Gauge-Higgs Unification

Yutaka Hosotani (Osaka University)

Gauge-Higgs Unification

Origin of the Higgs

Principle for the Higgs

4D Higgs is unified with gauge fields in higher dimensions

$$A_M = (A_\mu , A_{y_j})$$

gauge principle + background spacetime

Higgs interactions are predicted.

Fairlie-Manton's Gauge-Higgs Unification in EW

Fairlie 1979
Forgacs & Manton 1980
Manton 1979

$$A_M = (\ \underbrace{A_\mu}_{\gamma, W, Z}\ ,\ \underbrace{A_{y_j}}_{Higgs}\ )$$

$$\mu^2 = -\frac{1}{R^2} < 0$$

on $M^4 \times S^2$

$$G \longrightarrow SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}$$

Ad hoc ansatz: spherical sym. & flux

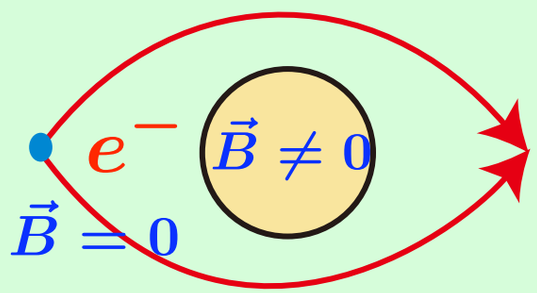
G	$\sin^2\theta_W$	M_W	M_Z	M_H
$SU(3)$	$3/4$	44 GeV	88	88
$O(5)$	$1/2$	54	76	76
G_2	$1/4$	76	88	88

Gauge-Higgs unification by AB phases

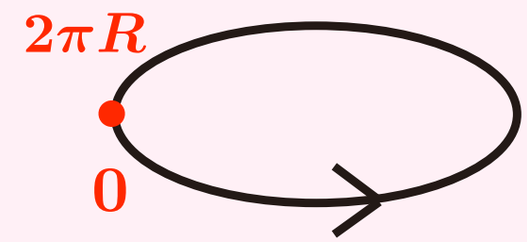
Hosotani 1983

AB phases in extra dimensions = 4D Higgs
(Wilson line)

$$\exp\left\{ ig \int A_y dy \right\} \sim e^{i\Theta_H(x)}$$



Aharonov-Bohm Effect



Extra dimension S^1

Gauge symmetry breaking

$$\exp\{\, i\Theta_H \,\} \sim \begin{pmatrix} \exp(iC) & & & & \\ & \exp(iC) & & & \\ & & \exp(iC) & & \\ & & & \exp(-\frac{3}{2}iC) & \\ & & & & \exp(-\frac{3}{2}iC) \end{pmatrix}$$

$SU(5) \to SU(3) \times SU(2) \times U(1)$ by quantum effects

Adjoint Higgs is unified with gauge fields.

