IMPLICATION OF RESULTS FROM HEAVY-ION EXPERIMENTS FOR COMPACT STARS

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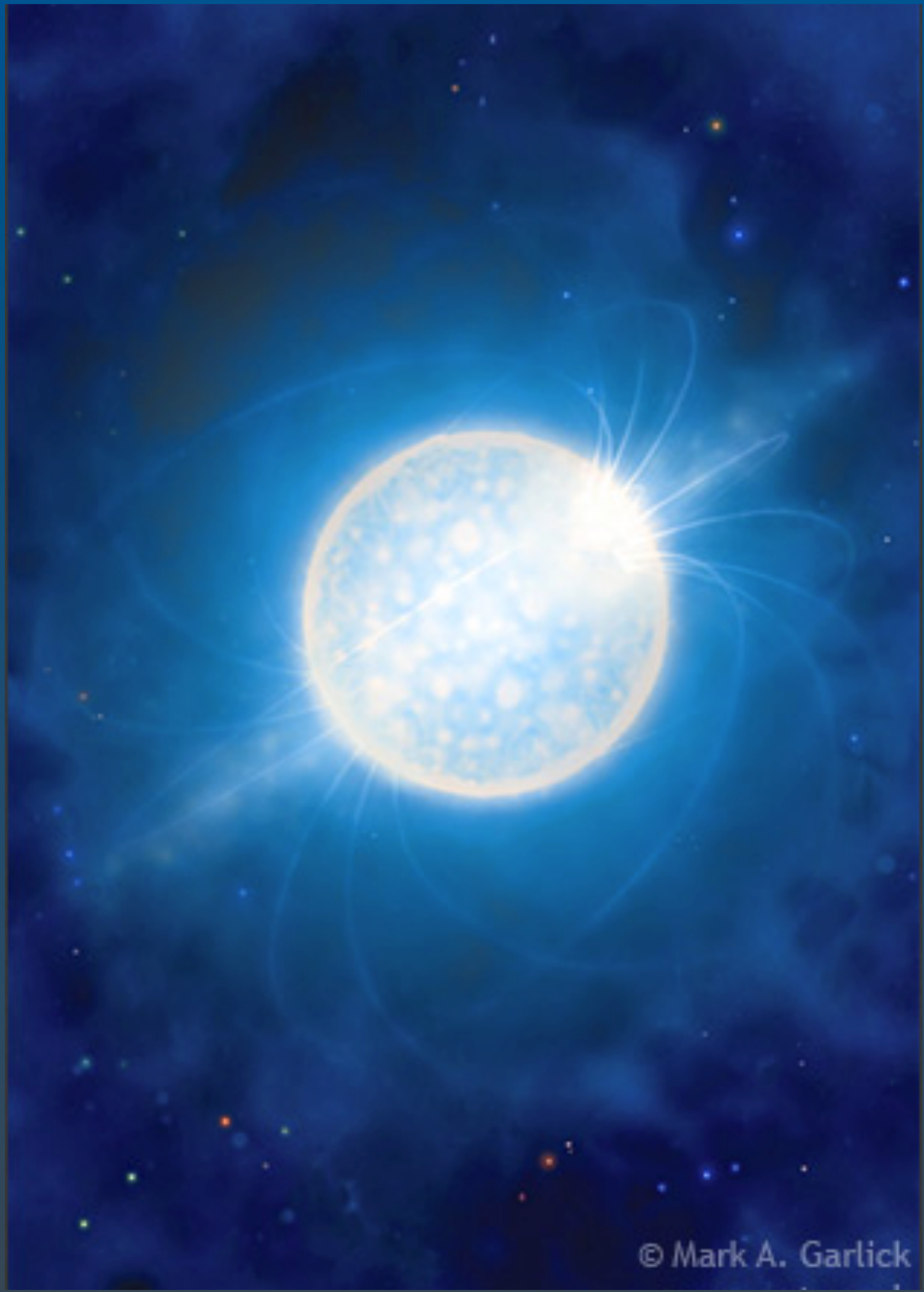
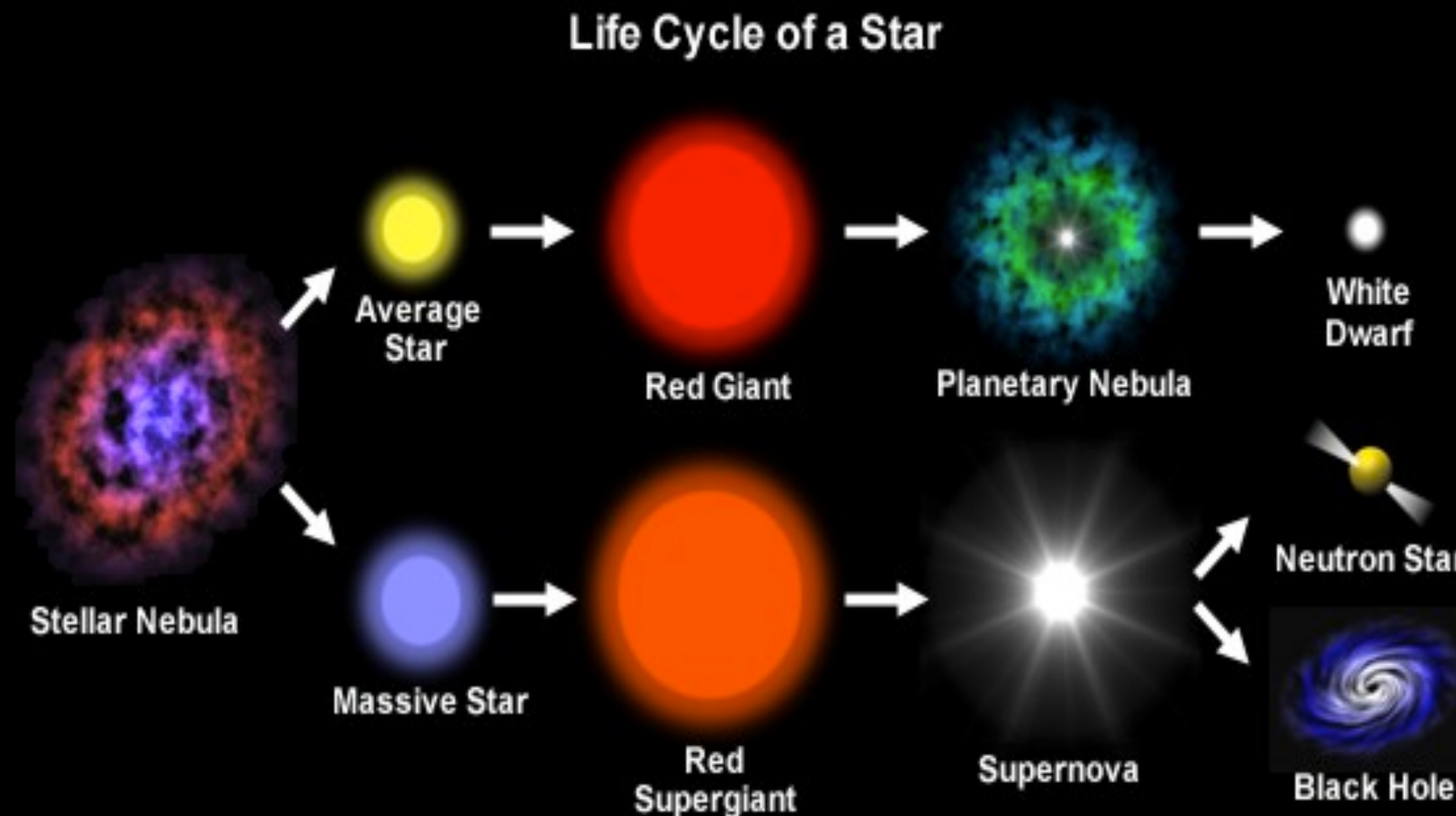
Cristian Sturm, GSI

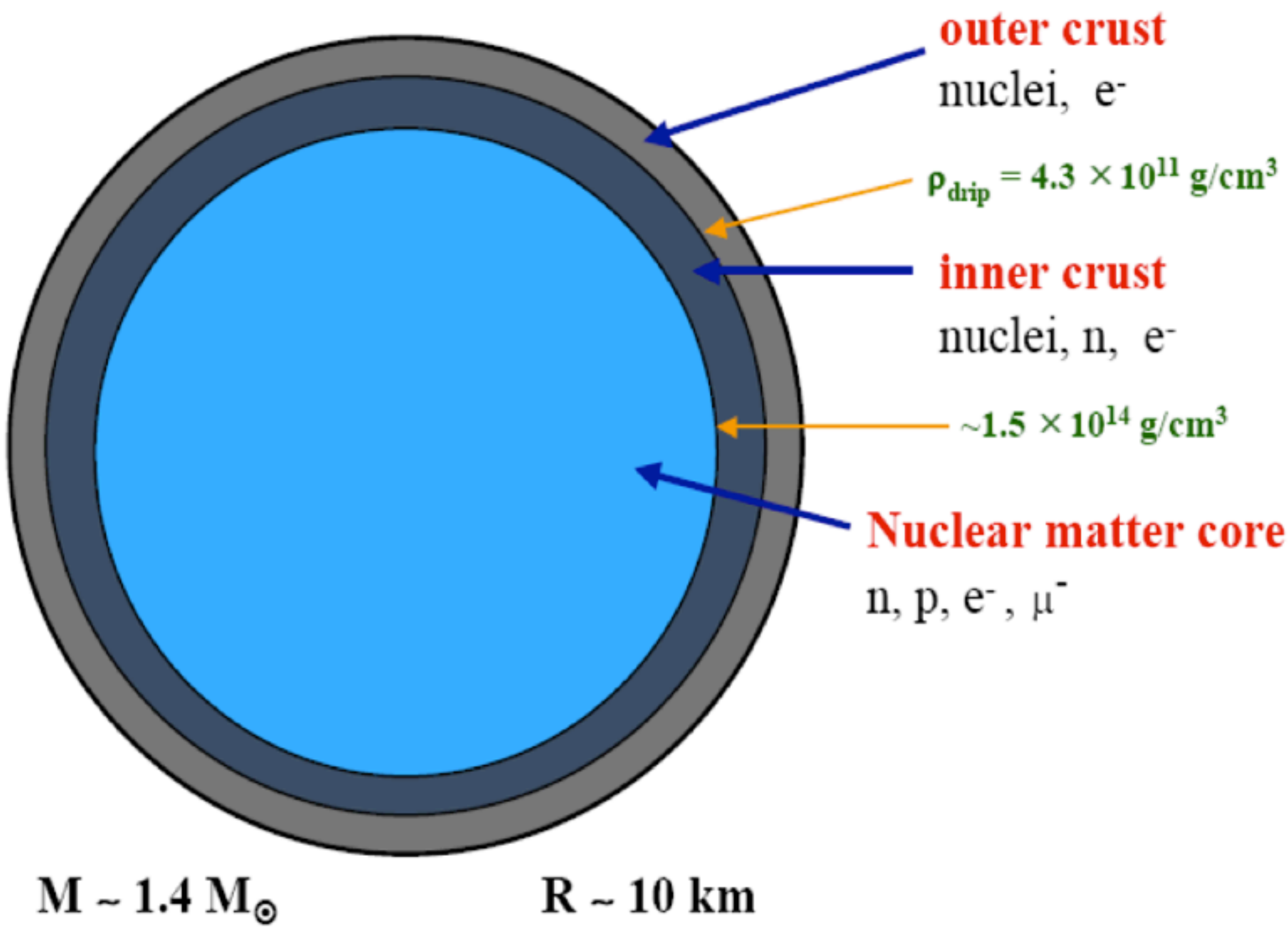


NEUTRON STARS



- *Produced in supernova explosions* (*Type II*)
- *Compact massive objects,* $M \sim 1\text{-}2\ M_{solar}$, $R \sim 10$ *km*

