GUT 2007 WORKSHOP, JAPAN

A 5-D SU(5) ORBIFOLD GUT MODEL

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PLAN OF THE TALK

- SU(5) Model, Proton decay problem, Doublet triplet splitting problem
- Fourier analysis of periodic functions
- Kawamura's five dimensional construction and our case with <u>different HIGGS choice</u>.
- Conclusions, speculations and outlook

SU(5) MULTIPLET STRUCTURE

1. A SU(N) group has $N^2 - 1$ generators and it's rank is $N-1$. Then SU(5) has 24 generators and 4 diagonal generators. $SU(3) \times SU(2) \times U(1)$ has 2+1+1=4 diagonal generators. Then SU(5) is the smallest group which can unify Standard Model group.

2. FERMION ASSIGNMENTS

$$
\begin{aligned}
5 &\supset (3,1,-1/3) + (1,2,1/2) \\
\overline{5} &\supset (\overline{3},1,1/3) + (1,2,-1/2) \\
5 \times 5 &= 10 + 15 \\
&= \underbrace{(3,2,1/6) + (1,1,1) + (\overline{3},1,-2/3)}_{10} \\
&+ \underbrace{(6,1,-2/3) + (3,2,1/6) + (1,3,1)}_{15}
\end{aligned}
$$

STANDARD FERMIONS IN THE SU(5) MATRIX

$$\overline{5}_F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu \end{pmatrix}$$

$$10_F = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix}$$

ADVANTAGES OF UNIFICATION OVER STANDARD MODEL

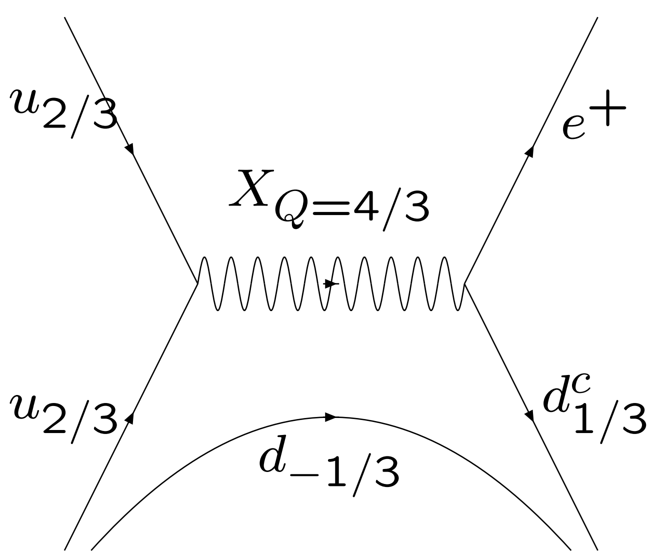
1. Charge quantization: $Q = T_L^3 + Y$. Y can take any value in SM. But in SU(5) Y is a diagonal generator with Tr(Y)=0. It can take only quantized values. Why ?

2. Gauge couplings approximately unify at a high scale around 10^{14-15} GeVs. But looking closely they do not really meet. This can be rectified by introducing supersymmetry.

PROTON DECAY PROBLEM AND PROTON LIFE-TIME

- Proton decay is a robust prediction of SU(5) model. Root cause being that SU(5) multiplet structure allows quarks and leptons to share big unified multiplets. So heavy gauge bosons can connect them.

- It is easy to see $p \to \pi^0 + e^+$ process through the exchange of leptoquark gauge bosons.
  $24 \to (8,1,0) + (1,3,0) + (1,1,0) + (\overline{3},2,\frac{5}{6}) + (3,2,-\frac{5}{6})$



PROTON DECAY LIFETIME

- Gauge boson mediated proton decay

  Amplitude $\sim \frac{\alpha_G}{M_X^2}$ and then $\boxed{\tau_P \sim \frac{M_X^4}{\alpha_G^2 m_P^5} \text{ GeV}^{-1}}$

  $\Delta E \Delta t \sim \hbar \sim 6.58 \times 10^{-25}$ GeV-Sec

- Because gauge couplings do not meet, one can have a rough estimate only.