Massless at the tree level --- flat direction

θ_H *: phase (large gauge inv.)*

$V_{\rm eff}(\theta_H)$ *--- non-trivial at the quantum level.*

--- finite; independent of a cutoff Λ

Finite Higgs mass : m_H *naturally light*

Minimal SU(5) model on $M^4 \times S^1$

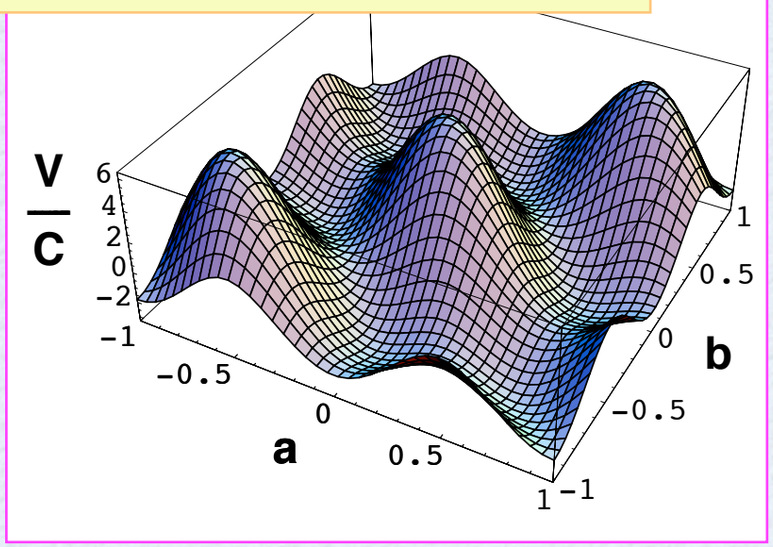
SU(5) symmetry unbroken.

Fermions in the adj. rep. etc can lead to sym. breaking.

Effective potential in SU(3) on $M^4 \times S^1$

$$(\theta_1, \theta_2, \theta_3) = (\pi a, \pi b, -\pi(a+b))$$

With $N^{\mathrm{F}}_{\mathrm{fund}} = 0\ ,\ N^{\mathrm{F}}_{\mathrm{ad}} = 1$

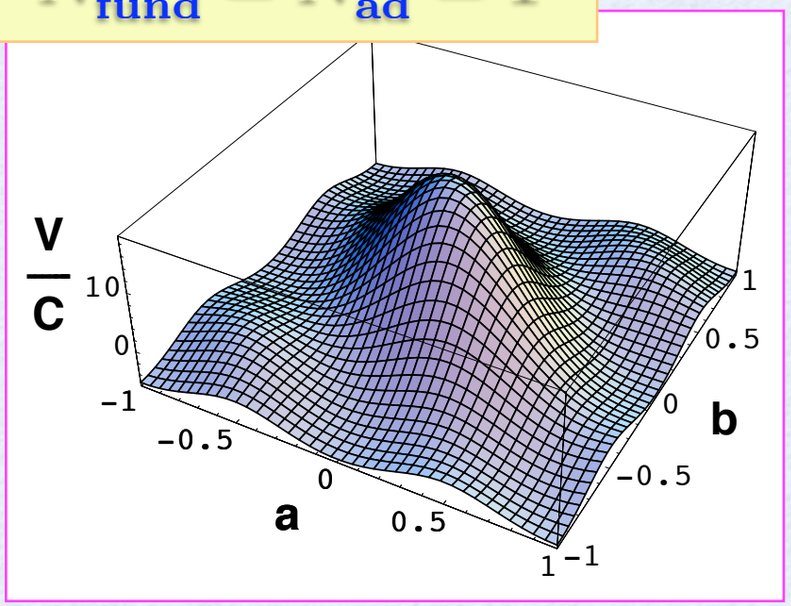


Minimum at

$$(\theta_1, \theta_2, \theta_3) = (+\frac{2\pi}{3}, -\frac{2\pi}{3},\ 0)$$

$$SU(3) \to U(1) \times U(1)$$

With $N^{\mathrm{F}}_{\mathrm{fund}} = N^{\mathrm{F}}_{\mathrm{ad}} = 1$



$$(\theta_1, \theta_2, \theta_3) = (\pi, \pi, 0)$$

$$SU(3) \to SU(2) \times U(1)$$

on S^1, T^n

Adjoint Higgs is naturally unified with gauge fields.

