# Higgsless breaking of Grand Unification

# Higgs effective potential in the warped Gauge-Higgs Unification

Toshifumi Yamashita

(SISSA → Osaka Univ.)

2007/12/19 @GUT07

in preparation

with N. Haba (Osaka Univ.)
S. Matsumoto (Tohoku Univ.)
Y. Sakamura (Osaka Univ.)
N. Okada (KEK)

# Plan

- Introduction
- Warped GHU
- Gauge-Higgs Condition
- Summary

# Introduction

- Gauge-Higgs Unification

D.B. Fairlie (1979)
N.S. Manton (1979)

5D theory

↓ compactification

4D theory

with KK modes

gauge field

$$A_M = (A_\mu, A_5)$$

$A_\mu$ — gauge field

$A_5$ — scalar field ⇒ Higgs

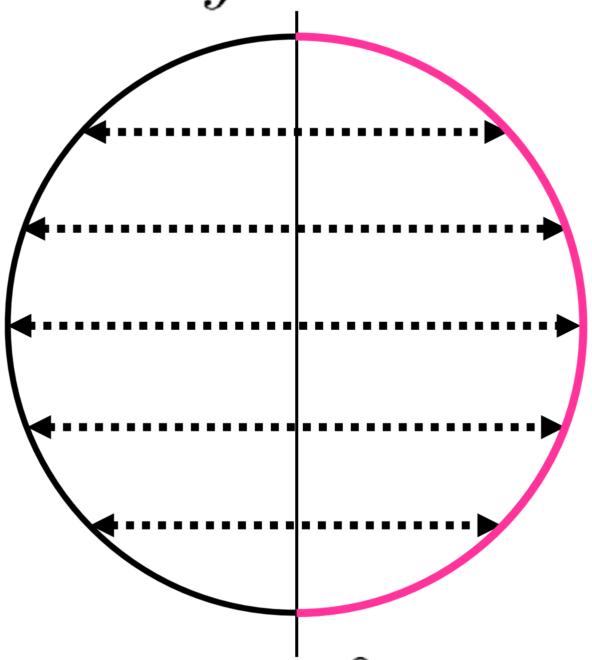
5D gauge invariance protects the Higgs mass!!

H.Hatanaka, T.Inami and C.S.Lim (1998)

# Introduction

- Orbifold

ex) $S^1/Z_2$




$$\mathcal{L}(x^\mu, y) = \mathcal{L}(x^\mu, y + 2\pi R) \quad : \hat{T}$$
$$= \mathcal{L}(x^\mu, -y) \quad : \hat{P}$$
$$\mathcal{L}(x^\mu, \pi R - y) = \mathcal{L}(x^\mu, \pi R + y) \quad : \hat{P}'$$
$$\hat{P}' = \hat{P}\hat{T}^{-1}, \ \hat{T} = \hat{P}\hat{P}'$$

Fields may not be invariant !

ex)
$$A_\mu(x^\mu, -y) = P A_\mu(x^\mu, y) P^\dagger$$
$$A_5(x^\mu, -y) = -P A_5(x^\mu, y) P^\dagger$$
$$\Psi(x^\mu, -y) = \eta_\Psi P \gamma_5 \Psi(x^\mu, y)$$

$P$ : Symm. transformation

# Introduction

- Orbifold breaking

Y.Kawamura (2000)

ex) $\mathrm{SU}(3) \rightarrow \mathrm{SU}(2)\times\mathrm{U}(1)$

$$T = \mathbf{1},\ \ P = \mathrm{diag.}(+,-,-) = P'$$

$$(P,P')A_\mu = \left(\begin{array}{c|cc} (+,+) & (-,-) & (-,-) \\ \hline (-,-) & (+,+) & (+,+) \\ (-,-) & (+,+) & (+,+) \end{array}\right)$$

$$(P,P')A_5 = \left(\begin{array}{c|cc} (-,-) & (+,+) & (+,+) \\ \hline (+,+) & (-,-) & (-,-) \\ (+,+) & (-,-) & (-,-) \end{array}\right)$$

Doublet Higgs!

flat directions ⇨ It is important to calculate effective potential.

