

Three Site Higgsless Model

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Outline

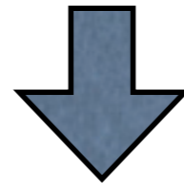
- 1. Introduction
- 2. Three Site Higgsless Model
- 3. Constraints and Signatures
- 4. Comparison to Continuum (5D) Models
- 5. Summary

1. Introduction

- Higgsless model is proposed as a new scenario of the EWSB
[Csaki, Grojean, Murayama, Pilo, Terning, PRD 69, 055006 \(2004\)](#)
- Higgsless models are based on five-dimensional gauge theories compactified on an interval
- EW symmetry is broken by boundary conditions of the gauge fields
- The spectrum includes states identified with the photon, W and Z bosons, and also an infinite tower of additional massive vector bosons
- Unitarization of the longitudinal W and Z boson scattering is ensured by the exchange of heavy gauge bosons
[Chivukula et. al. PLB 525, 175 \(2002\), PLB 532, 121 \(2002\), PLB 562, 109 \(2003\)](#)

Deconstruction (Arkani-Hamed - Cohen - Georgi, Hill - Pokorski - Wang)

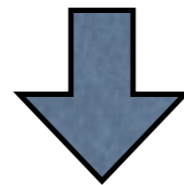
an alternative approach to study Higgsless Models



- Arbitrary 5D geometry
- Arbitrary position dependent couplings
- Brane localized kinetic terms

Deconstruction (Arkani-Hamed - Cohen - Georgi, Hill - Pokorski - Wang)

an alternative approach to study Higgsless Models



Higgsless models with **localized fermions** cannot simultaneously

- satisfy perturbative unitarity bound
- provide acceptably small EW corrections

Chivukula, Simmons, He, Kurachi, Tanabashi, Phys. Rev. D 71, 015016 (2005)

Fermion delocalization reduce the EW corrections

Cacciapaglia, Csaki, Grojean, Terning, Phys. Rev. D 71, 035015 (2005)

Foadi, Gopalakrishna, Schmidt, Phys. Lett. B 606, 157 (2005)

Ideal delocalization

For an arbitrary Higgsless models, choosing the probability distribution of the fermions to be related to the wave function of the W boson makes the S and T parameters vanish at tree-level

Chivukula, Simmons, He, Kurachi, Tanabashi, Phys. Rev. D 72, 015008 (2005)

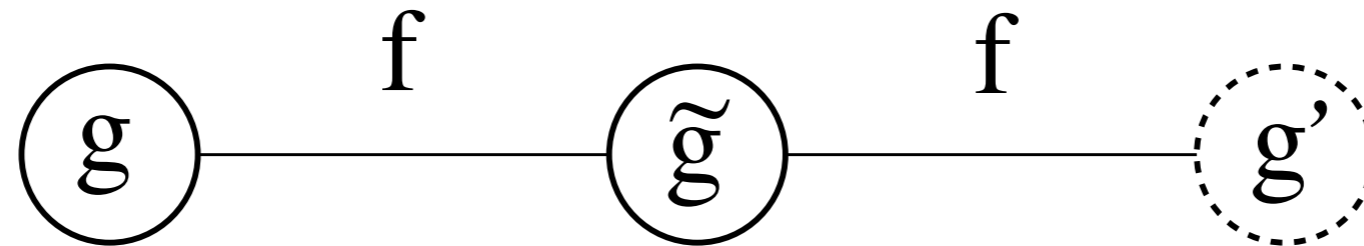
2. Three Site Higgsless Model

Chivukula, Coleppa, Di Chiara, Simmons, He, Kurachi, Tanabashi
PRD 74, 075001 (2006)

Many issues of current interest, such as ideal fermion delocalization and the generation of fermion masses (including the top quark mass) can be usefully illustrated in a Higgsless model **deconstructed to just three sites**

Three Site Higgsless Model (Gauge Sector)

Chivukula, Coleppa, Di Chiara, Simmons, He, Kurachi, Tanabashi, PRD 74, 075001 (2006)



$SU(2) \otimes SU(2) \otimes U(1)$ gauge invariant
non-linear sigma model

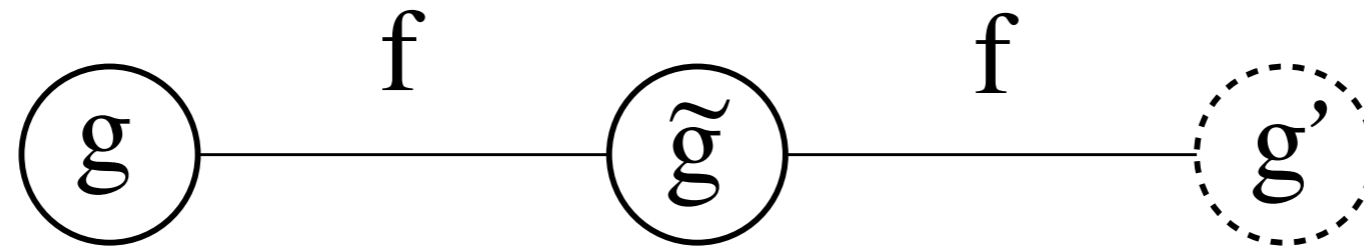
$$\mathcal{S} = - \int d^4x \sum_{j=0}^{N+1} \frac{1}{2g_j^2} \text{tr} (F_{\mu\nu}^j F^{j\mu\nu}) + \int d^4x \sum_{j=1}^{N+1} \frac{f_j^2}{4} \text{tr} ((D_\mu U_j)^\dagger (D^\mu U_j))$$

$$D_\mu U_j = \partial_\mu U_j - iA_\mu^{j-1} U_j + iU_j A_\mu^j$$

$$(N = 1) \quad g_0 \equiv g, \quad g_1 \equiv \tilde{g}, \quad g_2 \equiv g'$$

Three Site Higgsless Model (Gauge Sector)

