Family Symmetry and GUTs

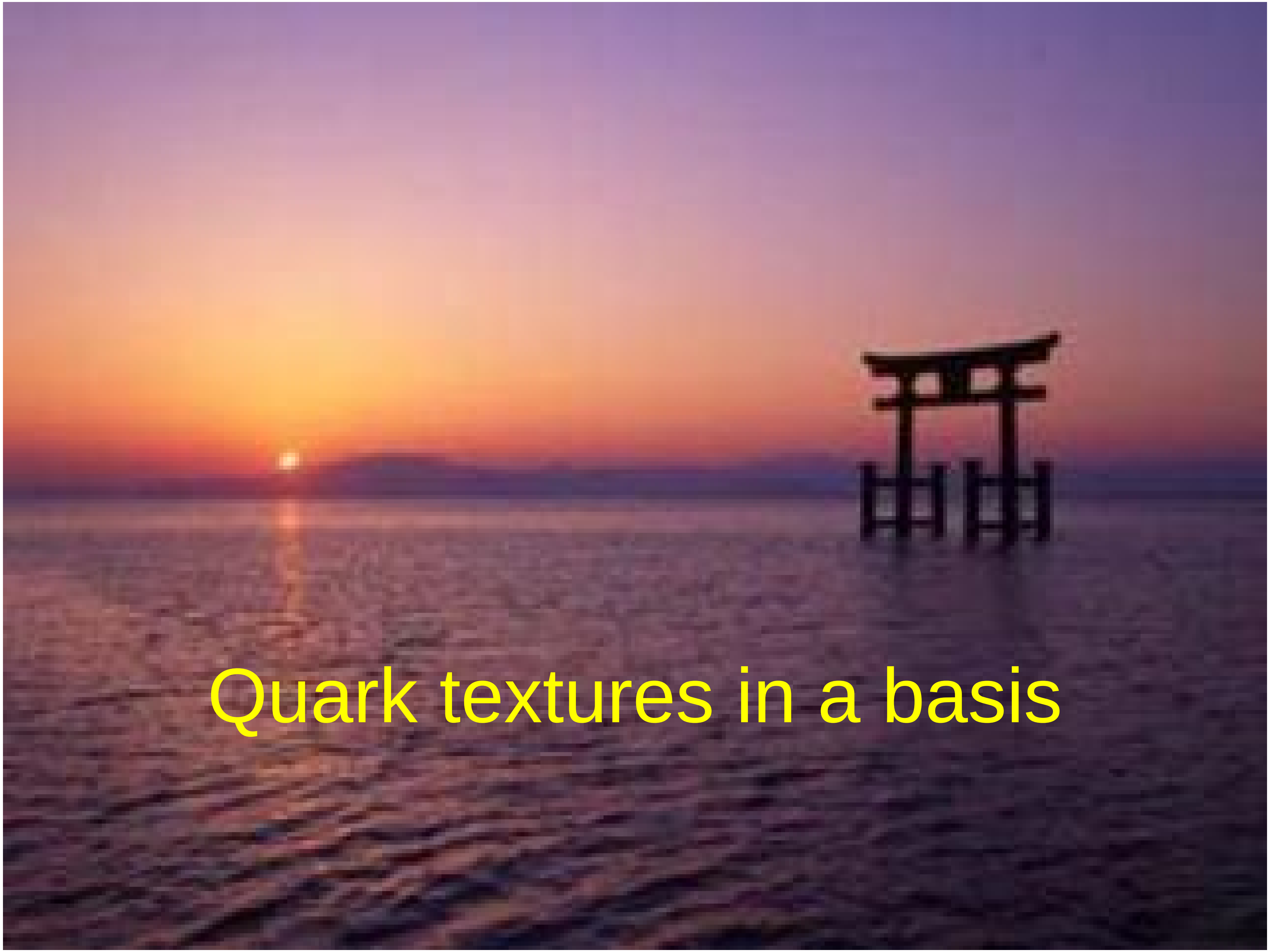
GUT'07

Ritsumeikan University

Japan

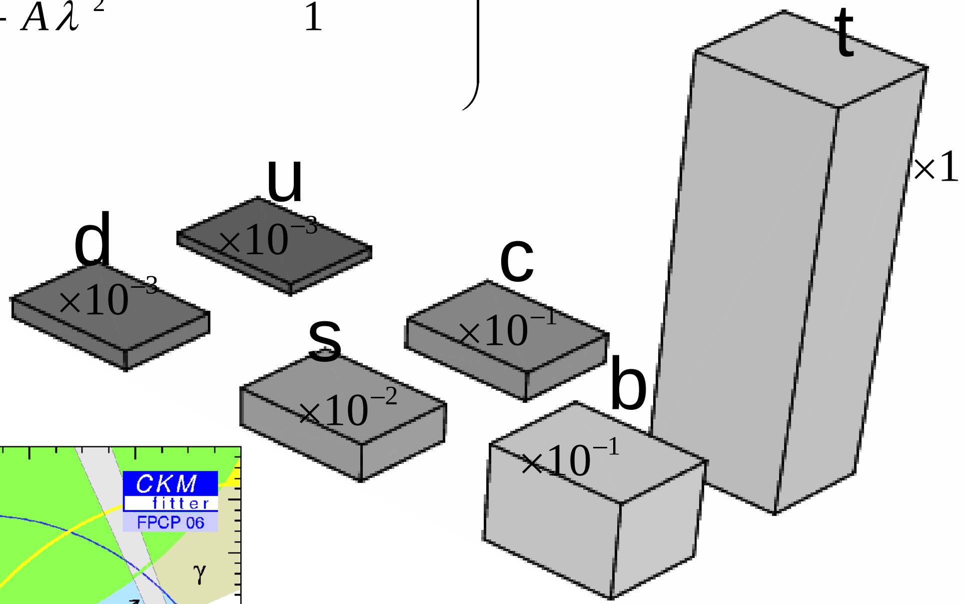
Steve King

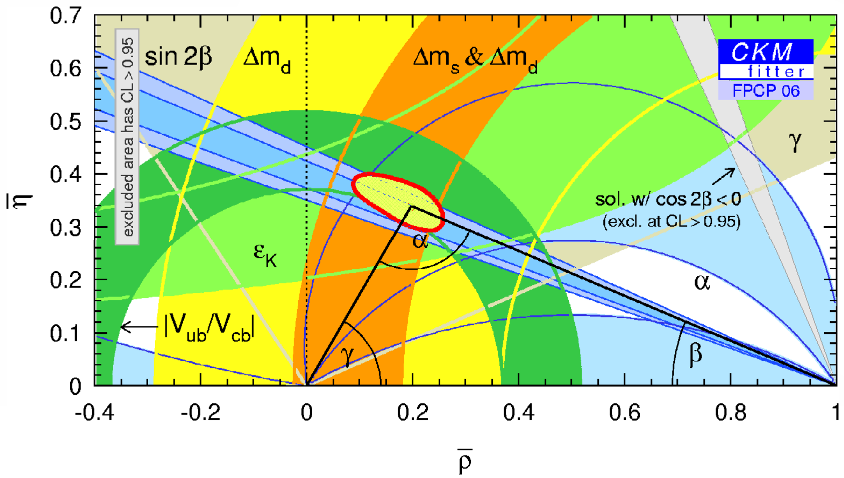
University of Southampton


Quark textures in a basis

Quark Masses and Mixings

$$V \approx \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$





Charged fermion data at GUT scale

Ross, Serna

SUSY thresholds

Parameters	Input SUSY Parameters					
$\tan\beta$	1.3	10	38	50	38	38
γ_b	0	0	0	0	-0.22	$+0.22$
γ_d	0	0	0	0	-0.21	$+0.21$
γ_t	0	0	0	0	0	-0.44
Parameters	**Corresponding GUT-Scale Parameters with Propagated Uncertainty**					
$y^t(M_X)$	6^{+1}_{-5}	0.48(2)	0.49(2)	0.51(3)	0.51(2)	0.51(2)
$y^b(M_X)$	$0.0113^{+0.0002}_{-0.01}$	0.051(2)	0.23(1)	0.37(2)	0.34(3)	0.34(3)
$y^\tau(M_X)$	0.0114(3)	0.070(3)	0.32(2)	0.51(4)	0.34(2)	0.34(2)
$(m_u/m_c)(M_X)$	0.0027(6)	0.0027(6)	0.0027(6)	0.0027(6)	0.0026(6)	0.0026(6)
$(m_d/m_s)(M_X)$	0.051(7)	0.051(7)	0.051(7)	0.051(7)	0.051(7)	0.051(7)
$(m_e/m_\mu)(M_X)$	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)	0.0048(2)
$(m_c/m_t)(M_X)$	$0.0009^{+0.001}_{-0.00006}$	0.0025(2)	0.0024(2)	0.0023(2)	0.0023(2)	0.0023(2)
$(m_s/m_b)(M_X)$	0.014(4)	0.019(2)	0.017(2)	0.016(2)	0.018(2)	0.010(2)
$(m_\mu/m_\tau)(M_X)$	0.059(2)	0.059(2)	0.054(2)	0.050(2)	0.054(2)	0.054(2)
$A(M_X)$	$0.56^{+0.34}_{-0.01}$	0.77(2)	0.75(2)	0.72(2)	0.73(3)	0.46(3)
$\lambda(M_X)$	0.227(1)	0.227(1)	0.227(1)	0.227(1)	0.227(1)	0.227(1)
$\bar\rho(M_X)$	0.22(6)	0.22(6)	0.22(6)	0.22(6)	0.22(6)	0.22(6)
$\bar\eta(M_X)$	0.33(4)	0.33(4)	0.33(4)	0.33(4)	0.33(4)	0.33(4)
$J(M_X)\times 10^{-5}$	$1.4^{+2.2}_{-0.2}$	2.6(4)	2.5(4)	2.3(4)	2.3(4)	1.0(2)
Parameters	**Comparison with GUT Mass Ratios**					
$(m_b/m_\tau)(M_X)$	$1.00^{+0.04}_{-0.4}$	0.73(3)	0.73(3)	0.73(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	$0.70^{+0.8}_{-0.05}$	0.69(8)	0.69(8)	0.69(8)	0.9(1)	0.6(1)
$(m_d/3\,m_e)(M_X)$	0.82(7)	0.83(7)	0.83(7)	0.83(7)	1.05(8)	0.68(6)
$(\frac{\det Y^d}{\det Y^e})(M_X)$	$0.57^{+0.08}_{-0.26}$	0.42(7)	0.42(7)	0.42(7)	0.92(14)	0.39(7)

Hierarchical Symmetric Textures

Symmetric hierarchical matrices with 11 texture zero motivated by

$$m_{LR} = \begin{pmatrix} 0 & m_{12} \\ m_{12} & m_{22} \end{pmatrix} \longrightarrow |V_{us}| \approx \left| \sqrt{\frac{m_d}{m_s}} \right| \approx \lambda$$

Gatto et al

This motivates the symmetric down texture at GUT scale of form

$$Y_{LR}^{d} \sim \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad |V_{cb}| \approx \left| \frac{(m_{LR}^{D})_{23}}{m_b} \right| \approx \lambda^2 \qquad |V_{ub}| \approx \left| \frac{(m_{LR}^{D})_{13}}{m_b} \right| \approx \lambda^3$$

$\lambda \approx 0.2$ is the Wolfenstein Parameter

Roberts, Romanino, Ross

Up quarks are more hierarchical than down quarks

This suggests different expansion parameters for up and down

$$\frac{m_{LR}^{D}}{m_b} \sim \begin{pmatrix} 0 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & 1 \end{pmatrix} \quad \bar{\varepsilon} \sim 0.15$$

$$\frac{m_{LR}^{U}}{m_t} = \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix} \quad \varepsilon \sim 0.05$$

$$m_d : m_s : m_b = \bar{\varepsilon}^4 : \bar{\varepsilon}^2 : 1$$

$$m_u : m_c : m_t = \varepsilon^4 : \varepsilon^2 : 1$$

Detailed fits require numerical (order unity) coefficients

$$Y^d(M_X) = y_{33}^d \begin{pmatrix} d\,\epsilon_d^4 & b\,\epsilon_d^3 & c\,\epsilon_d^3 \\ b\,\epsilon_d^3 & f\,\epsilon_d^2 & a\,\epsilon_d^2 \\ c\,\epsilon_d^3 & a\,\epsilon_d^2 & 1 \end{pmatrix} \quad Y^u(M_X) = y_{33}^u \begin{pmatrix} d'\,\epsilon_u^4 & b'\,\epsilon_u^3 & c'\,\epsilon_u^3 \\ b'\,\epsilon_u^3 & f'\,\epsilon_u^2 & a'\,\epsilon_u^2 \\ c'\,\epsilon_u^3 & a'\,\epsilon_u^2 & 1 \end{pmatrix}$$