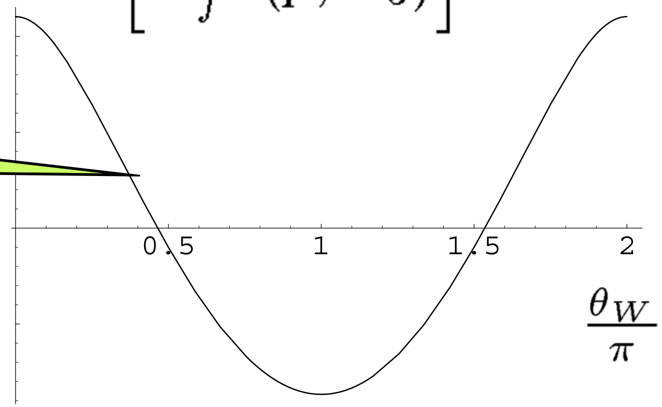
# Introduction

- 1-loop effective potential

M.Kubo, C.S.Lim and H.Yamashita (2002)
N.Haba and T.Y. (2004)

$$V_{\rm eff}(A_5) \sim \sum_w \frac{1}{w^5} \cos(w\theta_W) {\rm tr} \ln \left[\Delta_f^{-1}(p; A_5)\right]$$

finite!!




$$\frac{\theta_W}{\pi} \equiv gRA_5$$

too small!!

When $\theta_W \sim 1$, we get $1/R \sim \langle A_5 \rangle = 246\mathrm{GeV}$.

⇨ We need $\theta_W \ll 1$ ⇦ tuning

# Introduction

- Higgs mass

$$V_H = -\mu^2 H^2 + \lambda (H^2)^2 \Rightarrow m_H = \sqrt{\lambda} \langle H \rangle < m_Z$$

The quartic coupling is zero at the tree level.

⇒ The Higgs mass tends to be too small.

- Top Yukawa

Top Yukawa is larger than the gauge coupling.

In the flat GHU scenario, typically
too small KK scale, Higgs mass & top mass.

# Introduction

- warped GHU

The problems are solved (almost) automatically.

⇨ A lot of studies have been made.

- R. Contino, Y. Nomura and A. Pomarol (2003)
- K. Oda and A. Weiler (2005)
- K. Agashe, R. Contino and A. Pomarol (2005)
- Y. Hosotani and M. Mabe (2005)
- Y. Hosotani, S. Noda, Y. Sakamura and S. Shimasaki (2006)
- Y. Sakamura and Y. Hosotani (2007)

…

The eff. potential is studied less exhaustively.

⇨ We investigate it.

# Plan

- Introduction
- Warped GHU
- Gauge-Higgs Condition
- Summary

# Warped Gauge-Higgs Unification

- Randall-Sundrum

Randall & Sundrum (1999)

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \qquad \sigma(y) = k|y|$$

$$-\pi R \leq y \leq \pi R$$

$$\mathcal{L} = \sqrt{G}\left[-\frac{1}{4}F^{MN}F_{MN} + \cdots\right]$$

$$= -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}e^{-2\sigma}F_{\mu 5}F_{\mu 5} + \cdots$$

$$\mathcal{L}_{GF} = -\frac{1}{2}\left(\partial_\mu A_\mu - \partial_5\left(e^{-2\sigma}A_5\right)\right)^2$$

