Discrete Gauge Symmetries and Proton Stability in GUTs

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Ritsumeiken GUT workshop, 2007

(M. Ratz and RNM-PRD76, 095003 (2007))

Nature of Beyond the Standard model physics

- Two approaches:

- Top Down: String theories

- Bottom-Up: Supersymmetry, Extra Dimensions etc. (TeV scale physics), GUT theories.

Typical bottom up theories have a symmetry G above a scale M_0 .

Questions for Generic Bottom-Up Theories

- These are effective field theories; once we include effect of new physics, one can write:

$$L = L_0 + \sum_n \frac{O^{(n)}}{M^{n-4}}$$

L_0 and $O^{(n)}$ G inv. and M new physics scale.

- One known new physics is gravity, which can induce operators with $M = M_P$!!

  Are such operators experimentally acceptable ? If not how to make these theories ***NATURAL***?

- For acceptable operators, what is the lowest M ? that could indicate a new physics threshold !

Naturalness

- **Definition: Small parameters must be understandable in terms of some symmetries !**
- **E.g. electron mass, QCD theta parameter, neutrino masses etc.**
- **Many popular theories need help to satisfy "natural stability" requirement.**
- **One source of help are extra discrete gauge symmetries- subject of this talk.**

SUSY and SUSY GUTs

- Many reasons to think SUSY at TeV scale and possibly SUSY GUTs at superheavy scales !!
- Gauge hierarchy problem
- Gauge Coupling unification
- Seesaw model for Neutrino mass
- Dark matter

How natural are these models ?

Minimal SUSY Model- ν –MSSM

- **MSSM based on SU(3)xSU(2)xU(1) with $m_\nu \neq 0$:**

$$
\begin{aligned}
Q &= \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \sim (3, 2, \frac{1}{6}) \\
u^c &= (u_1^c \quad u_2^c \quad u_3^c) \sim (\bar{3}, 1, \frac{-2}{3}) \\
d^c &= (d_1^c \quad d_2^c \quad d_3^c) \sim (\bar{3}, 1, \frac{1}{3}) \\
L &= \begin{pmatrix} \nu \\ e^- \end{pmatrix} \sim (1, 2, \frac{-1}{2}) \\
e^c & \quad \sim (1, 1, +1) \\
\nu^c & \quad \sim (1, 1, 0)
\end{aligned}
$$

MSSM has too many unexplained small parameters !

- SM has stable proton- but MSSM has R-parity breaking terms that make proton decay in an instant:
- Culprit: R-parity breaking terms

$$W' = LLe^c + QLd^c + u^c d^c d^c$$

$$\lambda \qquad \lambda' \qquad \lambda''$$

- Limits on these couplings severe:

$$\lambda', \lambda'' \leq 10^{-12}$$

R-parity as a Cure

- **Demand R-parity invariance to restore naturalness to MSSM:**

$$R = (-1)^{3(B-L)+2S}$$

- **Can arise from GUT theories e.g. SO(10) with 126 Higgs fields.**

(RNM, 86; Font, Ibanez, Quevedo; 89; Martin,92)

- **Implies** $\lambda, \lambda', \lambda'' = 0$ **; their smallness related to R-parity breaking !**

New Naturalness Problem even with R-P

- Planck scale RP-conserving gravity induced proton decay operator:
- κ QQQL/M_Pl. Present limit on it κ

$\kappa \leq 10^{-7}$; **Why so small ?**

Problem persists in GUT theories-

In SO(10), operator form $\frac{\kappa}{M_P}[16_m]^4$

In SU(5), it is 10.10.10.5 operator;

No such operator in E_6 but a similar one not that small-(27_m)^4(27-bar-H)/M^2_P

SM Has None of These Problems !