Chivukula, Coleppa, Di Chiara, Simmons, He, Kurachi, Tanabashi, PRD 74, 075001 (2006)



$$f_1 = f_2 (= f) = \sqrt{2}v$$

$$g, g' \ll \tilde{g} \implies M_W, M_Z \ll M_{W'}, M_{Z'}$$

$$g/\tilde{g} \equiv x \ (\ll 1)$$

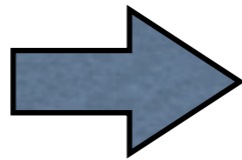
Numerically, g and g' are approximately equal to the standard model $SU(2)_W$ and $U(1)_Y$ couplings \implies We define an angle θ such that

$$\frac{g'}{g} = \frac{\sin \theta}{\cos \theta} \equiv \frac{s}{c} \equiv t$$

Gauge boson mass squared matrices

Charged gauge bosons

$$\frac{\tilde{g}^2 v^2}{2} \begin{pmatrix} x^2 & -x \\ -x & 2 \end{pmatrix}$$

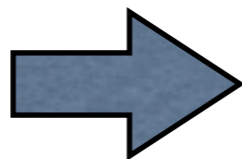


$$M_W^2 = \frac{g^2 v^2}{4} \left[1 - \frac{x^2}{4} + \dots \right]$$

$$M_{W'}^2 = \tilde{g}^2 v^2 \left[1 + \frac{x^2}{4} + \dots \right]$$

Neutral gauge bosons

$$\frac{\tilde{g}^2 v^2}{2} \begin{pmatrix} x^2 & -x & 0 \\ -x & 2 & -xt \\ 0 & -xt & x^2 t^2 \end{pmatrix}$$



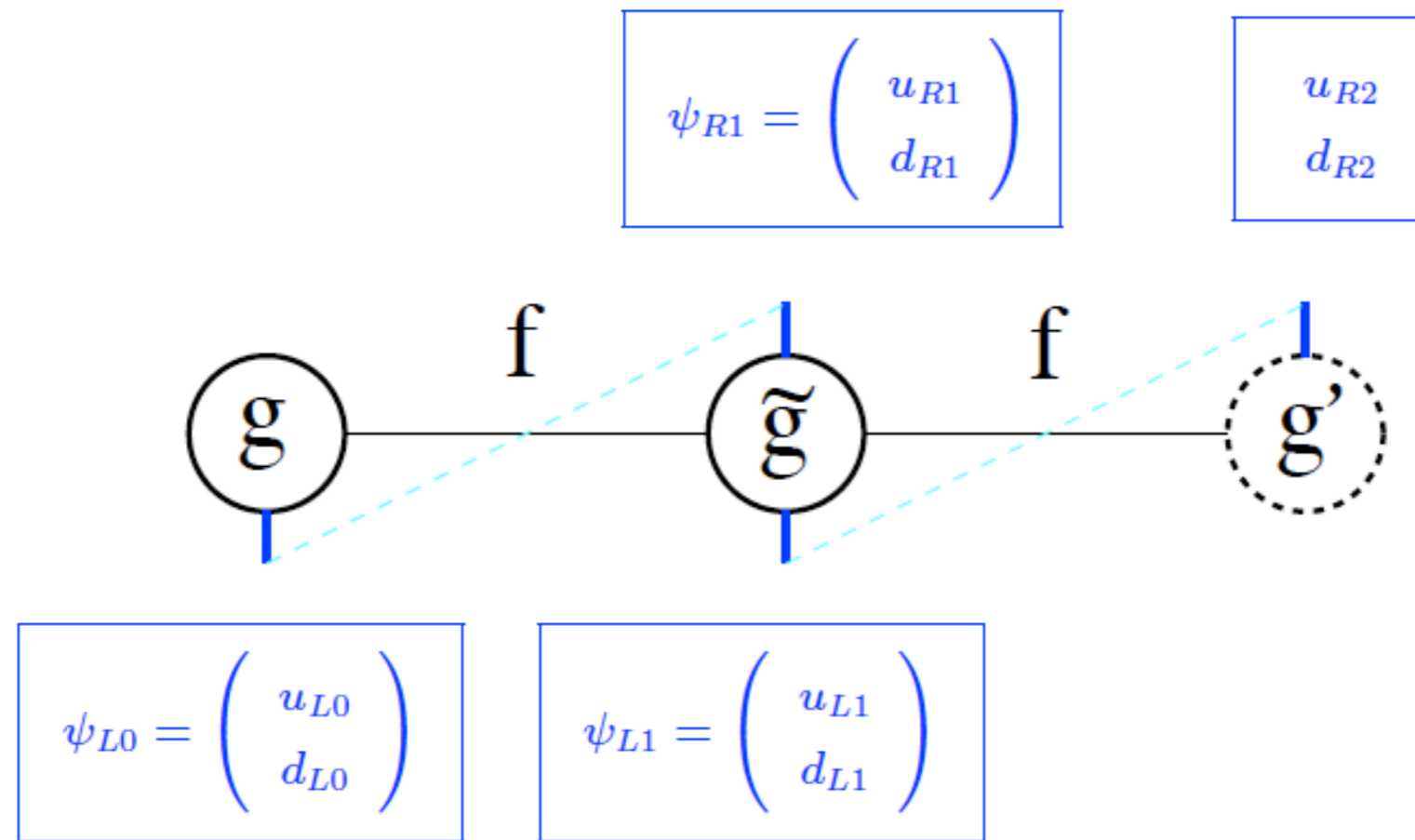
$$M_\gamma^2 = 0 \quad \left(\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{\tilde{g}^2} + \frac{1}{g'^2} \right)$$

$$M_Z^2 = \frac{g^2 v^2}{4c^2} \left[1 - \frac{x^2}{4} \frac{(c^2 - s^2)^2}{c^2} + \dots \right]$$

$$M_{Z'}^2 = \tilde{g}^2 v^2 \left[1 + \frac{x^2}{4c^2} + \dots \right]$$

Three Site Higgsless Model (Fermion Sector)

