of the standard model unless the model breaks the $SU(2)_L$ symmetry.

(YK, Phys.Rev. D71 (2005) 016010)

However, we should not consider this theorem to be negative.

We must take this theorem seriously, but **we should utilize this theorem positively to investigate the origin of the flavor mass spectra.**

In the present talk, I would like to talk about how to evade this No-Go theorem in order to build a realistic mass matrix model.

2 Masses and mixings in the standard model

In the standard model, the fermion masses are generated from the VEV of the Higgs scalar:

(We will denote a case in the quark sectors as an example.)

$$H_Y = Y_{ij}^u \overline{Q}_{Li} H_u u_{Rj} + Y_{ij}^d \overline{Q}_{Li} H_d d_{Rj} + h.c. \tag{2.1}$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad H_u = \begin{pmatrix} H_u^0 \\ H_u^- \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix}, \tag{2.2}$$

$$(M_u)_{ij} = Y_{ij}^u \langle H_u^0 \rangle, \quad (M_d)_{ij} = Y_{ij}^d \langle H_d^0 \rangle \tag{2.3}$$

The requirement of a flavor symmetry means as follows: **Under the transformation of the flavor basis**

$$
\begin{aligned}
Q_L &\rightarrow Q'_L = T_L Q_L \\
u_R &\rightarrow u'_R = T^u_R u_R \\
d_R &\rightarrow d'_R = T^d_R d_R
\end{aligned}
\qquad (2.4)
$$

the Hamiltonian is invariant.

Therefore, the requirement imposes on the Yukawa coupling constants Y^u and Y^d as the following constraints:

$$
T_L^\dagger Y^u T_R^u = Y^u, \quad T_L^\dagger Y^d T_R^d = Y^d \qquad (2.5)
$$

On the other hand, the CKM mixing matrix V is given by

$$V = (U_L^u)^\dagger U_L^d \qquad (2.6)$$

where

$$(U_L^u)^\dagger M_u U_R^u = D_u \equiv \mathrm{diag}(m_u, m_c, m_t)$$
$$(U_L^d)^\dagger M_d U_R^d = D_d \equiv \mathrm{diag}(m_d, m_s, m_b) \qquad (2.7)$$

Now we attention to $Y^f(Y^f)^\dagger$, and we obtain

$$T_L^\dagger Y^u (Y^u)^\dagger T_L = Y^u (Y^u)^\dagger$$
$$T_L^\dagger Y^d (Y^d)^\dagger T_L = Y^d (Y^d)^\dagger$$

(2.8)

$$(U_L^u)^\dagger Y^u (Y^u)^\dagger U_L^u = D_u (D_u)^\dagger$$
$$(U_L^d)^\dagger Y^d (Y^d)^\dagger U_L^d = D_d (D_d)^\dagger$$

(2.9)

3 No-Go theorem in flavor symmetries

[Theorem] When we introduce a flavor symmetry into a model within the framework of the standard model, the flavor mixing matrix (CKM matrix and/or neutrino mixing matrix) cannot describe a mixing among 3 families, and only a mixing between 2 families is allowed.

$$V = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(YK, Phys.Rev. D71 (2005) 016010)

Such a strong constraint comes from the relations

(2.8): Flavor symmetry requirement
$$T_L^\dagger Y^u (Y^u)^\dagger T_L = Y^u (Y^u)^\dagger$$
$$T_L^\dagger Y^d (Y^d)^\dagger T_L = Y^d (Y^d)^\dagger$$

and (2.9) : Diagonalization relation
$$(U_L^u)^\dagger Y^u (Y^u)^\dagger U_L^u = D_u (D_u)^\dagger$$
$$(U_L^d)^\dagger Y^d (Y^d)^\dagger U_L^d = D_d (D_d)^\dagger$$

When we define the operator
$$T_f = (U_L^f)^\dagger T_L U_L^f \qquad (3.1)$$

we can obtain the relation
$$T_f^\dagger D_f^2 T_f = D_f^2 \qquad (3.2)$$

from (2.8) and (2.9) .