Set $c' = d' = d = 0$ and $f = f' = 1$

Detailed fits at the GUT Scale

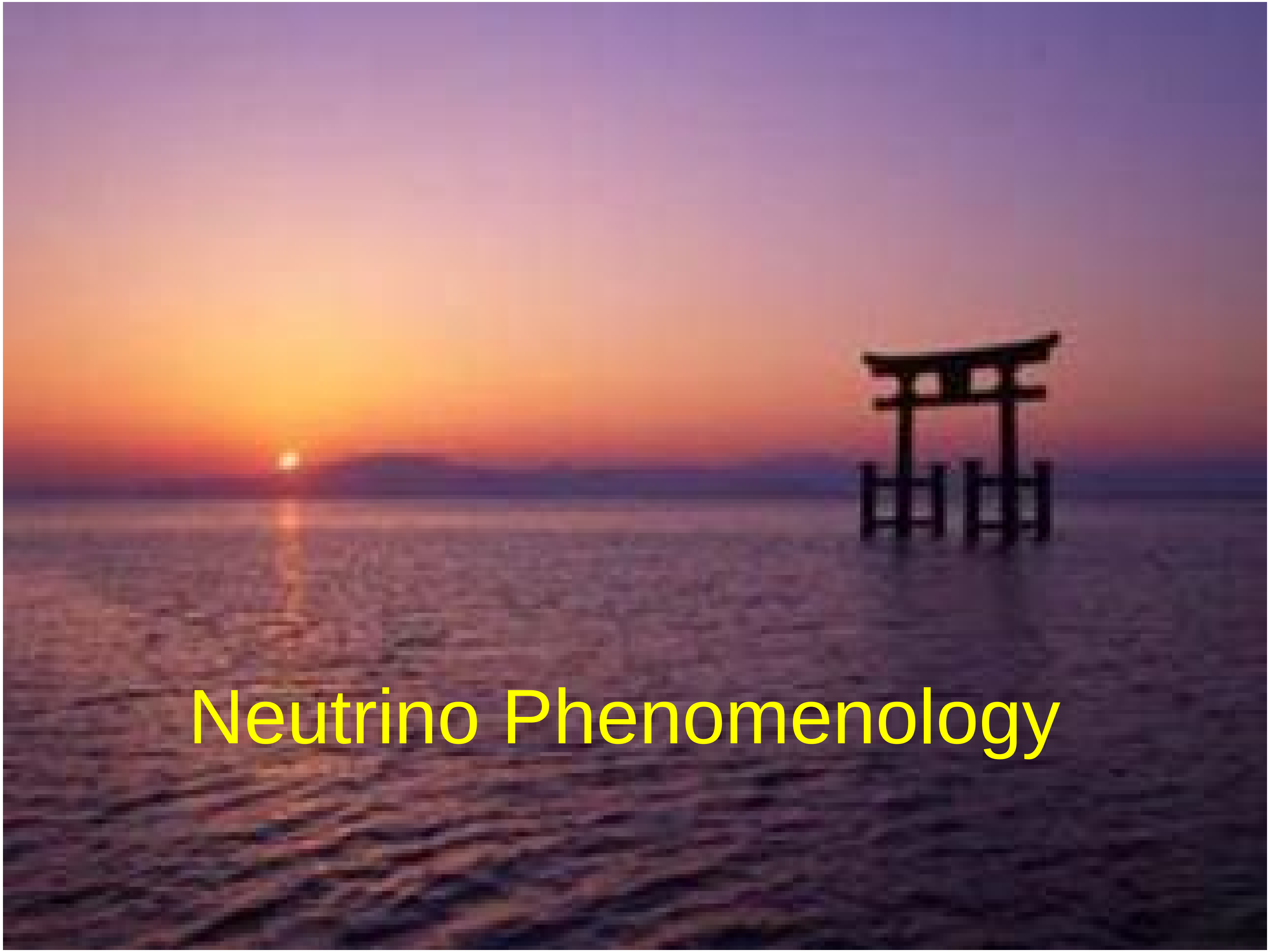
Ross and Serna

Parameter	A	B	C	B2	C2
$\tan\beta$	30	38	38	38	38
γ_b	0.20	-0.22	$+0.22$	-0.22	$+0.22$
γ_t	-0.03	0	-0.44	0	-0.44
γ_d	0.20	-0.21	$+0.21$	-0.21	$+0.21$
a'	0.0	0.0	0.0	-2	-2
ϵ_u	0.0495(17)	0.0483(16)	0.0483(18)	0.0485(17)	0.0485(18)
ϵ_d	0.131(7)	0.128(7)	0.102(9)	0.127(7)	0.101(9)
$\lvert b'\rvert$	1.04(12)	1.07(12)	1.07(11)	1.05(12)	1.06(10)
$\arg(b')$	$90(12)^o$	$91(12)^o$	$93(12)^o$	$95(12)^o$	$95(12)^o$
a	2.17(24)	2.27(26)	2.30(42)	2.03(24)	1.89(35)
b	1.69(13)	1.73(13)	2.21(18)	1.74(10)	2.26(20)
$\lvert c\rvert$	0.80(16)	0.86(17)	1.09(33)	0.81(17)	1.10(35)
$\arg(c)$	$-41(18)^o$	$-42(19)^o$	$-41(14)^o$	$-53(10)^o$	$-41(12)^o$
Y^u_{33}	0.48(2)	0.51(2)	0.51(2)	0.51(2)	0.51(2)
Y^d_{33}	0.15(1)	0.34(3)	0.34(3)	0.34(3)	0.34(3)
Y^e_{33}	0.23(1)	0.34(2)	0.34(2)	0.34(2)	0.34(2)
$(m_b/m_\tau)(M_X)$	0.67(4)	1.00(4)	1.00(4)	1.00(4)	1.00(4)
$(3m_s/m_\mu)(M_X)$	0.60(3)	0.9(1)	0.6(1)	0.9(1)	0.6(1)
$(m_d/3m_e)(M_X)$	0.71(7)	1.04(8)	0.68(6)	1.04(8)	0.68(6)
$\left\lvert \frac{\det Y^d(M_X)}{\det Y^e(M_X)} \right\rvert$	0.3(1)	0.92(14)	0.4(1)	0.92(14)	0.4(1)

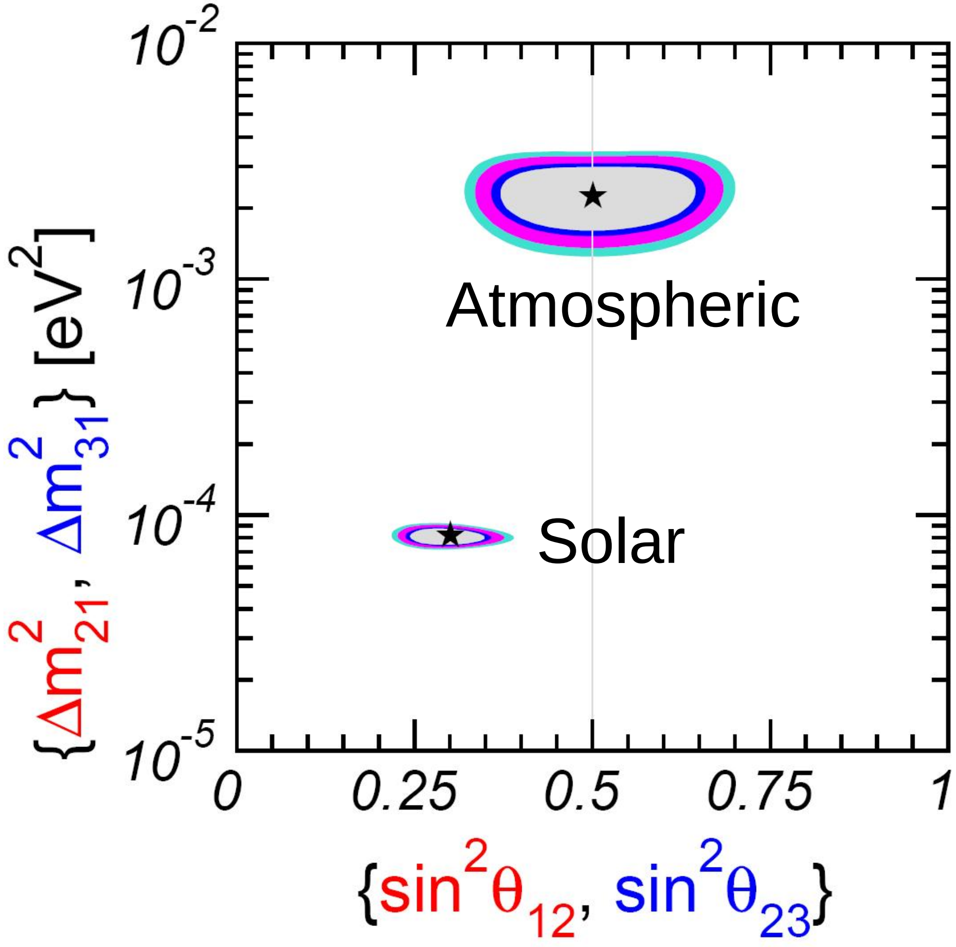
$$Y^d(M_X) = y^d_{33} \begin{pmatrix} d\,\varepsilon_d^4 & 1.7\,\varepsilon_d^3 & e^{-i\pi/4}\,\varepsilon_d^3 \\ 1.7\,\varepsilon_d^3 & \varepsilon_d^2 & 2\,\varepsilon_d^2 \\ e^{-i\pi/4}\,\varepsilon_d^3 & 2\,\varepsilon_d^2 & 1 \end{pmatrix}$$

Georgi-Jarlskog

$$\frac{m_b}{m_\tau}(M_{GUT}) = 1, \quad \frac{m_s}{m_\mu}(M_{GUT}) = \frac{1}{3}, \quad \frac{m_d}{m_e}(M_{GUT}) = 3$$


Neutrino Phenomenology

Latest global fit for atmospheric & solar oscillations



. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle,

Oct 07

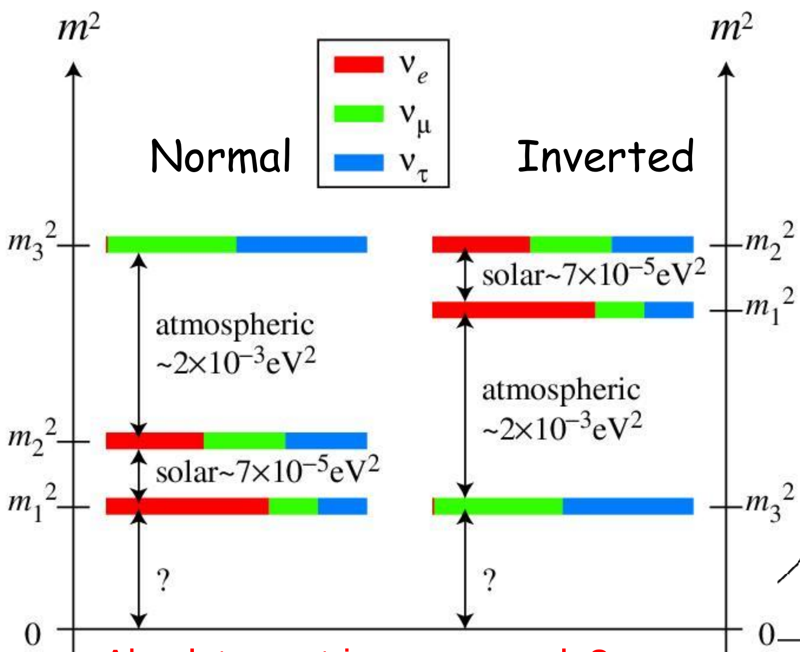
- Latest SSM
- SNO salt data
- K2K
- Latest MINOS results

Neutrino mass squared splittings and angles

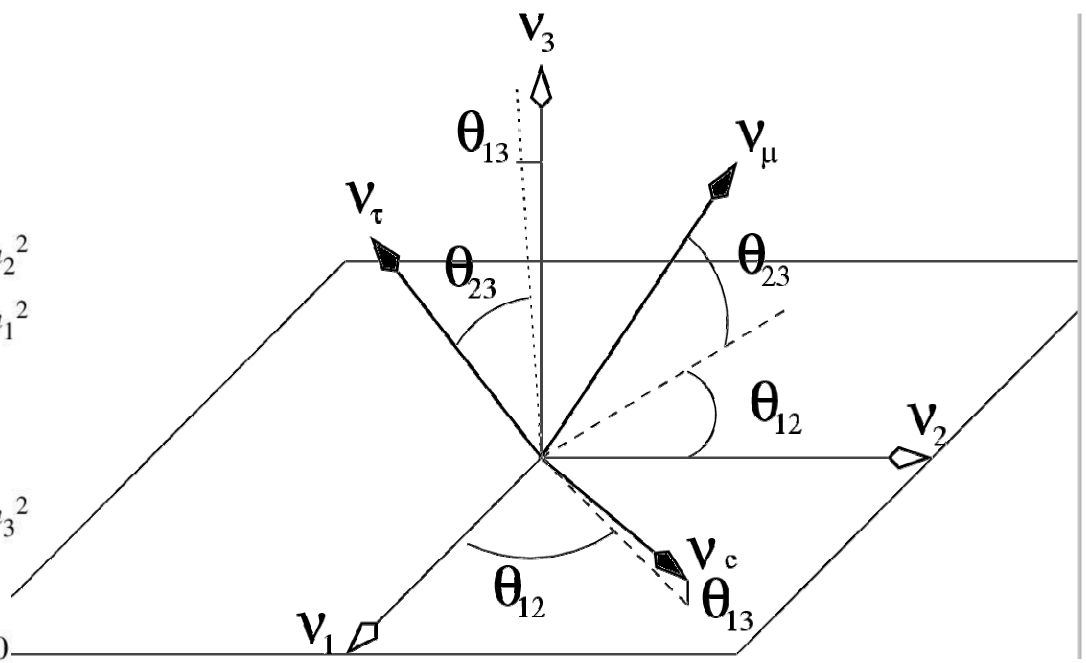
parameter	best fit	3σ range
Δm^2_{21} [10^{-5} eV^2]	7.9	7.1–8.9
Δm^2_{31} [10^{-3} eV^2]	2.6	2.0–3.2

$$\theta_{12} = 33^\circ \pm 5^\circ$$
$$\theta_{23} = 45^\circ \pm 10^\circ$$
$$\theta_{13} < 13^\circ$$

3 σ errors



Absolute neutrino mass scale?