- **What can be done to bring MSSM and SUSY GUTs to the same level of naturalness as SM as far as baryon non-conservation is concerned and still keep models realistic?**

Discrete Gauge symmetries as a cure

- Could it be that both R-parity breaking terms and R-conserving operators QQQL are suppressed by a single extra symmetry ?
- If so, the discrete symmetries better be gauge symmetries because

A. Gauge symmetries cannot receive nonperturbative gravitational corrections;

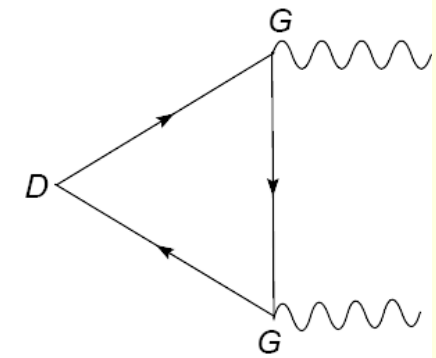
B. There will be no domain wall problem.

(Kibble, Lazaridis, Shafi,83; Preskill, Wilczek, Trivedi;)

How to ensure a discrete gauge symmetry ?

- **Must satisfy anomaly constraints !!**
- **Dgg, DGG, DDD =0**

$$D \subset U(1)$$



- **Previously explored in the context of MSSM:**
  - **(Ibanez,Ross; Banks, Dine; Hinchliffe, Kaeding; Babu, Gogoladze, Wang; Dreiner, Luhn and Thormier; Murayama, Dreiner et al., Maru, Kurosawa, Yanagida…)**

A typical MSSM CASE

- MSSM example from Babu et al :

	Q	u^c	d^c	ℓ	e^c	ν^c	H
Z_6	6	5	1	2	5	3	1
Z_3	3	2	1	2	2	3	1
Z_2	2	1	1	2	1	1	1

- Note that such an assignment is incompatible with GUTs;
- Are GUTs doomed to suffer from

p-decay problem from QQQL operator ?

Leading proton decay operator

- **Proton decay operator:**

$$(u^c d^c d^c)^3 L H_u \frac{1}{M^8}$$

- **Leads to processes:**

$$D \rightarrow \overline{n}\,\overline{\nu}\pi^+$$

- **M > 10 TeV i.e. with this discrete symmetry there could be “safe” new physics above 10 TeV.**

Only P-decay suppressed

- If only p-decay suppressed but R-parity broken, the constraints on λ', λ much weaker.
- DGS for this case is Z_9 with charge assignment Q given by:
- Q=6B and M> 10 TeV or so.
- the leading order proton decay operators

$$\frac{1}{M^9}(u^c d^c d^c)^3 LLe^c \quad \Longrightarrow \quad D \to \overline{n}\pi^+ e^- e^+ \nu$$

And $(u^c d^c d^c)^3 \quad \Longrightarrow \quad D \to \overline{n}\pi^+$ etc

Higher Unified Theories

- Seesaw models for neutrino mass generally imply extended gauge symmetries to understand why

$$M_R << M_{Pl}$$

- A suitable symmetry is B-L; we study the proton decay problem in models with B-L gauge sym e.g. LR and SO(10). Compatibility with GUTs ?

Results

- Simple Discrete symmetries for both LR and SO(10) models which suppress proton decay to desired level.
- In SO(10) with 16_H, the minimum number of gen for which a solution exists is three. Model not realistic; further extensions needed.
- Minimal SO(10) with 126 is also not realistic .
- Comments on non-abelian groups.

SUSY LR symmetric models

- **Gauge group**: $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$
- **Fermion assignment**

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \overset{P}{\Leftrightarrow} \begin{pmatrix} u_R \\ d_R \end{pmatrix} \qquad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \overset{P}{\Leftrightarrow} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

- **Two Higgs choices:** $\phi(2,2,0)$ **+**
- **(i)** $\chi(2,1,-1);\ \chi^c(1,2,+1) + bars$
- **(ii)** $;\Delta_R(1,3,+2) \oplus \Delta_L(3,1,+2)$

