Holographic dilepton and photon production in a thermalizing plasma

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- If a strongly interacting plasma (say, quark-gluon-plasma...) couples weakly to leptons via a photon, then the plasma produces virtual photons, which then decay into fermion-antifermion pairs.
- The dileptons can propagate relatively easily through the strongly coupled plasma, since they are coupled to the plasma not via the strong interaction, but a relatively weak electromagnetic interaction.
- Thus dileptons can provide an observational window into the thermalization process of the plasma.
- Calculating the dilepton production rate in a strongly interacting plasma is of course in general very difficult: only the leading order result is known even in the weak coupling.

- The AdS/CFT duality offers a unique approach to describing strongly interacting systems:

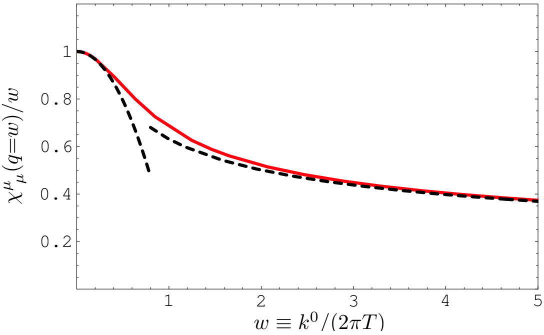
4+1 dimensional AdS weakly coupled gravity	$\Longleftrightarrow$	3+1 dimensional Minkowski strongly coupled CFT

- In an ideal world we would like to calculate predictions in QCD, an ongoing project called AdS/QCD.

- We calculate the dilepton production in $\mathcal{N} = 4$ SYM, from which we hope to learn something qualitative about strongly coupled theories.

- Even though it is not QCD, it is a non-abelian strongly interacting theory, where the temperature breaks both SUSY and the conformal invariance.

- The dilepton production rate in finite-T AdS/CFT was first calculated by Caron-Huot et al. [hep-th/0607237].



- We are interested in finding out how does this change when out of equilibrium – can we use the dileptons as probes of the thermalization process?

- AdS/CFT conjecture relates $4+1$-dimensional classical gravity in AdS to a strongly interacting conformal field theory in Minkowski.

"center" of AdS — $r = 0$

boundary — $r = \infty$

- The AdS/CFT-dictionary then tells us how to relate quantities in the gravity side and the CFT side.

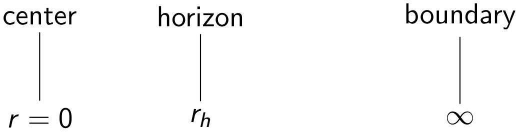
- This AdS-metric describes the vacuum state of a CFT living in the boundary.

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\mathbf{x}^2$$

- If one adds a black hole to the AdS-metric,

$$ds^2 = -(1 - \frac{r_h^4}{r^2} + r^2)dt^2 + \frac{dr^2}{1 - \frac{r_h^4}{r^2} + r^2} + r^2 d\mathbf{x}^2 \ ,$$

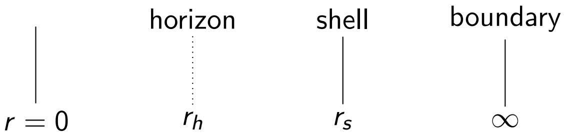
  this corresponds to the CFT having the same temperature as the Hawking temperature of the black hole, $T = r_h/\pi$.



- One can now repeat calculations done in flat AdS producing the quantities but now in finite-T!

- To describe a thermalizing plasma, we need to have a setup where the AdS-metric approaches a black hole solution.
  $\Rightarrow$ Gravitational collapse

- Easiest to do is to put a thin shell at some value of $r = r_s$:
  - Outside the metric looks like the black hole solution.
  - Inside, just like with 3-dimensional shells then, doesn't feel the gravity of the shell and thus the metric is flat AdS.



- To relate these two *a priori* unrelated patches, one has to map the coordinates from one patch to the other. It turns out that the radial coordinate is continuous, while the time coordinate is different in both of the patches.