Chivukula, Coleppa, Di Chiara, Simmons, He, Kurachi, Tanabashi, PRD 74, 075001 (2006)



$$\mathcal{L}_{mass} = M \left[\epsilon_L \bar{\psi}_{L0} U_1 \psi_{R1} + \bar{\psi}_{R1} \psi_{L1} + \bar{\psi}_{L1} U_2 \begin{pmatrix} \epsilon_{uR} & \\ & \epsilon_{dR} \end{pmatrix} \begin{pmatrix} u_{R2} \\ d_{R2} \end{pmatrix} \right] + h.c.$$

Fermion mass matrix

$$M_u = M \begin{pmatrix} \varepsilon_L & 0 \\ 1 & \varepsilon_{uR} \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} m_u &= M \frac{\varepsilon_L \varepsilon_{uR}}{\sqrt{1 + \varepsilon_{uR}}} \left[1 - \frac{\varepsilon_L^2}{2(\varepsilon_{uR} + 1)^2} + \dots \right] \\ m_U &= M \sqrt{1 + \varepsilon_{uR}} \left[1 + \frac{\varepsilon_L^2}{2(\varepsilon_{uR} + 1)^2} + \dots \right] \end{aligned}$$

- **Ideal delocalization :** $\varepsilon_L^2 \simeq \frac{1}{2} \frac{g^2}{\tilde{g}^2} \left(\equiv \frac{x^2}{2} \ll 1 \right)$

a *choice* we make in building the model in order to minimize precision EW corrections

- For d, c, s, t, b quarks, $\varepsilon_{uR} \rightarrow \varepsilon_{dR}, \varepsilon_{cR}, \varepsilon_{sR}, \varepsilon_{tR}, \varepsilon_{bR}$
 e, μ, τ , leptons, $\rightarrow \varepsilon_{eR}, \varepsilon_{\mu R}, \varepsilon_{\tau R}$

3. Constraints and Signatures

SM fermion couplings to W and Z bosons

Coupling to W : $g_L^W = \frac{e}{\sqrt{1 - \frac{M_W^2}{M_Z^2}}} [1 + O(x^4)]$

T₃ coupling to Z : $g_{3L}^Z = \frac{eM_W}{M_Z \sqrt{1 - \frac{M_W^2}{M_Z^2}}} [1 + O(x^4)]$

Hypercharge coupling to Z : $g_{Y_L}^Z = -\frac{eM_Z}{M_W} \sqrt{1 - \frac{M_W^2}{M_Z^2}} [1 + O(x^4)]$

Fermion couplings to W and Z bosons are
of very nearly the SM form

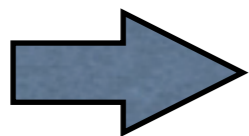
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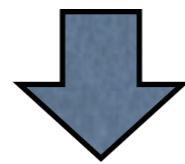
Three site Higgsless model with ideal fermion delocalization is consistent with EW precision tests

Triple gauge boson vertices

Hagiwara-Peccei-Zeppenfeld-Hikasa notation (Nucl. Phys. B 282, 253 (1987))

$$\begin{aligned}\mathcal{L}_{TGV} &= -ie \frac{c_Z}{s_Z} [1 + \Delta\kappa_Z] W_\mu^+ W_\nu^- Z^{\mu\nu} - ie [1 + \Delta\kappa_\gamma] W_\mu^+ W_\nu^- A^{\mu\nu} \\ &- ie \frac{c_Z}{s_Z} [1 + \Delta g_1^Z] (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) Z_\nu \\ &- ie (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) A_\nu ,\end{aligned}$$

$$g_{ZWW} = e \frac{c_Z}{s_Z} \left(1 + \frac{1}{8c^2} x^2 + \dots \right)$$



$$\Delta\kappa_Z = \Delta g_1^Z = \frac{1}{8c^2} x^2 \quad (\Delta\kappa_\gamma = 0)$$

Triple gauge boson vertices

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$$\Delta\kappa_Z = \Delta g_1^Z = \frac{1}{8c^2} x^2$$

95% C.L. upper limit from LEP-II: $\Delta g_1^Z < 0.028$

$$x \leq 0.42 \implies M_{W'} \simeq \frac{2}{x} M_W \geq 380 \text{ GeV}$$

Triple gauge boson vertices

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$$\Delta\kappa_Z = \Delta g_1^Z = \frac{1}{8c^2} x^2$$

95% C.L. upper limit from LEP-II: $\Delta g_1^Z < 0.028$

$$x \leq 0.42 \implies \varepsilon_L \simeq \frac{x}{\sqrt{2}} \simeq 0.3 \times \left(\frac{380 \text{ GeV}}{M_{W'}} \right)$$

LHC Signatures

He, Kuang, Qi, Zhang, Belyaev, Chivukula, Christensen, Pukhov and Simmons
arXiv.0708.2588 [hep-ph]

Belyaev, arXiv.0711.1919 [hep-ph]

Big advantage :