Therefore, the operator T_f must be
$$T_f = P_f \equiv \mathrm{diag}(e^{i\delta_1^f}, e^{i\delta_2^f}, e^{i\delta_3^f}) \qquad (3.3)$$

so that T_L is expressed as
$$T_L = U_L^u P_u (U_L^u)^\dagger = U_L^d P_d (U_L^d)^\dagger \qquad (3.4)$$
$$\Rightarrow \quad P_u = (U_L^u)^\dagger U_L^d P_d (U_L^d)^\dagger U_L^u = V_{CKM} P_d V_{CKM}^\dagger \qquad (3.5)$$

Eq.(3.5) leads to

$$P_u V_{CKM} - V_{CKM} P_d = 0 \qquad (3.6)$$

$$\Rightarrow \quad (e^{i\delta_i^u} - e^{i\delta_j^d})(V_{CKM})_{ij} = 0 \qquad (3.7)$$

Only when $\delta_i^u = \delta_j^d$, we can obtain $(V_{CKM})_{ij} \neq 0$

We do not consider the case with $\delta_1^u = \delta_2^u = \delta_3^u$ and $\delta_1^d = \delta_2^d = \delta_3^d$ which corresponds to $T_L = 1$

For a non-trivial flavor transformation T_L, we must choose, at least, one of δ_i^f differently from others. For example, for the case with $\delta_1^f = \delta_2^f \neq \delta_3^f$ we can obtain only a two-family mixing

$$V = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (3.8)$$

We summarize the premises to derive the theorem:

(i) The $SU(2)_L$ symmetry is unbroken.

(ii) There is only one Higgs scalar in each sector.

(iii) 3 eigenvalues of Y^f in each sector are non-zero and no-degenerate.

If one of them in a model is not satisfied, the model can evade the theorem.

Example which is ruled out by the theorem

(i) We consider a flavor symmetry at GUT scale.

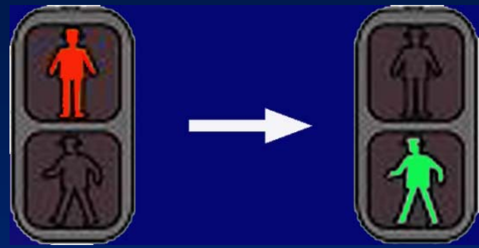
(Of course the $SU(2)_L$ symmetry is unbroken at the scale.)

(ii) There is only one Higgs scalar in each sector, e.g. H_u and H_d.

(iii) 3 eigenvalues of Y^f in each sector are completely different from each other and not zero at the GUT scale.

Then, such the model is ruled out by the theorem.

4 How to evade the no-go theorem



-- Three allowed ways to the mass matrix models

4.1 Multi-Higgs model

$$M_{ij} = Y_{ij}^a \langle H_a \rangle + Y_{ij}^b \langle H_b \rangle + Y_{ij}^c \langle H_c \rangle$$

4.2 Model with an explicitly broken symmetry

In other words, there is no flavor symmetry from the beginning

4.3 Model in which Y's are fields

$$M_{ij} = \frac{1}{\Lambda} \langle Y_{ij} \rangle \langle H^0 \rangle$$

4.1 Model with multi-Higgs scalars

Multi-Higgs models can evade the No-Go theorem, where the Higgs scalars have different transformation properties for the flavor symmetry.

$$M_{ij} = Y^a_{ij}\langle H_a\rangle + Y^b_{ij}\langle H_b\rangle + Y^c_{ij}\langle H_c\rangle \qquad (4.1.1)$$

However, generally, such a multi-Higgs model induces the so-called FCNC problem.

We must make those Higgs scalars heavy except for one of the linear combinations of those scalars:

$$U_H \begin{pmatrix} H_a \\ H_b \\ H_c \end{pmatrix} = \begin{pmatrix} H_0 \\ H_1 \\ H_2 \end{pmatrix} \begin{matrix} 10^2\mathrm{GeV} \\ 10^{16}\mathrm{GeV} \\ 10^{16}\mathrm{GeV} \end{matrix} \qquad (4.1.2)$$

However, at present, models which give a reasonable mechanism are few. The mechanism must be proposed in the framework of the exact flavor symmetry. However, in most models, the suppression of unwelcome components are only assumptions by hand.

4.2 Model with an explicitly broken symmetry

We consider a model in which the symmetry is broken explicitly from the beginning.

Example: N.Haba & YK, hep-ph/078.3913, PLB (2007)

We assume a U(3) flavor symmetry.

The symmetry U(3) is broken by the parameter Y^f explicitly:

$$W_Y = \sum_{i,j} Y^e_{ij} L_j E_i H_d \qquad (4.2.1)$$

(For convenience, hereafter, we drop the index "e".)

