A MODEL BUILDING BY COSET SPACE DIMENSIONAL REDUCTION USING 10 DIMENSIONAL COSET SPACES

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Outline

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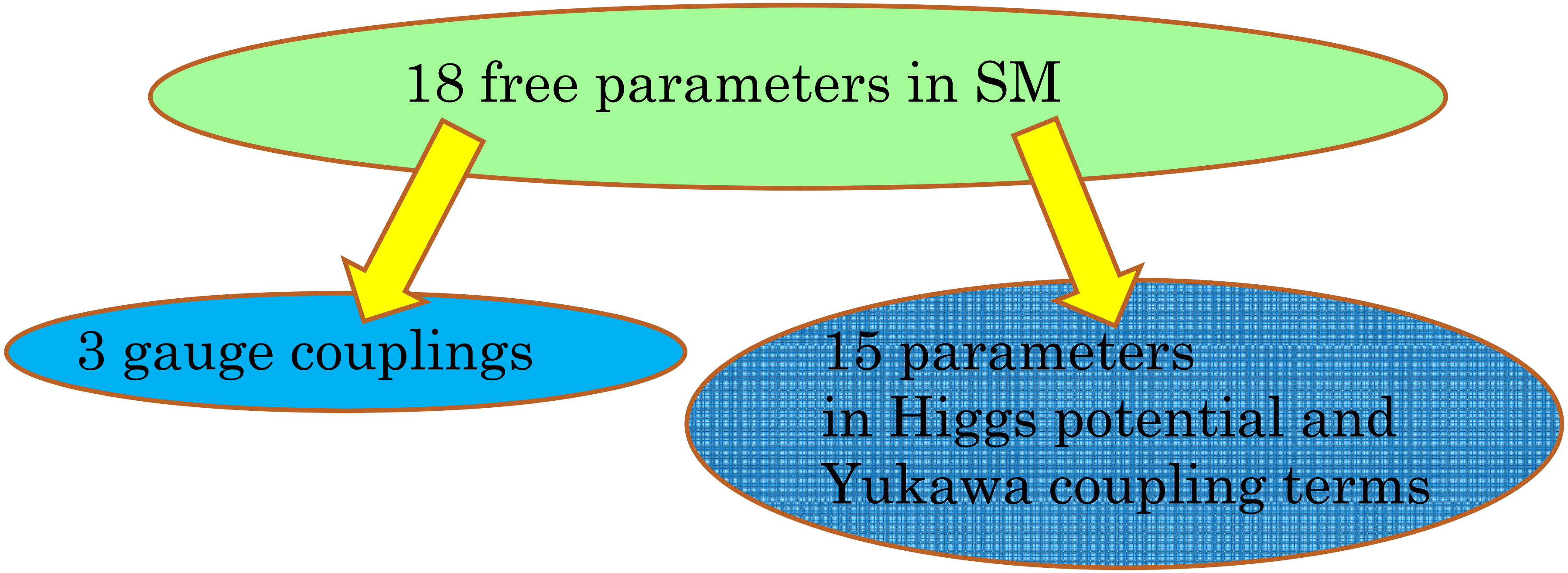
1. Introduction

Free parameters in standard model(SM)



Most of the free parameters are associated with Higgs particle

A model which describe origin of Higgs sector would be interesting!

1. Introduction

What kind of model would have origin of Higgs?

→ A model with extra dimensional spaces

Extra dimensional components of gauge field can be seen as scalar field in four dimensions

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Extra dimensional components of gauge field would be origin of Higgs field

→ The idea of gauge-Higgs unification model

Among these models we are interested in models based on coset space dimensional reduction(CSDR) scheme

(D. Kapetanakis and G. Zoupanos, Phys. Rept. 219(1992) 1.)

