Towards a Realistic Grand Gauge-Higgs Unification

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Outline

1: Introduction

2: 5D SU(6) Model

- 2-1: Gauge group structure
- 2-2: SM fermion embedding
- 2-3: Electroweak symmetry breaking
- 2-4: Proton decay @tree level

3:Summary

Introduction

Gauge-Higgs unification is a very attractive scenario since the quantum correction to Higgs mass is finite by the higher dimensional gauge symmetry in spite of nonrenormalizable theories

↓

Possibility to solve the gauge hierarchy problem without SUSY

There has been much progress from various viewpoints and the progress is still now going on

It is meaningful to consider the grand unified version of gauge-Higgs unification scenario ("Grand Gauge-Higgs unification")
since
the gauge hierarchy problem was originally addressed in the GUT framework and
GUT physics in the context of gauge-Higgs unification has not been studied so much

In this talk,
we discuss an attempt towards
a realistic grand gauge-Higgs unification

⇒ 5D SU(6) model on S^1/Z_2

Parity assignments

$$P_0 = diag\left(+,+,+,+,+,-\right) @\ y = 0$$

$$\Rightarrow SU\left(6\right) \rightarrow SU\left(5\right) \times U\left(1\right) @\ y = 0$$

$$P_1 = diag\left(+,+,-,-,-,-\right) @\ y = \pi R$$

$$\Rightarrow SU\left(6\right) \rightarrow SU\left(2\right) \times SU\left(4\right) \times U\left(1\right) @\ y = \pi R$$

For gauge field

$$A_\mu = \begin{pmatrix} (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \end{pmatrix}$$

KK mode expansions

$$\Phi_{(+,+)}(x,y)=\frac{1}{\sqrt{2\pi R}}\left[\phi^{(0)}_{(+,+)}(x)+\sqrt{2}\sum_{n=1}^{\infty}\phi^{(n)}_{(+,+)}(x)\cos\left(\frac{n}{R}y\right)\right]$$

$$\Phi_{(+,-)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi^{(n)}_{(+,-)}(x)\cos\left(\frac{n+\frac{1}{2}}{R}y\right)$$

$$\Phi_{(-,+)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi^{(n)}_{(-,+)}(x)\sin\left(\frac{n+\frac{1}{2}}{R}y\right)$$

$$\Phi_{(-,-)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\phi^{(n)}_{(-,-)}(x)\sin\left(\frac{n}{R}y\right)$$

KK mode expansions

Only (+,+) mode has a massless mode

$$\Phi_{(+,+)}(x,y)=\frac{1}{\sqrt{2\pi R}}\left[\phi^{(0)}_{(+,+)}(x)+\sqrt{2}\sum_{n=1}^{\infty}\phi^{(n)}_{(+,+)}(x)\cos\left(\frac{n}{R}y\right)\right]$$

$$\Phi_{(+,-)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi^{(n)}_{(+,-)}(x)\cos\left(\frac{n+\frac{1}{2}}{R}y\right)$$

$$\Phi_{(-,+)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi^{(n)}_{(-,+)}(x)\sin\left(\frac{n+\frac{1}{2}}{R}y\right)$$

$$\Phi_{(-,-)}(x,y)=\frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\phi^{(n)}_{(-,-)}(x)\sin\left(\frac{n}{R}y\right)$$

Focusing on 0 modes,

$$A_\mu = \begin{pmatrix} (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \end{pmatrix}$$

Gauge symmetry breaking

$SU(6) \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$

Weinberg angle

$$\sin^2\theta_W = \frac{3}{8}$$

Same as the Georgi-Glashow SU(5) GUT

This gauge symmetry breaking pattern is NOT NEW

SUSY gauge-Higgs ⇒ Burdman & Nomura, NPB656 (2003) 3
Non SUSY gauge-Higgs
⇒Haba, Hosotani, Kawamura & Yamashita, PRD70 (2004) 015010

This symmetry breaking structure was also considered in the pseudo NG boson scenario of Higgs boson as global symmetry breaking

Inoue, Kakuto & Komatsu, PTP75(1986) 664 etc...

↓

Very natural from the viewpoint of AdS/CFT

∵ global sym of 4D ⇔ gauge sym in 5D

Focusing on 0 modes,

$$A_\mu = \begin{pmatrix} (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \end{pmatrix}$$

A_5 has a zero mode transforming as an $SU(2)_L$ doublet

Higgs in the Standard Model!!

Furthermore,

$$A_\mu = \begin{pmatrix} (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (+,-)(+,-)(+,+)(+,+)(+,+)(-,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,-)(-,-)(-,+)(-,+)(-,+)(+,+) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (-,+)(-,+)(-,-)(-,-)(-,-)(+,-) \\ (+,+)(+,+)(+,-)(+,-)(+,-)(-,-) \end{pmatrix}$$

Colored Higgs has NO zero mode

↓

Doublet-Triplet mass splitting works
(doublet Higgs: massless, Triplet Higgs: 1/2R)

Kawamura, PTP105 (2001) 691, 999

1 generation of SM q & l are elegantly embedded into the following representations (including the RH ν) NEW!!

$$6^* = \begin{cases} 6^*_L = \underbrace{\left(3^*,1\right)^{(+,-)}_{(1/3,-1)} \oplus l_L\left(1,2\right)^{(+,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\left(1,1\right)^{(-,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{\left(3^*,1\right)^{(-,+)}_{(1/3,-1)} \oplus \left(1,2\right)^{(-,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\nu_R\left(1,1\right)^{(+,+)}_{(0,5)}}_{1} \end{cases}$$