Tri-bimaximal mixing (TBM)

Harrison, Perkins, Scott

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\theta_{12} = 35^\circ, \qquad \theta_{23} = 45^\circ, \qquad \theta_{13} = 0^\circ.$$

c.f. data $\theta_{12} = 33^\circ \pm 5^\circ, \theta_{23} = 45^\circ \pm 10^\circ, \ \theta_{13} < 13^\circ$

- Current data is consistent with TBM
- But no convincing reason for exact TBM – expect deviations

Useful to Parametrize lepton mixing matrix in terms of deviations from tri-bimaximal mixing

SFK **arXiv:0710.0530**

$$s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1+s), \quad s_{23} = \frac{1}{\sqrt{2}}(1+a)$$

$$0 < r < 0.22, \ -0.11 < s < 0.04, \ -0.12 < a < 0.13.$$

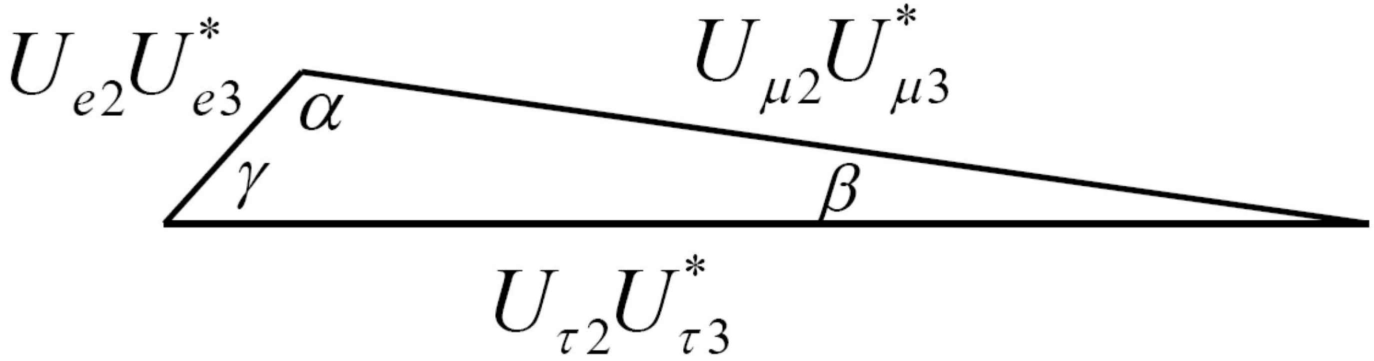
r = reactor | **s = solar** | a = atmospheric

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$

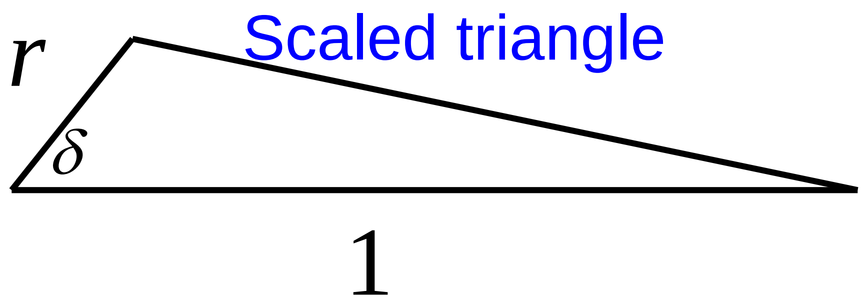
Present data is consistent with r,s,a=0 →tri-bimaximal

Lepton unitarity triangle undetermined

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$



$\nu_2 . \nu_3$



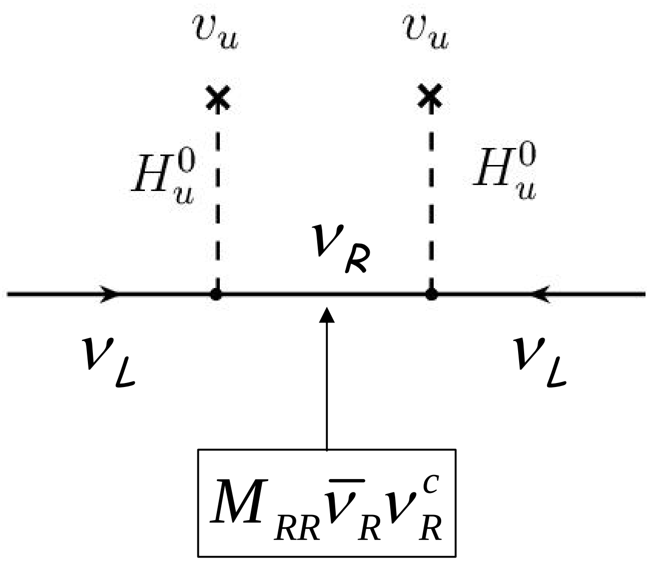
Since neither r nor δ is measured – UT could be a straight line!

See-saw mechanisms

Two types of see-saw mechanism

Type I see-saw mechanism

P. Minkowski (1977), Gell-Mann, Glashow, Mohapatra, Ramond, Senjanovic, Slanski, Yanagida (1979/1980)

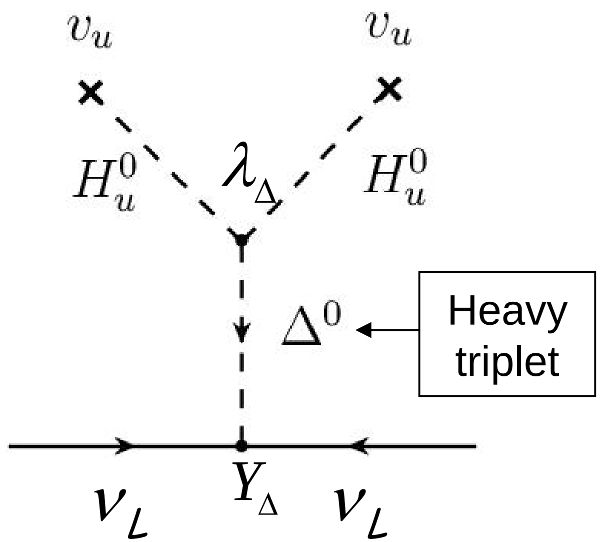


$$m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}$$

Type I

Type II see-saw mechanism (SUSY)

Lazarides, Magg, Mohapatra, Senjanovic, Shafi, Wetterich (1981)



$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

Type II

The Type I See-Saw Mechanism

Possible type II contribution
(ignored here)

Dirac matrix

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

Heavy Majorana matrix

Diagonalise to give effective mass $\rightarrow m_{LL}^{\nu} \overline{\nu}_L \nu_L^c$

Light Majorana matrix $\longrightarrow$ $m_{LL}^{\nu} = m_{LR} M_{RR}^{-1} m_{LR}^T \sim m_{LR}^2 / M_{RR}$

A very natural and appealing mechanism!

Neutrinos are so light because RH neutrino get heavy Majorana masses (L number violated at HE)

Neutrinos are a probe of physics at high energy scales up to M_{GUT}!

Sequential dominance can account for large neutrino mixing

Diagonal RH nu basis

$$M_{\rm RR} = \begin{pmatrix} X & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & Z \end{pmatrix}$$

columns

$$Y^{\nu}_{\rm LR} = \begin{pmatrix} A & B & C \end{pmatrix}$$

See-saw ⇒ $$m^{\nu}_{LL} = \frac{AA^T}{X} + \frac{BB^T}{Y} + \frac{CC^T}{Z}$$

Sequential dominance ⇒ Dominant m_3 Subdominant m_2 Decoupled m_1

$$\left.\begin{aligned} |A_1| &= 0, \\ |A_2| &= |A_3|, \\ |B_1| &= |B_2| = |B_3|, \\ A^\dagger B &= 0 \end{aligned}\right\} \Rightarrow \quad V^{\nu_L \dagger} \approx \begin{pmatrix} -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Constrained SD

Tri-bimaximal


Family symmetry

Introduction to Family Symmetry

We would like to account for the hierarchies embodied in the textures

$$Y^u \approx \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix} \qquad Y^d \approx \begin{pmatrix} 0 & \overline{\varepsilon}^3 & \overline{\varepsilon}^3 \\ \overline{\varepsilon}^3 & \overline{\varepsilon}^2 & \overline{\varepsilon}^2 \\ \overline{\varepsilon}^3 & \overline{\varepsilon}^2 & 1 \end{pmatrix} \qquad Y^e \approx \begin{pmatrix} 0 & \overline{\varepsilon}^3 & \overline{\varepsilon}^3 \\ \overline{\varepsilon}^3 & 3\overline{\varepsilon}^2 & 3\overline{\varepsilon}^2 \\ \overline{\varepsilon}^3 & 3\overline{\varepsilon}^2 & 1 \end{pmatrix} \qquad \begin{array}{l} \varepsilon \approx 0.05, \\ \overline{\varepsilon} \approx 0.15 \end{array}$$

SUSY GUTs can describe but not explain such hierarchies

To understand such hierarchies we shall introduce a family symmetry that distinguishes the three families

It must be spontaneously broken since we do not observe massless gauge bosons which mediate family transitions

The Higgs which break family symmetry are called flavons ϕ

The flavon VEVs introduce an expansion parameter $\varepsilon = \langle\phi\rangle/M$ where M is a high energy mass scale. Idea is to use ε to explain the textures.

Possible family symmetries

In SM the largest family symmetry possible is the symmetry of the kinetic terms

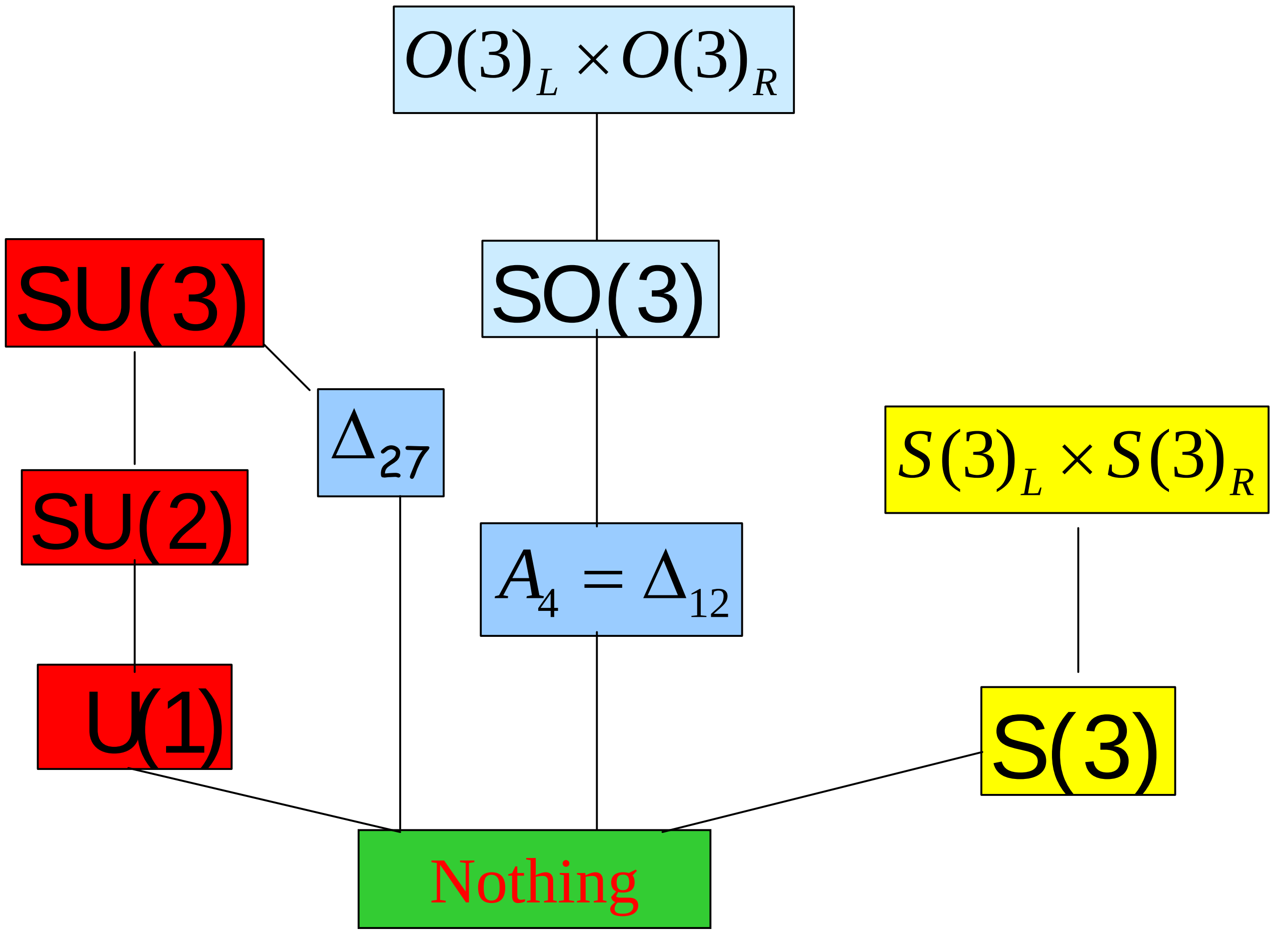
$$\sum_{i=1}^{3} \bar{\psi}_i \gamma_\mu D^\mu \psi_i \,, \quad \psi = Q, L, U^c, D^c, E^c, N^c \to U(3)^6$$

In SO(10) , ψ = 16, so the family largest symmetry is U(3)

Candidate continuous symmetries are U(1), SU(2), SU(3) or SO(3) …

N.B. If family symmetries are gauged and broken at high energies then no direct low energy signatures

Candidate Family Symmetries (very incomplete)



Large lepton mixing motivates non-Abelian family symmetry

Need

$$Y^{\nu}_{LR} = \begin{pmatrix} 0 & B_1 & - \\ A_2 & B_2 & - \\ A_3 & B_3 & - \end{pmatrix}$$

with CSD

$$\begin{aligned} |A_1| &= 0, \\ |A_2| &= |A_3|, \\ |B_1| &= |B_2| = |B_3|, \\ A^{\dagger}B &= 0 \end{aligned}$$

$2 \leftrightarrow 3$ symmetry (from maximal atmospheric mixing)

$1 \leftrightarrow 2 \leftrightarrow 3$ symmetry (from tri-maximal solar mixing)

Suitable non-Abelian family symmetries must span all three families e.g.