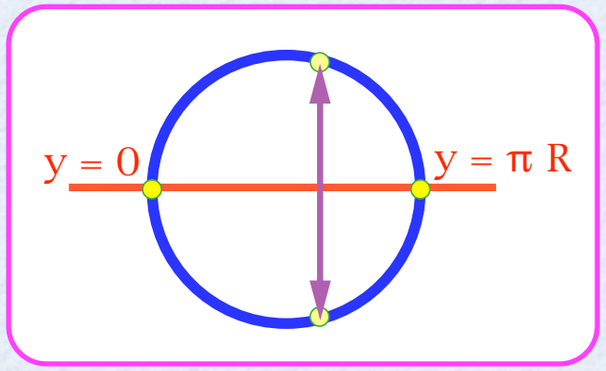
Problems on S^1

Chiral fermions

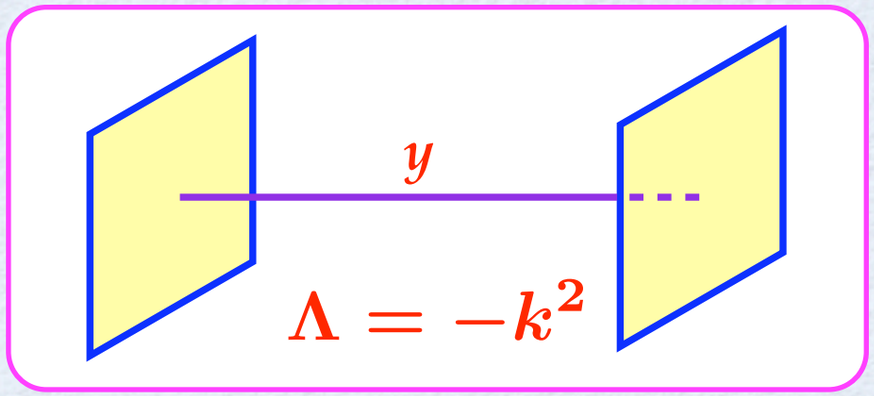
. . .

Orbifold

$M^4 \times (S^1/Z_2)$



RS spacetime



$$ds^2 = e^{-2k|y|} dx_\mu dx^\mu + dy^2$$

Single-valuedness of physics demands only

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix} (x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix} (x, y_j + y) P_j^\dagger$$

Orbifold BC : P_0 , P_1 **Parity / Twist**

Orbifold conditions : P_0 , P_1

Orbifold symmetry breaking

$SU(3)$ model

$$P_0 = P_1 = \begin{pmatrix} -1 & & \\ & -1 & \\ & & +1 \end{pmatrix}$$

$$\Rightarrow \quad A_\mu = \begin{pmatrix} \square & \\ & \square \end{pmatrix} \qquad A_y = \begin{pmatrix} & \boxed{\Phi} \\ \square & \end{pmatrix}$$

$\Phi(x)$ *(Higgs)*

$$SU(3) \to SU(2) \times U(1)$$

Chiral fermions in the bulk

$$\psi(x, -y) = P_0 \gamma^5 \psi(x, +y)$$
$$\psi(x, \pi R - y) = P_1 \gamma^5 \psi(x, \pi R + y)$$

$$\Rightarrow \quad \begin{pmatrix} \nu_L & \tilde{\nu}_R \\ e_L & \tilde{e}_R \\ \tilde{e}_L & e_R \end{pmatrix} \begin{pmatrix} \tilde{\nu}_L & \nu_R \end{pmatrix}$$

$S^1,\ T^n$

Orbifold

Adjoint Higgs
is naturally unified
with gauge fields.

Orbifold BC : $P_0\ ,\ P_1$

Chiral fermions
emerge.

We need

Dynamics ?

Fundamental Higgs
can be unified
with gauge fields.

Equivalence classes of boundary conditions

Partial dynamics for B.C.

Hosotani 1989
Haba, Harada, Hosotani, Kawamura 2003

Two distinct sets of boundary conditions can be equivalent.

$$\{P_0' \,,\, P_1'\} \sim \{P_0 \,,\, P_1\}$$

$$P_0' = \Omega(x, -y) P_0 \, \Omega(x, y)^\dagger$$
$$P_1' = \Omega(x, \pi R - y) P_1 \, \Omega(x, \pi R + y)^\dagger$$

Guaranteed by the dynamics of Wilson line phases Θ_H

$SU(N)$	S^1	**Only 1 equivalence class**
	S^1/Z_2	$(N+1)^2$ **equivalence classes**