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$$20 = \begin{cases} 20_L = \underbrace{q_L\left(3,2\right)^{(+,+)}_{(1/6,-3)} \oplus \left(3^*,1\right)^{(+,-)}_{(-2/3,-3)} \oplus \left(1,1\right)^{(+,-)}_{(1,-3)}}_{10} \oplus \underbrace{\left(3^*,2\right)^{(-,+)}_{(-1/6,3)} \oplus \left(3,1\right)^{(-,-)}_{(2/3,3)} \oplus \left(1,1\right)^{(-,-)}_{(-1,3)}}_{10^*} \\ 20_R = \underbrace{\left(3,2\right)^{(-,-)}_{(1/6,-3)} \oplus \left(3^*,1\right)^{(-,+)}_{(-2/3,-3)} \oplus \left(1,1\right)^{(-,+)}_{(1,-3)}}_{10} \oplus \underbrace{\left(3^*,2\right)^{(+,-)}_{(-1/6,3)} \oplus u_R\left(3,1\right)^{(+,+)}_{(2/3,3)} \oplus e_R\left(1,1\right)^{(+,+)}_{(-1,3)}}_{10^*} \end{cases}$$

The difference between L & R components are only relative parity sign

In 2nd 6* rep, the signs of parity @y=0 are flipped

1 generation of SM q & l are elegantly embedded into the following representations (including the RH ν)

NEW!!

$$6^* = \begin{cases} 6^*_L = \underbrace{(3^*,1)^{(+,-)}_{(1/3,-1)} \oplus l_L\,(1,2)^{(+,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(-,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{(3^*,1)^{(-,+)}_{(1/3,-1)} \oplus (1,2)^{(-,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\nu_R\,(1,1)^{(+,+)}_{(0,5)}}_{1} \end{cases}$$

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No Massless Exotics!!
(zero modes are just SM)

No SM Anomalies!!
(U(1)x broken by anomalies)

Electroweak symmetry breaking

Electroweak symmetry is broken by ‹A_5› (Hosotani mechanism)
Whether the desired pattern of breaking is obtained depends on the matter content
Here we just give an example

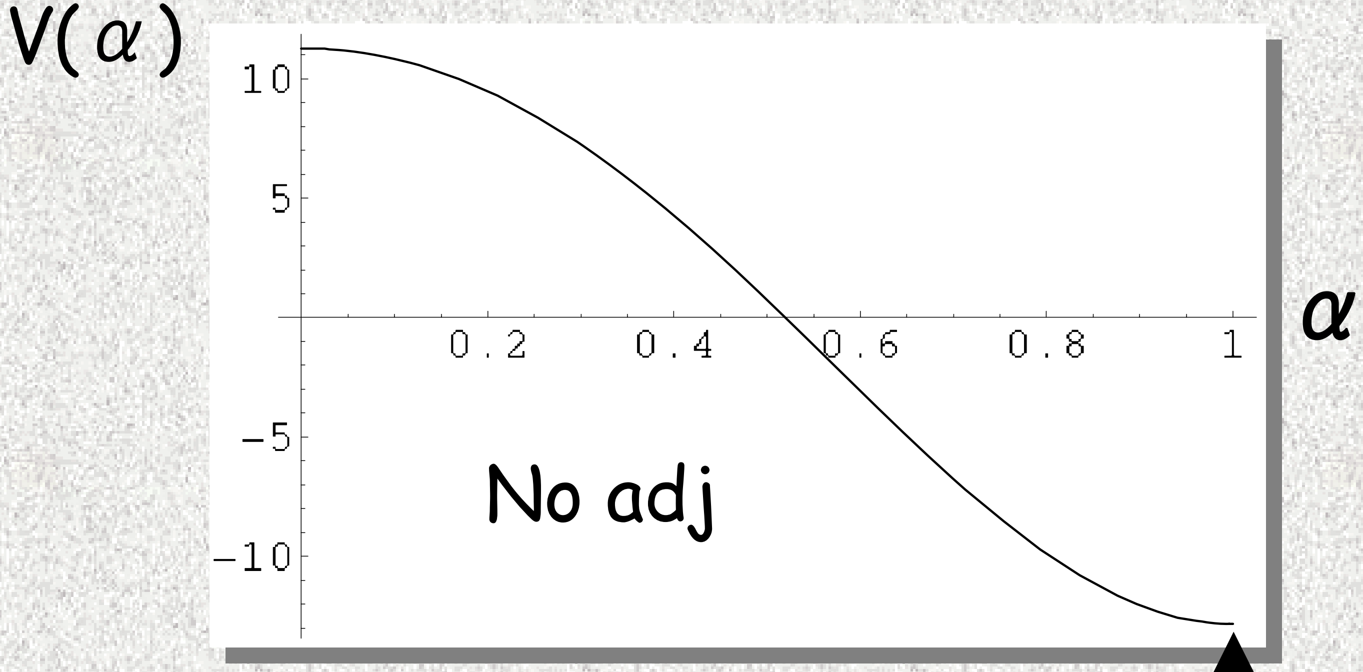
Consider as the matter fermion

N_{ad} x 35 + 3 x (6*+6*+20)

$$V(\alpha)=\frac{3}{128\pi^7R^5}\left[\underbrace{(4N_{ad}-3)\sum_{n=1}^{\infty}\frac{1}{n^5}\left\{\cos(2\pi n\alpha)+2\cos(\pi n\alpha)+6(-1)^n\cos(\pi n\alpha)\right\}}_{\text{Gauge + Ghost + }N_{ad}\times 35\text{ fermions}}\right.$$

$$\left.+48\underbrace{\sum_{n=1}^{\infty}\frac{1+(-1)^n}{n^5}\cos(\pi n\alpha)}_{3\times(6^*+6^*+20)\text{ fermions}}\right],\ \langle A_5\rangle\equiv\frac{\alpha}{gR}\frac{\lambda_{27}}{2}$$

