Clustering of Color Sources
&
Equation of State of the QGP

Brijesh K Srivastava
Department of Physics
Purdue University
USA

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Exploration of Hot QCD Matter
The Next Decade

Berndt Muller
CERN Theory Institute (HIC10)
Aug.16- Sept.10, 2010

Which properties of hot QCD matter can we hope to determine from relativistic heavy ion data (RHIC and LHC, maybe FAIR) ?

Easy for LQCD

$$T_{\mu\nu} \Leftrightarrow \varepsilon, p, s$$

Equation of state: spectra, coll. flow, fluctuations

$$c_S^2 = \partial p / \partial \varepsilon$$

Speed of sound: multiparticle correlations

$$\eta = \frac{1}{T} \int d^4x \left\langle T_{xy}(x) T_{xy}(0) \right\rangle$$

Shear viscosity: anisotropic collective flow

Hard for LQCD

$$\hat{q} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \left\langle F^{a+i}(y^-) F_i^{a+}(0) \right\rangle$$

$$\hat{e} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \left\langle i\partial^- A^{a+}(y^-) A^{a+}(0) \right\rangle$$

$$\hat{e}_2 = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \left\langle F^{a+-}(y^-) F^{a+-}(0) \right\rangle$$

Momentum/energy diffusion: parton energy loss, jet fragmentation

Easy for LQCD

$$m_D = -\lim_{|x| \to \infty} \frac{1}{|x|} \ln \left\langle E^a(x) E^a(0) \right\rangle$$

Color screening: Quarkonium states

Color Strings

Multiparticle production at high energies is currently described in terms of color strings stretched between the projectile and target. Hadronizing these strings produce the observed hadrons.

The no. of strings grow with energy and the no. of participating nuclei and one expects that interaction between them becomes essential.

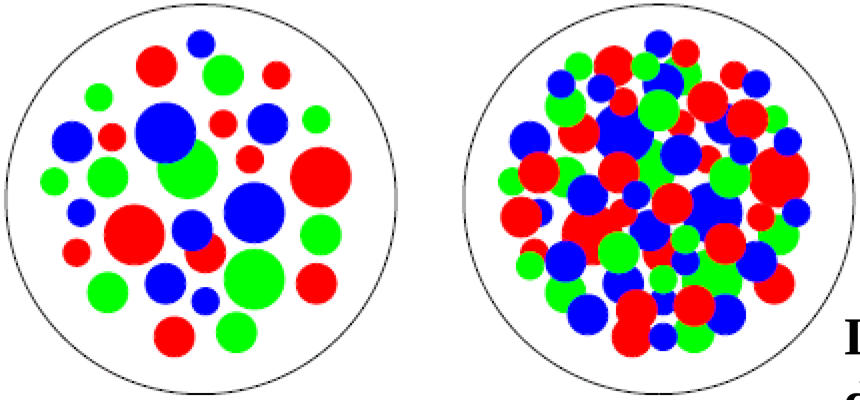
This problem acquires even more importance, considering the idea that at very high energy collisions of heavy nuclei (RHIC) may produce Quark-gluon Plasma (QGP).

The interaction between strings then has to make the system evolve towards the **QGP** state.

Clustering of Color Sources

De-confinement is expected when the density of quarks and gluons becomes so high that it no longer makes sense to partition them into color-neutral hadrons, since these would overlap strongly.

We have clusters within which color is not confined : De-confinement is thus related to cluster formation very much similar to cluster formation in percolation theory and hence a connection between percolation and de-confinement seems very likely.



Parton distributions in the transverse plane of nucleus-nucleus collisions

In two dimensions, for uniform string density, the percolation threshold for overlapping discs is: $\xi_c = 1.18$

Critical Percolation Density

H. Satz, Rep. Prog. Phys. 63, 1511(2000).
H. Satz , hep-ph/0212046

Multiplicity and $<p_T^2>$ of particles produced by a cluster of *n* strings

Multiplicity (μ_n)

$$\mu_n = F(\xi) N^s \mu_1$$

Average Transverse Momentum

$$< p_T^2 >_n = < p_T^2 >_1 / F(\xi)$$

$$\xi = \frac{N^s S_1}{S_N}$$

$$F(\xi) = \sqrt{\frac{1-e^{-\xi}}{\xi}}$$

= **Color suppression factor** (due to overlapping of discs).

N^s = # of strings
S_1 = disc area
S_N = total nuclear overlap area

ξ **is the percolation density parameter**.

M. A. Braun and C. Pajares, Eur.Phys. J. C16,349 (2000)
M. A. Braun et al, Phys. Rev. C65, 024907 (2002)

Percolation and Color Glass Condensate

Both are based on parton coherence phenomena.

Percolation : Clustering of strings

CGC : Gluon saturation

- **Many of the results obtained in the framework of percolation of strings are very similar to the one obtained in the CGC.**

- **In particular , very similar scaling laws are obtained for the product and the ratio of the multiplicities and transverse momentum.**

- **Both provide explanation for multiplicity suppression and $<p_t>$ scaling with dN/dy.**

$$Q_s^2 = \frac{k < p_t^2 >_1}{F(\xi)}$$

Monte Carlo model for nuclear collisions from SPS to LHC energies

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N.S. Amelin[1,a], N. Armesto[2,b], C. Pajares[3,c], D. Sousa[4,d] Eur. Phys. J. C 22, 149–163 (2001)

Elementary partonic collisions
Partonic wave functions of the colliding hadrons
Formation of Color String
Transverse momentum of both partons at the string ends
Given by Gaussian-> pt -broadening
Collectivity ->string fusion/percolation
SU(3) random summation of charges
Reduction in color charge
Increase in the string tension
String breaking leads to formation of secondaries
Probability rate ->Schwinger
Fragmentation proceeds in an iterative way

1. **Multiplicity**
2. **pt distribution**
3. **Particle ratios**
4. **Elliptic flow**
5. **Suppression of high pt particles R_{AA}**
6. **J/ψ production**
7. **Forward-Backward Multiplicity Correlations at RHIC**

Color String Percolation Model for Nuclear Collisions from SPS-RHIC-LHC

Using the p_T spectrum to calculate ξ

To compute the p_T distribution, a parameterization of the pp data is used:

$$\frac{dN}{dpt^2} = \frac{a}{(p_0 + pt)^n}$$

a, p_0 and n are parameters fit to the data.