- In general any (continuous) spacetime is a solution of the Einstein equations given a weird enough energy momentum tensor

- Both of the patches are vacuum solutions, so thus the only (gravitating) matter can be on the boundary $r = r_s$.

- To solve the dynamics of the background, e.g. the time dependence of r_s, one would need to specify the stuff that the shell is composed of, i.e. the equation of state of the shell.

- We don't do this, but rather assume that whatever the shell is composed of, it moves slowly compared to the timescales we are interested in.

- The dilepton production rate of a thermal plasma is given in general by the Wightman function of the photon field,

$$\frac{d\Gamma}{d^4q} = -\frac{\alpha\eta^{\mu\nu}\Pi^{<}_{\mu\nu}}{24\pi^4 Q^2} \, .$$

- To calculate the photon Wightman function, we add a U(1) gauge field in the bulk, which has the usual equation of motion

$$\partial_\mu \left(\sqrt{-g} g^{\mu\nu} g^{\alpha\beta} F_{\nu\beta}\right) = 0 \, ,$$

  plus EM couplings to the strongly interacting plasma and the leptons.

- We can then use the magic of AdS/CFT to calculate the Wightman function of the photon field in the boundary by solving classical equations in the bulk.

Literature[1] tells us how to calculate the retarded Green's function:

- Solve the EOM in the bulk and then write it in terms of a power series expansion around the singular point of its equation of motion, in our case in the boundary. Using the notation $u = \frac{r_h^2}{r^2}$,

$$E_{\text{outside}}(u) \overset{u \to 0}{=} \mathcal{A}[1 + h\, u \ln u + \ldots] + \mathcal{B}[u + \ldots] \, .$$

- The retarded Green's function is then given by

$$\Pi = -\frac{N_c^2 T^2}{8} \frac{\mathcal{B}}{\mathcal{A}} \, .$$

[1] See Son & Starinets [hep-th/0205051] and Kovtun & Starinets [hep-th/0506184]

- But the production rate was given by the Wightman function, not the retarded Green's function!

- The two can be related by using the fluctuation-dissipation theorem,

$$\eta^{\mu\nu} \Pi^{<}_{\mu\nu} = -2n \, \mathrm{Im} \, \Pi^{\mu}_{\mu} \; .$$

- However, the fluctuation-dissipation theorem doesn't generally apply when out-of-equilibrium (for an illustrative example, see e.g. Chesler & Teaney [1112.6196]).

- Fortunately one can do a calculation similar to that of Herzog & Son [hep-th/0212072] to show that the relation between the Wightman function and the retarded Green's function still applies!

- We are interested in virtual photons that decay into dileptons, and thus we choose the four-momentum of the photon to be $k = (\omega, \vec{0})$.
- Using the u variable, $u = r_h^2/r^2$, the equation of motion reads as
$$\partial_u^2 E + \frac{\partial_u f}{f}\partial_u E + \frac{\hat{\omega}^2}{u\, f^2}E = 0 \; ,$$
where f is given by $f \simeq r_h^2/u$ for inside and $f \simeq r_h^2(1/u - u)$ outside.
- Here we have defined the rescaled $\hat{\omega} = \omega/2\pi T$. Note also that we have Fourier transformed this in 4-space, but not in the radial AdS direction.

- The EOM has two solutions outside the shell, which we call
  - the infalling mode, E_{in}, since constant wavefronts propagate towards the horizon
  - the outgoing mode, E_{out}, since constant wavefronts propagate towards the boundary
- In the finite-T/equilibrium case, there can be no information escaping from the black hole, and thus there should be no outgoing mode present at the horizon.
- This defines the solution uniquely and fixes $\mathcal{A}$ and $\mathcal{B}$ giving the correlator.

