“Search for a Realistic Orbifold Grand Unification”

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2007. 12. 18 @ GUT07

References

"Triplet-doublet splitting, Proton Stability and the Extra dimension S^1/Z_2"

Y.K. *Prog.Theor.Phys.* 105 (2001), 999 (hep-ph/0012125).

"Dynamical rearrangement of gauge symmetry on the orbifold S^1/Z_2"

M. Harada, N. Haba, Y. Hosotani and Y.K.

Nucl. Phys. B657 (2003), 169 (hep-ph/0212035).

"Classification and Dynamics of Equivalence Classes in SU(N) Gauge Theory on the Orbifold S^1/Z_2"

N. Haba, Y. Hosotani and Y.K.

Prog. Theor. Phys. 111 (2004), 265 (hep-ph/0309088).

"Orbifold family unification"

Y. K., T. Kinami and K. Oda

Phys. Rev. D76 (2007), 035001 (hep-ph/0703195)

Contents

1. Introduction

☆ Problems in the Standard Model

☆ Problems in Supersymmetric (SUSY) GUTs

☆ Standpoint and Goal

☆ Problems in the Standard Model

The Standard Model (SM)

➔ Effective theory below the weak scale

"The SM cannot be an ultimate theory of nature."

P1. Charge quantization

P2. Origin of anomaly free set

P3. Parameters

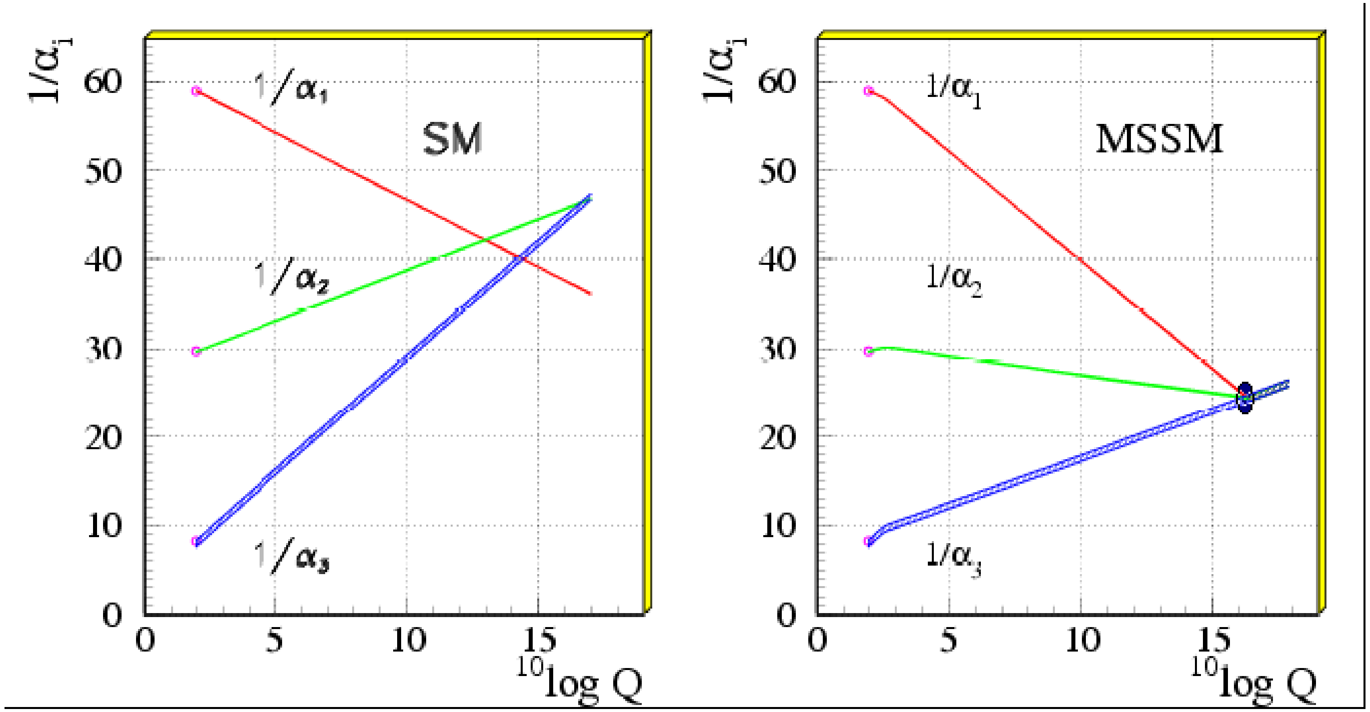
} Grand Unification

P4. Naturalness problem ➔ Supersymmetry (SUSY)

P5. No gravity

Grand Unification and SUSY have been paid much attention to as physics beyond (the minimal) SUSY SM.

← Gauge couplings at *Mz* and RGEs



From HP of PDG

☆ Problems in SUSY GUT

P1. Breaking mechanism of GUT symmetry －Triplet-doublet Higgs mass splitting－

P2. Proton stability

P3. Origin of fermion mass hierarchy and mixing

➔ Extension of Higgs sector

➔ Extension of Space-time

P4. Origin of family (generation)

☆ Standpoint and Goal

Standpoint : Grand Unification & SUSY

Goal : To construct a realistic grand unified theory (or underlying theory)

➔ A long way to go there.

Strategy : To introduce Extra dimension

Goal of today's talk : To introduce orbifold GUT and to discuss topics related to boundary conditions on orbifold(s) and suggest an origin of family, which will help us in a realistic model building.

Contents

2. Orbifold Grand Unified Theory

☆ What is Orbifold?

☆ Orbifold Breaking Mechanism

☆ Orbifold SUSY GUT

☆ What is Orbifold?