NEUTRON STAR STRUCTURE



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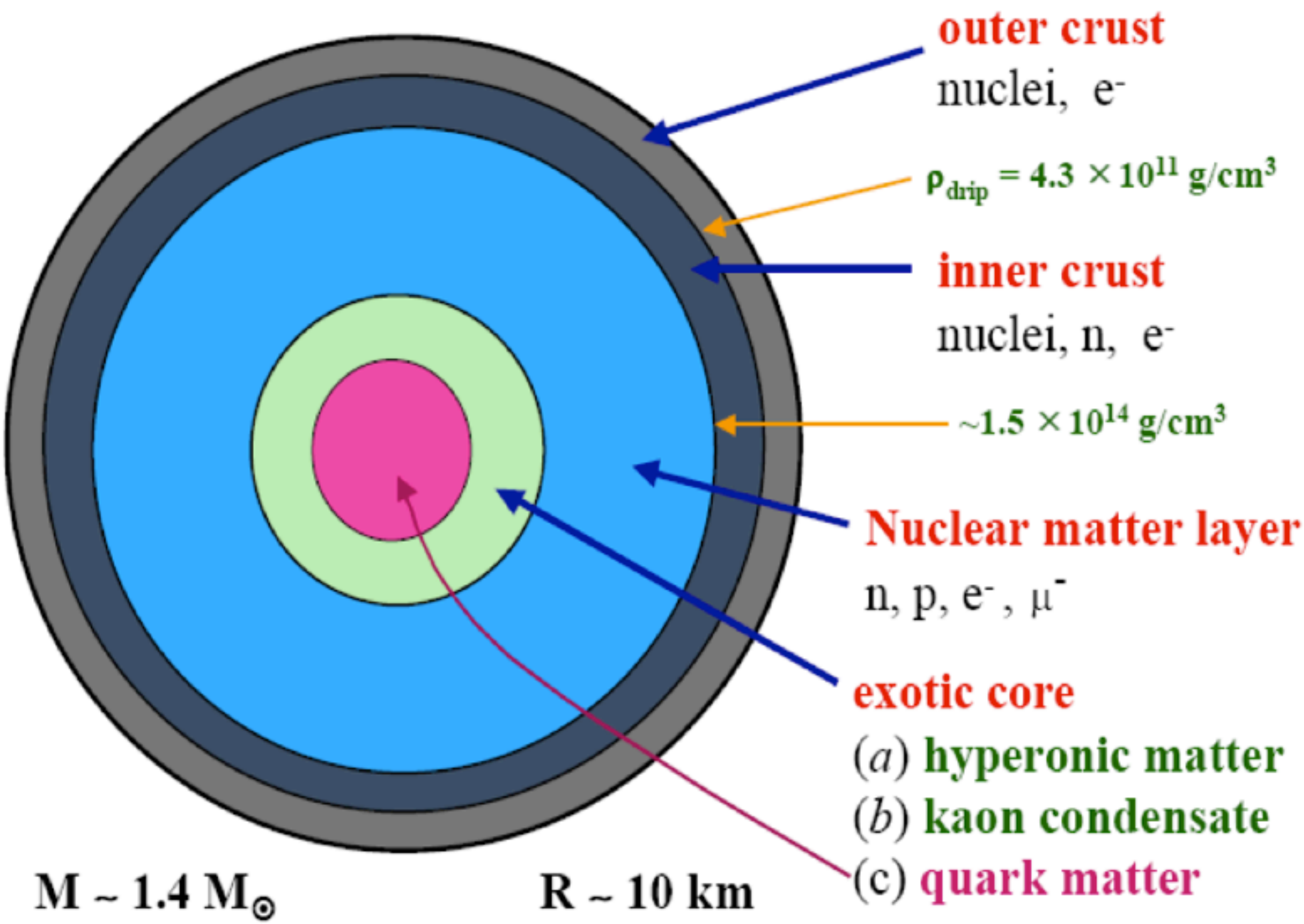
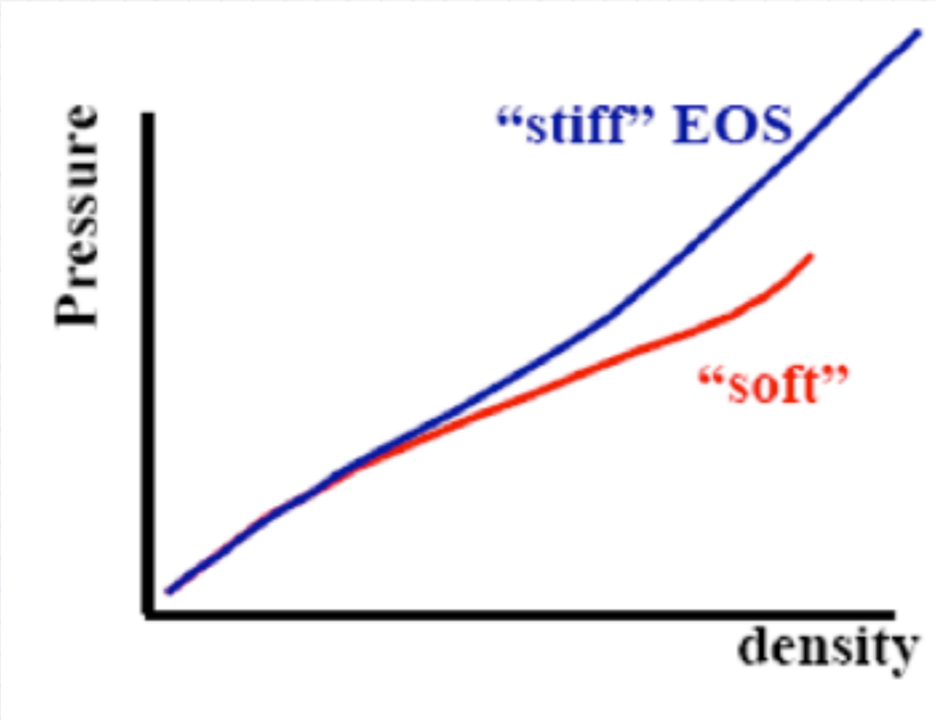


Equation of state (EoS)



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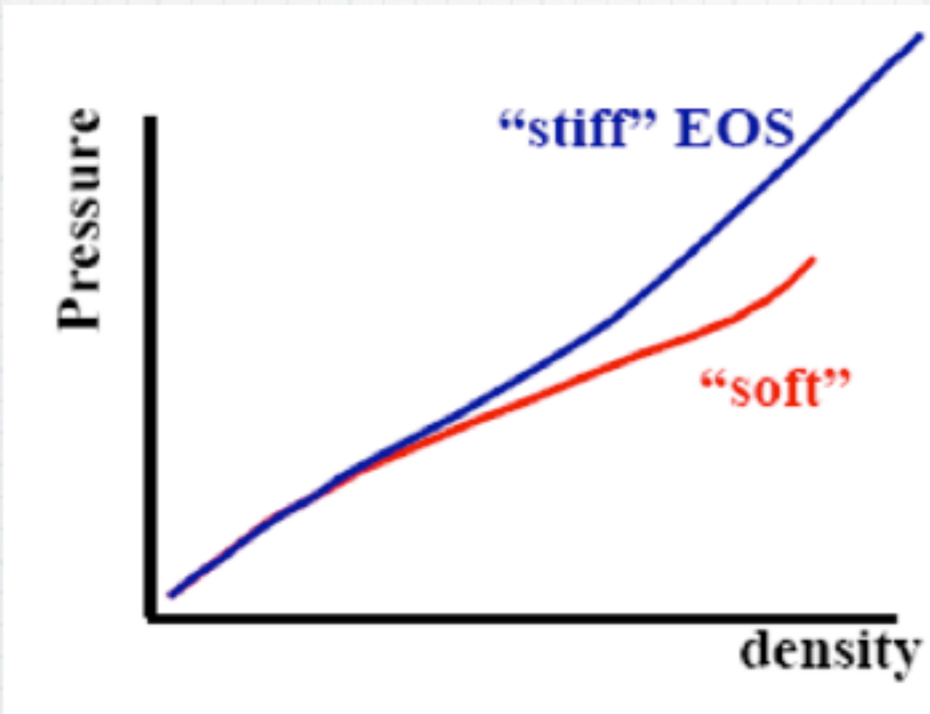

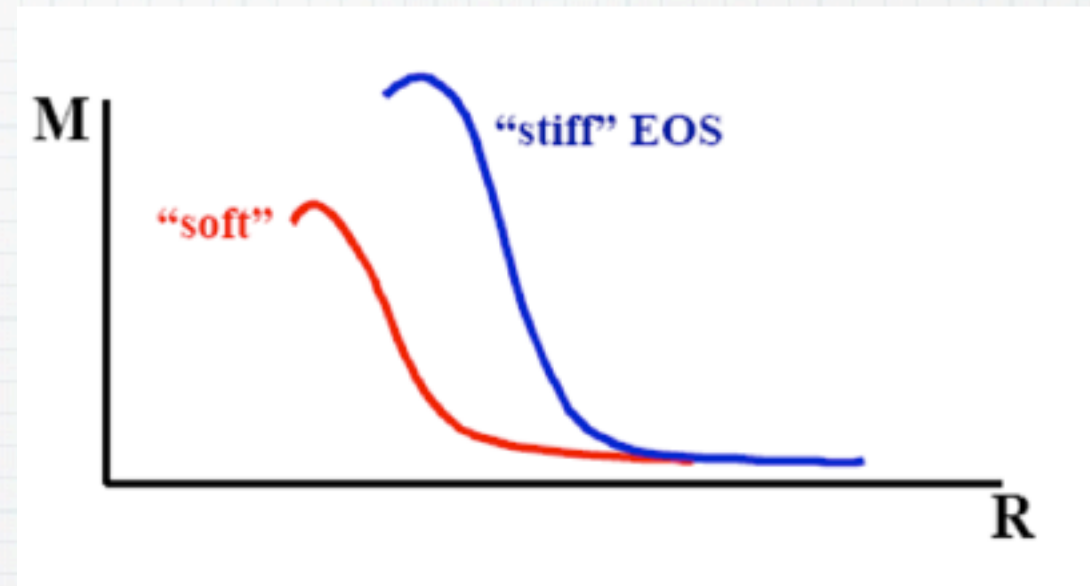

Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi \frac{\epsilon}{c^2} r^2$$

