Primordial scalar perturbations via conformal mechanisms

M. Libanov, S. R., V. Rubakov, G. Rubtsov

ICFP'12

June 2012

Properties of primordial scalar perturbations

- Nearly Gaussian primordial perturbations: obey Wick theorem.
- Nearly flat power spectrum $\zeta(\mathbf{x}) = \int d\mathbf{k} e^{i\mathbf{kx}} \zeta(\mathbf{k})$.

$$\langle \zeta^2(\mathbf{x}) \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\zeta, \quad \mathcal{P}_\zeta \approx 2.5 \times 10^{-9}.$$

Some epoch preceding conventional Hot Big Bang is required!
Inflation: approximate de Sitter symmetry.
Novel idea: flat spectrum of primordial perturbations is created by the phase of complex conformal field $\phi = |\phi| e^{i\theta}$,

$$\zeta \sim \delta\theta.$$

Conformal rolling

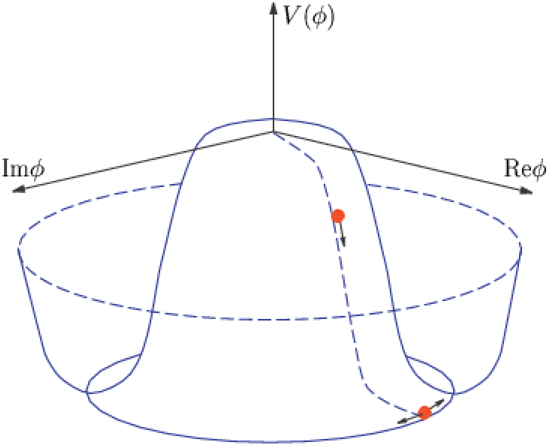
V. Rubakov, 2009

$$S_\phi = \int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^\star\partial_\nu\phi + \frac{R}{6}|\phi|^2 - \left(-h^2|\phi|^4\right)\right] .$$

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$$

- Long evolution before Big Bang starts.
- No other special assumptions about the background evolution.
- Conformal symmetry is explicitly broken at large $|\phi| \sim f$.
- Field ϕ is a spectator.
- Interaction with gravitational degrees of freedom is neglected.

Conformal rolling



Classical evolution

$$\chi = a\phi, \quad S[\chi] = \int d^3x d\eta \left[\eta^{\mu\nu}\partial_\mu^\star \chi \partial_\nu \chi - \left(-h^2|\chi|^4\right)\right]$$

Consider spatially homogeneous classical field,

$$-\chi'' + 2h^2|\chi^2|\chi = 0, \quad \chi = \rho e^{i\theta}.$$

$$\rho_0(\eta) = \frac{1}{h(\eta_\star - \eta)}$$

The phase θ freezes out,

$$\theta_\eta \to 0, \quad \theta \to \theta_0.$$

Using U(1) symmetry, $\theta_0 = 0$.

Fluctuations above the classical background,

$$\rho = \rho_0 + \delta\rho, \quad \theta = \delta\theta$$

Turn off interaction between phase and radial fluctuations.
Study the evolution of θ on the background $\rho_0 = \frac{1}{h(\eta_\star - \eta)}$.
Standard initial conditions of the free bosonic field on the Minkowskian background.
At late times $k(\eta_\star - \eta) \ll 1$,

$$\mathcal{P}_\theta = \frac{h^2}{4\pi^2}\,, \quad \partial_\eta \theta = 0\,.$$

If horizon problem is solved, then flat spectrum for all cosmologically interesting models: ekpyrosis, inflation, Galileon Genesis etc.
Remind: phase fluctuations are the most relevant for us, $\zeta \sim \theta$.
Conversion mechanisms (isocurvature $\theta \to$ adiabatic ζ):
K. Dimopoulos *et al.*, 2003, G. Dvali *et al.*, 2003

Radial perturbations

Radial field developes perturbations with red spectrum,

$$\rho = \frac{1}{h[\eta_\star(\mathbf{x}) - \eta]}, \quad \eta_\star(\mathbf{x}) = \eta_\star + \delta\eta_\star(\mathbf{x}) \ ,$$

