Review of phenomenological models
(of mass matrices)

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The mysteries in the mass matrices

- Quark sector
  - Quark masses

Same order

$m_u(M_X) = 1.04^{+0.19}_{-0.20}$ [MeV] $\qquad m_d(M_X) = 1.33^{+0.17}_{-0.19}$ [MeV]

$m_c(M_X) = 302^{+25}_{-27}$ [MeV] $\qquad m_s(M_X) = 26.5^{+3.3}_{-3.7}$ [MeV]

$m_t(M_X) = 129^{+196}_{-40}$ [GeV] $\qquad m_b(M_X) = 1.00 \pm 0.04$ [GeV]

Very different

  - Flavor mixings and phase

$|(U_{CKM})_{12}| = 0.2226 - 0.2259 \qquad |(U_{CKM})_{13}| = 0.0024 - 0.0038$

$|(U_{CKM})_{23}| = 0.0295 - 0.0387 \qquad \delta_q = 46^\circ - 74^\circ$

- Lepton sector
  - Lepton masses

Very small

$\sum_i m_{\nu_i}(m_Z) < 0.71$ [eV]

$m_e(M_X) = 0.325\cdots$ [MeV]

$m_\mu(M_X) = 68.6\cdots$ [MeV]

$m_\tau(M_X) = 1.17$ [GeV]

  - Flavor mixings and phases

$|(U_{MNS})^2_{12}| = 0.25 - 0.38 \qquad |(U_{MNS})^2_{13}| = 0.00 - 0.03$

$|(U_{MNS})^2_{23}| = 0.35 - 0.65 \qquad \delta_\ell, \beta, \gamma = ?$ Unknown

Very different

Bi-large mixings

The mysteries of the mass matrices

- There are many empirical rules that are difficult to explain by using mass matrix
  - **Barut's formula** (Barut, PRL42,1251(1979))

$$m_n(m_n) = m_e(m_e)\left\{1 + \frac{3}{2}\alpha^{-1}\left[1^4 + 2^4 + \cdots + (n-1)^4\right]\right\}$$

$$m_\mu(m_\mu) = m_e(m_e)\left\{1 + \frac{3}{2}\alpha^{-1}\left[1^4\right]\right\} = 105.55\,[\mathrm{MeV}]$$

$$m_\tau(m_\tau) = m_e(m_e)\left\{1 + \frac{3}{2}\alpha^{-1}\left[1^4 + 2^4\right]\right\} = 1786.2\,[\mathrm{MeV}]$$

cf. $m_e^{exp}(m_e) = 0.51099892 \pm 0.00000004$

$m_\mu^{exp}(m_\mu) = 105.658369 \pm 0.000009$

$m_\tau^{exp}(m_\tau) = 1776.99^{+0.29}_{-0.26}$

This formula says there is the relation between the charged lepton masses and the EW coupling.

The mysteries of the mass matrices

- There are many empirical rules that are difficult to explain by using mass matrix.
  - **Barut's formula** (Barut, PRL42,1251(1979))
  - **Quark Lepton Complementarity**
    (A. Y. Smirnov, hep-ph/0402264 (1994))

$$\theta_{us} + \theta_{12} = (46.7 \pm 2.4)^\circ \sim 45^\circ, \quad \theta_{cb} + \theta_{23} = (43.9^{+5.1}_{-3.6})^\circ \sim 45^\circ$$

$$\text{cf.} \quad \theta_{us} = (12.8 \pm 0.15)^\circ, \quad \theta_{12} = (33.8 \pm 2.2)^\circ, \quad \theta_{cb} \sim 0^\circ, \quad \theta_{23} \sim 45^\circ$$

This formula says there is the relation between the quark and lepton flavor mixings.

The mysteries of the mass matrices

- There are many empirical rules that are difficult to explain by using mass matrix.
  - **Barut's formula** (Barut, PRL42,1251(1979))
  - **Quark Lepton Complementarity**
    (A. Y. Smirnov, hep-ph/0402264 (1994))
  - **Koide's Relation** (Koide, PRD28,252(1983))

$$\frac{2}{3}\,\frac{\left\{\sqrt{m_e(m_e)}+\sqrt{m_\mu(m_\mu)}+\sqrt{m_\tau(m_\tau)}\right\}^2}{m_e(m_e)+m_\mu(m_\mu)+m_\tau(m_\tau)}=1-0{,}000002\,^{+0{,}000024}_{-0{,}000022}$$

We don‘t know whether they are accidental coincidence or not.

However Koide relation is consistent with extremely high accuracy.

So it is difficult to regard Koide’s Relation as accident.

When in building a model, it is important to consider the origin of Koide’s relation.

Goal of phenomenological model

- Final goal is to find the unified model of quark and lepton masses.
- However, there are many mass matrices which are consist with experiments.
  - Whenever we multiply the up and down mass matrices by unitary matrix *A, Bu* and *Bd* as follows

$$M_u \to M_u' = A^\dagger M_u B_u, \quad M_d \to M_d' = A^\dagger M_d B_d,$$

the experimental values (quark masses and the CKM matrix) do not change.

Therefore,

- The mass matrix must be constructed from as few parameters as possible.
- The mass matrix must be simple and beautiful.
- The mass matrix must suggest the new physics and new mechanism.
  - NNI, Texture zero, Democratic, GUT, Froggatt-Nielsen, S3, D4, Q6 ...

The approach to the mystery

- There are many mass matrix models which approach to the mysteries.
- They each have their own characteristic.
- They may be classified as follows.
  - The texture can explain either quark or lepton or both.
  - The textures of the quark and the lepton differ completely or not.
  - The texture can predict precisely or order of magnitude.
  - The texture is taken GUT into consideration or not.
  - etc…

The texture can explain either quark or lepton or both.

- The textures of the quark and the lepton differ completely or not.
  - Mass matrix of the quark and the lepton differ completely.

$$M_u \sim \mathrm{diag}(m_u, m_c, m_t), \quad M_d \sim \mathrm{diag}(m_d, m_s, m_b)$$

$$M_e \sim \mathrm{diag}(m_e, m_\mu, m_\tau), \quad M_\nu \simeq \begin{pmatrix} D_\nu & A_\nu & A_\nu \\ A_\nu & B_\nu & C_\nu \\ A_\nu & C_\nu & B_\nu \end{pmatrix}$$

  - CKM & MNS mixing matrices become

$$U_{CKM} \sim \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \quad U_{MNS} \sim \begin{pmatrix} c & s & 0 \\ -s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \\ -s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

  - This is most simple way and very familiar to derive the difference between quark and lepton flavor mixings!
  - Where does this difference come from?
    Seesaw mech., Discrete Sym (such as D4, Q8,...).

The texture can explain either quark or lepton or both.

- The textures of the quark and the lepton differ completely or not.
  - The structure is common to all the mass matrices.
    For example,

$$M_u = \begin{pmatrix} D_u & A_u & A_u \\ A_u' & B_u & C_u \\ A_u' & C_u & B_u \end{pmatrix},\ M_d = \begin{pmatrix} D_d & A_d & A_d \\ A_d' & B_d & C_d \\ A_d' & C_d & B_d \end{pmatrix},\ M_e = \begin{pmatrix} D_e & A_e & A_e \\ A_e' & B_e & C_e \\ A_e' & C_e & B_e \end{pmatrix},\ M_\nu = \begin{pmatrix} D_\nu & A_\nu & A_\nu \\ A_\nu' & B_\nu & C_\nu \\ A_\nu' & C_\nu & B_\nu \end{pmatrix}$$

  - Choose the up and down mass matrices which cancel Large mixing.
    - Large mixing – Large mixing = Small mixing … the CKM matrix
    - Large mixing + Large mixing = Large mixing … the MNS matrix
  - This is congenial to some of GUT models. In SO(10)-GUT,

$$M_u = S + \delta' A + \varepsilon S',\quad M_d = \alpha S + \delta A + S',\quad M_e = \alpha S + A - 3S'$$

$$M_D = S + \delta'' A - 3\varepsilon S',\quad M_L = \beta S',\quad M_R = \gamma S' \rightarrow m_\nu = M_L - M_D M_R^{-1} M_D^T$$

S, S' … Sym, A … Anti Sym

$\underset{10}{S}$, $\underset{126}{S'}$, $\underset{120}{A}$ ← Higgs dim.

The approach to the mystery

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The texture can predict precisely or order of magnitude.

- The texture can predict precisely or order of magnitude.
  - One of Froggatt-Nielsen mech. is written as follows

$$M_u = \begin{pmatrix} a_u\lambda^6 & b_u\lambda^5 & c_u\lambda^3 \\ d_u\lambda^5 & e_u\lambda^4 & f_u\lambda^2 \\ g_u\lambda^3 & h_u\lambda^2 & 1 \end{pmatrix},\ M_d = \begin{pmatrix} a_d\lambda^4 & b_d\lambda^3 & c_d\lambda^3 \\ d_d\lambda^3 & e_d\lambda^2 & f_d\lambda^2 \\ g_d\lambda & h_d & 1 \end{pmatrix} = M_e^T,\ M_\nu = \frac{1}{\Lambda_R}\begin{pmatrix} a_\nu\lambda^6 & b_\nu\lambda^5 & c_\nu\lambda^3 \\ b_\nu\lambda^5 & e_\nu\lambda^4 & f_\nu\lambda^2 \\ c_\nu\lambda^3 & f_\nu\lambda^2 & 1 \end{pmatrix},$$

Bando-Kugo-Yoshioka (2000)

  - Here, λ denotes a number of the order of the Cabibbo angle $\sin\theta_C$ = 0.22, and all the coefficients, $a_f, \cdots, h_f$, are assumed to be order 1.
  - Because we cannot determine the exact values of $a_f, \cdots, h_f$ by the theory, this model predict order of magnitude only.

$$m_u : m_c : m_t \sim m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \lambda^6 : \lambda^4 : 1,$$
$$m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \lambda^4 : \lambda^2 : 1$$

$$U_u \sim U_d \sim U_e \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},\quad U_e \sim \begin{pmatrix} 1 & \lambda & \lambda \\ \lambda & 1/\sqrt{2} & 1/\sqrt{2} \\ \lambda & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

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The texture is taken GUT into consideration or not.

- The texture is taken GUT into consideration or not.
  - For example, Koide's relation come from the S_3 invariant Higgs potential at low-energy scale. (Koide, 1990)

$$\frac{2}{3}\,\frac{\left\{\sqrt{m_e(m_e)}+\sqrt{m_\mu(m_\mu)}+\sqrt{m_\tau(m_\tau)}\right\}^2}{m_e(m_e)+m_\mu(m_\mu)+m_\tau(m_\tau)}=1-0{,}000002$$

  - Koide's relation becomes misaligned at GUT scale.
    And this relation can not be applied to Quark masses.

$$\frac{2}{3}\,\frac{\left\{\sqrt{m_u}+\sqrt{m_c}+\sqrt{m_t}\right\}^2}{m_u+m_c+m_t}=1-0{,}26\,,\quad \frac{2}{3}\,\frac{\left\{\sqrt{m_d}+\sqrt{m_s}+\sqrt{m_b}\right\}^2}{m_d+m_s+m_b}=1-0{,}067$$

$$\frac{2}{3}\,\frac{\left\{\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau}\right\}^2}{m_e+m_\mu+m_\tau}=1-0{,}0026 \qquad \text{for} \quad \mu=2\times10^{16}$$

Li and Ma (2006)

Summary

- There are many mysteries and models in the mass matrices!
  I cannot review all within a short time.
  - Froggatt-Nielsen mech.
    C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B 147 (1979) 277., …
  - Nearest-Neighbor Interactions
    H. Fritzsch, Phys. Lett. 73B, 317 (1978), …
  - Democratic type, (S3 discrete sym.)
    H. Harari, H. Haut and J. Weyers, Phys. Lett. 78B, 459 (1978), …
  - GUT
    H. Georgi and S.L. Glashow, PRD32, 438 (1974), …
  - Zero texture
    F. Wilczek, A. Zee, Phys.Lett.B70:418(1977),
    H. Fritzsch, Phys.Lett.B70:436,1977., …
  - Discrete symmetry
    There are many models such as S_n, D_n, A_n …
  - Etc…
- I am also looking forward to listen and discuss the many exciting models in this workshop.