Orbifold

= {Manifold/Discrete transformation group; with Fixed Point}

Fixed Points (y_{fp}) are points that transform into themselves under the discrete transformation,

$$y \to y' = k(y), \quad k(y_{fp}) = y_{fp}, \qquad (k \neq I)$$

For 4D string model building using 6D orbifold, see Ref. L. Dixon, J. Harvey, C. Vafa and E. Witten, *Nucl. Phys.* B261 (1985), 678; B274 (1986) 285.
For 4D string model building and phenomenology, see the excellent textbook "Quark and Leptons From Orbifolded Superstring" (Springer, 2006) by K.-S. Choi and J. E. Kim.

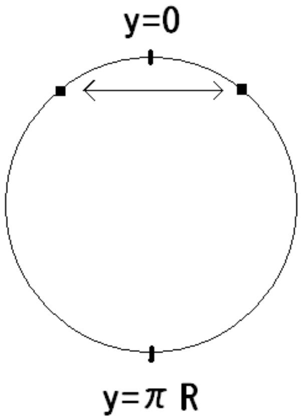
Orbifold S^1/Z_2

S^1 : **Circle with radius *R***

$$y \sim y + 2\pi R n \quad (n \in Z) \qquad y : [0, 2\pi R]$$

S^1/Z_2 : $y \sim y + 2\pi R n \quad (n \in Z)$ & $y \sim -y$

$$Z_2 : y \to y' = -y \qquad y : [0, \pi R]$$

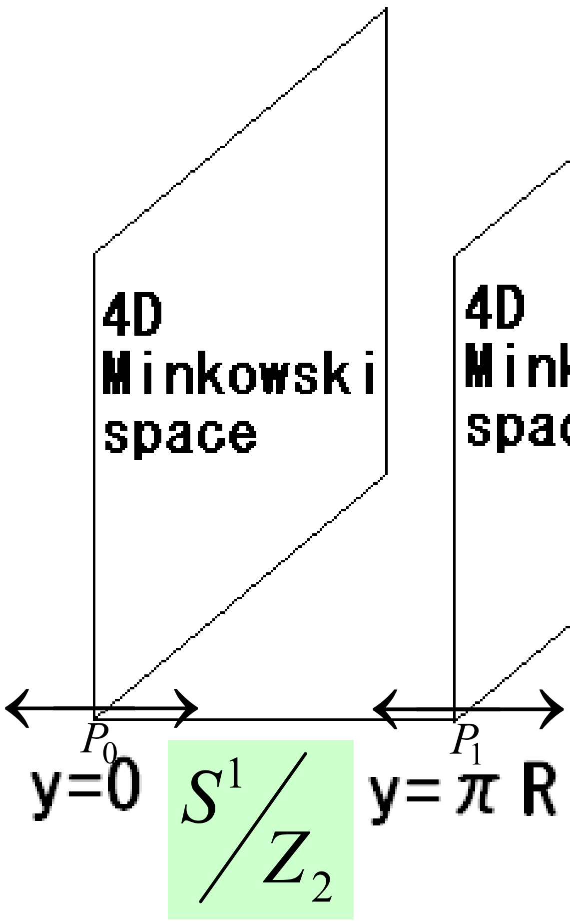


➔ $y \sim -y$

Fixed Points are $y = 0$ and $y = \pi R$.

$$(\because\ 0 \to -0 = 0, \pi R \to -\pi R \sim -\pi R + 2\pi R = \pi R)$$

Brane world scenario with the help of orbifold fixed points



$$x^\mu (= x) \qquad x^5 = y$$

$$(\mu = 0,1,2,3)$$

$$U : y \to y + 2\pi R$$

$$P_0 : y \to -y$$

$$P_1 : y \to 2\pi R - y$$

$$U = P_1 P_0$$

Feature on S^1/Z_2

On the orbifold, $y \sim -y \sim 2\pi R - y$,

but a value of field does not necessarily take an identical value at these points.

The Lagrangian density takes a single-value.

$$\rightarrow \begin{cases} \Phi(x,-y) = T_\Phi[P_0]\Phi(x,y), \\ \Phi(x,2\pi R - y) = T_\Phi[P_1]\Phi(x,y), \\ \Phi(x,y+2\pi R) = T_\Phi[U]\Phi(x,y) \end{cases} \quad (T_\Phi[P_a]^2 = I) \quad (a = 0,1)$$

$T_\Phi[P_a]$ → Z_2 parities on the 5-th coordinate

☆ Mode Expansion

Z_2 Parities $P_0^2 = P_1^2 = 1$ ➔ Eigenvalue =+1 or −1

$$\phi^{(P_0P_1)}(x,y) \to \phi^{(P_0P_1)}(x,-y) = P_0\phi^{(P_0P_1)}(x,y)$$

$$\phi^{(P_0P_1)}(x,y) \to \phi^{(P_0P_1)}(x,2\pi R - y) = P_1\phi^{(P_0P_1)}(x,y)$$

Fourier Expansions

+1 ➔ + , −1 ➔ −

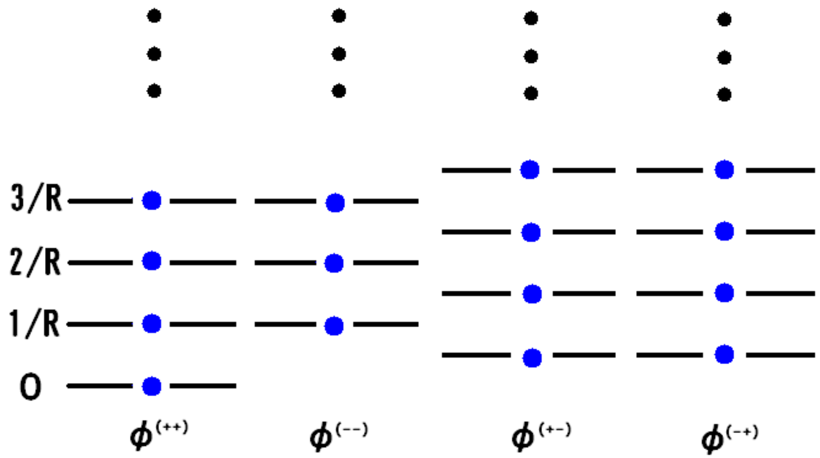
zero mode

$$\phi^{(++)}(x,y) = \underline{\frac{1}{\sqrt{\pi R}}\phi_0^{(++)}(x)} + \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty}\phi_n^{(++)}(x)\cos\frac{ny}{R}$$