Without adjoint fermion, SU(2) x U(1) → U(1) x U(1)

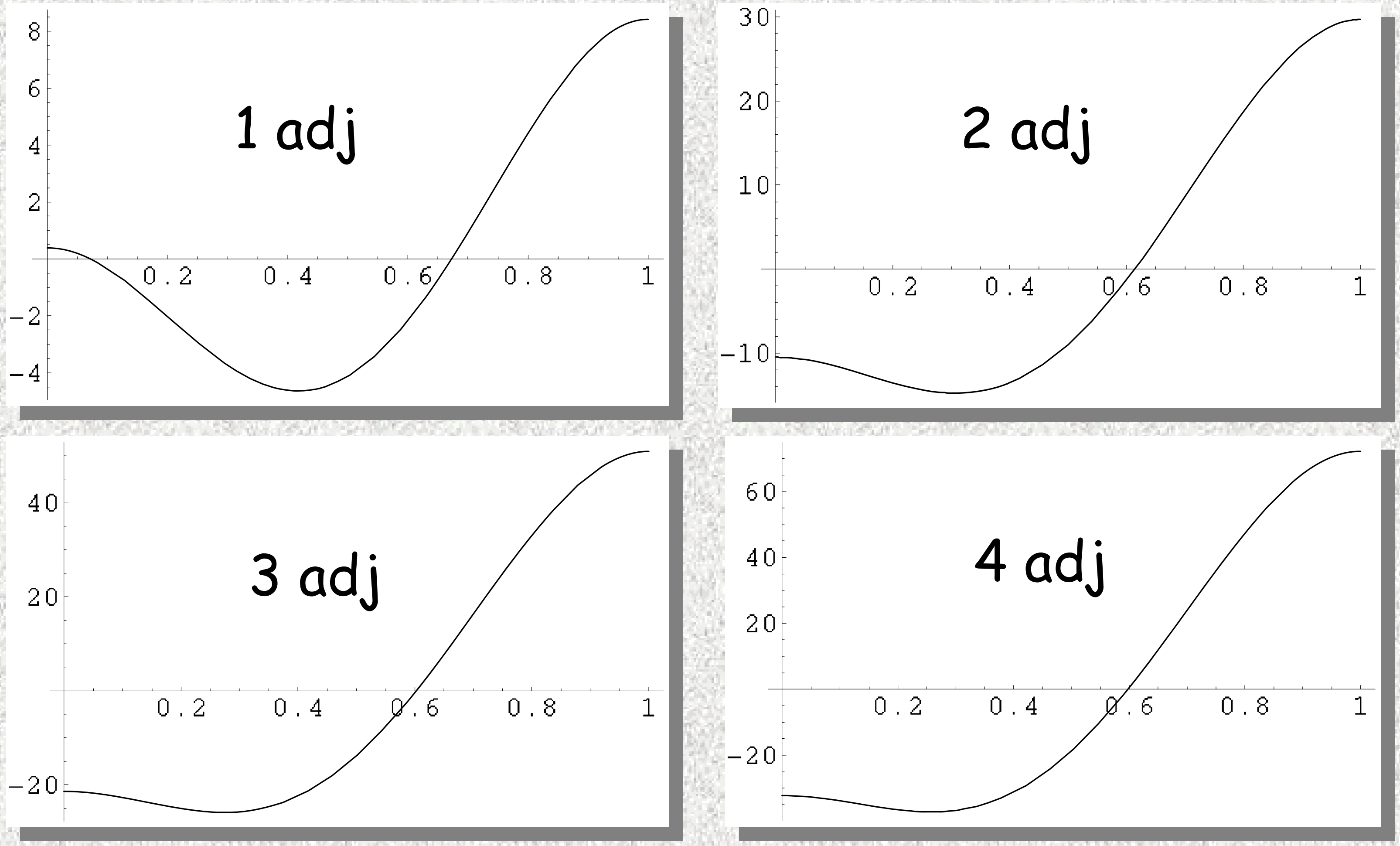


Minimum@ α =1

Wilson line phase:

$$W = \mathcal{P}\exp\left(ig\oint_{S^1} dyA_y\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi\alpha) & i\sin(\pi\alpha) \\ 0 & i\sin(\pi\alpha) & \cos(\pi\alpha) \end{pmatrix} \quad (\alpha \bmod 2) \rightarrow \begin{cases} SU(2)\times U(1) \text{ for } \alpha = 0 \\ U(1)\times U(1) \text{ for } \alpha = 1 \\ U(1)_{em} \text{ for other cases} \end{cases}$$