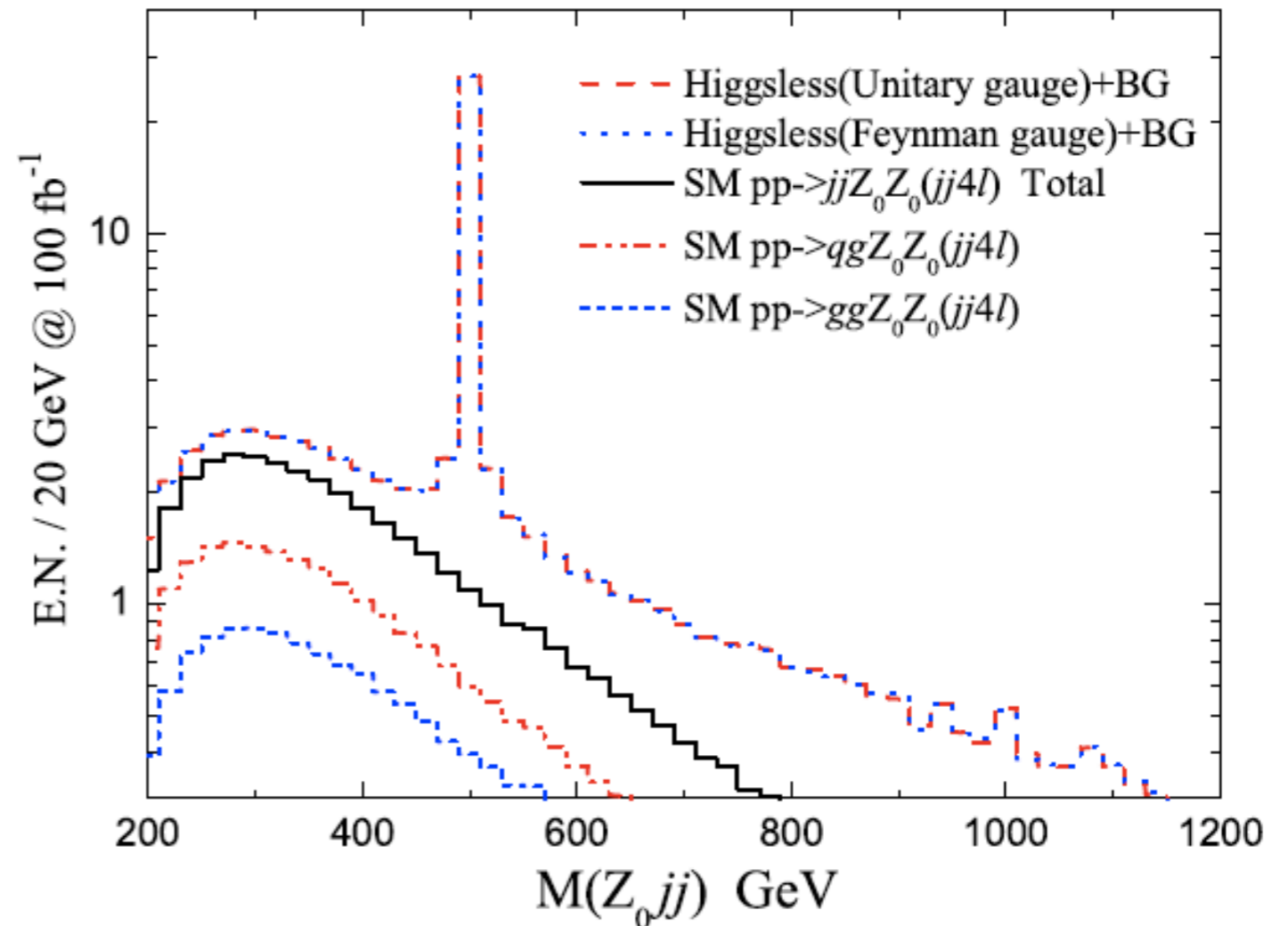
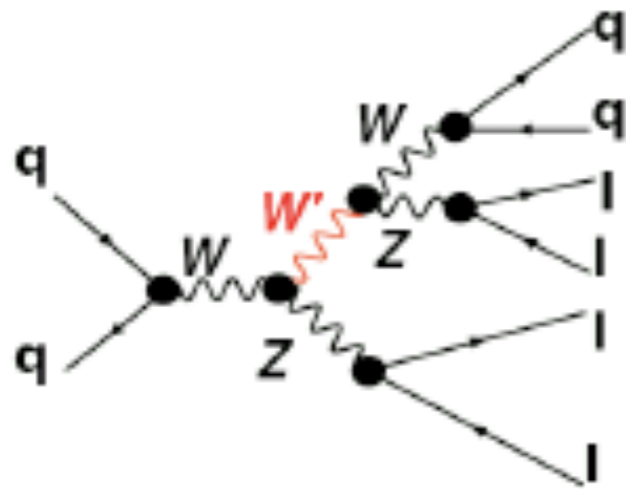
Three-site model is a complete gauge-invariant effective theory
⇒ it is possible to do a full calculation of signal and background
in a self-consistent way

LHC Signatures

He, Kuang, Qi, Zhang, Belyaev, Chivukula, Christensen, Pukhov and Simmons
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Discovering the W' boson via $pp \rightarrow (W)^* \rightarrow (W')^* Z \rightarrow W Z Z$

