Private review of the present status of GUT

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Why GUT ?

Beyond Standard model is not necessarily GUT.
GUT does not necessarily mean ultra high energy.

- Experimental observations

Asymptotic freedom in strong interactions

-->natural to extend Electro-weak to incorpolate strong interaction

Neutrino oscillations (high energy scales via Seesaw)

Theoretical implications (aesthetic displeasure of STD model)

The unification of so many parameters in the Standard model

Higgs mass hierarchy problem->new physics above TeV

Intermediate energy scales of axion and Heavy (right-handed) neutrino etc.

Confrontation with Inflation model

Why SUSY?

- Gauge coupling unification (MSSM)

(Non-SUSY with Pati-Salam, threshold corrections)

- Mass hierarchy problem

(Little Higgs, Extra dimension,..)

- Dark Matters (Modified gravity, axion)
- SUSY resolves some problems but gives rise to other problems

Susy breaking mechanism, Mu-problem, gravitino problem, …..

What is the gauge group ?

- SU(5)
- SO(10)=5bar+10+1, anomaly free*
- E_6, E_8

* Anomalous<->unrenormalizable, nonunitary

T.F and K.Kamimura 1988 Phys.Lett.) argued agains the anomalous but consistent theory (Jackiew-Rajaraman 1985).

- These are the (vertical) symmetries of one family (?).

How about the (horizontal) symmetry of inter-families ?

Can GUT be consistent in 4 D ?

- The relation with the alternative approach
- Extra dimension is not opposed to SUSY

but may be necessary for its consistency via SUSY breaking mechanism and others.

How about Hosotani mechanism ?

GUT in warped extra dimensions (Randall-Sandrum)

Orbifold GUT a la Kawamura

What does the string theory impose on GUT ?

GUT is testable from the wide regions

The relation with the Cosmology, sepecially with Inflation model.

GUT and Collider phenomenology.

Hope for rich arguments.

SO(10) GUT

- Renormalizable SUSY SO(10)

Based on the works in collaboration with

T. Kikuchi, N. Okada, **A.Ilakovac** and **S. Meljanac,** References:

JHEP 0211 (2002); Phys. Rev. D 68 (2003); Int. J. Mod. Phys. A19 (2004); JHEP 0409 (2004); J. Math. Phys. 46 (2005); JHEP 0505 (2005); **Phys.Rev.D75 (2007)**

With N.Okada: works in preparation. Earlier works: with

K. Matsuda, Y.Koide, H.Nishiura.

The other groups' works: Mohapatra et.al., Senjanovic et.al. and…

Perturbative (to the Planck scale) SUSY SO(10)

- Raby, Albright, Babu-Barr,…..

$$
\begin{aligned}
W_Y &= 16_i 16_j (Y_{10}^{ij} 10_H \\
&+ Y_{45}^{ij} 10_H 45_H / M + Y_{16}^{ij} 16_H 16_H / M + \ldots)
\end{aligned}
$$

Why renoramalizable ?

- Renoramalizability is the guiding principle for the SM. Remember anomalous free condition.
- For SU(3)xU(1)_em Fermi interaction is the effective interaction.
- For SM model it becomes renoramalizable intearaction but neutrino mass is the effective mass.
- At GUT nu_R becomes massless and it is described by the renoramalizable Yukawa coupling.

Renormalizable Minimal SUSY SO(10) model

(Babu-Mohapatra (93'); Fukuyama-Okada (01'))

- Two kinds of symmetric Yukawa couplings

$$W_Y = Y_{10}^{ij} \mathbf{16}_i H_{10} \mathbf{16}_j + Y_{126}^{ij} \mathbf{16}_i H_{126} \mathbf{16}_j$$

- Two Higgs fields are decomposed to

$$\begin{aligned} \mathbf{10} &\to (\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{2}) \\ \overline{\mathbf{126}} &\to (\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{15},\mathbf{2},\mathbf{2}) \\ &+ (\overline{\mathbf{10}},\mathbf{3},\mathbf{1}) + (\mathbf{10},\mathbf{1},\mathbf{3}) \end{aligned}$$

- SU(4) adjoint **15** have a basis, $\mathrm{diag}(1,1,1,-3)$so as to satisfy the traceless condition. Putting leptons into the 4th color, we get, so called, 'Georgi-Jarslkog' factor, -3 for leptons.