M_X	α_X	τ_P (yes)
10^{14}	$1/40$	3×10^{28}
10^{15}	$1/40$	3×10^{32}
10^{16}	$1/40$	3×10^{36}

- Experimental number:
  $\Gamma[P \to e^+\pi^0] > 10^{33}$ Years

GENERIC DOUBLET-TRIPLET SPLITTING PROBLEM

$$V(H,\phi) = V(H) + V(\phi) + \lambda\ (TrH^2)(\phi^\dagger\phi) + \lambda'(\phi H^2 \phi^\dagger)$$

Then we give VEV to H which breaks GUT symmetry

$$< H >= v \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

Then through cross terms between H and ϕ we will get large masses for the doublet as well as triplet components.

4-D SOLUTIONS

- Sliding singlet mechanism
- Dimopoulos-Wilczek mechanism
- Low energy color triplet models

PERIODIC FIELDS

$\phi(y) = \phi(y + 2\pi R)$

$$\phi(y) \;=\; \sum_{n=0}^{\infty} [a^n \cos(\frac{ny}{R}) + b^n \sin(\frac{ny}{R})]$$

Under y reflection, even and odd modes are

$$\phi_+(y) \;=\; \sum_{n=0}^{\infty} \phi_+^n \cos[\frac{ny}{R}] \qquad \boxed{a^n = \phi_+^n}$$

$$\phi_-(y) \;=\; \sum_{n=0}^{\infty} \phi_-^n \sin[\frac{ny}{R}] \qquad \boxed{b^n = \phi_-^n}$$

We can rewrite $\phi_-(y)$ as

$$\phi_-(y) \;=\; \sum_{n=0}^{\infty} \phi_-^{n+1} \sin[\frac{(n+1)y}{R}]$$

We make a change of variable now $y' = y + \frac{\pi R}{2}$

$$\phi_+(y')$$
$$= \sum_{n=0}^{\infty} \phi_+^n \cos[\frac{ny'}{R} - \frac{n\pi}{2}]$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$
$$= \sum_{n=0}^{\infty} [\phi_+^n \cos\frac{n\pi}{2}] \cos\frac{ny'}{R} + \sum_{n=0}^{\infty} [\phi_+^n \sin\frac{n\pi}{2}] \sin\frac{ny'}{R}$$

First (second) term vanishes for n=1,3,5,(0,2,4..)

$\phi(y') =$
$\sum_{n=0}^{\infty} [\phi_+^{2n} \cos\frac{2n\pi}{2}] \cos\frac{2ny'}{R} +$
$\sum_{n=0}^{\infty} [\phi_+^{2n+1} \sin\frac{2n+1\pi}{2}] \sin\frac{(2n+1)y'}{R}$

We identity even and odd terms under $y' \to -y'$

Then we get famous expressions for Kaluza-Klein modes.

$$\begin{aligned} \phi_{++}(y) &= \sum_{n=0}^{\infty} \phi_{++}^{2n} \cos \frac{2ny}{R} \\ \phi_{+-}(y) &= \sum_{n=0}^{\infty} \phi_{+-}^{2n+1} \sin \frac{(2n+1)y}{R} \end{aligned}$$

Now similarly let us consider $\phi_-(y')$

$$\phi_-(y') = \sum_{n=0}^{\infty} \phi_-^n \sin[\frac{(n+1)y'}{R} - \frac{(n+1)\pi}{2}]$$

Let us simplify RHS

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$\sum_{n=0}^{\infty} \phi_-^n \{\sin\frac{(n+1)y'}{R}\cos\frac{(n+1)\pi}{2} - \cos\frac{(n+1)y'}{R}\sin\frac{(n+1)\pi}{2}\}$$

$$\sum_{n=0}^{\infty} [\phi_-^n \cos\frac{(n+1)\pi}{2}] \sin\frac{(n+1)y'}{R} - \sum_{n=0}^{\infty} [\phi_-^n \sin\frac{(n+1)\pi}{2}] \cos\frac{(n+1)y'}{R}$$

Replace $n \to 2n+1$ in the first term and $n \to 2n$ in the second term to get rid of zeros.

$$\phi_-(y') = \sum_{n=0}^{\infty} \phi_{--}^{2n+2} \sin\frac{(2n+2)y'}{R} + \sum_{n=0}^{\infty} \phi_{-+}^{2n+1} \cos\frac{(2n+1)y'}{R}$$

$$
\begin{aligned}
\phi_{-+}(y) &= \sum_{n=0}^{\infty} \phi_{-+}^{2n+1} \cos \frac{(2n+1)y}{R} \\
\phi_{--}(y) &= \sum_{n=0}^{\infty} \phi_{--}^{2n+2} \sin \frac{(2n+2)y}{R}
\end{aligned}
$$

Question: What are the normalization factors sitting in front of these modes ?