$\delta\eta_\star(\mathbf{x})$ is the random Gaussian field,

$$\delta\eta_\star(\mathbf{x}) = \frac{3h}{4\sqrt{2}\pi^{3/2}} \int \frac{d^3p}{p^{5/2}} \left(e^{i\mathbf{px}} B_{\mathbf{p}} + h.c. \right) .$$

Interesting: derivatives of $\delta\eta_\star$, $v_i = -\partial_i \eta_\star, \quad \partial_i\partial_j\eta_\star$ ..

$$\theta|_{\eta=\eta_\star} = \theta(\mathbf{x}, v_i, \partial_i\partial_j\eta_\star, v^2) \ .$$

After $|\phi| = f$,

$$S_\theta = \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu\theta\partial_\nu\theta \ .$$

The end of conformal rolling

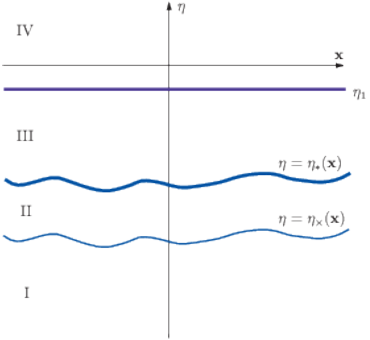
Two possible regimes:

1. Modes of interest are already superhorizon, $\frac{k}{a} < H$.

Considered in Libanov&Rubakov, Libanov&Mironov&Rubakov.

2. They are still subhorizon, $\frac{k}{a} > H$.

In the latter case phase perturbations proceed to evolve at the intermediate stage, which ends as phase perturbations freeze out.



Assumptions

- The duration of the intermediate stage is long enough,

$$k(\eta_1 - \eta_\star) \gg 1$$

- Nearly Minkowskian evolution: $\theta'' - \partial_i^2 \theta = 0$
  (to do not spoil the flat power spectrum).

Background evolution:

1. Galileon Genesis.
2. Ekpyrosis, $p \gg \rho$.

Initial conditions defined at the curved hypersurface $\eta = \eta_\star(\mathbf{x})$,

$$\theta|_{\eta=\eta_\star(\mathbf{x})} = \theta(\mathbf{x}, v_i, \partial_i \partial_j \eta_\star, v^2) \qquad \partial_N \theta = 0$$

$$\theta(\mathbf{k}) = F[\mathbf{v}(\pm \mathbf{n}_k r)] A_{\mathbf{k}} + \text{h.c.}, \quad r = \eta_1 - \eta_\star \; .$$

Non-Gaussianity

Since $\theta = -\theta$, intrinsic bispectrum vanishes.
It may appear due to the conversion mechanism.

Intrinsic non-Gaussianity arises in trispectrum,

$$\langle \zeta(\mathbf{k})\zeta(\mathbf{k}')\zeta(\mathbf{q})\zeta(\mathbf{q}')\rangle = \frac{\mathcal{P}_\zeta(k)}{4\pi k^3}\frac{\mathcal{P}_\zeta(q)}{4\pi q^3}\delta(\mathbf{k}+\mathbf{k}')\delta(\mathbf{q}+\mathbf{q}')\left[1 + F_{NG}(\mathbf{n}_k, \mathbf{n}_q)\right]$$

$$F_{NG} = \frac{3h^2}{\pi^2}\ln\frac{\text{const}}{|\mathbf{n}_k - \mathbf{n}_q|}$$

Logarithmically amplified in the folded limit.

Tilt

Small negative tilt

$$n_s - 1 = -\frac{3h^2}{4\pi^2}$$

This is not a strong result.

Other sources of scalar negative tilt are possible (Rubakov&Osipov, 2010)
What if the preferrable value of tilt was due to this contribution?

$$n_s - 1 \approx -0.04$$

Then, $h^2 \sim 1$. However, disfavored by observations of statistical anisotropy.