Yukawa couplings

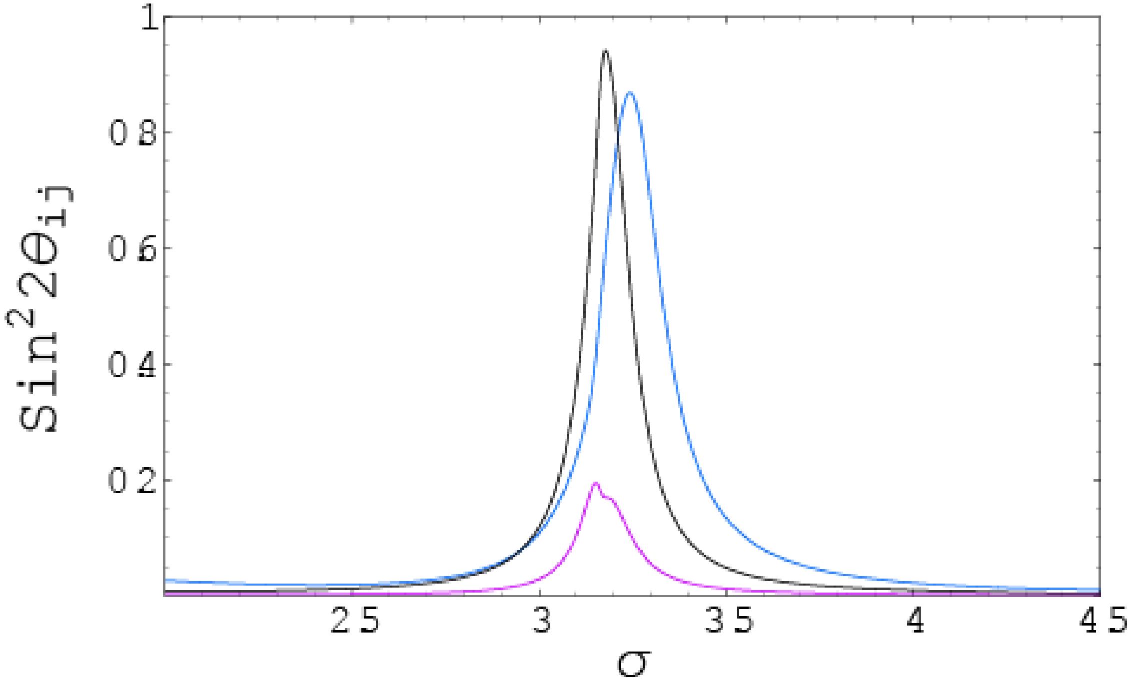
- After the symmetry breakings, we have

$$\begin{aligned} W_Y &= (u_R^c)_i \left(Y_{10}^{ij} H_{10}^u + Y_{126}^{ij} H_{126}^u\right) q_j \\ &+ (d_R^c)_i \left(Y_{10}^{ij} H_{10}^d + Y_{126}^{ij} H_{126}^d\right) q_j \\ &+ (\nu_R^c)_i \left(Y_{10}^{ij} H_{10}^u - 3Y_{126}^{ij} H_{126}^u\right) \ell_j \\ &+ (e_R^c)_i \left(Y_{10}^{ij} H_{10}^d - 3Y_{126}^{ij} H_{126}^d\right) \ell_j \\ &+ (\nu_R^c)_i \left(Y_{126}^{ij}\, v_R\right) (\nu_R^c)_j \end{aligned}$$

- Below the GUT scale, we assume MSSM is realized, and we have two Higgs doublet which are linear combinations of original fields.

Predictions in neutrino sector

- We have only one parameter $\sigma = \arg(c_d)$, left free. So, we can make definite predictions.



For $\sigma = 3.198$ [rad] :

$\sin^2 2\theta_{12} \sim 0.72, \ \sin^2 2\theta_{23} \sim 0.90, \ \sin^2 2\theta_{13} \sim 0.16$

Yukawa’s are determined!

- Now, all the mass matrices have been determined!
- For example, Neutrino Dirac Yukawa coupling matrix (in the basis where charged lepton mass matrix is diagonal):

$$Y_\nu = \begin{pmatrix} -0.000135-0.00273i & 0.00113+0.0136i & 0.0339+0.0580i \\ 0.00759+0.0119i & -0.0270-0.00419i & -0.272-0.175i \\ -0.0280+0.00397i & 0.0635-0.0119i & 0.491-0.526i \end{pmatrix}$$

- We must check this model by proving the other phenomena related to the Yukawa couplings! (LFV, muon g-2, EDM, neutrino magnetic dipole moment (MDM), proton decay, etc.)

Soft SUSY breaking terms

- We consider the general soft SUSY breaking mass parameters at low-energy

$$\begin{aligned}
-\mathcal{L}_{\mathsf{soft}} &= \tilde{\ell}_i^\dagger \left(m^2_{\tilde{\ell}}\right)_{ij} \tilde{\ell}_j + \tilde{\nu}^\dagger_{Ri} \left(m^2_{\tilde{\nu}}\right)_{ij} \tilde{\nu}_{Rj} + \tilde{e}^\dagger_{Ri} \left(m^2_{\tilde{e}}\right)_{ij} \tilde{e}_{Rj} \\
&+ m^2_{H_u} H_u^\dagger H_u + m^2_{H_d} H_d^\dagger H_d \\
&+ \left(B\mu H_d H_u + \frac{1}{2} B_\nu M_{R_{ij}} \tilde{\nu}^\dagger_{Ri} \tilde{\nu}_{Rj} + h.c. \right) \\
&+ \left(A_\nu^{ij} \tilde{\nu}^\dagger_{Ri} \tilde{\ell}_j H_u + A_e^{ij} \tilde{e}^\dagger_{Ri} \tilde{\ell}_j H_d + h.c. \right) \\
&+ \left(\frac{1}{2} M_1 \tilde{B}\tilde{B} + \frac{1}{2} M_2 \tilde{W}^a \tilde{W}^a + \frac{1}{2} M_3 \tilde{G}^a \tilde{G}^a + h.c. \right)
\end{aligned}$$

Lepton Flavor Violation (LFV)

- Because we really see LFV as neutrino oscillations, we naturally expect LFV can also be seen in the charged lepton sector!
- Current experimental bound:

$$\mathrm{BR}(\mu \to e\gamma) < 1.2 \times 10^{-11}$$

$$\mathrm{BR}(\tau \to \mu\gamma) < 0.6 \times 10^{-6}$$

- How well motivated from theoretical point of view?

➢ In the Standard Model (+ Right-handed neutrinos): too small rate (∵GIM suppression well works).