Symmetry breaking

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

$$\downarrow \quad <\Delta_R> \neq 0 \quad or \quad <\chi^c> \neq 0 \qquad M_{W_R}, M_{Z'} \neq 0$$

$$SU(2)_L \otimes U(1)_Y$$

$$\downarrow \quad <\phi> = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}$$

$$U(1)_{em} \qquad M_{W_L}, M_Z \neq 0; m_{q,l} \neq 0$$

Constraints on the DGS

- ## Doublet Higgs case:
- **Anomaly constraints:**

$$N_g\,[6\,(q_Q + q_{Q^c}) + 2\,(q_L + q_{L^c})] + 4\,q_\Phi + 2\,(q_\chi + q_{\chi^c} + q_{\overline{\chi}} + q_{\overline{\chi}^c}) \;=\; 0 \quad \mathrm{mod}\ N'$$

$$N_g\,[2\,(q_Q + q_{Q^c})] \;=\; 0 \quad \mathrm{mod}\ N$$

$$N_g\,[3\,q_Q + q_L] + 2\,q_\Phi + q_\chi + q_{\overline{\chi}} \;=\; 0 \quad \mathrm{mod}\ N$$

Super-potential terms: Wanted and Unwanted

- **Wanted:**

$$W = \mathrm{i}\, h\, Q^T \tau_2 \Phi\, Q^c + \mathrm{i}\, h'\, L^T \tau_2 \Phi L^c + \mathrm{i}\, f_c\, L^{cT} \tau_2 \overline{\chi^c}\, \overline{\chi^c}^T \tau_2 L^c + \mathrm{i}\, f_c L^T \tau_2 \overline{\chi} L^T \tau_2 \overline{\chi} + S\,(\chi^c \overline{\chi^c} + \chi\, \overline{\chi} - v_{\mathrm{R}}^2) + \mu\, \mathrm{Tr}(\Phi^2)\,.$$

- **Unwanted:**

$$W_{\mathrm{unwanted}} = Q^3 L + Q^{c3} L^c + Q^3 \chi + Q^{c3} \chi^c + L \overline{\chi} + L^c \overline{\chi}^c + L\, Q\, Q^c \chi^c + Q\, \chi\, Q^c L^c + L^2 L^c \chi^c + \overline{\chi}\, L\, \Phi^2 + \overline{\chi}^c L^c \Phi^2 + L\, \Phi\, \chi^c + L^c \Phi\, \chi\,.$$

Result

- Smallest number of Gen. for which all constraints are satisfied is

$$N_g = 3$$

- Smallest group is Z_6

q_L	q_Q	q_{L^c}	q_{Q^c}	q_Φ	q_χ	q_{χ^c}	$q_{\overline{\chi}}$	$q_{\overline{\chi}^c}$	q_S
1	1	5	5	0	0	2	0	4	0

Leading order proton decay

- **Leading order proton decay operator in this theory:**

$$\frac{1}{M^{17}} Q^{c15} L^{c} \chi^{c4}$$

- **Highly suppressed even for M=TeV.**

Results for the Triplet case

- **Requirements are much weaker since R-parity is automatically conserved: Forbid QQQL +Qc….**
- **Solutions exist for** $N_g \geq 2$
- **And Z_2 group for 2 gen and Z_3 for three.**

G_224 Model and N-N-bar Oscillation

- Model based on gauge group:

$$SU(2)_L \times SU(2)_R \times SU(4)_c$$

- Fermions: ψ, ψ^c = $\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}$
- Higgs fields: (2,2,1) ϕ_1 (3 of them)
- (2,2,15) ϕ_{15} $S = (1,1,15)$
- (1,3,10) Δ^c +(1,3,10*) $\overline{\Delta}^c$ +(1,3,1) Ω

$$\Delta(1,3,10) \supset \Delta_{q^c q^c}, \Delta_{q^c l^c}, \Delta_{l^c l^c}$$

Superpotential and Discrete SymMetry

- Renormalizable part:

$$W = M\Delta^c\overline{\Delta}^c + M'\Omega^2 + \lambda\Delta^c\Omega\overline{\Delta}^c$$

$$+\psi\phi_{1,15}\psi^c + Tr\phi^2 S + S^3$$

- Discrete Gauge Sym: **Z_3**
- $\psi, \psi^c \quad \Delta^c \quad \phi \; S$ Q=+1; $\overline{\Delta}^c$ Q=-1
- All other fields Q=0; S has weak scale vev. And induces mu-term.

