# Scalar non-degeneracy and flavor unification

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Based on: K. Inoue, K. K, and K. Yoshioka

"Probing flavor structure in unified theory with scalar spectroscopy"

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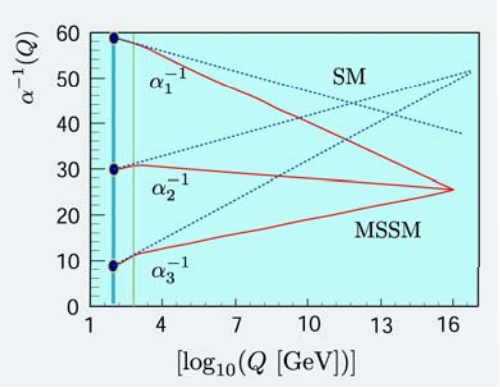
# Introduction

Successful gauge coupling unification in MSSM

$$G_{\rm SM} \subset G_{\rm simple} \text{ at } M_G \simeq 10^{16}\ \text{GeV}$$




However, unification of flavor is not simply realized

- CKM vs. MNS: naively conflict with quark-lepton unification
- Yukawa (mass) unification: possible only for third generation

It is important to obtain suggestions of
GUT flavor structure from low-energy physics

Origin of MSSM matter in GUT ⇒ distinctive low-energy sparticle spectrum

# Origin of MSSM matter fields in unified theory ??

Conventional assumption: ~~generation independent matter embedding~~

e.g. $SO(10)$ GUT

$$\begin{pmatrix} 16_1 \\ 16_2 \\ 16_3 \end{pmatrix} \ni \begin{pmatrix} (Q,\bar{u},\bar{e})_1 \oplus (L,\bar{d})_1 \oplus \bar{\nu}_1 \\ (Q,\bar{u},\bar{e})_2 \oplus (L,\bar{d})_2 \oplus \bar{\nu}_2 \\ (Q,\bar{u},\bar{e})_3 \oplus (L,\bar{d})_3 \oplus \bar{\nu}_3 \end{pmatrix}$$

A class of models: un-parallel generation structure [Sato, Yanagida, Nomura, Bando, Kugo, maekawa, et. al]

"generation dependent matter embedding into GUT multiplets"

several GUT multiplets includes MSSM matter fields

e.g. $G_U = SO(10)$ $\quad 16_i = 10_i \oplus 5^{*\prime}_i \oplus 1_i, \quad 10_i = 5_i \oplus 5^{*\prime\prime}_i$

e.g. $G_U = E_6$ $\quad 27_i = 10_i \oplus 5^{*\prime}_i \oplus 5^{*\prime\prime}_i \oplus 5_i \oplus 1'_i \oplus 1''_i$

General situation:

MSSM field GUT multiplets

$$\phi_i = \phi'_i \cos\theta^\phi_i + \phi''_i \sin\theta^\phi_i$$

$\theta^\phi_i$ : generation twist angles for φi

($E_6$ example:)

$$L_1(\text{and } \bar{d}_1) = 5^{*\prime}_1 \cos\theta^L_1 + 5^{*\prime\prime}_1 \sin\theta^L_1$$
$$L_2(\text{and } \bar{d}_2) = 5^{*\prime}_2 \cos\theta^L_2 + 5^{*\prime\prime}_2 \sin\theta^L_2$$
$$L_3(\text{and } \bar{d}_3) = 5^{*\prime}_3 \cos\theta^L_3 + 5^{*\prime\prime}_3 \sin\theta^L_3$$

# Generation twisting and MSSM scalar mass spectrum

$$\phi_i = \phi'_i \cos\theta^{\phi}_i + \phi''_i \sin\theta^{\phi}_i$$

⇒ Remnants of the twisting appear in low-energy squark and slepton masses

## MSSM scalar masses

$$m^2_{\phi i}(M_S) \simeq m^2_{\phi i}(M_G) + \Delta^{(\mathrm{gauge})}_{\phi} + \delta^{(\mathrm{Yukawa})}_{\phi i}$$