Gauge-Higgs unification in electroweak interactions

$SO(5) \times U(1)$ *model*

Agashe, Contino, Pomarol 2005
Hosotani, Sakamura 2006
Medina, Shah, Wagner 2007

$$P_0 = P_1 = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & +1 \end{pmatrix} \Rightarrow A_y \sim \begin{pmatrix} & & \phi_1 \\ & & \phi_2 \\ & & \phi_3 \\ & & \phi_4 \\ \boxed{} & & \end{pmatrix} \quad \Phi = \begin{bmatrix} \phi_1 + i\phi_2 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

$$SO(5) \to SO(4) \simeq SU(2)_L \times SU(2)_R$$

Spinor

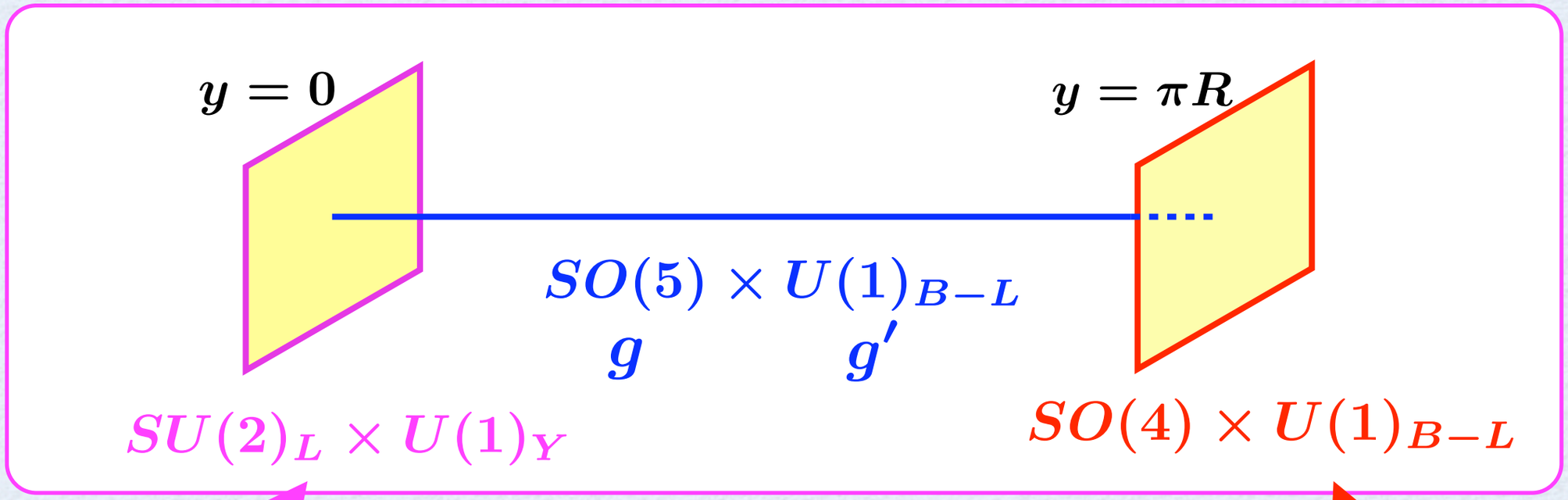
$$\begin{pmatrix} u_L & \tilde{u}_R \\ d_L & \tilde{d}_R \\ \tilde{u}_L & u_R \\ \tilde{d}_L & d_R \end{pmatrix}$$