Koide's relations

- The texture of mass matrix may be classified as follows.
  - The texture is taken GUT into consideration or not.
  - For example, Koide's relation come from the S_3 invariant Higgs potential. (Koide, 1990)

$$V = \mu^2 \sum \bar{\Phi}_i \phi_i + \frac{\lambda_1}{2} \{ \sum \bar{\Phi}_i \phi_i \}^2 + \lambda_2 (\bar{\Phi}_\sigma \phi_\sigma)(\bar{\Phi}_\pi \phi_\pi + \bar{\Phi}_\eta \phi_\eta)$$

$$\phi_\pi = \frac{1}{\sqrt{2}} \{ \phi_1 - \phi_2 \}, \quad \phi_\eta = \frac{1}{\sqrt{6}} \{ \phi_1 + \phi_2 - 2\phi_3 \}, \quad \phi_\sigma = \frac{1}{\sqrt{3}} \{ \phi_1 + \phi_2 + \phi_3 \}$$

  - This potential become minimum at

$$\bar{\Phi}_\sigma \phi_\sigma = \bar{\Phi}_\pi \phi_\pi + \bar{\Phi}_\eta \phi_\eta = \frac{-\mu^2}{2\lambda_1 + \lambda_2}$$

So, we get

$$\sum \bar{\Phi}_i \phi_i = \bar{\Phi}_\sigma \phi_\sigma + \bar{\Phi}_\pi \phi_\pi + \bar{\Phi}_\eta \phi_\eta$$

$$= \bar{\Phi}_1 \phi_1 + \bar{\Phi}_2 \phi_2 + \bar{\Phi}_3 \phi_3 = \{ v_1^2 + v_2^2 + v_3^2 \}$$

$$= 2 \bar{\Phi}_\sigma \phi_\sigma = \frac{2}{3} \overline{\{ \phi_1 + \phi_2 + \phi_3 \}} \{ \phi_1 + \phi_2 + \phi_3 \} = \frac{2}{3} (v_1 + v_2 + v_3)^2$$

Here, $\langle \phi_1 \rangle = v_1$, $\langle \phi_2 \rangle = v_2$, $\langle \phi_3 \rangle = v_3$

$$V = \mu^2 \sum \bar{\Phi}_i \phi_i + \frac{\lambda_1}{2}\left\{\sum \bar{\Phi}_i \phi_i\right\}^2 + \lambda_2 (\bar{\Phi}_\sigma \phi_\sigma)(\bar{\Phi}_\pi \phi_\pi + \bar{\Phi}_\eta \phi_\eta)$$

$$\phi_\pi = \frac{1}{\sqrt{2}}\{\phi_1 - \phi_2\},\quad \phi_\eta = \frac{1}{\sqrt{6}}\{\phi_1 + \phi_2 - 2\phi_3\},\quad \phi_\sigma = \frac{1}{\sqrt{3}}\{\phi_1 + \phi_2 + \phi_3\}$$

$$\bar{\Phi}_\sigma \phi_\sigma = \bar{\Phi}_\pi \phi_\pi + \bar{\Phi}_\eta \phi_\eta = \frac{-\mu^2}{2\lambda_1 + \lambda_2}$$

$$\sum \bar{\Phi}_i \phi_i = \bar{\Phi}_\sigma \phi_\sigma + \bar{\Phi}_\pi \phi_\pi + \bar{\Phi}_\eta \phi_\eta$$

$$= \bar{\Phi}_1 \phi_1 + \bar{\Phi}_2 \phi_2 + \bar{\Phi}_3 \phi_3 = \{v_1^2 + v_2^2 + v_3^2\}$$

$$= 2\bar{\Phi}_\sigma \phi_\sigma = \frac{2}{3}\{\overline{\phi_1 + \phi_2 + \phi_3}\}\{\phi_1 + \phi_2 + \phi_3\} = \frac{2}{3}(v_1 + v_2 + v_3)^2$$

Here, $\langle \phi_1 \rangle = v_1$, $\langle \phi_2 \rangle = v_2$, $\langle \phi_3 \rangle = v_3$

Our Strategy for the Four-Zero-Texture (FZT) mass matrix model

Many people have discussed the FZT mass matrix model;

H. Nishiura, K. M, and T. Fukuyama, PRD **60**,013006 (1999).

Four-zero-texture (FZT) mass matrix model

- We would like to discuss the following Four-Zero-Texture mass matrix model.

$$M_f = \begin{pmatrix} 0 & a_f e^{+i\tau_f} & 0 \\ a_f e^{-i\tau_f} & b_f & c_f e^{+i\sigma_f} \\ 0 & c_f e^{-i\sigma_f} & d_f \end{pmatrix}$$

for $f = u, d, e, D, L, R$

- If we assume the Hermite matrix, we get

$$a_f = \sqrt{-\frac{m_{f1} m_{f2} m_{f3}}{d_f}}, \quad c_f = \sqrt{-\frac{(d_f - m_{f1})(d_f - m_{f2})(d_f - m_{f3})}{d_f}}$$

$$b_f = (m_{f1} + m_{f2} + m_{f3}) - d_f$$

Here $0 < m_{f1} < -m_{f2} < m_{f3}$ for $|m_{f1}| < d_f < |m_{f2}|$

$0 < -m_{f1} < m_{f2} < m_{f3}$ for $|m_{f2}| < d_f < |m_{f3}|$

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And they applied the FZT mass matrix model to SO(10)GUT;
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modify

Recently, some people say "the FZT in quark sector is dying";

We check this opinion.
K. M and H. Nishiura, PRD 74, 033014 (2006)

the solutions is remained

Apply the results

Recently, some people say "the some FZT in the SO(10) GUT is dying";

We check the our model.
hep-ph/0702284

Is there solutions?

General SO(10)

- Each SM family + a right-handed neutrino in a single 16-dim rep.

$$W^{Y}_{SO(10)} = Y^{10}_{ij}\, 16_i 16_j 10_H + Y^{120}_{ij}\, 16_i 16_j 120_H + Y^{126}_{ij}\, 16_i 16_j 126_H$$

  Here the matrices Y^{10}, Y^{126} are symmetric, and Y^{120} is anti-symmetric.

- Each terms include the following mass terms, respectively.

$$16\,16\,10 \supset 5(uu^c + \nu\nu^c) + \bar{5}(dd^c + ee^c)$$
$$16\,16\,120 \supset 5\,\nu\nu^c + 45\,uu^c + \bar{5}(dd^c + ee^c) + \overline{45}(dd^c - 3ee^c)$$
$$16\,16\,126 \supset 1\,\nu\nu^c + 15\,\nu\nu + 5(uu^c - 3\nu\nu^c) + \overline{45}(dd^c - 3ee^c)$$

The relations from the SO (10) GUT

- The resulting tree level mass matrices as follows

$$
\begin{aligned}
M_u &= S + \delta' A + \varepsilon S' \\
M_d &= \alpha S + \delta A + S' \\
M_e &= \alpha S + A - 3 S' \\
M_D &= S + \delta'' A - 3\varepsilon S' \\
M_L &= \beta S' \\
M_R &= \gamma S'
\end{aligned}
$$

$$
\begin{array}{l}
\\ \text{Higgs}:
\end{array}
\underbrace{\begin{array}{ccc}
\uparrow & \uparrow & \uparrow \\
10 & 120 & 126 \\
\text{sym} & \text{anti-sym} & \text{sym}
\end{array}}_{\text{from } SO(10)\text{ GUT}}
$$

This unification gives more stringent constraints than the case where only FZT is considered.

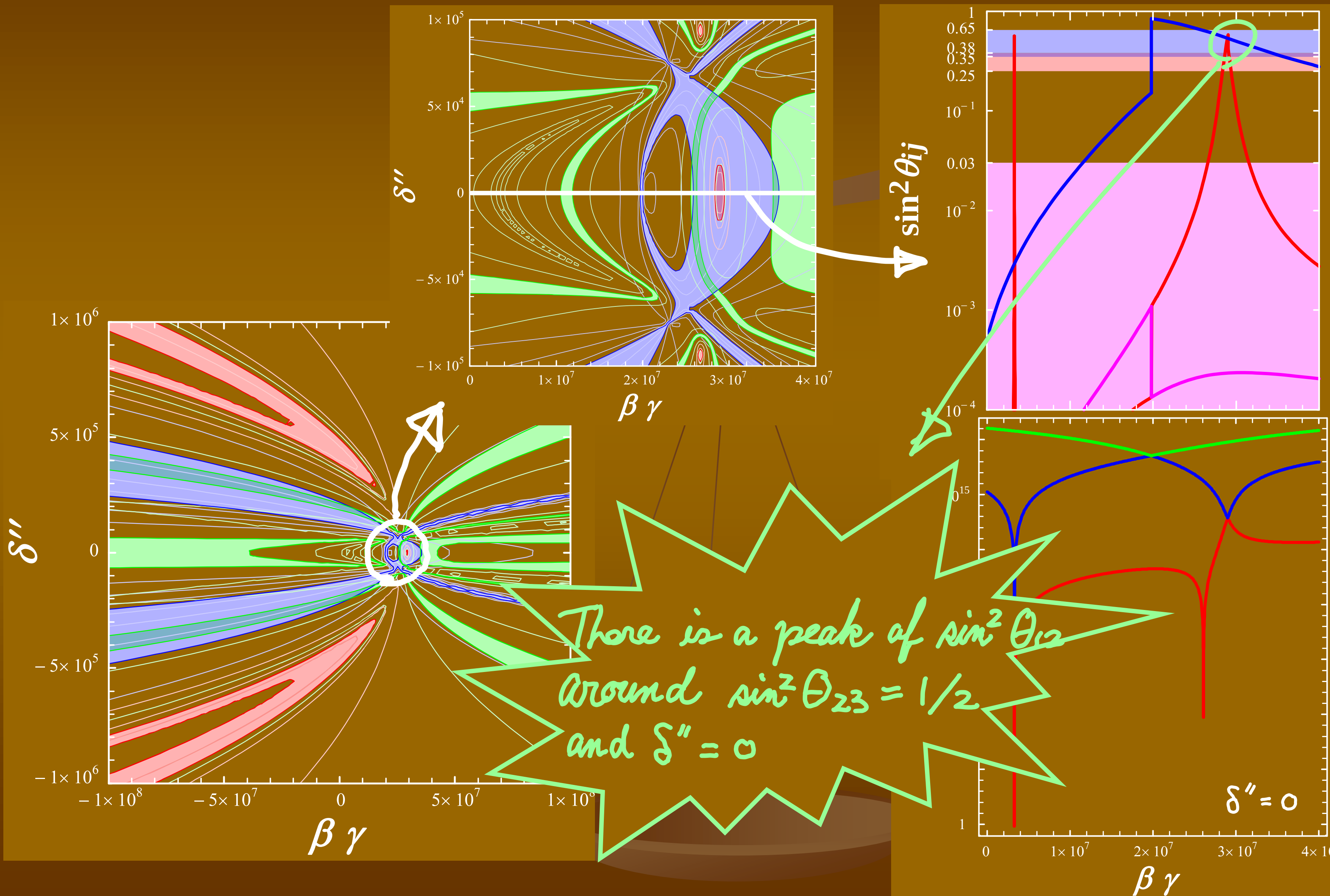
The numerical results

- We show the best fit values as a example

$\Delta\tau = \pi/2,\ \Delta\sigma = -0.121,\ x_u = 0.9560,\ x_d = 0.9477$

	Our results	The values estimated from exp data in MSSM ($\tan\beta=10$)
$\lvert m_u(M_x)\rvert$	$= 1.04$ [MeV]	$1.04^{+0.19}_{-0.20}$ [MeV]
$m_c(M_x)$	$= 302$ [MeV]	302^{+25}_{-27} [MeV]
$m_t(M_x)$	$= 129$ [GeV]	129^{+196}_{-40} [GeV]
$\lvert m_d(M_x)\rvert$	$= 1.33$ [MeV]	$1.33^{+0.17}_{-0.19}$ [MeV]
$m_s(M_x)$	$= 26.5$ [MeV]	$26.5^{+3.3}_{-3.7}$ [MeV]
$m_b(M_x)$	$= 1.00$ [GeV]	1.00 ± 0.04 [GeV]
$\lvert (U_{CKM})_{12}\rvert$	$= 0.2251$	$0.2226 - 0.2259$
$\lvert (U_{CKM})_{12}\rvert$	$= 0.0340$	$0.0295 - 0.0387$
$\lvert (U_{CKM})_{12}\rvert$	$= 0.0032$	$0.0024 - 0.0038$
δ_ε	$= 58.86$	$46^\circ - 74^\circ$

- The allowed regions of neutrino masses and mixing angles in the case of the normal hierarchy.