In CSDR scheme…

4-dim theory obtained from higher dimensional theory is strongly restricted

⬇

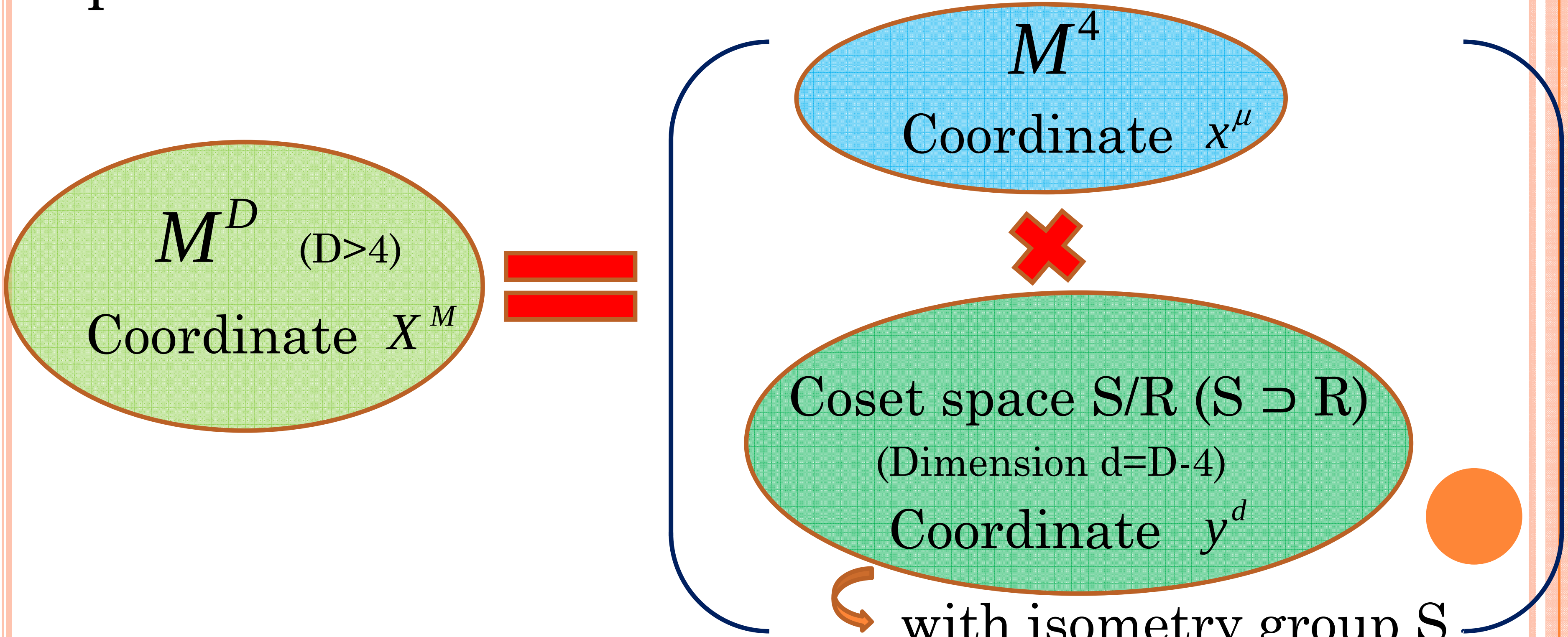
It is interesting to investigate models based on CSDR scheme

2. CSDR scheme

Basic theory

Gauge theory(gauge group G)
defined on D-dimensional space-time M^D

Space-time structure



Action in higher dimensional space-time

$$\int L(x,y)dX^D = \int [-\frac{1}{8}Tr(F^{MN}F^{KL})g_{MK}g_{NL} + \frac{1}{2}i\overline{\Psi}D^M\Gamma_M\Psi]dX^D$$

(Gauge group G)

