QCD thermodynamics and quark number susceptibilities at intermediate coupling

Nan Su

Collaborators:

Jens O. Andersen & Lars E. Leganger (Trondheim), Michael Strickland (Gettysburg)

Phys. Rev. Lett. 104, 122003 (2010); JHEP 1008, 113 (2010);

Phys. Lett. B 696, 468 (2011); JHEP 1108, 053 (2011)

Jens O. Andersen (Trondheim), Sylvain Mogliacci & Aleksi Vuorinen (Bielefeld), forthcoming

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Universität Bielefeld

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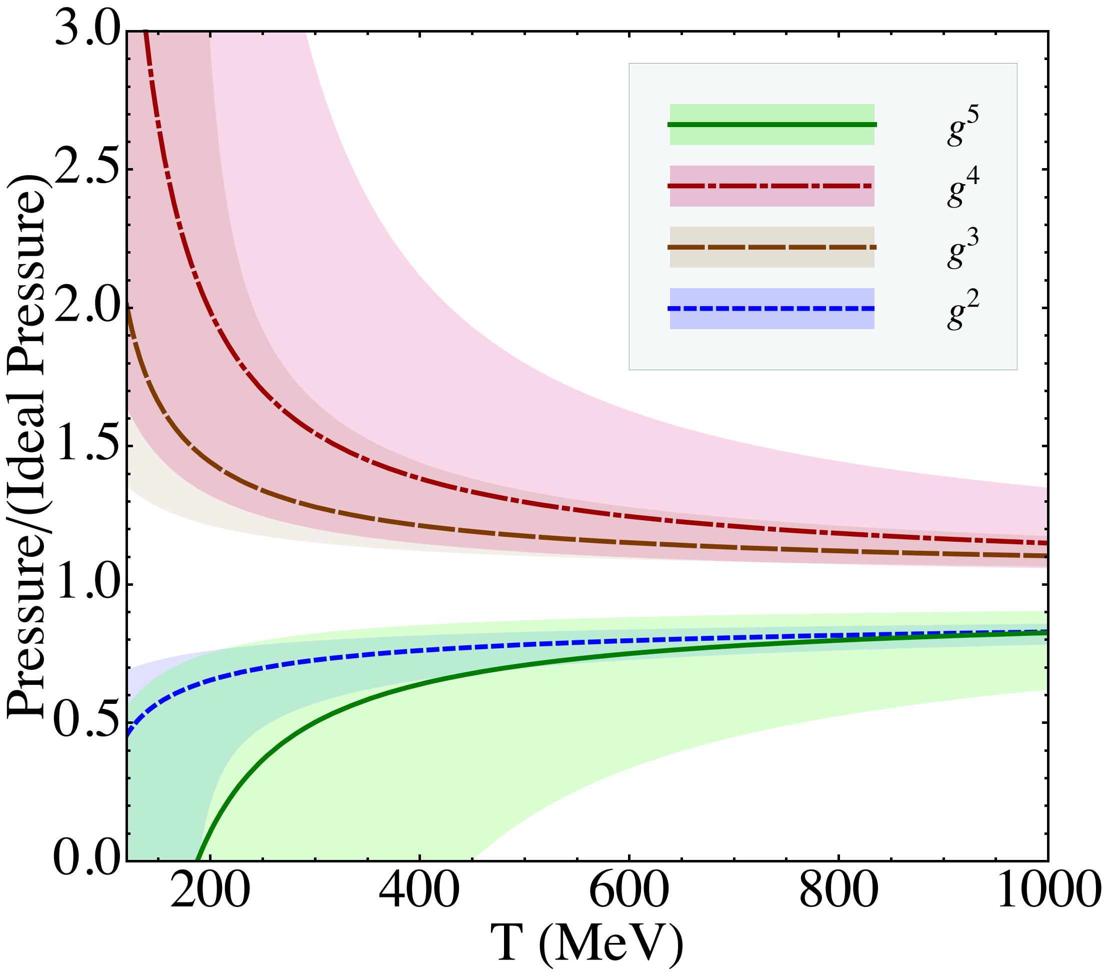
Introduction: Heavy ion collisions $\rightarrow$ QGP or QGL?

- RHIC: $T_0 \sim$ 350 MeV $\sim 2\,T_c$.
- LHC: $T_0 \sim$ 800 - 1000 MeV ~ 4 - $6\,T_c$.
- Quark Gluon Plasma (QGP) or Quark Gluon Liquid (QGL) at LHC?
- Running coupling expected is $g \sim 2$ or $\alpha_s \sim 0.3$.
- Neither infinitesimally small, nor infinitely large: intermediate coupling.
- Can pQCD methods reproduce lattice data for thermodynamic functions at such “intermediate” couplings ($g \sim 2$)?
- More importantly the resulting machinery should be able to address dynamics as well.

Intro: Nonconvergence of canonical thermal QCD



Weak-coupling expansion QCD free energy with $N_c = 3$ and $N_f = 3$ vs temperature.

($\pi T \leq \Lambda \leq 4\pi T$ & $\alpha_s = g^2/4\pi$)

- Weak-coupling expansion of the QCD free energy, $\mathcal{F}$, is known to order $g^6 \log g$.[1,2,3,4,5,6,7]
- At temperatures expected at RHIC energies, $T \sim 0.35$ GeV, the running coupling constant α_s is approximately 0.3, or $g \sim 2$.
- Successive terms contributing to $\mathcal{F}$ can strictly only form a decreasing series if $\alpha_s \lesssim 1/20$ which corresponds to $T \sim 10^5$ GeV.

[1] Shuryak, 78.

[2] Kapusta, 79.

[3] Toimela, 85.

[4] Arnold and Zhai, 94/95.

[5] Kastening and Zhai, 95.

[6] Braaten and Nieto, 96.

[7] Kajantie, Laine, Rummukainen and Schröder, 02.

Anharmonic oscillator: Small coupling $\stackrel{?}{=}$ perturbative

- Consider the perturbation series for the ground state energy, E, of a simple anharmonic oscillator with potential

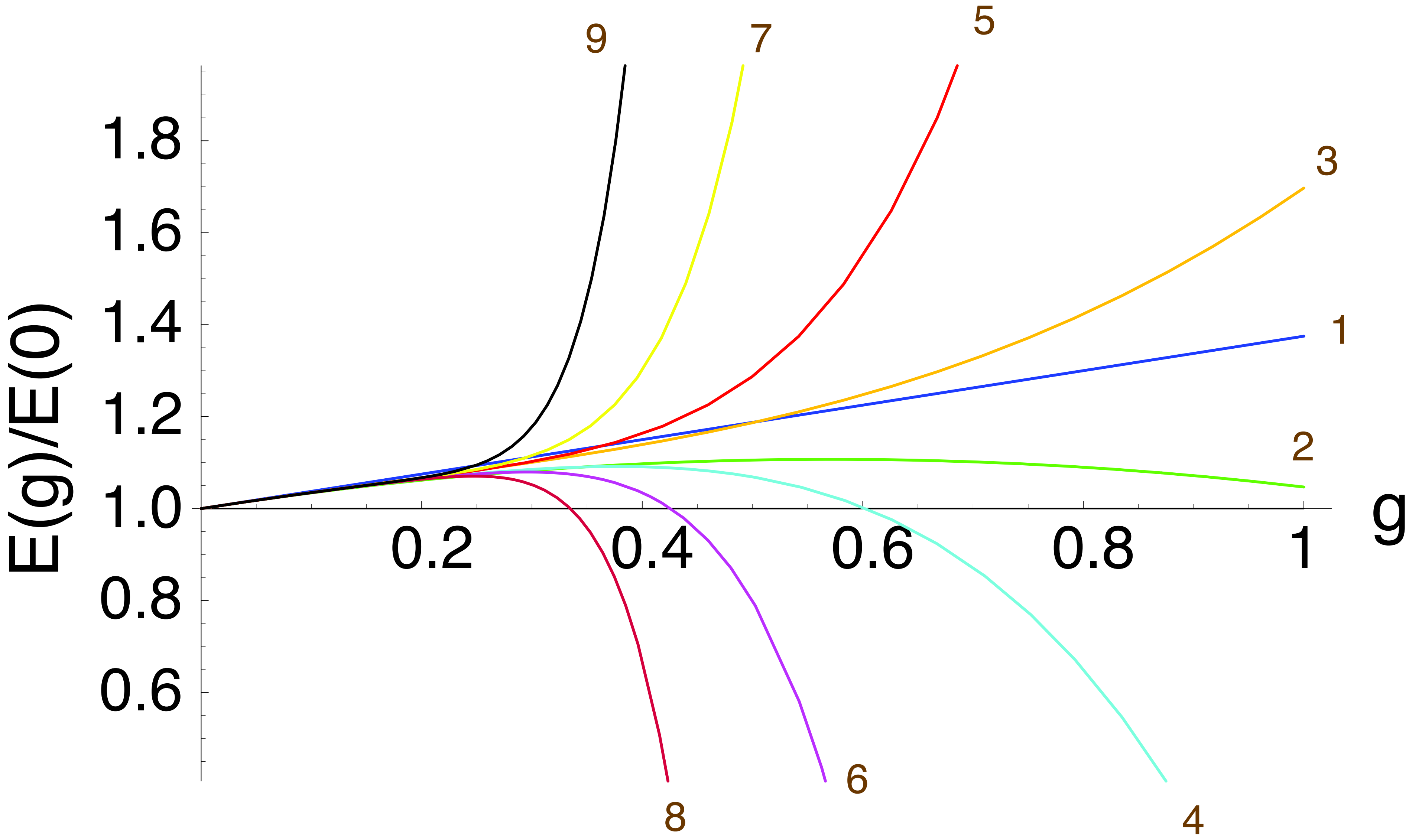
$$V(x) = \frac{1}{2}\omega^2 x^2 + \frac{g}{4}x^4 \qquad (\omega^2, g > 0)$$

- Weak-coupling expansion of the ground state energy $E(g)$ is known to all orders (Bender and Wu, 69/73)

$$E(g) = \omega \sum_{n=0}^{\infty} c_n^{\mathrm{BW}} \left(\frac{g}{4\omega^3}\right)^n, \quad c_n^{\mathrm{BW}} = \left\{\frac{1}{2}, \frac{3}{4}, -\frac{21}{8}, \frac{333}{16}, -\frac{30885}{16}, \ldots\right\}$$

- $\lim_{n\to\infty} c_n^{\mathrm{BW}} = (-1)^{n+1}\sqrt{\frac{6}{\pi^3}}3^n(n-\frac{1}{2})!$
- Because of the factorial growth, the expansion is an asymptotic series with zero radius of convergence!

Anharmonic oscillator: Small coupling $\neq$ perturbative!!!



Variational perturbation theory (Janke and Kleinert, 95/97)

- Split the harmonic term into two pieces (with $r \equiv \frac{2}{g}\left(\omega^2 - \Omega^2\right)$)

$$\omega^2 \to \Omega^2 + \left(\omega^2 - \Omega^2\right) \quad \Longrightarrow \quad V_{\text{int}}(x) = \frac{g}{4}\left(rx^2 + x^4\right)$$