Also, we consider a U(3) nonet field Φ

and we denote the superpotential for Φ as

$$W_\Phi = m_1 \mathrm{Tr}[\Phi\Phi] + m_2(\mathrm{Tr}[\Phi])^2$$
$$+\lambda_1 \mathrm{Tr}[\Phi\Phi\Phi] + \lambda_2 \mathrm{Tr}[\Phi\Phi]\mathrm{Tr}[\Phi] + \lambda_3(\mathrm{Tr}[\Phi])^3 \qquad (4.2.2)$$

We assume that **the symmetry is also broken by a tadpole term with the same symmetry breaking parameter Y** as follows:

$$W = W_\Phi - \mu^2 \mathrm{Tr}[Y\Phi] + W_Y \qquad (4.2.3)$$

Then, we obtain

$$\frac{\partial W}{\partial \Phi} = 0 = \frac{\partial W_\Phi}{\partial \Phi} - \mu^2 Y \qquad (4.2.4)$$

$$\frac{\partial W_\Phi}{\partial \Phi} = 3\lambda_1 \Phi\Phi + c_1(\Phi)\Phi + c_0(\Phi)\mathbf{1} \qquad (4.2.5)$$

$$c_1(\Phi) = 2(m_1 + \lambda_2 \mathrm{Tr}[\Phi]) \qquad (4.2.6)$$

$$c_0(\Phi) = 2m_2 \mathrm{Tr}[\Phi] + \lambda_2 \mathrm{Tr}[\Phi\Phi] + 3\lambda_3 (\mathrm{Tr}[\Phi])^2 \qquad (4.2.7)$$

Now, we put an ansatz that our vacuum is given by a specific solution of Eq.(4.2.4):

$$3\lambda_1 \Phi\Phi - \mu^2 Y = 0 \qquad (4.2.8)$$

and

$$c_1(\Phi)\Phi + c_0(\Phi)\mathbf{1} = 0 \qquad (4.2.9)$$

Eq.(4.2.8) leads to a bilinear mass formula

$$Y_{ij} = \frac{3\lambda_1}{\mu^2}\sum_k \langle\Phi_{ik}\rangle\langle\Phi_{kj}\rangle \qquad (4.2.10)$$

For non-zero and non-degenerate eigenvalues v_i, Eq.(4.2.9) leads to $c_1=0$ and $c_0=0$.

For example, when we assume

$$W_\Phi = m\mathrm{Tr}[\Phi\Phi] + \lambda\mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] \qquad (4.2.11)$$

where $\Phi^{(8)} = \Phi - \frac{1}{3}\mathrm{Tr}[\Phi]\mathbf{1}$, we obtain

$$\mathrm{Tr}[\Phi\Phi] = \frac{2}{3}(\mathrm{Tr}[\Phi])^2 \qquad (4.2.12)$$

from $c_0=0$ because

$$\mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] = \mathrm{Tr}[\Phi\Phi\Phi] - \mathrm{Tr}[\Phi]\left(\mathrm{Tr}[\Phi\Phi] - \frac{2}{9}(\mathrm{Tr}[\Phi])^2\right) \qquad (4.2.13)$$

Eq.(4.2.12) leads to

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2 \quad (4.2.14)$$

in the diagonal basis of $\langle \Phi_{ij} \rangle = \delta_{ij} v_i$.

Therefore, from Eqs.(4.2.10) and (4.2.14), we obtain the charged lepton mass formula

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 \quad (4.2.15)$$

which can give an excellent prediction $m_\tau = 1776.97$ MeV from the observed values of m_e and m_μ .

(The observed value is $m_\tau^{obs} = 1776.99^{+0.29}_{-0.26}$ MeV)

4.3 Model in which Y's are fields

We consider that Y's are fields in the Yukawa interactions

$$H_Y = \frac{Y^u_{ij}}{M}\overline{Q}_{Li} H_u u_{Rj} + \frac{Y^d_{ij}}{M}\overline{Q}_{Li} H_d d_{Rj} + h.c. \tag{4.3.1}$$

Since the fields Y`s are transformed as

$$Y_f \to Y'_f = T_L Y_f (T^f_R)^\dagger \tag{4.3.2}$$

under the transformation

$$\begin{array}{lll} Q_L & \to & Q'_L = T_L Q_L \\ u_R & \to & u'_R = T^u_R u_R \\ d_R & \to & d'_R = T^d_R d_R \end{array} \tag{4.3.3}$$

the constraints (2.8) for $Y^f(Y^f)^\dagger$ disappear, so that we can again evade the No-Go theorem.