On adding adjoint fermions, $SU(2)\times U(1) \rightarrow U(1)$ is realized



# of adj fermions is larger, α becomes smaller
$\Rightarrow$ useful to make Higgs mass heavy

Higgs mass

$$m_H^2 = g^2 R^2 \frac{d^2 V(\alpha)}{d\alpha^2}\bigg|_{\alpha=\alpha_0} \qquad m_W = \frac{\alpha}{R}, g_4^2 = \frac{g^2}{2\pi R}$$

$$= -\frac{3 g_4^2 m_W^2}{32\pi^4 \alpha^2}\left[(4N_{ad}-3)\sum_{n=1}^{\infty}\frac{1}{n^3}\left\{2\cos(\pi n\alpha) + \cos(\pi n\alpha) + 3(-1)^n \cos(\pi n\alpha)\right\} + 24\sum_{n=1}^{\infty}\frac{1+(-1)^n}{n^3}\cos(\pi n\alpha)\right]\Bigg|_{\alpha=\alpha_0}$$

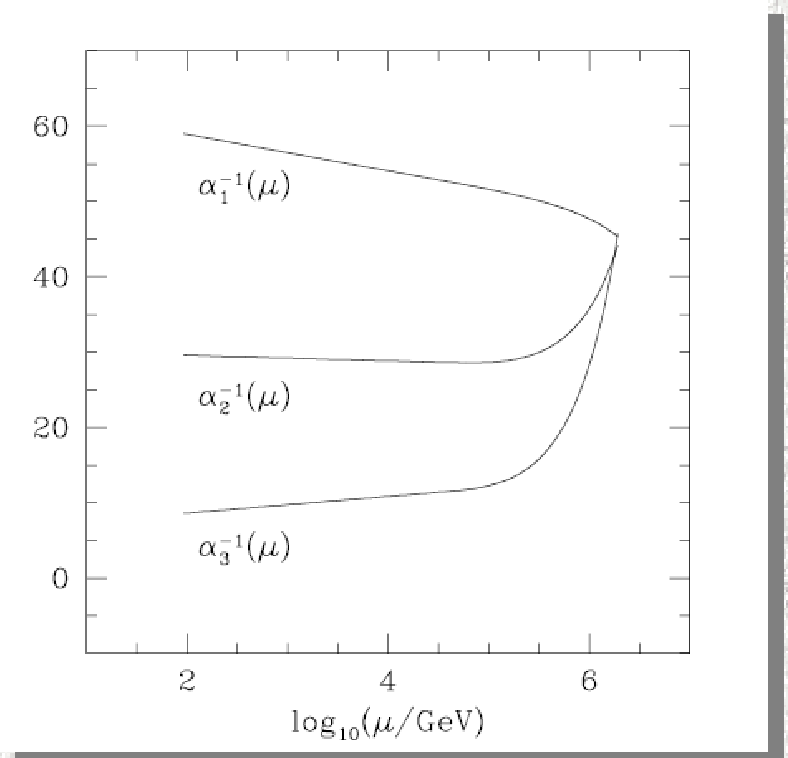
$$m_H^2 \approx \frac{g^2}{16\pi^2 R^3} \approx \frac{g_4^2}{16\pi^2\alpha^2} m_W^2 \ll m_W^2 \left(\alpha \sim \mathcal{O}(1)\right)$$

Highly nontrivial to obtain the Higgs mass > 114GeV in the gauge-Higgs unification

Ex.

N_{ad}	α_0	Higgs mass (GeV)
20	0.216557	113.9 g_4
21	0.216083	116.9 g_4

In the extra dimensional theory (large extra dimension), GUT scale is expected to be lowered because of the power-law gauge coupling running



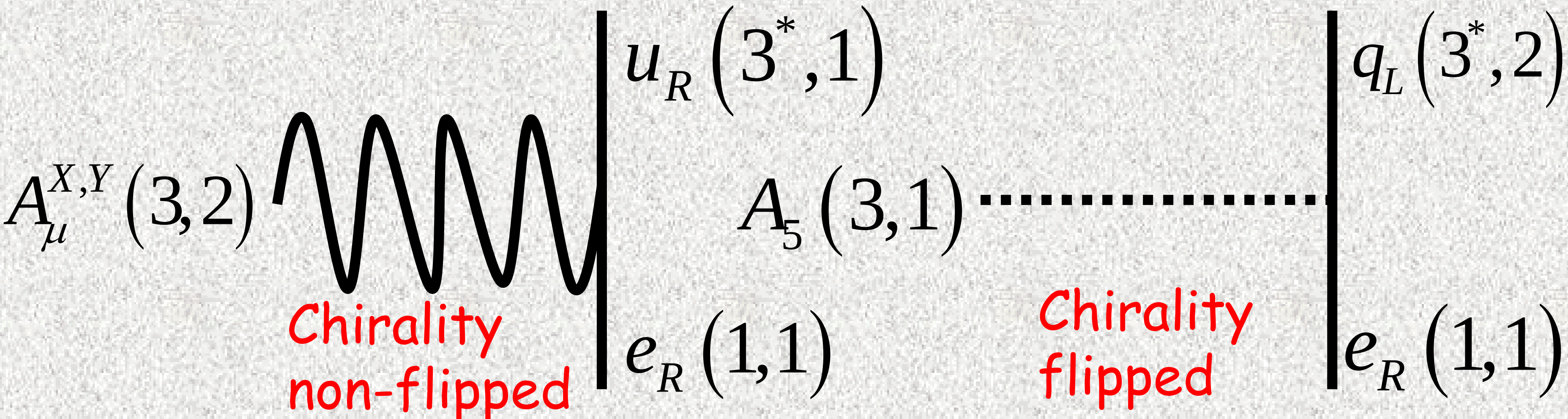
5D MSSM with 1/R∼100TeV, vector, Higgs only in the bulk (Dienes, Dudas & Gherghetta)

Low GUT scale
↓
Rapid proton decay!!