$SU(3)$ Δ_{27} SFK, Ross; Velasco-Sevilla; Varzelias

$SO(3)$ A_4 SFK, Malinsky

Gauged SO(3) family symmetry

Suppose that left handed leptons are triplets under SO(3) family symmetry and right handed leptons are singlets

$$L^i = \begin{pmatrix} \nu \\ e \end{pmatrix}_L^i = 3, \quad e_R, \nu_R = 1$$

To break the family symmetry introduce three flavons ϕ_3, ϕ_{23}, ϕ_{123}

Real vacuum alignment
(a,b,c,e,f,h real)

$$<\phi_{23}> = \begin{pmatrix} 0 \\ e \\ f \end{pmatrix} \qquad <\phi_{123}> = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad <\phi_3> = \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix}$$

If each flavon is associated with a particular right-handed neutrino

$$\frac{1}{M}\phi_{23}{}^{i} H L_i \nu_R^1 + \frac{1}{M}\phi_{123}{}^{i} H L_i \nu_R^2 + \frac{1}{M}\phi_{3}{}^{i} H L_i \nu_R^3$$

then the following Yukawa matrix results

$$<\phi_{23}>=\begin{pmatrix}0\\ e\\ f\end{pmatrix} \qquad <\phi_{123}>=\begin{pmatrix}a\\ b\\ c\end{pmatrix} \qquad <\phi_{3}>=\begin{pmatrix}0\\ 0\\ h\end{pmatrix}$$

$$Y_{LR}^{\nu} \sim \begin{pmatrix} 0 & ae^{i\delta_2} & 0 \\ ee^{i\delta_1} & be^{i\delta_2} & 0 \\ fe^{i\delta_1} & ce^{i\delta_2} & he^{i\delta_3} \end{pmatrix} \frac{1}{M}$$

$$L.\phi_{23}\nu_R^1 h \qquad L.\phi_{123}\nu_R^2 h \qquad L.\phi_3 \nu_R^1 h$$

But this is not sufficient to account for tri-bimaximal neutrino mixing

For tri-bimaximal neutrino mixing we need

$$<\phi_{23}> \propto \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \qquad <\phi_{123}> \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad <\phi_{3}> \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

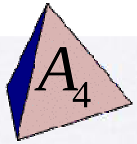
$$Y^{\nu}_{LR} \sim \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

This requires a delicate vacuum alignment of flavon vevs

Ma; Altarelli, Feruglio; Varzeilas, Ross, SFK, Malinsky

Vacuum alignment prefers discrete A_4 subgroup

Symmetry group of the tetrahedron



	SO(3)	A_4
quadratic:	$\phi.\phi$	$\phi.\phi$
cubic:	$(\phi \times \chi).\psi$	$(\phi \times \chi).\psi,\ (\phi * \chi).\psi$
quartic:	$(\phi.\phi)^2, (\phi \times \psi)^2, \ldots$	$(\phi.\phi)^2, \sum_{i=1}^{3} \phi_i\phi_i\phi_i\phi_i,\ (\phi \times \psi)^2, \ldots$

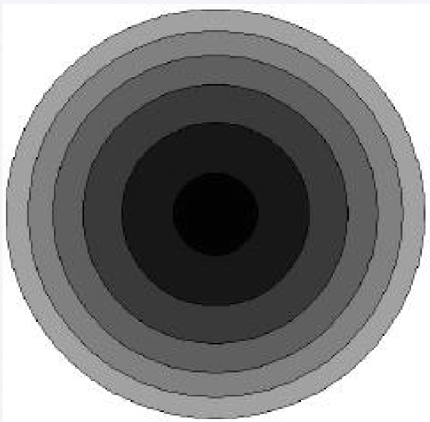
The (single) flavon scalar potential can in A(4) case contain terms like:

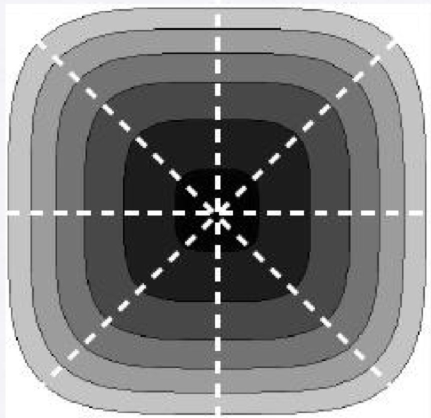
$$V \ni -M_\phi^2(\phi^\dagger\phi) + \Lambda(\phi^\dagger\phi)^2 + \lambda\, \phi_i^\dagger\phi_i\phi_i^\dagger\phi_i + \ldots$$

Discrete set of possible vacua

$$\lambda > 0 : \langle\vec{\phi}\rangle \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\lambda < 0 : \langle\vec{\phi}\rangle \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ or perms.}$$





Gauged SU(3) family symmetry

Now suppose that the fermions are triplets of SU(3) $\psi_i = 3$

i.e. each SM multiplet transforms as a triplet under a gauged SU(3)

$\psi_i = Q_i, L_i, U_i^c, D_i^c, E_i^c, N_i^c \sim 3$ with the Higgs being singlets $H \sim 1$

This “explains” why there are three families c.f. three quark colours in $SU(3)_c$

The family symmetry is spontanously broken by antitriplet flavons $\phi^i = \overline{3}$

Again need flavons with vacuum alignments (up to phases):

$\langle\phi_3\rangle \propto (0,0,1)$, $\langle\phi_{23}\rangle \propto (0,1,1)$, $\langle\phi_{123}\rangle \propto (1,1,1)$ in family space

In SU(3) with ψ_i=3 and H=1 all tree-level Yukawa couplings $H\psi_i\ \psi_j$ are forbidden.

$$Y_{tree-level}^{SU(3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In SU(3) with flavons $\phi^i = \overline{3}$ the lowest order Yukawa operators allowed are:

$$\frac{1}{M^2}\phi^i\phi^j H\psi_i\psi_j$$

We first consider a flavon $\phi_3{}^i$ with VEV $\langle\phi_3{}^i\rangle = (0,0,1)u_3$ then this generates a (3,3) Yukawa coupling

$$\frac{1}{M^2}\phi_3{}^i\phi_3{}^j H\psi_i\psi_j \to \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\frac{u_3{}^2}{M^2}$$

Note that we label the flavon $\phi_3{}^i$ with a subscript 3 which denotes the direction of its VEV in the i=3 direction.

Next suppose we consider a flavon $\phi_{23}{}^{i}$ with VEV $\langle \phi_{23}{}^{i} \rangle = (0,1,1)u_2$ then this generates (2,3) block Yukawa couplings

$$\frac{1}{M^2}\phi_{23}{}^{i}\phi_{23}{}^{j}H\psi_i\psi_j \to \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}\frac{u_2{}^2}{M^2}$$

To complete the texture we introduce another flavon $\langle \phi_{123}{}^{i} \rangle = (1,1,1)u_1$

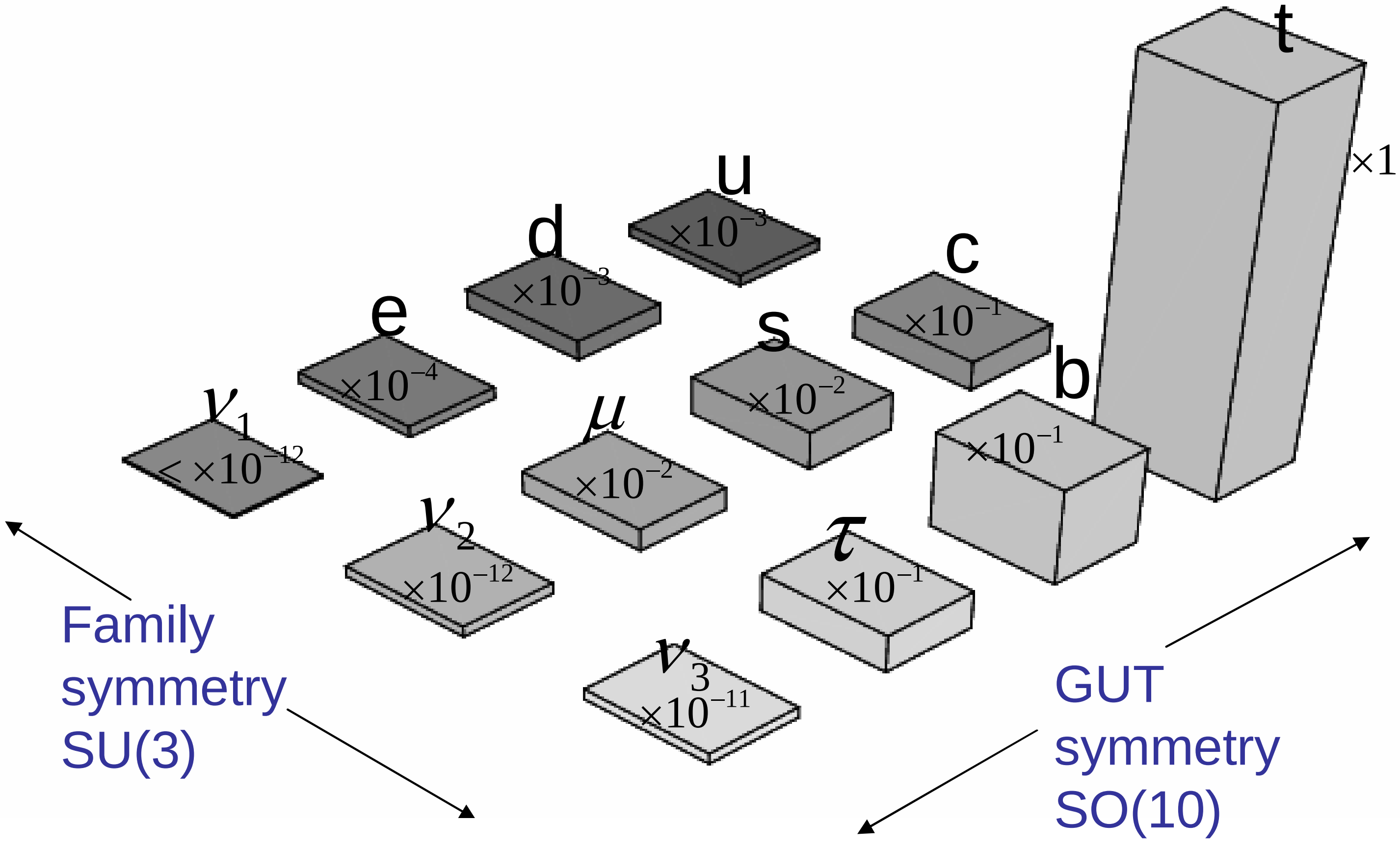
$$\frac{1}{M^2}\phi_{123}{}^{i}\phi_{23}{}^{j}H\psi_i\psi_j + \frac{1}{M^2}\phi_{23}{}^{i}\phi_{123}{}^{j}H\psi_i\psi_j \to \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{pmatrix}\frac{u_1 u_2}{M^2}$$

Writing $\varepsilon_3 = \frac{u_3}{M} > \varepsilon_2 = \frac{u_2}{M} > \varepsilon_1 = \frac{u_2}{M}$ these flavons generate Yukawa couplings

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{\langle \phi_3 \rangle \neq 0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_3^2 \end{pmatrix} \xrightarrow{\langle \phi_{23} \rangle \neq 0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon_2{}^2 & \varepsilon_2{}^2 \\ 0 & \varepsilon_2{}^2 & \varepsilon_3{}^2 \end{pmatrix} \xrightarrow{\langle \phi_{123} \rangle \neq 0} \begin{pmatrix} 0 & \varepsilon_1\varepsilon_2 & \varepsilon_1\varepsilon_2 \\ \varepsilon_1\varepsilon_2 & \varepsilon_2{}^2 & \varepsilon_2{}^2 \\ \varepsilon_1\varepsilon_2 & \varepsilon_2{}^2 & \varepsilon_3{}^2 \end{pmatrix}$$


Family symmetry and GUTs

SU(3)xSO(10)



u
$\times 10^{-3}$
d
$\times 10^{-3}$
c
$\times 10^{-1}$
t
$\times 1$
e
$\times 10^{-4}$
s
$\times 10^{-2}$
b
$\times 10^{-1}$
ν_1
$< \times 10^{-12}$
μ
$\times 10^{-2}$
ν_2
$\times 10^{-12}$
τ
$\times 10^{-1}$
ν_3
$\times 10^{-11}$
Family symmetry SU(3)
GUT symmetry SO(10)

Varzielas,SFK,Ross

Realistic SU(3)× SO(10) Model

Field	$SU(3)_f$	$SU(4)_{PS}$	$SU(2)_L$	$SU(2)_R$	R	$U(1)$	Z_2
ψ	$\mathbf{3}$	$\mathbf{4}$	$\mathbf{2}$	$\mathbf{1}$	1	0	1
ψ^c	$\mathbf{3}$	$\bar{\mathbf{4}}$	$\mathbf{1}$	$\mathbf{2}$	1	0	1
θ	$\bar{\mathbf{3}}$	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{2}$	0	0	-1
H	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	0	0	1
Σ	$\mathbf{1}$	$\mathbf{15}$	$\mathbf{1}$	$\mathbf{3}$	0	2	1
ϕ_{123}	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	-1	1
ϕ_3	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	3	1
ϕ_1	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	-4	-1
$\bar{\phi}_3$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}\oplus\mathbf{1}$	0	0	-1
$\bar{\phi}_{23}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	-1	-1
$\bar{\phi}_{123}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	1	-1

Model also works with SU(3) replaced by its Δ_{27} subgroup

Yukawa Operators

$$P_Y \sim \frac{1}{M^2}\bar{\phi}_3^i\psi_i\bar{\phi}_3^j\psi_j^c H$$

$$+\frac{1}{M^3}\bar{\phi}_{23}^i\psi_i\bar{\phi}_{23}^j\psi_j^c H\,\Sigma$$

$$+\frac{1}{M^2}\bar{\phi}_{23}^i\psi_i\bar{\phi}_{123}^j\psi_j^c H$$

$$+\frac{1}{M^2}\bar{\phi}_{123}^i\psi_i\bar{\phi}_{23}^j\psi_j^c H$$

Majorana Operators

$$P_M \sim \frac{1}{M}\theta^i\psi_i^c\theta^j\psi_j^c$$

$$+\frac{1}{M^5}\bar{\phi}_{23}^i\psi_i^c\bar{\phi}_{23}^j\psi_j^c\theta^k\phi_{123_k}\theta^l\phi_{3_l}$$

$$+\frac{1}{M^5}\bar{\phi}_{123}^i\psi_i^c\bar{\phi}_{123}^j\psi_j^c\theta^k\phi_{123_k}\theta^l\phi_{123_l}$$