$dX^D = dx^4 dy^d$

$F^{MN}(x,y)$: **gauge field strength(M, N, … = 0-D(>4))**

g^{MN} : **metric** $(D,M,N,\ldots=\{(\mu,\nu,\ldots=0,..3),(a,b,..4,..D-d)\})$

$\Psi(x,y)$: **fermion(in representation F of G)**

Γ^M **:higher dimensional gamma matrix**

D^M **:covariant derivative**

We impose "symmetric condition" on the fields to carry out dimensional reduction

Symmetric condition

We can write symmetric condition for each fields as

$$A_\mu(x,y) = g(s)A_\mu(x,s^{-1}y)g^{-1}(s)$$

$$A_a(x,y) = g(s)J_\alpha{}^\beta A_\beta(x,s^{-1}y)g^{-1}(s) + g(s)\partial_\alpha g^{-1}(s)$$

$$\psi(x,y) = f(s)\Omega(y,s)\psi(x,s^{-1}y)f^{-1}(s)$$

$g(s)$:gauge transformation for gauge field (adjoint rep of G)
$f(s)$:gauge transformation for fermion (representation F of G)
$\Omega(s,y)$:rotation of spinor field for coordinate transformation
$J_\alpha{}^\beta$:Jacobian for coordinate transformation

→ It connects transformation of S/R coordinates
$(x,y) \to (x,sy)$ s(∈S)
and gauge transformation.

Coordinate transformation is compensated by gauge transformation

As a consequence of symmetric condition…

Lagrangian in higher dimensions becomes independent of extra dimensional space coordinate y

$$L(x, y^s) = L(x, y)$$

$$\Rightarrow \quad L(x, y) \longrightarrow L(x)$$

Extra dimension can be integrated out

$$\int L(x) dx^4 dy^d = \int L^{eff}(x) dx^4$$

Structure of this 4-dim Lagrangian is restricted by constraints derived from symmetric condition

2. CSDR scheme

Constraints for each field in higher dimension

Gauge field

- **4-dim components** A_μ

$$\partial_a A_\mu = 0$$

$$[J_i, A_\mu] = 0$$

(**J : R generators in Lie algebra of G**)

- **S/R components** A_a

$$\partial_a A_b - \partial_b W_a = 1/2 f^c{}_{ab} A_c$$

$$[J_i, A_a] = f^c{}_{ia} A_c$$

(W : Lie algebra of G,
f : structure const of S)

- **fermion** ψ

$$\partial_a \psi = 0$$

$$T_i \psi = W_i \psi$$

(**T:generator of SO(d)**)

These constraints give rules for identifying 4-dim gauge symmetry group H and its representations under which fields transform.

Rules for identifying gauge group and field contents in 4-dim

Gauge group H in 4-dim

⇒ Centralizer of R in G （G＝H×R)

Scalar contents

$$G \supset R_G \times H, adjG = (adjR, 1) + (1, adjH) + \sum (r_i, h_i)$$

$$S \supset R, adjS = adjR + \sum s_i$$

r_i, s_i :Irreducible representation of R
h_i :Irreducible representation of H

h_i is Higgs representation in 4-dim when $r_i = s_i$

fermion

$$G \supset R_G \times H, F = \sum (r_i, h_i)$$

$$S \supset R, \sigma_d = \sum \sigma_i$$

F :Fermion's representation of G
σ_d :SO(d) spinor
r_i, σ_i :Irreducible representation of R

h_i is fermion representation in 4-dim when $r_i = \sigma_i$

Set up and results in CSDR scheme

Set up(in higher dimension)

Lagrangian

$L(x,y)$

Gauge theory with fermion

- Number of dimension D
- Gauge group G
- Coset space S/R
- Fermion representtions F
- Embedding of R in G

Results(in 4-dimension)

Lagrangian

$L^{eff}(x,y)$

Gauge theory with fermion and Higgs

Gauge group H
Fermion representations
Higgs representations
Structure of the Lagrangian
are determined.