No D=6 #B violating operators @tree level without relying on any additional symmetry

Proton decay analysis@tree level

Possible B-number violating gauge interactions: only from 20



$$6^* = \begin{cases} 6_L^* = \underbrace{(3^*,1)^{(+,-)}_{(1/3,-1)} \oplus l_L\,(1,2)^{(+,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(-,-)}_{(0,5)}}_{1} \\ 6_R^* = \underbrace{(3^*,1)^{(-,+)}_{(1/3,-1)} \oplus (1,2)^{(-,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\nu_R\,(1,1)^{(-,-)}_{(0,5)}}_{1} \end{cases}, \quad 6^* = \begin{cases} 6_L^* = \underbrace{(3^*,1)^{(-,-)}_{(1/3,-1)} \oplus (1,2)^{(-,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(+,-)}_{(0,5)}}_{1} \\ 6_R^* = \underbrace{d_R\,(3^*,1)^{(+,+)}_{(1/3,-1)} \oplus (1,2)^{(+,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(-,+)}_{(0,5)}}_{1} \end{cases}$$

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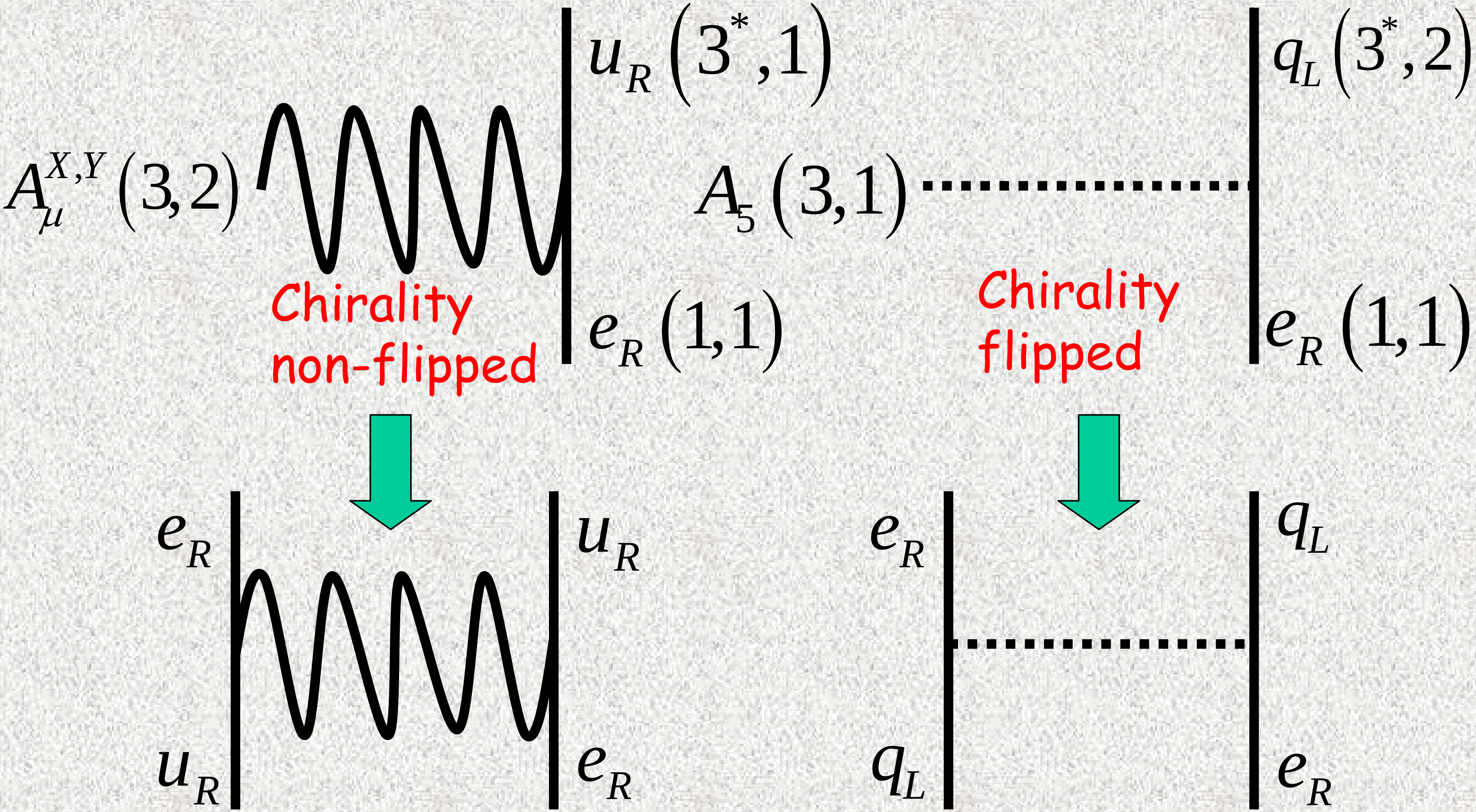
Proton decay analysis@tree level

Possible B-number violating gauge interactions: only from 20



NO net B-number violation!!

What is the difference from the conventional GUT???