$(M_S \sim 1\ \mathrm{TeV})$ $(M_G \sim 10^{16}\ \mathrm{GeV})$ MSSM RG effects Ms⇔MG (calculable)

$\Delta^{(\mathrm{gauge})}_{\phi}$ : generation independent

$\delta^{(\mathrm{Yukawa})}_{\phi i}$ : negligible for i =1, 2

If $m^2_{\phi i}(M_G) = m^2_{\phi}(M_G)$ e.g. CMSSM, mSUGRA, …

- degenerate 1st and 2nd generation scalars
- suppression of sparticle-mediated FCNC

**Now:** $\phi'$ and $\phi''$ have different $G_U/G_{\mathrm{SM}}$ charges (interactions)

if Mmed >> MG (e.g. gravity), scalar non-degeneracy is induced by generation dependent Gu/Gsm interactions above the GUT scale

With the universal hypothesis for scalar masses,
we can probe into the origin of matter through the scalar non-degeneracy

# Induced non-universality

- D-term corrections to scalar soft masses [Drees, '86; Kawamura, Murayama, Yamaguchi, '94]

$$U(1) \times U(1) \times \cdots \times U(1) \subset G_U/G_{SM}$$

$$V \ni \frac{1}{2}D^2 \ni q_{\phi i}^{U(1)} \langle D_{U(1)} \rangle |\phi_i|^2 \quad ; \qquad \langle D_{U(1)} \rangle \sim 0 + m_S^2$$

multiple D-term correction is parametrized by Cartan generators in $G_U/G_{SM}$

e.g. $G_U = E_6$ case: $E_6 \supset SO(10) \times U(1)_Z \supset SU(5) \times U(1)_X \times U(1)_Z$

$$\Delta m_{\phi i}^2 = q_{\phi i}^X D_X + q_{\phi i}^Z D_Z$$

(independent of breaking chains above MG)

- RG effects above the GUT scale [Kawamura, Murayama, Yamaguchi, '94, Polonsky, Pomarol, '95]

$$\frac{d}{d\ln Q} m_{\phi i}^2 \ni \frac{-8}{16\pi^2} g^2 C^{(2)}(\phi_i) M_g^2$$

characterized by quadratic Casimir invariant

Induced non-universality is tightly connected to Gu/Gsm charges ⟺ Low-energy scalar mass spectrum has generation dependent non-degeneracy

via generation twisting with θi

# Scalar masses at the GUT scale in E6 example

e.g. $G_U = E_6$: three 27-plets as the fundamental matter multiplets

$$E_6 \to \text{any} \to SU(5) \xrightarrow{M_G} G_{\rm SM}$$

$$27_i = 10_i \oplus 5^{*\prime}_i \oplus 5^{*\prime\prime}_i \oplus 5_i \oplus 1'_i \oplus 1''_i \quad \text{(SU(5) decomposition)}$$

$$L_i \ (\text{and } \bar{d}_i) = 5^{*\prime}_i \cos\theta_i + 5^{*\prime\prime}_i \sin\theta_i$$

induced non-universality

$$\begin{aligned}
\Delta m^2_{10} &= D_Z - D_X + \Delta_{10}, \\
\Delta m^2_{\bar{5}'} &= D_Z + 3D_X + \Delta_{\bar{5}'}, \\
\Delta m^2_{\bar{5}''} &= -2D_Z - 2D_X + \Delta_{\bar{5}''},
\end{aligned}$$

## GUT scale MSSM scalar masses

$$m^2_{\bar{e}}(M_G) = m^2_{\bar{u}}(M_G) = m^2_Q(M_G) = m_0^2 + D_Z - D_X + \Delta_{10},$$

$$\begin{aligned}
m^2_{L_i}(M_G) = m^2_{\bar{d}_i}(M_G) &= m_0^2 + (\cos^2\theta_i - 2\sin^2\theta_i) D_Z \\
&\quad + (3\cos^2\theta_i - 2\sin^2\theta_i) D_X + \cos^2\theta_i \Delta_{\bar{5}'} + \sin^2\theta_i \Delta_{\bar{5}''},
\end{aligned}$$