Vector

$$\begin{pmatrix} u'_L & \tilde{u}'_R \\ u''_L & \tilde{u}''_R \\ d'_L & \tilde{d}'_R \\ d''_L & \tilde{d}''_R \\ \tilde{d}_L & d_R \end{pmatrix}$$

in the RS warped space

$$ds^2 = e^{-2k|y|} dx_\mu dx^\mu + dy^2$$



$$P_0 = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & +1 \end{pmatrix}$$

+ brane dynamics

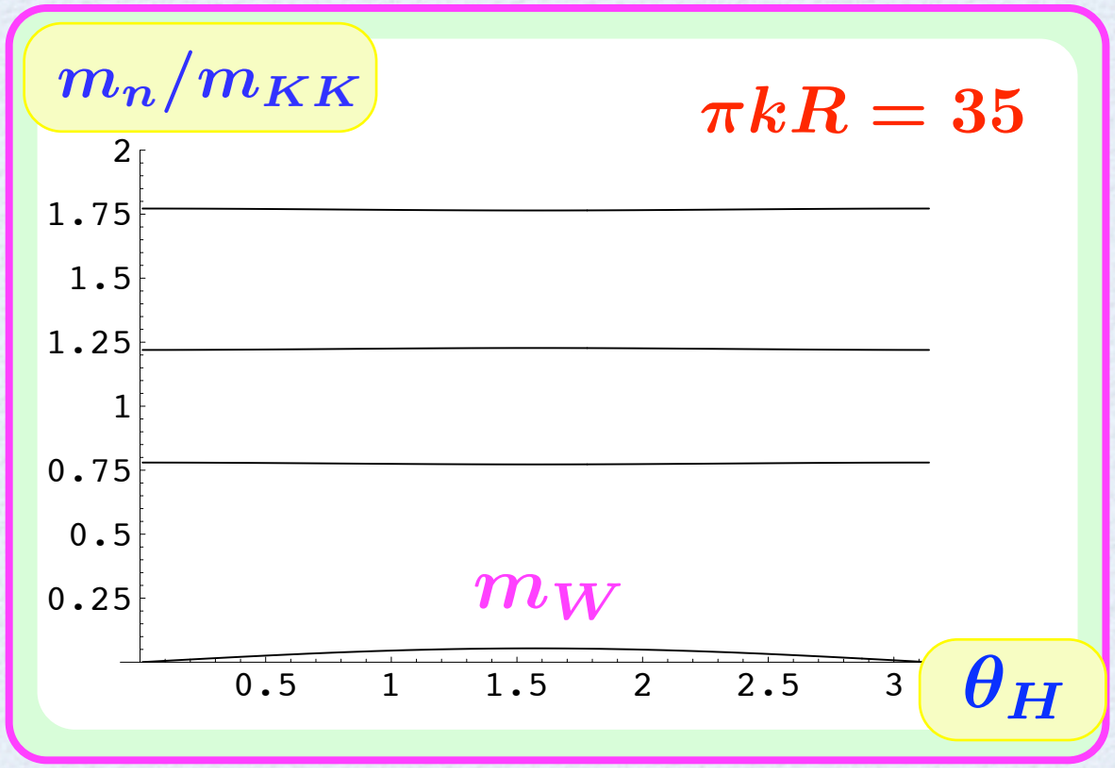
Hosotani mechanism

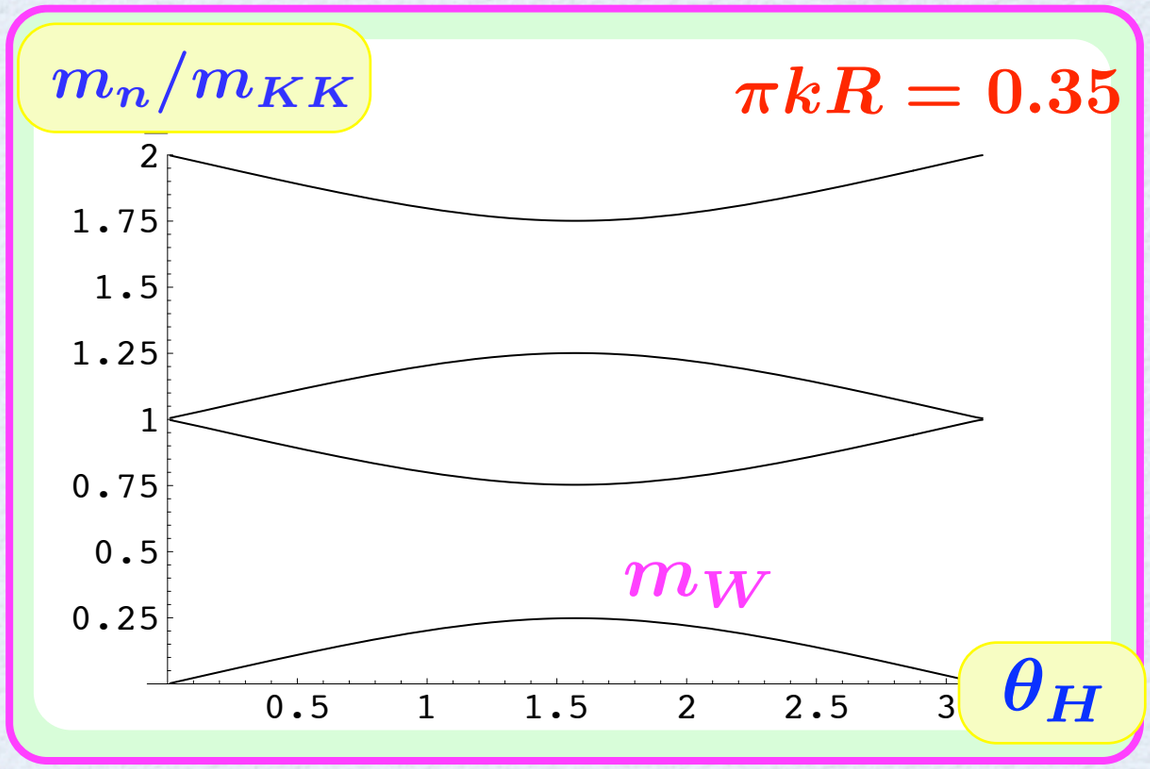
$$\theta_H \neq 0$$

$$P_1 = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & +1 \end{pmatrix}$$

$$U(1)_{EM}$$

KK tower of W-boson





$m_W \ll m_{KK}$ **in RS**

$m_W \sim (0.1 \sim 0.2)\, m_{KK}$
in flat space

too small
m_{KK}

Wave functions

γ

$$\sin\theta_W (T_L^3 + T_R^3) + \sqrt{1 - 2\sin^2\theta_W}\; T_B$$

W

$$h_L^+(y)\, T_L^+ + h_R^+(y)\, T_R^+ + \hat{h}^+(y)\, \hat{T}^+$$

$SU(2)_L$ $SU(2)_R$ $SO(5)/SO(4)$

$$h_L^+ \sim 1 + \cos\theta_H$$

$$h_R^+ \sim 1 - \cos\theta_H$$

$$\hat{h}^+ \sim \sqrt{2}\sin\theta_H \left(1 - \frac{e^{2ky}}{e^{2k\pi R}}\right)$$

Almost flat in RS !

Higgs

$$h_H \sim e^{2ky}$$

localized near TeV brane.