- Weak-coupling expansion in g

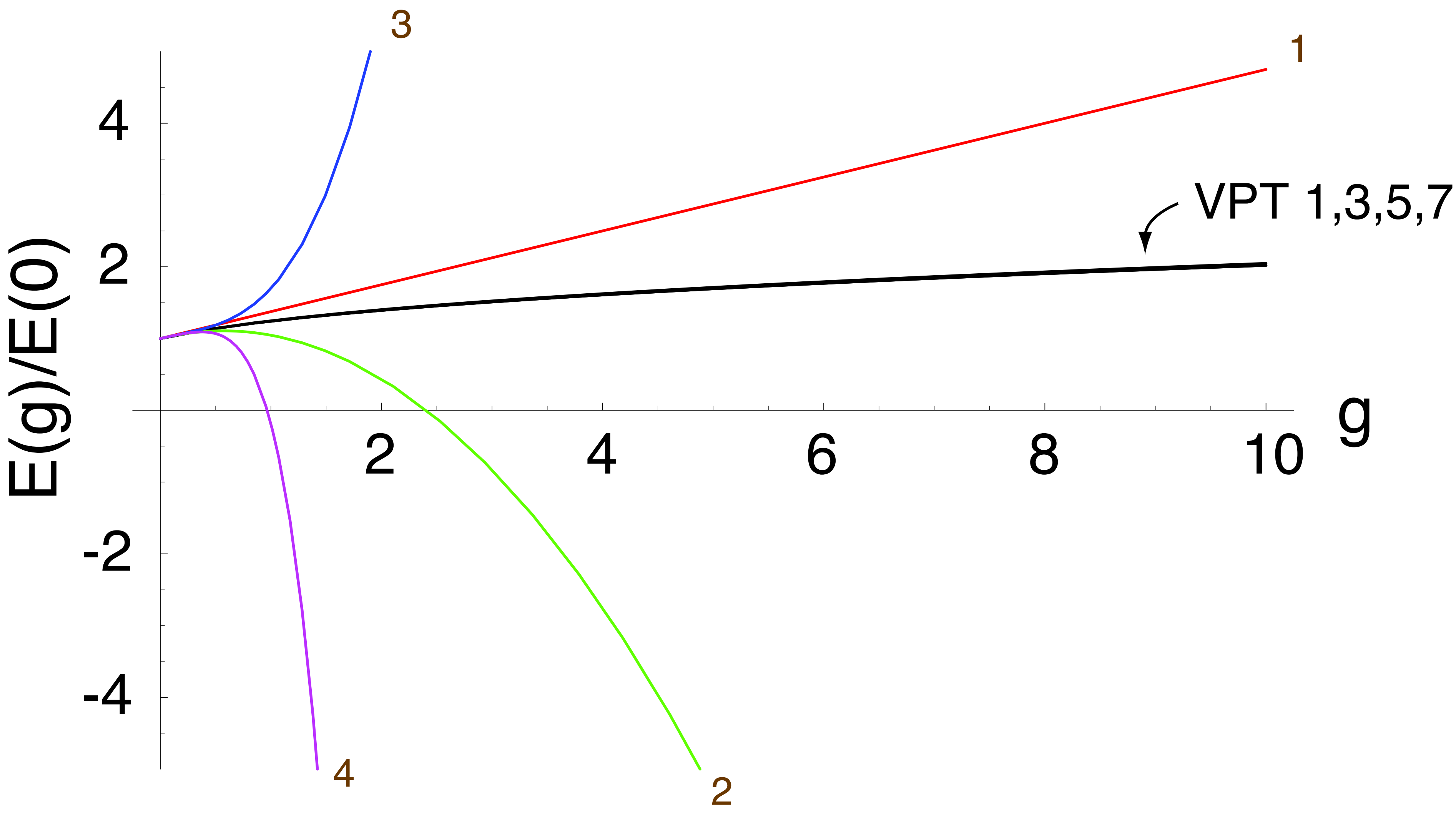
$$E_N(g,r) = \Omega \sum_{n=0}^{N} c_n(r) \left(\frac{g}{4\Omega^3}\right)^n$$

- New coefficients c_n can be obtained by

$$c_n(r) = \sum_{j=0}^{n} c_j^{\text{BW}} \begin{pmatrix} (1-3j)/2 \\ n-j \end{pmatrix} (2r\Omega)^{n-j}$$

- Fix Ω_N by requiring that at each order N: $\left.\frac{\partial E_N}{\partial \Omega}\right|_{\Omega=\Omega_N} = 0$

VPT: PT can tackle strong coupling!!!



Hard-thermal-loop perturbation theory (HTLpt)

- HTLpt is a reorganization of thermal pQCD in spirit to VPT
  (Andersen, Braaten and Strickland, 99)

$$\mathcal{L}_{\rm HTLpt} = \left(\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL} - \delta\mathcal{L}_{\rm HTL}\right)\Big|_{g\to\sqrt{\delta}g}$$

- Hard-Thermal-Loop (HTL) effective action reads

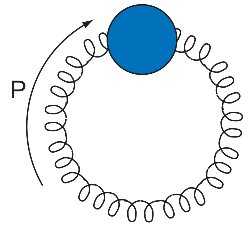
$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}m_D^2 \,{\rm Tr}\left(G_{\mu\alpha}\left\langle \frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y G^\mu{}_\beta\right)$$

- Adding $\mathcal{L}_{\rm HTL}$ SHIFTS the expansion to an ideal gas of massive quasiparticles – appropriate d.o.f. at high T.
- δ: # of HTL dressed loops. Interested in $T > 2-3\,T_c$.

HTLpt: LO free energy for Yang-Mills

- Separation into hard and soft contributions ($d = 3 - 2\epsilon$)

$$\mathcal{F}_g = -\frac{1}{2}\sum\!\!\!\!\!\!\!\int_P \left\{(d-1)\ln[-\Delta_T^{\rm HTL}(P)] + \ln[\Delta_L^{\rm HTL}(P)]\right\}$$



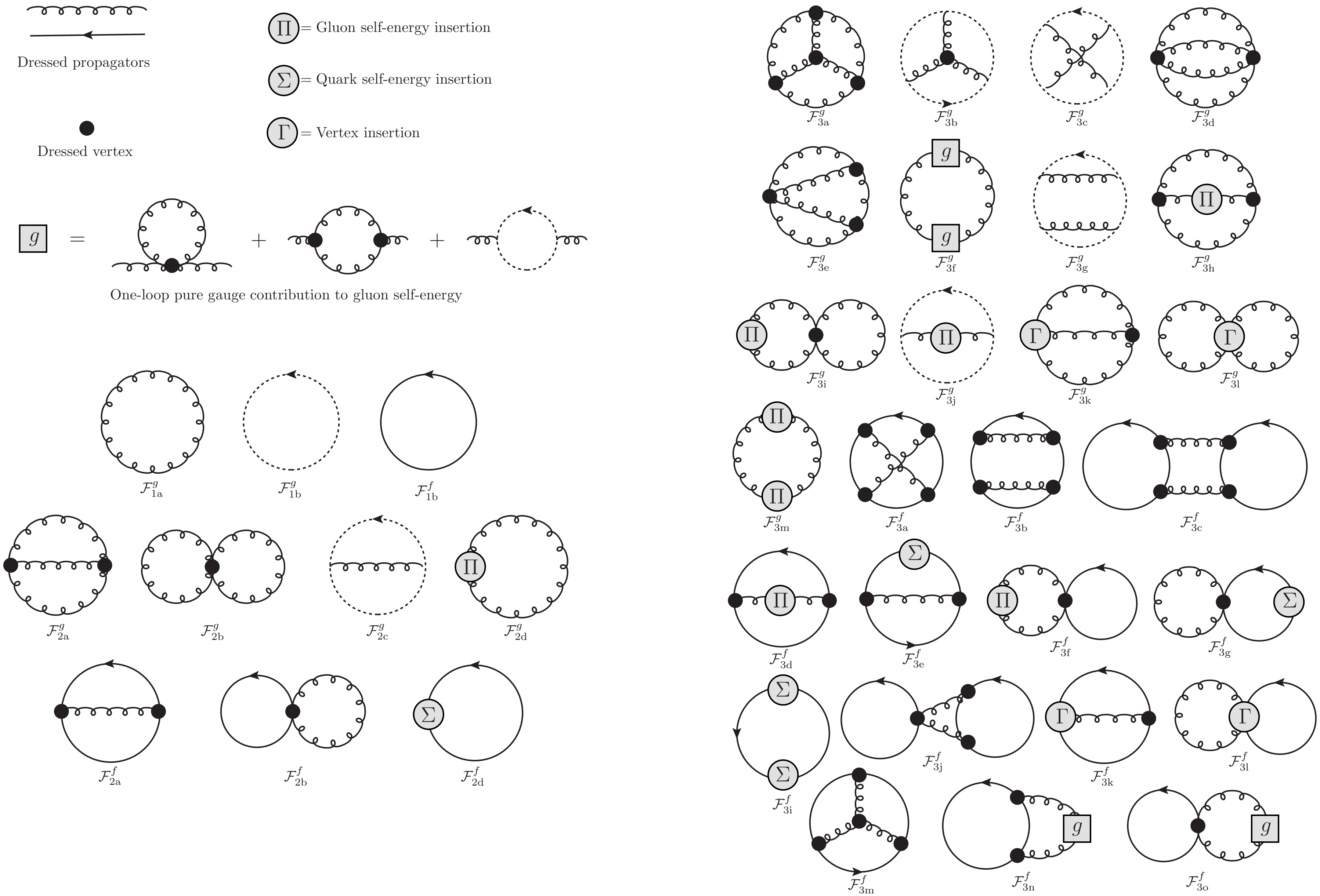
  - Hard momenta $(\omega, \mathbf{p} \sim T)$

$$\mathcal{F}_g^{(h)} = \frac{d-1}{2}\sum\!\!\!\!\!\!\!\int_P \ln(P^2) + \frac{1}{2}m_D^2 \sum\!\!\!\!\!\!\!\int_P \frac{1}{P^2} - \frac{1}{4(d-1)} m_D^4 \sum\!\!\!\!\!\!\!\int_P \left[\frac{1}{(P^2)^2} - 2\frac{1}{p^2 P^2} - 2d\frac{1}{p^4}\mathcal{T}_P + 2\frac{1}{p^2 P^2}\mathcal{T}_P + d\frac{1}{p^4}(\mathcal{T}_P)^2\right] + \mathcal{O}(m_D^6)$$

  - Soft momenta $(\omega, \mathbf{p} \sim gT)$

$$\mathcal{F}_g^{(s)} = \frac{1}{2}T\int_{\mathbf{p}} \ln(p^2 + m_D^2)$$

HTLpt: QCD diagrams through NNLO



HTLpt: NNLO thermodynamic potential for QCD

- For QCD with general N_c and N_f ($\mathcal{F}_{\rm ideal} \equiv -\frac{(N_c^2-1)\pi^2 T^4}{45}$, $\hat{x}_D \equiv \frac{x}{2\pi T}$)

$$
\begin{aligned}
\frac{\Omega^T_{\rm NNLO}}{\mathcal{F}_{\rm ideal}} = \; & 1+\frac{7}{4}\frac{d_F}{d_A}-\frac{15}{4}\hat{m}_D^3+\frac{c_A\alpha_s}{3\pi}\left[-\frac{15}{4}+\frac{45}{2}\hat{m}_D-\frac{135}{2}\hat{m}_D^2-\frac{495}{4}\left(\log\frac{\hat{\Lambda}}{2}+\frac{5}{22}+\gamma_E\right)\hat{m}_D^3\right] \\
& +\frac{s_F\alpha_s}{\pi}\left[-\frac{25}{8}+\frac{15}{2}\hat{m}_D+15\left(\log\frac{\hat{\Lambda}}{2}-\frac{1}{2}+\gamma_E+2\log 2\right)\hat{m}_D^3-90\hat{m}_q^2\hat{m}_D\right] \\
& +\left(\frac{c_A\alpha_s}{3\pi}\right)^2\left[\frac{45}{4}\frac{1}{\hat{m}_D}-\frac{165}{8}\left(\log\frac{\hat{\Lambda}}{2}-\frac{72}{11}\log\hat{m}_D-\frac{84}{55}-\frac{6}{11}\gamma_E-\frac{74}{11}\frac{\zeta'(-1)}{\zeta(-1)}\right.\right. \\
& \left.\left.+\frac{19}{11}\frac{\zeta'(-3)}{\zeta(-3)}\right)+\frac{1485}{4}\left(\log\frac{\hat{\Lambda}}{2}-\frac{79}{44}+\gamma_E+\log 2-\frac{\pi^2}{11}\right)\hat{m}_D\right] \\
& +\left(\frac{c_A\alpha_s}{3\pi}\right)\left(\frac{s_F\alpha_s}{\pi}\right)\left[\frac{15}{2}\frac{1}{\hat{m}_D}-\frac{235}{16}\left(\log\frac{\hat{\Lambda}}{2}-\frac{144}{47}\log\hat{m}_D-\frac{24}{47}\gamma_E+\frac{319}{940}+\frac{111}{235}\log 2\right.\right. \\
& \left.\left.-\frac{74}{47}\frac{\zeta'(-1)}{\zeta(-1)}+\frac{1}{47}\frac{\zeta'(-3)}{\zeta(-3)}\right)+\frac{315}{4}\left(\log\frac{\hat{\Lambda}}{2}-\frac{8}{7}\log 2+\gamma_E+\frac{9}{14}\right)\hat{m}_D+90\frac{\hat{m}_q^2}{\hat{m}_D}\right] \\
& +\left(\frac{s_F\alpha_s}{\pi}\right)^2\left[\frac{5}{4}\frac{1}{\hat{m}_D}+\frac{25}{12}\left(\log\frac{\hat{\Lambda}}{2}+\frac{1}{20}+\frac{3}{5}\gamma_E-\frac{66}{25}\log 2+\frac{4}{5}\frac{\zeta'(-1)}{\zeta(-1)}-\frac{2}{5}\frac{\zeta'(-3)}{\zeta(-3)}\right)\right. \\
& \left.-15\left(\log\frac{\hat{\Lambda}}{2}-\frac{1}{2}+\gamma_E+2\log 2\right)\hat{m}_D+30\frac{\hat{m}_q^2}{\hat{m}_D}\right]+s_{2F}\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{15}{64}(35-32\log 2)-\frac{45}{2}\hat{m}_D\right]
\end{aligned}
$$

PURELY ANALYTIC!!!