Measurement of neutron star masses :
Relativistic binaries

Keplerian parameters

- Orbital period P_b
- Projected semi-major axis $x = (a_p \sin i) / c$
- Orbital eccentricity e
- Longitude of periastron ω
- Epoch of periastron passage T_o

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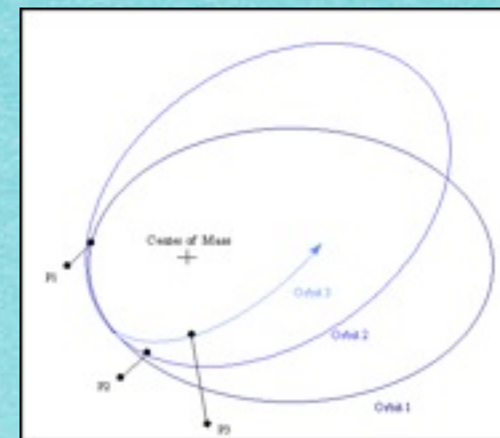

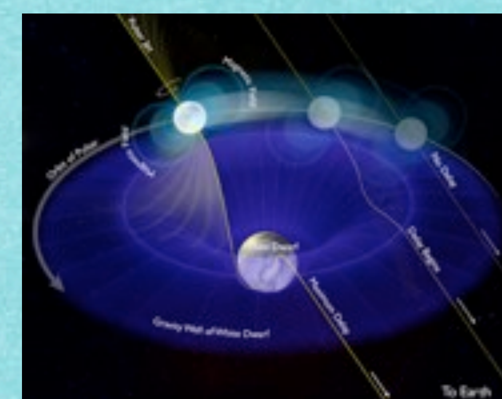

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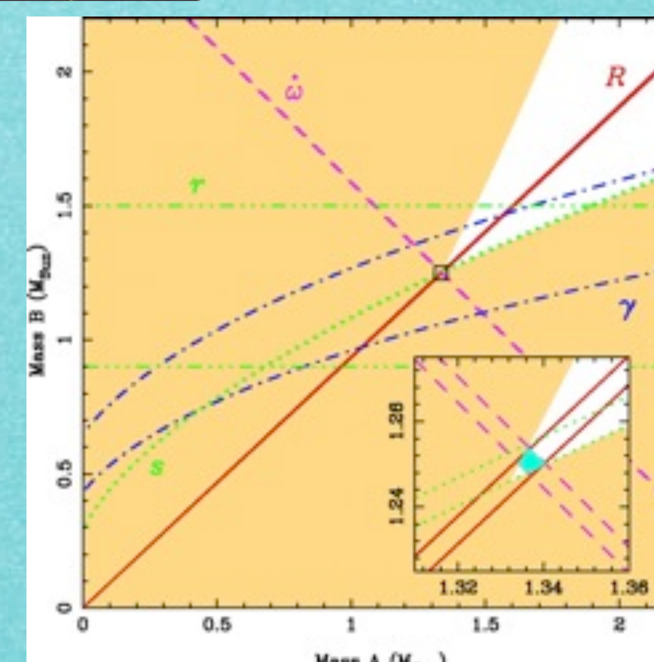
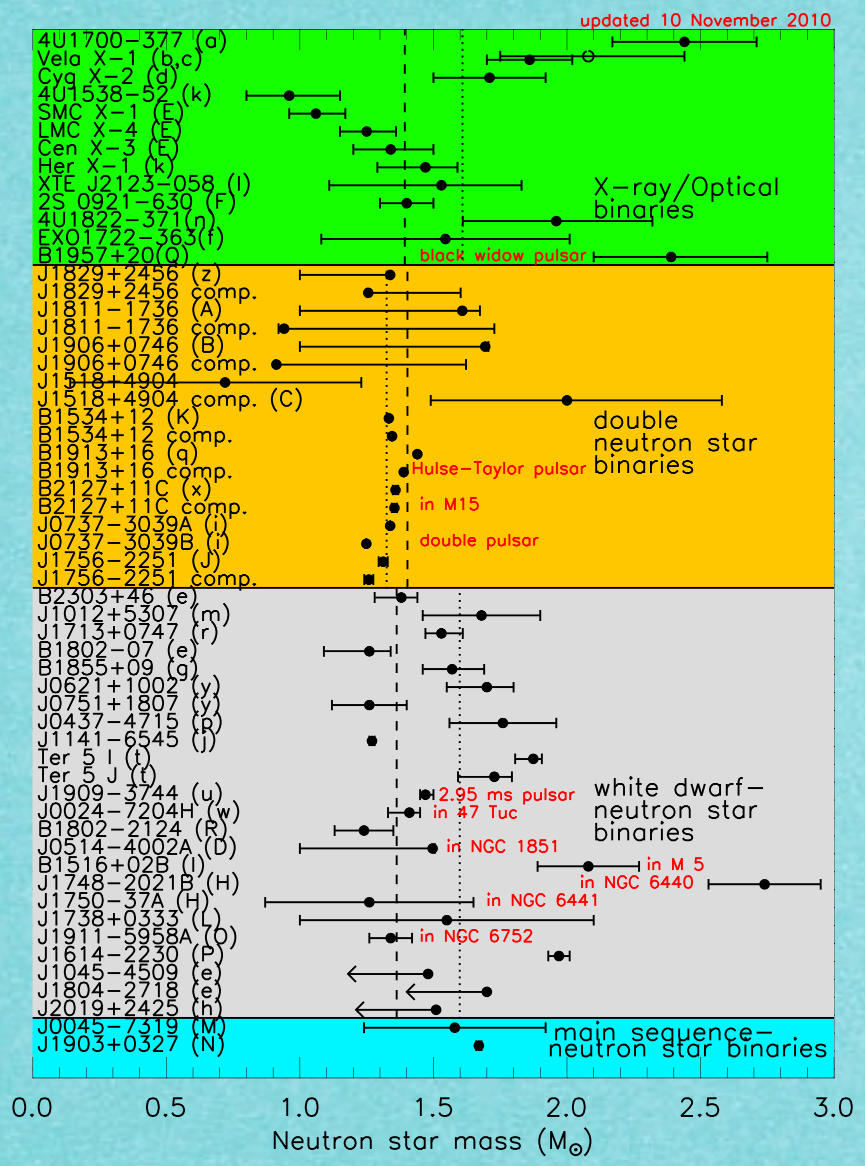
Post-Keplerian Parameters

- *Relativistic advance of periastron* $\dot{\omega}$
- *Gravitational redshift and time dilation* γ
- *Orbital decay change in period* $\dot{P}_b$
- *Shapiro delay range r and shape s*