$$\phi^{(--)}(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty}\phi_n^{(--)}(x)\sin\frac{ny}{R}$$

$$\phi^{(+-)}(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty}\phi_n^{(+-)}(x)\cos\frac{(2n-1)y}{2R}$$

$$\phi^{(-+)}(x,y) = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty}\phi_n^{(-+)}(x)\sin\frac{(2n-1)y}{2R}$$



Mass spectrum in 4-dimension

☆ Orbifold Breaking

For N-plet

$$\Phi(x,y)=\begin{pmatrix}\phi_1(x,y)\\ \phi_2(x,y)\\ \vdots \\ \phi_N(x,y)\end{pmatrix}$$

$$\Phi(x,-y)=T_\Phi[P_0]\Phi(x,y)$$

$$\Phi(x,2\pi R-y)=T_\Phi[P_1]\Phi(x,y)$$

$T_\Phi[P_a]: N\times N$ **matrix**

$$T_\Phi[P_a]^2=I$$

Unless all components have common Z_2 parities, a symmetry reduction occurs upon compactification! Because zero modes are absent in fields with an odd parity. **(Orbifold Breaking Mechanism)**

In the framework of field theory, for the breakdown of SUSY, see E.A. Mirabelli and M.E. Peskin, *Phys. Rev.* D58 (1998), 065002. For the reduction of gauge symmetry, Y. K., *Prog. Theor. Phys.* 103 (2000), 613 (hep-ph/9902423).

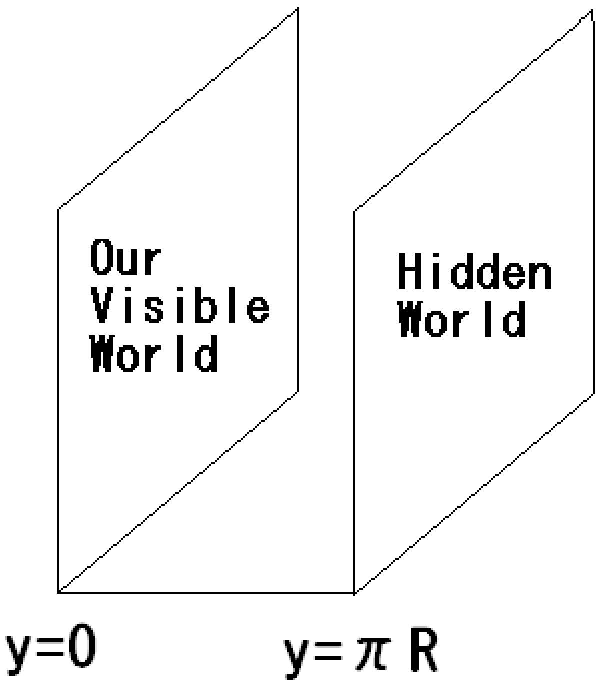
☆ Orbifold SUSY GUT

Y.K. *Prog. Theor. Phys.* 105 (2001), 999 (hep-ph/0012125)

☆Framework (Assumptions)

$M^4 \times S^1 / Z_2 \times Z'_2$

Space-time $M^4 \times S^1 / Z_2$



Bulk fields consist of

$SU(5)$ Gauge supermultiplet
& 2 Higgs Hypermultiplets
with fundamental rep.

Brane fields consist of

3 families of matter chiral supermultiplets.

(This assumption can be relaxed.)

Z_2 parity assignment

$$P_0 = \mathrm{diag}(+1,+1,+1,+1,+1), \quad P_1 = \mathrm{diag}(-1,-1,-1,+1,+1)$$

$$\begin{cases} A_\mu^\alpha(x,-y)T^\alpha = P_0 A_\mu^\alpha(x,y)T^\alpha P_0^{-1} = A_\mu^\alpha(x,y)T^\alpha, \\ A_y^\alpha(x,-y)T^\alpha = -P_0 A_y^\alpha(x,y)T^\alpha P_0^{-1} = -A_y^\alpha(x,y)T^\alpha \end{cases}$$

$$\begin{cases} A_\mu^\alpha(x,2\pi R-y)T^\alpha = P_1 A_\mu^\alpha(x,y)T^\alpha P_1^{-1}, \\ A_y^\alpha(x,2\pi R-y)T^\alpha = -P_1 A_y^\alpha(x,y)T^\alpha P_1^{-1} \end{cases}$$

$$\begin{cases} T^a : \text{SM generators}, \\ T^{\hat{a}} : SU(5)/G_{SM} \text{ generators} \end{cases}$$

$$A_\mu^{a(++)}(x,y),\ A_\mu^{\hat{a}(+-)}(x,y),\ A_y^{a(--)}(x,y),\ A_y^{\hat{a}(-+)}(x,y)$$

$$\lambda_1^{a(++)}(x,y),\ \lambda_1^{\hat{a}(+-)}(x,y),\ \lambda_2^{a(--)}(x,y),\ \lambda_2^{\hat{a}(-+)}(x,y) \qquad (\because)\ \ i\lambda_2^{c\dagger}\partial_y\lambda_1$$

$$\sigma^{a(--)}(x,y),\ \sigma^{\hat{a}(-+)}(x,y)$$

Zero modes are *N*=1 SM gauge supermultiplets.