⇒ These fields belong to the same multiplets in the usual GUT

$$6^* = \begin{cases} 6^*_L = \underbrace{(3^*,1)^{(+,-)}_{(1/3,-1)} \oplus l_L\,(1,2)^{(+,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(-,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{(3^*,1)^{(-,+)}_{(1/3,-1)} \oplus (1,2)^{(-,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\nu_R\,(1,1)^{(-,-)}_{(0,5)}}_{1} \end{cases}, \quad 6'^* = \begin{cases} 6^*_L = \underbrace{(3^*,1)^{(-,-)}_{(1/3,-1)} \oplus (1,2)^{(-,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(+,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{d_R\,(3^*,1)^{(+,+)}_{(1/3,-1)} \oplus (1,2)^{(+,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{(1,1)^{(-,+)}_{(0,5)}}_{1} \end{cases}$$

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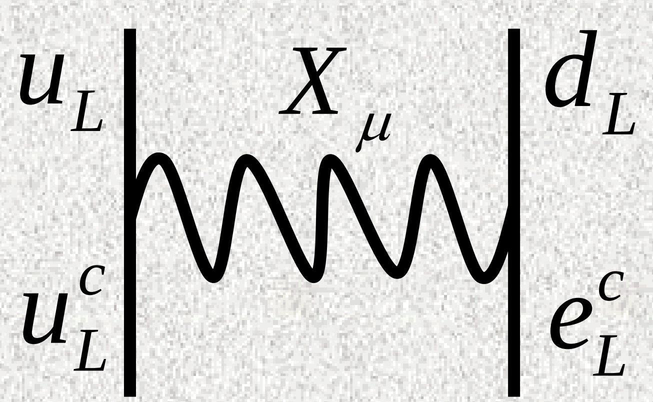
*

Ex.



ΔB = 1 process allowed

Summary

5D SU(6) grand gauge-Higgs unification is studied:

- SM quarks and leptons (including RH ν's) are embedded in 3 x (6* + 6* + 20) w/o massless exotics ⇒ No SM anomalies
- Electroweak symmetry breaking is realized simply by introducing adj fermions
- Higgs mass > 114 GeV if more than 20 adj fermions added
- Vanishing proton decay @tree level without relying on any additional symmetry

The attempt towards a realistic grand gauge-Higgs unification has just started, many issues remain to be investigated...

- Simplifying the matter content generating Higgs mass > 114GeV
- Proton decay analysis @quantum level
- Gauge coupling unification
- Fermion mass hierarchy & Flavor physics
- More...

Hope to report good results for the issues@GUT08!

Backup slides

Selection rule?

X,Y gauge bosons & Colored Higgs: $\cos\left[\left(n+\frac{1}{2}\right)\frac{y}{R}\right]$

Their partners: $\sin\left[\left(n+\frac{1}{2}\right)\frac{y}{R}\right]$

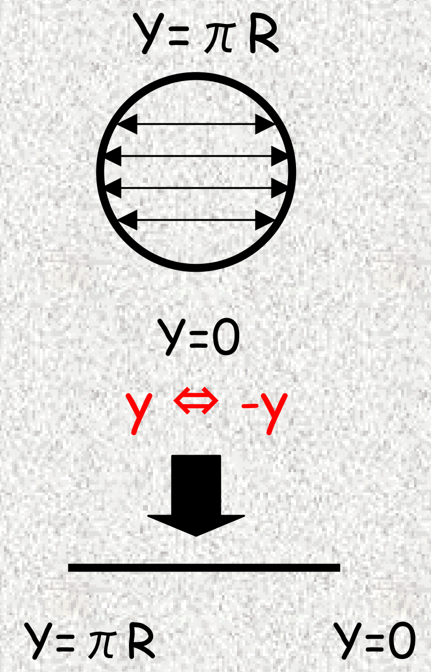
Gauge interactions for zero modes are proportional to the integral of the wave function for the gauge fields

$$\int_0^{2\pi R} dy\cos\left[\left(n+\frac{1}{2}\right)\frac{y}{R}\right] \propto \left[\sin\left[\left(n+\frac{1}{2}\right)\frac{y}{R}\right]\right]_0^{2\pi R} = 0$$

$$\int_0^{2\pi R} dy\sin\left[\left(n+\frac{1}{2}\right)\frac{y}{R}\right] \propto \left[-\cos\left[\left(n+\frac{1}{2}\right)\frac{y}{R}\right]\right]_0^{2\pi R} \neq 0$$

The gauge interaction vertices from X,Y gauge bosons and colored Higgs are vanishing

For 5th dimension compactified on S^1/Z_2



$$A_M\left(x, y+2\pi R\right)=UA_M\left(x,y\right)U^\dagger \;(M=\mu, y)$$

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}\left(x, y_i - y\right)=P_i\begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}\left(x, y_i + y\right)P_i^\dagger \;(\mathrm{i}=0,1)$$

$$\psi\left(y+2\pi R\right)=U\psi\left(y\right)$$

$$\psi\left(x, y_i - y\right)=\eta_i P_i \gamma^5 \psi\left(x, y_i + y\right)\left(\eta_i=\pm 1\right)$$

R : radius of 5th dimension, $y_0=0, y_1=\pi R$

$$U^\dagger=U^{-1}, P_i^\dagger=P_i^{-1}=P_i=diag\left(1,1,-1\right)$$

- Minus sign of A_y is required from the gauge invariance

$$F_{\mu y}=\partial_\mu A_y-\partial_y A_\mu+\left[A_\mu, A_y\right]:\text{"}-\text{"}$$