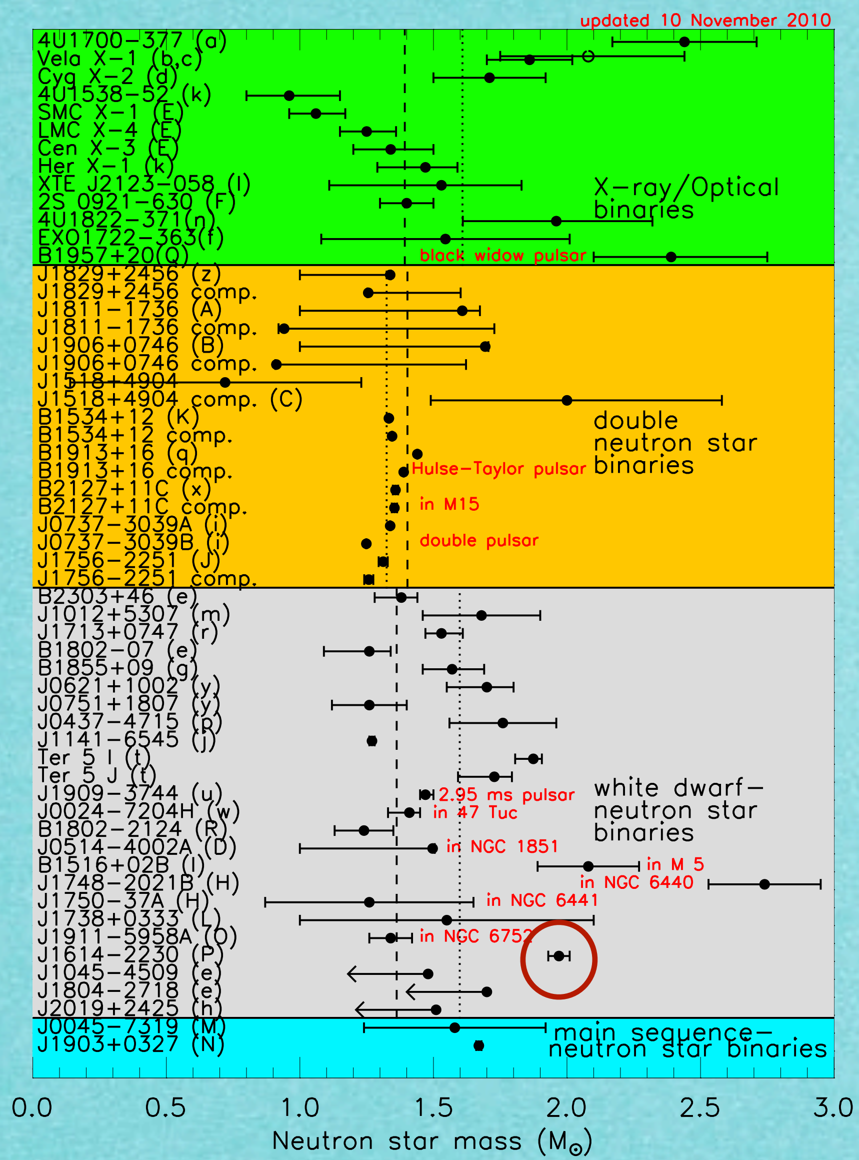


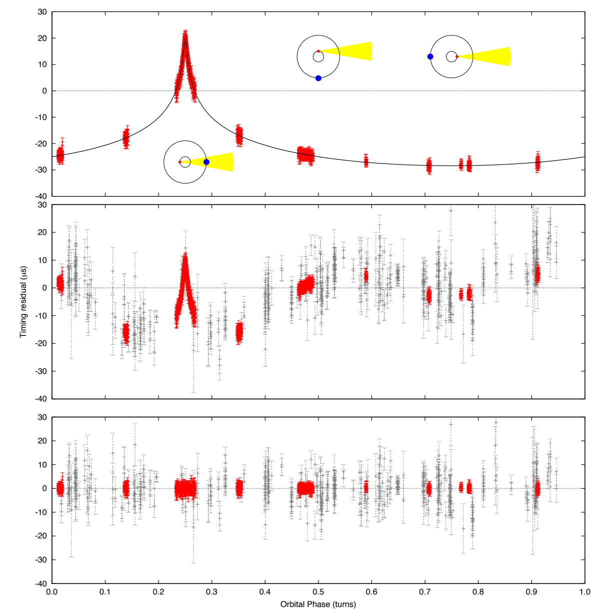
Mass measurements



Lattimer and Prakash, arXiv1012.3208

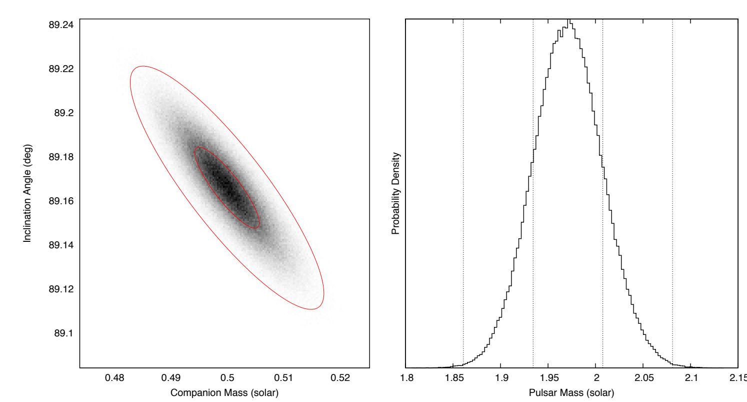
Highest mass measurement : J1614-2230





Timing residual as a function of pulsar's orbital phase

$1.97 \pm 0.04\ M_{\odot}$

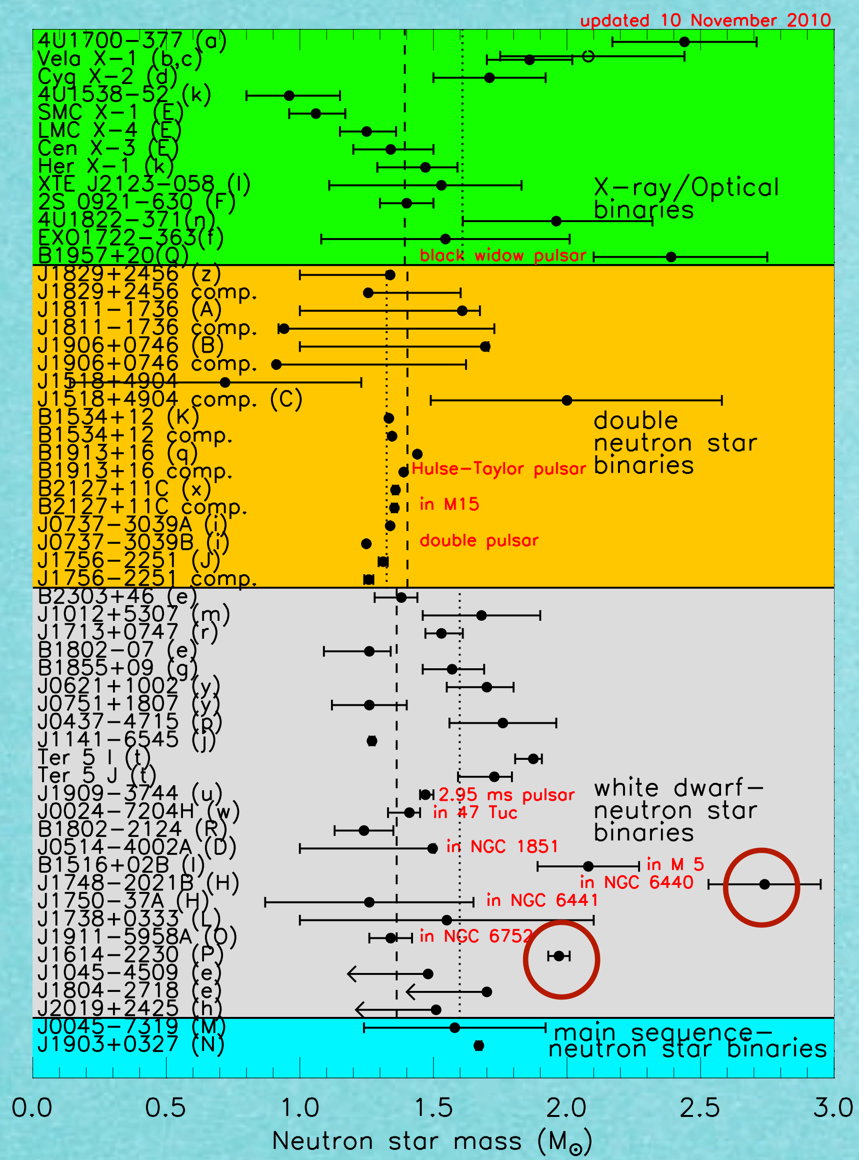


Monte Carlo analysis: Probability density function

Lattimer and Prakash, arXiv1012.3208

Demorest et al (Nature 2010)

Mass measurements



Freire et al (Ap. J. 2008)

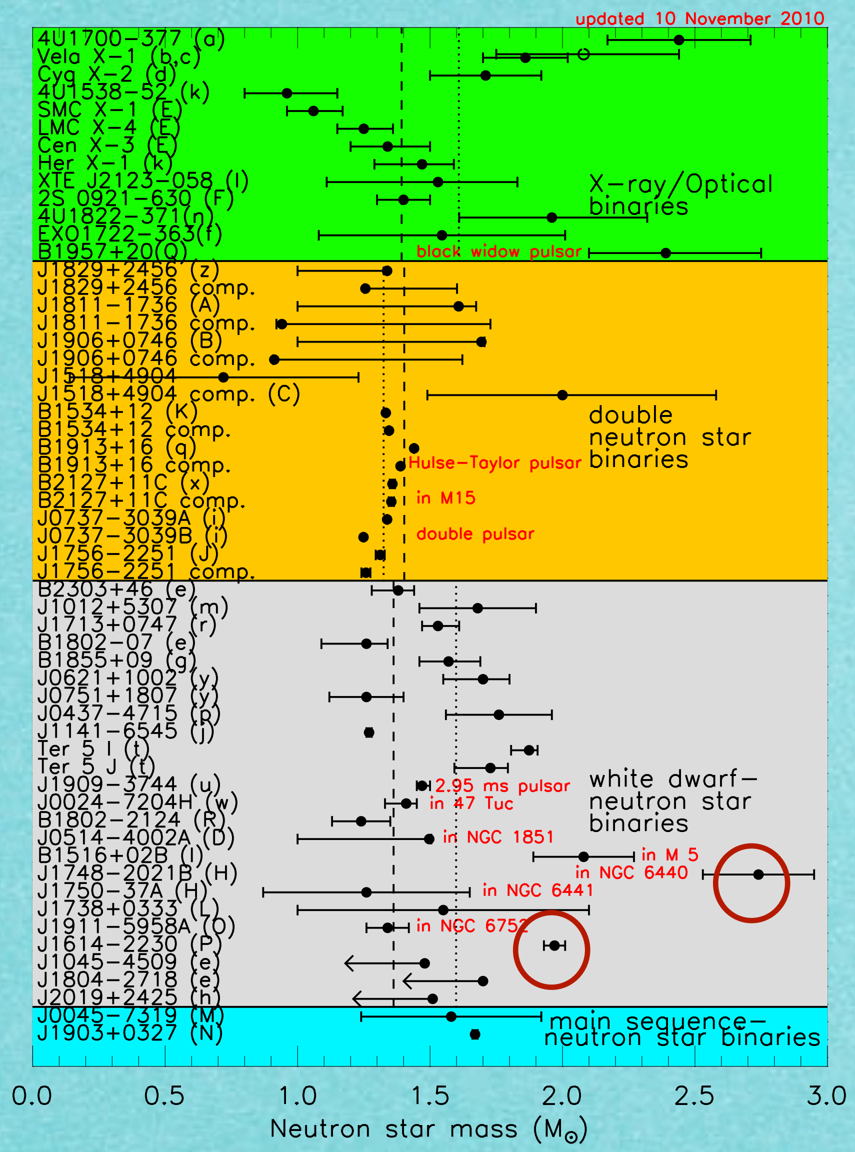
$2.74 \pm 0.2\ M_{sol}$

$1.97 \pm 0.04\ M_{\odot}$

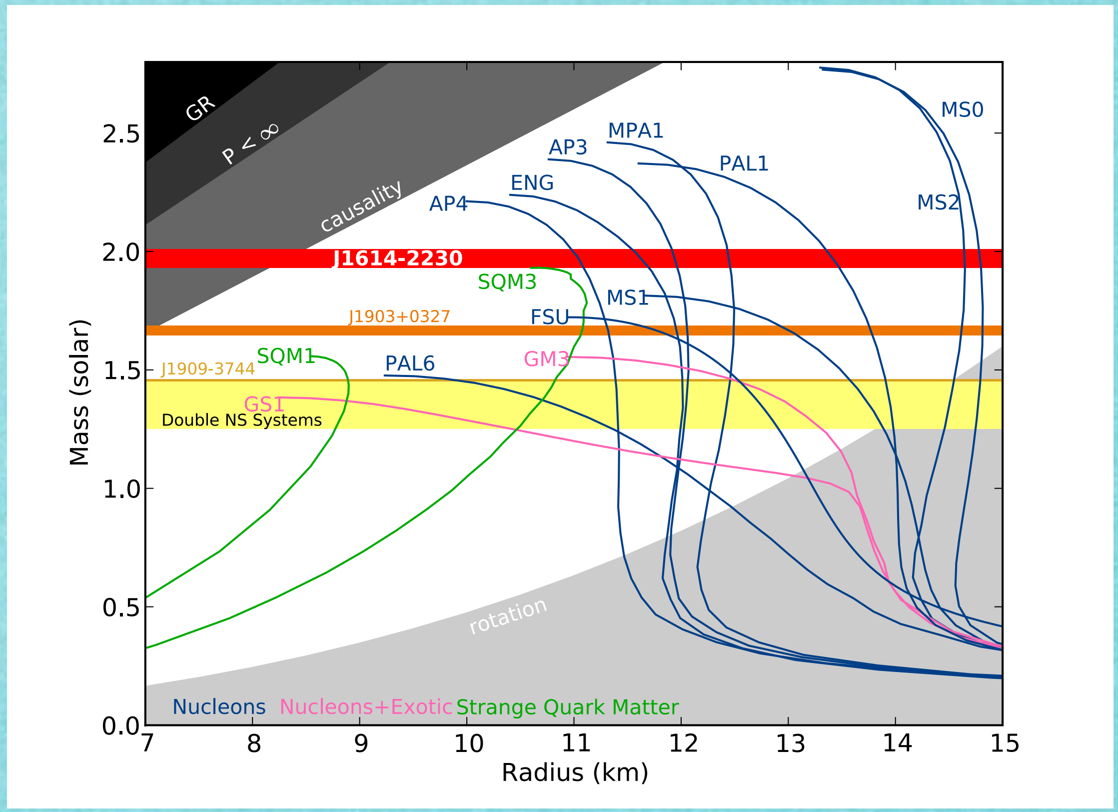
Demorest et al (Nature 2010)