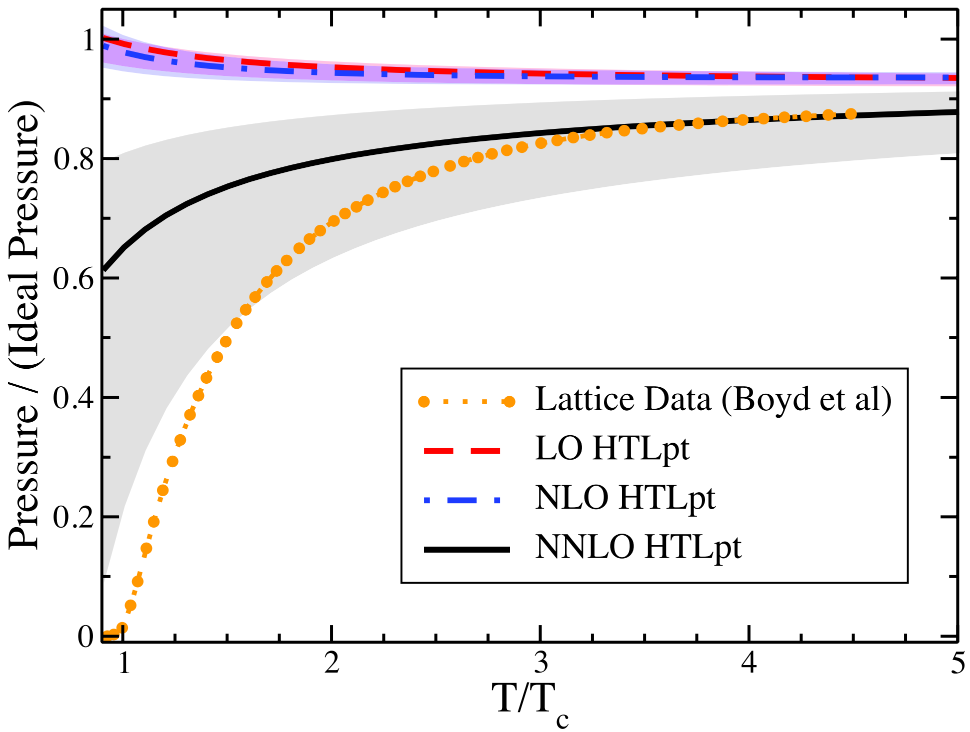
HTLpt: Mass prescriptions for QCD

- NNLO gap equations give complex variational m_D and $m_q = 0$.
- Weak-coupling expansion of Debye mass involves the nonpert. magnetic scale $g^2 T$ (Linde, 80; Gross, Pisarski and Yaffe, 81) beyond LO and therefore is IR divergent (Rebhan, 94; Arnold and Yaffe, 95).
- Use the gauge-invariant NLO electric mass from dimensional reduction: hard contribution (from the scale T) to Debye mass and well defined to all orders (Braaten and Nieto, 96)

$$m_D^2 = \frac{4\pi\alpha_s}{3}T^2 \left\{ c_A + s_F + \frac{c_A^2\alpha_s}{3\pi}\left(\frac{5}{4} + \frac{11}{2}\gamma_E + \frac{11}{2}\log\frac{\hat{\Lambda}}{2}\right) \right.$$
$$+ \frac{c_A s_F \alpha_s}{\pi}\left(\frac{3}{4} - \frac{4}{3}\log 2 + \frac{7}{6}\gamma_E + \frac{7}{6}\log\frac{\hat{\Lambda}}{2}\right)$$
$$\left. + \frac{s_F^2\alpha_s}{\pi}\left(\frac{1}{3} - \frac{4}{3}\log 2 - \frac{2}{3}\gamma_E - \frac{2}{3}\log\frac{\hat{\Lambda}}{2}\right) - \frac{3}{2}\frac{s_{2F}\alpha_s}{\pi}\right\}$$

HTLpt: Yang-Mills and QCD free energies

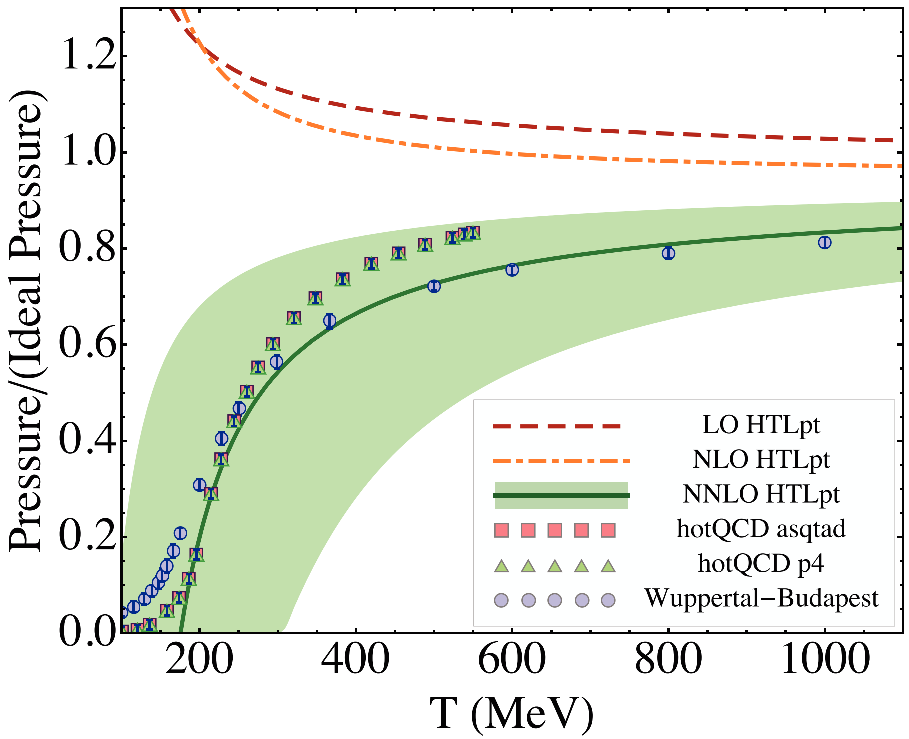
Yang-Mills



Andersen, Strickland and Su,

PRL 104, 122003 (2010) & JHEP 1008, 113 (2010)

QCD with $N_f = 3$

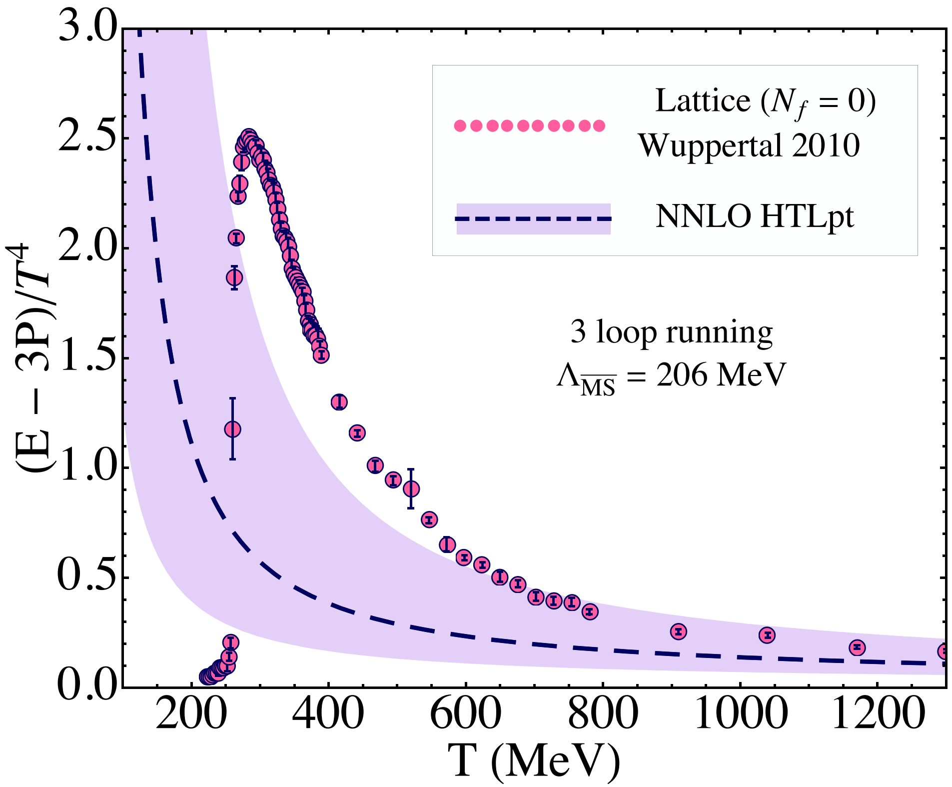


Andersen, Leganger, Strickland and Su,

PLB 696, 468 (2011) & JHEP 1108, 053 (2011)

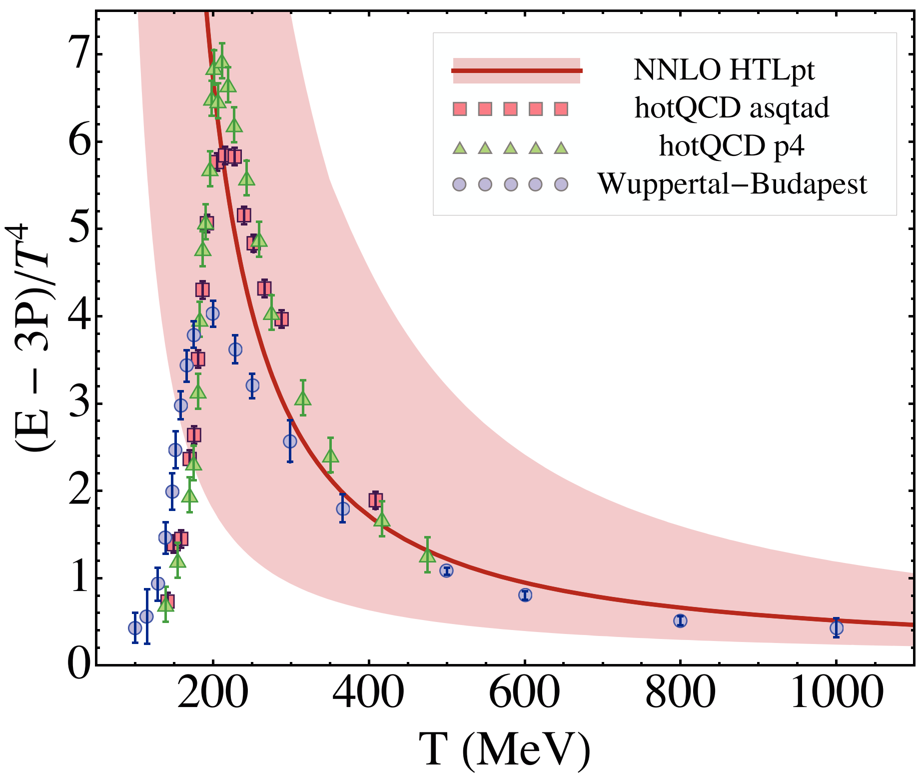
HTLpt: Y-M and QCD trace anomalies scaled by T^4

Yang-Mills



Andersen, Strickland and Su, JHEP 1008, 113 (2010)

QCD with $N_f = 3$



Andersen, Leganger, Strickland and Su,

PLB 696, 468 (2011) & JHEP 1108, 053 (2011)

HTLpt: LO quark number susceptibility

- Including chemical potential: $\omega_n^{(f)} = (2n+1)\pi T - i\mu$
- LO thermodynamic potential with both T and μ reads

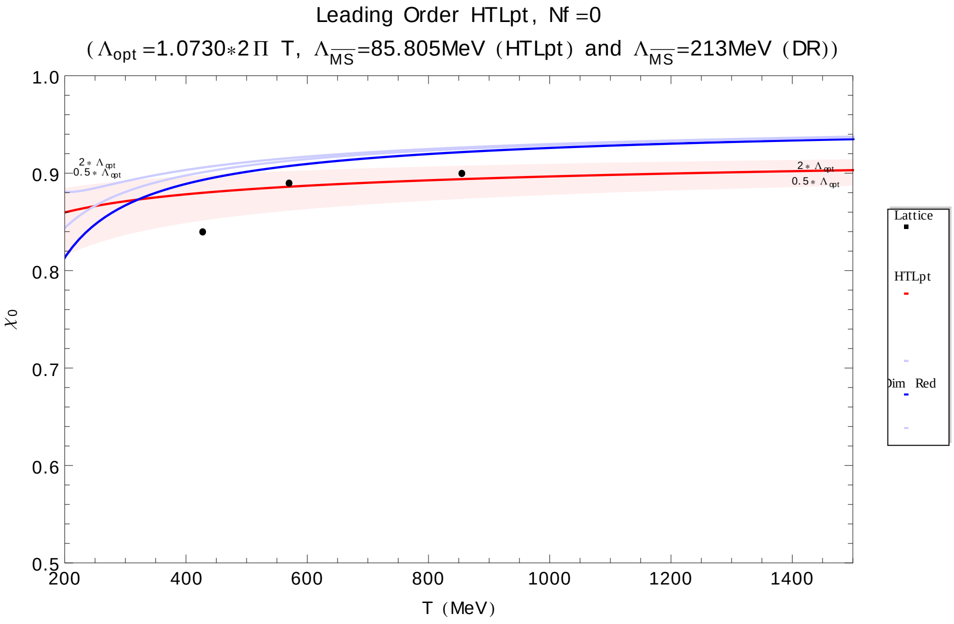
$$\frac{\Omega_{\rm LO}^{T,\mu}}{\mathcal{F}_{\rm ideal}} = 1+\frac{7}{4}\frac{d_F}{d_A}\left(1+\frac{120}{7}\hat{\mu}^2+\frac{240}{7}\hat{\mu}^4\right)-\frac{15}{2}\hat{m}_D^2+30\hat{m}_D^3+\frac{45}{4}\left(\gamma_E-\frac{7}{2}+\frac{\pi^2}{3}+\log\frac{\hat{\Lambda}}{2}\right)\hat{m}_D^4$$
$$-30\frac{d_F}{d_A}\left(1+12\hat{\mu}^2\right)\hat{m}_q^2+60\frac{d_F}{d_A}\left(6-\pi^2\right)\hat{m}_q^4+\mathcal{O}(m^6)$$