Inserting flavon VEVs gives Yukawa couplings

$$Y^f_{ij} = y^f_0 \frac{\langle(\phi_3)_i(\phi_3)_j\rangle}{M_f^2} + y^f_1 \frac{\langle(\phi_{123})_i(\phi_{23})_j\rangle}{M_f^2} + y'^f_1 \frac{\langle(\phi_{23})_i(\phi_{123})_j\rangle}{M_f^2} + y^f_\Sigma \frac{\langle(\phi_{23})_i(\phi_{23})_j\Sigma\rangle}{M_f^3} + \ldots$$

After vacuum alignment the flavon VEVs are

$$\langle\phi_{123}\rangle = \begin{pmatrix} 1 \\ e^{i\phi_1} \\ e^{i\phi_2} \end{pmatrix} u_1 \,, \quad \langle\phi_{23}\rangle = \begin{pmatrix} 0 \\ 1 \\ e^{i\phi_3} \end{pmatrix} u_2 \,, \quad \langle\phi_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} u_3^u & 0 \\ 0 & u_3^d \end{pmatrix}$$

Writing $\epsilon^f_3 \equiv u^f_3/M_f,\ \epsilon^f_1 \equiv u_1/M_f,\ \epsilon^f_2 \equiv u_2/M_f$ and $\sigma^f \equiv \langle\Sigma\rangle/M_f$

Yukawa matrices become:

$$Y^f = y^f_0(\epsilon^f_3)^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y^f_\Sigma(\epsilon^f_2)^2\sigma^f \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & e^{i\phi_3} \\ 0 & e^{i\phi_3} & e^{2i\phi_3} \end{pmatrix}$$

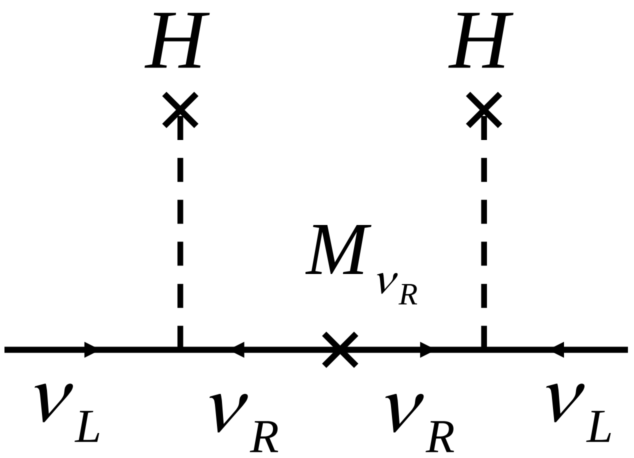
$$+ \epsilon^f_1\epsilon^f_2 \left[y^f_1 \begin{pmatrix} 0 & 1 & e^{i\phi_3} \\ 0 & e^{i\phi_1} & e^{i(\phi_3+\phi_1)} \\ 0 & e^{i\phi_2} & e^{i(\phi_3+\phi_2)} \end{pmatrix} + y'^f_1 \begin{pmatrix} 0 & 0 & 0 \\ 1 & e^{i\phi_1} & e^{i\phi_2} \\ e^{i\phi_3} & e^{i(\phi_1+\phi_3)} & e^{i(\phi_2+\phi_3)} \end{pmatrix} \right]$$

Clebsch factors $\quad y^f_\Sigma = y_\Sigma C^f \qquad C^{u,d,e,\nu} = -2, 1, 3, 0$

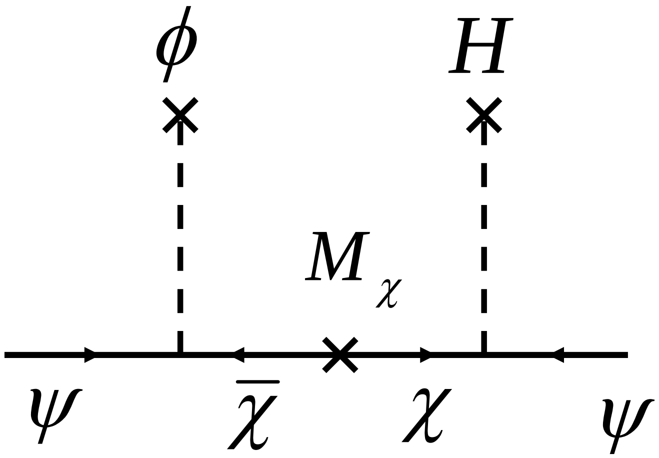
Froggatt-Nielsen Mechanism

What is the origin of the higher order operators?

Froggat and Nielsen took their inspiration from the see-saw mechanism



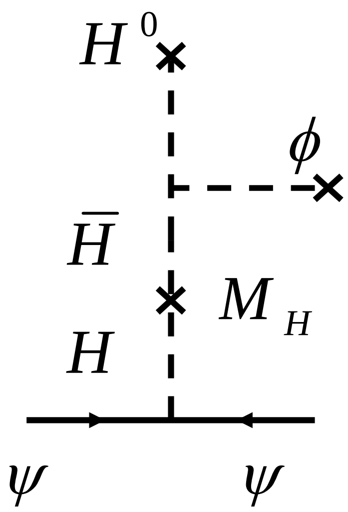
$$\rightarrow \frac{H^2}{M_{\nu_R}} \nu_L \nu_L$$

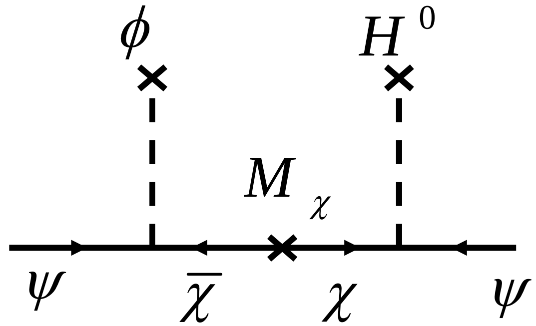


$$\rightarrow \frac{\phi}{M_\chi} H \psi \, \psi$$

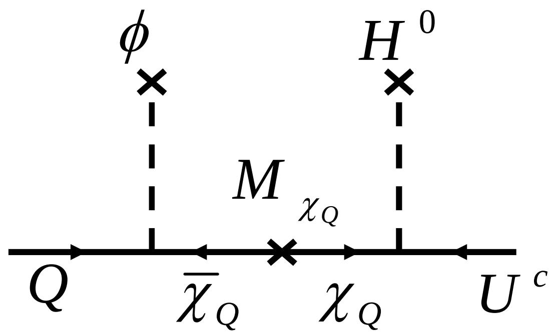
Where χ are heavy fermion messengers
c.f. heavy RH neutrinos

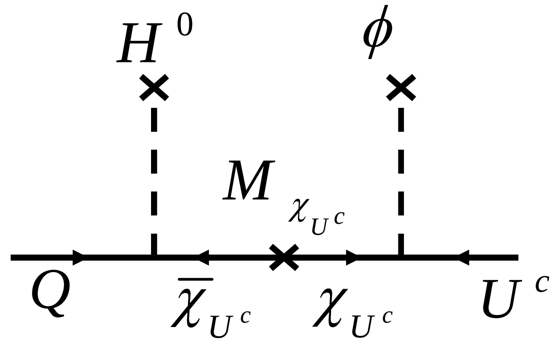
There may be Higgs messengers or fermion messengers





Fermion messengers may be $SU(2)_L$ doublets or singlets





Assume fermion messengers whose mass scales M_f satisfy

$$M_Q = M_u \gg M_d \,, \quad M_L = M_\nu \gg M_e = M_d$$

Then write
$$\frac{u_2}{M_u} = \frac{u_2}{M_Q} = \varepsilon \,, \quad \frac{u_2}{M_d} = \bar{\varepsilon} \,, \quad \frac{u_1}{M_u} = \frac{u_1}{M_Q} = \varepsilon^2 \,, \quad \frac{u_1}{M_d} = \bar{\varepsilon}^2 \,,$$

$$\frac{u_2}{M_\nu} = \frac{u_2}{M_L} = \varepsilon \,, \quad \frac{u_2}{M_e} = \bar{\varepsilon} \,, \quad \frac{u_1}{M_\nu} = \frac{u_1}{M_L} = \varepsilon^2 \,, \quad \frac{u_1}{M_e} = \bar{\varepsilon}^2$$

Yukawa matrices become, ignoring phases:

$$Y^u = \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & 2\varepsilon^2 & 2\varepsilon^2 \\ \varepsilon^3 & 2\varepsilon^2 & 1 \end{pmatrix} , \quad Y^d = \begin{pmatrix} 0 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & 1 \end{pmatrix} , \quad Y^e = \begin{pmatrix} 0 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ \bar{\varepsilon}^3 & 3\bar{\varepsilon}^2 & 3\bar{\varepsilon}^2 \\ \bar{\varepsilon}^3 & 3\bar{\varepsilon}^2 & 1 \end{pmatrix} .$$

Where
$$\varepsilon = \epsilon_2^u = \epsilon_2^\nu \,, \quad \bar{\varepsilon} = \epsilon_2^d = \epsilon_2^e \,,$$
$$\varepsilon^2 = \epsilon_1^u = \epsilon_1^\nu \,, \quad \bar{\varepsilon}^2 = \epsilon_1^d = \epsilon_1^e \,.$$

$$\varepsilon \approx 0.05, \bar{\varepsilon} \approx 0.15$$
$$\varepsilon_3^u = \varepsilon_3^d = \varepsilon_3^e \approx 1$$

The Neutrino Sector

In theory basis the see-saw matrices have the form

$$Y^{\nu} = \begin{pmatrix} 0 & B & C_1 \\ A & Be^{i\phi_1} + Ae^{i\phi_1} & C_2 \\ Ae^{i\phi_3} & Be^{i\phi_2} + Ae^{i(\phi_1+\phi_3)} & C_3 \end{pmatrix}, \quad M = \begin{pmatrix} M_A & M_A e^{i\phi_1} & 0 \\ M_A e^{i\phi_1} & M_A e^{2i\phi_1} + M_B & 0 \\ 0 & 0 & M_C \end{pmatrix}$$

See-saw mechanism is invariant under non-unitary transformations (SFK hep-ph/0610239)

$$Y^{\nu} \rightarrow Y^{\nu} S^{-1}, \quad M \rightarrow S^{T^{-1}} M S^{-1}, \quad M^{-1} \rightarrow S M^{-1} S^T$$

$$S^{-1} = \begin{pmatrix} 1 & -e^{i\phi_1} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : \quad Y^{\nu} \rightarrow \begin{pmatrix} 0 & B & C_1 \\ A & Be^{i\phi_1} & C_2 \\ Ae^{i\phi_3} & Be^{i\phi_2} & C_3 \end{pmatrix} \quad M \rightarrow \begin{pmatrix} M_A & 0 & 0 \\ 0 & M_B & 0 \\ 0 & 0 & M_C \end{pmatrix}$$

Has the CSD form and predicts tri-bimaximal neutrino mixing


Predictions

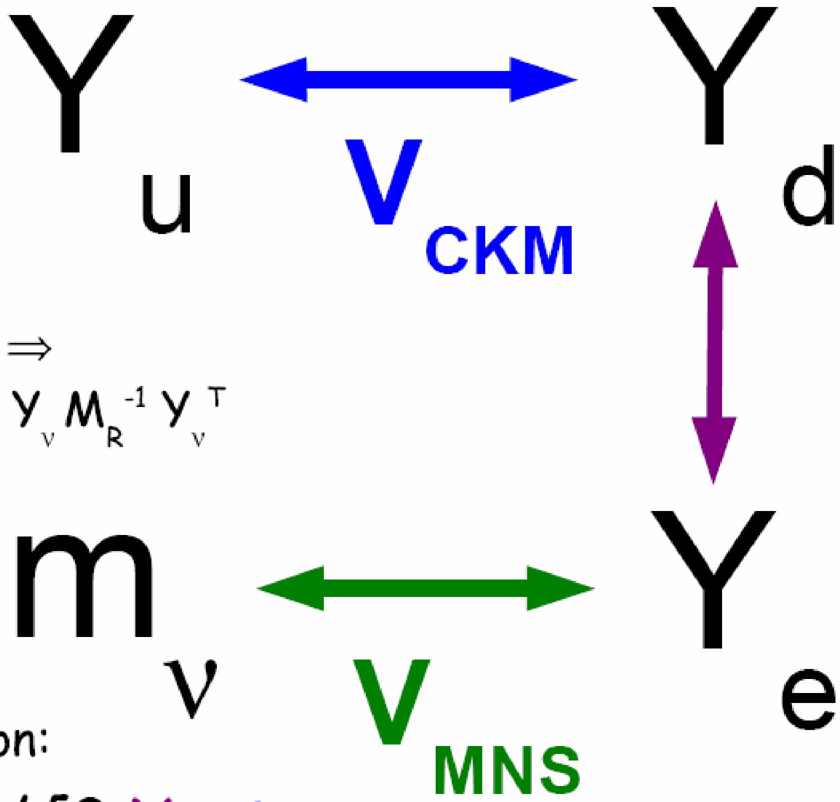
Quark-Lepton Connections

Class of models: $\theta_{12}^{d} \gg \theta_{12}^{u}$

$\theta_{12}^{d} \approx \theta_{C}$

See-saw $\Rightarrow$

$m_{\nu} = v_{EW}^{2} \, Y_{\nu} M_{R}^{-1} Y_{\nu}^{T}$



GUT relation

$\theta_{12}^{e} \approx X_{CG}\, \theta_{C} \approx \dfrac{\theta_C}{3}$

(X_{CG}: from symmetry)

Prediction:

$\theta_{13} \approx 1/\surd 2 \; X_{CG}\, \theta_{C}$

(for small θ_{13}^{ν})

TBM in the neutrino sector becomes corrected by charged lepton corrections, leading to a sum rule

Charged Lepton Corrections and ν sum rule

SFK,Antusch; Masina,....