With particular ratios of linear combinations, we can extract θi

# Relations between observables and twist angles

e.g. $G_U = E_6$: three 27-plets as the fundamental matter multiplets

$$27_i = 10_i \oplus 5^{*\prime}_i \oplus 5^{*\prime\prime}_i \oplus 5_i \oplus 1'_i \oplus 1''_i \quad \text{(SU(5) decomposition)}$$

$$L_i \ (\text{and } \bar{d}_i) = 5^{*\prime}_i \cos\theta_i + 5^{*\prime\prime}_i \sin\theta_i$$

$E_6 \supset SO(10) \times U(1)_Z$
$\supset SU(5) \times U(1)_X \times U(1)_Z$

| | | |
|---|---|---|
| $10_i$ | $-1$ | $+1$ |
| $5^{*\prime}_i$ | $+3$ | $+1$ |
| $5^{*\prime\prime}_i$ | $-2$ | $-2$ |

- Dominant U(1)x D-term case

$$\frac{\cos^2\theta_1 - \cos^2\theta_2}{\cos^2\theta_1 + \cos^2\theta_2 - 2/5} = \frac{m^2_{L1}(M_S) - m^2_{L2}(M_S)}{m^2_{L1}(M_S) + m^2_{L2}(M_S) - 2m^2_{\bar{e}1}(M_S) + K}$$

$$K = -2(\xi_L - \xi_e)M^2_{1/2} - 2\eta S(M_S) \qquad \xi_\phi = \sum_a \frac{2}{b_a} C_2^{(a,\phi)}(1 - g_a^4(M_S)/g_G^4)$$

$$\eta = \frac{3}{5b_1}(1 - g_G^2/g_1^2(M_S)) \qquad \textbf{(MSSM gauge dependent RG effects)}$$

- General case (including D-term corrections and RG effects)

$$\frac{\cos^2\theta_1 - \cos^2\theta_2}{\cos^2\theta_1 - \cos^2\theta_2 + 2\cos^2\vartheta_3} = \frac{m^2_{L_1}(M_S) - m^2_{L_2}(M_S)}{m^2_{L_1}(M_S) + m^2_{L_2}(M_S) - 2(m^2_{L_3}(M_S) + \delta^Y)}$$

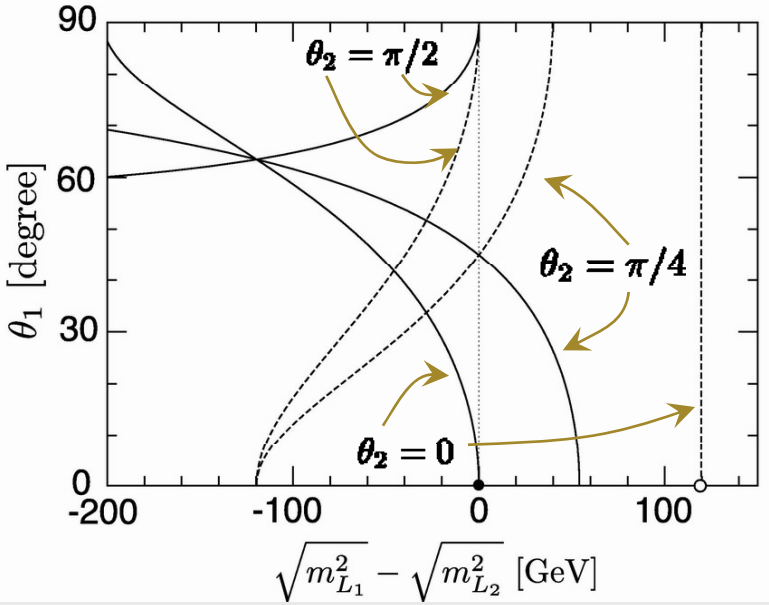
**Yukawa dependent MSSM RG effects**

- negligible for small tanβ
- numerically calculable

RHS: observables (independent of high-energy mass parameters)