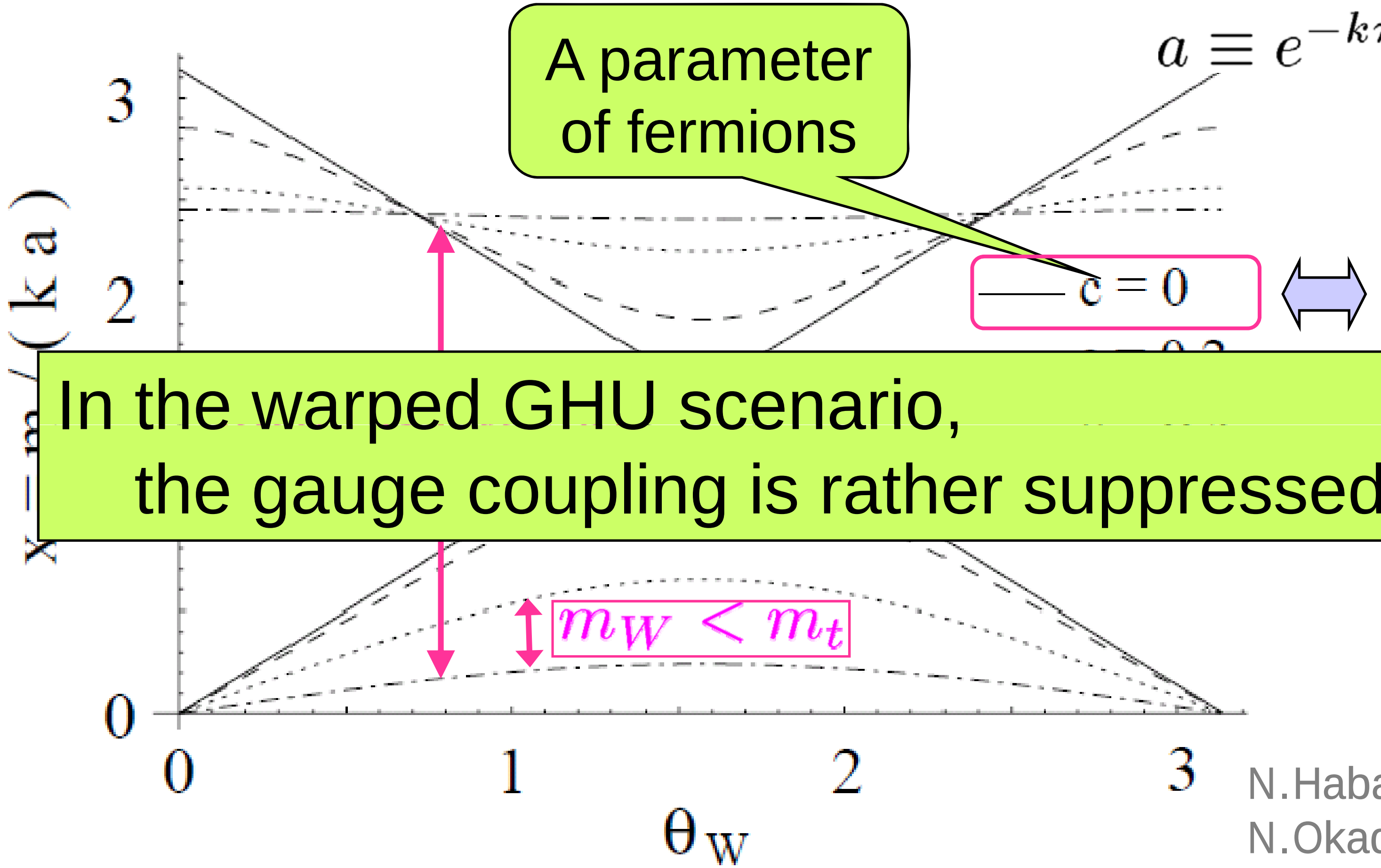
⇨ $\left(e^{2\sigma}D_\mu^2 - D_5^2\right)\left(e^{-2\sigma}A_5\right) = 0$ ⇨ $A_5^{(0)} \propto e^{2\sigma}$

⇨ natural scale : TeV

# Warped Gauge-Higgs Unification

- Weak scale vs. KK & top mass

$a \equiv e^{-k\pi R} = 10^{-15}$




In the warped GHU scenario,
the gauge coupling is rather suppressed.

N.Haba, S.Matsumoto, N.Okada and T.Y.
in preparation

# Warped Gauge-Higgs Unification

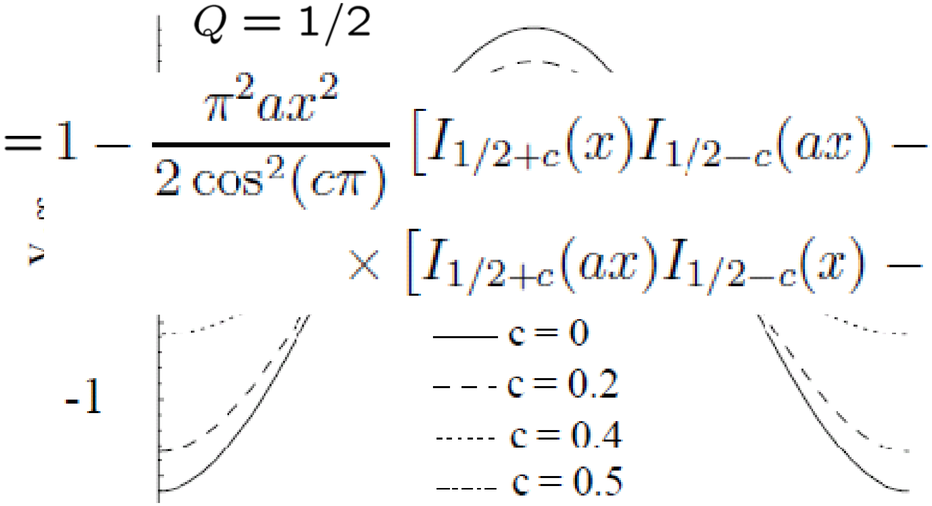
- Effective Potential

+ : periodic
- : anti-periodic

$$V_{\rm eff}(\theta_W; c, Q) = \sum_i \frac{1}{2}\frac{(ka)^4}{(4\pi)^2} v_{\rm eff}(\theta_W; c, Q)$$

$$v_{\rm eff}(\theta_W; c, Q) = 2\int_0^\infty {\rm d}x\; x^3 \ln\left[1 \pm \frac{\cos(2Q\theta_W)}{\bar{N}_c(x;c)}\right]$$

$$\bar{N}_c(x;c) = 1 - \frac{\pi^2 a x^2}{2\cos^2(c\pi)}\left[I_{1/2+c}(x)I_{1/2-c}(ax) - I_{-1/2+c}(ax)I_{-1/2-c}(x)\right] \times \left[I_{1/2+c}(ax)I_{1/2-c}(x) - I_{-1/2+c}(x)I_{-1/2-c}(ax)\right]$$