➢ In SUSY models: New source of LFV, soft SUSY breaking terms (with No GIM suppression, in general) exist.

LFV processes are important sources for low-energy SUSY search!

Soft SUSY breaking terms

- We consider the general soft SUSY breaking mass parameters at low-energy

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{soft}} &= \tilde{\ell}_i^\dagger \left(m_{\tilde{\ell}}^2\right)_{ij} \tilde{\ell}_j + \tilde{\nu}_{Ri}^\dagger \left(m_{\tilde{\nu}}^2\right)_{ij} \tilde{\nu}_{Rj} + \tilde{e}_{Ri}^\dagger \left(m_{\tilde{e}}^2\right)_{ij} \tilde{e}_{Rj} \\
&+ m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d \\
&+ \left(B\mu H_d H_u + \frac{1}{2} B_\nu M_{R_{ij}} \tilde{\nu}_{Ri}^\dagger \tilde{\nu}_{Rj} + h.c. \right) \\
&+ \left(A_\nu^{ij} \tilde{\nu}_{Ri}^\dagger \tilde{\ell}_j H_u + A_e^{ij} \tilde{e}_{Ri}^\dagger \tilde{\ell}_j H_d + h.c. \right) \\
&+ \left(\frac{1}{2} M_1 \tilde{B}\tilde{B} + \frac{1}{2} M_2 \tilde{W}^a \tilde{W}^a + \frac{1}{2} M_3 \tilde{G}^a \tilde{G}^a + h.c. \right)
\end{aligned}
$$

mSUGRA boundary condition

- We impose the universal boundary conditions at the GUT scale

$$
\begin{aligned}
&\left(m^2_{\tilde{\ell}}\right)_{ij} = \left(m^2_{\tilde{\nu}}\right)_{ij} = \left(m^2_{\tilde{e}}\right)_{ij} = m_0^2 \delta_{ij} \ , \\
&m^2_{H_u} = m^2_{H_d} = m_0^2 \ , \\
&A_\nu^{ij} = A_0 Y_\nu^{ij} \ , \ \ A_e^{ij} = A_0 Y_e^{ij} \ , \\
&M_1 = M_2 = M_3 = M_{1/2}
\end{aligned}
$$

- So we have three free parameters in the soft mass terms,

  $m_0, \ M_{1/2}$ and A_0.

RG induced Flavor mixings

- Flavor mixing is induced by RG running from the GUT scale to the EW scale

$$\begin{aligned}\mu\frac{d}{d\mu}\left(m^2_{\tilde{\ell}}\right)_{ij} &= \mu\frac{d}{d\mu}\left(m^2_{\tilde{\ell}}\right)_{ij}|_{\mathrm{MSSM}} \\ &+ \frac{1}{16\pi^2}\left(m^2_{\tilde{\ell}}Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m^2_{\tilde{\ell}} + 2Y_\nu^\dagger m^2_{\tilde{\nu}}Y_\nu\right. \\ &+ \left. 2m^2_{H_u}Y_\nu^\dagger Y_\nu + 2A_\nu^\dagger A_\nu\right)_{ij}\end{aligned}$$

- In the basis where the charged lepton mass matrix is diagonal, LFV is induced through the neutrino Dirac Yukawa coupling matrix.

$$\left(\Delta m^2_{\tilde{\ell}}\right)_{ij} \sim -\frac{3m_0^2 + A_0^2}{8\pi^2}\left(Y_\nu^\dagger L Y_\nu\right)_{ij}$$

where $L_{ij} = \log\left(M_{R_i}/M_G\right)\,\delta_{ij}$

First part summary

- We estimated neutrino oscillation parameters, LFV rates, Muon g-2 and Proton decay in the **Minimal SO(10) Model**

- Results:

  Neutrino masses & mixings can be well much described.
  LFV rates can be well exceed the future bounds
  Muon g-2 can be suitable for BNL E821 result
  Combining with WMAP data
  → Cosmologically allowed region exists

  Using these unambiguous Yukawa couplings,
  we have also analyzed the proton decay rate in detail.

- Proton Decay Results: The decay rate in our model is within the experimental upper bound if we take the small tan β = 2.5, assuming the color triplet Higgs mass to be at the usual GUT scale.

$$\tau(p \to K^+ \bar{\nu}) \leq 2 \times 10^{33} \left(\frac{10}{\tan\beta} \right)^2 \left(\frac{m_S}{10\,[\mathrm{TeV}]} \right)^2 \times \left(\frac{M_C}{2 \times 10^{16}\,[\mathrm{GeV}]} \right)^2 [\mathrm{years}].$$

- Note added: It’s possible to completely cancel out the proton decay rate by tuning the parameters in the Higgs sector (3 +1 or 5+1 parameters).

Our SO(10) Model is very testable in the near future experiments!