For example, recently, Haba has suggested that the effective Yukawa interaction originates in a higher mass-dimensional term in Kähler potential K

$$K \sim \frac{1}{M^2} y_A A^{\dagger}_{ij} L_j E_j H_d \qquad (4.3.4)$$

$$\Rightarrow \quad (K)_D \sim \frac{1}{M^2} y_A (F_A^{\dagger})_{ij} L_j E_i H_d \qquad (4.3.5)$$

(N.Haba, private communication)

A similar idea in the neutrino masses has been proposed by Arkani-Hamed, Hall, Murayama, Smith and Weiner [PRD64 (2001) 115011]

When we adopt a O'Raifeartaigh-type SUSY breaking mechanism [PL B429 (1998) 263]

$$W = W_\Phi(\Phi) + \lambda_A \mathrm{Tr}[A\Phi\Phi] + \lambda_B \mathrm{Tr}[B\Phi\Phi] - \mu^2 \mathrm{Tr}[\xi A] \qquad (4.3.6)$$

where ξ (3x3 matrix) is a flavor breaking parameter.

We can again obtain a bilinear form for the effective Yukawa coupling constant as follows:

$$-F_A^\dagger = \frac{\partial W}{\partial A} = \lambda_A \Phi\Phi - \mu^2 \xi = \lambda_B \frac{\lambda_B}{\lambda_A} \Phi\Phi \neq 0 \qquad (4.3.7)$$

$$-F_B^\dagger = \frac{\partial W}{\partial B} = \lambda_B \Phi\Phi \neq 0 \qquad (4.3.8)$$

where the VEV spectrum of Φ is determined from the equation

$$\frac{\partial W}{\partial \Phi} = \frac{\partial W_\Phi}{\partial \Phi} = 0 \qquad (4.3.9)$$

More details will appear in hep-ph soon.

5 Summery

The no-go theorem in flavor symmetries tells us that **we cannot bring any flavor symmetry, at least, into a mass matrix model based on the standard model**.

We have demonstrated 3 ways to evade the no-go theorem in the flavor symmetries:

(A) Model with multi-Higgs scalars

(B) Model with an explicit broken symmetry

(C) Model in which Y's are fields

- Models based on the scenario A have been proposed by many authors.
  - However, current most models have not given a plausible mechanism which makes Higgs scalars heavy except for one.
- In the scenario B, there is no flavor symmetry from the beginning. The "flavor symmetry" is a faked one for convenience.
  - However, if we once suppose a flavor symmetry, rather, we would like to consider that the symmetry is exact, and then it is broken spontaneously. Therefore, we are still unsatisfactory to the scenario B.

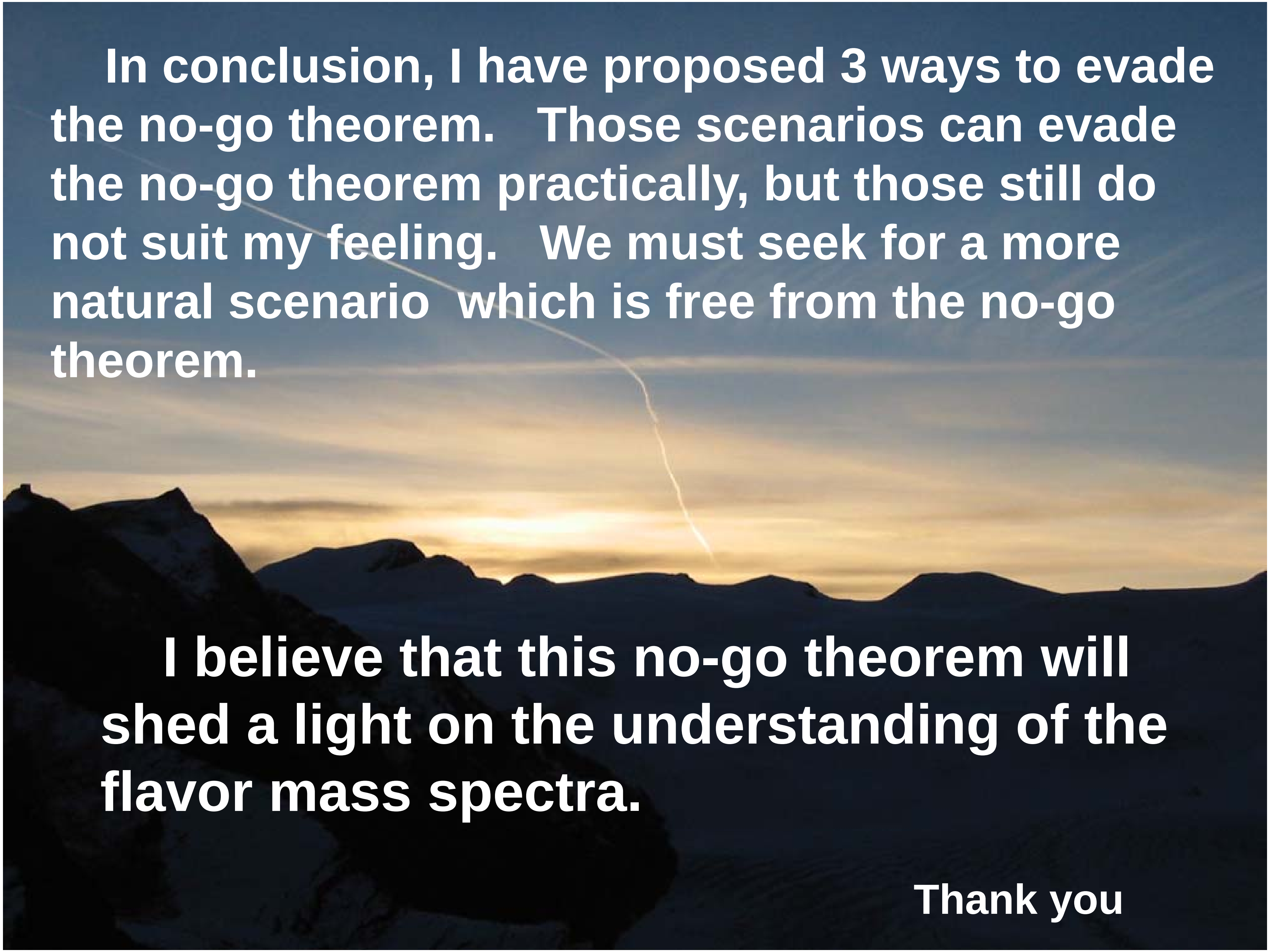
Models based on the scenario C are interesting. However, in order to give an effective Yukawa interaction, we need a term with higher mass dimension