ORTHOGONALITY RELATIONS

$$\int_{-\pi}^{\pi} \sin(nx)\sin(mx)\ dx = \delta_{mn}\pi$$
$$\int_{-\pi}^{\pi} \cos(nx)\cos(mx)\ dx = \delta_{mn}\pi$$
$$\int_{-\pi}^{\pi} \sin(nx)\cos(mx)\ dx = 0$$

Let us normalize $\phi_{++}(x,y)$. If we were working with y we would have cared for $\boxed{y = [0 - \pi R]}$. But because we have already changed to $\boxed{y' = y - \frac{\pi R}{2}}$ we will integrate in the interval $\boxed{y = [0 - (+\frac{\pi R}{2})]}$.

$$\begin{aligned}
I &= \int_{-\infty}^{+\infty} dx\ \phi_{++}^{2m}\ \phi_{++}^{2n} \int_{0}^{\frac{\pi R}{2}} dy\ \cos\frac{2ny}{R}\cos\frac{2my}{R} \\
&= \frac{R}{2}\int_{-\infty}^{+\infty} dx\ \phi_{++}^{2m}\ \phi_{++}^{2n} \underbrace{\int_{0}^{\pi} d\xi\ \cos n\xi \cos m\xi}_{\pi/2} \\
&= \frac{\pi R}{4}\delta_{mn}\int_{-\infty}^{+\infty} dx\ \phi_{++}^{2m}\ \phi_{++}^{2n} \\
&= \frac{\pi R}{4}\ \text{ for m = n}
\end{aligned}$$

Because we have taken $\int_{-\infty}^{+\infty} dx \; \phi_{++}^{2m} \; \phi_{++}^{2n} = 1$. So these fields do have components in ordinary 4-dimensions for experimental search.

Question: What are the masses of KK states

MASSES OF KK STATES

Example: Scalars

$$[\partial_y^2 + m_0^2] f(x) \cos \frac{2ny}{R} \;\; = \;\; 0$$

$$[\partial_y^2 + m_0^2 + (\frac{2n}{R})^2] f(x) \cos \frac{2ny}{R} \;\; = \;\; 0$$

Example: Fermions

Consider Kinetic Energy density

$\overline{\psi}(i \not\partial)\psi$

Expand,

$\xi(x,y) = \sum_n \frac{1}{\sqrt{2\pi R}} \xi_n e^{iny/R}$

Taking all the derivatives and rearranging we get

$$\boxed{\sum_n \overline{\xi_n}(i \not\partial - \frac{n}{R})\xi_n}$$

MASSES OF FOURIER MODES

$$\phi_{++}(y) = \sqrt{\frac{4}{\pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(2n)} \cos \frac{2ny}{R}$$

$M_n = \frac{2n}{R}$

$$\phi_{+-}(y) = \sqrt{\frac{4}{\pi R}} \sum_{n=0}^{\infty} \phi_{+-}^{(2n+1)} \sin \frac{(2n+1)y}{R}$$

$M_n = \frac{2n+1}{R}$

$$\phi_{-+}(y) = \sqrt{\frac{4}{\pi R}} \sum_{n=0}^{\infty} \phi_{-+}^{(2n+1)} \cos \frac{(2n+1)y}{R}$$

$M_n = \frac{2n+1}{R}$

$$\phi_{--}(y) = \sqrt{\frac{4}{\pi R}} \sum_{n=0}^{\infty} \phi_{--}^{(2n+2)} \sin \frac{(2n+2)y}{R}$$

$M_n = \frac{2n+2}{R}$

Only $\phi_{++}(y)$ has a zero mode.