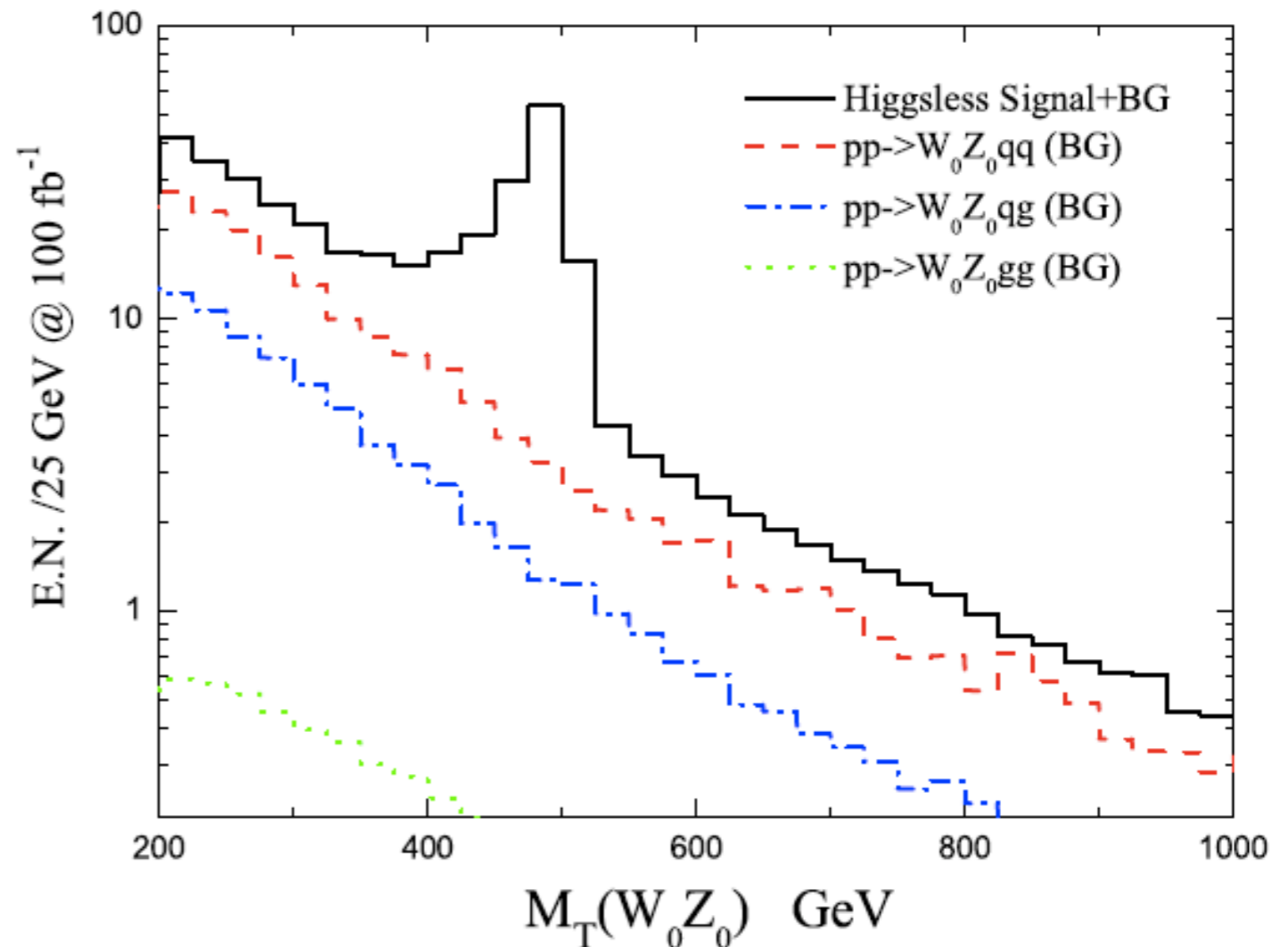
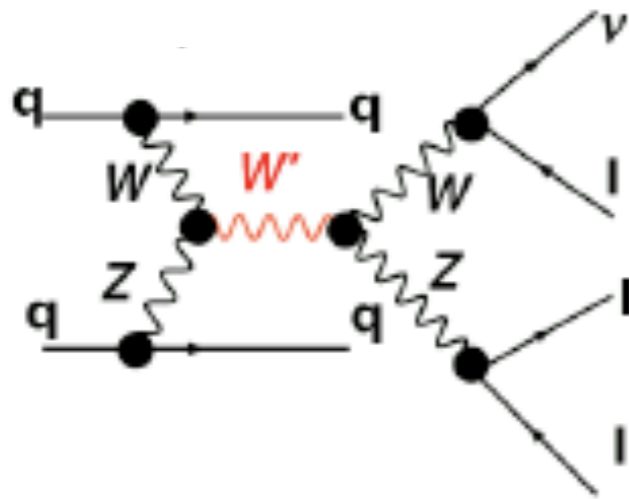


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Belyaev, arXiv.0711.1919 [hep-ph]