4D gauge and Higgs couplings

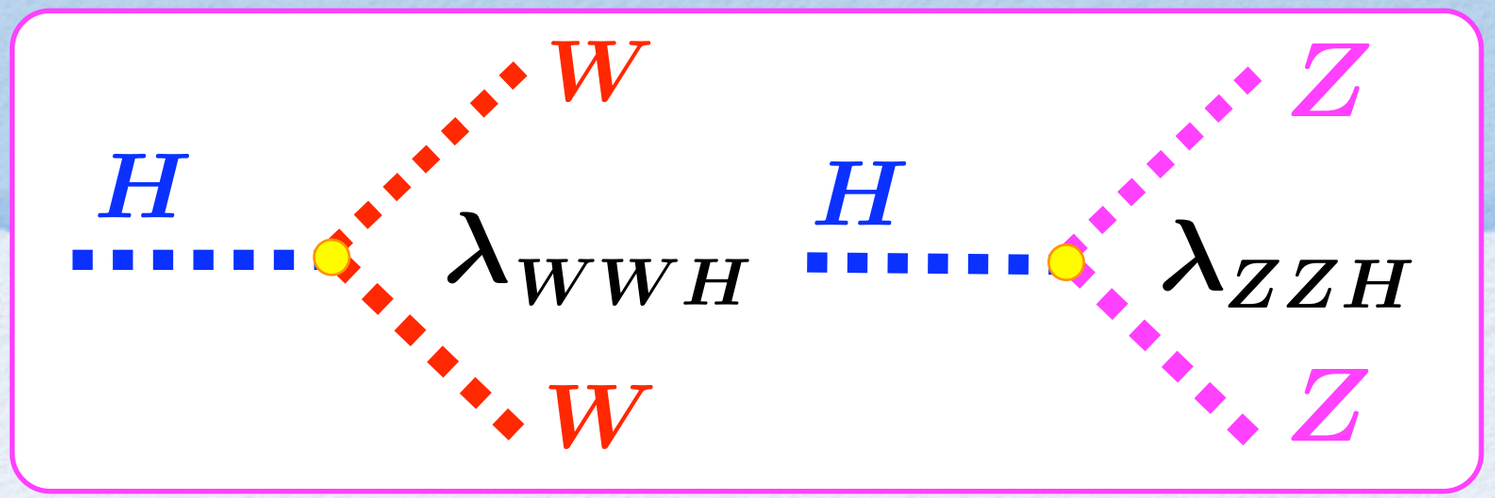
$$F_{MN}{}^2 \sim (\partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu])^2$$

WWZ
WWZZ
WWWW

$$+(\partial_\mu A_y - \partial_y A_\mu + g[A_\mu, A_y])^2$$

WWH
ZZH
WWHH
ZZHH

WWH & ZZH coupling



$$\lambda_{WWH} = g_A k \int \frac{dz}{z} h_H^{\hat{4}} \left\{ h_W^{\hat{+}} \partial_z (h_W^{+R} - h_W^{+L}) - \partial_z h_W^{\hat{+}} (h_W^{+R} - h_W^{+L}) \right\}$$

behavior near TeV brane $(z = e^{ky} = e^{\pi k R})$

$$\lambda_{WWH} \simeq g m_W \cdot \cos\theta_H$$

$$\lambda_{ZZH} \simeq \frac{g m_Z}{\cos\theta_W} \cdot \cos\theta_H$$

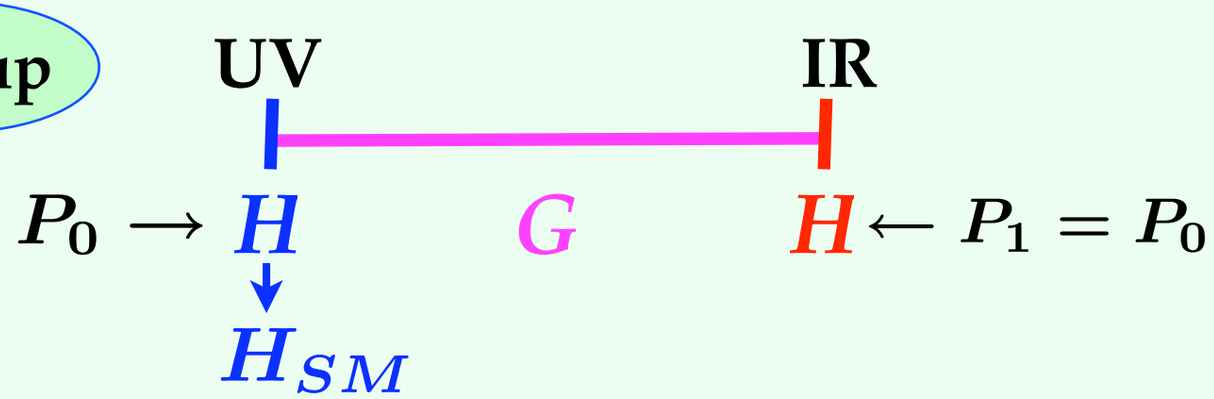
suppression

Holographic description

Effective theory at low energies

Sakamura 2007
(Panico, Wulzer 2007)