Assume I: charged lepton mixing angles are small

$$V_{MNS} = V^{E_L} V^{\nu_L \dagger}$$

$$s_{23}e^{-i\delta_{23}} \approx s_{23}^{\nu}e^{-i\delta_{23}^{\nu}} - \theta_{23}^{E}c_{23}^{\nu}e^{-i\delta_{23}^{E}}$$

$$\theta_{13}e^{-i\delta_{13}} \approx \cancel{\theta_{13}^{\nu}e^{-i\delta_{13}^{\nu}}} - \cancel{\theta_{13}^{E}c_{23}^{\nu}e^{-i\delta_{13}^{E}}} - \theta_{12}^{E}s_{23}^{\nu}e^{-i(\delta_{12}^{E}+\delta_{23}^{\nu})}$$

$$s_{12}e^{-i\delta_{12}} \approx s_{12}^{\nu}e^{-i\delta_{12}^{\nu}} + \cancel{\theta_{13}^{E}c_{12}^{\nu}s_{23}^{\nu}e^{i(\delta_{23}^{\nu}-\delta_{13}^{E})}} - \theta_{12}^{E}c_{23}^{\nu}c_{12}^{\nu}e^{-i\delta_{12}^{E}}$$

Assume II: all 13 angles are very small

$$\rightarrow \theta_{13} \approx \frac{\theta_{12}^{E}}{\sqrt{2}}$$

$$\rightarrow \theta_{12} \approx \theta_{12}^{\nu} + \frac{\theta_{12}^{E}}{\sqrt{2}}\cos\delta$$

In a given model we can predict θ_{12}^{E} and θ_{12}^{ν}.

Note the sum rule

$$\theta_{12} \approx \theta_{12}^{\nu} + \theta_{13}\cos\delta$$

Two generic predictions

Cabibbo

$$\theta_{13}^{o} \approx \frac{\theta_C^{o}}{3\sqrt{2}} \approx 3^{\circ}$$

$$\sin^2 2\theta_{13} \approx 10^{-2}$$

Bjorken,Pakvasa,SFK…

$$\theta_{12}^{o} - 35^{o} \approx \theta_{13}^{o} \cos\delta$$

Sum rule

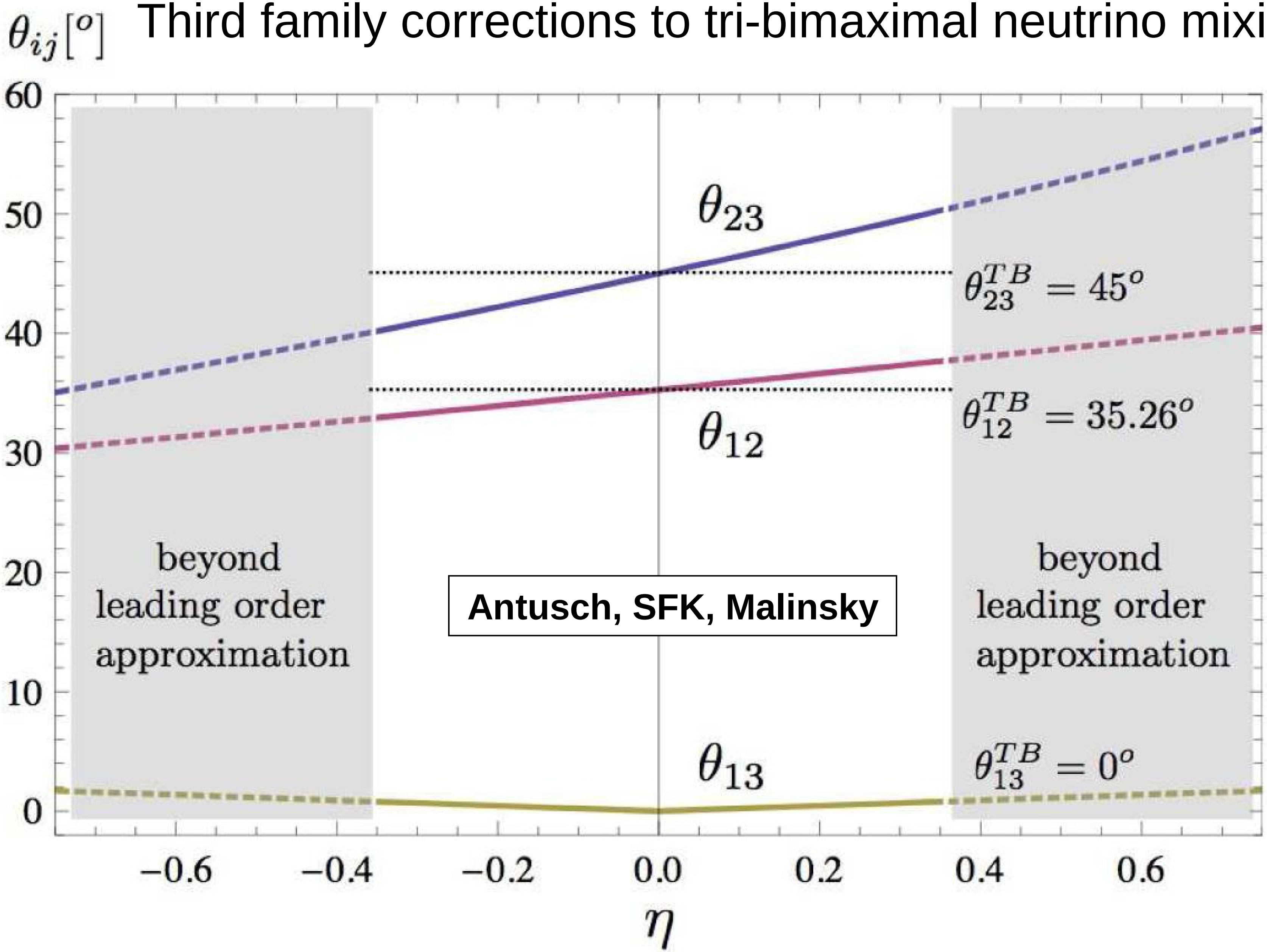
SFK, Antusch,Masina,…

In the TBM parametrization these are recast as

$$s \approx r \cos\delta \qquad\qquad r \approx \lambda / 3$$

Solar Reactor CP phase Reactor Wolfenstein

Third family corrections to tri-bimaximal neutrino mixing



η encodes RG and CN corrections

Theoretical Corrections

Antusch,SFK,Malinsky

Including third family wavefunction corrections (RG and CN) we find a theoretically stable sum rule:

$$s \approx r\cos\delta + \frac{2}{3}a$$

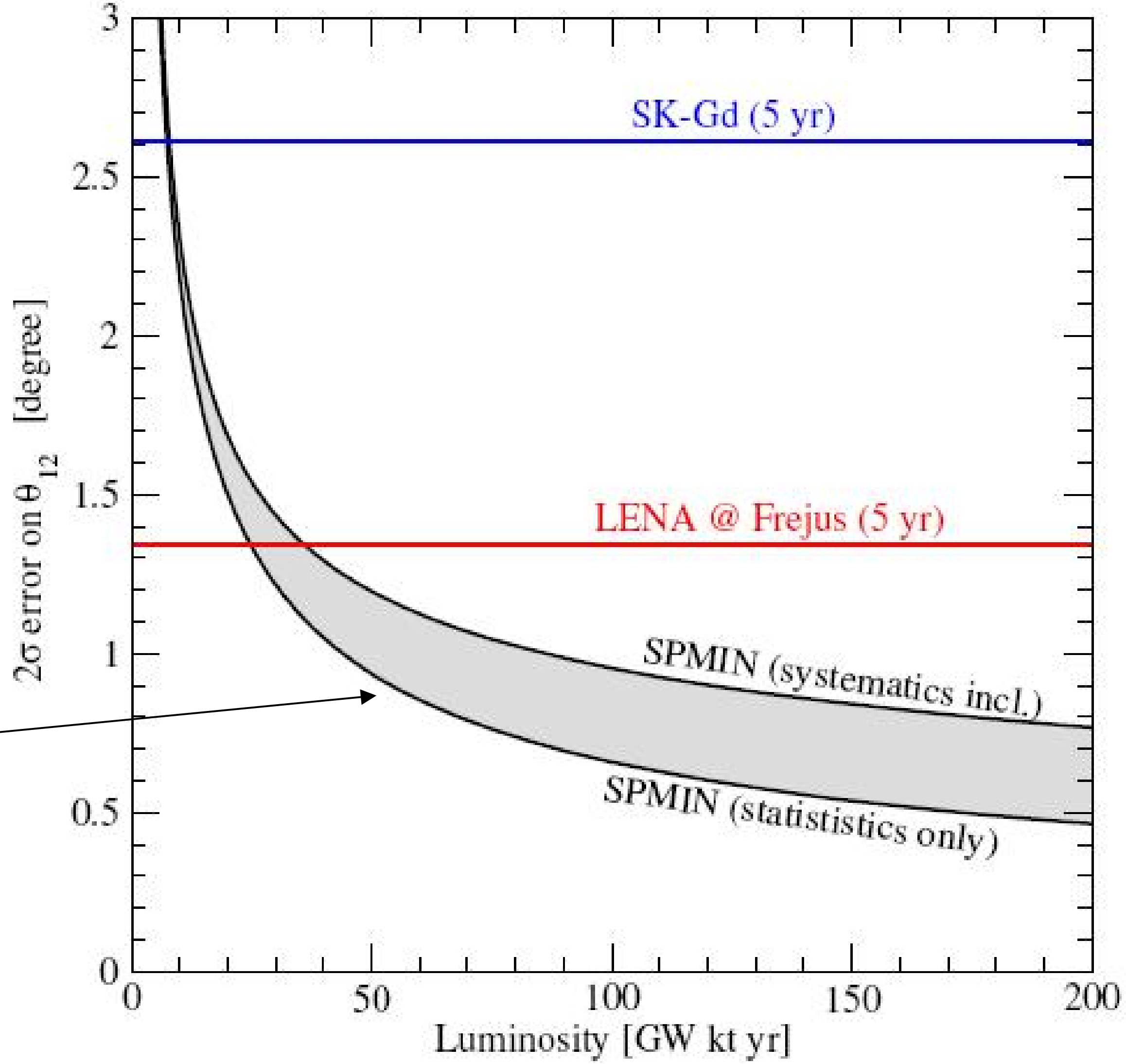
Solar Reactor CP phase Atmospheric

Where s,r,a parametrize the deviations of the solar,reactor, atmospheric mixing angles from their tri-bimaximal values

$$s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1+s), \quad s_{23} = \frac{1}{\sqrt{2}}(1+a)$$