This parameterization can be used for nucleus-nucleus collisions, accounting for percolation by:

$$p_0 \rightarrow p_o \left(\frac{\left\langle \frac{nS_1}{S_n} \right\rangle_{Au-Au}}{\left\langle \frac{nS_1}{S_n} \right\rangle_{pp}} \right)^{1/4}$$

In **pp** at low energy, $\langle nS_1/S_n \rangle_{pp} = 1 \pm 0.1$, **due to low string overlap probability in pp collisions.**

M. A. Braun, et al. hep-ph/0208182.

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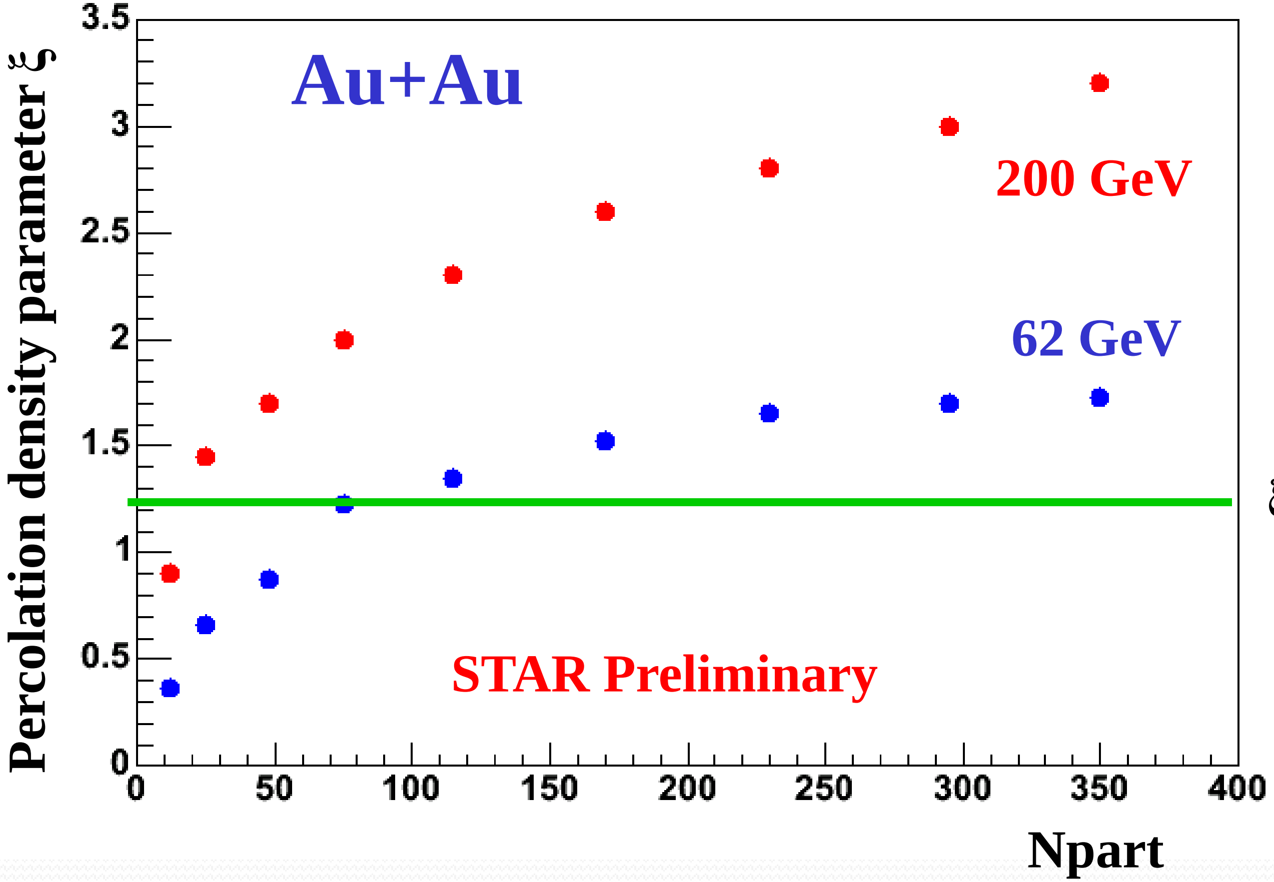
STAR Collaboration, Nukleonika 51, S109 (2006)



$$F(\xi) = \sqrt{\frac{1-e^{-\xi}}{\xi}}$$

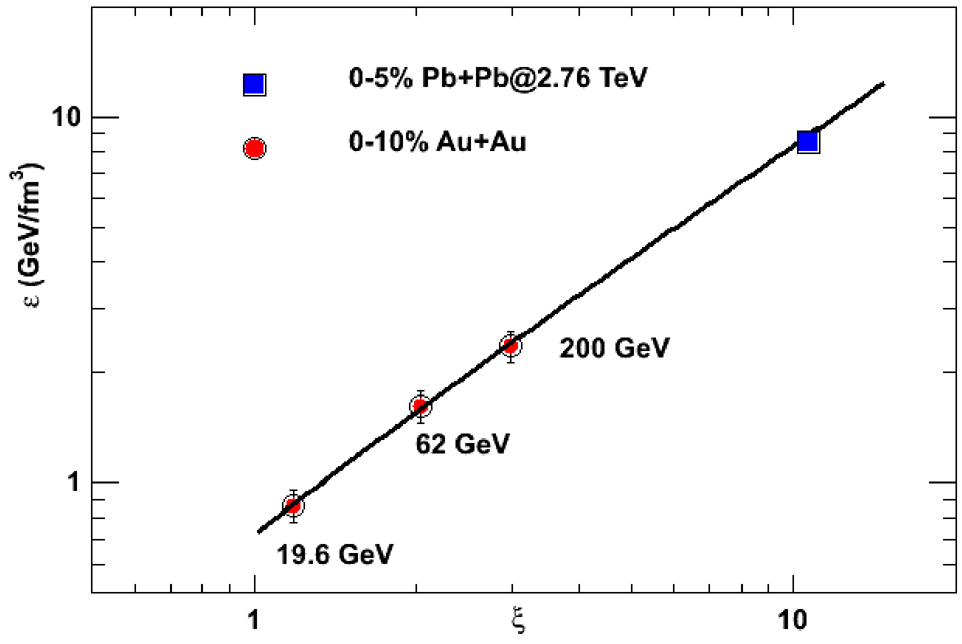
$\xi_c = 1.2$

Now the aim is to connect F(ξ) with Temperature and Energy density

Energy Density

STAR Collaboration has published charged particle multiplicities and transverse momentum of particles in for Au+Au collisions at various energies. It is found that

$$\varepsilon \propto \xi$$



1. D. Cebra , STAR Collaboration, QM08
2. B. I. Abelev et al, STAR Collaboration, Phys. Rev.C79, 34909 (2009)
3. B. I. Abelev et al, STAR Collaboration, Phys. Rev.C81, 24911(2010)