LET US CHOOSE $M_{gut} = 1/R$

This means that if ϕ_{++} will have vanishing mass.

$\phi_{+-}, \phi_{-+}, \phi_{--}$ will have mass of the order of M_{GUT}.

Kawamura has applied this property

to doublet-triplet splitting problem

THE KAWAMURA MODEL

There are two 4-D walls located at $y = 0$ and $\frac{\pi R}{2}$.

- There is a SU(5) group in 5-D.
- There are $5_H, \overline{5}_H, 10_H, \overline{10}_H$ Higgs scalars.
- A_μ has + parity and A_5 has - parity under Z_2. Further more (3,2) and ($\overline{3}$,2) components of A_μ has negative Z_2' parity.

Components	Z_2	Z_2'
$A_\mu \to$ (1,1)+(1,3)+(8,1)	+	+
$A_\mu \to$ (3,2)+($\overline{3}$,2)	+	-
$A_5 \to$ (1,1)+(1,3)+(8,1)	-	-
$A_5 \to$ (3,2)+($\overline{3}$,2)	-	+

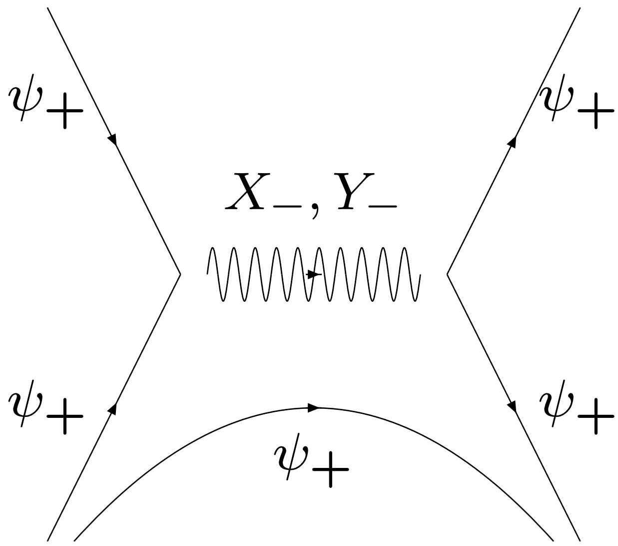
- In other words Z_2 can distinguish between usual 4-D and the extra fifth component of A^M, whereas Z_2' can distinguish between SM gauge bosons and the extra SU(5) gauge bosons. Thus

  Z_2' assignments break SU(5) symmetry.

Components	Z_2	Z_2'
5 $\to$ (1,2)	+	+
5 $\to$ (3,1)	+	-
10 $\to$ (3,2)	+	+
10 $\to$ (1,3)+(1,1)	+	-

- It was shown by Murayama and Yanagida Mod.Phys.Lett.A7:147-152,1992. That this split multiplet scenario leads to unification of gauge couplings.

FORBIDDEN PROTON DECAY



- K. R. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. B537: 47-108, 1999 Fermions are restricted to orbifold 'fixed-points'. All Z_2-odd type wave functions vanish at orbifold fixed points. Thus there is no coupling of X *and* Y gauge bosons to low energy quarks and leptons forbidding proton decay

- G. Altarelli, F. Feruglio, Phys.Lett. B511: 257-264, 2001 Absence of tree level amplitudes can provide an explanation of present negative experimental results. Idea of forbidding proton decay by a suitable discrete symmetry is not new, but its physical origin is clear in the present context

- A. Hebecker, John March-Russell, Phys.Lett. B539: 119-125, 2002 Dimension-4 and dimension-5 proton decay operators are absent. They have analyzed dimension-6 proton decay operators in a specific scenario. In their scenario first and second generations are in one brane and the third generation is in a separate brane.

- L. J. Hall and Y. Nomura, Phys. Rev. D 66, 075004 (2002)

  Have done $b-\tau$ unification in the supersymmetric case. They have got $mb(MZ) =$ 3.3$\pm$0.2 GeV. This is a bit high at the scale m_Z, but there are radiative corrections to the bottom quark mass, which, when added to this can give us a satisfactory result.