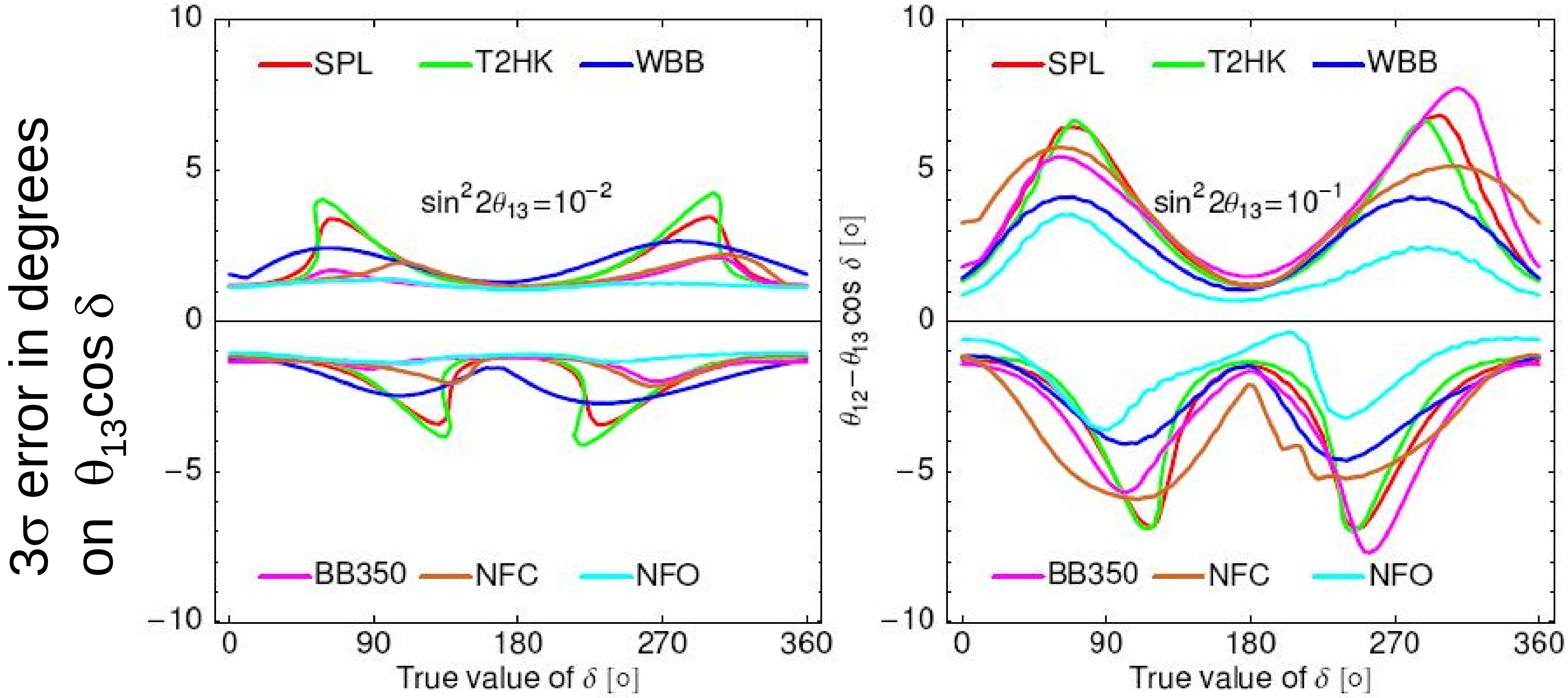
Present 2 σ error

Prospects to measure the deviation of θ_{12} from 35^o



SPMIN=reactor experiment at the first Survival Probability Minimum

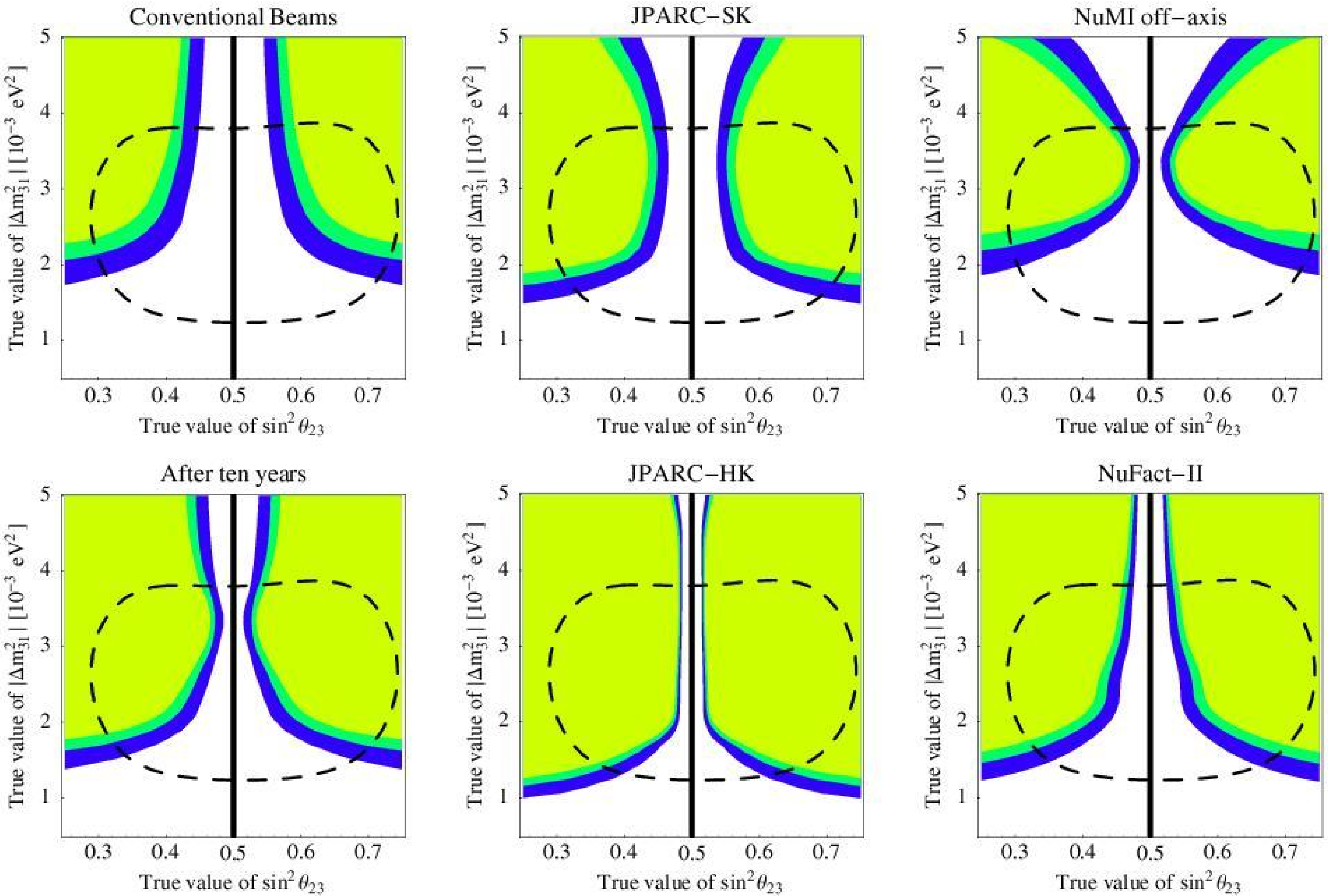
Prospects to measure sum rule combination $\theta_{13} \cos \delta$



Antusch, Huber, SFK, Schwetz

Prospects to measure a = s_{23}^2- 0.5

S. Antusch, M. Huber, J. Kersten, T. Schwetz, W. Winter




Supersymmetry

The SUSY Flavour and CP Problems

- What is the pattern of the soft SUSY breaking masses?
- Why are SUSY FCNCs so small?
- Why is SUSY CP violation so small?

-- plus all of the SM questions

-- plus other questions (μ problem, p-decay) not discussed here

SU(3) Family Symmetry

An old observation: SU(3) family symmetry predicts universal soft mass matrices in the symmetry limit

$$m_Q^2 \propto m_{u^c}^2 \propto m_{d^c}^2 \propto m_L^2 \propto m_{e^c}^2 \propto m_{N^c}^2 \propto \mathbb{1}$$

Perhaps CP is also conserved in this limit

Then SU(3) Family Symmetry can address:

- SM Flavour & CP Problems
- SUSY Flavour & CP Problems

The SU(3) symmetry limit

- $SU(3)\times SO(10)$ $\psi=(3,16)$ $H=(1,10)$
- Yukawa $\psi\ \psi$ H forbidden by SU(3)
- Soft trilinears A $\psi\ \psi$ H also forbidden
- Soft masses $\psi^*\ m^2\ \psi$ allowed but $m^2 \propto 1$
- Postulate CP conserved in SU(3) limit

Not very realistic → SU(3) must be broken!

SU(3) breaking

By anti-triplet flavon vevs $<\phi>$ resulting in:

- Effective Yukawas $\psi\ \psi\ \phi\ \phi\ H \rightarrow Y \propto \phi\ \phi$
- Effective trilinears $A_0\ \psi\ \psi\ \phi\ \phi\ H \rightarrow A \propto Y$
- Effective non-universal soft masses $m^2 \propto 1 + \phi^*\ \phi$
- CP spontaneously broken by $<\phi>$ phases

Antusch,SFK,Malinský

Operator expansions for soft masses

SU(3) breaking effects

$$\mathcal{L}^{m^2}_{\text{soft}} = \frac{1}{M^2_{\text{hid.}}} \int \mathrm{d}^4\theta\, \hat{X}^\dagger \hat{X} \sum_f \hat{f}^\dagger \left(b^f_0 \mathbb{1} + \sum_\phi b^f_\phi \frac{\hat{\phi} \otimes \hat{\phi}^\dagger}{M^{m2}_f} + \dots \right) \hat{f}$$

Unit matrix in the SU(3) symmetry limit $\phi = 0$

$$m^2_Q \propto m^2_{u^c} \propto m^2_{d^c} \propto m^2_L \propto m^2_{e^c} \propto m^2_{N^c} \propto \mathbb{1}$$

Recall Yukawa matrices, ignoring phases:

$$Y^u = \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & 2\varepsilon^2 & 2\varepsilon^2 \\ \varepsilon^3 & 2\varepsilon^2 & 1 \end{pmatrix}, \quad Y^d = \begin{pmatrix} 0 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & 1 \end{pmatrix}, \quad Y^e = \begin{pmatrix} 0 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ \bar{\varepsilon}^3 & 3\bar{\varepsilon}^2 & 3\bar{\varepsilon}^2 \\ \bar{\varepsilon}^3 & 3\bar{\varepsilon}^2 & 1 \end{pmatrix}.$$

Where $\varepsilon \approx 0.05, \bar{\varepsilon} \approx 0.15$

Under similar assumptions we predict (at the GUT scale):

$$m_Q^2 \approx m_0^2 \begin{pmatrix} 1+\varepsilon^4 & \varepsilon^4 & \varepsilon^4 \\ \varepsilon^4 & 1+\varepsilon^2 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1+\mathcal{O}(1) \end{pmatrix}, \quad m_{d^c}^2 \approx m_0^2 \begin{pmatrix} 1+\bar{\varepsilon}^4 & \bar{\varepsilon}^4 & \bar{\varepsilon}^4 \\ \bar{\varepsilon}^4 & 1+\bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^4 & \bar{\varepsilon}^2 & 1+\mathcal{O}(1) \end{pmatrix}$$

$$m_{u^c}^2 \approx m_0^2 \begin{pmatrix} 1+\varepsilon^4 & \varepsilon^4 & \varepsilon^4 \\ \varepsilon^4 & 1+\varepsilon^2 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1+\mathcal{O}(1) \end{pmatrix},$$

$$m_L^2 \approx m_0^2 \begin{pmatrix} 1+\varepsilon^4 & \varepsilon^4 & \varepsilon^4 \\ \varepsilon^4 & 1+\varepsilon^2 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1+\mathcal{O}(1) \end{pmatrix}, \quad m_{e^c}^2 \approx m_0^2 \begin{pmatrix} 1+\bar{\varepsilon}^4 & \bar{\varepsilon}^4 & \bar{\varepsilon}^4 \\ \bar{\varepsilon}^4 & 1+\bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^4 & \bar{\varepsilon}^2 & 1+\mathcal{O}(1) \end{pmatrix}$$

$$(\delta^d_{LL})_{12} \approx \frac{\tilde{b}^d_{12}}{b_Q}\varepsilon^2\bar{\varepsilon}\,, \quad (\delta^d_{LL})_{13} \approx \frac{\tilde{b}^d_{13}}{b_Q}\varepsilon^2\bar{\varepsilon}\,, \quad (\delta^d_{LL})_{23} \approx \frac{\tilde{b}^d_{23}}{b_Q}\bar{\varepsilon}^2$$

$$(\delta^d_{RR})_{12} \approx \frac{\tilde{b}^{d^c}_{12}}{b_{d^c}}\bar{\varepsilon}^3\,, \quad (\delta^d_{RR})_{13} \approx \frac{\tilde{b}^{d^c}_{13}}{b_{d^c}}\bar{\varepsilon}^3\,, \quad (\delta^d_{RR})_{23} \approx \frac{\tilde{b}^{d^c}_{23}}{b_{d^c}}\bar{\varepsilon}^2$$

where $\varepsilon^2 \approx 2.5 \times 10^{-3}$, $\varepsilon^2\bar{\varepsilon} \approx 4 \times 10^{-4}$ and $\varepsilon^3 \approx 1 \times 10^{-4}$

$$\bar{\varepsilon}^2 \approx 2 \times 10^{-2} \text{ and } \bar{\varepsilon}^3 \approx 3 \times 10^{-3}.$$

Theoretical predictions for mass insertion parameters δ^d

δ^d	LL	LR/RL	RR	source	
$\lvert\delta_{12}\rvert$	$LL: 1.4 \times 10^{-2}$	$LR: 9.0 \times 10^{-5}$	$RR: 9.0 \times 10^{-3}$	[32]	$\Delta m_K, \varepsilon, \ldots$
$\lvert\mathrm{Re}\delta^2_{12}\rvert^{\frac{1}{2}}$	$LL^2: 4.0 \times 10^{-2}$	$LR^2: 4.4 \times 10^{-3}$	$LLRR: 2.8 \times 10^{-3}$	[31, 33]	Δm_K
$\lvert\mathrm{Im}\delta^2_{12}\rvert^{\frac{1}{2}}$	$LL^2: 3.2 \times 10^{-3}$	$LR^2: 3.5 \times 10^{-4}$	$LLRR: 2.2 \times 10^{-4}$	[31, 33]	ε
$\lvert\mathrm{Im}\delta_{12}\rvert$	$LL: 4.8 \times 10^{-1}$	$LR: 2.0 \times 10^{-5}$	–	[31, 33]	ε'/ε
$\lvert\delta_{13}\rvert$	$LL: 9.0 \times 10^{-2}$	$LR: 1.7 \times 10^{-2}$	$RR: 7.0 \times 10^{-2}$	[32]	$\Delta m_{B_d}, 2\beta$
$\lvert\mathrm{Re}\delta^2_{13}\rvert^{\frac{1}{2}}$	$LL^2: 9.8 \times 10^{-2}$	$LR^2: 3.3 \times 10^{-2}$	$LLRR: 1.8 \times 10^{-2}$	[31]	Δm_{B_d}
$\lvert\mathrm{Re}\delta_{13}\rvert$	$LL: 1.4 \times 10^{-1}$	$LR: 5.2 \times 10^{-2}$	$RR: 2.1 \times 10^{-2}$	[34]	Δm_{B_d}
$\lvert\mathrm{Im}\delta_{13}\rvert$	$LL: 3.0 \times 10^{-1}$	$LR: 2.3 \times 10^{-2}$	$RR: 9.0 \times 10^{-3}$	[34]	$B_d - \overline{B}_d$
$\lvert\delta_{23}\rvert$	$LL: 1.6 \times 10^{-1}$	$LR: 4.5 \times 10^{-3}$	$RR: 2.2 \times 10^{-1}$	[32]	Δm_{B_s}
$\mathrm{Re}\delta_{23}$	$LL: 5.0 \times 10^{-1}$	$LR: 2.5 \times 10^{-2}$	$RR: 5.0 \times 10^{-1}$	[35]	$b \to s\gamma$
$\lvert\mathrm{Im}\delta_{23}\rvert$	–	$LR: 1.5 \times 10^{-2}$	–	[35]	$b \to s\gamma$
$\lvert\mathrm{Re}\delta_{11}\rvert$	–	$LR: 1.6 \times 10^{-3}$	–	[31]	Δm_d
$\lvert\mathrm{Im}\delta_{11}\rvert$	–	$LR: 3.0 \times 10^{-6}$	–	[31]	d_n
	–	$LR: 1.1 \times 10^{-6}$	–	[36]	d_n
	–	$LR: 6.7 \times 10^{-8}$	–	[36]	d_{Hg}
$\lvert\mathrm{Re}\delta_{22}\rvert$	–	$LR: 2.4 \times 10^{-2}$	–	[31]	Δm_s
$\lvert\mathrm{Im}\delta_{22}\rvert$	–	$LR: 6.6 \times 10^{-6}$	–	[36]	d_n
	–	$LR: 5.6 \times 10^{-6}$	–	[36]	d_{Hg}
$\lvert\mathrm{Re}\delta_{33}\rvert$	–	$LR: 7.3 \times 10^{-1}$	–	[31]	Δm_b