Temperature & Thermalization

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Schwinger
p_t distribution of the produced quarks

$$\frac{dn}{d^2 p_\perp} \sim \exp\left(-\frac{\pi p_t^2}{k^2}\right)$$

1. It is shown that quantum fluctuations of the string tension can account for the thermal distributions of hadrons created in the decay of color string.

2. The tension of the macroscopic cluster fluctuates around its mean value because the chromoelectric field is not constant . Assuming a Gaussian form for these fluctuations one arrives at the probability distribution of transverse momentum.

1. Fluctuations of the string and transverse mass distribution
   A. Bialas, Phys. Lett., B 466 (1999) 301.
2. Percolation of color sources and critical temperature
   J. Dias de Deus and C. Pajares, Phys.Lett , B 642 (2006) 455

Temperature

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Schwinger

p_t distribution of the produced quarks

$$\frac{dn}{d^2p_\perp} \sim \exp(-\frac{\pi p_t^2}{k^2})$$

Thermal Distribution

$$\frac{dn}{d^2p_\perp} \sim \exp(-\frac{\pi p_t}{T})$$

$$P(k)dk = \sqrt{\frac{2}{\pi\langle k^2\rangle}}\exp\left(-\frac{k^2}{2\langle k^2\rangle}\right)dk$$

which gives rise to thermal distribution

$$\frac{dn}{d^2p_\perp} \sim \exp\left(-p_\perp\sqrt{\frac{2\pi}{\langle k^2\rangle}}\right)$$

$$T = \sqrt{\frac{\langle k^2\rangle}{2\pi}}$$

Temperature of primary partons/cluster

$$\sqrt{< p_t^2 >} = \sqrt{\frac{< k^2 >}{\pi}} = \sqrt{\frac{< p_t^2 >_1}{F(\xi)}}$$

$$T = \sqrt{\frac{\langle p_t^2\rangle_1}{2F(\xi)}}$$

Temperature

$$T = \sqrt{\frac{\langle p_t^2 \rangle_1}{2F(\xi)}}$$

At the critical percolation density

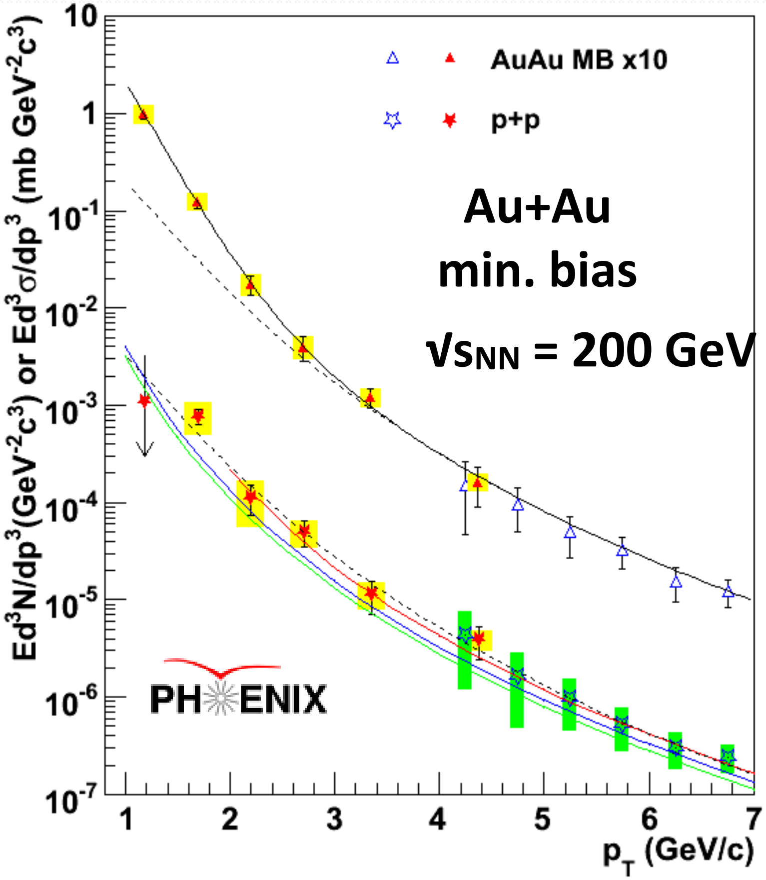
$\xi_c = 1.2$ T_c **= 167 MeV**

For Au+Au@ 200 GeV

0-10% centrality ξ = 2.88 T ~ 195 MeV

1. Chemical freeze out temperature
P. Braun-Munzinger et al., Phys. Lett. B596, 61 (2006)

2. Universal chemical freeze out temperature
F. Becattini et al, Eur. Phys. J. C66, 377 (2010).



Direct photon excess in Au+Au above *p+p* spectrum

Exponential (consistent with thermal) Inverse slope = 220 ± 20 MeV

PRL 104, 132301 (2010)

Thermodynamic Relations
Equation of State

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The QGP in the clustering of color sources is born in local thermal equilibrium because the temperature is determined at string level. This can be coupled to hydrodynamics to obtain energy density, pressure , entropy density and sound velocity