Lagrangian in 4-dimensions

$$L^{eff}(x) = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(D_\mu\phi_a)(D^\mu\phi^a)^* + V(\phi) + \frac{1}{2}i\overline{\Psi}\gamma^\mu D_\mu\Psi - \frac{1}{2}i\overline{\Psi}\Gamma^a D_a\Psi$$

$$V(\phi) = -\frac{1}{8}g^{ac}g^{bd}Tr[(f_{ab}{}^{C}\phi_C - [\phi_a,\phi_b])(f_{cd}{}^{D}\phi_D - [\phi_c,\phi_d])] \qquad (A_a \equiv \phi_a)$$

3 . Model building-our result

By CSDR scheme…

Possibilities of model building were well investigated for D≤10 cases.

(D. Kapetanakis and G. Zoupanos, Phys. Rept. 219(1992) 1.)

↓

SM could not be obtained in these approaches

↓

We studied a model building for D=14

(To obtain chiral fermion we required D=4n+2)

using 10-dimensional coset spaces

(To obtain chiral fermion we require rankR=rankS)

To specify a model we need to fix in higher dimensions

⇒ coset space S/R, gauge group G, fermion representation F and embedding of R in G

•restrictions

- We need S/R with rankS=rankR
- Gauge group in higher dimension G should have complex or pseudreal representations
- We only consider regular subgroup of G
- rankG≦8
- We consider candidates of gauge group in 4-dim as

$$E_6, SO(10), SU(5)(\times U(1))$$

$$SU(3)\times SU(2)\times U(1)(\times U(1))$$

Classification of 10 dimensional S/R(rankS=rankR)

(1)$SU(4)/SU(2) \times U(1) \times U(1)$
(2)$G_2/SU(2) \times U(1)$
(3)$Sp(6)/Sp(4) \times U(1)$
(4)$Sp(4) \times SU(2)/[U(1) \times U(1)] \times U(1)$
(5)$G_2 \times (SU(2))^2/SU(3) \times (U(1))^2$
(6)$SU(3) \times (SU(2))^2/[U(1) \times U(1)] \times (U(1))^2$
(7)$Sp(4) \times (SU(2))^2/[SU(2) \times U(1)]_{non-max} \times (U(1))^2$
(8)$G_2 \times SU(3)/SU(3) \times [SU(2) \times U(1)]$
(9)$SU(3) \times SU(3)/[U(1) \times U(1)] \times [SU(2) \times U(1)]$
(10)$Sp(4) \times SU(3)/[SU(2) \times U(1)]_{non-max} \times [SU(2) \times U(1)]$
(11)$G_2 \times Sp(4)/SU(3) \times [SU(2) \times SU(2)]$
(12)$SU(3) \times Sp(4)/[U(1) \times U(1)] \times [SU(2) \times SU(2)]$
(13)$Sp(4) \times Sp(4)/[SU(2) \times U(1)]_{non-max} \times [SU(2) \times SU(2)]$
(14)$G_2 \times SU(2)/SU(2) \times SU(2) \times U(1)$
(15)$SU(4) \times SU(2)/SU(2) \times SU(2) \times U(1) \times U(1)$
(16)$SO(11)/SO(10)$
(17)$SU(6)/SU(5) \times U(1)$
(18)$SU(5) \times SU(2)/[SU(4) \times U(1)] \times U(1)$
(19)$SO(9) \times SU(2)/SO(8) \times U(1)$
(20)$Sp(6) \times SU(2)/[Sp(4) \times SU(2)] \times U(1)$
(21)$SU(4) \times (SU(2))^2/[SU(3) \times U(1)] \times (U(1))^2$
(22)$SO(7) \times (SU(2))^2/SO(6) \times (U(1))^2$