Discovering the W' boson via $pp \rightarrow WZjj$



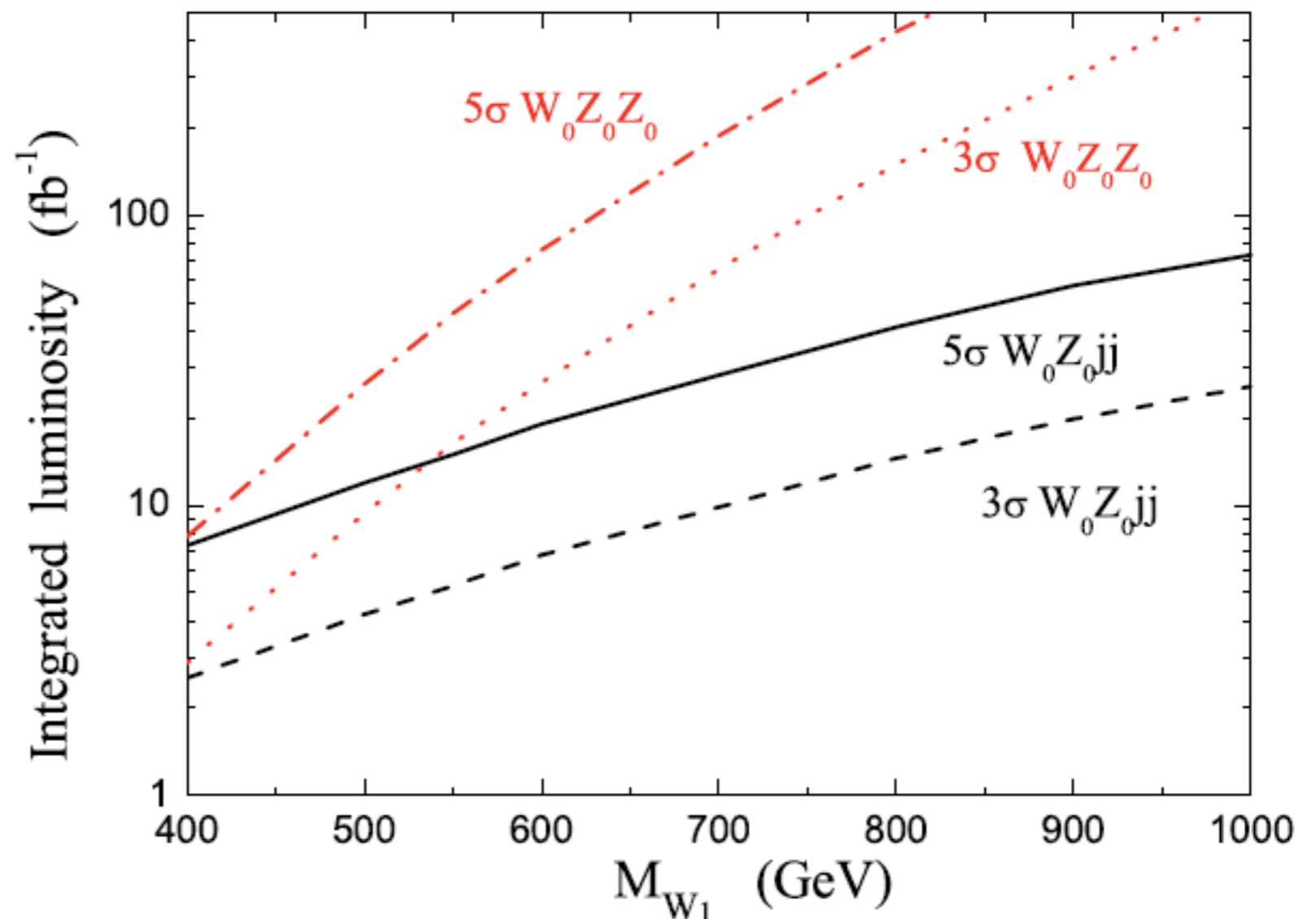
LHC Signatures

He, Kuang, Qi, Zhang, Belyaev, Chivukula, Christensen, Pukhov and Simmons
arXiv.0708.2588 [hep-ph]

Belyaev, arXiv.0711.1919 [hep-ph]

- Integrated luminosities required for 3σ and 5σ detection of W' signals as a function of $M_{W'}$

Heavy gauge boson W' can be discovered within the first few years' run at the LHC