N.Haba, S.Matsumoto,
N.Okada and T.Y.
in preparation

# Warped Gauge-Higgs Unification

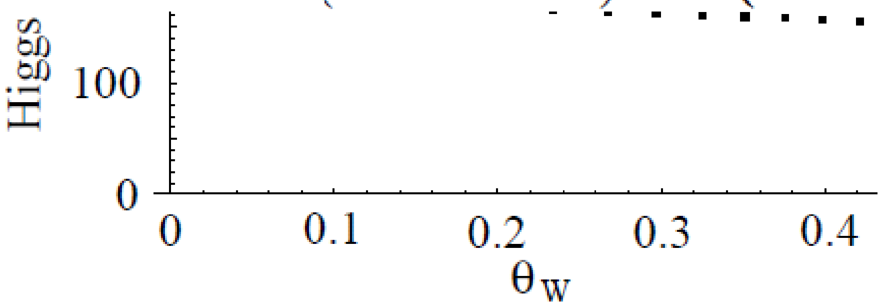
N.Haba, S.Matsumoto,
N.Okada and T.Y.
in preparation

- Effective Potential

**SU(3)→SU(2)×U(1) model**

w/ 2 anti-periodic fundamental fermions w/ $c_f$
1 periodic adjoint fermion w/ $c_a = 0.48$

$$V_{\rm eff}(\theta_W) = \frac{(ka)^4}{2(4\pi)^2}\left[3\left\{v_{\rm eff}\left(\theta_W;\frac{1}{2},1\right)+2v_{\rm eff}\left(\theta_W;\frac{1}{2},\frac{1}{2}\right)\right\} -8v_{\rm eff}\left(\theta_W+\pi;c_f,\frac{1}{2}\right)-4\left\{v_{\rm eff}\left(\theta_W;c_a,1\right)+2v_{\rm eff}\left(\theta_W;c_a,\frac{1}{2}\right)\right\}\right]$$




N. Haba and T.Y. (2004)

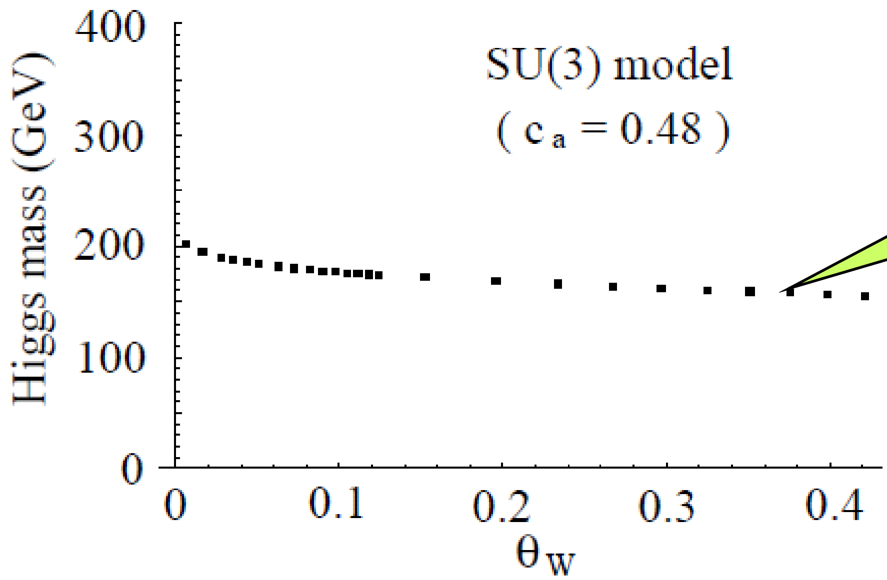
# Warped Gauge-Higgs Unification

N.Haba, S.Matsumoto,
N.Okada and T.Y.
in preparation

- Effective Potential

**SU(3)→SU(2)×U(1) model**

w/ 2 anti-periodic fundamental fermions w/ $c_f$
1 periodic adjoint fermion w/ $c_a = 0.48$




A large Higgs mass

# Plan

- Introduction
- Warped GHU
- Gauge-Higgs Condition
- Summary

# Gauge-Higgs Condition

- 4D effective theory = SM + GH condition

  Theory of zero-modes.