SO(10) with 16_H and 126_H

- Two classes of SO(10) models:
- (i) B-L broken by 16-H: R-parity not automatic
- (ii) B-L broken by 126-H: R-parity automatic.
- Case (i) 16_m;10+10',45,54,16+16-bar
- Case (ii) 16_m; 10+10'; 210; 126+126-bar

Proton decay in SO(10)

- 3 Contributions (with R_p)
- (i) Gauge exchange
- (ii) Color Higgsino exchange
- (iii) Planck scale induced
- (ii) and (iii) potentially fatal:
- Discrete symmetry as a way to control them.
- Other ways to suppress (ii): Mimura talk
- (Babu, Barr-(16 models); Dutta, Mimura, RNM (05,07)-126 models)

SO(10) with 16-Higgs

- **Anomaly constraints:**

$$16\,(N_g\,q_{\psi_m} + q_{\psi_H} + q_{\overline{\psi}_H}) + 10\,(q_H + q_{H'}) + 45\,q_A + 54\,q_S \;=\; 0 \quad \mathrm{mod}\ N'$$

$$2N_g\,q_{\psi_m} + 2\,q_{\psi_H} + q_{\overline{\psi}_H} + q_H + q_{H'} + 8q_A + 12q_S \;=\; 0 \quad \mathrm{mod}\ N$$

- **Superpotential constraints:**

$$2q_{\psi_m} + q_H = 0 \quad \mathrm{mod}\ N\,, \qquad 2q_{\psi_m} + 2q_{\overline{\psi}_H} = 0 \quad \mathrm{mod}\ N\,,$$

$$q_{\psi_H} + q_{\overline{\psi}_H} = 0 \quad \mathrm{mod}\ N\,, \qquad q_H + q_{H'} = 0 \quad \mathrm{mod}\ N\,,$$

$$2q_A = 0 \quad \mathrm{mod}\ N\,, \qquad 2q_S = 0 \quad \mathrm{mod}\ N\,,$$

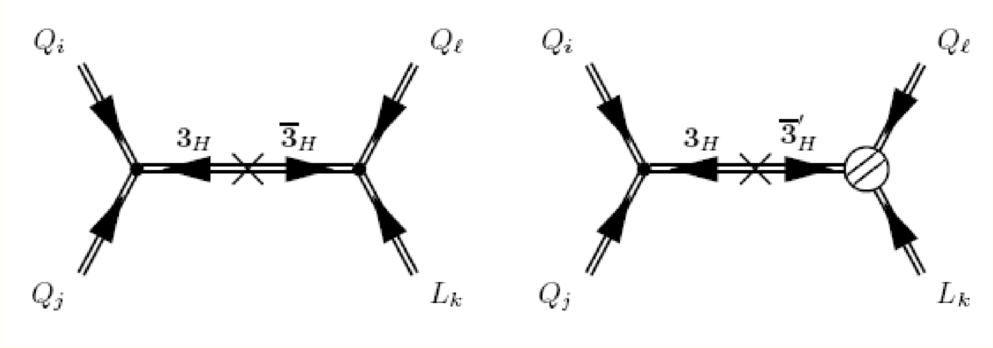
$$3q_S = 0 \quad \mathrm{mod}\ N\,, \qquad 2q_A + q_S = 0 \quad \mathrm{mod}\ N\,.$$

Constraints from proton decay and R-parity

- **Undesirables constraints**

$$
\begin{aligned}
2\,q_{\psi_m} + q_{H'} &\neq 0 \mod N\,, & 4\,q_{\psi_m} &\neq 0 \mod N\,,\\
q_{\psi_m} + q_{\overline{\psi}_H} &\neq 0 \mod N\,, & 3\,q_{\psi_m} + q_{\psi_H} &\neq 0 \mod N\,,\\
q_{\psi_m} + q_{H',H} + q_{\psi_H} &\neq 0 \mod N\,, & q_{\psi_m} + q_{\overline{\psi}_H} + q_A &\neq 0 \mod N\,.
\end{aligned}
$$

- **Forbid color Triplet p-decay**



Solution

- Smallest number of generations is three and group Z_6.
- Z_6 forbids both QQQL + RP breaking terms.
- Charge assignment

(a) MSSM part.