- U, P_0, P_1 are not independent because

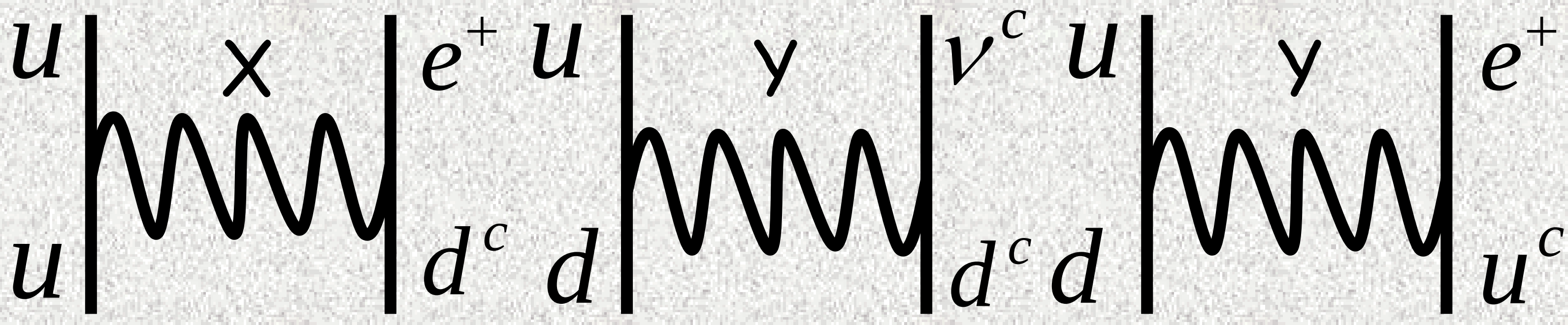
$$P_1:\pi R+y\xrightarrow{P_0}-\left(\pi R+y\right)\xrightarrow{U}\pi R-y\Rightarrow U=P_1P_0=\mathrm{diag}\left(1,1,1\right)$$

Proton decay in the minimal SU(5) GUT

X,Y gauge boson vertex:

$$\bar{\psi}\left(5^*\right)\gamma^\mu A_\mu \psi\left(5\right) + Tr\bar{\psi}\left(10^*\right)\gamma^\mu\left\{A_\mu,\psi\left(10\right)\right\} \supset X^{a}_{\mu\alpha}\left[\varepsilon^{\alpha\beta\gamma}\bar{u}^c_\gamma\gamma^\mu q_{\beta a} + \varepsilon^{ab}\left(\bar{q}_{\alpha b}\gamma^\mu e^+ - \bar{l}_b\gamma^\mu d^c_\alpha\right)\right]$$

$$X^\mu{}_{a\alpha} = \left(X^\mu{}_\alpha, Y^\mu{}_\alpha\right),\ q_{a\alpha} = \left(u_\alpha, d_\alpha\right),\ l = \left(\nu_e, e\right)$$



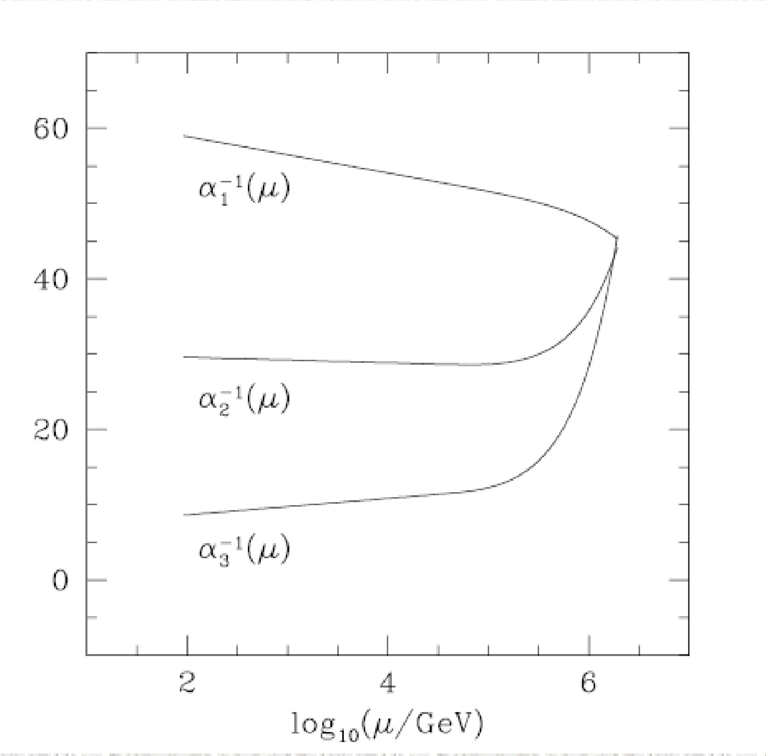
Δ B=1 proton decay diagrams

$$\mathcal{L}_{\Delta B=1} = \frac{g^2}{2M_X^2}\varepsilon^{\alpha\beta\gamma}\varepsilon^{ab}\left(\bar{u}^c_\gamma\gamma^\mu q_{\beta a}\right)\left(\bar{d}^c_\alpha\gamma_\mu l_b + \bar{e}^+\gamma_\mu q_{\alpha b}\right)$$

In the extra dimensional theory, GUT scale is expected to be lowered because of the power-law gauge coupling running