(23)$Sp(4) \times (SU(2))^2/[SU(2) \times U(1)]_{max} \times (U(1))^2$
(24)$SU(4) \times SU(3)/[SU(3) \times U(1)] \times [SU(2) \times U(1)]$
(25)$SO(7) \times SU(3)/SO(6) \times [SU(2) \times U(1)]$
(26)$Sp(4) \times SU(3)/[SU(2) \times U(1)]_{max} \times [SU(2) \times U(1)]$
(27)$SU(4) \times Sp(4)/[SU(3) \times U(1)] \times [SU(2) \times SU(2)]$
(28)$SO(7) \times Sp(4)/SO(6) \times [SU(2) \times SU(2)]$
(29)$Sp(4) \times Sp(4)/[SU(2) \times U(1)]_{max} \times [SU(2) \times SU(2)]$
(30)$(SU(3))^2 \times SU(2)/[SU(2) \times U(1)]^2 \times U(1)$
(31)$(Sp((4))^2 \times SU(2)/[SU(2) \times SU(2)]^2 \times U(1)$
(32)$Sp(4) \times SU(3) \times SU(2)/[SU(2) \times SU(2)] \times [SU(2) \times U(1)] \times U(1)$
(33)$SU(3) \times (SU(2))^3/[SU(2) \times U(1)] \times (U(1))^3$
(34)$Sp(4) \times (SU(2))^3/[SU(2) \times SU(2)] \times (U(1))^3$
(35)$(SU2/U(1))^5$

35 coset spaces

We construct a model with these coset spaces

Pairs of (S/R,G) satisfying our restrictions

S/R\H	SO(10) × U(1)	SU(5) × U(1)	SU(3) × SU(2) × U(1)	SU(3) × SU(2) × U(1) × U(1)
(1)				SU(7), SO(12), SO(13), Sp(12), E_6
(2)	SO(14)	SU(7), SO(13), Sp(12)	SO(10), SO(11), Sp(10)	SU(7), SO(12), SO(13), Sp(12), E_6
(3)		Sp(14)	Sp(12)	Sp(16)
(5)				SU(8), Sp(14)
(8)				Sp(16)
(10)				SO(14), Sp(14)
(12)				SO(14), Sp(14)
(13)		Sp(16)	SO(14), Sp(14)	Sp(16)
(14)		SO(14), Sp(14)	SO(13), Sp(12)	SO(14), Sp(14)
(15)				SO(14), Sp(14)
(18)				SU(9), Sp(16)
(20)		Sp(16)	Sp(14)	Sp(16)
(22)				SU(9), Sp(16)
(24)				Sp(16)
(26)				SO(14), Sp(14)
(29)		Sp(16)	SO(14), Sp(14)	Sp(16)
(31)			Sp(16)	
(32)				Sp(16)

- Each block has G corresponding to each S/R and H
- Numbers for S/R correspond to numbers in S/R table

3. Model building

We used these fermion representation F for each gauge group G in higher dimensions

$$G = SU(7) : F = 21,28,35,84$$

$$G = SO(12) : F = 32,352$$

$$G = SO(13) : F = 64,768$$

$$G = Sp(12) : F = 208,364$$

$$G = E_6 : F = 27,351$$

$$G = SO(14) : F = 64,832$$

$$G = Sp(14) : F = 350$$

$$G = Sp(16) : F = 544$$

We investigated models based on above (G,S/R,F) combinations

Results(1)

- **The cases in which GUT-like gauge group is obtained in 4-dimensions**

S/R\H	SO(10) × U(1)	SU(5) × U(1)	SU(3) × SU(2) × U(1)	SU(3) × SU(2) × U(1) × U(1)
(1)				SU(7), SO(12), SO(13), Sp(12), E_6
(2)	SO(14)	SU(7), SO(13), Sp(12)	SO(10), SO(11), Sp(10)	SU(7), SO(12), SO(13), Sp(12), E_6
(3)		Sp(14)	Sp(12)	Sp(16)
(5)				SU(8), Sp(14)
(8)				Sp(16)
(10)				SO(14), Sp(14)
(12)				SO(14), Sp(14)
(13)		Sp(16)	SO(14), Sp(14)	Sp(16)
(14)		SO(14), Sp(14)	SO(13), Sp(12)	SO(14), Sp(14)
(15)				SO(14), Sp(14)
(18)				SU(9), Sp(16)
(20)		Sp(16)	Sp(14)	Sp(16)
(22)				SU(9), Sp(16)
(24)				Sp(16)
(26)				SO(14), Sp(14)
(29)		Sp(16)	SO(14), Sp(14)	Sp(16)
(31)			Sp(16)	
(32)				Sp(16)