J. D. Bjorken, Phys. Rev. D27, 140 (1983).

$$\varepsilon = G(T)\frac{\pi^2}{30}T^4$$

$$Ts = (\varepsilon + p)$$

$$p = C_s^2 \varepsilon$$

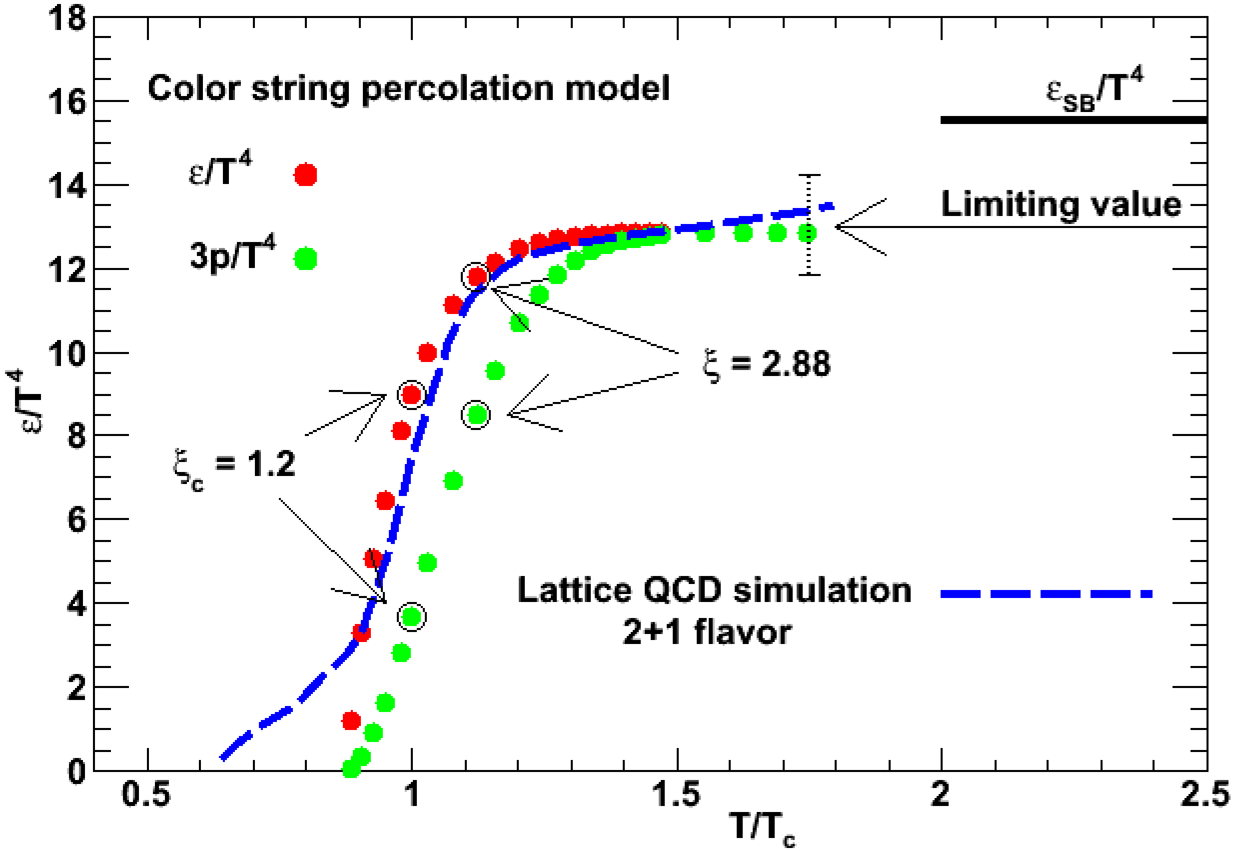
An analytic function of ξ for the EOS of QGP for $T \geq T_c$

$$C_s^2 = (1 + C_s^2)(-1/4)\left(\frac{\xi e^{-\xi}}{1 - e^{-\xi}} - 1\right)$$

Finally, we have energy density, temperature and sound velocity as a function of color suppression factor F(ξ) or percolation density parameter ξ.