  ⇨ renormalization condition

⇩

$$\lambda\left(\frac{1}{2\pi R}\right) \sim 0$$ : Gauge-Higgs condition

$$\left.\frac{\mathrm{d}^4 V_{\text{eff}}}{\mathrm{d}h^4}\right|_{h=\mu} = \lambda(\mu)$$

S.Coleman and E.Weinberg

# Gauge-Higgs Condition

- 4D effective theory = SM + GH condition

  Theory of zero-modes.

  ⇨ renormalization condit[ion]

  little corrections by anti-periodic modes

  ⇩

  $\lambda\left(\frac{1}{2\pi R}\right) \sim 0$ : Gauge-Higgs condition

- 5D theory appears from $\Lambda_{\mathrm{UV}}$.

  In 5D theory, higgs is unified with gauge.

  ⇨ Vanishing potential.

# Gauge-Higgs Condition

- 4D effective theory = SM + GH condition

  Theory of zero-modes.

  ⇨ renormalization condition

  ⇩

  $$\lambda\left(\frac{1}{2\pi R}\right) \sim 0$$ : Gauge-Higgs condition

- We can calculate everything more easily.
  - complicated models warped GHU?
  - RG improved analysis
  - Higher loop corrections

# Gauge-Higgs Condition

- 4D effective theory = SM + GH condition

  Theory of zero-modes.

  ⇨ renormalization condition

⇩

$$\lambda\left(\frac{1}{2\pi R}\right) \sim 0$$ : Gauge-Higgs condition

$$\left.\frac{\mathrm{d}^4 V_{\mathrm{eff}}}{\mathrm{d}h^4}\right|_{h=\mu} = \lambda(\mu)$$