Superpotential was fully analyzed

- Fukuyama et.al. hep-ph/0401213 v1

gave the symmetry breaking pattern

from minimal SO(10) to Standard Model,starting from

$$W = m_1\Phi^2 + m_2\Delta\overline{\Delta} + m_3H^2 + \lambda_1\Phi^3 + \lambda_2\Phi\Delta\overline{\Delta} + \lambda_3\Phi\Delta H + \lambda_4\Phi\overline{\Delta}H.$$

$$H = 10, \Delta = 126, \Phi = 210$$

Many vacua (STD singlets)

- (1234)
- (5678+5690+7890) $\mathbf{210}$
- (1256+1278+1290+3456+3478+3490)

- (13579) $\overline{\mathbf{126}}$

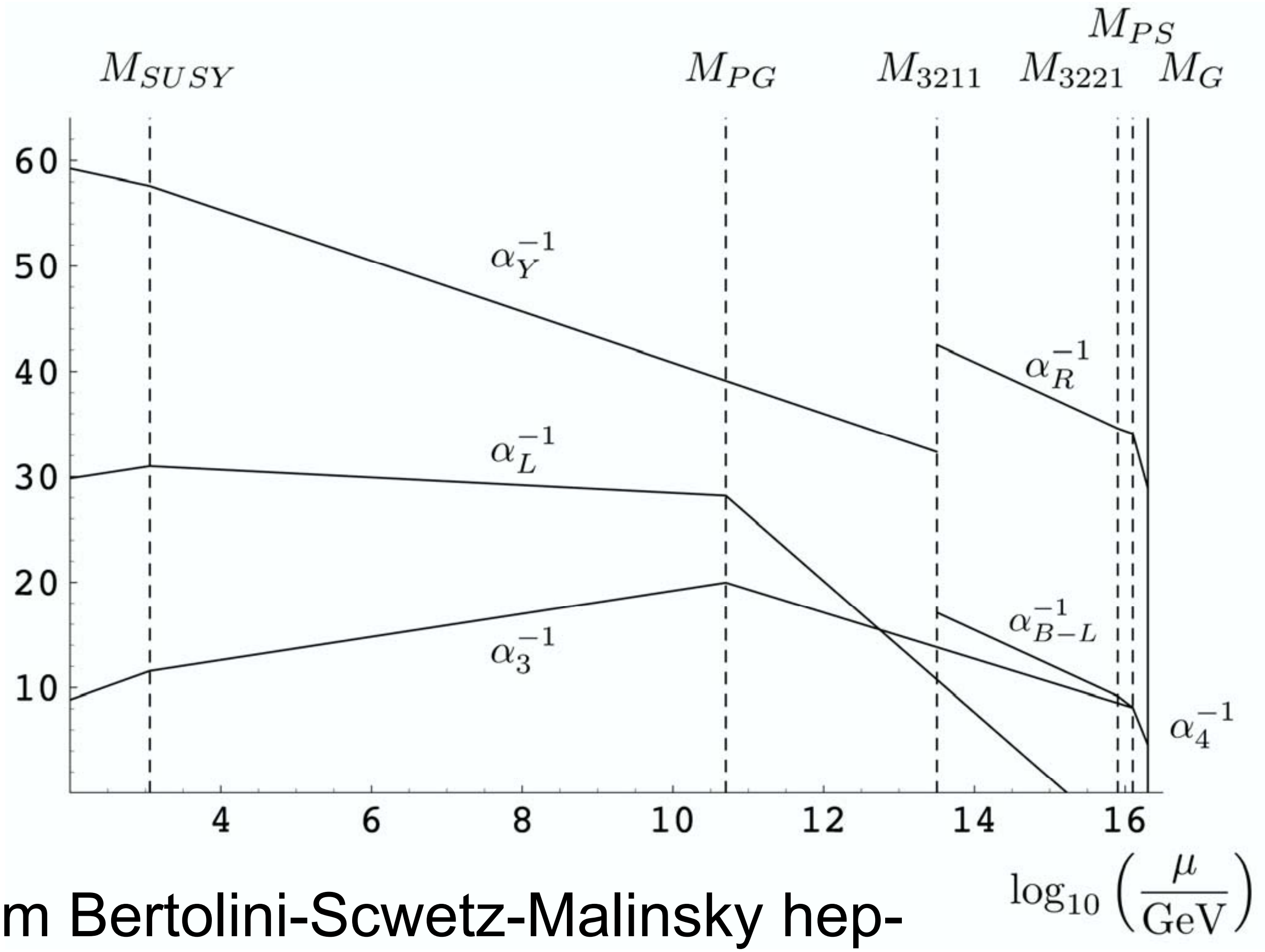
- (24680) $\mathbf{126}$

２．Problems of 4D SO(10) GUT

- Thus the theory enter into the precision calculation phase, where some problems give arise:
- Fast Proton Decay.
- Many intermediate energy scales break the gauge coupling unification since we have five directions which are singlet under G_{321}.

- We adjust this with neutrino oscillation data to detail, we are faced with the problem (next page diagram).
- However, this does not imply that the renormalizable SO(10) GUT is ruled out.
- We can add 120_H in Yukawa without changing the essential scheme, which may change the situation.

The gauge coupling unification



From Bertolini-Scwetz-Malinsky hep-ph/0605006

Solution by warped extra dimension -First approach-

1. A variety of Higgs mass spectra destroys the successful gauge coupling unification in the MSSM. Especially, the existence of the intermediate mass scale for the right-handed neutrino cause a problem.

$$M_R \ll M_G \;\; \Leftrightarrow \text{ gauge coupling unification}$$

2. This model has a cut off scale at the GUT scale. It means that a concrete UV completion of the model is necessary to be considered.

$$\alpha_G \gg 1 \;\; \Leftrightarrow \;\; \Lambda_{UV} = M_G \text{ ; UV completion}$$

We explore to solve these problems by changing the 4D flat space to the 5D Randall-Sundrum type warped background.

1. This model can easily provide a natural suppression for the Yukawa couplings by a wave function localization.
2. UV completion is provided by a strong gravity. (cf. AdS/CFT)

In order to realize the gauge coupling unification, the simplest way is to put all the VEV's at the GUT scale.