Lattimer and Prakash, arXiv1012.3208

Constraining the EoS



$M^{max}(theo) > M^{max}(obs)$



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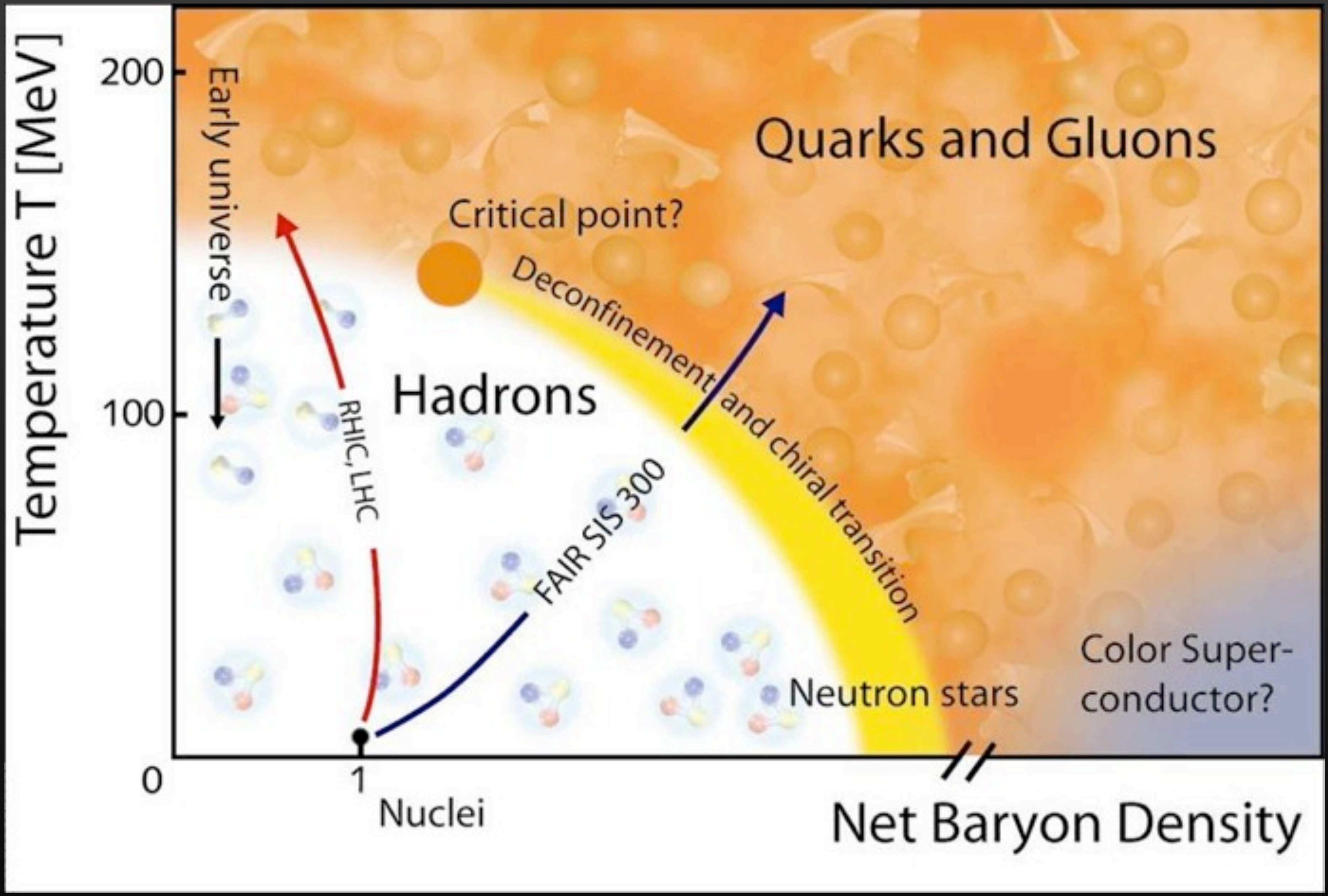
CONSTRAINTS FOR NUCLEAR EOS



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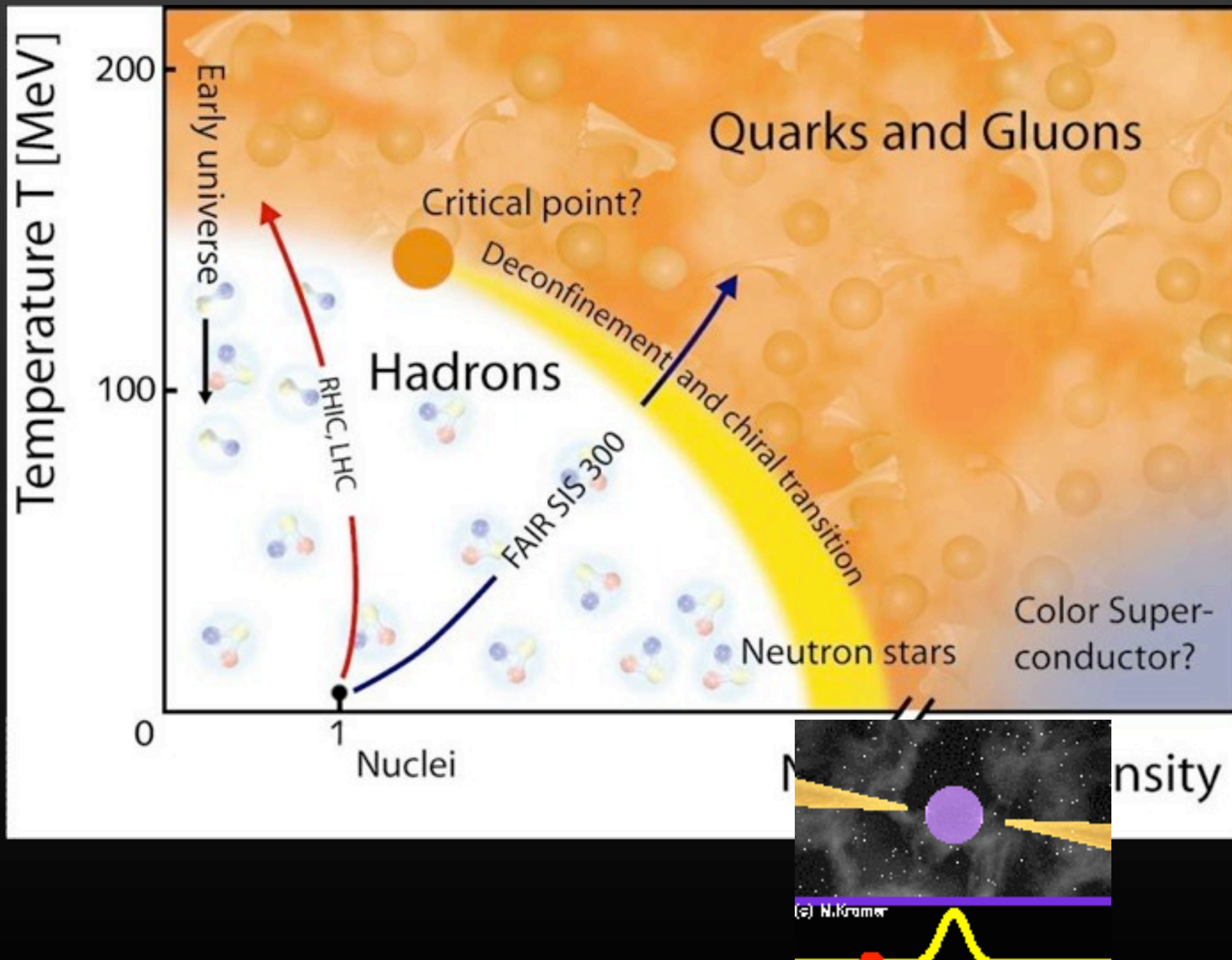