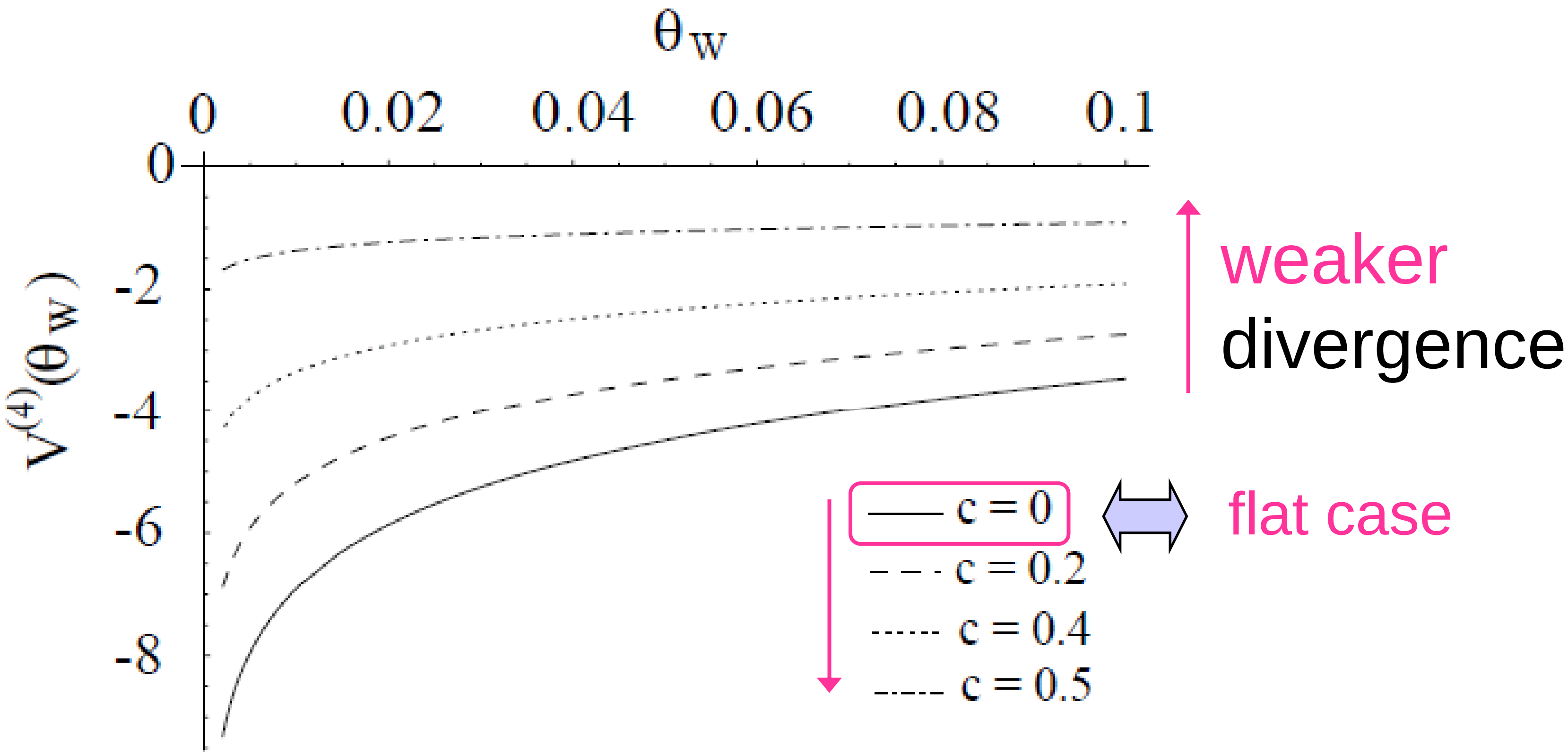
S.Coleman and E.Weinberg

$$\xrightarrow[\mu \to 0]{} \frac{b}{2} y^4 \ln\left(\frac{m_1^2}{\Lambda_{\mathrm{UV}}^2}\right)$$

w/ IR log. divergence.

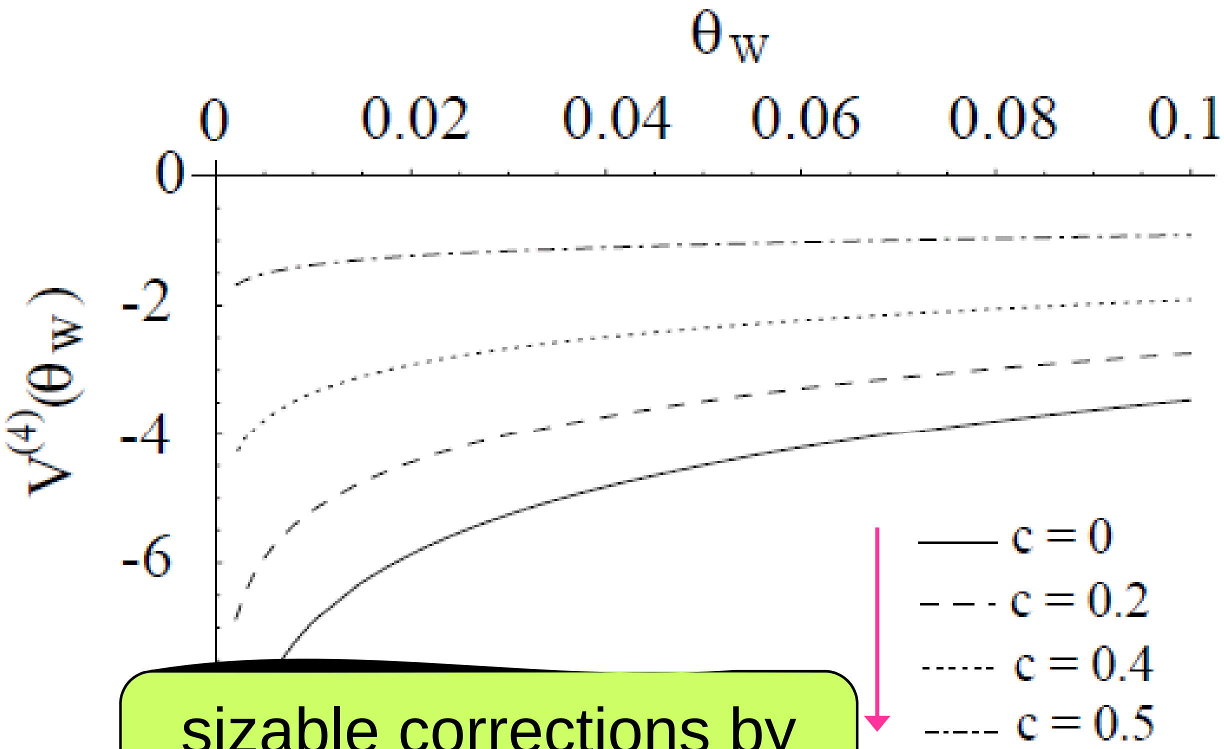
# Gauge-Higgs Condition

- In warped GHU



# Gauge-Higgs Condition

- In warped GHU




sizable corrections by anti-periodic modes

weaker divergence

- larger $\Lambda_{\rm UV}$
- threshold at $\Lambda_{\rm UV} \sim M_{\rm KK}$

not easy to evaluate…

# Summary

- warped Gauge-Higgs unification
  - large KK scale, top & Higgs mass
- Effective potential is calculated

N.Haba, S.Matsumoto, N.Okada and T.Y. in preparation

  - Gauge-Higgs condition is studied.

    ⇨ not goes well.