- Quark number susceptibility is obtained from

$$\chi(T,\alpha_s) \equiv -\frac{\partial^2}{\partial\mu^2}\Omega(T,\mu,\alpha_s)\bigg|_{\mu=0}$$

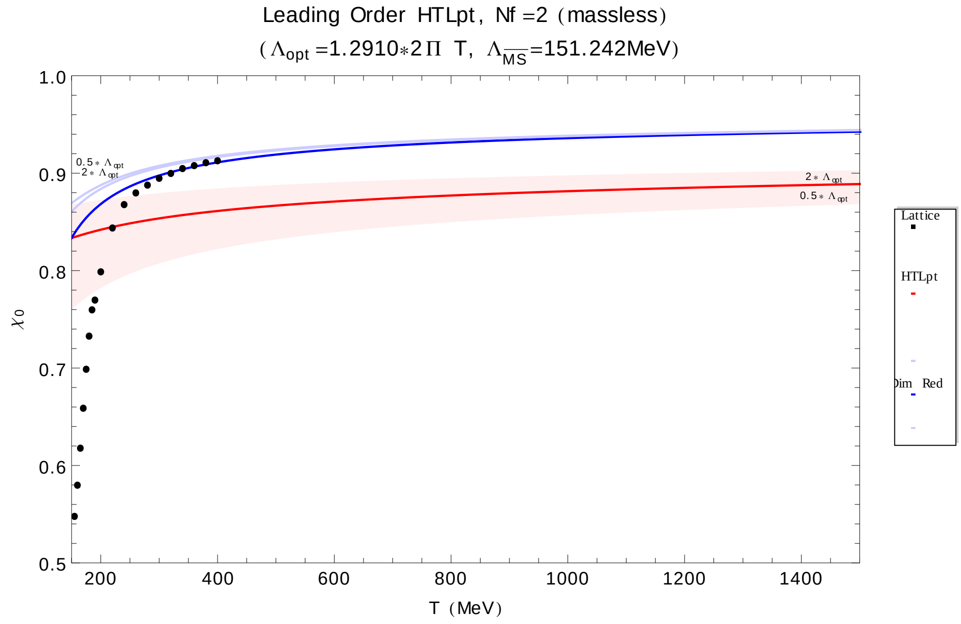
HTLpt: Preliminary results of LO susceptibilities

$N_f = 0$



Lattice data from Gavai and Gupta, PRD 67, 03450 (2003)

$N_f = 2$



Lattice data from the Wuppertal-Budapest collaboration, JHEP 1201, 138 (2012)

Andersen, Mogliacci, Su and Vuorinen (forthcoming)

Conclusions

- Poor convergence of weak-coupling expansion is generic: not just in field theory, but even in quantum mechanics.
- Generalized from VPT, HTLpt can improve the convergence of perturbative calculations in a gauge-invariant manner.
- NNLO HTLpt results for YM look very good for $T > 2 - 3\,T_c$. After including quarks the QCD ones are even better! Especially considering that there are no free parameters to fit.
- LO HTLpt susceptibilities are in reasonable agreement with lattice results, since it is only the free gas limit.
- In HTLpt, the nonpert. magnetic scale g^2T and center symmetry $Z(N_c)$ are not incorporated. They are important ingredients when approaching T_c.

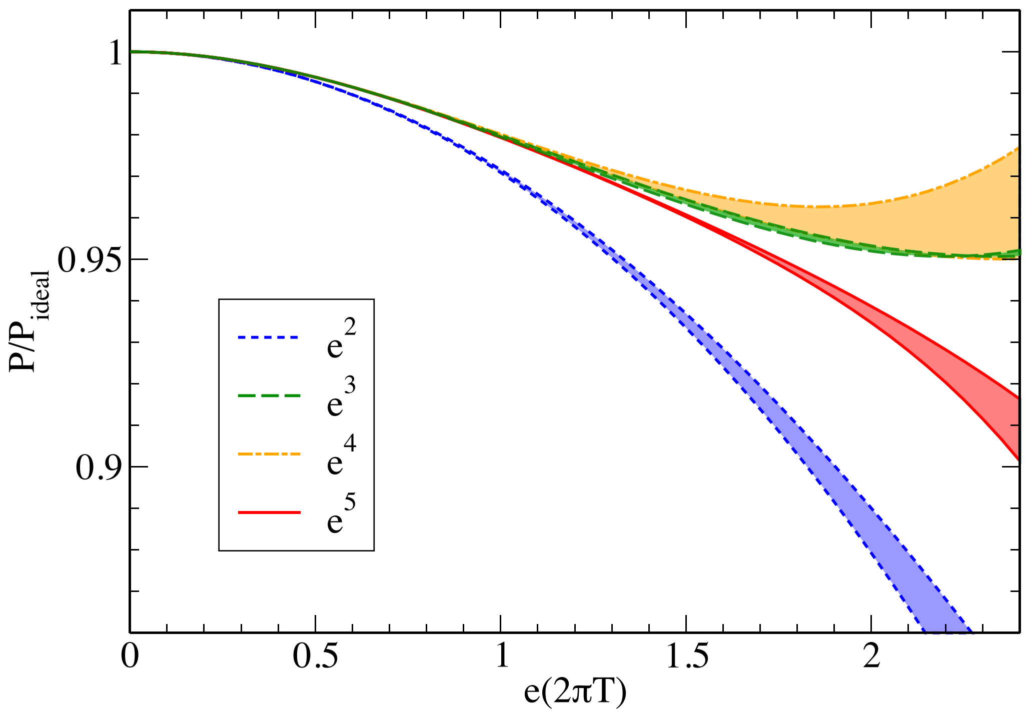
Outlook

- Weak-coupling expansion of realtime dynamics is as badly convergent as thermodynamics.
- Since HTLpt is formulated in Minkowski space, it provides a systematic and selfconsistent calculation scheme for both thermodynamics and dynamics.
- NNLO QCD thermodynamics calculation sets the stage of generalizing HTLpt to dynamic quantities, such as jet energy loss, viscosities, momentum diffusion, etc, at LHC relevant T.
- Exploring the applicability of HTLpt/SPT to other systems:
  - Ultracold quantum gases and BCS-BEC crossover
  - Quark stars and astrophysics
  - Strong laser field and QED plasma
  - . . . . . .

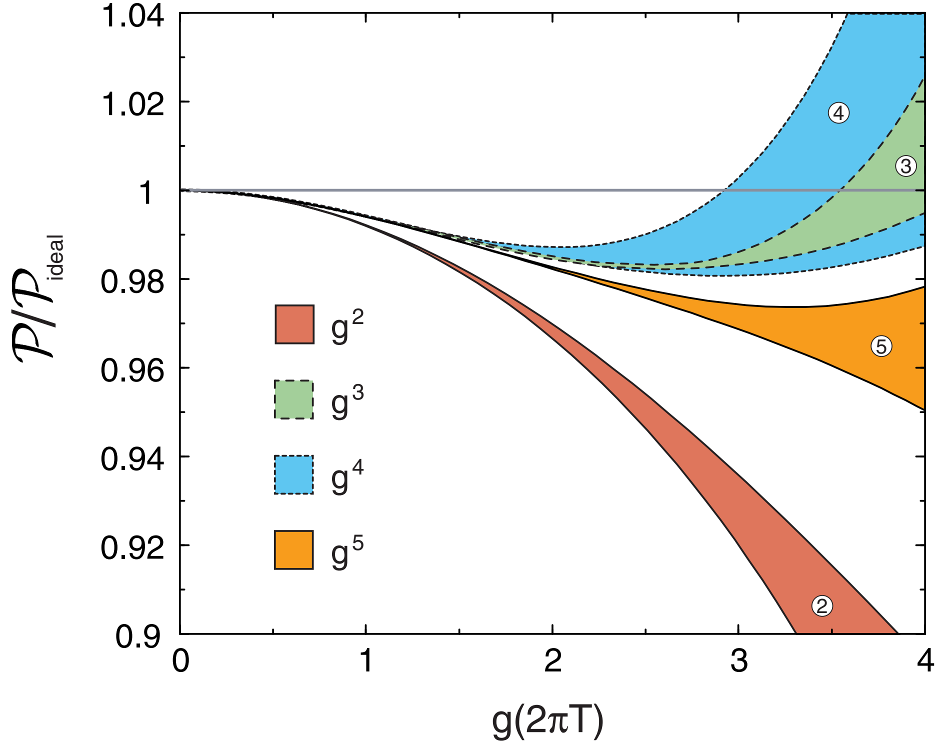
Backup

Perturbative QED and scalar ϕ^4 thermodynamics

QED



Scalar ϕ^4



Same nonconvergence pattern as QCD!

Finite temperature field theory primer

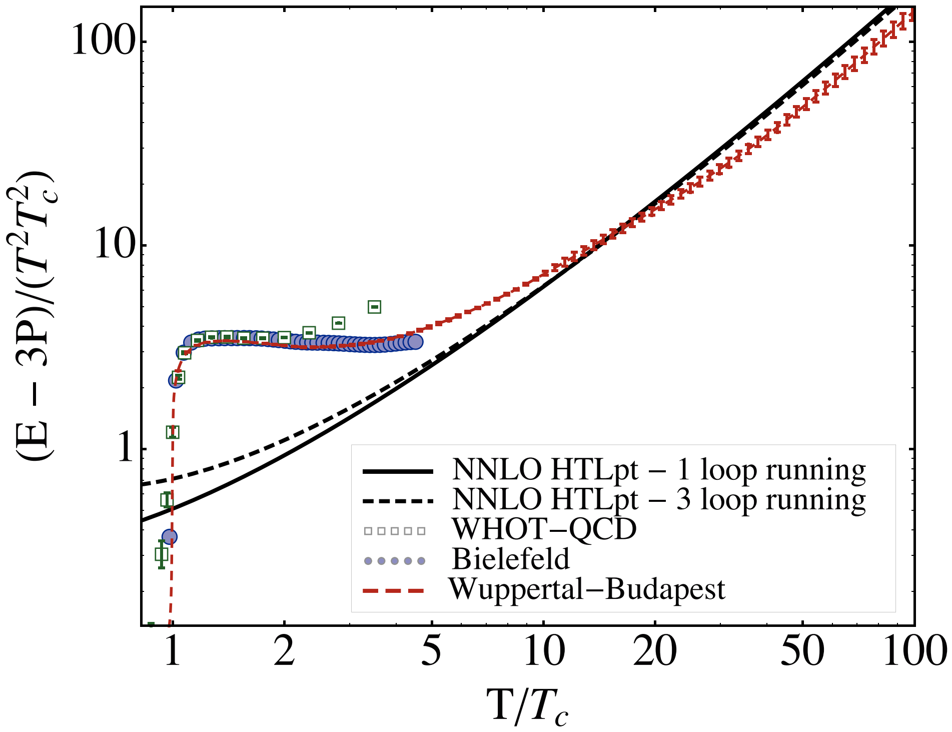
- T much bigger than particle's bare masses: massless particles.
- More scales than in the vacuum. Nonanalytic contributions to the free energy from the medium.

a_0	$a_2 g^2$		$a_4 g^4$		$a_6 g^6$	$\ldots$	(T, hard)
		$b_3 g^3$	$b_4 g^4$	$b_5 g^5$	$b_6 g^6$	$\ldots$	(gT, soft or electric, HTL)
					$c_6 g^6$	$\ldots$	($g^2 T$, ultrasoft or magnetic)
							Linde problem, nonpert.!
d_0	$d_2 g^2$	$d_3 g^3$	$d_4 g^4$	$d_5 g^5$	$d_6 g^6$	$\ldots$	