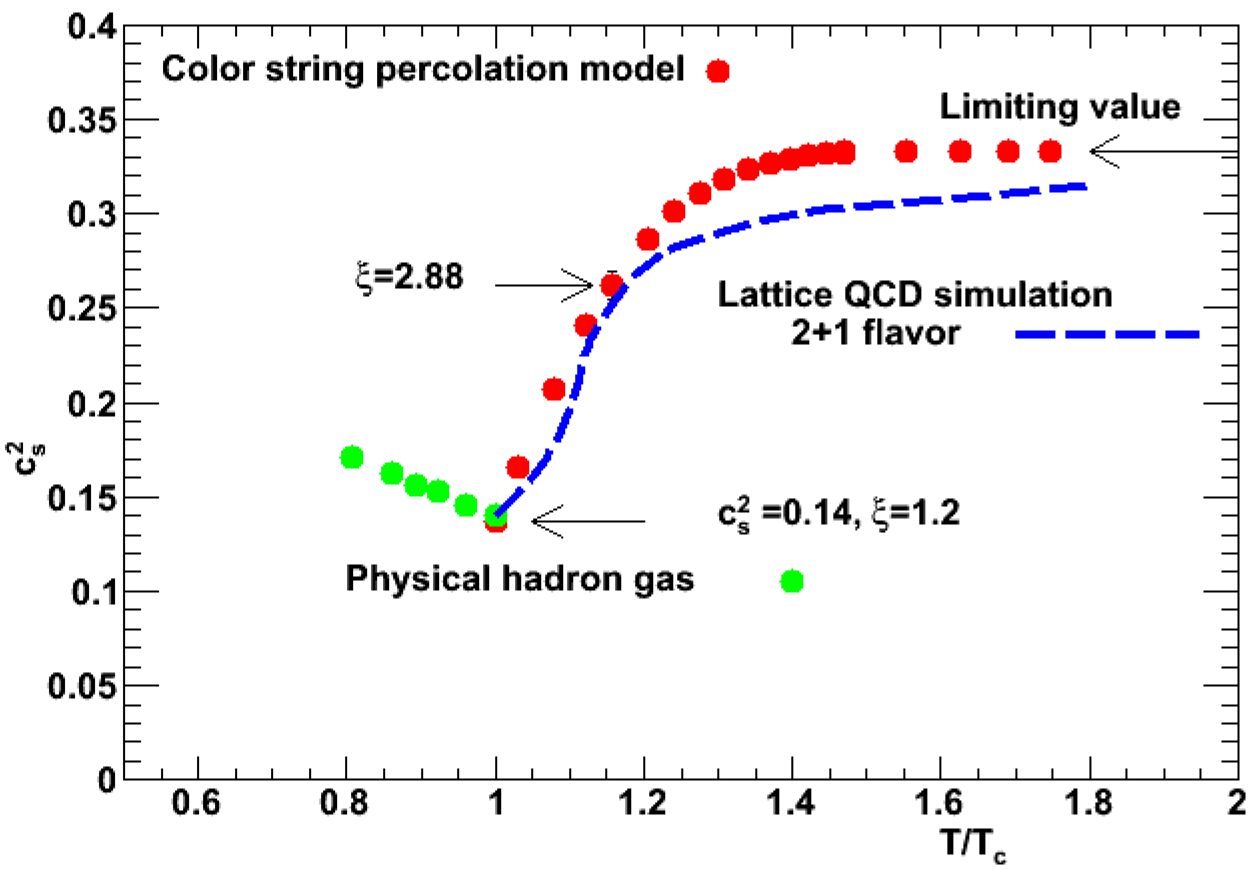
Energy Density

0-10% centrality



Lattice QCD : Bazavov et al, Phys. Rev. D 80, 014504(2009).

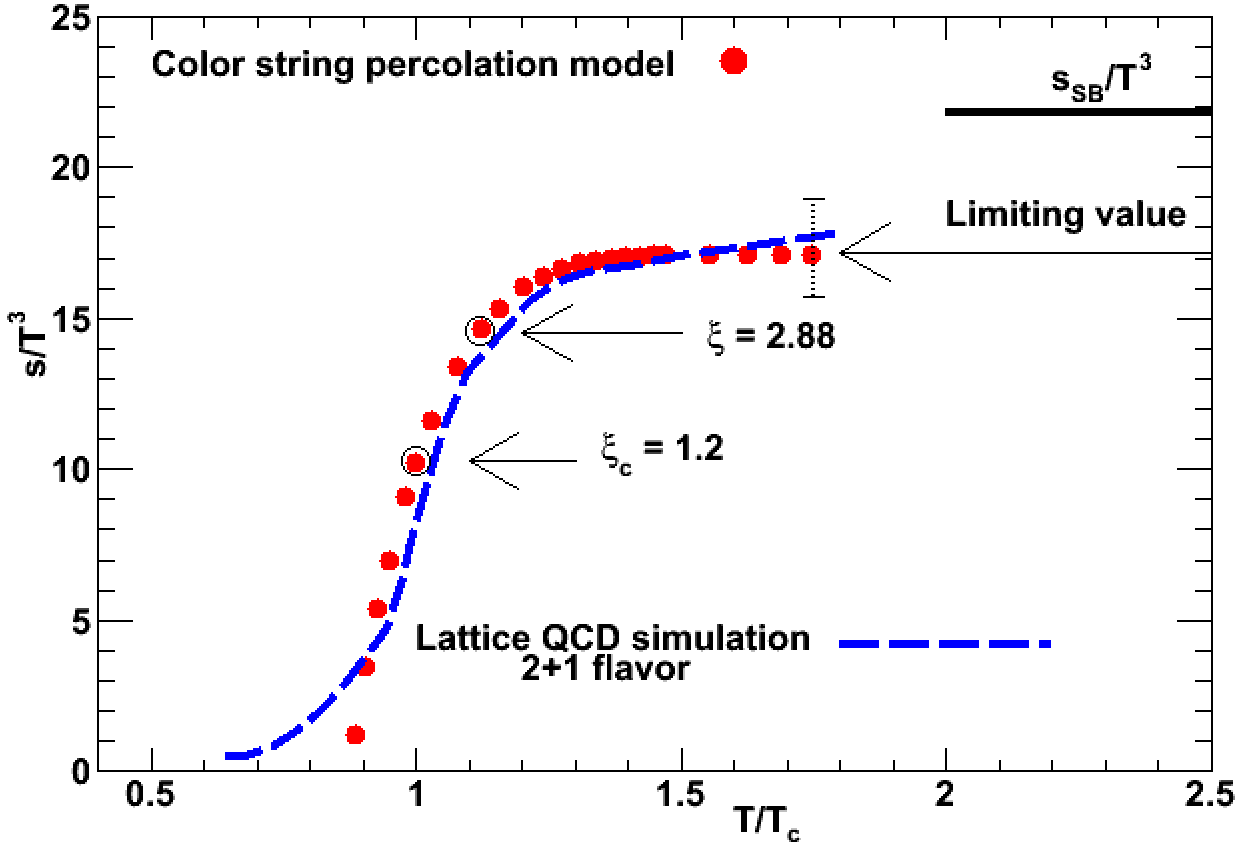
Velocity of Sound expressed as C_s^2



Lattice QCD : Bazavov et al, Phys. Rev. D 80, 014504(2009).
Physical hadron gas: Castorina et al, arXiv:0906.2289/hep-ph

Entropy Density

0-10% centrality



Lattice QCD : Bazavov et al, Phys. Rev. D 80, 014504(2009).