Gauge symmery reduction

$$SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$$

For Higgs multiplets $(H, H'; \overline{H}, \overline{H}')$

$$P_0 = \mathrm{diag}(+1,+1,+1,+1,+1), \quad P_1 = \mathrm{diag}(-1,-1,-1,+1,+1)$$

$$\begin{cases} H(x,-y) = \eta_{0H} P_0 H(x,y) = H(x,y), \\ H'(x,-y) = -\eta_{0H} P_0 H'(x,y) = -H'(x,y) \end{cases} \qquad (\because) \quad i\widetilde{H}'\partial_y \widetilde{H}$$

$$\begin{cases} H(x, 2\pi R - y) = \eta_{1H} P_1 H(x,y), \\ H'(x, 2\pi R - y) = -\eta_{1H} P_1 H'(x,y) \end{cases}$$

$$H(x,y) = \begin{pmatrix} H_C(x,y) \\ H_W(x,y) \end{pmatrix}$$

'Intrinsic Z_2 parity' $\eta_{0H} = \eta_{1H} = +1$

$$H_C^{(+-)}(x,y), \underline{H_W^{(++)}(x,y)}, H_C'^{(-+)}(x,y), H_W'^{(--)}(x,y),$$

$$\overline{H}_C^{(+-)}(x,y), \underline{\overline{H}_W^{(++)}(x,y)}, \overline{H}_C'^{(-+)}(x,y), \overline{H}_W'^{(--)}(x,y),$$

Zero modes are Weak Higgs supermultiplets.

Triplet-doublet splitting!

For triplet-doublet splitting by Wilson line mechanism in 4D heterotic orbifold models, see Ref. L. E. Ibanez, J. E. Kim, H. P. Nilles and F. Quevedo, Dixon, *Phys. Lett.* B191 (1987), 282.

Z_2 parity assignment

$$P_0 = \mathrm{diag}(+1,+1,+1,+1,+1),\ P_1 = \mathrm{diag}(-1,-1,-1,+1,+1)$$

Zero modes in Bulk fields

= SM gauge supermultiplet

& 2 weak Higgs chiral supermultiplets

Our 4D Brane fields

➔ The MSSM fields !

- The Kaluza-Klein modes have heavy mass of O(1/R).
- It is possible to derive some chiral matter fields as zero modes of bulk hypermultiplets.

☆ Problems in SUSY GUT

P1. Breaking mechanism of GUT symmetry −Triplet-doublet Higgs mass splitting ➔ Orbifold breaking

➔ Section. 3

P2. Proton stability

G. Altarelli & F. Feruglio, *Phys. Lett.* B511 (2001) 257 (hep-ph/0102301).

L. J. Hall & Y. Nomura, *Phys. Rev.* D64 (2001) 055003 (hep-ph/0103125), *Annals. Phys.* 306 (2003) 132 (hep-ph/0212134)

P3. Origin of fermion mass hierarchy and mixing

Y. Nomura, *Phys. Rev.* D65 (2002) 085036 (hep-ph/0108170)

P4. Origin of family (generation)

➔ Section. 4

☆ Problems in SUSY GUT

P1. Breaking mechanism of GUT symmetry －Triplet-doublet Higgs mass splitting ➔ Orbifold breaking

$$P_0 = \mathrm{diag}(1,1,1,1,1), \quad P_1 = \mathrm{diag}(-1,-1,-1,1,1)$$

➔ What is the origin of non-trivial Z_2 parities ?

＝ What is the principle to determine BCs ?

"Arbitrariness Problem"

3. Dynamical Rearrangement of Gauge Symmetry and Equivalence Classes of Boundary Conditions

← What is the principle to determine BCs ?

"Dynamical rearrangement of gauge symmetry on the orbifold S^1/Z_2"
M. Harada, N. Haba, Y. Hosotani and Y.K.
Nucl. Phys. B657 (2003), 169 (hep-ph/0212035).

"Classification and Dynamics of Equivalence Classes in SU(N) Gauge Theory on the Orbifold S^1/Z_2"
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Prog. Theor. Phys. 111 (2004), 265 (hep-ph/0309088).

$$P_0^2 = P_1^2 = I$$

$$
\begin{array}{cc}
P_0 & P_1 \\
(+1,+1,+1,+1,+1) & (+1,+1,+1,+1,+1) \\
(+1,+1,+1,+1,+1) & (+1,+1,+1,+1,-1) \\
(+1,+1,+1,+1,+1) & (+1,+1,+1,-1,-1) \\
(+1,+1,+1,+1,+1) & (+1,+1,-1,-1,-1) \\
\cdots\cdots\cdots & \\
(-1,-1,-1,-1,-1) & (-1,-1,-1,-1,-1)
\end{array}
$$

& Non-diagonal ones

Ex.

$$
\begin{pmatrix}
-\cos \pi p & 0 & 0 & -i\sin \pi p & 0 \\
0 & -\cos \pi q & 0 & 0 & -i\sin \pi q \\
0 & 0 & -1 & 0 & 0 \\
i\sin \pi p & 0 & 0 & \cos \pi p & 0 \\
0 & i\sin \pi q & 0 & 0 & \cos \pi q
\end{pmatrix}
$$

Some of them are gauge equivalent because they are related to by BCs-changing gauge transformations.