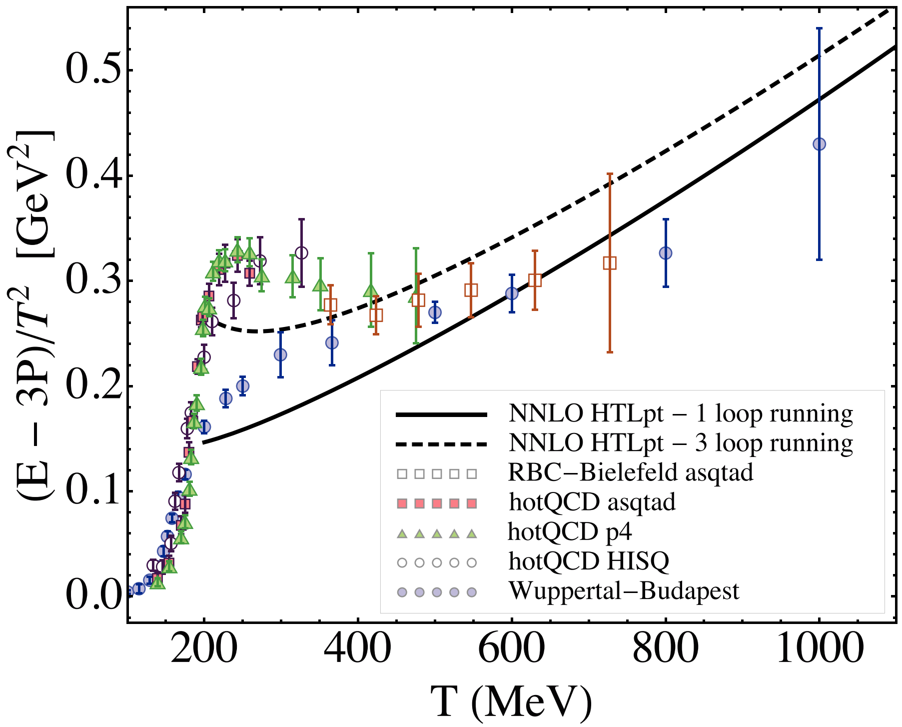
- gT defines the (LO) Debye mass.
- Massless particles become massive quasiparticles.
- Weak-coupling expansion expands around an ideal gas of massless particles which is NOT the appropriate d.o.f. at high T.

HTLpt: Y-M and QCD trace anomalies scaled by T^2

Yang-Mills



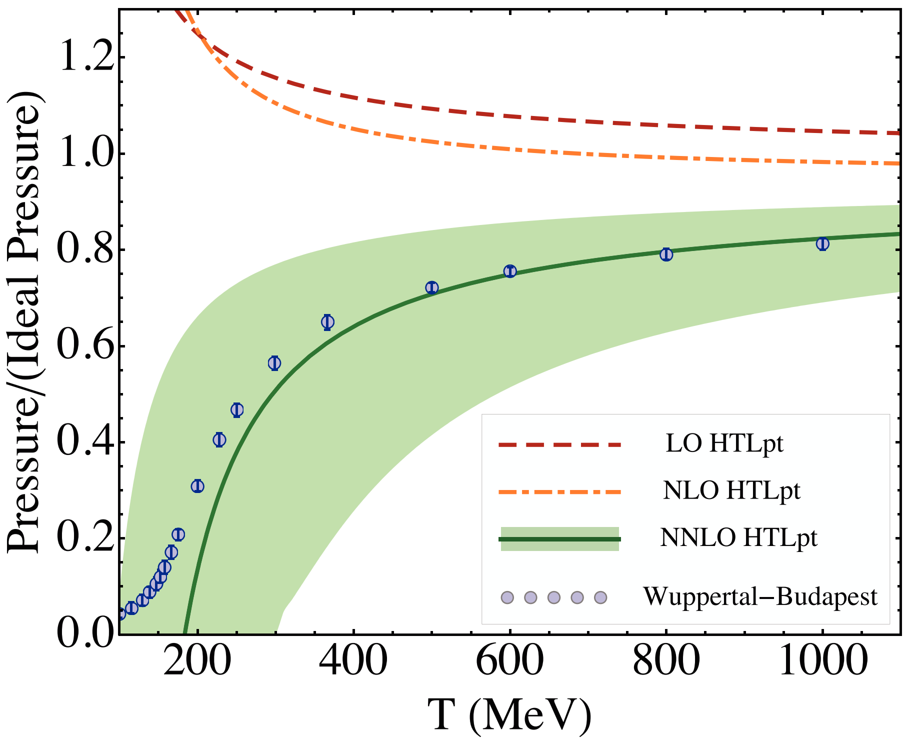
QCD with $N_f = 3$



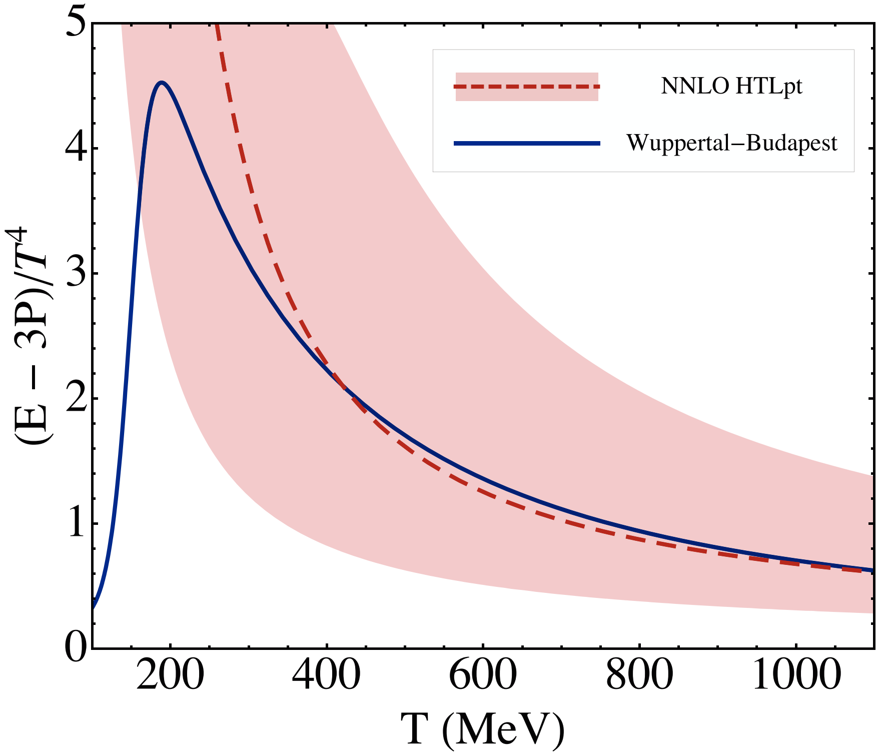
Andersen, Leganger, Strickland and Su, PRD 84, 087703 (2011)

HTLpt: $N_f = 4$ QCD thermodynamics

Free energy

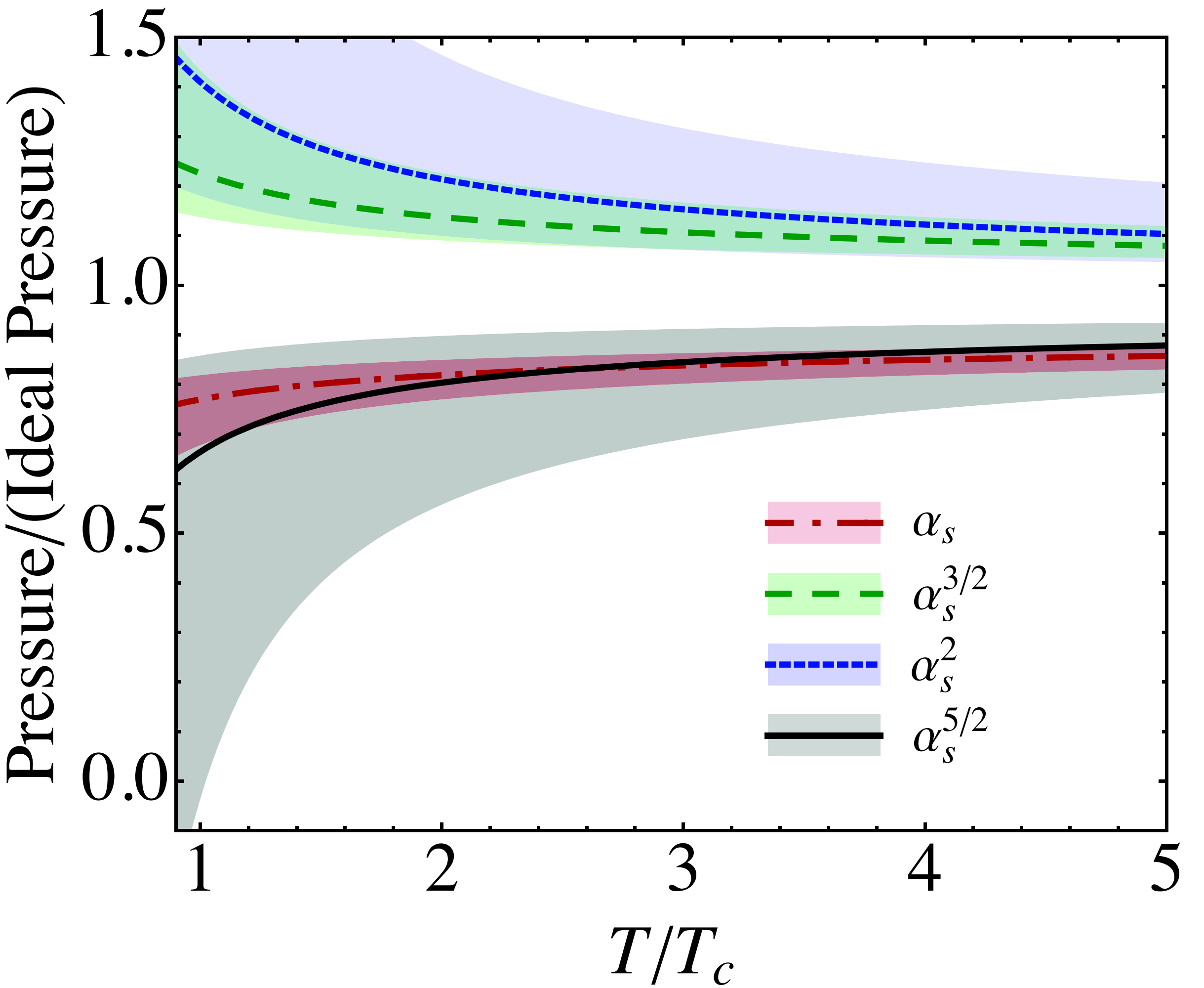


Trace anomaly



Andersen, Leganger, Strickland and Su, PLB 696, 468 (2011)