Transport Coefficient of QCD Matter

Shear Viscosity/Entropy η/s

PRL 97, 152303 (2006) PHYSICAL REVIEW LETTERS week ending 13 OCTOBER 2006

Strongly Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions

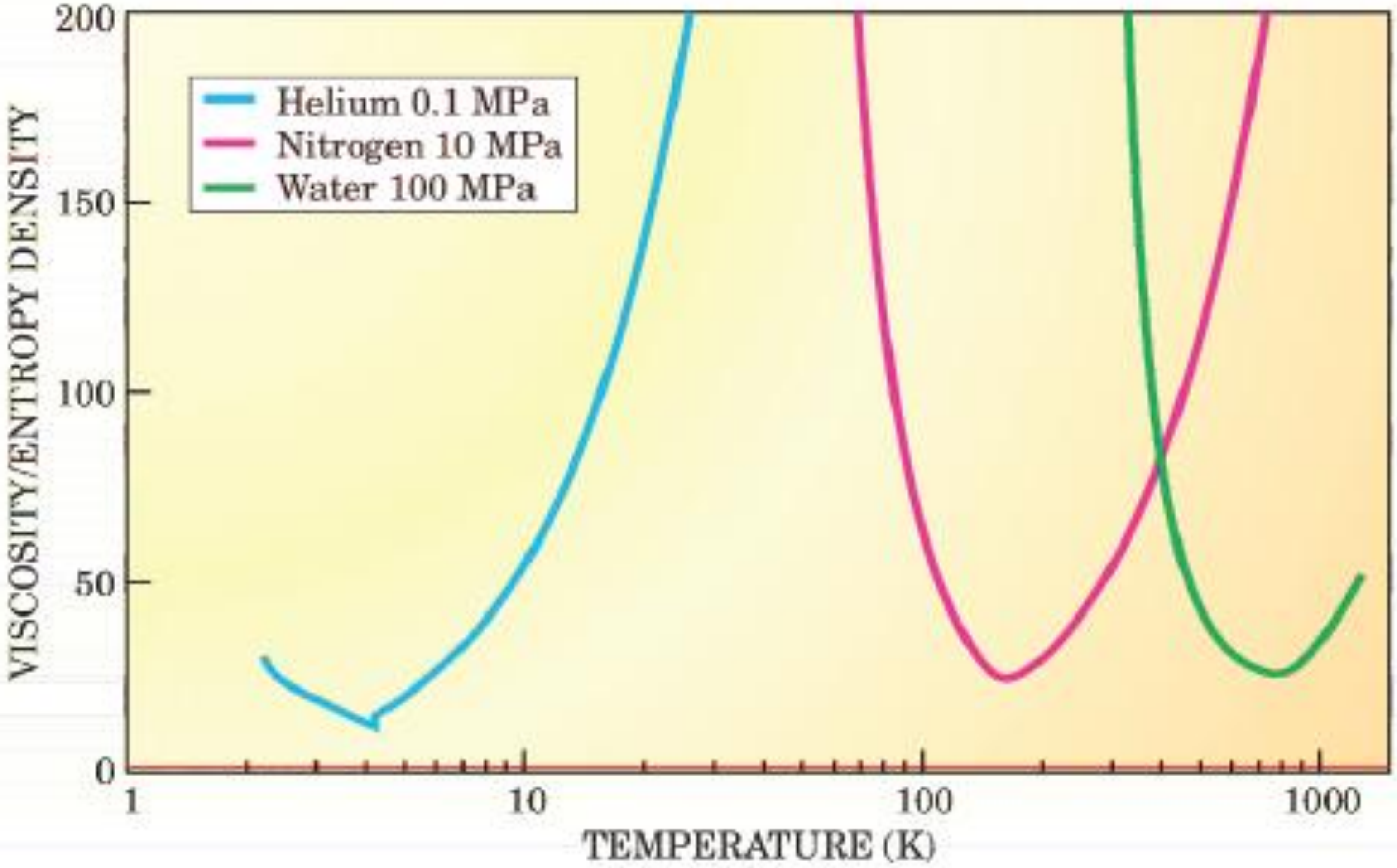
Laszlo P. Csernai,[1,2] Joseph I. Kapusta,[3] and Larry D. McLerran[4]

[1]*Section for Theoretical Physics, Department of Physics, University of Bergen, Allegaten 55, 5007 Bergen, Norway*
[2]*MTA-KFKI, Research Institute of Particle and Nuclear Physics, 1525 Budapest 114, P. O. Box 49, Hungary*
[3]*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*
[4]*Nuclear Theory Group and Riken Brookhaven Center, Brookhaven National Laboratory, Bldg. 510A, Upton, New York 11973, USA*

(Received 12 April 2006; published 12 October 2006)

Substantial collective flow is observed in collisions between large nuclei at BNL RHIC (Relativistic Heavy Ion Collider) as evidenced by single-particle transverse momentum distributions and by azimuthal correlations among the produced particles. The data are well reproduced by perfect fluid dynamics. A calculation of the dimensionless ratio of shear viscosity η to entropy density s by Kovtun, Son, and Starinets within anti-de Sitter space/conformal field theory yields $\eta/s = \hbar/4\pi k_B$, which has been conjectured to be a lower bound for any physical system. Motivated by these results, we show that the transition from hadrons to quarks and gluons has behavior similar to helium, nitrogen, and water at and near their phase transitions in the ratio η/s. We suggest that experimental measurements can pinpoint the location of this transition or rapid crossover in QCD.

η/s for He, N_2 and H_2O



The viscosity can be estimated from kinetic theory to be

$$\eta \approx \frac{4}{15}\varepsilon(T)\lambda_{mfp} \approx \frac{1}{5}\frac{T}{\sigma_{tr}}\frac{s(T)}{n(T)}$$

$$\varepsilon(T) = \frac{3}{4}Ts$$

$$\lambda_{tr} = \frac{1}{(n\sigma_{tr})}$$

$$\frac{n}{s} \approx \frac{T\lambda_{mfp}}{5}$$

Hirano & Gyulassy, Nucl. Phys. A769, 71(2006)