# Probing into GUT flavor structure with scalar spectroscopy

- D-term dominated case

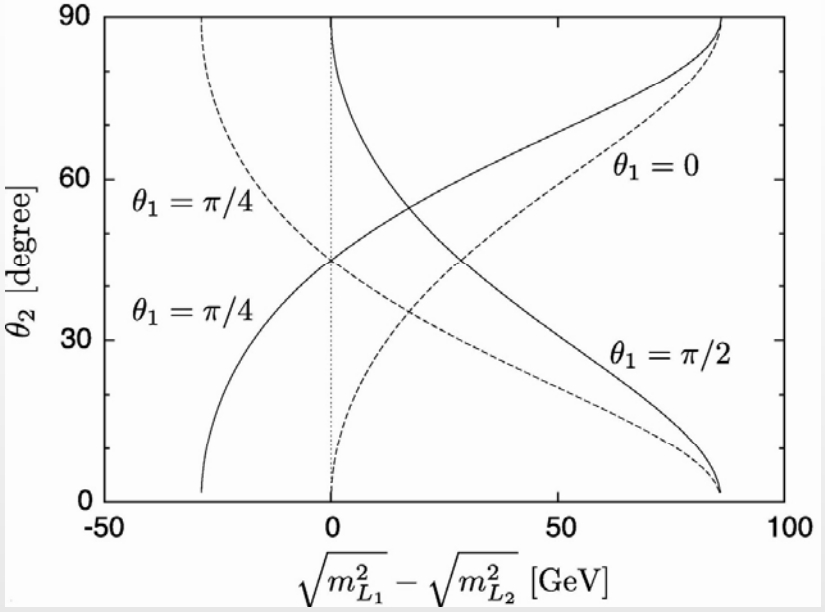



solid: $U(1)_X$, dashed: $U(1)_V$

$M_{1/2} = 300$ GeV $\quad m_{\tilde{e}1}^2 = (400\ \text{GeV})^2$

$m_{L1}^2 + m_{L2}^2 = 2 \times (500\ \text{GeV})^2$

- General case




solid: $\theta_3 = 0$, dashed: $\theta_3 = \pi/2$

$M_{1/2} = 300$ GeV $\quad m_{\tilde{e}1}^2 = (500\ \text{GeV})^2$

$m_{L1}^2 + m_{L2}^2 = 2 \times (700\ \text{GeV})^2$

$m_{L3}^2 = (650\ \text{GeV})^2 \quad \tan\beta = 10$

scalar non-degeneracy ⇒ non-trivial generation twisting

A key ingredient of flavor origin in unified theory

# Implications of flavor violation process

non-degenerate scalars

⇒ superparticle mediated flavor violation would become large

e.g. $G_U = E_6$ with three 27-plets

slepton mediated flavor violation $\ell_j \to \ell_i + \gamma$

$$m^2_{L_{ij}} = \begin{pmatrix} m^2_{L_1} & & \\ & m^2_{L_2} & \\ & & m^2_{L_3} \end{pmatrix} \to m^{2\,(\mathrm{SCKM})}_{L_{ij}} = V^\dagger_{eL} \begin{pmatrix} m^2_{L_1} & & \\ & m^2_{L_2} & \\ & & m^2_{L_3} \end{pmatrix} V_{eL}$$