- C. D. Carone and J. M. Conroy, Phys. Rev. D 70, 075013 (2004). They consider this mechanism in models with gauge trinification. They determine the boundary conditions necessary to break the trinified gauge group directly down to that of the standard model. Working in an effective theory for the gauge-symmetry-breaking parameters on a boundary, they examine the limit in which the grand-unified theory-breaking-sector is Higgsless and show how one may obtain the low-energy particle content of the minimal supersymmetric standard model

OUR MODEL OF ONLY 5-PLETS

WE HAVE CONSTRUCTED A SIMPLE SU(5) GUT MODELS WITH ONLY 5 PLETS OF HIGGS AT THE GUT SCALE.

WE ASSUME THAT THERE ARE ‘n’ OF THEM. THEN USING THE RENORMALISATION GROUP EQUATIONS WE CALCULATE THE NUMBER ‘n’

IN THIS MODEL, ONLY GAUGE BOSONS AND HIGGS SCALARS PROPAGATE IN THE FIFTH DIMENSION, FERMIONS REMAIN IN FOUR DIMENSIONS.

GAUGE STRUCTURE IS EXACTLY THE SAME AS THAT OF THE KAWAMURA MODEL

PARITY CALCULATIONS FOR SCALARS

$SU(5) \supset$	$SU(3)_c \times SU(2)_L \times U(1)_Y$
$\mathbf{5} \supset$	$(1,2,1/2)_{+\,+} + (3,1,-1/3)_{+\,-}$
$\overline{\mathbf{5}} \supset$	$(1,2,-1/2)_{+\,+} + (\overline{3},1,1/3)_{+\,-}$
$\mathbf{10} \supset$	$(1,1,1)_{+\,+} + (\overline{3},1,-2/3)_{+\,+} + (3,2,1/6)_{+\,-}$
$\mathbf{15} \supset$	$(1,3,1)_{+\,+} + (3,2,1/6)_{+\,-} + (6,1,-2/3)_{+\,+}$
$\mathbf{24} \supset$	$(1,1,0)_{+\,+} + (1,3,0)_{+\,+}$ $+(3,2,-5/6)_{+\,-} + (\overline{3},2,5/6)_{+\,-} + (8,1,0)_{+\,+}$
$\mathbf{35} \supset$	$(1,4,-3/2)_{+\,+} + (\overline{3},3,-2/3)_{+\,-} + (\overline{6},2,1/6)_{+\,+}$ $+(\overline{10},1,1)_{+\,-}$
$\mathbf{40} \supset$	$(1,2,-3/2)_{+\,+} + (3,2,1/6)_{+\,+} + (\overline{3},1,-2/3)_{+\,-}$ $+(\overline{3},3,-2/3)_{+\,-}$ $+(8,1,1)_{+\,-} + (\overline{6},2,1/6)_{+\,+}$
$\mathbf{45} \supset$	$(1,2,1/2)_{+\,+} + (3,1,-1/3)_{+\,-} + (3,3,-1/3)_{+\,-}$ $+(\overline{3},1,4/3)_{+\,-}$ $+(\overline{3},2,-7/6)_{+\,+} + (\overline{6},1,-1/3)_{+\,-} + (8,2,1/2)_{+\,+}$
$\mathbf{50} \supset$	$(1,1,-2)_{+\,+} + (3,1,-1/3)_{+\,+} + (\overline{3},2,-7/6)_{+\,-}$ $+(\overline{6},3,-1/3)_{+\,-}$ $+(6,1,4/3)_{+\,+} + (8,2,1/2)_{+\,-}$
$\mathbf{70} \supset$	$(1,2,1/2)_{+\,+} + (1,4,1/2)_{+\,+} + (3,1,-1/3)_{+\,-}$ $+(3,3,-1/3)_{+\,-} + (\overline{3},3,4/3)_{+\,-} + (6,2,-7/6)_{+\,+}$ $+(8,2,1/2)_{+\,+} + (15,1,-1/3)_{+\,-}$
$\mathbf{70'} \supset$	$(1,5,-2)_{+\,+} + (\overline{3},4,-7/6)_{+\,-}$ $+(\overline{6},3,-1/3)_{+\,+} + (\overline{10},2,1/2)_{+\,-}$ $+(\overline{15},1,4/3)_{+\,+}$
$\mathbf{75} \supset$	$(1,1,0)_{+\,+} + (3,1,5/3)_{+\,+} + (3,2,-5/6)_{+\,-}$ $+(\overline{3},1,5/3)_{+\,+} + (\overline{3},2,5/6)_{+\,-} + (\overline{6},2,-5/6)_{+\,-}$ $+(6,2,5/6)_{+\,-} + (8,1,0)_{+\,+}$ $+(8,3,0)_{+\,+}$