Field	quantum numbers
ψ_m	$\mathbf{16}_1$
H	$\mathbf{10}_{-2}$
H'	$\mathbf{10}_2$

(b) **16**-Higgs model.

Field	quantum numbers
ψ_H	$\mathbf{16}_{-2}$
$\overline{\psi}_H$	$\overline{\mathbf{16}}_2$
A	$\mathbf{45}_0$
S	$\mathbf{54}_0$

Doublet triplet splitting

- **Doublets are in H, H' and $\psi_H, \overline{\psi}_H$**
  - Azatov and RNM,07
- **Mass matrix for Doublets**

$$(H_d, H'_d, \psi_{Hd})$$

$$\begin{pmatrix} H_u \\ H'_u \\ \psi_{H_d} \end{pmatrix} \begin{pmatrix} 0 & b-\frac{3}{2}s+M & c_1 \\ -b-\frac{3}{2}s+M & 0 & 0 \\ 0 & c_2 & M_\psi \end{pmatrix}$$

- **Color Triplets**

$$\begin{pmatrix} 0 & a+s+M & c_1 \\ -a+s+M & 0 & 0 \\ 0 & c_2 & M_\psi \end{pmatrix}$$

Fine tuning to get light doublets:

- Choose:

$$c_1 c_2 = M_\psi (b - \frac{3}{2} s + M)$$

- One pair of MSSM doublets but no light color triplets. However,

$$H_d^{\ mssm} \propto (-H'_d + \alpha\, \psi_{H_d})$$

- Down quark masses zero in leading orders. Model unrealistic.

Fermion Masses

- Fermion Yukawa couplings limited:

$$W_Y = h\psi_m\psi_m H + \frac{f}{M_P}\psi_m\psi_m\overline{\psi}_H\overline{\psi}_H$$

- But no $\psi_m\psi_m\psi_H{}^2$ term.
- Up-quark and seesaw OK but in down-charged lepton sector gives
- $M_d = M_l$ =0

Minimal Realistic model for 16_H case:

- Three 10-H: H(-2); H'(+2); H"(0)
- Three 45's: A(0); A'(-2); A"(+2)
- One 54-H (S): $< S >= I \times (s, s, s, -\frac{3}{2} s, -\frac{3}{2} s)$
- VeV pattern: $< A >= i\tau_2 \times (a, a, a, b', b')$
- Consistent with $< A' >= diag(0,0,0,b,b) \times i\tau_2$

  F=0 condition. $< A'' >= 0; < \psi_H >=< \overline{\psi}_H >= v_R$
- Need one fine tuning to keep a linear comb. Of H and H" doublets light.

$$W = HH'(M + \lambda_1 A) + H' A' H'' + M' H'' H''$$

Doublet mass matrix and MSSM doublets:

$$\begin{pmatrix} 0 & M+S+A & 0 & c \\ M+S-A & 0 & A_2 & 0 \\ 0 & -A_2 & 0 & 0 \\ 0 & c & \delta & M_\psi \end{pmatrix}$$

$$H_d = H_{1d}$$

Choose M+S-A=0; Gives MSSM pair:

$$H_u = \Sigma x_i H_{iu} + x_4 \overline{\psi}_{H,u}$$

Works for fermion massses.