➔ Equivalence classes of BCs

Under the gauge transformation,

$$\Phi(x,y) \to \Phi'(x,y) = T_\Phi[\Omega]\Phi(x,y)$$

BCs of $\Phi'(x,y)$ **are**

$$\begin{cases} \Phi'(x,-y) = T_\Phi[P'_0]\Phi'(x,y) \\ \Phi'(x,2\pi R - y) = T_\Phi[P'_1]\Phi'(x,y) \\ \Phi'(x,y+2\pi R) = T_\Phi[U']\Phi'(x,y) \end{cases}$$

$$\begin{cases} P'_0 = \Omega(x,-y)P_0\Omega^{-1}(x,y) \\ P'_1 = \Omega(x,2\pi R - y)P_1\Omega^{-1}(x,y) \\ U' = \Omega(x,y+2\pi R)U\Omega^{-1}(x,y) \end{cases}$$

$(P_0, P_1, U) \neq (P'_0, P'_1, U')$ for a singular $\Omega(x,y)$

Ex. $SU(2)$

$$\Omega = \exp\left(-i\frac{\tau_2}{2R}y\right)$$

$$P_0 = P_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow P_0' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; P_1' = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = -P_1$$

$(\because)$

$$P_0' = \Omega(x,-y)P_0\Omega^\dagger(x,y) = e^{-i\frac{\tau_2}{2R}(-y)}\tau_3 e^{i\frac{\tau_2}{2R}y}$$

$$= e^{i\frac{\tau_2}{2R}y}e^{-i\frac{\tau_2}{2R}y}\tau_3 = \tau_3 \qquad (\tau_3\tau_2 = -\tau_2\tau_3)$$

$$P_1' = \Omega(x, 2\pi R - y)P_1\Omega^\dagger(x,y) = e^{-i\frac{\tau_2}{2R}(2\pi R - y)}\tau_3 e^{i\frac{\tau_2}{2R}y}$$

$$= e^{-i\frac{\tau_2}{2R}(2\pi R - y)}e^{-i\frac{\tau_2}{2R}y}\tau_3 = e^{-i\pi\tau_2}\tau_3 = -\tau_3$$

Ex. $SU(2)$

$$\Omega = \exp\left(-i\frac{\tau_2}{2R}y\right)$$

$$P_0 = P_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow P_0' = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; P_1' = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = -P_1$$

$$\Rightarrow (P_0 = \tau_3, P_1 = \tau_3, U = I) \sim (P'_0 = \tau_3, P'_1 = -\tau_3, U' = -I)$$

$$(U = P_1 P_0)$$

For $SU(5)$

$$\begin{array}{cc} P_0 & P_1 \\ (+1,\underline{+1,-1},-1,-1) & (+1,\underline{+1,-1},-1,-1) \\ \tau_3 & \tau_3 \\ \sim (+1,\underline{+1,-1},-1,-1) & (+1,\underline{-1,+1},-1,-1) \\ \tau_3 & -\tau_3 \\ \sim (+1,+1,-1,-1,-1) & (-1,-1,+1,+1,-1) \end{array}$$

Let us check whether this equivalence holds.

a. The theories should be equivalent if they are related to by gauge transformation.

$$\begin{array}{cc} P_0 & P_1 \\ (+1,+1,-1,-1,-1) & (+1,+1,-1,-1,-1) \\ \sim (+1,+1,-1,-1,-1) & (+1,-1,+1,-1,-1) \end{array}$$

b. The symmetry of BCs differs from each other, with different mode expansions.

The equivalence is guaranteed by the *Hosotani mechanism* or understood as the gauge invariance of effective potential.

Y. Hosotani, *Phys. Lett.* B126 (1983), 309; *Ann. of Phys.* 190 (1989), 233.

$(Ex.1)$ $\quad (\ 1,\ 1,-1,-1,-1,)(\ 1,\ 1,-1,-1,-1)$

$$\int dy\left|\partial_y h\right|^2 \Rightarrow \sum_{n=0}^{\infty}\left(\frac{n}{R}\right)^2\left(\left|h_n^{(++)1}\right|^2+\left|h_n^{(++)2}\right|^2\right)$$

$$\int dy\left|(\partial_y - igA_y)h\right|^2 \quad + \sum_{n=1}^{\infty}\left(\frac{n}{R}\right)^2\left(\left|h_n^{(--)3}\right|^2+\left|h_n^{(--)4}\right|^2+\left|h_n^{(--)5}\right|^2\right)$$

⇕

Min. of effective potential → $\left\langle A_y \right\rangle$

→ Same mass spectrum!

(Hosotani mechanism)

$(Ex.2)$ $\quad (\ 1,\ 1,-1,-1,-1,)(\ 1,\underline{-1,\ 1},-1,-1)$

$$\int dy\left|\partial_y h\right|^2 \Rightarrow \sum_{n=0}^{\infty}\left(\frac{n}{R}\right)^2\left|h_n^{(++)1}\right|^2$$

$$\int dy\left|(\partial_y - igA_y)h\right|^2 \quad + \sum_{n=1}^{\infty}\left(\frac{n}{2R}\right)^2\left(\left|h_n^{(+-)2}\right|^2+\left|h_n^{(-+)3}\right|^2\right)$$

$$+ \sum_{n=1}^{\infty}\left(\frac{n}{R}\right)^2\left(\left|h_n^{(--)4}\right|^2+\left|h_n^{(--)5}\right|^2\right)$$

"Dynamical rearrangement of gauge symmetry!"

Different BCs ➔ Some of them are gauge equivalent because they are related to by BCs-changing gauge transformation.

➔ Equivalence Classes of BCs

"What is a principle to select a realistic one?"