Statistical anisotropy

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_\zeta(k)\left(1 + n_{k_i}\left[v_i(\mathbf{n}_k r) - v_i(-\mathbf{n}_k r)\right]\right) \ .$$

$$= \sum_{LM} q_{LM} Y_{LM}(\mathbf{n}_k)$$

Statistical anisotropy of all even multipoles.

$$\langle q_{LM} q^\star_{L'M'} \rangle = \frac{3}{\pi} \frac{h^2}{(L-1)(L+2)} \delta_{LL'} \delta_{MM'},$$

- Statistical isotropy is favored by inflation.
- In extended inflation statistical anisotropy is of special quadrupole type, (Ackerman *et al.*, 2007; Soda *et al.*, 2008).

$$\mathcal{P}_\zeta = \mathcal{P}_0 \left(1 + g(\mathbf{n}_k \mathbf{d})^2\right) \ .$$

Imprint on CMB spectrum

Having fluctuations $\delta T(\mathbf{n})$ of the CMB sky, we define

$$a_{lm} = \int d\mathbf{n} \delta T(\mathbf{n}) Y^{\star}_{lm}(\mathbf{n}), \quad C_{lm;l'm'} = \langle a_{lm} a^{\star}_{l'm'} \rangle$$

In case of statistical anisotropy off-diagonal elements appear,

$$C_{lm;l'm'} = C_l \delta_{ll'} \delta_{mm'} + \sum_{LM} q_{LM} D^{LM}_{lm;l'm'}$$

$$\langle a_{lm} a^{\star}_{l+2,m} \rangle, \quad \langle a_{lm} a^{\star}_{l+4,m} \rangle$$

are non zero.

How to constrain the model?

$$\langle |q_{LM}|^2 \rangle = \frac{3}{\pi} \frac{h^2}{(L-1)(L+2)} \; .$$

Quadratic maximum likelihood estimation

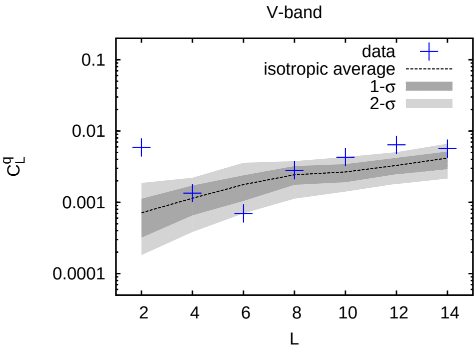
$$\mathcal{L}(\mathbf{a}|\mathbf{q}) = \ln \mathcal{W}(\mathbf{a}|\mathbf{q}), \quad \frac{\partial \mathcal{L}(\mathbf{a}|\mathbf{q})}{\partial q_{LM}} = 0 \, .$$

Assume small statistical anisotropy, $q_{LM} \ll 1$

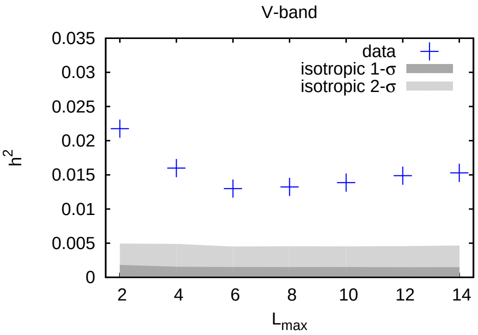
$$\mathcal{L}(\mathbf{a}|\mathbf{q}) = \mathcal{L}_0 + \sum_{LM} \left(\frac{\partial \mathcal{L}}{\partial q_{LM}} \right)_0 q_{LM} + \frac{1}{2} \sum_{LM;L'M'} \left(\frac{\partial^2 \mathcal{L}}{\partial q_{LM} \partial q^\star_{L'M'}} \right)_0 q_{LM} q^\star_{L'M'} \, .$$

Hirata&Seljak (2002), Hanson&Lewis (2009)
One estimates q_{LM} and $C^q_L = \frac{1}{2L+1} |q_{LM}|^2$.
To estimate h^2,

$$\frac{\partial \mathcal{L}(\mathbf{a}|h^2)}{\partial h^2} = 0, \quad h^2 = f(C^q_2, C^q_4, ...).$$



- The quadrupole preferred direction is aligned with the poles of the ecliptic plane
- The result is frequency dependent.
- Beam asymmetries? (Challinor&Hanson&Lewis, 2010).



$h^2 < 0.045$ at the 95% CL.

Conclusions

- Flat spectrum of primordial scalar perturbations.
- Statistical anisotropy of all even multipoles.
- Non-Gaussianity in trispectrum.
- Upper constraint on the paramater $h^2 < 0.045$ at the 95 % CL.