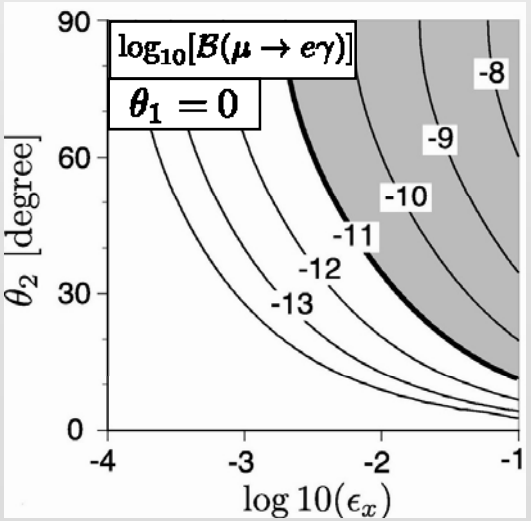
diagonalizes Ye

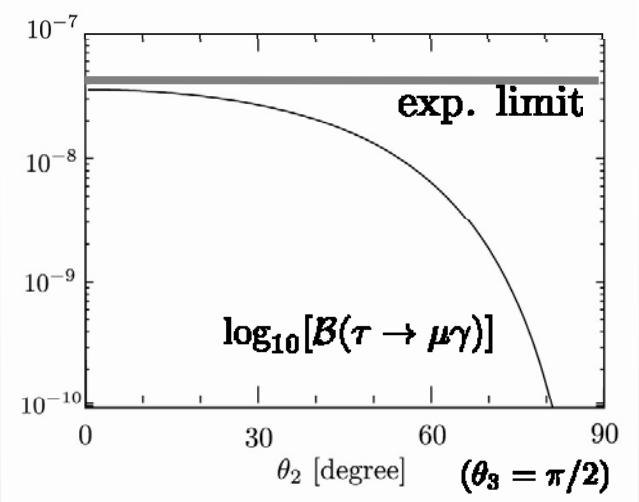
$$V_{\mathrm{MNS}} = V^\dagger_{eL} V_\nu$$

constraints from FCNC processes rather depend on the forms of Yukawa matrices

e.g. $$V_{eL} = \begin{pmatrix} 1 & \epsilon_a & \epsilon_c \\ -\epsilon_a & 1 & \epsilon_b \\ -\epsilon_c & -\epsilon_b & 1 \end{pmatrix},$$

generation twisting is consistent with FCNC






$$V_{eL} = \begin{pmatrix} 1 & \epsilon_x & \epsilon_z \\ \frac{-\epsilon_x+\epsilon_z}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{-\epsilon_x-\epsilon_z}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

(e.g. from lopsided $Y_e = Y_d^{\mathrm{T}}$)

$(\epsilon_x, \epsilon_z) = (10^{-3}, 10^{-2})$

$M_{1/2} = 300$ GeV $\quad m^2_{\tilde{e}_1} = (500\ \mathrm{GeV})^2$

$m^2_{L_1} + m^2_{L_2} = 2 \times (700\ \mathrm{GeV})^2$

$m^2_{L_3} = (650\ \mathrm{GeV})^2 \quad \tan\beta = 10$

The generation twisting and large 2-3 mixing in charged lepton sector enhance $\tau \to \mu\gamma$ process as reachable in near future experiments

Scalar non-degeneracy would enhances FCNC contributions and flavor violation searches may give us implications of generation twisting

# Summary

generation dependent matter embedding into GUT multiplets

$$\phi_i = \phi'_i \cos\theta^{\phi}_i + \phi''_i \sin\theta^{\phi}_i$$

$\phi'_i$ and $\phi''_i$ different $G_U/G_{SM}$ interaction

scalar non-degeneracy is induced through gauge interactions above $M_G$

near future experiments (LHC, ILC):

superparticle spectrum

Flavor structure above GUT scale

MSSM scalar masses

$$\frac{m^2_{\phi_1} - m^2_{\phi_2}}{m^2_{\phi_1} + m^2_{\phi_2} - 2m^2_{\phi_3}}$$

Generation independent contributions are canceled out

generation twisting angles:
Origin of generation
in unified theory