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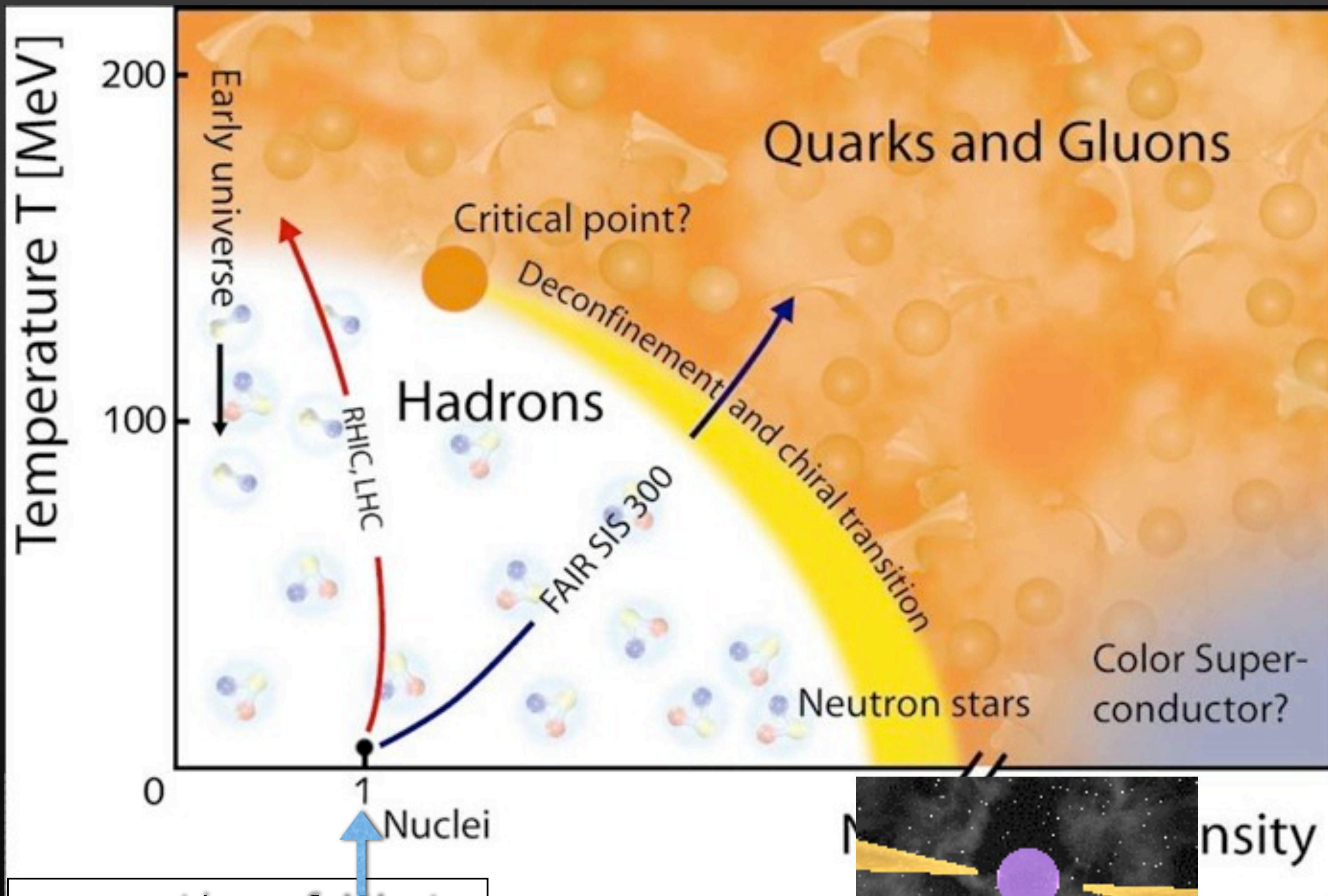

properties of NM at saturation n_0:
B/A, m*/m, K, a_{sym}

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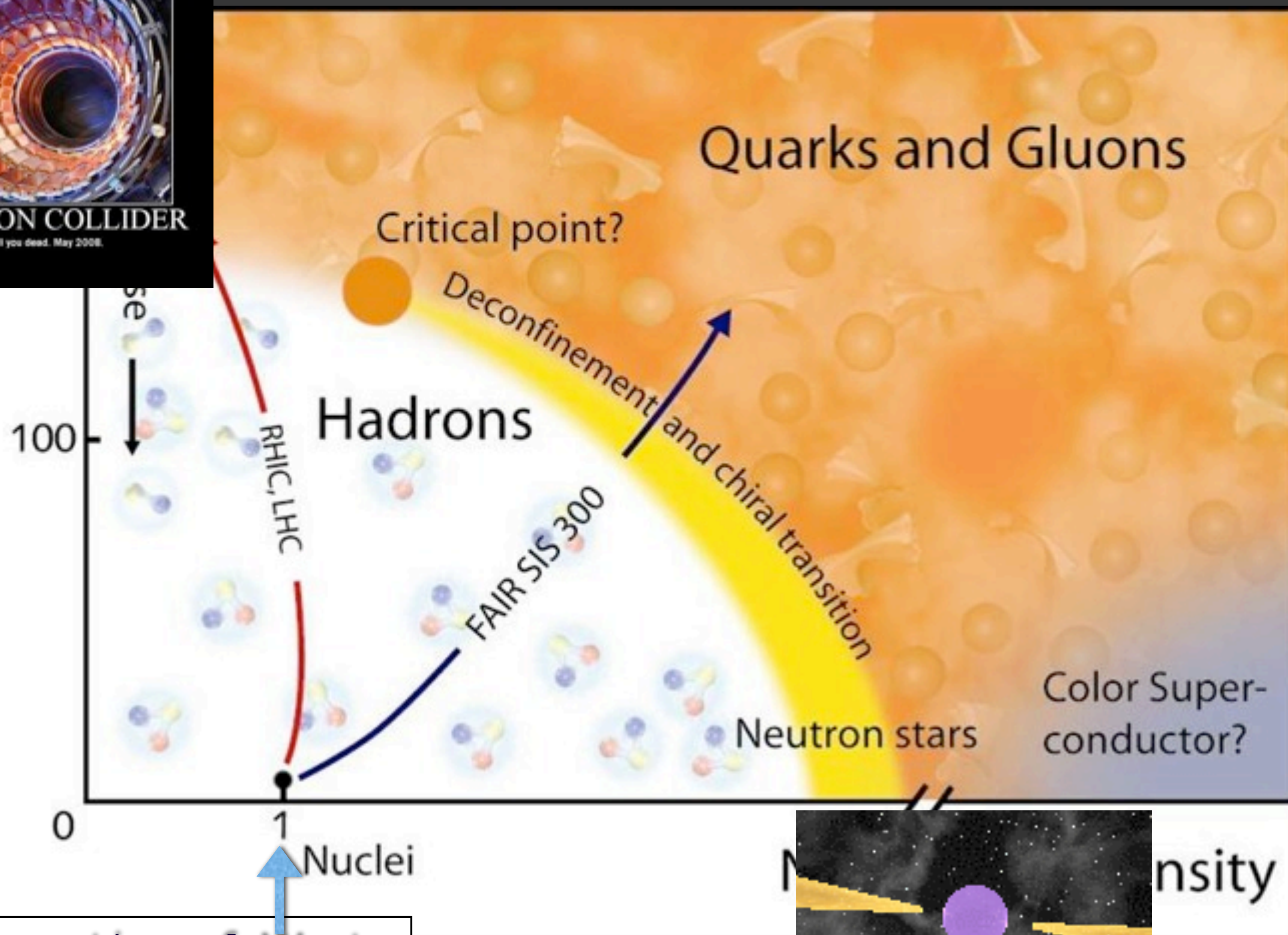

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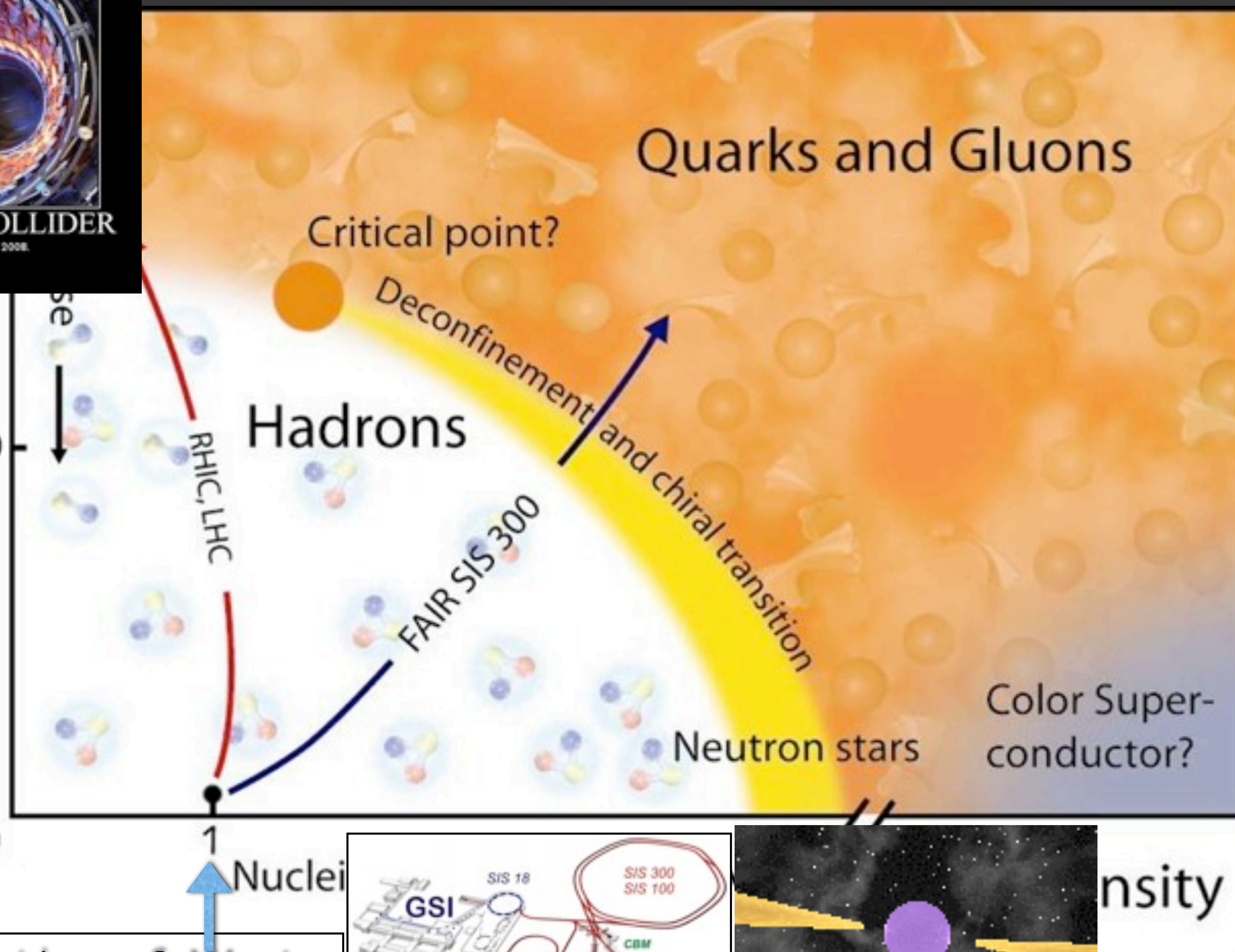