SCALAR CONTRIBUTIONS TO RGE

R	light scalar multiplets	T_s^3	T_s^2	T_s^1
5	(1,2,1/2)	0	1/2	3/10
10	(1,1,1)+($\overline{3}$, 1,- 2/3)	1/2	0	7/5
15	(1,3,1)+(6,1,-2/3)	5/2	2	17/5
24	(1,3,0)+(8,1,0)	3	2	0
35	(1,4,-3/2)+($\overline{6}$,2,1/6)	5	8	28/5
40	(1,2,-3/2)+(3,2,1/6)+($\overline{6}$,2,1/6)	6	5	3
45	(1,2,1/2)+($\overline{3}$,2,-7/6)+(8,2,1/2)	7	6	38/5

SEE: B.B and Amitava Raychaudhuri, J.Phys.G29:B5-B14,2003.

Define $m_{k,l} = \ln(M_k/M_l)$ and $b^i_{k,l}$ coefficients range $M_k \leftrightarrow M_l$

$$2\pi\alpha_i^{-1}(M_Z) = 2\pi\alpha_X^{-1} + b^i_{X,I} M_{X,I} + b^i_{I,Z} M_{I,Z}$$

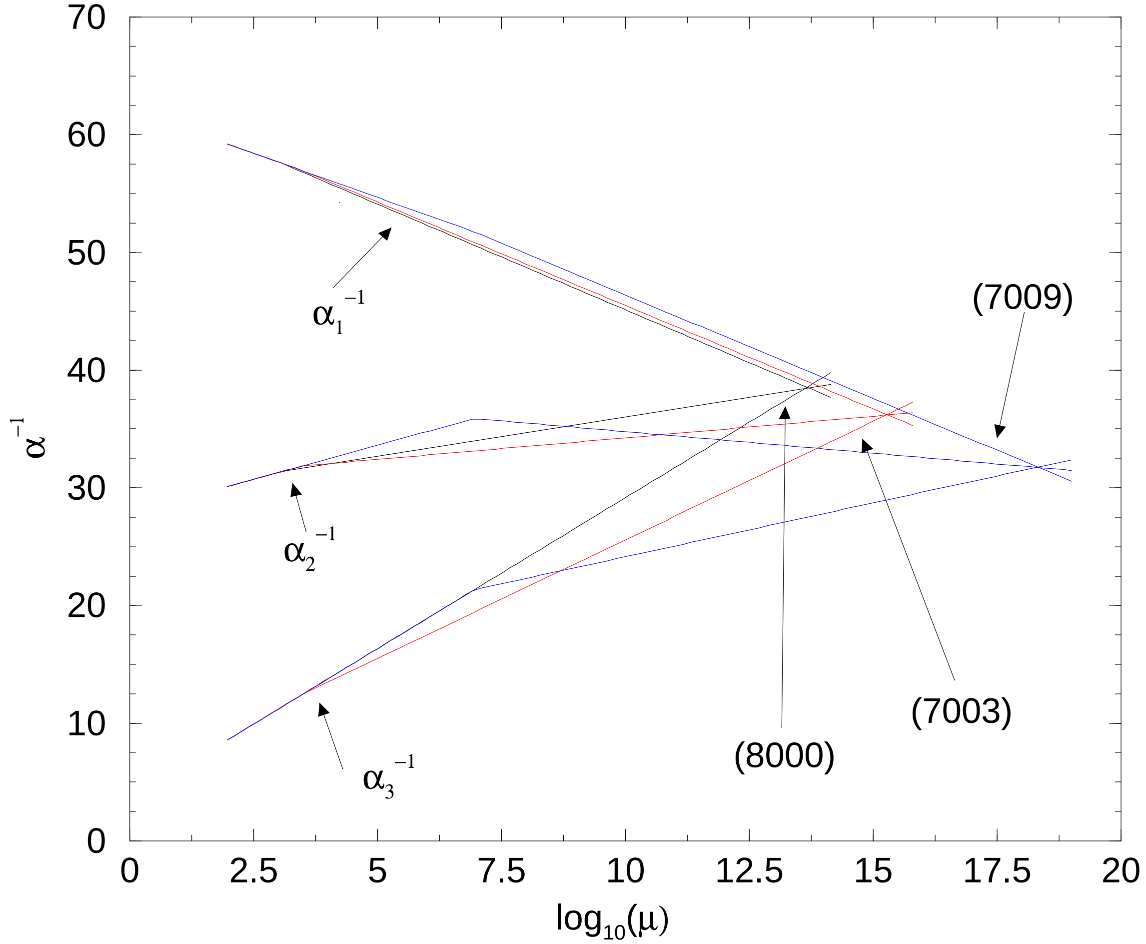
THREE EQUATIONS AND THREE UNKNOWNS

$$b^i_{X,I} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix} + \frac{n_5}{3} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}$$

$$\alpha_X^{-1} = 38.53$$
$$M_{I,Z} = 26.98 - 194.75/n_5$$
$$M_{X,I} = 194.75/n_5$$

Because $M_{I,Z} \geq 0$ we obtain $n_5 \geq 8$. For the case of $n_5 = 8$ we get,

$$M_I = 1.39 \text{ TeV} \quad M_X = 5.0 \times 10^{13} \text{ GeV}$$



Gauge coupling evolution and unification in a few cases. The curves are labeled by $(n_5, n_{10}, n_{15}, n_{24})$ We have considered only one intermediate scale which is the mass of the extra light scalars allowed by $S^1/Z_2 \times Z_2'$ compactification.

A NOTE ON 5-D PLANCK SCALE

$$M_{planck}^{5d} = M_P^{2/3} \times M_c^{1/3}$$

For our case

$$M_P = 10^{19} \quad GeV$$

and

$$M_c = M_{GUT} = 10^{13} \quad GeV$$

Therefore the 5-D Planck scale is at

$$M_{planck}^{5d} = 10^{16.87} \quad GeV$$

CONCLUSIONS AND OUTLOOK

- There is proton decay non-observation problem in conventional GUTs