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We wonder what is behind this hidden property.

Recently, some people say "the FZT in quark sector is dying";

modify

the solutions is remained

We check this opinion.
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Apply the results

Recently, some people say "the some FZT in the SO(10) GUT is dying";

We check the our model.
hep-ph/0702284

There is the hidden property in FZT which makes the peak of $\sin^2\theta_{12}$ around $\sin^2\theta_{23}$=1/2.

We find the solutions in the SO(10) GUT.

The remaining problems in the FZT with SO(10) GUT

- $\Delta m_{12}{}^2$ is small. We need the tiny perturbation term in neutrino mass matrix.
- We don't know the reasons or the mechanisms which always make the peak of $\sin^2\theta_{12}$ around $\sin^2\theta_{23}=1/2$ under some conditions.
- We don't know where the FZT come from.

The relations of quark and charged leptons in the SO (10) GUT

- First, we only discuss the mass matrices of up, down and charged lepton.

$$M_u = S + \delta' A + \varepsilon S'$$

$$M_d = \alpha S + \delta A + S'$$

$$M_e = \alpha S + A - 3 S'$$

Here, we define $r \equiv \delta'/\delta$ for the sake of later discussion.

The number of parameters in SO(10) GUT

- The number of parameters in our model.

$$S, A, S' \Rightarrow 4+2+4=10$$

$$M_f = (S_f + A_f) \Rightarrow 6\times 3 = 18$$

$$+)\ \alpha, \delta, \delta', \varepsilon \Rightarrow \quad = 4$$

$$N(pmt) \quad = 32$$

- The number of constraints from equations.

$$N(eqs) = 6\times 3 \quad = 18$$

- The number of constraints from experiments.

$$\text{masses} \Rightarrow 3\times 3 = 9$$

$$+)\ \text{CKM} \Rightarrow 3+1 = 4$$

$$N(exp) \quad = 13$$

- The number of free parameters in our model.

$$N(free) = N(pmt) - N(eqs) - N(exp) = 1$$

- After summarizing these eqs., two parameters d_e and r remain as free parameters in one equation.

$$F(r)^2\left[4\alpha a_u\cos(\Delta\tau+\tau_d)-(3+K)a_d\cos\tau_d\right]^2-\left[4\alpha c_u\cos(\Delta\sigma+\sigma_d)-(3+K)c_d\cos\sigma_d\right]^2$$
$$=(1-K)^2\left[a_e^2F(r)^2-c_e^2\right]$$

where $\Delta\tau\equiv\tau_u-\tau_d$, $\Delta\sigma\equiv\sigma_u-\sigma_d$

The parameters $d_u, d_d, \Delta\tau, \Delta\sigma$ are fixed by the CKM angles and phase in former discussion.

$$\alpha(d_u,d_d,d_e)=\frac{(3d_d+d_e)(\Sigma_d-\Sigma_e)-(d_d-d_e)(3\Sigma_d+\Sigma_e)}{4\{d_u(\Sigma_d-\Sigma_e)-(d_d-d_e)\Sigma_u\}}$$

$$K(d_u,d_d,d_e)=-\frac{(3d_d+d_e)\Sigma_u-d_u(3\Sigma_d+\Sigma_e)}{(d_d-d_e)\Sigma_u-d_u(\Sigma_d-\Sigma_e)}$$

where $\Sigma_u\equiv m_u+m_c+m_t$, $\Sigma_d\equiv m_d+m_s+m_b$, $\Sigma_e\equiv m_e+m_\mu+m_\tau$

$$a_f(d_f)=\sqrt{-\frac{m_{f1}m_{f2}m_{f3}}{d_f}},\quad c_f(d_f)=\sqrt{-\frac{(d_f-m_{f1})(d_f-m_{f2})(d_f-m_{f3})}{d_f}}\quad \text{for } f=u,d,e$$

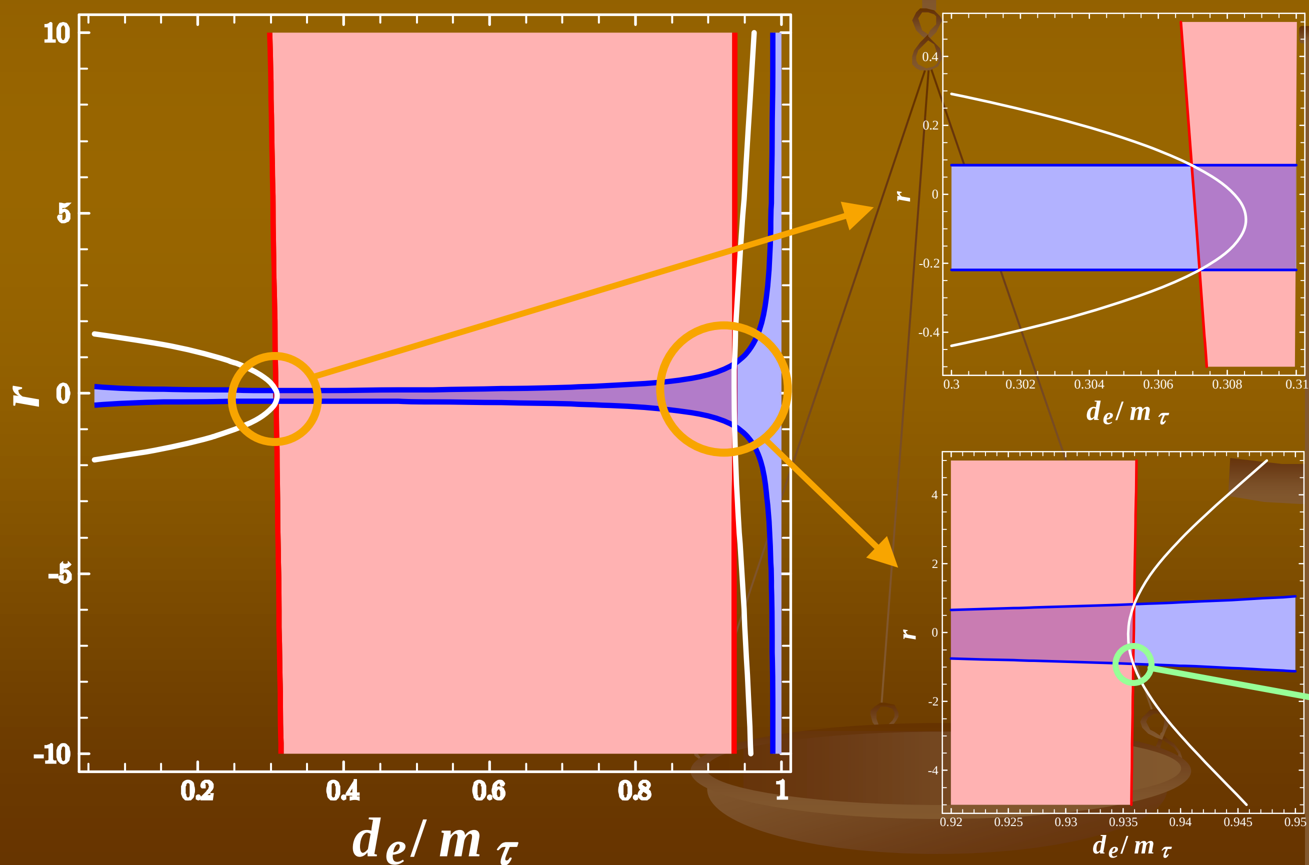
$$\tan\tau_d(r,d_u,d_d,\Delta\tau)=\frac{a_u\sin\Delta\tau}{r a_d-a_u\cos\Delta\tau},\quad \tan\sigma_d(r,d_u,d_d,\Delta\sigma)=\frac{c_u\sin\Delta\sigma}{r c_d-c_u\cos\Delta\sigma}$$

$$F(r,d_d)=\frac{c_d\sin\sigma_d}{a_d\sin\tau_d}$$

- The additional conditions which come from the phases in the charged lepton mass matrix.

$$-1 \le \cos\tau_e(d_e, r) = \frac{4\alpha\, a_u \cos(\Delta\tau + \tau_d) - (3+K)\, a_d \cos\tau_d}{(1-K)\, a_e} \le +1 \rightarrow$$

$$-1 \le \cos\sigma_e(d_e, r) = \frac{4\alpha\, C_u \cos(\Delta\sigma + \sigma_d) - (3+K)\, C_d \cos\sigma_d}{(1-K)\, C_e} \le +1 \rightarrow$$



Sol. (a)

Sol. (b)

In this talk, I will concentrate at this point. Namely, the contribution of 120 is small.

The neutrino mass matrix predicted from FZT in the SO (10) GUT

- As we have shown, the quark and charged lepton parts are OK.
- Next, Let's discuss the neutrino mass matrix.

$$M_u = S + \delta' A + \varepsilon S'$$
$$M_d = \alpha S + \delta A + S'$$
$$M_e = \alpha S + A - 3 S'$$

OK!
&
All parameters are determined by the former discussion

$$M_D = S + \delta'' A - 3 \varepsilon S'$$
$$M_L = \beta S'$$
$$M_R = \gamma S'$$
$$m_\nu = M_L - M_D M_R^{-1} M_D^T$$

There are three free pmt in the ν part.

The neutrino mass matrix predicted from FZT in the SO (10) GUT

- As we have shown, the quark and charged lepton parts are OK.
- Next, Let's discuss the neutrino mass matrix.
- We use the following global analysis of neutrino experiments.

$$0.25 < \sin^2\theta_{12} < 0.38$$

$$0.35 < \sin^2\theta_{23} < 0.65$$

$$\sin^2\theta_{13} < 0.03$$

$$\begin{cases} \Delta m^2_{21} = (7.2 - 8.9)\times 10^{-5}\ \mathrm{eV}^2 \\ |\Delta m^2_{32}| = (2.1 - 3.1)\times 10^{-3}\ \mathrm{eV}^2 \end{cases}$$

$$\to \frac{\Delta m^2_{21}}{|\Delta m^2_{32}|} = (2.3 - 4.2)\times 10^{-2}$$

at 99% CL

A. Strumia, F. Vissani, hep-ph/0606054

Summary of our FZT mass matrix model

- Many people have discussed the FZT mass matrix model;
  H. Nishiura, K. M, and T. Fukuyama, PRD **60**,013006 (1999).