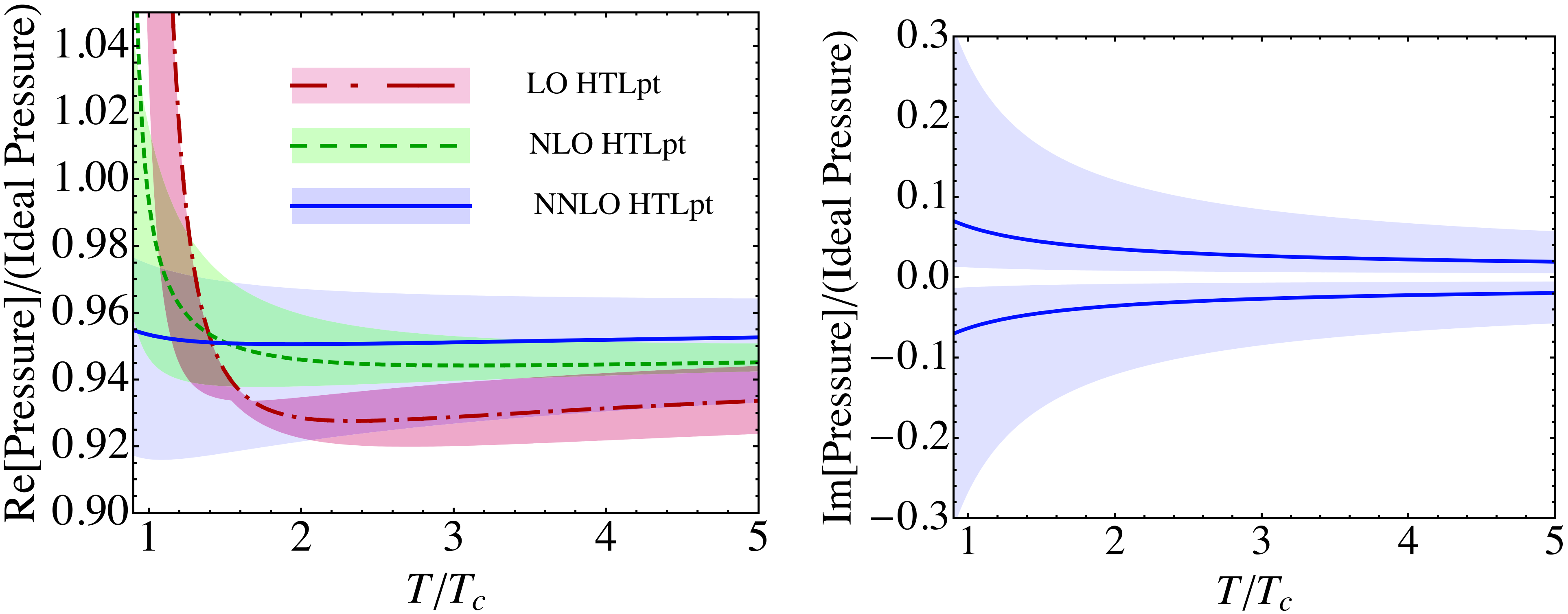
YM: Weak-coupling expansion free energy



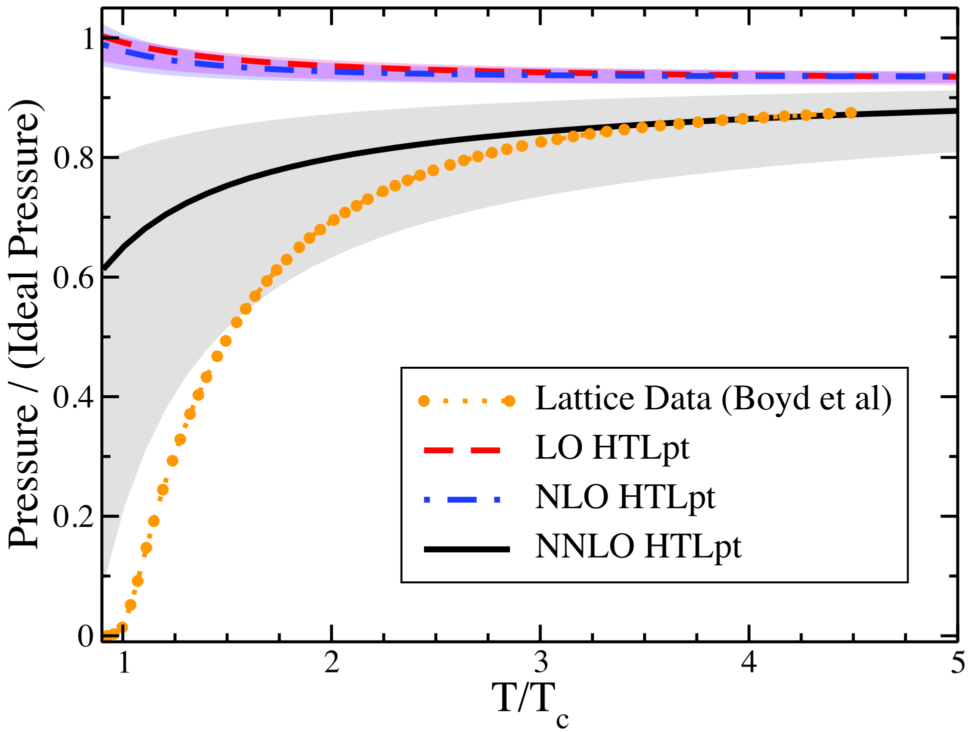
Perturbative pure-glue free energy (Kastening and Zhai, 95 & Braaten and Nieto, 96)

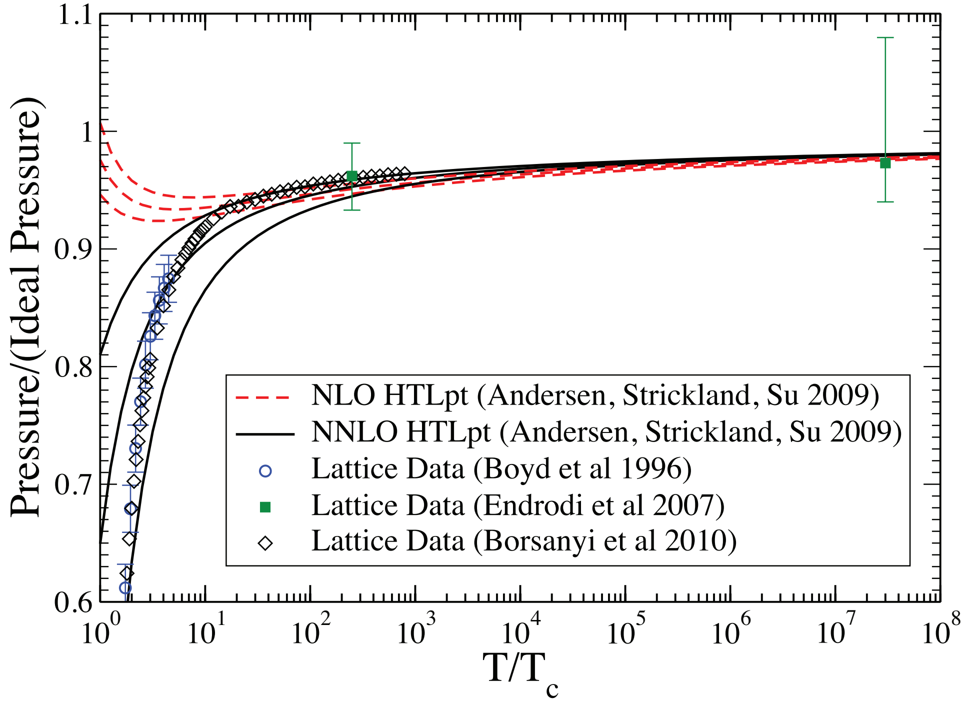
YM: NNLO variational mass convergence

Convergence is much better in pure glue using the variational mass. However, the result for the pressure in this case is complex and multivalued.



HTLpt: Yang-Mills free energy at arbitrary N_c and high T



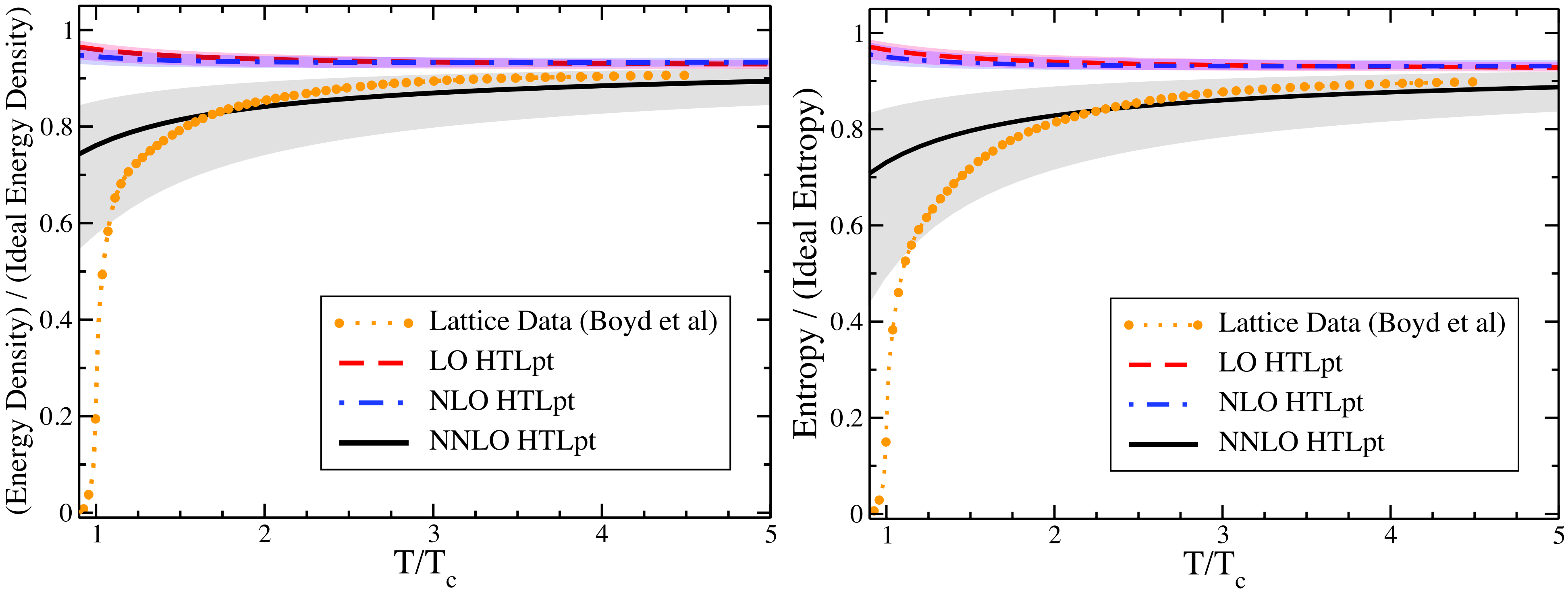


$\Omega/\mathcal{F}_{\rm ideal}$ is solely a function of the 't Hooft coupling λ. It has no N_c dependence after the substitution $g_s^2 \to \lambda/N_c$. The same applies the BN mass and running coupling. This is in accordance with recent lattice studies of Panero (2009), Datta and Gupta (2010).

Andersen, Strickland and Su, PRL, 104, 122003 (2010) & JHEP 1008, 113 (2010)

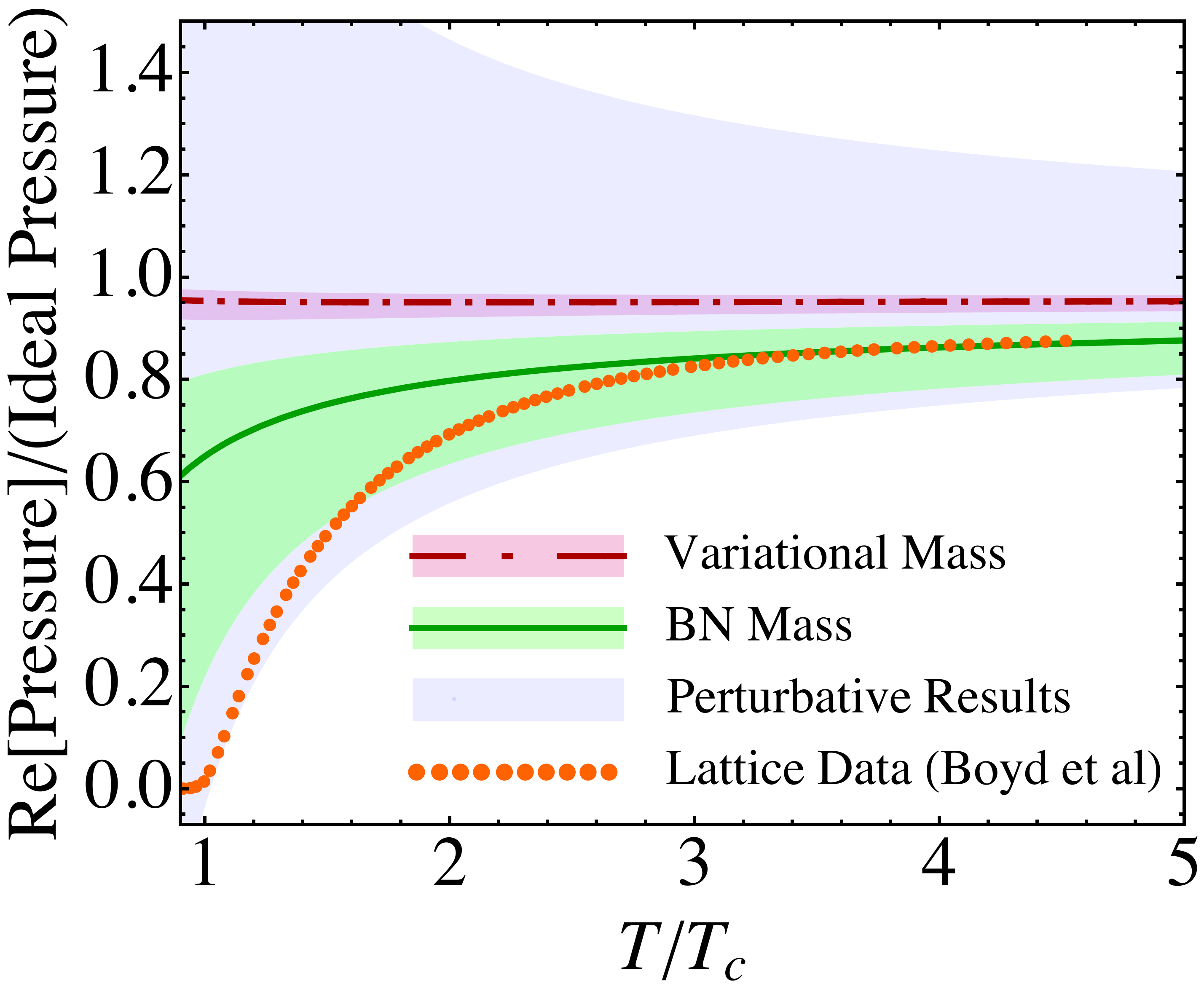
HTLpt: Yang-Mills energy and entropy

From the free energy we can evaluate other thermodynamic variables using standard relations: $\mathcal{P} = -\mathcal{F}, \mathcal{E} = \mathcal{F} - T\frac{d\mathcal{F}}{dT}, \mathcal{S} = -\frac{d\mathcal{F}}{dT}$.