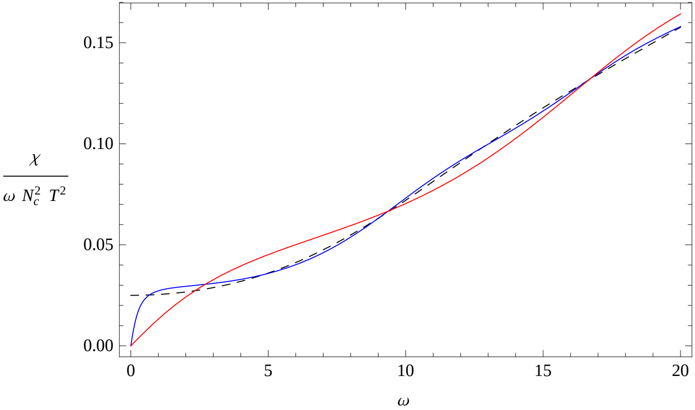
- Now the solution is a linear combination of both modes:

$$E_{\text{outside}} = c_+ E_{\text{in}} + c_- E_{\text{out}}$$

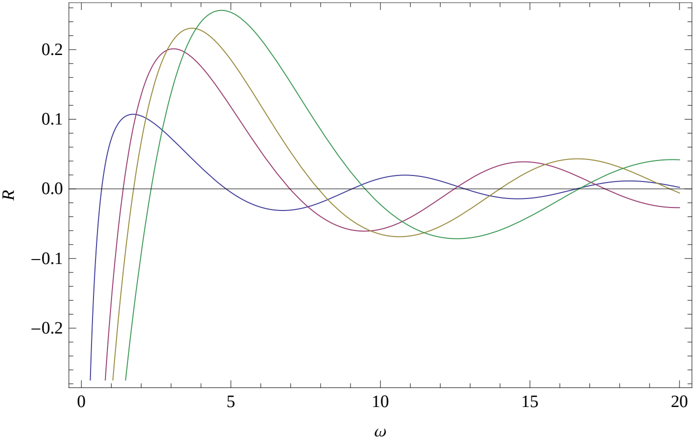
- The combination is fixed by matching the outside solution to the inside solution using the boundary conditions

$$E_{\text{inside}} = \sqrt{f}\, E_{\text{outside}} \qquad E'_{\text{inside}} = f\, E'_{\text{outside}} \qquad f \simeq 1 - r_h^4/r^4$$

- The boundary condition is essentially just requiring the continuity of the solution, and taking into account the difference in coordinates.

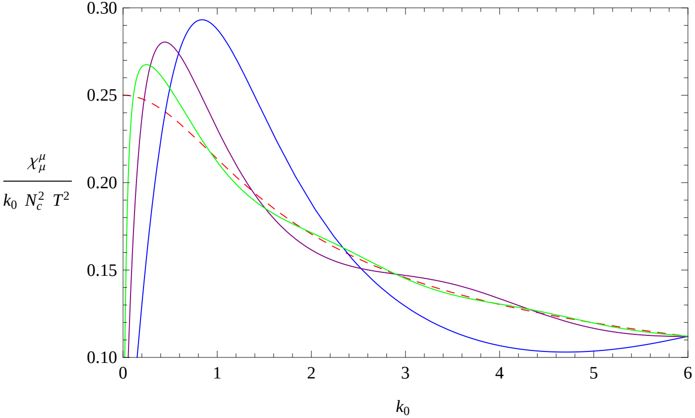


The spectral density as a function of ω for $r_s/r_h = 1$ (thermal case, black dashed), $r_s/r_h = 1.005$ (blue), and $r_s/r_h = 1.1$ (red).



$R = \frac{\chi - \chi_{\text{thermal}}}{\chi_{\text{thermal}}}$ as a function of ω for $r_s/r_h = 1.1$ (green), 1.04 (yellow), 1.02 (red) and 1.002 (blue).

- We can calculate the production rate of dileptons in a thermalizing $\mathcal{N} = 4$ SYM plasma in the quasistatic approximation, i.e. large ω.
- The spectral function features "signature" oscillations, and has the correct thermal limit.
- Trying to have "realistic" shell evolution would be interesting, but seems to make the calculation extremely difficult.
- Computing the production rate of photons can be done using a very similar computation: Instead of the photon having a momentum $(\omega, \vec{0})$, it should now have $(|\vec{k}|, \vec{k})$.



The spectral function for photons for $r_s/r_h = 1.1,\ 1.01,\ 1.001$ and 1.