- And they applied the FZT mass matrix model to SO(10)GUT;
  K. M, T. Fukuyama, and H. Nishiura, PRD **61**, 053001 (2000).

Recently, some people say "the FZT in quark sector is dying";

modified

the solutions is remained

We check this opinion.
K. M and H. Nishiura, PRD **74,** 033014 (2006)

Apply the results

Recently, some people say "the some FZT in the SO(10) GUT is dying";

We will check the our model.
hep-ph/0702284

There is the hidden property in FZT which always makes the peak of $\sin^2\theta_{12}$ around $\sin^2\theta_{23}=1/2$.

We find the solutions in the SO(10) GUT.

Sol. (b)

$$M_e = \alpha S + A - 3S' = \begin{pmatrix} 0 & 4.8-5.7\times10^{-2}i & 0 \\ 4.8+5.7\times10^{-2}i & 1.4\times10^{2} & 2.8\times10^{2}+2.6\times10^{-1}i \\ 0 & 2.8\times10^{2}-2.6\times10^{-1}i & 1.1\times10^{3} \end{pmatrix}$$

$$M_D = S + \underset{\underset{0}{\shortparallel}}{\delta''} A - 3\varepsilon S' = \begin{pmatrix} 0 & 1.4\times10^{3} & 0 \\ 1.4\times10^{3} & 3.5\times10^{4} & 5.3\times10^{4} \\ 0 & 5.3\times10^{4} & 1.9\times10^{5} \end{pmatrix}$$

$$M_L = \beta S', \quad M_R = \gamma S' = \gamma \begin{pmatrix} 0 & -7.7\times10^{-1} & 0 \\ -7.7\times10^{-1} & -1.6\times10 & -1.4\times10 \\ 0 & -1.5\times10 & -3.7\times10 \end{pmatrix}$$

$$m_\nu = M_L - M_D M_R^{-1} M_D = \frac{1}{\gamma}\left[\underbrace{\beta\gamma}_{= 2.85\times10^{7}} S' - M_D (S')^{-1} M_D^{T} \right]$$

$$= \frac{1}{\gamma} \begin{pmatrix} 0 & -2.0\times10^{7} & 0 \\ -2.0\times10^{7} & -3.9\times10^{8} & -2.1\times10^{8} \\ 0 & -2.1\times10^{8} & -1.2\times10^{8} \end{pmatrix}$$

$\sin^2\theta_{23} = 0.53$, $\sin^2\theta_{13} = 2.3\times10^{-4}$

$\sin^2\theta_{12} = 0.29$, $\Delta m_{12}^2/\Delta m_{13}^2 = \underline{4.4\times10^{-4}}$

It is too small.

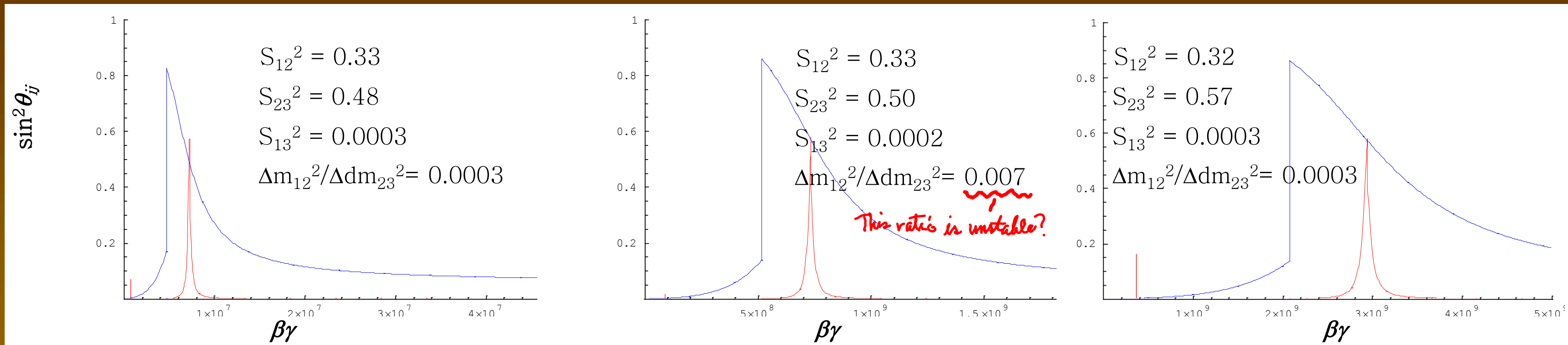
The uncertainty from the RG effects

- We don't know how much the true masses of fermions are in the GUT scale because nobody know what is really happened above TeV scale.
- However, in the most case, the ratio m_{f1}/m_{f2} is stable.
- Therefore, by changing only m_{f3} in the very wide range even where it is unreasonable value theoretically, we will check the stability of the bi-large mixing.

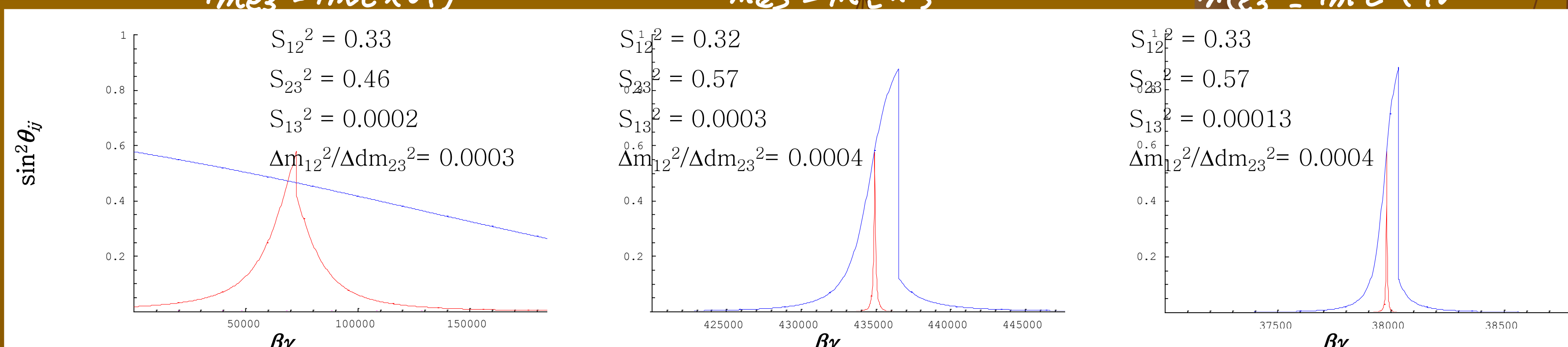
- I regret that the discussions about the uncertainty from the CKM matrix and so on are not finished.

- There is always the peak of $\sin^2\theta_{12}$ around $\sin^2\theta_{12}=1/2$, even if m_{u3} and m_{e3} are changed.

$m_{u3} = m_t \times 0.5$ $\quad$ $m_{u3} = m_t \times 5$ $\quad$ $m_{u3} = m_t \times 10$



$m_{e3} = m_\tau \times 0.5$ $\quad$ $m_{e3} = m_\tau \times 5$ $\quad$ $m_{e3} = m_\tau \times 10$



- The rates of change of each elements with respect to m_{e3} are incoherent as follows.

$$\frac{S_{ij}(m_{e3}=0.5\,m_\tau)}{S_{ij}(m_{e3}=10\,m_\tau)} =$$

Indeterminate	-10.7516	Indete
-10.7516	-5.03675	-3.1
Indeterminate	-3.10112	-2.5

$$\frac{S'_{ij}(m_{e3}=0.5\,m_\tau)}{S'_{ij}(m_{e3}=10\,m_\tau)} =$$

Indeterminate	1.17063	Indete
1.17063	0.0894184	-0.02
Indeterminate	-0.0262295	-0.04

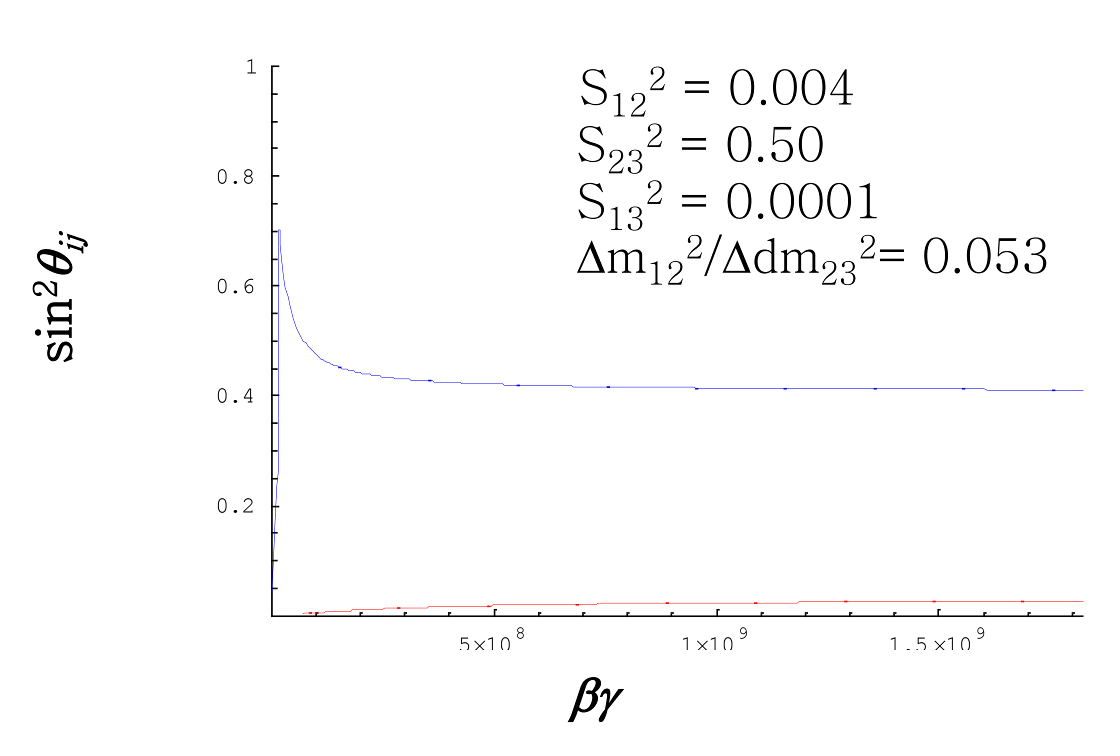
$$\frac{A_{ij}(m_{e3}=0.5\,m_\tau)}{A_{ij}(m_{e3}=10\,m_\tau)} =$$

Indeterminate	1.22501	Indete
1.22501	Indeterminate	1.2
Indeterminate	1.20363	Indete