ε Energy density

s Entropy density

n the number density

λ_{mfp} **Mean free path**

σ_{tr} **Transport cross section**

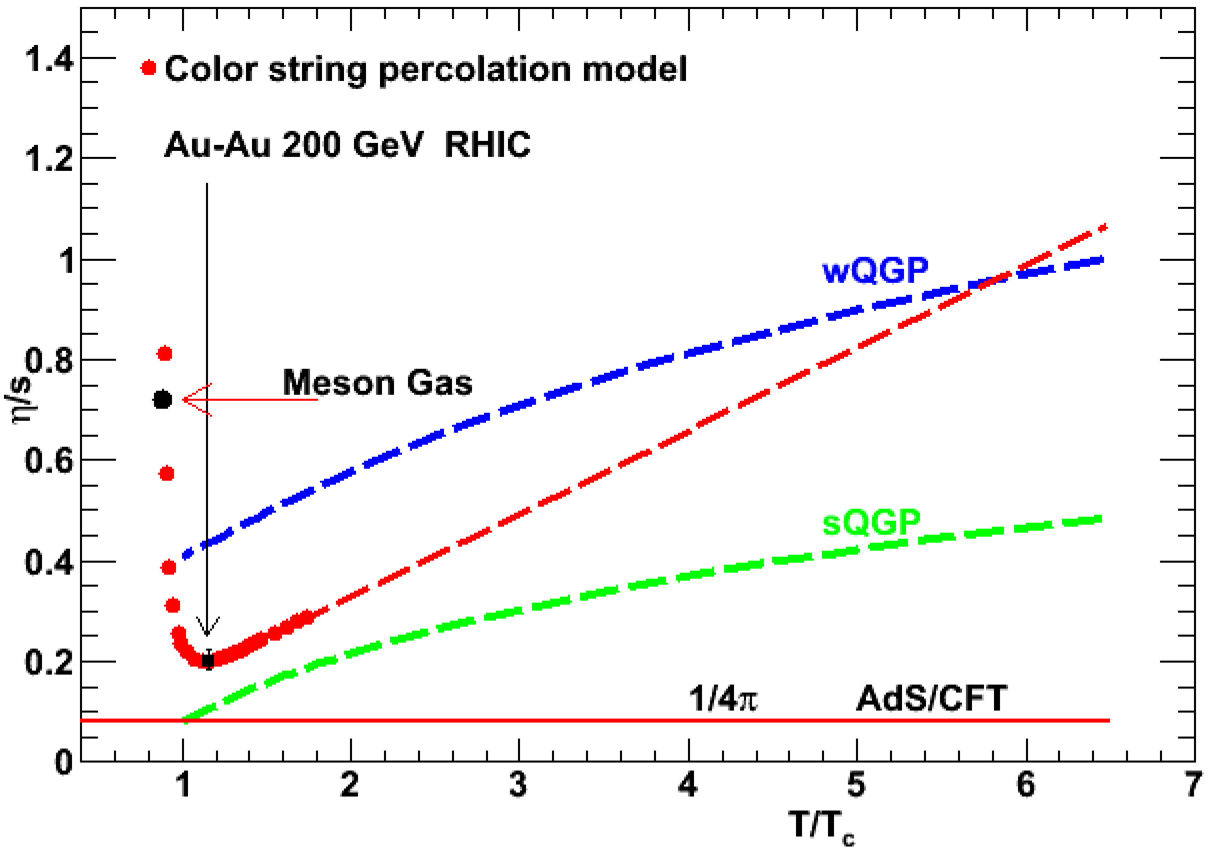
$\sqrt{<pt>_1^2}$ **Average transverse momentum of the single string**

L is Longitudinal extension of the source 1 *fm*

$$\lambda_{mfp} = \frac{L}{1-e^{-\xi}}$$

$$\frac{\eta}{s} \approx \frac{1}{5}\frac{L}{1-e^{-\xi}}T$$

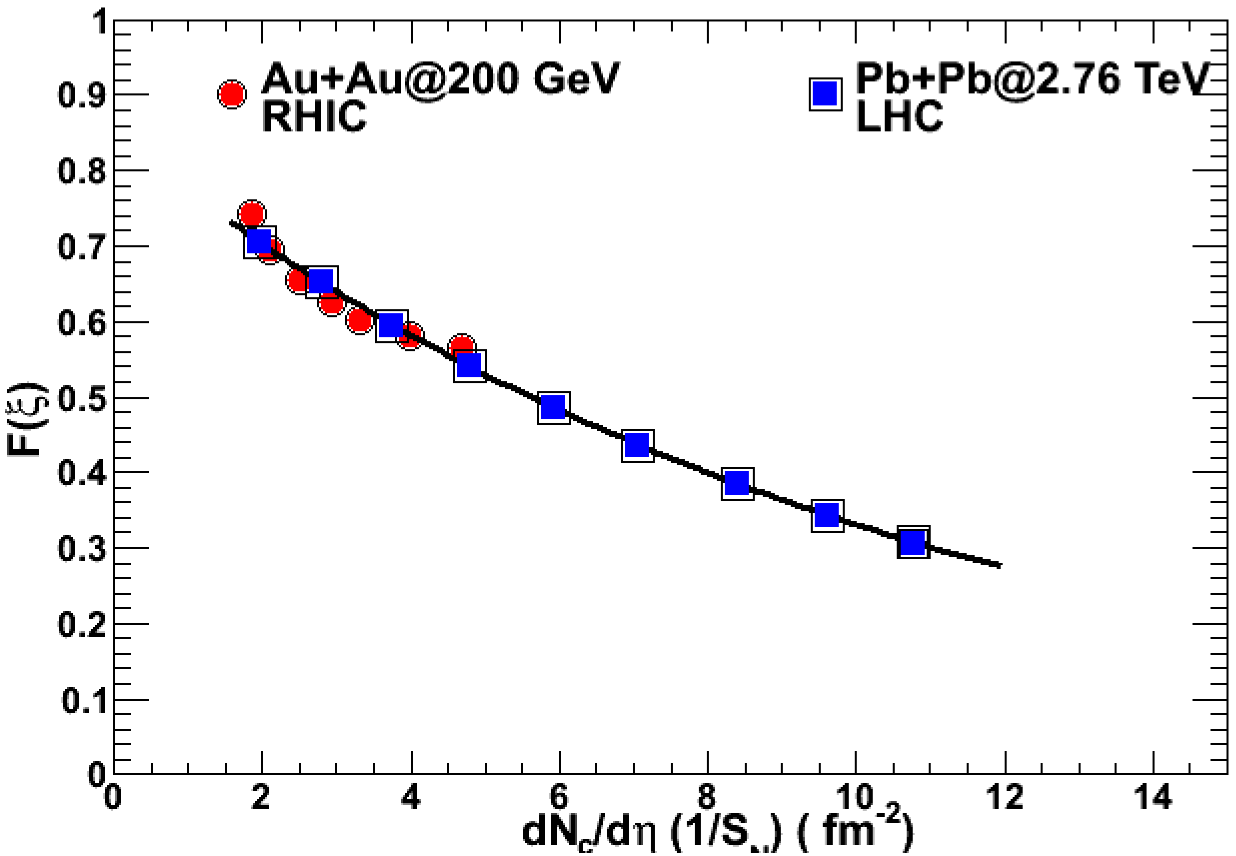
Shear viscosity to Entropy ratio



$$\frac{\eta}{s} \approx \frac{1}{5} \frac{L}{1-e^{-\xi}} T$$

Prediction for Pb+Pb at LHC Energy   2.76 TeV

$F(\xi)$



$$F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}}$$

PRL 106, 032301 (2011) PHYSICAL REVIEW LETTERS week ending 21 JANUARY 2011

Centrality Dependence of the Charged-Particle Multiplicity Density at Midrapidity in Pb-Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV