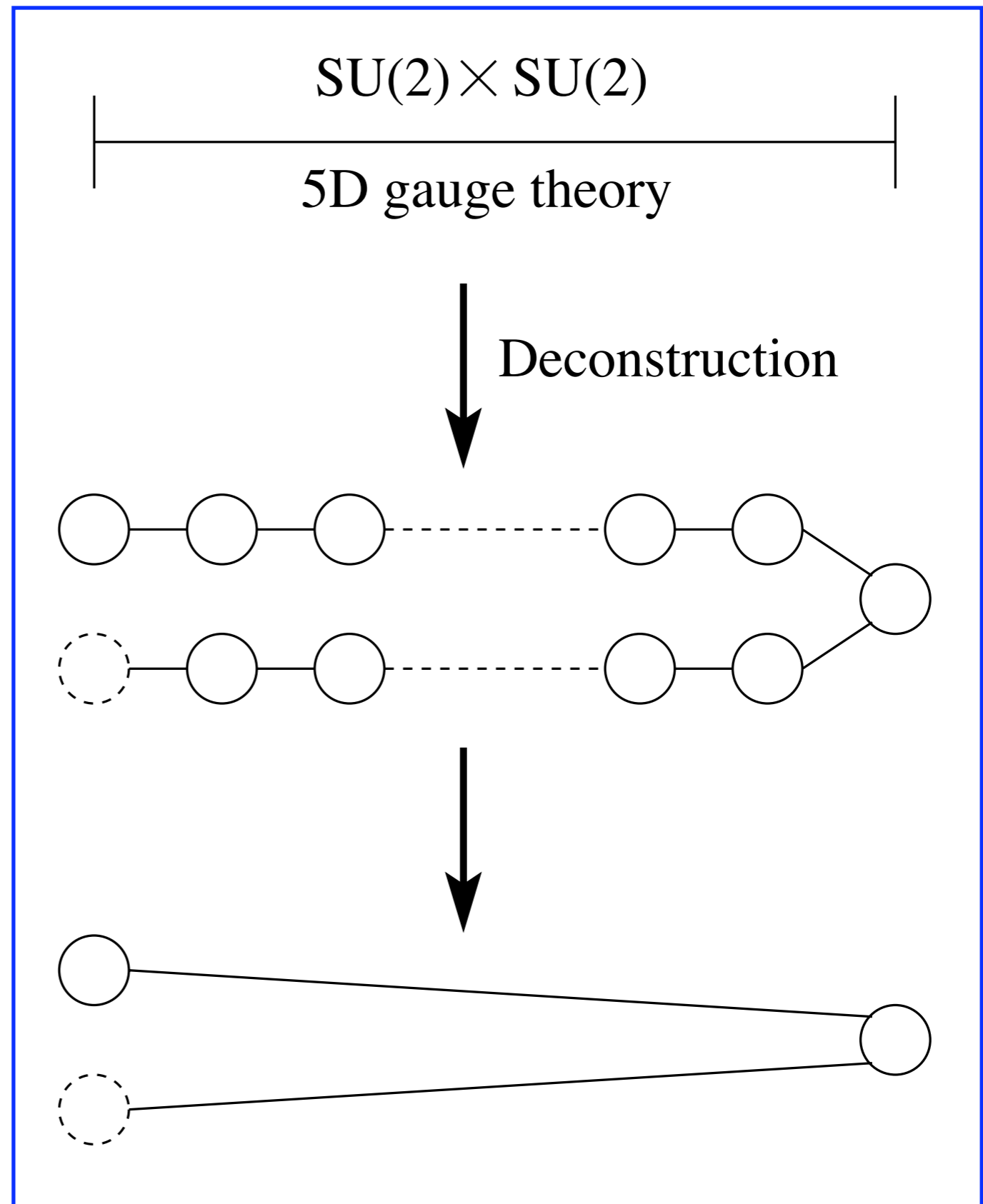


4. Comparison to Continuum Models

Three site model can be viewed as a highly deconstructed version of $SU(2)_L \times SU(2)_R$ 5D Higgsless models

Here, we discuss how well the three site model approximates 5D Higgsless models

We compare the model to 5D Higgsless model with a **flat** and a **warped** extra dimension



$SU(2)_L \times SU(2)_R$ Higgsless Model with a flat extra dimension

$$0 \leq z \leq \pi R$$

$$S_{5D} = \int_0^{\pi R} dz \int d^4x \frac{1}{g_5^2} \left[-\frac{1}{4} W_{\mu\nu}^{La} W_{\alpha\beta}^{La} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W_{\mu z}^{La} W_{\nu z}^{La} \eta^{\mu\nu} \right] \\ + \frac{1}{g_5^2} \left[-\frac{1}{4} W_{\mu\nu}^{Ra} W_{\alpha\beta}^{Ra} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W_{\mu z}^{Ra} W_{\nu z}^{Ra} \eta^{\mu\nu} \right]$$

We introduce brane localized kinetic terms for $SU(2)_L$ and $U(1)_Y$ at $z = 0$ to achieve $M_W/M_{W'} \ll 1$

$$S_{z=0} = \int_0^{\pi R} dz \int d^4x \delta(z - \epsilon) \left[-\frac{1}{4g_0^2} W_{\mu\nu}^{La} W_{\alpha\beta}^{La} \eta^{\mu\alpha} \eta^{\nu\beta} - \frac{1}{4g_Y^2} W_{\mu\nu}^{R3} W_{\alpha\beta}^{R3} \eta^{\mu\alpha} \eta^{\nu\beta} \right] \\ (\epsilon \rightarrow 0+)$$

Four free parameters : R, g_5, g_0, g_Y

$SU(2)_L \times SU(2)_R$ Higgsless Model with a **warped** extra dimension

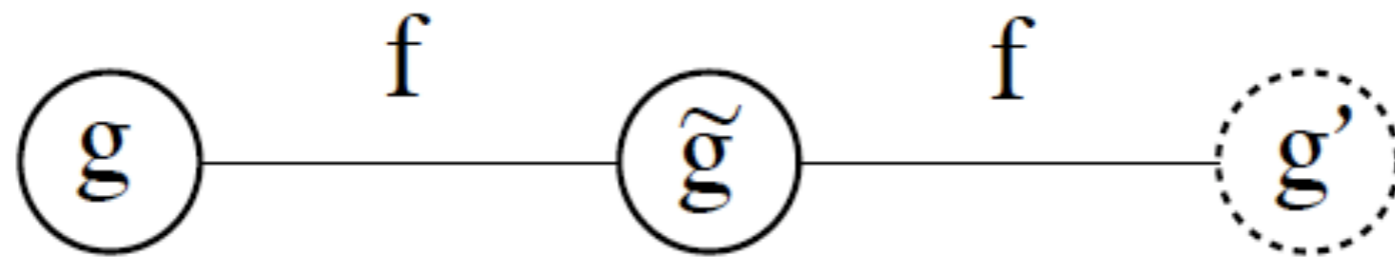
$$R \left(\equiv R' e^{-b/2} \right) \leq z \leq R'$$

$$S_{5D} = \int_R^{R'} dz \frac{R}{z} \int d^4x \frac{1}{g_5^2} \left[-\frac{1}{4} W_{\mu\nu}^{La} W_{\alpha\beta}^{La} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W_{\mu z}^{La} W_{\nu z}^{La} \eta^{\mu\nu} \right] \\ + \frac{1}{g_5^2} \left[-\frac{1}{4} W_{\mu\nu}^{Ra} W_{\alpha\beta}^{Ra} \eta^{\mu\alpha} \eta^{\nu\beta} + \frac{1}{2} W_{\mu z}^{Ra} W_{\nu z}^{Ra} \eta^{\mu\nu} \right]$$

We introduce brane localized kinetic terms for $U(1)_Y$ at $z = R$ in order to arrange non-trivial weak mixing angle ($M_W/M_{W'} \ll 1$ is achieved by taking $b \gg 1$)

$$S_{z=0} = \int_R^{R'} dz \int d^4x \delta(z - R - \epsilon) \left[-\frac{1}{4g_Y^2} W_{\mu\nu}^{R3} W_{\alpha\beta}^{R3} \eta^{\mu\alpha} \eta^{\nu\beta} \right] \\ (\epsilon \rightarrow 0+)$$