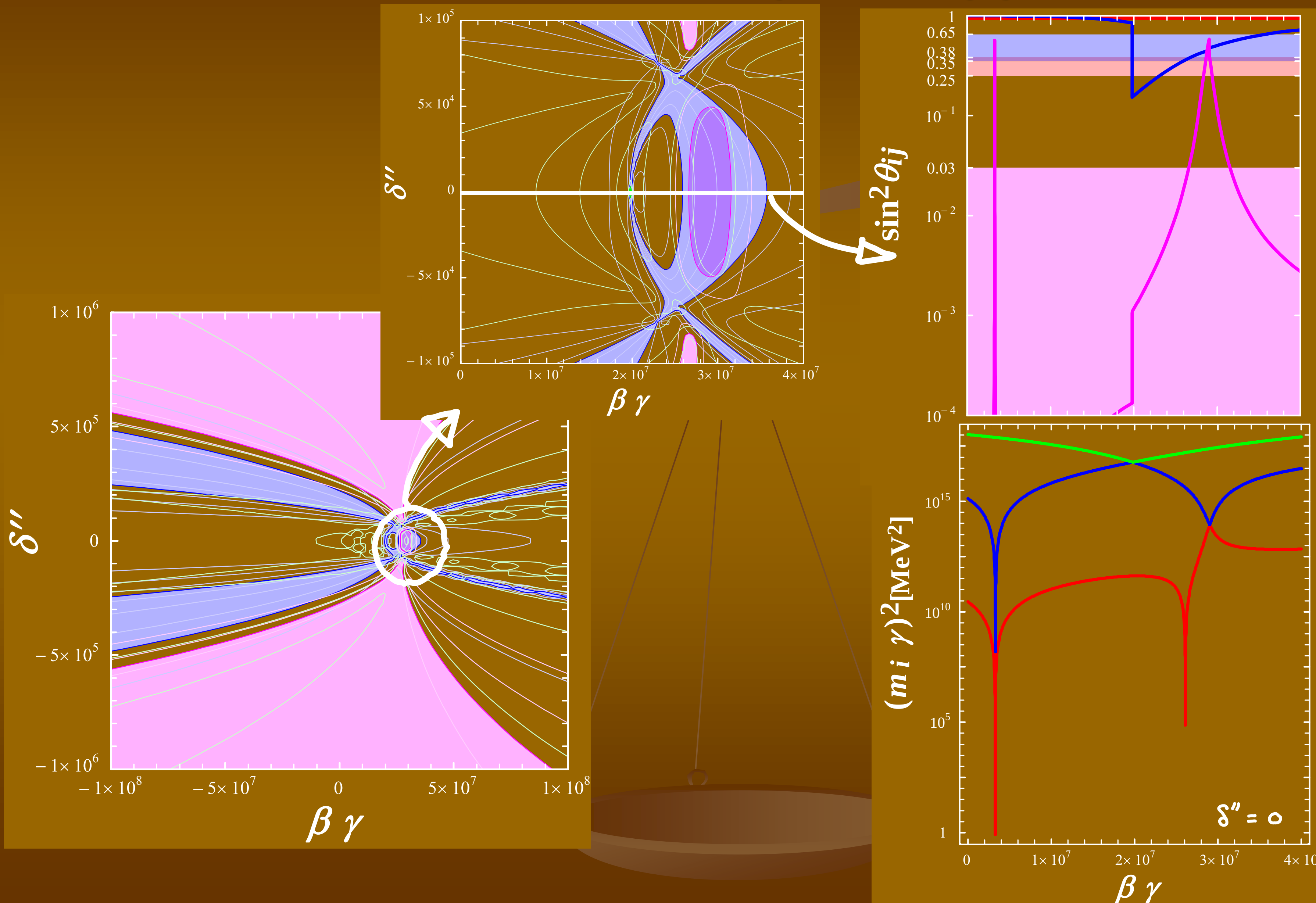
Because the CKM matrix is sensitive to m_{d3},
we must check more carefully when we change m_{d3}.
However I have no time to check….

- Note that we can not take the bi-large mixing for granted in the general FZT.
- If you change other masses, the bi-large mixing is sometimes forbidden. For example, if muon was more heavy, the large $\sin^2\theta_{12}$ can not be derived.

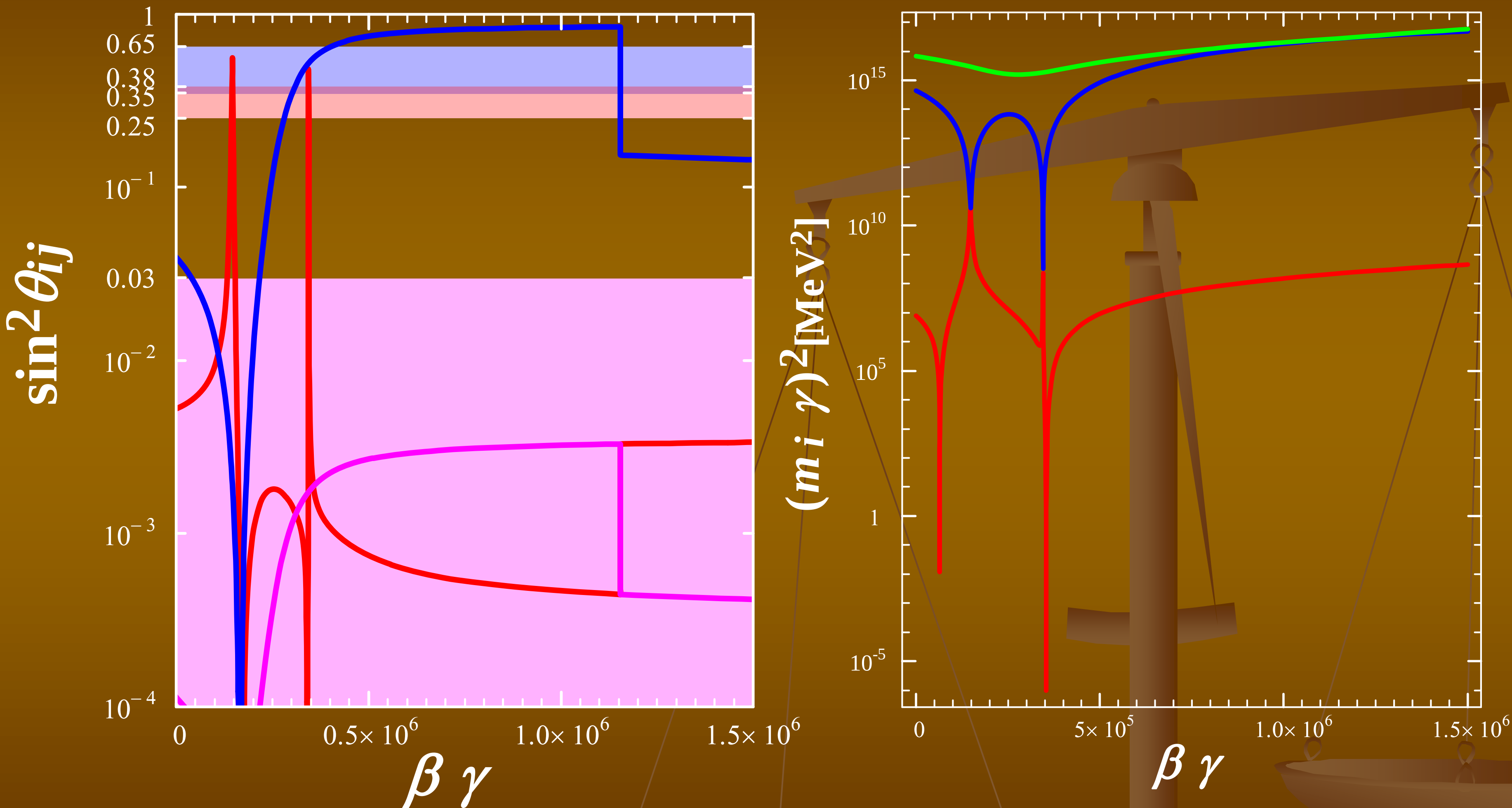
$m_{e2} = m_\mu \times 3$



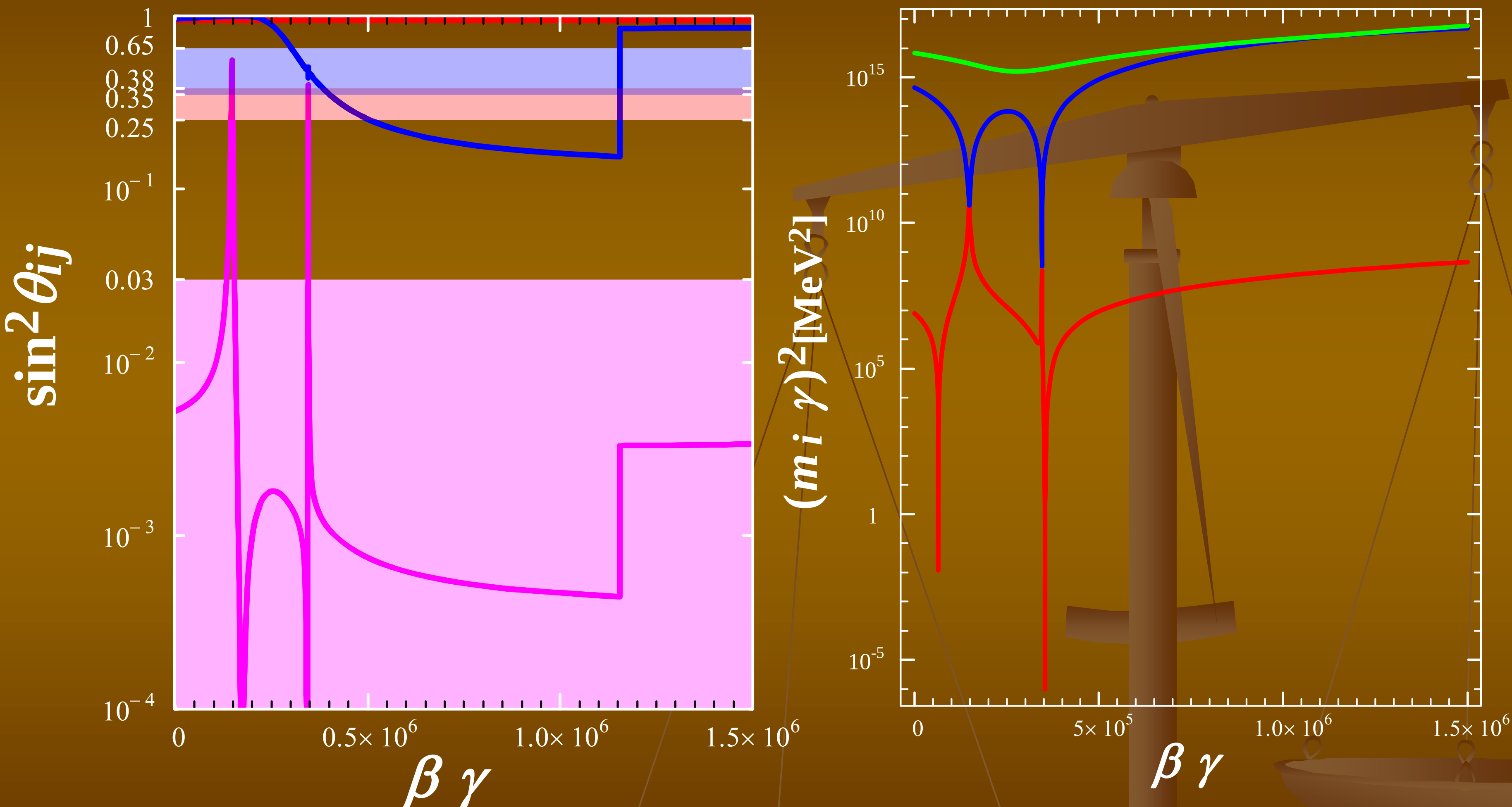
- The allowed regions of neutrino masses and mixing angles in the case of the inverse hierarchy at Sol. (b)



- The allowed regions of neutrino masses and mixing angles in the case of the normal hierarchy at Sol. (a)



- The allowed regions of neutrino masses and mixing angles in the case of the inverse hierarchy at Sol. (a)



By substituting "$s=\frac{3s_d+s_e}{4\alpha}$ and $s'=\frac{s_d-s_e}{4}$"

in the GUT relations, these relation is given

$$4\alpha s_u = (3s_d+s_e)+K(s_d-s_e)$$

where $K \equiv \alpha\varepsilon$.