On the other hand, neutrino oscillation data shows the existence of the intermediate mass scale, which may destabilize the successful gauge coupling unification in the MSSM

$$M_\nu = M_D^T M_R^{-1} M_D \Rightarrow M_R \sim 10^{11-14} GeV$$

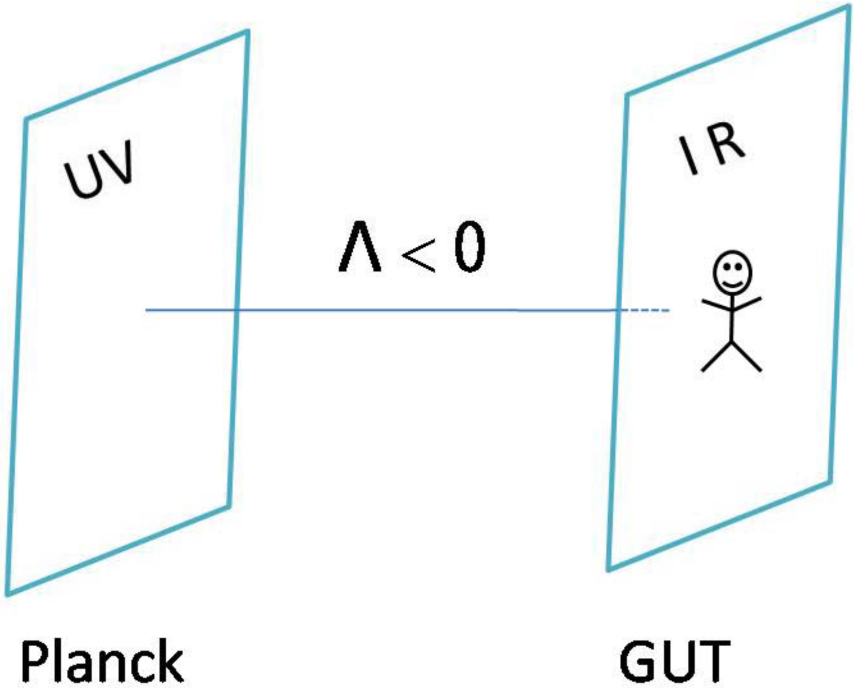
How to solve this mild hierarchy problem?

$$M_R \ll M_G$$

It is necessary to have an additional suppression as follows

$$M_R = c M_G, \; c = \mathcal{O}(10^{-2} - 10^{-5})$$

World of Extra Dimension



A solution to the mass hierarchy problem

In these days, there is a natural solution to provide a large mass hierarchy. In the Randall-Sundrum scenario, the exponentially warped metric can be used to explain any mass hierarchy.

$$c = \mathcal{O}(10^{-2} - 10^{-5}) = e^{-kr_c\pi} \Rightarrow kr_c = 1.5 - 3.7$$

The Setup We assume a 5D warped space:

$$ds^2 = e^{-2kr_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 dy^2$$

In SUSY models in 5D, any chiral multiplets become a part of hypermultiplet and vector multiplet is extended to include an adjoint scalar.

$$\mathbb{H} = (H, H^c),\ \mathbb{V} = (v_\mu, \lambda, \bar{\lambda}, \phi)$$

By assigning an *even/odd* parity for H/H^c, only H has a zero mode wave function. This assignment also allows a bulk mass term for them.

Effect of the VEV for the adjoint scalar

If we take into account of the VEV for the adjoint scalar in the vector multiplet, the bulk mass term is shifted as follows [Kitano-Li (03')] :

$$\mathcal{S} = \int d^5x \int d^2\theta e^{-3kr_c|y|} H^c \left[\partial_5 - \left(\frac{3}{2} - c\right) kr_C \epsilon(y) + g \langle\phi\rangle\right] H$$