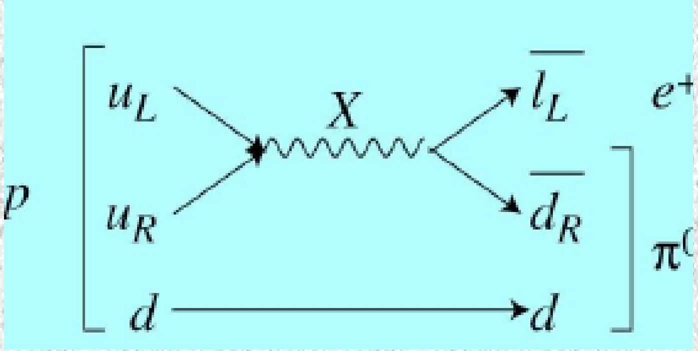
$$5D:\alpha^{-1}(\mu)=\alpha^{-1}(\mu_0)+c\left(\frac{\mu}{\mu_0}\right)\ (c:\text{const})$$

Low GUT scale
↓
Rapid proton decay!!



5D MSSM with 1/R~100TeV, vector, Higgs only in the bulk (Dienes, Dudas & Ghergheta)

X,Y gauge boson exchange
D=6 operator $p\rightarrow\pi^0+e^+$



$$M_X>10^{15-16}GeV\left(\tau_p>10^{31}\text{ years}\right)$$

Mixed Anomalies with $U(1)_X$

$$U(1)_X\left[SU(3)_C\right]^2 : (-1-3\times 2-3)\times 3C_2(3) = -30C_2(3)$$

$$U(1)_X\left[SU(2)_L\right]^2 : (-1-3\times 3)\times 3C_2(2) = -30C_2(2)$$

$$U(1)_X\left[U(1)_Y\right]^2 : \left(-\frac{1}{4}\times 2-\frac{1}{9}\times 3-3\times\frac{1}{36}\times 6-3\times\frac{4}{9}\times 3+3\right)\times 3 = -7$$

$$U(1)_Y\left[U(1)_X\right]^2 : \left(-\frac{1}{2}\times 2+\frac{1}{3}\times 3+\frac{1}{6}\times 9\times 6-\frac{2}{3}\times 9\times 3+1\times 9\right)\times 3 = 0$$

$$6^* = \begin{cases} 6^*_L = \underbrace{\left(3^*,1\right)^{(+,-)}_{(1/3,-1)} \oplus l_L\left(1,2\right)^{(+,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\left(1,1\right)^{(-,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{\left(3^*,1\right)^{(-,+)}_{(1/3,-1)} \oplus \left(1,2\right)^{(-,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\nu_R\left(1,1\right)^{(-,-)}_{(0,5)}}_{1} \end{cases}, 6^* = \begin{cases} 6^*_L = \underbrace{\left(3^*,1\right)^{(-,-)}_{(1/3,-1)} \oplus \left(1,2\right)^{(-,+)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\left(1,1\right)^{(+,-)}_{(0,5)}}_{1} \\ 6^*_R = \underbrace{d_R\left(3^*,1\right)^{(+,+)}_{(1/3,-1)} \oplus \left(1,2\right)^{(+,-)}_{(-1/2,-1)}}_{5^*} \oplus \underbrace{\left(1,1\right)^{(-,+)}_{(0,5)}}_{1} \end{cases}$$

$$20 = \begin{cases} 20_L = \underbrace{q_L\left(3,2\right)^{(+,+)}_{(1/6,-3)} \oplus \left(3^*,1\right)^{(+,-)}_{(-2/3,-3)} \oplus \left(1,1\right)^{(+,-)}_{(1,-3)}}_{10} \oplus \underbrace{\left(3^*,2\right)^{(-,+)}_{(-1/6,3)} \oplus \left(3,1\right)^{(-,-)}_{(2/3,3)} \oplus \left(1,1\right)^{(-,-)}_{(-1,3)}}_{10^*} \\ 20_R = \underbrace{\left(3,2\right)^{(-,-)}_{(1/6,-3)} \oplus \left(3^*,1\right)^{(-,+)}_{(-2/3,-3)} \oplus \left(1,1\right)^{(-,+)}_{(1,-3)}}_{10} \oplus \underbrace{\left(3^*,2\right)^{(+,-)}_{(-1/6,3)} \oplus u_R\left(3,1\right)^{(+,+)}_{(2/3,3)} \oplus e_R\left(1,1\right)^{(+,+)}_{(-1,3)}}_{10^*} \end{cases}$$

Mixed Anomalies with $U(1)_X$

$$U(1)_X\left[SU(3)_C\right]^2:(-1-3\times 2-3)\times 3C_2(3)=-30C_2(3)$$

$$U(1)_X\left[SU(2)_L\right]^2:(-1-3\times 3)\times 3C_2(2)=-30C_2(2)$$

$$U(1)_X\left[U(1)_Y\right]^2:\left(-\frac{1}{4}\times 2-\frac{1}{9}\times 3-3\times\frac{1}{36}\times 6-3\times\frac{4}{9}\times 3+3\right)\times 3=-7$$

$$U(1)_Y\left[U(1)_X\right]^2:\left(-\frac{1}{2}\times 2+\frac{1}{3}\times 3+\frac{1}{6}\times 9\times 6-\frac{2}{3}\times 9\times 3+1\times 9\right)\times 3=0$$

These mixed anomalies are canceled by Green-Schwarz mechanism