- There is doublet triplet splitting problem in conventional GUTs
- These two problems can be solved if SU(5) symmetry exists in a 5-D world
- Here the simplest model has nine doublets (1+8) at low energy. If we count the number of fermions in standard model, there are 6 quarks and 3 leptons. Therefore the total number of fermions are nine per generation. So may be we have one doublet for each of them. It is not a bad puzzle.

CONCLUSIONS AND OUTLOOK

- We have compactified the fifth dimension on a $S^1/Z_2 \times Z_2'$ orbifold. These extra symmetries can make the proton quite stable. This type of model being non-supersymmetric, there is no R-parity violating proton decay.

- There is no neutrino mass in this model. However we can add heavy right handed neutrinos in the GUT scale. This is a heavy right handed fermion of mass 10^{13} GeV or so. Then via see-saw mechanism one can produce a small majorana mass of the order of

$$m = \frac{m_D^2}{M_{GUT}}$$

  Here, we can see that, in our case $M_{GUT} \sim 10^{13}$. Therefore,

$$m \sim \frac{(10^2)^2}{10^{13}} \sim 10^{-9} \text{ GeV} \sim 1 \text{ eV}.$$

CONCLUSIONS AND OUTLOOK

- We should study Kaluza-Klein type dark matter in this model.

- It can be seen that Z_2 parity is the same for all multiplets of $scalars$. Only the Z_2' changes. Therefore is Z_2 redundant ? We have kept it here even though it is of no use $for\ scalars$. This is because if we want to construct the supersymmetric version of the theory, we may have to use it.

- One can study $b-\tau$ unification in this model. The simplest way is to couple only one doublet to Fermions. However all nine doublets contribute to gauge coupling unification.