To obtain a hint

☆ Classification

☆ Vacuum energy density

☆ Classification

Each equivalence class has a diagonal representative. The diagonal representatives are specified by three non-negative integers (p, q, r) .

$$P_0 = (\overbrace{+1, \cdots, +1, +1, \cdots, +1, +1, \cdots, +1, +1, \cdots +1}^{N})$$

$$P_1 = (\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{+1, \cdots, +1}_{r}, \underbrace{-1, \cdots -1}_{s=N-p-q-r})$$

\# of equivalence classes of BCs

= # of diagonal representatives

− # of equivalence relations

$= (N+1)^2$ for $SU(N)$ particle contents are fixed

☆ Vacuum energy density

← The min. of potential

$$V_{\text{eff}} = V_{\text{eff}}(A_M^0; P_0, P_1, U) = V_{\text{eff}}(A_M^0; p, q, r, \beta)$$

$$\begin{pmatrix} \phi_1(x, y+2\pi R) \\ \phi_2(x, y+2\pi R) \end{pmatrix} = \begin{pmatrix} \cos 2\pi\beta & -\sin 2\pi\beta \\ \sin 2\pi\beta & \cos 2\pi\beta \end{pmatrix} \begin{pmatrix} \phi_1(x, y) \\ \phi_2(x, y) \end{pmatrix}$$

If Higgs bosons form the Z_2 doublet (ϕ_1, ϕ_2) , Scherk-Schwarz SUSY breaking occurs!

$$V_{\text{eff}} = \sum \mp i \int \frac{d^4 p}{(2\pi)^4} \frac{1}{2\pi R} \sum_{n \in Z} \ln(-p^2 + M_n^2 - i\varepsilon)$$

− for bosons,

+ for fermions and ghosts

$M_n = M_n(n, \beta, R)$: Mass

The effective potential takes a finite value at the minima after soft SUSY breaking by Scherk-Schwarz mechanism.

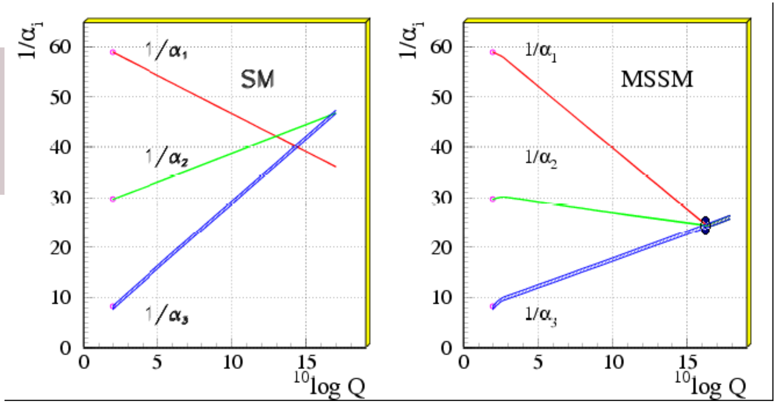
➔ Comparison of the vacuum energy density if it were allowed

- V_{eff} does not necessarily take the minimal value with the MSSM particles.
- Is the comparison among gauge-inequivalent theories meaningful?

➔ Bigger symmetry?
BCs are dynamically determined?

4. Orbifold Family Unification

Grand unification !



From HP of PDG

Grand unification of forces

Grand unification of gauge particles

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$

$$24 = (8,1)_0 + (3,1)_0 + (1,1)_0 + (3,2)_{-5} + (\bar{3},2)_5$$

Gauge particles in the SM — $(8,1)_0 + (3,1)_0 + (1,1)_0$

X, Y gauge bosons — $(3,2)_{-5} + (\bar{3},2)_5$

$$SO(10) \supset SU(5) \times U(1)$$

$$45 = 24 + 10 + \overline{10} + 1$$

Partial unification of matters

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$

$$\begin{cases} \bar{5} = (\bar{3},1)_2 + (1,2)_{-3} & \Rightarrow (d_R)^c + l_L \\ 10 = (3,2)_1 + (\bar{3},1)_{-4} + (1,1)_6 & \Rightarrow q_L + (u_R)^c + (e_R)^c \end{cases}$$

C : Charge conjugation

$$SO(10) \supset SU(5) \times U(1)$$

$$16 = \bar{5} + 10 + 1$$

One multiplet for
One family !

Unification of families ?

Unification of families ?

$SO(10) \supset SU(5) \times U(1)$ $\quad 16_1 (= \bar{5} + 10 + 1),\ 16_2,\ 16_3$

$SO(16) \supset SO(10) \times SU(4)$ $\quad 128 = (16,4) + (\overline{16},\bar{4})$

Mirror particles ?

$SU(8) \supset SU(5) \times SU(3) \times U(1)$

$$[8,3](={}_8C_3) = 56 = (10,3) + (5,\bar{3}) + (\overline{10},1) + (1,1)$$

Extra particles? Gauge anomalies ?

$$\begin{cases} (\bar{3},1)_2 + (1,2)_{-3} \\ (3,2)_1 + (\bar{3},1)_{-4} + (1,1)_6 \end{cases} \Leftrightarrow \begin{cases} (3,1)_{-2} + (1,2)_3 \\ (\bar{3},2)_{-1} + (3,1)_4 + (1,1)_{-6} \end{cases}$$

The SM matters $\qquad$ Mirror particles

☆ 4-dim. Quntum Field Theories

Anomaly free set of representations under a large gauge group

➔ Extra particles ?

How about QFTs in a higher-dimensional space-time ?

For complete family unification in orbifold GUT, K. S. Babu, S. M. Barr and B. Kyae, *Phys. Rev.* D65 (2002), 115008.

For a recent work on family unification, I. Gogoladze, C. A. Lee, Y. Mimura and Q. Shafi, *Phys. Lett.* B649 (2007), 212.

☆ 5 -dim. ($M^4 \times S^1/Z_2$) QFTs

Anomaly free in 5-dim. Bulk

➔ Bulk field with arbitrary representation

➔ Elimination of extra particles by orbifolding ! ➔ 3 families ?

Anomalous in 4 -dim. brane

➔ brane fields with suitable representations

For anomalies on the orbifold, H. D. Kim, J. E. Kim and H. M. Lee, *J. High Energy Phys.* 06 (2002), 48.

3 families including brane fields ?

Today's Theme

From 5-dim. ($M^4 \times S^1/Z_2$) QFTs

A fermion (a hypermultiplet in SUSY) with a large representation of $SU(N)$

→ Elimination of extra particles by orbifolding & addition of suitable brane fields !