$$4\alpha\, d_u = [\,3d_d+d_e\,]+K[\,d_d-d_e\,]$$

$$4\alpha\,\Sigma_u = [3\Sigma_d+\Sigma_e]+K[\Sigma_d-\Sigma_e]$$

where $\begin{cases} \Sigma_u \equiv m_u+m_c+m_t \\ \Sigma_d \equiv m_d+m_s+m_b \\ \Sigma_e \equiv m_e+m_\mu+m_\tau \end{cases}$

$$\alpha(d_u, d_d, d_e) = \frac{(3d_d+d_e)(\Sigma_d-\Sigma_e)-(d_d-d_e)(3\Sigma_d+\Sigma_e)}{4\{d_u(\Sigma_d-\Sigma_e)-(d_d-d_e)\Sigma_u\}}$$

$$K(d_u, d_d, d_e) = -\frac{(3d_d+d_e)\Sigma_u - d_u(3\Sigma_d+\Sigma_e)}{(d_d-d_e)\Sigma_u-d_u(\Sigma_d-\Sigma_e)}$$

α , Kは d_u, d_d. d_e の関数

From $\frac{1}{\delta}A_d = \frac{1}{\delta'}A_u$, we get

$$\frac{1}{\delta} a_d \sin\tau_d = \frac{1}{\delta'} a_u \sin(\tau_d + \Delta\tau)$$

$$= \frac{1}{\delta'} a_u \left[\sin\tau_d \cos\Delta\tau + \cos\tau_d \sin\Delta\tau\right]$$

$$\frac{1}{\delta} c_d \sin\sigma_d = \frac{1}{\delta'} c_u \sin(\sigma_d + \Delta\sigma)$$

$$= \frac{1}{\delta'} c_u \left[\sin\sigma_d \cos\Delta\sigma + \cos\sigma_d \sin\Delta\sigma\right]$$

Here, $a_f = \sqrt{-\frac{m_{f1}\, m_{f2}\, m_{f3}}{d_f}}$, $c_f = \sqrt{-\frac{(d_f - m_{f1})(d_f - m_{f2})(d_f - m_{f3})}{d_f}}$

$$\begin{cases} \tan\tau_d(r, d_u, d_d) = \dfrac{\frac{1}{\delta'} a_u \sin\Delta\tau}{\frac{1}{\delta} a_d - \frac{1}{\delta'} a_u \cos\Delta\tau} = \dfrac{a_u \sin\Delta\tau}{r\, a_d - a_u \cos\Delta\tau} \\[2ex] \tan\sigma_d(r, d_u, d_d) = \dfrac{\frac{1}{\delta'} c_u \sin\Delta\sigma}{\frac{1}{\delta} c_d - \frac{1}{\delta'} c_u \cos\Delta\sigma} = \dfrac{c_u \sin\Delta\sigma}{r\, c_d - c_u \cos\Delta\sigma} \end{cases}$$

where, $r \equiv \delta'/\delta$

残る方程式は

$$
\left.
\begin{aligned}
a_e \sin\tau_e &= \frac{1}{\delta} a_d \sin\tau_d \\
c_e \sin\sigma_e &= \frac{1}{\delta} c_d \sin\sigma_d \\
4\alpha a_u \cos(\Delta\tau+\tau_d) &= (3+K) a_d \cos\tau_d + (1-K) a_e \cos\tau_e \\
4\alpha c_u \cos(\Delta\sigma+\sigma_d) &= (3+K) c_d \cos\sigma_d + (1-K) c_e \cos\sigma_e
\end{aligned}
\right\} \text{\# of Eqs} = 4
$$

$d_u, d_d, \Delta\tau \equiv \tau_u-\tau_d, \Delta\sigma \equiv \sigma_u-\sigma_d$ はCKM決定の自由度に使われる

上の方程式を使って残るのは $\underbrace{d_e, \tau_e, \sigma_e, \delta, r}_{5コ}$ のどれか1つ

恐らく d_e を free pmt にするのが良い

以下 d_e を input して議論 → a_f, b_f, c_f, d_f は全て決まる。定数として取り扱う

$$\delta = \frac{a_d \sin\tau_d}{a_e \sin\tau_e} = \frac{c_d \sin\sigma_d}{c_e \sin\sigma_e} \Longleftrightarrow \frac{c_e \sin\sigma_e}{a_d \sin\tau_d}\,\delta = \frac{c_e \sin\sigma_e}{a_e \sin\tau_e} = \frac{c_d \sin\sigma_d}{a_d \sin\tau_d} \equiv F(r)$$

$$|\cos\sigma_e| \to 1 \Longleftrightarrow \delta \to \infty \Longleftrightarrow |\cos\tau_e| \to 1$$

$$\begin{cases} \sin\tau_e(r,\sigma_e) = \dfrac{c_e \sin\sigma_e}{a_e\, F(r)} \Rightarrow \left[a_e \cos\tau_e\right]^2 = a_e^2 - \left(\dfrac{c_e \sin\sigma_e}{F(r)}\right)^2 \\ \delta(r,\sigma_e) = \dfrac{1}{c_e \sin\sigma_e}\, a_d \sin\tau_d\, F(r) \equiv \dfrac{1}{c_e \sin\sigma_e}\, G(r) \end{cases}$$

残るpmtは r と $\sin\sigma_e$ のみ！

$$
\begin{cases}
4\alpha a_u \cos(\Delta\tau+\tau_d) = (3+K)a_d\cos\tau_d + (1-K)a_e\cos\tau_e \\
4\alpha c_u \cos(\Delta\sigma+\sigma_d) = (3+K)c_d\cos\sigma_d + (1-K)c_e\cos\sigma_e
\end{cases}
$$

$$
\begin{cases}
\left[4\alpha a_u \cos(\Delta\tau+\tau_d) - (3+K)a_d\cos\tau_d\right]^2 = (1-K)^2\left[a_e^2 - \left(\dfrac{c_e\sin\sigma_e}{F(r)}\right)^2\right] \\
\left[4\alpha c_u \cos(\Delta\sigma+\sigma_d) - (3+K)c_d\cos\sigma_d\right]^2 = (1-K)^2\left[c_e^2 - (c_e\sin\sigma_e)^2\right]
\end{cases}
$$

$$
F(r)^2\left[4\alpha a_u \cos(\Delta\tau+\tau_d) - (3+K)a_d\cos\tau_d\right]^2 - \left[4\alpha c_u \cos(\Delta\sigma+\sigma_d) - (3+K)c_d\cos\sigma_d\right]^2
$$

$= (1-K)^2\left[a_e^2 F(r)^2 - c_e^2\right]$ … d_e と r のみの方程式!

d_eを動かしつつ
この方程式を満す
rを求める

→ rが見つかる! ☺ → σ_eの決定

↓

全て決定 → ν-mass, MNS へ

to be continued

↓

The End ← rが見つからない ☹

Sol. (a)

$$S = \begin{pmatrix} 0 & 3.1\times10^{2} & 0 \\ 3.1\times10^{2} & 3.5\times10^{4} & 3.6\times10^{4} \\ 0 & 3.6\times10^{4} & 1.0\times10^{5} \end{pmatrix}$$

$$S' = \begin{pmatrix} 0 & -2.0 & 0 \\ -2.0 & -2.0\times10^{2} & -6.7\times10 \\ 0 & -6.7\times10 & 1.5\times10^{2} \end{pmatrix}, \quad A = i\begin{pmatrix} 0 & -2.5\times10^{-1} & 0 \\ 2.5\times10^{-1} & 0 & 1.1 \\ 0 & -1.1 & 0 \end{pmatrix}$$

$$\alpha = 7.9\times10^{-3}, \quad \delta = 2.3\times10, \quad \delta' = -5.2, \quad \varepsilon = 1.5\times10^{2}$$

$$M_u = \begin{pmatrix} 0 & 1.8\times10 + 1.3i & 0 \\ 1.8\times10 - 1.3i & 6.0\times10^{3} & 2.6\times10^{4} - 5.8i \\ 0 & 2.6\times10^{4} + 5.8i & 1.2\times10^{5} \end{pmatrix}$$

$$M_d = \begin{pmatrix} 0 & 4.4\times10^{-1} - 6.1i & 0 \\ 4.4\times10^{-1} + 6.1i & 7.7\times10 & 2.2\times10^{2} + 2.6\times10 i \\ 0 & 2.2\times10^{2} - 2.6\times10 i & 9.5\times10^{2} \end{pmatrix}$$

$$M_e = \begin{pmatrix} 0 & -8.5 - 2.6\times10^{-1} i & 0 \\ 8.5 + 2.6\times10^{-1} & 8.8\times10^{2} & 4.8\times10^{2} + 1.1i \\ 0 & 4.8\times10^{2} - 1.1i & 3.6\times10^{2} \end{pmatrix}$$

Sol. (b)

$$S = \begin{pmatrix} 0 & 3.6\times10^{2} & 0 \\ 3.6\times10^{2} & 1.3\times10^{4} & 3.3\times10^{4} \\ 0 & 3.3\times10^{4} & 1.4\times10^{4} \end{pmatrix}$$

$$S' = \begin{pmatrix} 0 & -7.7\times10^{-1} & 0 \\ -7.7\times10^{-1} & -1.6\times10 & -1.4\times10 \\ 0 & -1.5\times10 & -3.7\times10 \end{pmatrix}, \quad A = i\begin{pmatrix} 0 & 5.3 & 0 \\ -5.3 & 0 & -2.4\times10 \\ 0 & 2.4\times10 & 0 \end{pmatrix}$$

$$\alpha = 7.0\times10^{-3}, \quad \delta = 1.0\times10^{2}, \quad \delta' = -9.3\times10, \quad \varepsilon = 4.5\times10^{2}$$

$$M_u = \begin{pmatrix} 0 & 1.7\times10+5.3\,i & 0 \\ 1.7\times10-5.3\,i & 6.0\times10^{3} & 2.6\times10^{4}-2.4\times10\,i \\ 0 & 2.6\times10^{4}+2.4\times10\,i & 1.2\times10^{5} \end{pmatrix}$$

$$M_d = \begin{pmatrix} 0 & 1.8-5.8\,i & 0 \\ 1.8+5.8\,i & 7.7\times10 & 2.2\times10^{2}+2.6\times10\,i \\ 0 & 2.2\times10^{2}-2.6\times10\,i & 9.5\times10^{2} \end{pmatrix}$$

$$M_e = \begin{pmatrix} 0 & 4.8-5.7\times10^{-2}\,i & 0 \\ 4.8+5.7\times10^{-2}\,i & 1.4\times10^{2} & 2.8\times10^{2}+2.6\times10^{-1}\,i \\ 0 & 2.8\times10^{2}-2.6\times10^{-1}\,i & 1.1\times10^{3} \end{pmatrix}$$

The numerical results

$|m_u(M_X)| = 1.04^{+0.19}_{-0.20}$ [MeV]

$m_c(M_X) = 302^{+25}_{-27}$ [MeV]

$m_t(M_X) = 129^{+196}_{-40}$ [GeV]

$|m_d(M_X)| = 1.33^{+0.17}_{-0.19}$ [MeV]

$m_s(M_X) = 26.5^{+3.3}_{-3.7}$ [MeV]

$m_b(M_X) = 1.00 \pm 0.04$ [GeV]

$|m_e(M_X)| = 0.3250\ldots$ [MeV]

$m_\mu(M_X) = 68.598\ldots$ [MeV]

$m_\tau(M_X) = 1171.4 \pm 0.2$ [MeV]