Set-up



$H \qquad G/H$

$\{\, T^a \,,\, T^{\hat{a}} \,\}$

Step 1 *Take the gauge* $\tilde{A}_y = 0$ *by*

$$\Omega = P \exp\left\{ ig \int_y^{\pi R} dy' \, A_y(x, y') \right\} \sim \exp\left\{ i\theta^{\hat{a}}(x,y) T^{\hat{a}} \right\}$$

Step 2 *Wave functions of low energy modes*

$$\tilde{A}^a_\mu(x,y) \sim \tilde{A}^a_\mu(x,0)$$

$$\tilde{A}^{\hat{a}}_\mu(x,y) \sim \frac{e^{2\pi kR} - e^{2ky}}{e^{2\pi kR} - 1} \, \tilde{A}^{\hat{a}}_\mu(x,0)$$

Step 3 *Express* $\tilde{A}^a_\mu(x,0)$, $\tilde{A}^{\hat{a}}_\mu(x,0)$ *in terms of*

$$A^{4D,a}_\mu(x)$$

$$\Omega(x,0) = \exp\left\{ \frac{i}{f_H}\, H^{\hat{a}}(x)\, T^{\hat{a}} \right\} \qquad f_H = \sqrt{\frac{2}{\pi k R}}\, \frac{m_{KK}}{g_4}$$

Step 4

$$\mathcal{L}^{4D}_{\rm eff} \sim -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} - \frac{1}{2} \tilde{D}_\mu H^{\hat{a}} \tilde{D}^\mu H^{\hat{a}}$$

Approximation justified for $\pi k R \gg 1$

$\Longleftrightarrow$ *Holographic PGB picture*

$SO(5) \times U(1)$ model

$$\mathcal{L}_{\rm int}^{4D} \sim -\frac{1}{2} g_4^2 f_H^2 \sin^2\left(\theta_H + \frac{H}{\sqrt{2} f_H}\right) \left\{ W_\mu^\dagger W^\mu + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu \right\}$$

$$m_W = \frac{1}{\sqrt{2}} g_4 f_H \sin\theta_H \qquad m_Z = \frac{m_W}{\cos\theta_W}$$

WWH	$g_4 m_W \cos\theta_H$	ZZH	$\frac{g_4 m_Z}{\cos\theta_W} \cos\theta_H$
$WWHH$	$g_4^2 \cos 2\theta_H$	$ZZHH$	$\frac{g_4^2}{\cos^2\theta_W} \cos 2\theta_H$

suppression

EW Gauge-Higgs Unification in the Warped Space

	RS	flat
m_H	$140 \sim 280$ GeV	~ 10 GeV
m_{KK}	$1.5 \sim 3.5$ TeV	~ 800 GeV
WWZ $WWZZ$ $WWWW$	almost universal	deviation
$Wq\bar{q}'$ $W\ell\bar{\nu}$	tiny universality violation	
WWH ZZH	suppressed by $\cos\theta_H$	
$WWHH$, $ZZHH$	suppressed by $\cos 2\theta_H$	
Yukawa/m_F	suppressed by $\cdots$	
m_F	natural hierarchy	fine tuning

Summary

Gauge-Higgs unification in EW

Predictions to be tested at LHC/ILC.

GUT symmetry breaking

Hosotani mechanism **v.s.** **Equivalence classes of orbifold B.C.**

Spectrum and mixing of quarks and leptons

need to be explained.