→ 3 families ?

Cf. Derivation from gauge multiplet

Gauge Unification

$$V^{\alpha}(x,y) \Rightarrow G_{\mu}^{a}(x), W_{\mu}^{i}(x), B_{\mu}(x), \widetilde{G}^{a}(x), \widetilde{W}^{i}(x), \widetilde{B}(x)$$

Gauge-Higgs Unification

$$V^{\alpha}(x,y) \Rightarrow \begin{cases} G_{\mu}^{a}(x), W_{\mu}^{i}(x), B_{\mu}(x), \widetilde{G}^{a}(x), \widetilde{W}^{i}(x), \widetilde{B}(x) \\ H_{W}(x), \overline{H}_{W}(x), \widetilde{H}_{W}(x), \widetilde{\overline{H}}_{W}(x) \end{cases}$$

Gauge-Matter(Family) Unification

$$V^{\alpha}(x,y) \Rightarrow \begin{cases} G_{\mu}^{a}(x), W_{\mu}^{i}(x), B_{\mu}(x), \widetilde{G}^{a}(x), \widetilde{W}^{i}(x), \widetilde{B}(x) \\ q_{L}(x), \widetilde{q}_{L}(x), \cdots \end{cases}$$

Gauge-Higgs-Matter Unification

Two Questions

Orbifold Breaking

A large representation ➔ 3 families

Q1. $SU(N) \xrightarrow{Z_2} SU(5)\times\cdots,$

$$[N,k] = {}_N C_k \to 3\times(\bar{5}+10)?$$

Q2. $SU(N) \xrightarrow{Z_2} SU(3)_C\times SU(2)_L\times U(1)_Y\times\cdots,$

$$[N,k] = {}_N C_k \to 3\times\begin{pmatrix}(\bar{3},1)_2+(1,2)_{-3} \\ +(3,2)_1+(\bar{3},1)_{-4}+(1,1)_6\end{pmatrix}?$$

Z_2 Orbifold Breaking

$$P_0 = \text{diag}(\overbrace{+1,\cdots,+1}^{p},\overbrace{+1,\cdots,+1}^{q},\overbrace{-1,\cdots-1}^{r},\overbrace{-1,\cdots,-1}^{s=N-p-q-r})$$

$$P_1 = \text{diag}(+1,\cdots,+1,-1,\cdots,-1,+1,\cdots,+1,-1,\cdots,-1)$$

Rep. Matrix for fundamental rep. up to intrinsic Z_2 parity

$$SU(N) \to SU(p)\times SU(q)\times SU(r)\times SU(s)\times U(1)^n$$

$$\left(SU(1) \Rightarrow U(1), SU(0) \Rightarrow \text{Nothing}\right)$$

$$[N,k] = {}_N C_k \Rightarrow \sum_{l_1=0}^{k}\sum_{l_2=0}^{k-l_1}\sum_{l_3=0}^{k-l_1-l_2}({}_p C_{l_1},\ {}_q C_{l_2},\ {}_r C_{l_3},\ {}_s C_{k-l_1-l_2-l_3})$$

(Formula) $${}_{n+m}C_k = \sum_{l=0}^{k} {}_n C_l \cdot {}_m C_{k-l}$$

Z_2 Orbifold Breaking

$$P_0 = \mathrm{diag}(\overbrace{+1,\cdots,+1}^{p},\overbrace{+1,\cdots,+1}^{q},\overbrace{-1,\cdots-1}^{r},\overbrace{-1,\cdots,-1}^{s=N-p-q-r})$$
$$P_1 = \mathrm{diag}(+1,\cdots,+1,-1,\cdots,-1,+1,\cdots,+1,-1,\cdots,-1)$$

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$$[N,k] = {}_N C_k \Rightarrow \sum_{l_1=0}^{k}\sum_{l_2=0}^{k-l_1}\sum_{l_3=0}^{k-l_1-l_2}\left({}_p C_{l_1},\ {}_q C_{l_2},\ {}_r C_{l_3},\ {}_s C_{k-l_1-l_2-l_3}\right)$$

Values of Z_2 parity for $\left({}_p C_{l_1}, {}_q C_{l_2}, {}_r C_{l_3}, {}_s C_{l_4}\right)$?

$$l_4 \equiv k - l_1 - l_2 - l_3$$

Z_2 parity assignment

$$Z_2 : N = [N,1] \to \eta_{[N,1]} P_0 N,\ \ N = [N,1] \to \eta'_{[N,1]} P_1 N$$

For $[N,k] = {}_N C_k = \left(N \times \cdots \times N\right)_a$

$$\Rightarrow \sum_{l_1=0}^{k} \sum_{l_2=0}^{k-l_1} \sum_{l_3=0}^{k-l_1-l_2} \left({}_p C_{l_1},\ {}_q C_{l_2},\ {}_r C_{l_3},\ {}_s C_{k-l_1-l_2-l_3}\right),$$

$$\left(N \times \cdots \times N\right)_a \to \eta_{[N,k]} \left(P_0 N \times \cdots \times P_0 N\right)_a$$

$$\left(N \times \cdots \times N\right)_a \to \eta'_{[N,k]} \left(P_1 N \times \cdots \times P_1 N\right)_a$$

$\eta_{[N,k]}, \eta'_{[N,k]}$: intrinsic Z_2 parity, $+1$ or -1

Z_2 parity of $\left({}_p C_{l_1}, {}_q C_{l_2}, {}_r C_{l_3}, {}_s C_{l_4}\right)$ $\quad l_4 \equiv k - l_1 - l_2 - l_3$

$$\rho_0 = (-1)^{l_1+l_2} (-1)^k \eta_{[N,k]},\ \ \rho_1 = (-1)^{l_1+l_3} (-1)^k \eta'_{[N,k]}$$

Spin1/2 fermion on 5-dim.