Four free parameters : R' , b , g_5 , g_Y



EW gauge sector of the three site model also has four free parameters

We choose four free parameters in each model to fix the values of four physical quantities (three EW quantities + $M_{W'}$)

We calculate other quantities and compare the results in each model to see how reasonable the three site Higgsless model is as a low energy effective theory of each 5D Higgsless model

We focus on triple-gauge-boson couplings

Triple-gauge-boson couplings of SM gauge bosons

Hagiwara-Peccei-Zeppenfeld-Hikasa notation (Nucl. Phys. B 282, 253 (1987))

$$\begin{aligned} \mathcal{L}_{TGV} = & -ie \frac{c_Z}{s_Z} [1 + \Delta\kappa_Z] W_\mu^+ W_\nu^- Z^{\mu\nu} - ie [1 + \Delta\kappa_\gamma] W_\mu^+ W_\nu^- A^{\mu\nu} \\ & - ie \frac{c_Z}{s_Z} \left[1 + \Delta g_1^Z \right] (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) Z_\nu \\ & - ie (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) A_\nu, \end{aligned}$$

$$\Delta\kappa_\gamma = 0, \quad \Delta\kappa_Z = \Delta g_1^Z$$

	Three Site	5D Flat	5D Warped
Δg_1^Z	$\frac{1}{2} \frac{1}{c^2} \frac{M_W^2}{M_{W'}^2}$	$\frac{\pi^2}{12} \frac{1}{c^2} \frac{M_W^2}{M_{W'}^2}$	$\frac{3x_1^2}{16} \frac{1}{c^2} \frac{M_W^2}{M_{W'}^2}$

($x_1 \simeq 2.4048$)

$$\frac{\Delta g_1^Z|_{\text{three-site}}}{\Delta g_1^Z|_{\text{flat-5D}}} \simeq 0.61, \quad \frac{\Delta g_1^Z|_{\text{three-site}}}{\Delta g_1^Z|_{\text{warped-5D}}} \simeq 0.46$$

Difference appears only in the next to leading order term

Triple-gauge-boson couplings which involve a heavy gauge boson

	Three Site	5D Flat	5D Warped
$g_{Z'WW}$	$-\frac{1}{2} \frac{e}{s} \left(\frac{M_W}{M_{W'}} \right)$	$-\frac{4\sqrt{2}}{\pi^2} \frac{e}{s} \left(\frac{M_W}{M_{W'}} \right)$	$-0.36 \left(\frac{M_W}{M_{W'}} \right)$
$g_{ZW'W}$	$-\frac{1}{2} \frac{e}{sc} \left(\frac{M_W}{M_{W'}} \right)$	$-\frac{4\sqrt{2}}{\pi^2} \frac{e}{sc} \left(\frac{M_W}{M_{W'}} \right)$	$-0.36 \frac{1}{c} \left(\frac{M_W}{M_{W'}} \right)$

- Values of $g_{Z'WW}$, $g_{ZW'W}$ are suppressed by a factor of $M_W/M_{W'}$
- Values of $g_{Z'WW}$, $g_{ZW'W}$ in flat and warped 5D Higgsless models are almost the same

$$\frac{g_{Z'WW}|_{\text{warped-5D}}}{g_{Z'WW}|_{\text{flat-5D}}} \simeq \frac{g_{ZW'W}|_{\text{warped-5D}}}{g_{ZW'W}|_{\text{flat-5D}}} \simeq 1$$
- Values of $g_{Z'WW}$, $g_{ZW'W}$ in the three site model are only about 13% smaller than those in 5D Higgsless models

$$\frac{g_{Z'WW}|_{\text{three-site}}}{g_{Z'WW}|_{\text{warped-5D}}} \simeq \frac{g_{ZW'W}|_{\text{three-site}}}{g_{ZW'W}|_{\text{warped-5D}}} \simeq 0.87$$

Question

Why do $gz'WW$ and $gzW'W$ take similar values in different models?

Question

Why do $g_{Z'WW}$ and $g_{ZW'W}$ take similar values in different models?

- Sum rules
- Lowest KK mode dominance

Sum rules for the cancelation of E^4 term in $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$

$$\sum_i g_{Z^{(i)}WW}^2 = g_{WWW} - g_{ZWW}^2 - g_{\gamma WW}^2$$

Sum rules for the cancelation of E^2 term in $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$

$$3 \sum_i g_{Z^{(i)}WW}^2 M_{Z^{(i)}}^2 = 4g_{WWW} M_W^2 - 3g_{ZWW}^2 M_Z^2$$

Note : In the case of deconstructed models, additional contributions from NG boson scattering appear

(i =1 term of LHS)/RHS

	Three Site	5D Flat	5D Warped
$\frac{g_{Z'WW}^2}{g_{WWW} - g_{ZWW}^2 - g_{\gamma WW}^2}$	1	$\frac{960}{\pi^6} \simeq 0.996$	0.992
$\frac{3g_{Z'WW}^2 M_{Z'}^2}{4g_{WWW} M_W^2 - 3g_{ZWW}^2 M_Z^2}$	$\frac{3}{4}^*$	$\frac{96}{\pi^4} \simeq 0.986$	0.986

* Remaining 1/4 comes from the NG-boson scattering amplitude
Chivukula, Simmons He, Kurachi, Tanabashi, in preparation

5. Summary

- Three site Higgsless model is based on the 4-dimensional gauge invariant Lagrangian
- Three site Higgsless model is consistent with EW precision measurement, and main constraint comes from measurement on triple-gauge-boson couplings
- Heavy gauge boson W' can be discovered within the first few years' run at the LHC
- Due to the sum rules and lowest KK mode dominance, some phenomenologies of the three site Higgsless model and 5-dimensional Higgsless models are expected to be very similar