⇒ We can't obtain Higgs which break GUT gauge symmetry

(Representations obtained from adjG are limited)

⇒ We can't construct a realistic model by these cases

Results(2)

- **cases with H＝SU(3)×SU(2)×U(1)**

S/R\H	SO(10) × U(1)	SU(5) × U(1)	SU(3) × SU(2) × U(1)	SU(3) × SU(2) × U(1) × U(1)
(1)				SU(7), SO(12), SO(13), Sp(12), E_6
(2)	SO(14)	SU(7), SO(13), Sp(12)	SO(10), SO(11), Sp(10)	SU(7), SO(12), SO(13), Sp(12), E_6
(3)		Sp(14)	Sp(12)	Sp(16)
(5)				SU(8), Sp(14)
(8)				Sp(16)
(10)				SO(14), Sp(14)
(12)				SO(14), Sp(14)
(13)		Sp(16)	SO(14), Sp(14)	Sp(16)
(14)		SO(14), Sp(14)	SO(13), Sp(12)	SO(14), Sp(14)
(15)				SO(14), Sp(14)
(18)				SU(9), Sp(16)
(20)		Sp(16)	Sp(14)	Sp(16)
(22)				SU(9), Sp(16)
(24)				Sp(16)
(26)				SO(14), Sp(14)
(29)		Sp(16)	SO(14), Sp(14)	Sp(16)
(31)			Sp(16)	
(32)				Sp(16)

We can't obtain some of SM fermions

(U(1) charges which can be obtained in this cases are limited)

We can't construct a realistic model by these cases

Results(3)

- **cases with H=SU(3)×SU(2)×U(1)×U(1)**

S/R\H	SO(10) × U(1)	SU(5) × U(1)	SU(3) × SU(2) × U(1)	SU(3) × SU(2) × U(1) × U(1)
(1)				SU(7), SO(12), SO(13), Sp(12), E_6
(2)	SO(14)	SU(7), SO(13), Sp(12)	SO(10), SO(11), Sp(10)	SU(7), SO(12), SO(13), Sp(12), E_6
(3)		Sp(14)	Sp(12)	Sp(16)
(5)				SU(8), Sp(14)
(8)				Sp(16)
(10)				SO(14), Sp(14)
(12)				SO(14), Sp(14)
(13)		Sp(16)	SO(14), Sp(14)	Sp(16)
(14)		SO(14), Sp(14)	SO(13), Sp(12)	SO(14), Sp(14)
(15)				SO(14), Sp(14)
(18)				SU(9), Sp(16)
(20)		Sp(16)	Sp(14)	Sp(16)
(22)				SU(9), Sp(16)
(24)				Sp(16)
(26)				SO(14), Sp(14)
(29)		Sp(16)	SO(14), Sp(14)	Sp(16)
(31)			Sp(16)	
(32)				Sp(16)

Among these candidates only

$$(Sp(12), G_2/SU(2)\times U(1), F=364)$$

$$(SO(13), G_2/SU(2)\times U(1), F=768)$$

$$(Sp(16), G_2\times SU(3)/SU(3)\times SU(2)\times U(1), F=544)$$

cases can give SM field contents in four dimensions

Comparing models which give SM contents, we find…

$(Sp(12), G_2 / SU(2)\times U(1), F = 364)$

$(SO(13), G_2 / SU(2)\times U(1), F = 768)$

⇨ Too many extra field appear including colored Higgs

⇩

These cases are not suitable for model building

The best candidate for model building is the case of

$$G = Sp(16)$$
$$S / R = G_2 \times SU(3) / SU(3)\times SU(2)\times U(1)$$
$$F = 544$$