Experimental limits on δ^d

$$(\delta^e_{LL})_{12} \approx \frac{\tilde{b}^e_{12}}{b_L}\varepsilon^2\bar{\varepsilon}\,, \quad (\delta^e_{LL})_{13} \approx \frac{\tilde{b}^e_{13}}{b_L}\varepsilon^2\bar{\varepsilon}\,, \quad (\delta^e_{LL})_{23} \approx \frac{\tilde{b}^e_{23}}{b_L}\bar{\varepsilon}^2$$

$$(\delta^e_{RR})_{12} \approx \frac{\tilde{b}^{e^c}_{12}}{b_{e^c}}\bar{\varepsilon}^3\,, \quad (\delta^e_{RR})_{13} \approx \frac{\tilde{b}^{e^c}_{13}}{b_{e^c}}\bar{\varepsilon}^3\,, \quad (\delta^e_{RR})_{23} \approx \frac{\tilde{b}^{e^c}_{23}}{b_{e^c}}\bar{\varepsilon}^2$$

Theoretical predictions for δ^l

where $\varepsilon^2 \approx 2.5 \times 10^{-3}, \varepsilon^2\bar{\varepsilon} \approx 4 \times 10^{-4}$ and $\varepsilon^3 \approx 1 \times 10^{-4}$

$$\bar{\varepsilon}^2 \approx 2 \times 10^{-2} \text{ and } \bar{\varepsilon}^3 \approx 3 \times 10^{-3}.$$

δ^l	LL	LR/RL	RR	source	
$\lvert\delta_{12}\rvert$	$LL: 6.0 \times 10^{-4}$	$LR: 1.0 \times 10^{-5}$	$-$	[32]	$\mu \to e\gamma$
	$LL: 2.0 \times 10^{-3}$	$LR: 3.5 \times 10^{-5}$	$RR: 9.0 \times 10^{-2}$	[32]	$\mu \to eee$
	$LL: 2.0 \times 10^{-4}$	$LR: 3.5 \times 10^{-5}$	$-$	[32]	$\mu \to e$ in ^{22}Ti
$\lvert\delta_{13}\rvert$	$LL: 1.5 \times 10^{-1}$	$LR: 4.0 \times 10^{-2}$	$-$	[32]	$\tau \to e\gamma$
	$-$	$LR: 5.0 \times 10^{-1}$	$-$	[32]	$\tau \to eee$
$\lvert\delta_{23}\rvert$	$LL: 1.2 \times 10^{-1}$	$LR: 3.0 \times 10^{-2}$	$-$	[32]	$\tau \to \mu\gamma$
	$-$	$LR: 5.0 \times 10^{-1}$	$-$	[32]	$\tau \to \mu ee$
$\lvert\mathrm{Re}\delta_{11}\rvert$	$-$	$LR: 8.0 \times 10^{-3}$	$-$	[31]	Δm_e
$\lvert\mathrm{Im}\delta_{11}\rvert$	$-$	$LR: 3.7 \times 10^{-7}$	$-$	[31]	d_e
	$-$	$LR: 1.6 \times 10^{-7}$	$-$	[36]	d_e

Experimental limits on δ^l

SU(3) predictions for CP violation

$$|\mathrm{Im}(\delta^d_{LR})_{11}| \sim \frac{A_0 v}{\langle \tilde{m}_d \rangle^2_{LR}} \frac{1}{\sqrt{1+\tan^2\beta}} \bar{\varepsilon}^5 \sin\phi_1 \sim 10^{-6}$$

$$|\mathrm{Im}(\delta^l_{LR})_{11}| \sim \frac{A_0 v}{\langle \tilde{m}_e \rangle^2_{LR}} \frac{1}{\sqrt{1+\tan^2\beta}} \frac{1}{3} \bar{\varepsilon}^5 \sin\phi_1 \sim 10^{-6}$$

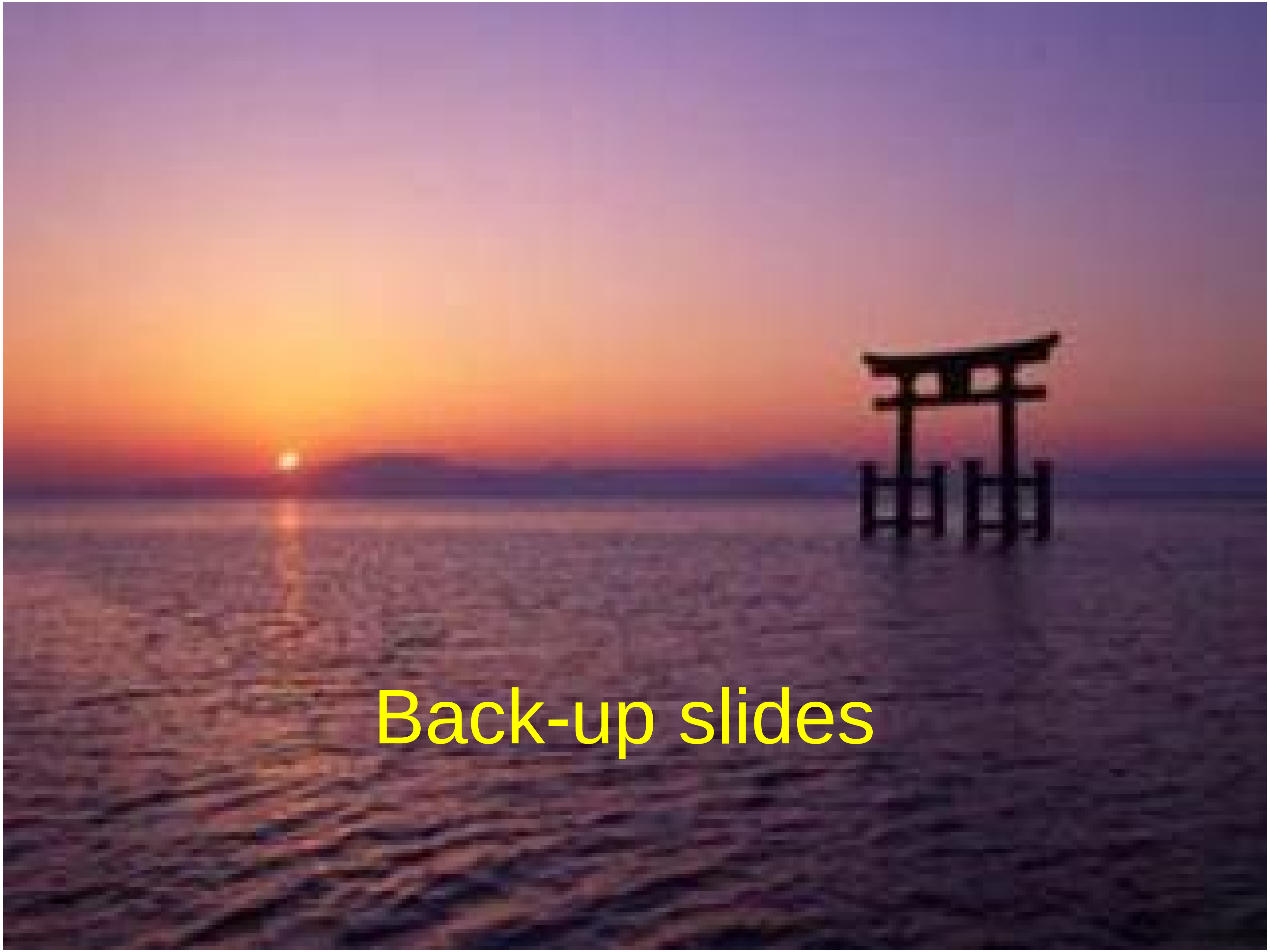
EDMs originate from phases in the (2,2) entry in the theory basis of order $\sim \varepsilon^3$ and require a further double Cabibbo suppression to reach the (1,1) entry in the SCKM basis

→ ε^5 suppressed EDMs! (more suppressed than CMSSM)

c.f. CMSSM $$|\mathrm{Im}(\delta^{d,l}_{11})_{LR}| \sim \frac{\mathrm{Im}(A_0) v}{\langle \tilde{m}_{d,l} \rangle^2} \frac{1}{\sqrt{1+\tan^2\beta}} \bar{\varepsilon}^4$$

Conclusion 結論する

- Quark spectrum fit by symmetric hierarchical textures
- MNS parametrized by deviations from tri-bimaximal mixing
- Tri-bimaximal neutrino mixing obtained from see-saw via CSD
- CSD obtained from SO(3) or SU(3) fam sym + vac alignment
- Discrete gauge A_4 or Δ_{27} are simpler and string motivated
- SU(3) or Δ_{27} is consistent with SO(10) GUT
- Predicts r=λ /3 and the stable sum rule $s \approx r\cos\delta + \frac{2}{3}a$
- The SUSY Flavour and CP problems solved by SU(3) or Δ_{27}


Back-up slides

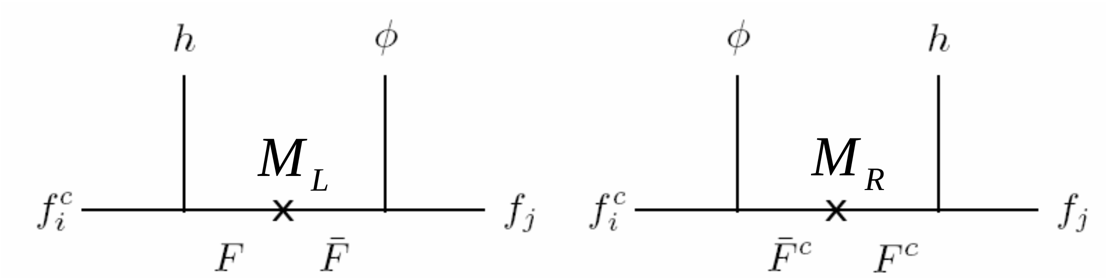
Family hierarchies without a family symmetry

Ferretti, SFK, Romanino;
Barr

$$W = M\bar{\Psi}\Psi + \alpha_i \bar{\Psi}\psi_i\phi + \lambda_i \Psi\psi_i h$$

One family of "messengers" dominates

Three families of quarks and leptons



$$M\bar{\Psi}\Psi \equiv M_Q\bar{Q}Q + M_U\bar{U}^cU^c + M_D\bar{D}^cD^c + M_L\bar{L}L + M_N\bar{N}^cN^c + M_E\bar{E}^cE^c$$

Suppose $M_Q \ll M_D \ll M_U$ then in a particular basis

$$Y^U_{LR} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & ab\varepsilon_U & ad\varepsilon_U \\ 0 & cb\varepsilon_U & 1 \end{pmatrix} \frac{<\phi>}{M_Q} \qquad Y^D_{LR} = \begin{pmatrix} 0 & ef\varepsilon_D & ek\varepsilon_D \\ 0 & gf\varepsilon_D & gk\varepsilon_D \\ 0 & hf\varepsilon_D & 1 \end{pmatrix} \frac{<\phi>}{M_Q}$$

$$\varepsilon_U = \frac{M_Q}{M_U}, \quad \varepsilon_D = \frac{M_Q}{M_D}, \quad a,b,c,d,e,f,g,h,k \sim 1$$

$m_u = m_d = 0 \rightarrow$ Accidental sym

$\frac{m_c}{m_t} \ll \frac{m_s}{m_b} \ll 1$ Not bad! But…

$\tan\beta \sim 50$ $\quad \tan\theta_C \sim 1$

$\frac{m_s}{m_b} \approx |V_{cb}|$

Need broken Pati-Salam…

Conclusion: partial success, but little predictive power esp. in neutrino sector

U(1) Family Symmetry

Simplest example is U(1) family symmetry spontaneously broken by a flavon vev $\langle\phi\rangle \neq 0$

For D-flatness we use a pair of flavons with opposite U(1) charges $Q(\phi) = -Q(\overline{\phi})$

Example: U(1) charges as Q (ψ_3)=0, Q (ψ_2)=1, Q (ψ_1)=3, Q(H)=0, Q(ϕ)=-1,Q($\overline{\phi}$)=1

Then at tree level the only allowed Yukawa coupling is H ψ_3 ψ_3 $\rightarrow Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The other Yukawa couplings are generated from higher order operators which respect U(1) family symmetry due to flavon ϕ insertions:

$$\overset{\boxed{-1+0+1+0=0}}{\frac{\phi}{M}} H\psi_2\psi_3 + \left(\frac{\phi}{M}\right)^2 H\psi_2\psi_2 + \left(\frac{\phi}{M}\right)^3 H\psi_1\psi_3 + \left(\frac{\phi}{M}\right)^4 H\psi_1\psi_2 + \left(\frac{\phi}{M}\right)^6 H\psi_1\psi_1$$

When the flavon gets its VEV it generates small effective Yukawa couplings in terms of the expansion parameter $\varepsilon = \dfrac{\langle\phi\rangle}{M}$

$$\rightarrow Y = \begin{pmatrix} \varepsilon^6 & \varepsilon^4 & \varepsilon^3 \\ \varepsilon^4 & \varepsilon^2 & \varepsilon \\ \varepsilon^3 & \varepsilon & 1 \end{pmatrix}$$

Shortcomings of U(1) Family Symmetry

A Problem with U(1) Models is that it is impossible to obtain

$$Y \approx \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}$$

For example consider Pati-Salam where there are effectively no constraints on the charges from anomaly cancellation

$$Y^f = \begin{pmatrix} \epsilon^{|l_1+e_1+h_f|} & \epsilon^{|l_1+e_2+h_f|} & \epsilon^{|l_1+e_3+h_f|} \\ \epsilon^{|l_2+e_1+h_f|} & \epsilon^{|l_2+e_2+h_f|} & \epsilon^{|l_2+e_3+h_f|} \\ \epsilon^{|l_3+e_1+h_f|} & \epsilon^{|l_3+e_2+h_f|} & \epsilon^{|l_3+e_3+h_f|} \end{pmatrix}$$

There is no choice of l_i and e_i that can give the desired texture

e.g. previous example $l_1=e_1=3$, $l_2=e_2=1$, $l_3=e_3=h_f=0$ gave:

$$Y = \begin{pmatrix} \varepsilon^6 & \varepsilon^4 & \varepsilon^3 \\ \varepsilon^4 & \varepsilon^2 & \varepsilon \\ \varepsilon^3 & \varepsilon & 1 \end{pmatrix}$$

The desired texture can be achieved with non-Abelian family symmetry. There is also an independent motivation for non-Abelian symmetry from neutrino physics…