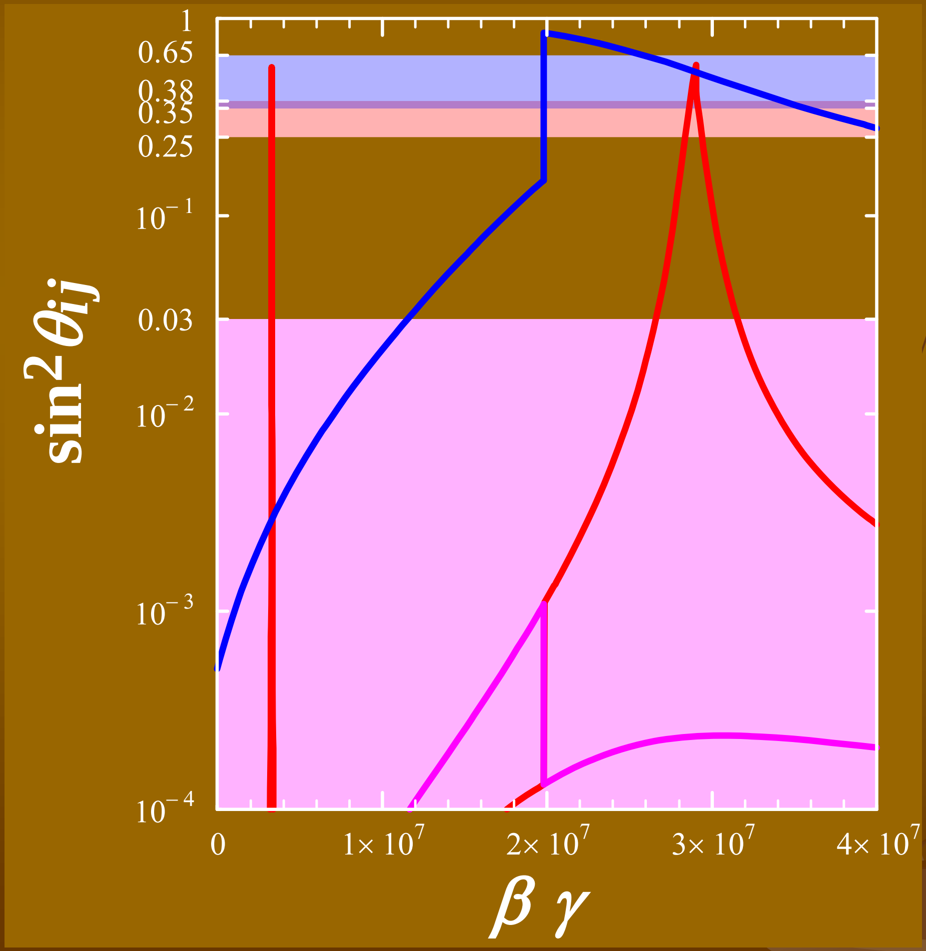
$|(U_{CKM})_{12}| = 0.2226 - 0.2259$

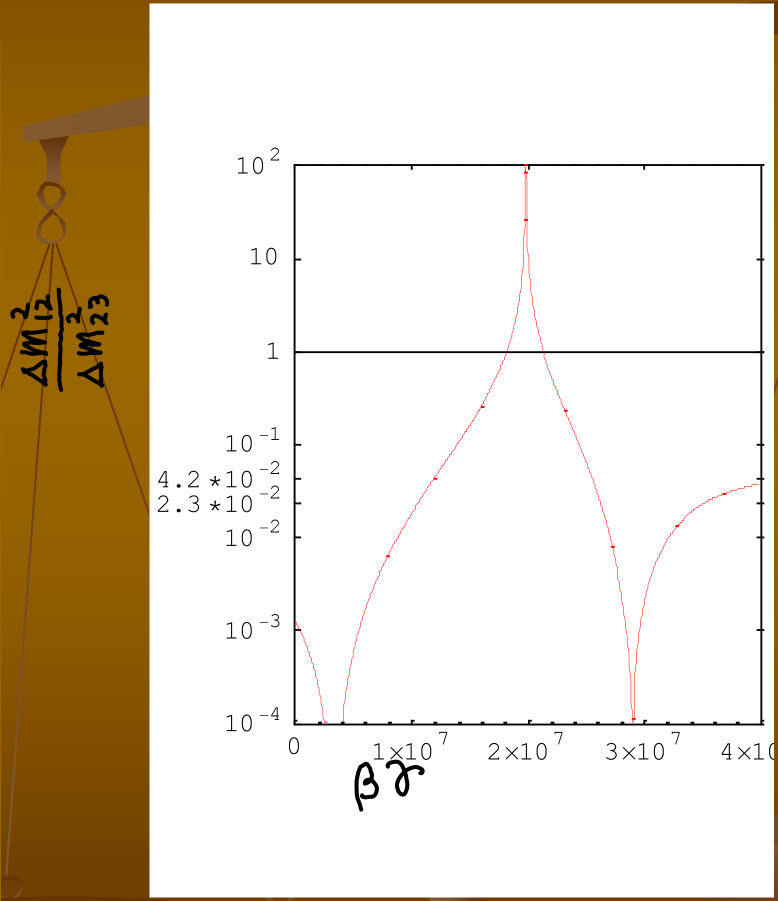
$|(U_{CKM})_{12}| = 0.0295 - 0.0387$

$|(U_{CKM})_{12}| = 0.0024 - 0.0038$

$\delta_\varepsilon = 46^\circ - 74^\circ$

The allowed regions of neutrino masses and mixing angles in the case of the normal hierarchy at Sol. (b)





The number of parameters in the quark sector

- The number of parameters in the quark sector.

$$\begin{array}{lll} & M_u \Rightarrow & 6 \\ +) & M_d \Rightarrow & 6 \\ \hline & N(pmt) & =12 \end{array}$$

- The number of constraints from experiments.

$$\begin{array}{lll} & \text{masses} \Rightarrow & 3\times 2 = 6 \\ +) & CKM \Rightarrow & 3+1=4 \\ \hline & N(exp) & =10 \end{array}$$

- The following two phase parameters can not be determined.

$$N(free) = N(pmt) - N(exp) = 12 - 10 = 2$$

$$\Rightarrow (\tau_u + \tau_d), (\sigma_u + \sigma_d)$$

Diagonalization

- The FZT matrix is diagonalized as follows

$$U_f^\dagger M_f U_f = \mathrm{diag}(m_1, m_2, m_3)$$

Here,

$$U_f \equiv P_f^\dagger O_f, \quad P_f \equiv (1, \tau_f, \sigma_f + \tau_f) \equiv (1, \alpha_{f2}, \alpha_{f3})$$

$$O_f \equiv \begin{pmatrix} \sqrt{\dfrac{(d_f - m_{f1}) m_{f2} m_{f3}}{R_{f1} d_f}} & \sqrt{\dfrac{(d_f - m_{f2}) m_{f3} m_{f1}}{R_{f2} d_f}} & \sqrt{\dfrac{(d_f - m_{f3}) m_{f1} m_{f2}}{R_{f3} d_f}} \\ -\sqrt{-\dfrac{(d_f - m_{f1}) m_{f1}}{R_{f1}}} & \sqrt{-\dfrac{(d_f - m_{f2}) m_{f2}}{R_f}} & \sqrt{-\dfrac{(d_f - m_{f3}) m_{f3}}{R_{f3}}} \\ \sqrt{\dfrac{m_{f1}(d_f - m_{f2})(d_f - m_{f3})}{R_{f1} d_f}} & -\sqrt{\dfrac{m_{f2}(d_f - m_{f3})(d_f - m_{f1})}{R_{f2} d_f}} & \sqrt{\dfrac{m_{f3}(d_f - m_{f1})(d_f - m_{f2})}{R_{f3} d_f}} \end{pmatrix}$$

$$R_{f1} = (m_{f1} - m_{f2})(m_{f1} - m_{f3}), \quad R_{f2} = (m_{f2} - m_{f3})(m_{f2} - m_{f1})$$

$$R_{f3} = (m_{f3} - m_{f1})(m_{f3} - m_{f2})$$

The CKM matrix

- The CKM quark mixing matrix $U_{\rm CKM} \equiv U_u^{\dagger} U_d$ is given by

$$(U_{CKM})_{12} \simeq \sqrt{\frac{|m_d|}{m_s}} - e^{i\alpha_2}\sqrt{\frac{|m_u|}{m_c}}\,x_u x_d - e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_c}}\,(1-x_u)(1-x_d)$$

$$(U_{CKM})_{23} \simeq \sqrt{\frac{|m_d|\,m_s}{m_b^2}} - e^{i\alpha_2}\sqrt{\frac{|m_u|}{m_c}}\,x_u(1-x_d) + e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_c}}\,(1-x_u)x_d$$

$$(U_{CKM})_{23} \simeq \sqrt{\frac{|m_u|}{m_c}\frac{|m_d|\,m_s}{m_b^2}\frac{(1-x_d)}{x_d}} + e^{i\alpha_2}\sqrt{x_u(1-x_d)} - e^{i\alpha_3}\sqrt{(1-x_u)x_d}$$

$$\delta_q \simeq \arg \frac{\left(e^{i\alpha_3}\sqrt{(1-x_u)(1-x_d)} + e^{i\alpha_2}\sqrt{x_u x_d}\right)^*}{\left(e^{i\alpha_3}\sqrt{(1-x_u)x_d} - e^{i\alpha_2}\sqrt{x_u(1-x_d)}\right)\left(e^{i\alpha_2}\sqrt{(1-x_u)x_d} - e^{i\alpha_3}\sqrt{x_u(1-x_d)}\right)^*}$$

where $x_f \equiv d_f/m_{f3}$,

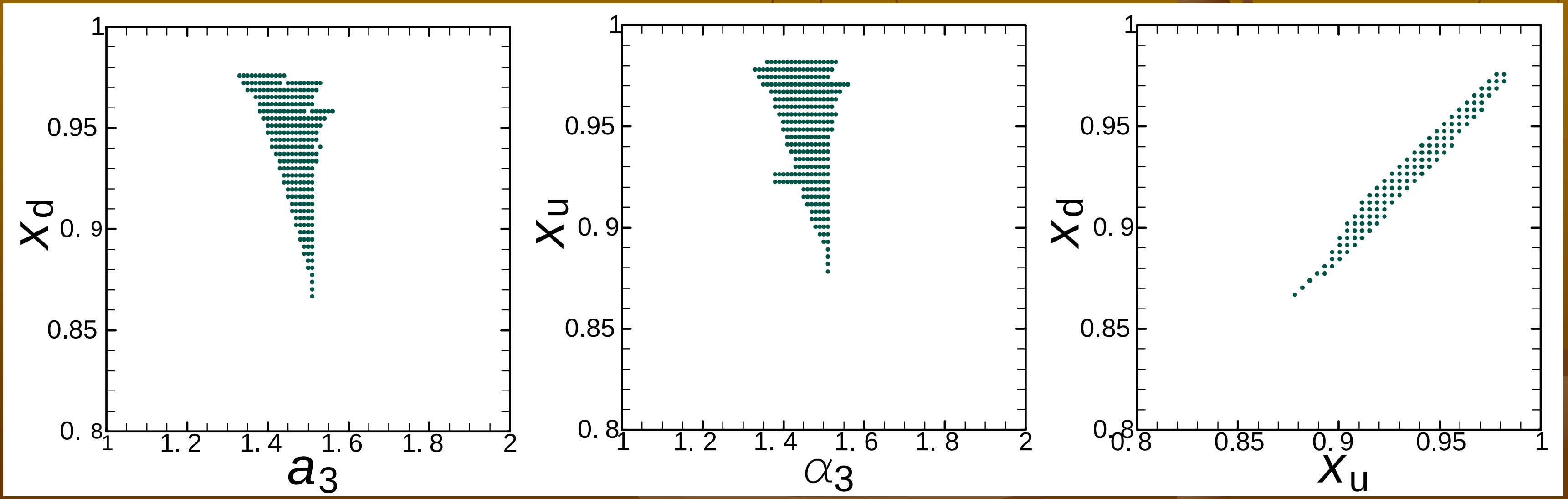
$$\alpha_2 \equiv \alpha_{u2} - \alpha_{d2} = \tau_u - \tau_d \equiv \Delta\tau,$$

$$\alpha_3 \equiv \alpha_{u3} - \alpha_{d3} = (\tau_u - \tau_d) + (\sigma_u - \sigma_d) \equiv \Delta\tau + \Delta\sigma.$$

Note that m_u/m_t and m_c/m_t are not sensitive to CKM matrix.