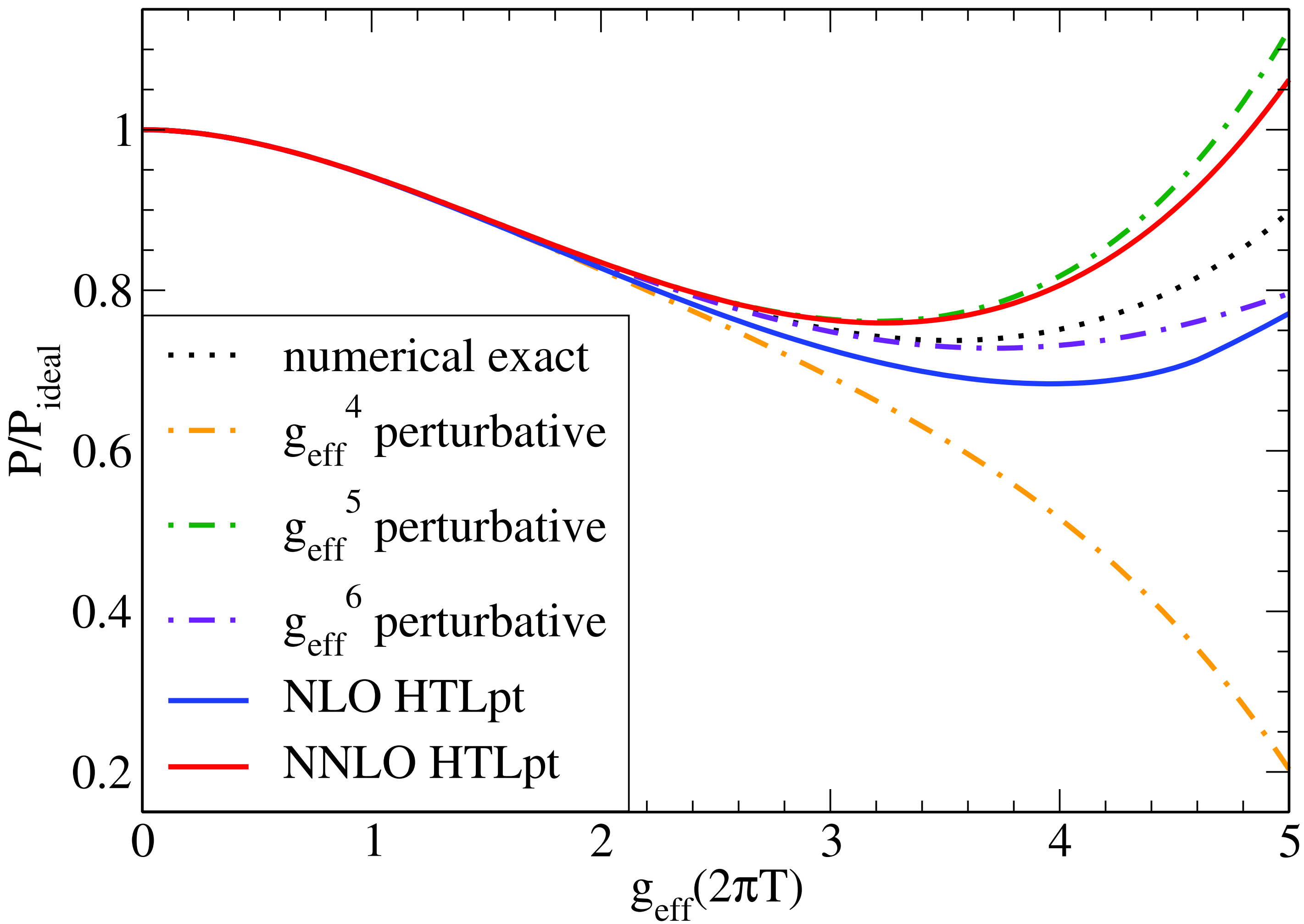


Andersen, Strickland and Su, PRL, 104, 122003 (2010) & JHEP 1008, 113 (2010)

YM: Compare with NNLO variational mass

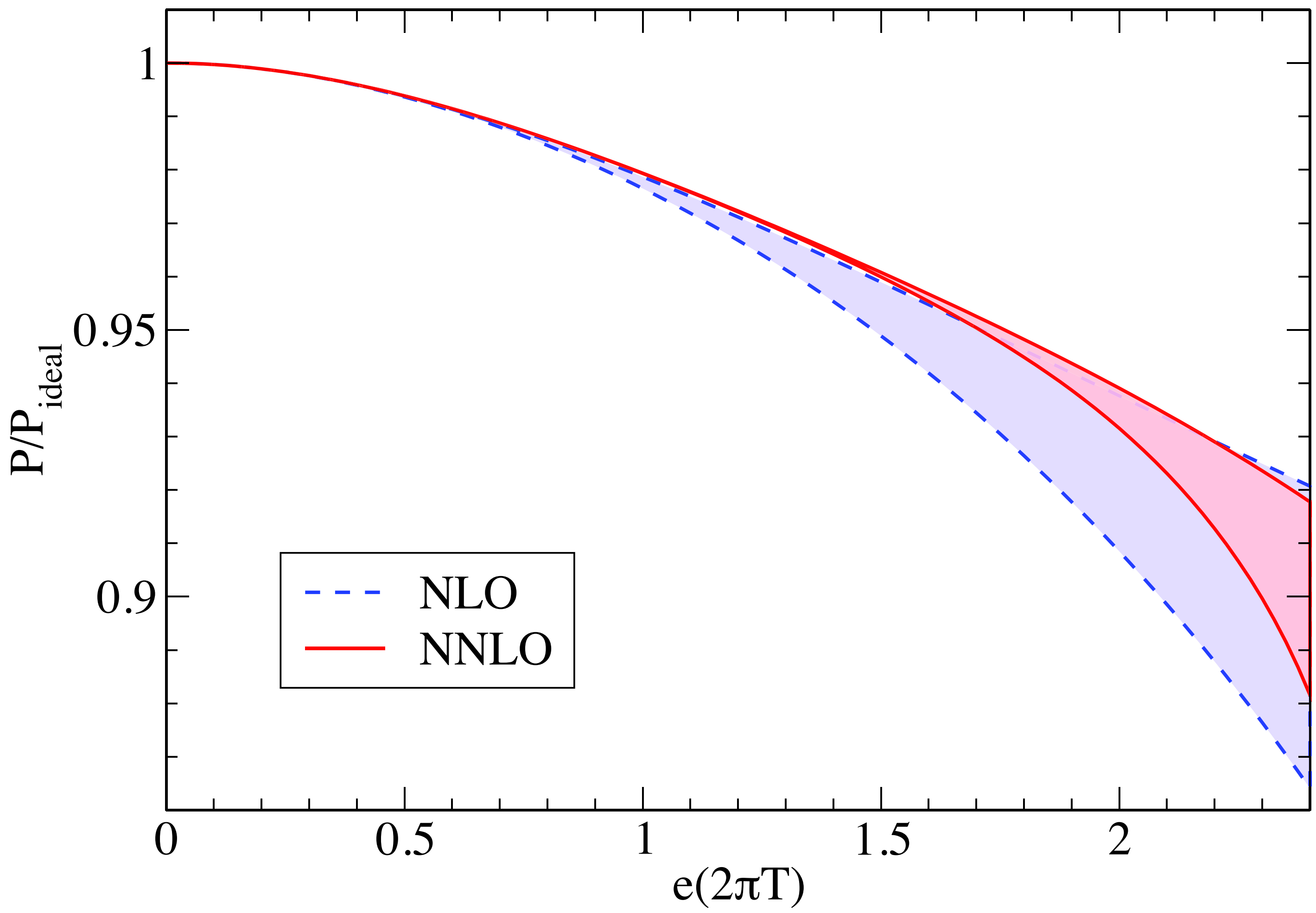


HTLpt: large N_f for QCD



Exact result from Moore, Ipp, Rebhan, 2003. g^6 perturbative results from Gynther, Kurkela, Vuorinen, 2009. NNLO HTLpt results from Andersen, Su, and Strickland calculation for QED, 2009.

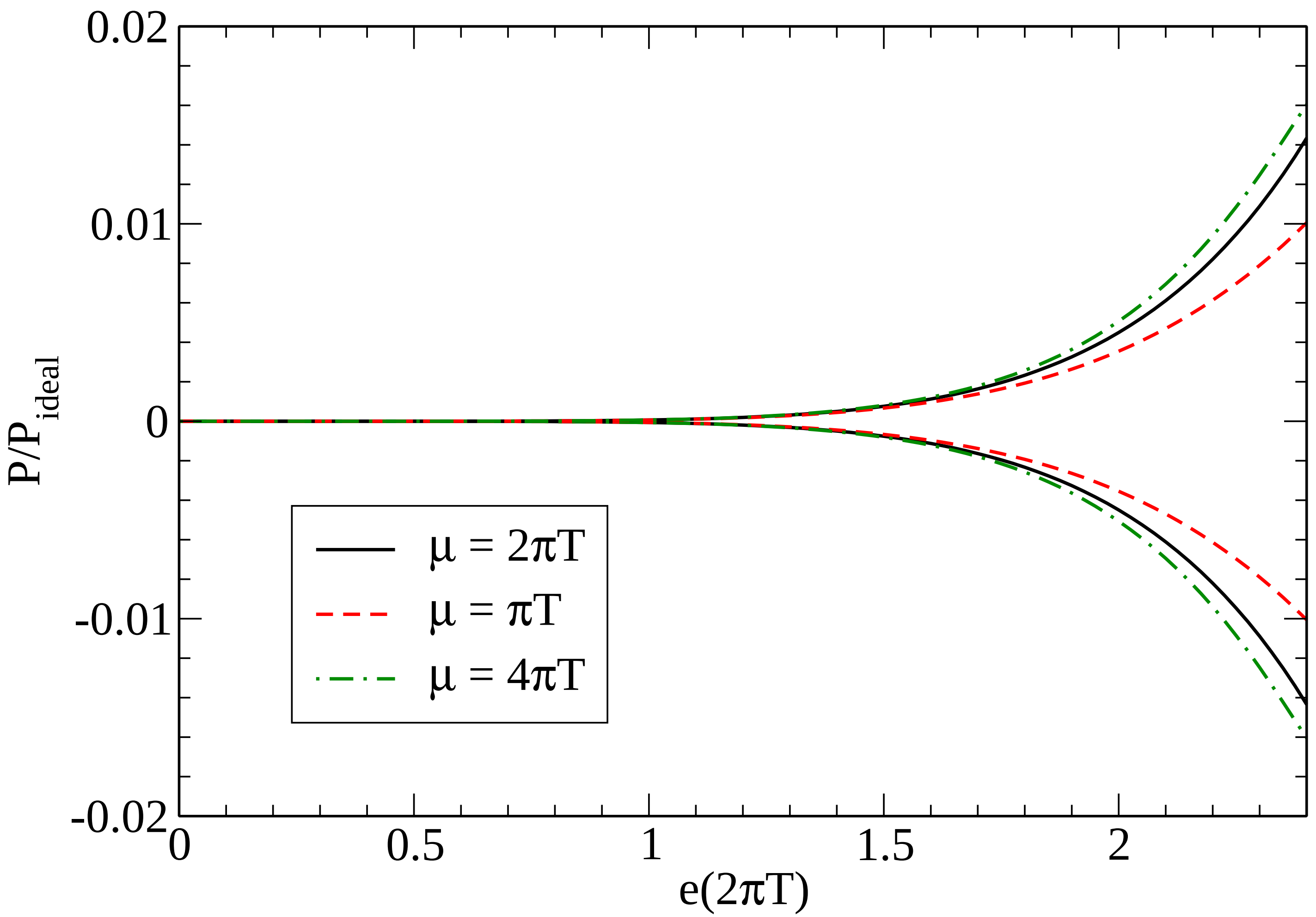
HTLpt: QED free energy as a test



Variational masses are solved from the gap equations $\frac{\partial\Omega(T,\alpha,m_D,m_e,\delta=1)}{\partial m_{D/e}} = 0$