$$+h.c.$$

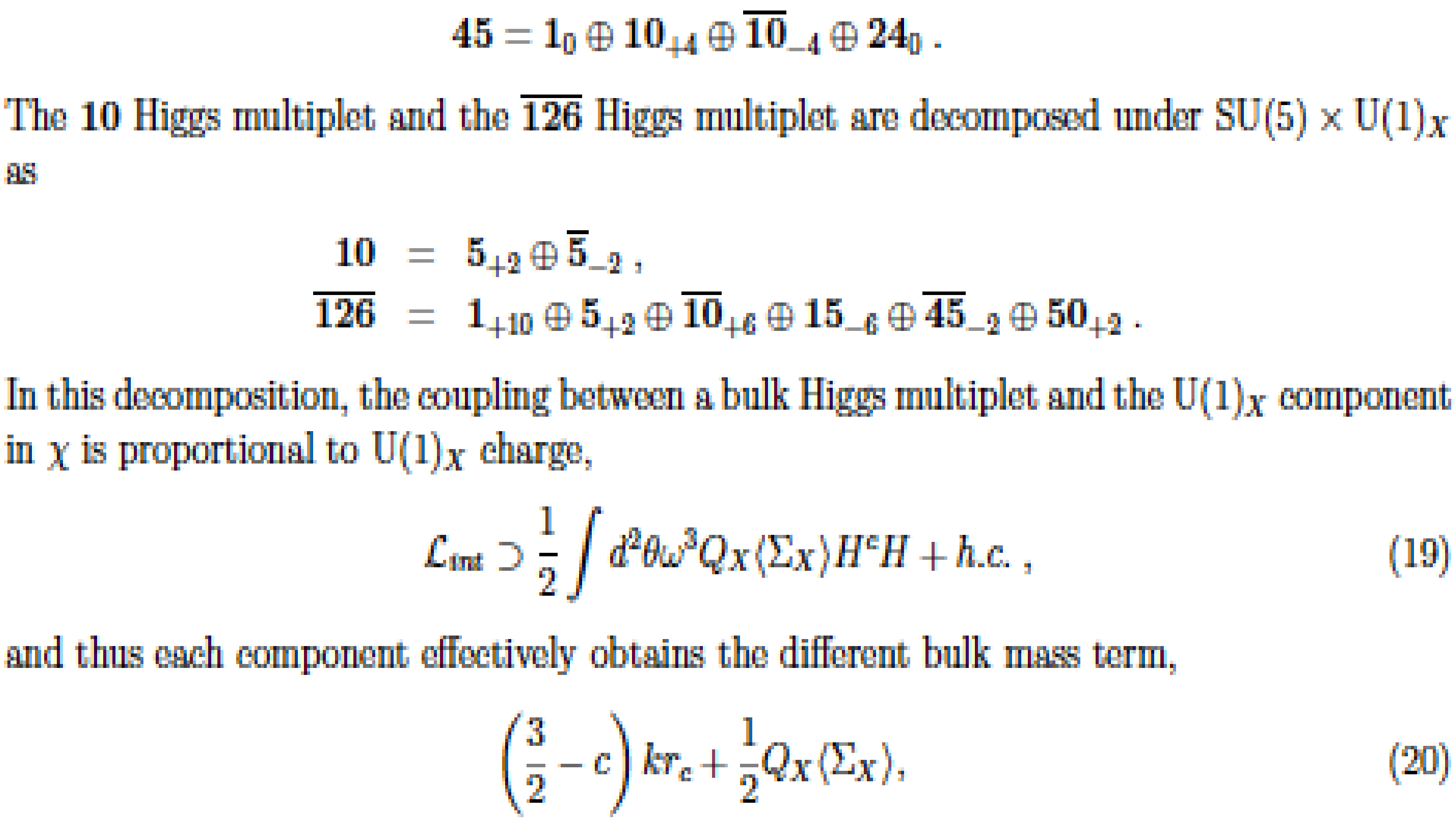
$$H = \frac{1}{\sqrt{N}} exp\{(\tfrac{3}{2} - c + \alpha) kr_C|y|\} h(x_\mu)$$

$$< \phi >= 2\alpha kr_C \epsilon(y)$$

In the context of the Left-Right symmetric gauge theory, providing the VEV only for the right-handed part provides an explanation for the mild hierarchy.

$$M_R/M_G = e^{-g\langle\phi\rangle\pi}, \ \ \langle\phi\rangle \propto SU(2)_R,$$

$$g\langle\phi\rangle = 1.5 - 3.7$$



$$\mathbf{45} = \mathbf{1}_0 \oplus \mathbf{10}_{+4} \oplus \overline{\mathbf{10}}_{-4} \oplus \mathbf{24}_0 \ .$$

The **10** Higgs multiplet and the $\overline{\mathbf{126}}$ Higgs multiplet are decomposed under SU(5) × U(1)$_X$ as

$$\begin{aligned} \mathbf{10} &= \mathbf{5}_{+2} \oplus \overline{\mathbf{5}}_{-2} \ , \\ \overline{\mathbf{126}} &= \mathbf{1}_{+10} \oplus \mathbf{5}_{+2} \oplus \overline{\mathbf{10}}_{+6} \oplus \mathbf{15}_{-6} \oplus \overline{\mathbf{45}}_{-2} \oplus \mathbf{50}_{+2} \ . \end{aligned}$$

In this decomposition, the coupling between a bulk Higgs multiplet and the U(1)$_X$ component in χ is proportional to U(1)$_X$ charge,

$$\mathcal{L}_{int} \supset \frac{1}{2} \int d^2\theta \omega^3 Q_X \langle \Sigma_X \rangle H^c H + h.c. \ , \tag{19}$$

and thus each component effectively obtains the different bulk mass term,

$$\left(\frac{3}{2} - c \right) k r_c + \frac{1}{2} Q_X \langle \Sigma_X \rangle, \tag{20}$$

So we can determine the profile of wave function from group propery.

Solution by orbifold

-Second Approah-

- Kawamura('01),Hall-Nomura('01),Dermisek-Mafi('01)