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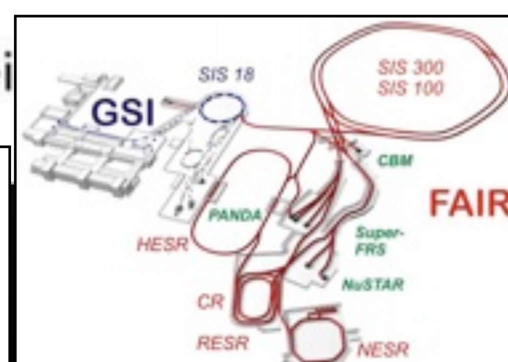
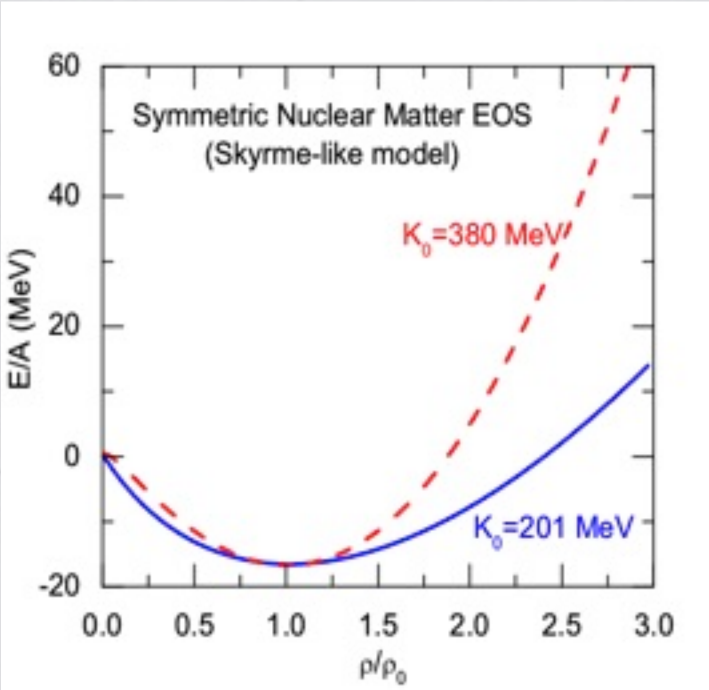


Properties of dense nuclear matter

Symmetric nuclear matter at saturation



- *saturation density* $n_0 = 0.17\ fm^{-3}$
- *binding energy per nucleon* $B/A = -16.3\ MeV$
- *effective nucleon mass* $m^*/m = 0.55\text{-}0.8$
- *incompressibility* $K_o = 235 \pm 14\ MeV$

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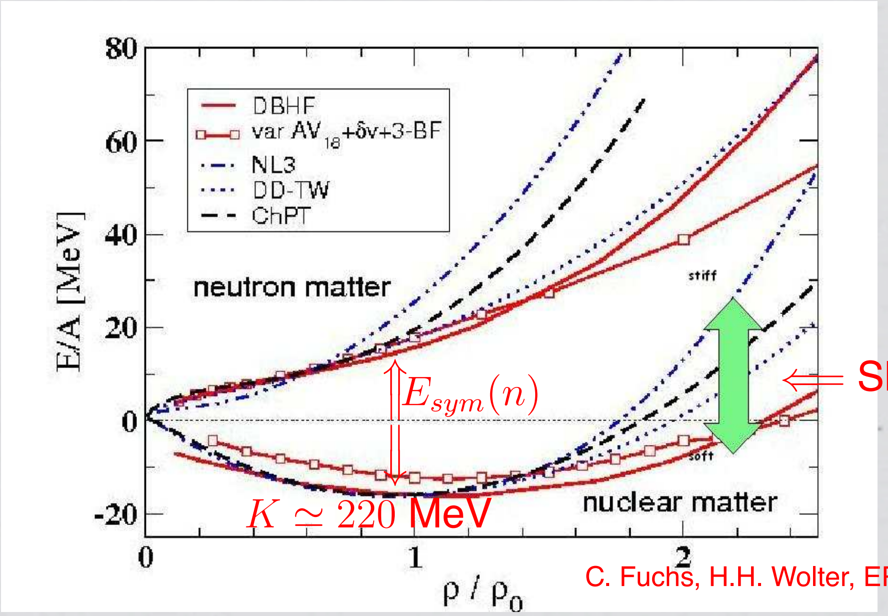
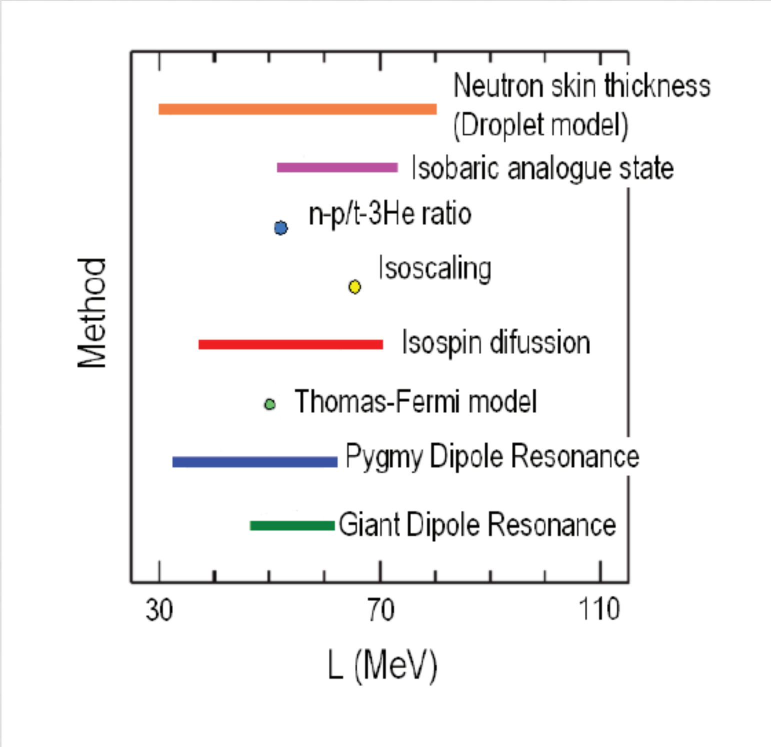
- *symmetry energy* $E_{sym} = 28\text{-}32\ MeV$



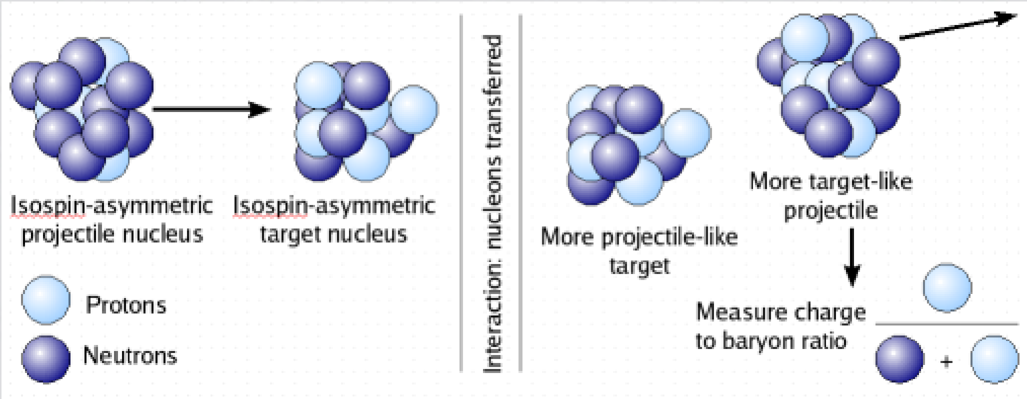
C. Fuchs, H.H. Wolter, EPJA 30(2006) 5

Density dependence of Symmetry Energy L

Asymmetric nuclear matter at saturation



- *the density dependence of symmetry energy is a crucial quantity in nuclear physics*
- nuclei $\Rightarrow$ $n < n_0$
- *Isospin diffusion data from intermediate energy HIC provide constraint on L only around* n_0
- *neutron skin thickness of heavy nuclei*
- *Giant dipole resonance in* ^{208}Pb
- *Pygmy dipole resonance in* ^{208}Pb

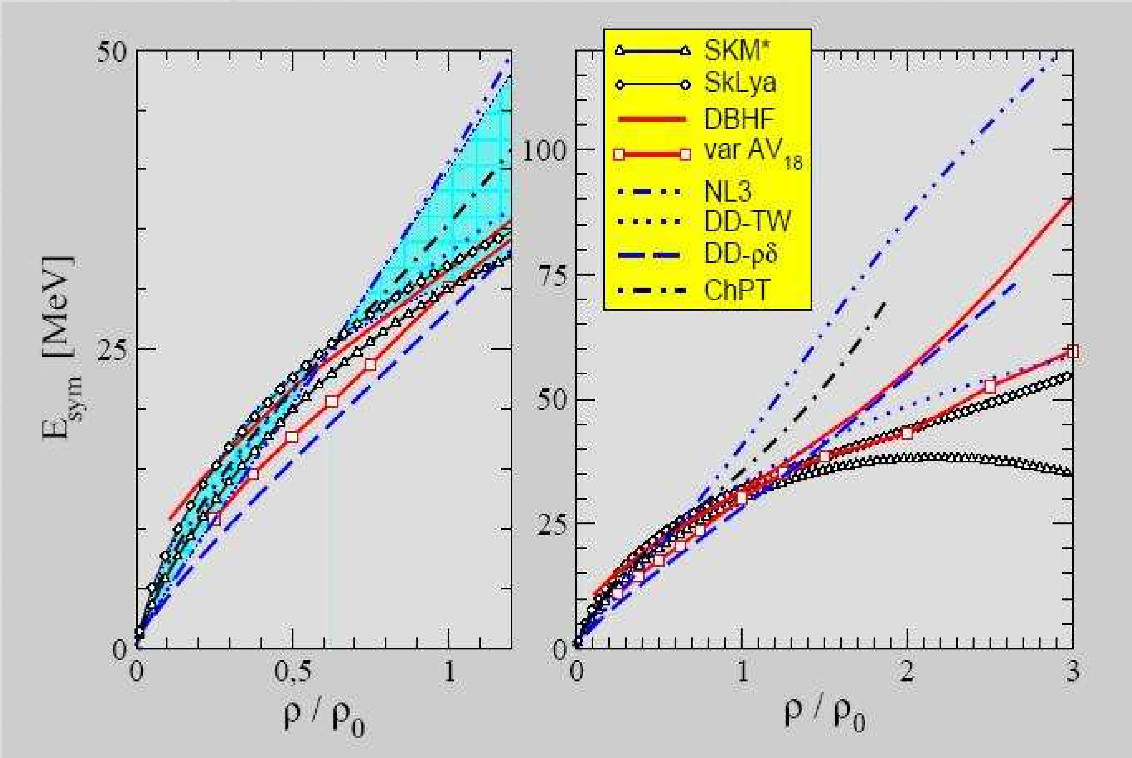


Density dependence of Symmetry Energy *L*

Nuclear matter beyond saturation

- *Density dependence of symmetry energy "L" becomes highly uncertain at $n >> n_0$*