  - Non-orbifold like fermions

    ⇨ Vanishing contributions

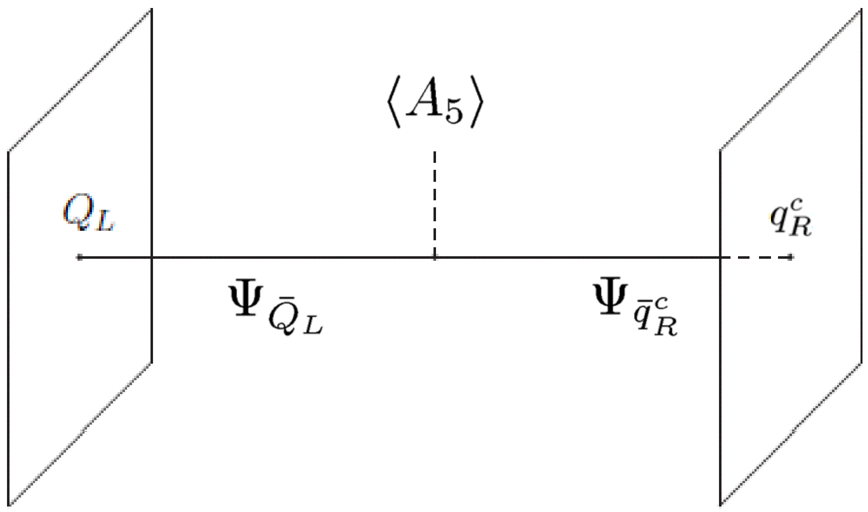
# Back-up Slides

# Yukawa Coupling

NPB **669** (2003)
Scrucca, Serone, and Silvestrini

- bulk-brane mass
  - chiral fermions on fixed points $Q_L \quad q_R^c$
  - bulk fermions as messengers $\Psi = \left(\Psi_{\bar{Q}_L}, \Psi_{\bar{q}_R^c}\right)$
  - vector-like partners of the bulk fermions

$$\bar{\Psi} = \left(\bar{\Psi}_{Q_L}, \bar{\Psi}_{q_R^c}\right)$$



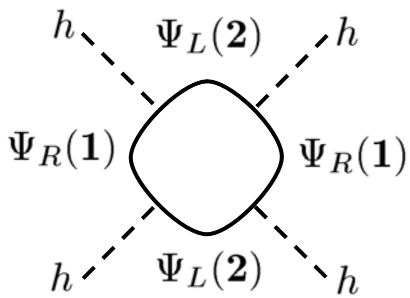
# Gauge-Higgs Condition

JHEP 0602(2006)
N.Haba, S.Matsumoto, N.Okada and T.Y.

- Zero-modes contributions

$$\Psi(\mathbf{3}) = \begin{cases} \Psi_L(\mathbf{2}) + \Psi_L(\mathbf{1}) \\ \Psi_R(\mathbf{2}) + \Psi_R(\mathbf{1}) \end{cases} \qquad \mathcal{L} = \frac{g_4}{2}\Psi_L(\mathbf{2})\Psi_R(\mathbf{1})h$$

- 1-loop correction




\+ appropriate renormalization

$$\left.\frac{\mathrm{d}^4 V_{\text{eff}}}{\mathrm{d}h^4}\right|_{h=\mu} = \lambda(\mu)$$

S.Coleman and E.Weinberg

$$V_{\text{eff}} = -\frac{1}{2}m^2h^2 + \frac{1}{4!}\left(\lambda(\mu) + \frac{b}{2}\left[\ln\left(\frac{h^2}{\mu^2}\right) - \frac{25}{6}\right]\right)h^4 \qquad b = -\frac{3}{\pi^2}$$

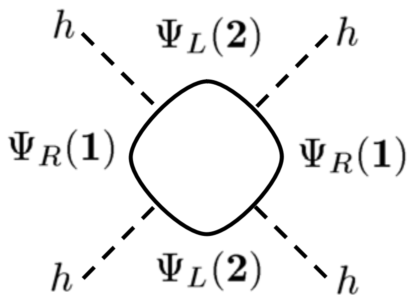
# Gauge-Higgs Condition

JHEP 0602(2006)
N.Haba, S.Matsumoto, N.Okada and T.Y.

- KK-modes v.s. Zero-modes

$$\sum_{w=1}^{\infty} \frac{1}{w^5} \cos(w\theta_W) \sim -\frac{1}{2}\zeta_R(3)\theta_W^2 + \boxed{\frac{1}{4!}\frac{1}{2}\left(\frac{25}{6} - \ln\left(x^2\right)\right)}\theta_W^4$$

$$\Updownarrow \qquad \theta_W = 2\pi R g_4 Q h = m_0/\Lambda_{\rm UV}$$




+ appropriate renormalization

$$\left.\frac{{\rm d}^4 V_{\rm eff}}{{\rm d}h^4}\right|_{h=\mu} = \boxed{\lambda(\mu) = 0 \quad @\mu = \frac{1}{2\pi R}}$$

$$V_{\rm eff} = -\frac{1}{2}m^2h^2 + \frac{1}{4!}\left(\lambda(\mu) + \frac{b}{2}\left[\ln\left(\frac{h^2}{\mu^2}\right) - \frac{25}{6}\right]\right)h^4$$