$$S^1/(Z_2 \times Z_2{}')$$

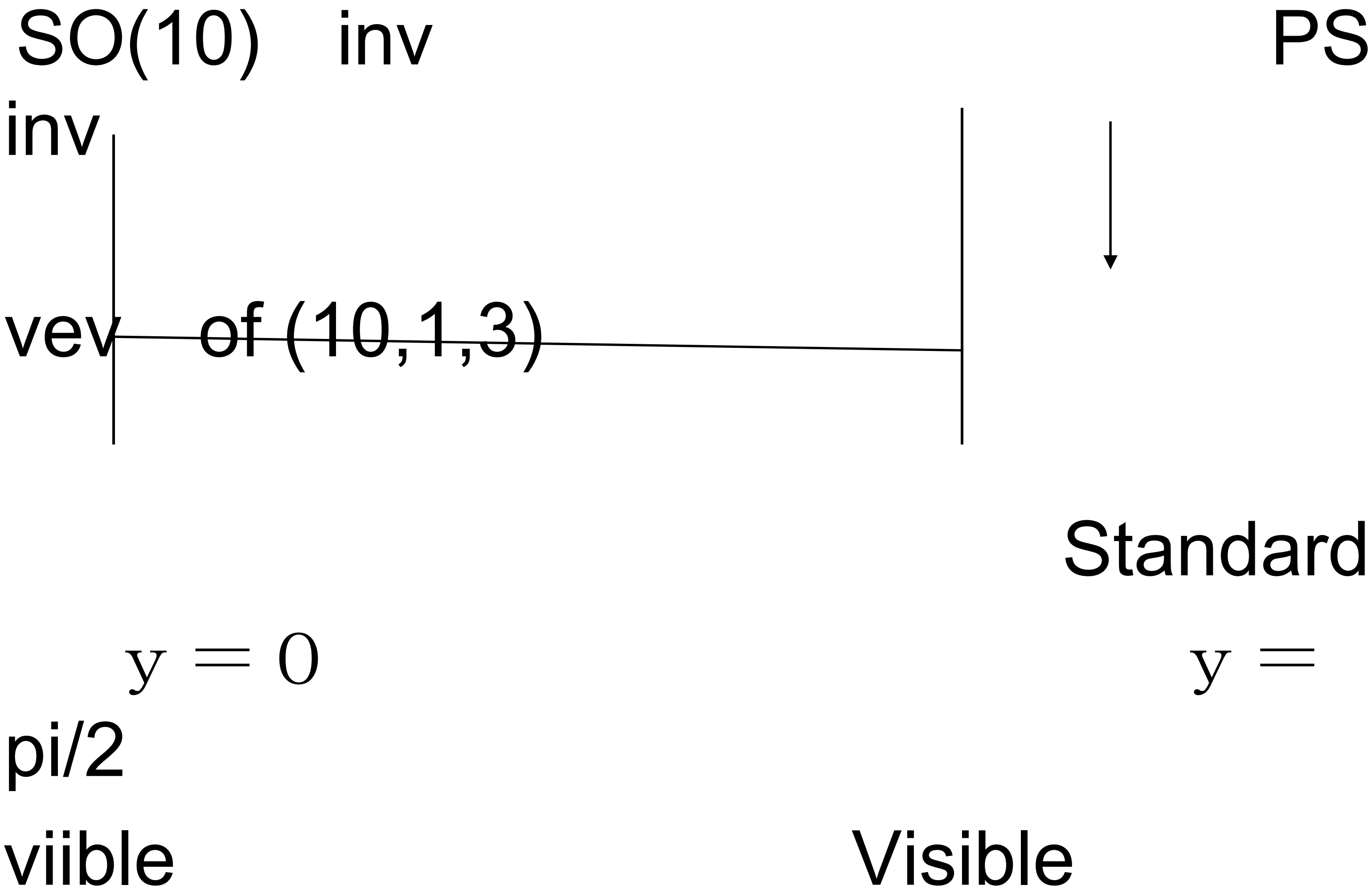
Proton decay is suppressed by BC.

BC breaks SU(5) into MSSM with direct no coupling with triplet, wheras SO(10) into Pati-Salam included it in general, inducing large proton decay ratio.

- For SO(10), there are many set ups even in $S^1/(Z_2 \times Z_2^{'})$.
- y=0 visible and

  vev of PS singlet at y=pi/2 brane destroys SUSY.
- y=pi/2 visible and vev of SO(10) singlet at y=0 branes destroys SUSY – today’s talk.
- In both cases, PS invariance play the crucial role to protect the fast proton decay.
- We can exclude (6,1,1) which was harmful which was included in full SO(10) invariant theory.

Our Setup



SO(10) inv

PS

inv

vev of (10,1,3)

Standard

$y = 0$

$y =$

pi/2

Inviible

Visible

	`bulk`	`brane` $y = \pi R/2$
`Matter fields`	No matter	$\psi_i = F_{Li} \oplus F_{Ri} \quad (i = 1, 2, 3)$
`Higgs sector`	$H_{10},\ H'_{10}$	$(\mathbf{15}, \mathbf{1}, \mathbf{1})$, $(\mathbf{10}, \mathbf{1}, \mathbf{3})$, $(\overline{\mathbf{10}}, \mathbf{1}, \overline{\mathbf{3}})$

Table 2: The contents of matters and Higgs with Z parity even. $F_{L.Ri}$ are $(\mathbf{4}, \mathbf{2}, \mathbf{2})$ of i'th generation and given in Eq.(3)

$$\begin{pmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e \end{pmatrix}_{L(R)} \equiv F_{L(R)1},$$

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) \times (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) = (\mathbf{15}, \mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}, \mathbf{2}),$$

From $\tau - b$ unification at GUT scale the third generation is described by H_{10}. The deviation of the first and second generation is complimented by the $(\mathbf{15}, \mathbf{2}, \mathbf{2})_H$. This term is constructed from the product of $H'(\mathbf{1}, \mathbf{2}, \mathbf{2})$ in the bulk and $(\mathbf{15},\mathbf{1},\mathbf{1})$ in the PS brane as $H'(\mathbf{1}, \mathbf{2}, \mathbf{2}) \times (\mathbf{15}, \mathbf{1}, \mathbf{1})/M_5$, and

(P, P')	field	mass
$(+,+)$	$V(15,1,1)$, V(1,3,1),V(1,1,3),$H(1,2,2)$	$\frac{2n}{R}$
$(+,-)$	$V(6,2,2)$, $H(6,1,1)$	$\frac{(2n+1)}{R}$
$(-,+)$	$\Phi(6,2,2)$, $\hat{H}(6,1,1)$	$\frac{(2n+1)}{R}$
$(-,-)$	$\Phi(15,1,1)$, $\Phi(1,3,1)$, $\Phi(1,1,3)$, $\hat{H}(1,2,2)$	$\frac{(2n+2)}{R}$