K. Aamodt *et al.**
(ALICE Collaboration)

(Received 8 December 2010; published 20 January 2011)

Temperature

$$T = \sqrt{\frac{\left\langle p_t^2 \right\rangle_1}{2F(\xi)}}$$

→

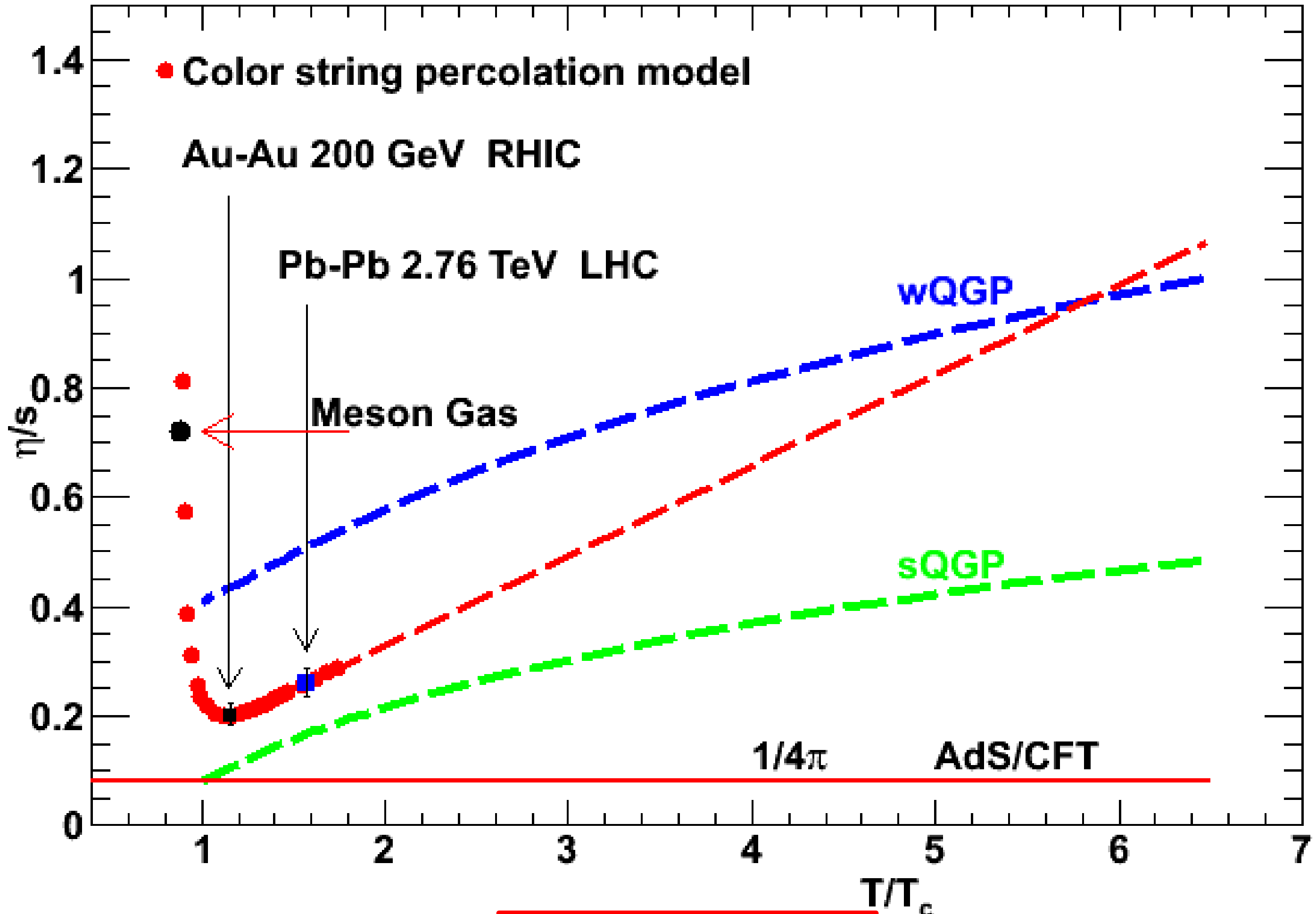
Pb+Pb @ 2.76TeV for 0-5%
T ~265 MeV
Temperature has increased by 35% from Au+Au @ 0.2 TeV

- **A "Little Bang " arrives at the LHC**
  E Shuryk
  Physics 3, 105 (2010)

$$\frac{T_{LHC}}{T_{RHIC}} = 1.3$$

- **First Results from Pb+Pb Collisions at the LHC**
  Muller, Schukraft and Wyslouch
  arXiv:1202.3233, To appear in Ann. Rev. Nucl. & Part. Sci, Oct. 2012
  The initial temp increases by 30% to T~300 MeV

Shear viscosity to Entropy ratio



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Color screening: Quarkonium states

Summary

- The Percolation analysis of Clustering of Color Sources at RHIC Energy provides a compelling argument that QGP is produced in all soft events.

- The minimum in η/s can be studied as a function of the beam energy at RHIC that could locate the critical point/crossover in QCD phase diagram as seen in substances like He, N_2 and H_2O.

  The STAR experiment is conducting this analysis.

- A further definitive test of clustering phenomena can be made at LHC energies by comparing *h-h* and A-A collisions.

H. Satz : Quantum Field Theory in Extreme Environments, Paris , April 2009
Clustering and percolation can provide a conceptual basis for the QCD phase diagram which is more general than symmetry breaking .

Many Thanks to Organizers
for arranging this Conference in
such a beautiful place

Color Strings + Percolation = CSPM

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