Representations obtained from

$$(Sp(14), G_2 \times SU(3) / SU(3) \times SU(2) \times U(1), F = 544)$$

Higgs

$(1,2)(-3,2), (1,2)(3,-2) \quad (SU(3), SU(2))(U(1)_Y, U(1)_{ex})$

→ SM Higgs is obtained(with extra U(1))

fermion

SM fermions	SM mirror	Extra fermions
(1,1)(6,-4), (1,2)(-3,2) (3*,1)(-4,7), (3,2)(1,-5) (3*,1)(2,3) **One generation SM fermion** **(3*,1)(2,3),(3*,1)(2,3)**	**(3,1)(-2,-3) (3,1)(-2,-3) (3,1)(-2,-3)**	**(1,1)(0,0), (3,1)(-8,1) (3*,2)(5,1), (8,1)(0,0) (6,1)(2,3),(6*,1)(-2,-3) (1,1)(0,0)**

→ One generation(+α) SM fermions is obtained(with extra U(1))

4. summary

We investigated model building with 10 dimensional coset space S/R

Restrictions
- rankG≤8
- considering only regular subgroup of G

The best candidate of set (G,S/R,F) that induce SM representations is

$$(Sp(14), G_2 \times SU(3) / SU(3) \times SU(2) \times U(1), F = 544)$$

But..
extra U(1)
extra fermions
} are left

Future works

- Extra U(1) should be dropped
- Extra fermions should be dropped

↓

- Imposing other symmetry

  (ex. imposing discrete symmetry Z2 etc)
- Changing symmetric condition

Analysis of Higgs potential is also needed to confirm constructed model

Appendix

How to get chiral fermions

If D=4n+2, D-dimensional Weyl spinor $\sigma_D(\overline{\sigma}_D)$ is decomposed for $\underline{(SU(2)\times SU(2))}\times SO(d)$ as

Lorentz group in SU(2) basis

These are SO(d) spinor and are same rep unless rankS=rankR

$$\sigma_D = (2,1;\sigma_d) + (1,2;\overline{\sigma}_d)$$
$$\overline{\sigma}_D = (2,1;\overline{\sigma}_d) + (1,2;\sigma_d)$$

(2,1):left handed in 4-dim
(1,2):right handed in 4-dim

We impose Weyl condition in higher dimension and take one of above two.

Fermion representations of H is decided by the matching condition.

Different representation can be obtained for left-handed and right-handed fermions in 4-dimension.

Ex)

$$S/R = G_2/SU(3)$$

D=10, d=6

Taking only this by Weyl conditon

$$\sigma_{10} = (2,1;\sigma_6) + (1,2;\bar{\sigma}_6)$$
$$\bar{\sigma}_{10} = (2,1;\bar{\sigma}_6) + (1,2;\sigma_6)$$

$$16 = (2,1;4) + (1,2;\bar{4})$$
$$\overline{16} = (2,1;\bar{4}) + (1,2;4)$$

$$G_2 \supset SU(3)$$

$$4 = 3 + 1$$
$$\bar{4} = \bar{3} + 1$$

Decomposition of SO(d) spinor under R

$$G = SU(5)$$
$$F = 5$$

decomposition

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$
$$5 = (3,1)(-2) + (1,2)(3)$$

(SU(3),SU(2)(U(1))

Applying CSDR constraints and matching condition

H=SU(2) $\times$ U(1), remaining fermion is $1(-2)_L$ SU(2)(U(1))

If we chose real (pseudo real) representation as fermion representation F

By matching condition with

$$\sigma_D = (2,1;\sigma_d) + (1,2;\overline{\sigma}_d)$$

Fermion representations will be doubled

To eliminate the doubling of the fermion spectrum

We impose Majorana condition

To impose Majorana condition

D=8n+2 :F must be real representation

D=8n+6 :F must be pseudo real representation