Table 1: P and P' assignment and masses ($n \geq 0$) of fields in the vector multipet (V, Φ) and hypermultiplets (H, $\hat{H}$) under the PS group. P' even V contains the PS gauge bosons; P' odd V contains the SO(10)/PS gauge bosons.

	bulk	brane $y = \pi R/2$
Matter fields	No matter	$\psi_i = F_{Li} \oplus F_{Ri} \quad (i = 1, 2, 3)$
Higgs sector	H_{10}	$(\mathbf{15}, \mathbf{1}, 1)$, $(\mathbf{1}, 2, \mathbf{2})$, $(4, \mathbf{1}, \mathbf{2})$, $(\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}})$

Gauge coupling unification

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M)} + \frac{1}{2\pi} b_i \ln\left(\frac{M}{\mu}\right). \quad (i = 3, 2.1) \tag{9}$$

In the PS stage $\mu > M_c$, the threshold corrections Δ_i due to KK mode in the bulk are added. b_i at G_{321} are

$$b_3 = -7,\ b_2 = -19/6,\ b_1 = 41/10 \tag{10}$$

at $M_{SUSY} > \mu > M = M_Z$ and

$$b_3 = -3,\ b_2 = 1,\ b_1 = 33/5 \tag{11}$$

at $M_c > \mu > M = M_{SUSY}$ The PS symmetry is recovered at $\mu = M_c = 1/(\pi R)$ and the matching condition holds

$$\begin{aligned} \alpha_3^{-1}(M_c) &= \alpha_4^{-1}(M_c) \\ \alpha_2^{-1}(M_c) &= \alpha_{2L}^{-1}(M_c) \qquad (12) \\ \alpha_1 - 1(M_c) &= [2\alpha_4^{-1}(M_c) + 3\alpha_{2R}^{-1}]/5 \qquad (13) \end{aligned}$$

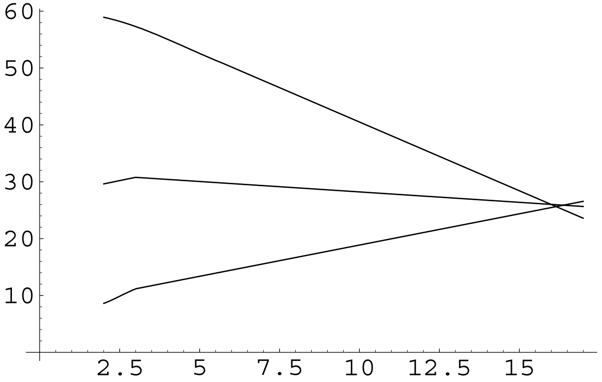
at M_c. In the PS stage $\mu > M_c$, the threshold corrections Δ_i due to KK mode in the bulk are added,

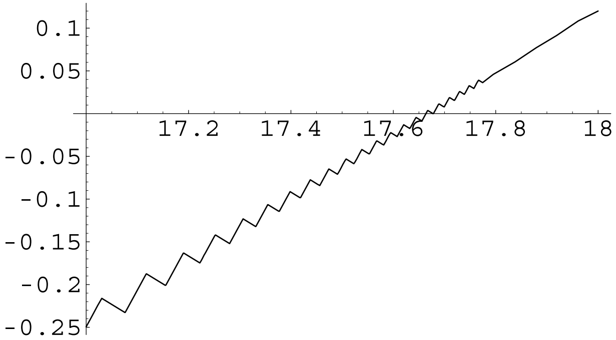
$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_c)} + \frac{1}{2\pi} b_i \ln\left(\frac{M_c}{\mu}\right) + \Delta_i. \quad (i = 4, 2_L, 2_R) \tag{14}$$

The beta functions of the PS gauge coupling constants are

$$b_4 = 5,\ b_{2L} = b_{2R} = 9 \tag{15}$$

Blow up problem is not serious





Mu-problem

$$\begin{aligned} L &= \delta(y)\{\int d^2\theta (S W_a W_a / M_* + S H_{10} H_{10} / M_*) \\ &+ \int d^4\theta S S^\dagger H_{10} H_{10} / M_*^2 \} \end{aligned}$$

$\sqrt{F} = O(10^{10} TeV), \quad M_{1/2} = \mu = F/M_*, \quad B\mu = F^2/M_*^2$

- Doublet-doublet problem appears

The relation with the other world

- Inflation theory in the framework of GUT
- Gauge Higgs unification-Hosotani Mechanism
- Landau’s conjecture:

QED may be consistent iff it is incorporated with gravitation.

Summary

4D

- The explicit decompositions of Higgs superpotetial revealed the precise spectra of intermediate enrgy specta to standard.
- It led to the destruction of gauge coupling unification.
- It may lead to the incompatibility with proton stability
- In order to circumvent these pathologies, we are forced to put it in extra dimensions.

5D

- We considered SO(10) in 5 D in two ways.
- One is to break SO(10) by the vev of bulk, which may explain tha mass spectra.
- Another is to break SO(10) by using both the boundary conditions and vev. This preserves the advantageous points of SO(10) (mass relations of Dirac fermions and saves the disadvantageous points of SO(10) (Proton decay and gauge unification) , though still many things are left unsolved.