$$\frac{1}{M}(Y_e)_{ij} L_j E_i H_d \quad \text{in W}$$

or

$$\frac{1}{M^2}(Y_e^\dagger)_{ij} L_j E_i H_d \quad \text{in K}$$

However, we want a model without such higher mass dimensional terms as possible.


In conclusion, I have proposed 3 ways to evade the no-go theorem. Those scenarios can evade the no-go theorem practically, but those still do not suit my feeling. We must seek for a more natural scenario which is free from the no-go theorem.
I believe that this no-go theorem will shed a light on the understanding of the flavor mass spectra.
Thank you

Proof of the theorem

We define Hermitian matrices $H_f = Y_f Y_f^\dagger \ (f = u, d)$

$$(U_L^f)^\dagger H_f U_L^f = D_f^2 \equiv \frac{1}{v_f^2}\text{diag}(m_{f1}^2, m_{f2}^2, m_{f3}^2) \quad \text{(A.1)}$$

$$\hookrightarrow H_f = T_L^\dagger H_f T_L \quad \text{(A.2)}$$

$$\hookrightarrow H_f = U_L^f D_f^2 (U_L^f)^\dagger$$

We obtain $$T_f^\dagger D_f^2 T_f = D_f^2 \quad \text{(A.3)}$$

where $$T_f = (U_L^f)^\dagger T_L U_L^f \quad \text{(A.4)}$$

If m_{fi}^2 are non-zero and non-degenerate, T_f must be

$$T_f = P_f \equiv \text{diag}(e^{i\delta_1^f}, e^{i\delta_2^f}, e^{i\delta_3^f}) \quad \text{(A.5)}$$

so that $$T_L = U_L^u P_u (U_L^u)^\dagger = U_L^d P_d (U_L^d)^\dagger \quad \text{(A.6)}$$

$$\Rightarrow \ P_u = (U_L^u)^\dagger U_L^d P_d (U_L^d)^\dagger U_L^u = V_{CKM} P_d V_{CKM}^\dagger \quad \text{(A.7)}$$

Eq.(A.7) leads to

$$P_u V_{CKM} - V_{CKM} P_d = 0 \qquad \text{(A.8)}$$

$$\Rightarrow \quad (e^{i\delta_i^u} - e^{i\delta_j^d})(V_{CKM})_{ij} = 0 \qquad \text{(A.9)}$$

Only when $\delta_i^u = \delta_j^d$, we can obtain $(V_{CKM})_{ij} \neq 0$

We do not consider the case with $\delta_1^u = \delta_2^u = \delta_3^u$ and $\delta_1^d = \delta_2^d = \delta_3^d$ which corresponds to $T_L = 1$

For a non-trivial flavor transformation T_L, we must choose, at least, one of δ_i^f differently from others. For example, for the case with $\delta_1^f = \delta_2^f \neq \delta_3^f$ we can obtain only a two-family mixing

$$V = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{(A.10)}$$