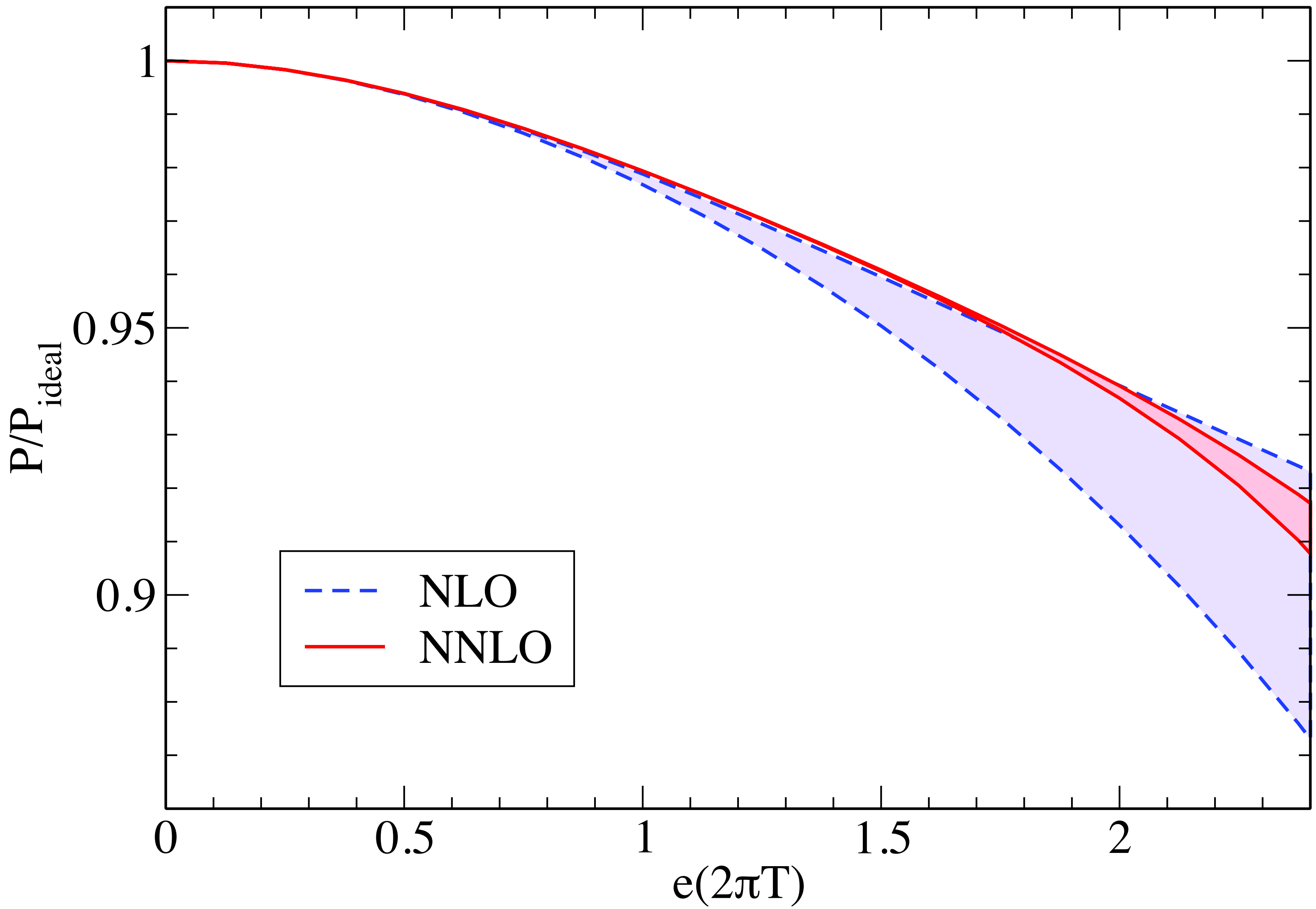
Andersen, Strickland and Su, PRD 80, 085015 (2009)

HTLpt: 3-loop free energy for QED



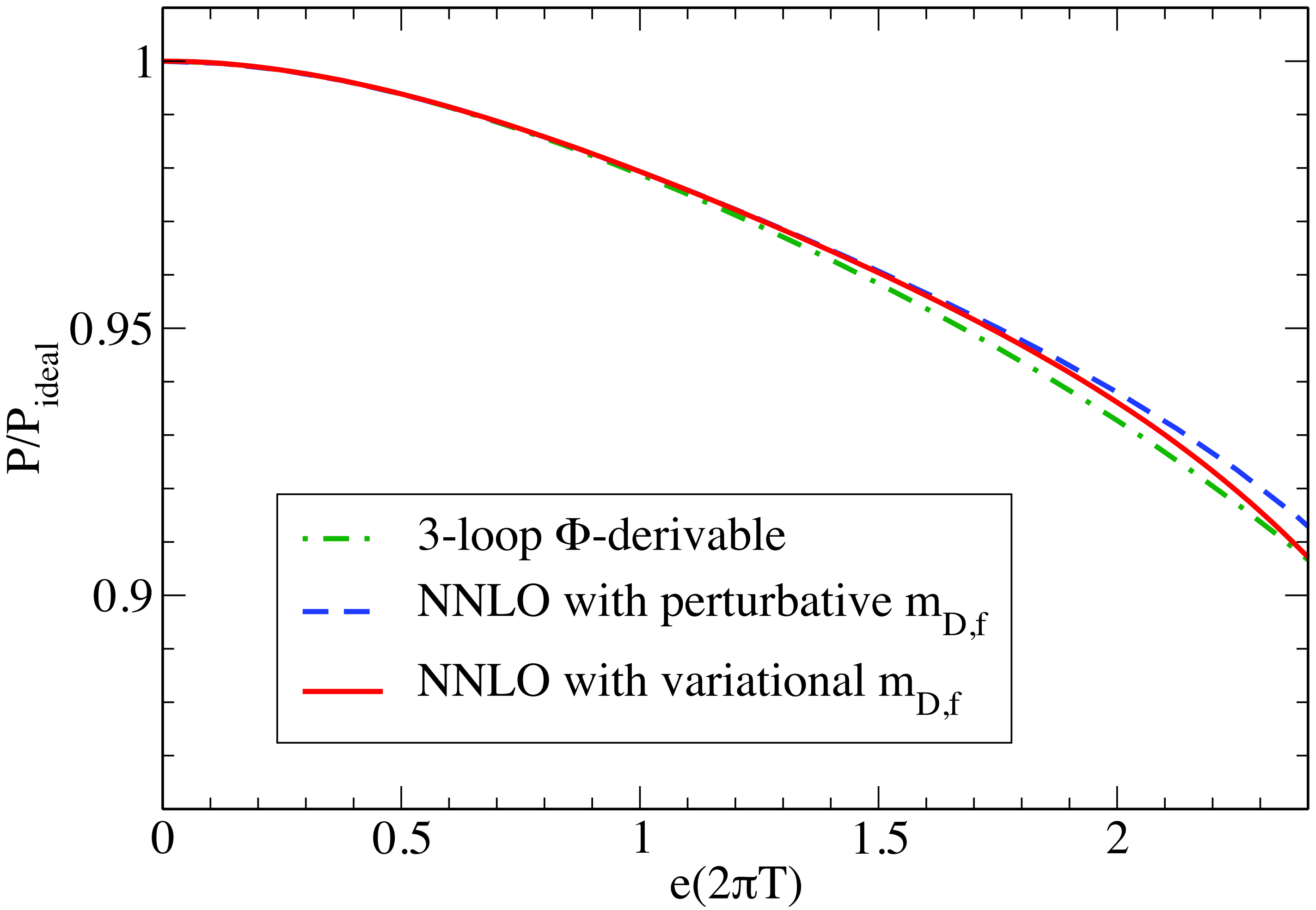
The imaginary part of NNLO HTLpt predictions for QED free energy

HTLpt: 3-loop free energy for QED



NLO and NNLO HTLpt thermodynamic potentials with perturbative thermal masses

HTLpt: Comparison of different schemes for QED



Comparison of three different predictions for QED free energy at $\mu = 2\pi T$

3-loop Φ-derivable result is taken from Andersen and Strickland, 05

Screened perturbation theory (SPT)

- Within screened perturbation theory, a mass, which can be treated as a variational parameter, is added to the Lagrangian and the loop expansion is re-computed. (Karsch, Patkós, and Petreczky, 97; Chiku and Hatsuda, 98; Andersen, Braaten and Strickland, 00; Andersen and Strickland, 01)

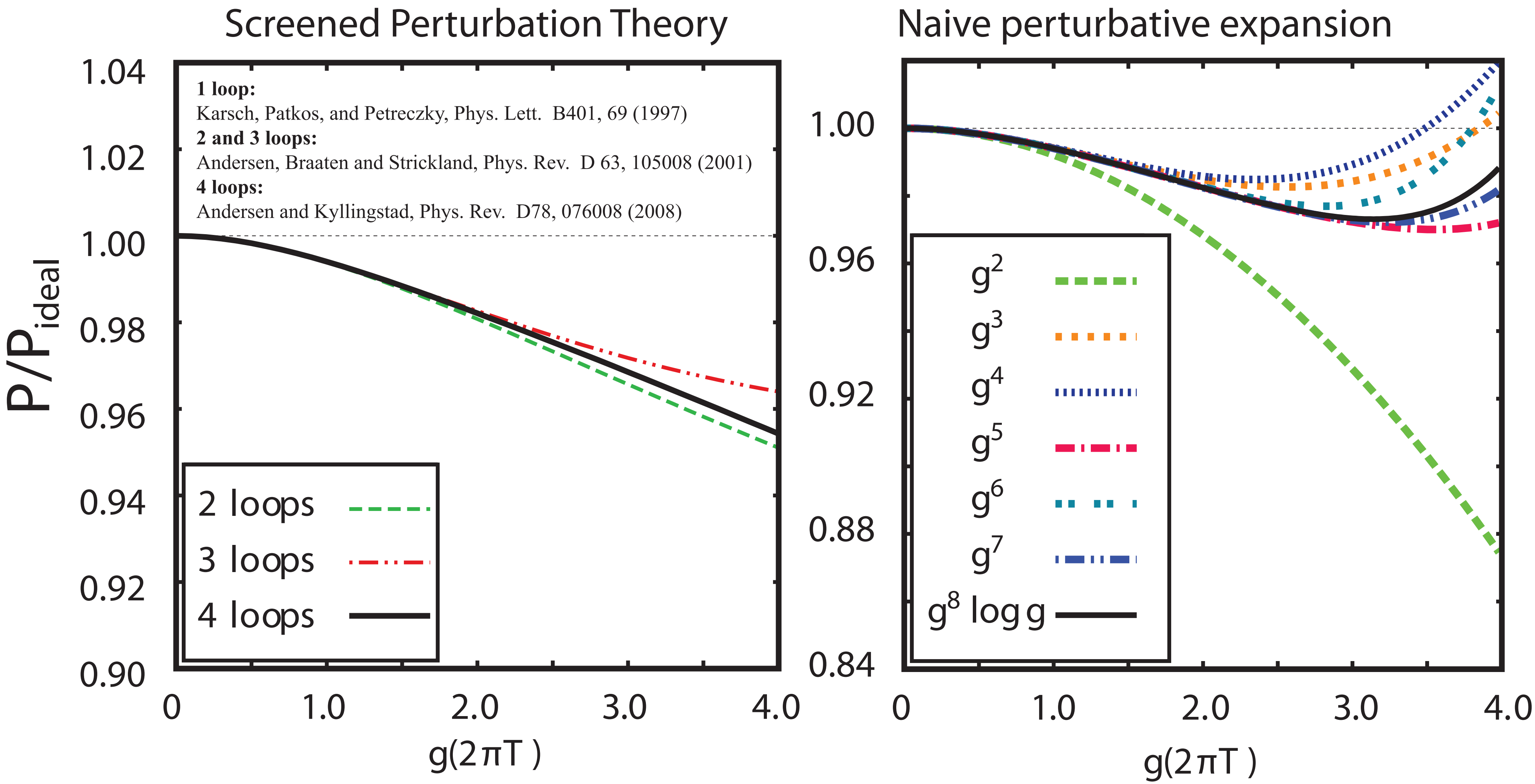
$$\mathcal{L}_{\text{SPT}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2(1-\delta)\phi^2 - \frac{\delta}{24}g^2\phi^4 + \Delta\mathcal{L} + \Delta\mathcal{L}_{\text{SPT}}$$

We can split this into free, interaction, and counterterm parts

$$\begin{aligned}
\mathcal{L}_{\text{free}} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 \\
\mathcal{L}_{\text{int}} &= \delta\left(-\frac{1}{24}g^2\phi^4 + \frac{1}{2}m^2\phi^2\right) \\
\mathcal{L}_{\text{ct}} &= \Delta\mathcal{L} + \Delta\mathcal{L}_{\text{SPT}}
\end{aligned}$$

N^{3}LO SPT free energy for massless ϕ^4 theory

Scalar theory has more combination of scales than QED!



4-loop SPT pressure vs weak-couping pressure