→ 4-dim. Dirac fermion or
a pair of 4-dim. Weyl fermion

$$\psi(x,y) = \begin{pmatrix} \psi_R(x,y) \\ \psi_L(x,y) \end{pmatrix}$$

$$\bar{\psi} i\gamma^M \partial_M \psi = \bar{\psi} i\gamma^\mu \partial_\mu \psi + \bar{\psi} i\gamma^5 \partial_y \psi$$

$$\bar{\psi} i\gamma^5 \partial_y \psi = i\left(\psi_L^\dagger \partial_y \psi_R - \psi_R^\dagger \partial_y \psi_L\right)$$

$$\partial_y \xrightarrow{Z_2} -\partial_y$$

$\psi_R(x,y)$ has a different Z_2 parity of $\psi_L(x,y)$.

Results

"Orbifold family unification"
Y. K., T. Kinami and K. Oda,
Phys. Rev. D76 (2007), 035001
(hep-ph/0703195)

Q1. $SU(N) \xrightarrow{Z_2} SU(5) \times \cdots,$

$[N,k] = {}_N C_k \rightarrow 3 \times (\overline{5} + 10)\,?$

$[N,k]$	(p,q,r,s)	$(-1)^k \eta_{[N,k]}$	$(-1)^k \eta'_{[N,k]}$
$[9,3]$	$(5,0,3,1)$	$+1$	-1
$[9,3]$	$(5,3,0,1)$	-1	$+1$
$[9,6]$	$(5,0,3,1)$	$+1$	$+1$
$\vdots$			

$$\text{Q1. } SU(N) \xrightarrow{Z_2} SU(5)\times\cdots,$$
$$[N,k] = {}_N C_k \to 3\times(\bar{5}+10)\,?$$

(Ex. 1) $SU(9)$ (SUSY) GUT

$$P_0 = \mathrm{diag}(\overbrace{+1,+1,+1,+1,+1}^{p=5},\overbrace{}^{q=0}\overbrace{-1,-1,-1}^{r=3},\overbrace{-1}^{s=1})$$
$$P_1 = \mathrm{diag}(+1,+1,+1,+1,+1,+1,+1,+1,-1)$$

$$SU(9) \to SU(5)\times SU(3)\times U(1)^2$$

$$[9,6] = {}_9 C_6 \Rightarrow \sum_{l_1=0}^{6}\sum_{l_3=0}^{6-l_1}\left({}_5 C_{l_1},\ {}_3 C_{l_3},\ {}_1 C_{6-l_1-l_3}\right)$$

$$\text{Q2. } SU(N) \xrightarrow{Z_2} SU(3)_C \times SU(2)_L \times U(1)_Y \times \cdots,$$

$$[N,k] = {}_N C_k \to 3 \times \begin{pmatrix} (\bar{3},1)_2 + (1,2)_{-3} \\ + (3,2)_1 + (\bar{3},1)_{-4} + (1,1)_6 \end{pmatrix} ?$$

No solution satisfying $n_{\bar{d}} = n_l = n_{\bar{u}} = n_{\bar{e}} = n_q = 3$

Table: Flavor number of each chiral fermion from [9, 6]

Representation	$\eta_{[9,6]}$	$\eta'_{[9,6]}$	$n_{\bar{d}}$	n_l	$n_{\bar{u}}$	$n_{\bar{e}}$	n_q	$n_{\bar{\nu}}$
[9,6]	$+1$	$+1$	3	3	3	3	2	1
[9,6]	$+1$	-1	3	3	2	2	3	3

$$P_0 = \text{diag}(\overbrace{+1,+1,+1}^{p=3},\overbrace{+1,+1}^{q=2},\overbrace{-1,-1,-1}^{r=3},\overbrace{-1}^{s=1})$$

$$P_1 = \text{diag}(+1,+1,+1,-1,-1,+1,+1,+1,-1)$$

Subject 1 ：Derive it from more fundamental theory such as superstring theory ?

Subject 2 ：Construct a realistic model

$[N,1] \Rightarrow H_W,\ [N,1] \Rightarrow \overline{H}_W$

Breakdown of extra gauge group, weak Higgs doublets

Realistic fermion mass matrices, CKM matrix, MNS matrix

Model-dependent predictions

Realistic fermion mass matrices, CKM matrix, MNS matrix

Extra $U(1)$ ➔ Froggatt-Nielsen mechanism

Bulk field ➔ Volume suppression

K. Yoshioka "On fermion mass hierarchy with extra dimensions" *Mod. Phys. Lett.* A15 (2000), 29

$$f_{ij}\left(\frac{\phi_\alpha}{\Lambda}\right)^{q_{i\alpha}+u_{j\alpha}+h_{u\alpha}}\left(\frac{1}{\sqrt{M_* R}}\right)^{n_{ij}} Q_i U_j^c H_u$$

Model-dependent predictions

Sum rules among sparticles masses can be useful to select high-energy physics including orbifold family unification model.

"More about Superparticle Sum Rules …" 0705.1014 [hep-ph], *Int. J. Mod. Phys.* A22 (2007), 4671 (with T. Kinami)

"Sfermion Mass Relations in Orbifold Family Unification" 0709.1524 [hep-ph], (with T. Kinami)

5. Summary

Orbifold SUSY GUTs

➔ Reduction of Gauge symmetry

Triplet-doublet Higgs mass splitting

"Arbitrariness Problem"

What is an origin of non-trivial Z_2 parities?

➔Theories are classified into equivalent classes of boundary conditions.

➔What is a principle to select a realistic one?

An underlying theory must tell something.

“Orbifold family unification”

What is an origin of three families?

Many models with three families of $SU(5)$ GUT

No model with only three families of the SM as zero modes from a unique representation

An underlying theory must tell something.

Thank you for your attention!