Effective Proton decay operator in the model:

- Dominant proton decay operator in the model:

$$\frac{\kappa}{M_P{}^3}[\psi_m]^4 A'^2 \rightarrow \left(\frac{b}{M_P}\right)^2 [\psi_m]^4 / M_P$$

- For $b \approx 10^{15} GeV$, coefficient $\approx 10^{-6}$

- **Much less fine tuning to suppress proton decay.**

SO(10) with 126-H

- Constraints are much weaker:
- Simplest model for 3 gen is Z 3:

field	q_{ψ_m}	H	H'	Δ	Δ
$\mathbb{Z}_3$	1	1	-1	-1	1

- Same model in fermion sector as has been discussed in literature:
  - Babu, RNM (92);
  - Fukuyama, Okada (01), Bajc, Senjanovic, Vissani (02);
  - Dutta, Mimura, RNM; Grimus, Kuhbock,..

Same problem as in the 16_H case

- There are two solutions: either $M_u=0$ or $M_d=M_l=0$.

- **Minimal model with naturally suppressed Planck scale induced proton decay unrealistic. Need extension !**

Important point

- No dim = 5 proton decay operator in both classes of models before or after SO(10) breaking !
- One can carry out Doublet-triplet splitting in these models using earlier papers: (Fukuyama et al; Aulakh et al.)

OUTLOOK

- Search for nonabelian symmetries as well as product symmetries;

- May provide a guide to string model building;

Some initial comments on Non-abelian groups

- **Look for groups that have 3-dim irreps:**
- **Some groups such as Q8, D4, D5,D6,Q6,D7 all have max dim =2 and allow 16^4 operator.**
- **A4, S4 have d=3 rep butnot acceptable.**
- **Minimal acceptable group:** $\Delta(3n^2)$
- **Typically have several 3-dim reps and stop dim 4 proton decay operator in SO(10)**

Some details

- For D4 and A4,
- 3X3 $\supset$ 1; hence problem !
- But for $\Delta(3n^2)$, n=3
- 3X3 = (3-bar) and
- 3X3-bar $\supset$ 1 ; So 16^4 is not invariant. **Hence no proton decay operator till 16^6.**

Model building with SO(10)x$\Delta(27)$

- Irreps of $\Delta(27)$: 9 {1}+3+3-bar;
- **16_H models:**
- Need $16_m \subset 3; \overline{16}_H \subset \overline{3}$ for seesaw;
- This implies that there is large R-parity violation from $16_m \overline{16}_H$ type operator + many others.

----Problematic !

Model Building contd.

- **126- models: No such problem:**
- **Assign {10} and 126-bar $\subset 3$ so that Yukawa couplings are allowed.**
- **No R-parity violating terms.**

Fermion masses in SO(10)x $\Delta(27)$ with 126_H

- Construction of invariants:
- Assign: {16}-m; H(10), {126-bar}: {120} belong to {3} of $\Delta(27)$; Generic Yukawa couplings:

$$h^{10} = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}, \begin{pmatrix} d & & \\ & e & \\ & & f \end{pmatrix} \sim f^{126}$$

- Similarly one for 120 (one matrix): 19 parameters for fermions. Realistic !

Anomalies:

- Only anomaly D^3:
- Need massive heavy sector to cancel anomalies.

Conclusion

- Conventional GUT models as well as partial Unif models suffer from a naturalness problem from proton decay life times. We construct models that cure these problem and are realistic !
- Partial Unif. G_224 models lead to observable NN-bar osc while keeping proton stable from Higher order operators.
- Minimal models work only for SO(10) with both 126-H; and 16-H Higgs- with R_p;
- Promising Nonabelian group is $\Delta(27)$

Application to SM

- In SM $\Delta(B-L)=0$ naturally true to all orders. First example of $O^{(n)}$ is $QQQL\,/\,M^2$ and is OK till $10^{15}GeV$.
- (i) $QQQL\,/\,M_P^{\,2}$ safe;
- (ii) One can expect new physics that gives proton decay only above $10^{15}GeV$. Example are non-susy GUT theories.

(iii) Another operator is $(LH)^2/M$; implies M= $10^{14}GeV$ implying there could be new physics above this scale as long as they not give p-decay. Examples LR sym. Models.