# Plan

- Introduction
- Warped GHU
- Gauge-Higgs Condition
- Interval v.s. orbifold
- Summary

# Interval v.s. orbifold

- Orbifold

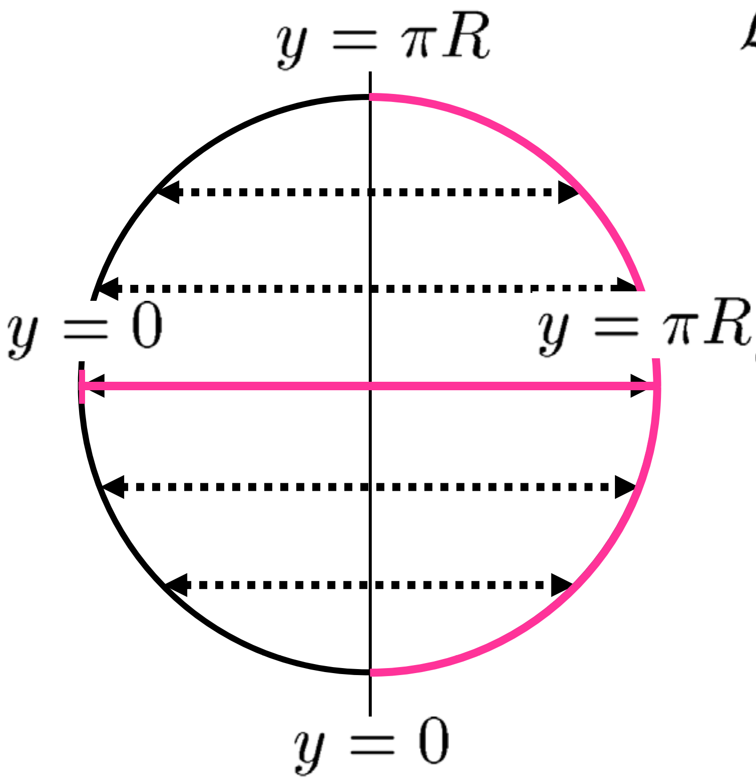
ex) $S^1/Z_2$ = interval (?)

$$\mathcal{L}(x^\mu, y) = \mathcal{L}(x^\mu, y + 2\pi R) \quad : \ \hat{T}$$

$$= \mathcal{L}(x^\mu, -y) \quad : \ \hat{P}$$

$$\mathcal{L}(x^\mu, \pi R - y) = \mathcal{L}(x^\mu, \pi R + y) \ : \ \hat{P}'$$

$$\hat{P}' = \hat{P}\hat{T}^{-1}, \ \hat{T} = \hat{P}\hat{P}'$$




Fields may not be invariant !

ex)

$$A_\mu(x^\mu, -y) = P A_\mu(x^\mu, y) P^\dagger$$

$$A_5(x^\mu, -y) = -P A_5(x^\mu, y) P^\dagger$$

$$\Psi(x^\mu, -y) = \eta_\Psi P \gamma_5 \Psi(x^\mu, y)$$

$P$ : Symm. transformation

# Interval v.s. orbifold

N.Haba, S.Matsumoto,
N.Okada and T.Y.
in preparation

- Orbifold

ex) $S^1/Z_2$ = interval (?)

+ : Neumann

- : Dirichlet

**wider class of BCs**

C. Csaki et al (2004)
N. Sakai and N. Uekusa (2007)

$y = 0$ $y = \pi R$

$SU(2) \to U(1)$ model

- In the orbifold picture

$$\underbrace{\begin{pmatrix} +,+ \\ -,- \end{pmatrix}, \begin{pmatrix} -,- \\ +,+ \end{pmatrix}}_{\text{Periodic}}, \underbrace{\begin{pmatrix} +,- \\ -,+ \end{pmatrix}, \begin{pmatrix} -,+ \\ +,- \end{pmatrix}}_{\text{Anti-Periodic}}$$

$(P, P')$ $\begin{pmatrix} \Psi^u \\ \Psi^d \end{pmatrix}$

- In the interval picture

$$\begin{pmatrix} +,+ \\ +,+ \end{pmatrix}, \begin{pmatrix} +,+ \\ +,- \end{pmatrix}, \ldots, \begin{pmatrix} -,- \\ \ldots \end{pmatrix}, \begin{pmatrix} -,- \\ -,- \end{pmatrix}$$

All vanishing !!

Preliminary