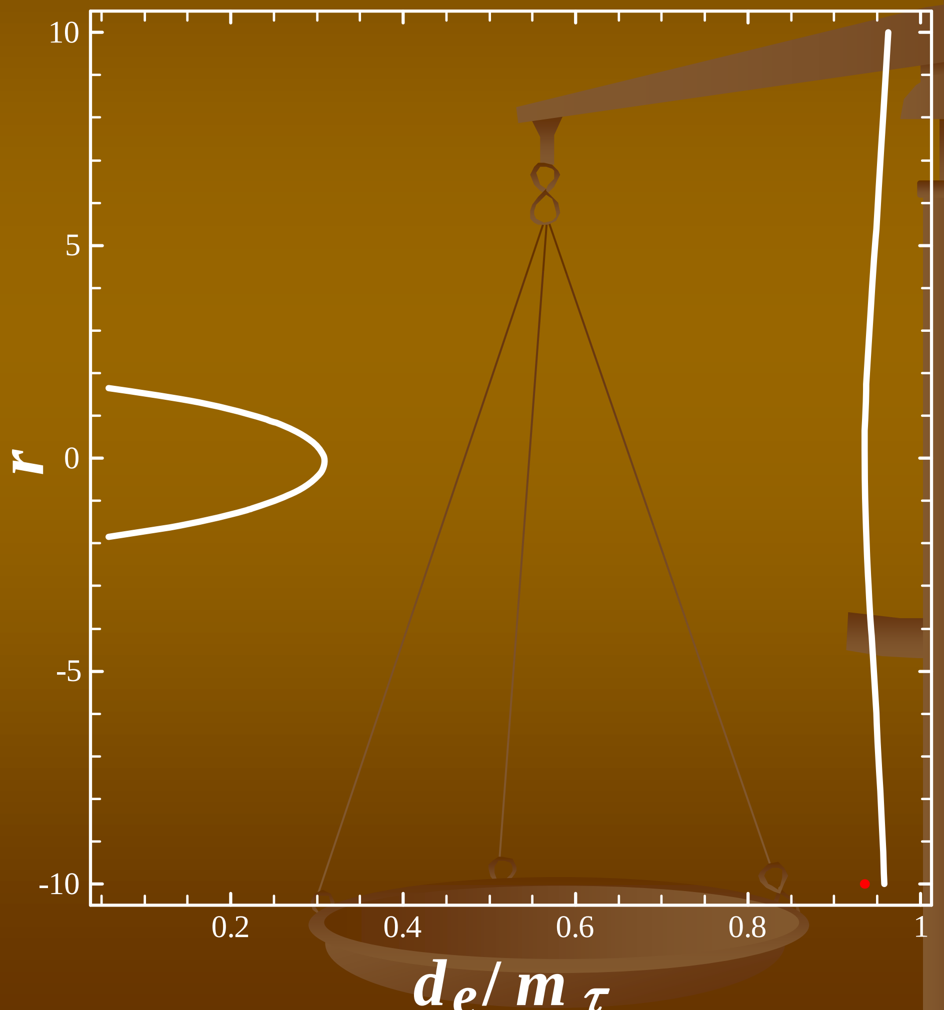
Numerical estimation

- We fix the quark masses by the observed masses.
- Two component parameters x_u and x_d and two phase parameters α_2 and α_3 are left as free parameters.
- In our former paper, we find that if α_2 takes a value as $\alpha_2 \cong \pi/2$, there are the allowed region in the dotted regions.



- The contour lines on which the following equation is satisfied.

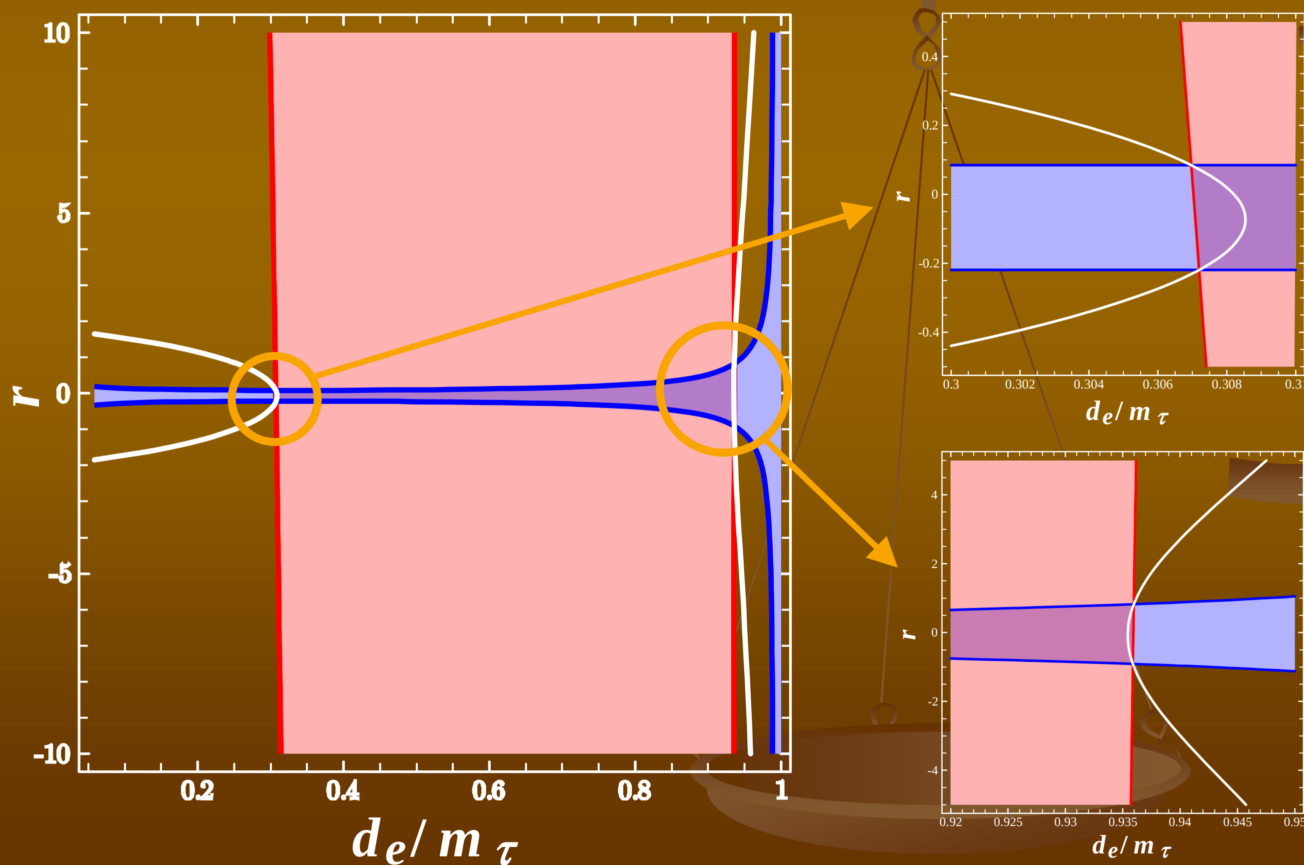
$$F(r)^2\left[4\alpha a_u \cos(\Delta\tau+\tau_d) - (3+K)a_d\cos\tau_d\right]^2 - \left[4\alpha c_u \cos(\Delta\sigma+\sigma_d) - (3+K)c_d\cos\sigma_d\right]^2$$
$$= (1-K)^2\left[a_e^2 F(r)^2 - c_e^2\right]$$



- The additional conditions which come from the phases in the charged lepton mass matrix.

$$-1 \le \cos\tau_e(d_e, r) = \frac{4\alpha a_u \cos(\Delta\tau + \tau_d) - (3+K)\, a_d \cos\tau_d}{(1-K)\, a_e} \le +1 \rightarrow$$

$$-1 \le \cos\sigma_e(d_e, r) = \frac{4\alpha C_u \cos(\Delta\sigma + \sigma_d) - (3+K)\, C_d \cos\sigma_d}{(1-K)\, C_e} \le +1 \rightarrow$$



Sol. (a)

Zoomed view

Sol. (b)

Zoomed view

Sol. (a)

$$\underset{\sim}{S} = \begin{pmatrix} 0 & 3{,}1\times10^{2} & 0 \\ 3{,}1\times10^{2} & 3{,}5\times10^{4} & 3{,}6\times10^{4} \\ 0 & 3{,}6\times10^{4} & 1{,}0\times10^{5} \end{pmatrix}$$

$$\underset{\sim}{S}' = \begin{pmatrix} 0 & -2{,}0 & 0 \\ -2{,}0 & -2{,}0\times10^{2} & -6{,}7\times10 \\ 0 & -6{,}7\times10 & 1{,}5\times10^{2} \end{pmatrix}, \quad A = i\begin{pmatrix} 0 & -2{,}5\times10^{-1} & 0 \\ 2.5\times10^{-1} & 0 & 1{,}1 \\ 0 & -1{,}1 & 0 \end{pmatrix}$$

$$\alpha = 7{,}9\times10^{-3}, \quad \delta = 2{,}3\times10, \quad \delta' = -5{,}2, \quad \varepsilon = 1{,}5\times10^{2}$$

Sol. (b)

$$\underset{\sim}{S} = \begin{pmatrix} 0 & 3{,}6\times10^{2} & 0 \\ 3{,}6\times10^{2} & 1{,}3\times10^{4} & 3{,}3\times10^{4} \\ 0 & 3{,}3\times10^{4} & 1{,}4\times10^{4} \end{pmatrix}$$

$$\underset{\sim}{S}' = \begin{pmatrix} 0 & -7{,}7\times10^{-1} & 0 \\ -7{,}7\times10^{-1} & -1{,}6\times10 & -1{,}4\times10 \\ 0 & -1{,}5\times10 & -3{,}7\times10 \end{pmatrix}, \quad A = i\begin{pmatrix} 0 & 5{,}3 & 0 \\ -5{,}3 & 0 & -2{,}4\times10 \\ 0 & 2{,}4\times10 & 0 \end{pmatrix}$$

$$\alpha = 7{,}0\times10^{-3}, \quad \delta = 1.0\times10^{2}, \quad \delta' = -9{,}3\times10, \quad \varepsilon = 4.5\times10^{2}$$

Our Strategy for new physics on phenomenology

- The relationship of the quark and lepton mass matrices
  - Can four-zero-texture mass matrix model reproduce the quark and lepton mixing angles and CP violating phases?
    - PRD74:033014(2006)
    - "Topical Seminar on Frontier of Particle Physics", Beijing, China, Aug 2006
  - Zero texture model and SO(10) GUT
    - hep-ph/0702284. *Submitted to JHEP.*
    - "International Workshop on Neutrino Mass and Mixings", Univ. of Shizuoka, Japan, Dec 2006 (The talk title is different)
  - A possible origin of the CP violation due to Froggatt-Nielsen fields
    - We will submit it soon.

I will concentrate this.

- The nature of the Higgs sector
  - New physics search by helicity decomposition of heavy fermion pair-production from W-boson fusion at the ILC
    - "9th ACFA ILC Physics and Detector Workshop", IHEP, Beijing, Feb 2007.

- The nature of neutrino
  - The phenomenology of high energy neutrino at IceCube.