The Uncertain $E_{sym}(n)$



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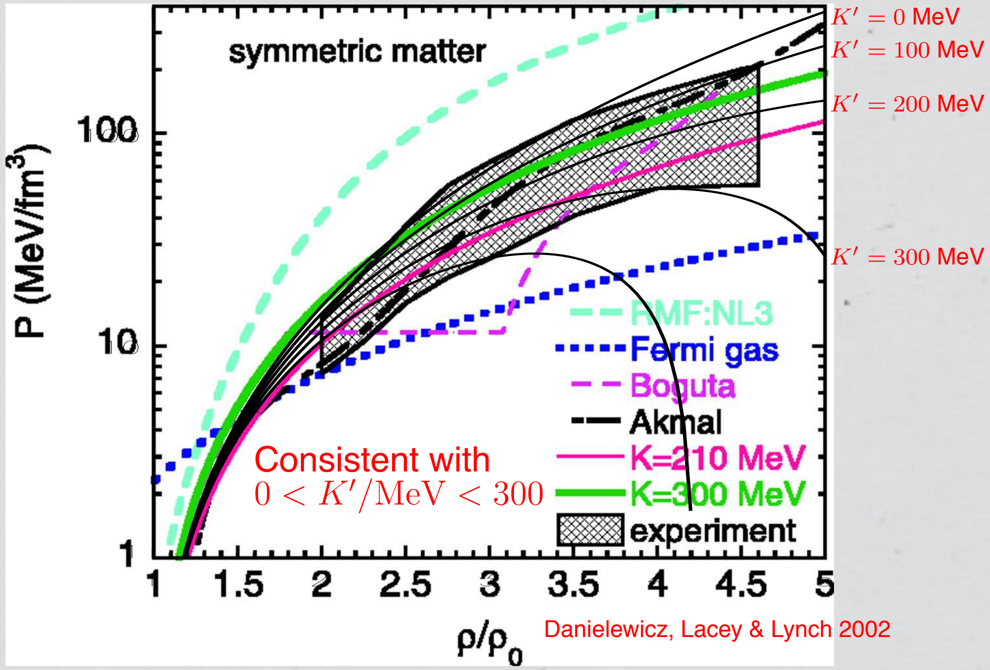
  - (not conclusive, new degrees of freedom at high energies, momentum dependence of interaction, model dependence of analysis)

Flow Constraint From Heavy Ions



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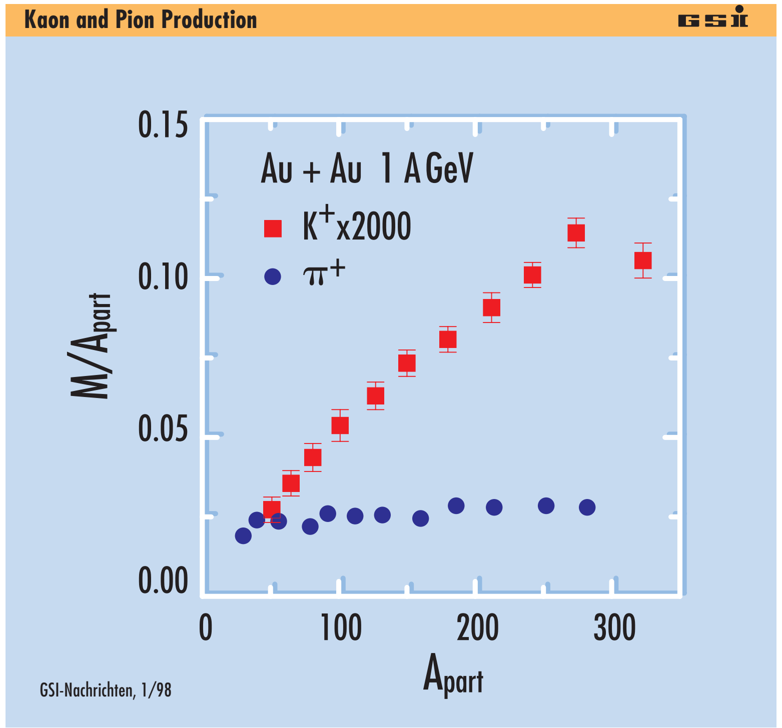
- *Elliptic flow of nucleons in non-central nucleus-nucleus collisions*
  - (not conclusive, new degrees of freedom at high energies, momentum dependence of interaction, model dependence of analysis)
- *K^+ meson production in nuclear collisions* ✓

K^+ meson production in heavy-ion collisions

KaoS experiment,
GSI Darmstadt

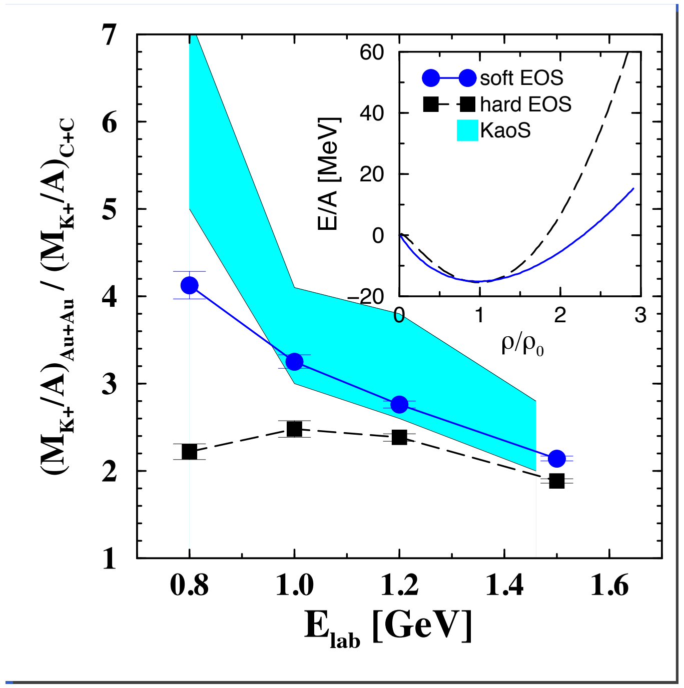
NN → NΔ → NYK	
↳ πN → YK	



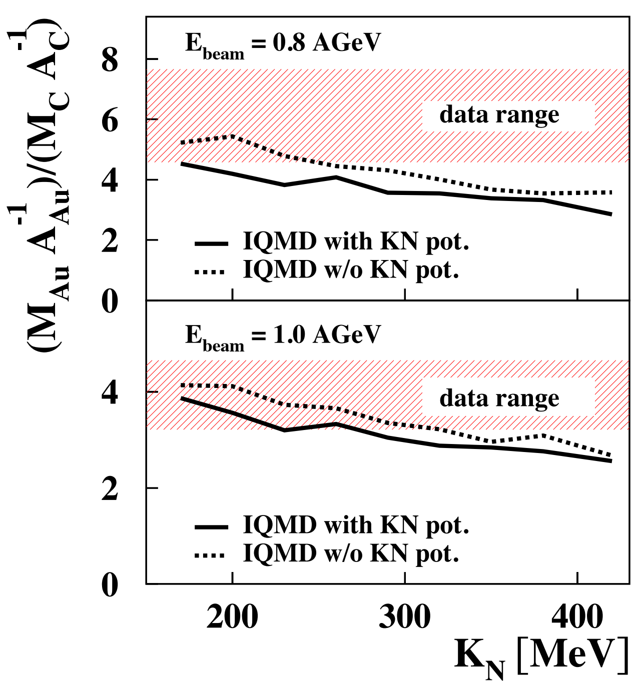
Subthreshold production of K+ particles

- *K^+ particles produced by multiple NN collisions ($NN \rightarrow N\Lambda K$, $NN \rightarrow NNK\bar{K}$) or secondary collisions ($\pi N \rightarrow \Lambda K$, $\pi\Lambda \rightarrow N\bar{K}$)*
- *Nuclear matter compressed up to ~ 2-3 n_0*
- *Production of K+ particles sensitive to the nuclear EoS ⇒ tool to probe compressibility of nuclear matter at ~ 2-3 n_0*

Soft equation of state from heavy-ion data



Sturm et al. (KaoS collaboration), PRL 2001



Hartnack, Oeschler, Aichelin, PRL 2006

- *K^+ multiplicity ratio in Au+Au and C+C collisions at 0.8 AGeV and 1.0 AGeV is sensitive to the compression modulus of matter*
- *transport model calculations performed: Skyrme-type nucleon potential with 2BF, 3BF were applied, with parameters to reproduce a soft EoS (with K =200 MeV) and a stiff one (with K =380 MeV).*
- *transport models agree, confirm that matter in the collision zone reaches densities up to 2-3 n_0*
- *only K~ 200 MeV can describe the data (KaoS collaboration, 2007)*

⇒ ***the nuclear EoS is soft***

Testing soft EoS with Neutron Stars

properties of NM at saturation n_0:
B/A, m*/m, K, E_{sym}

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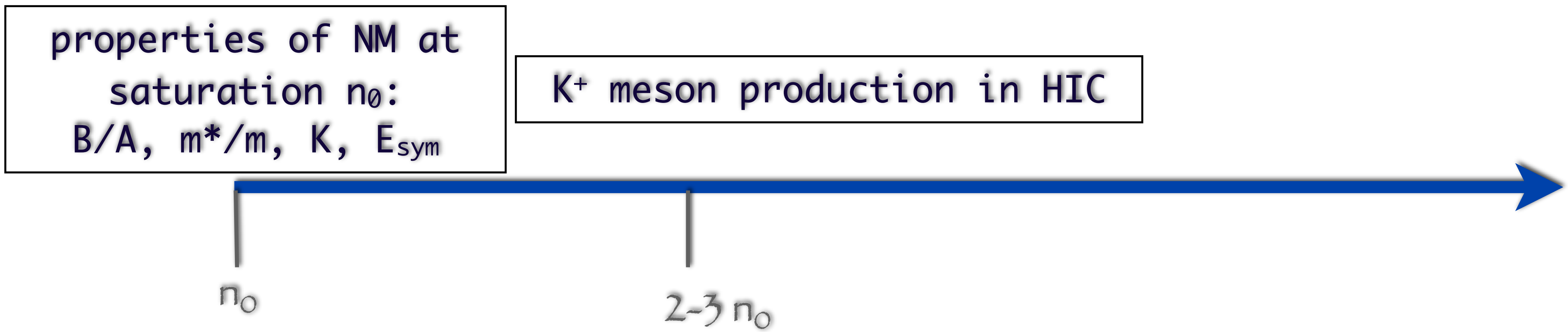

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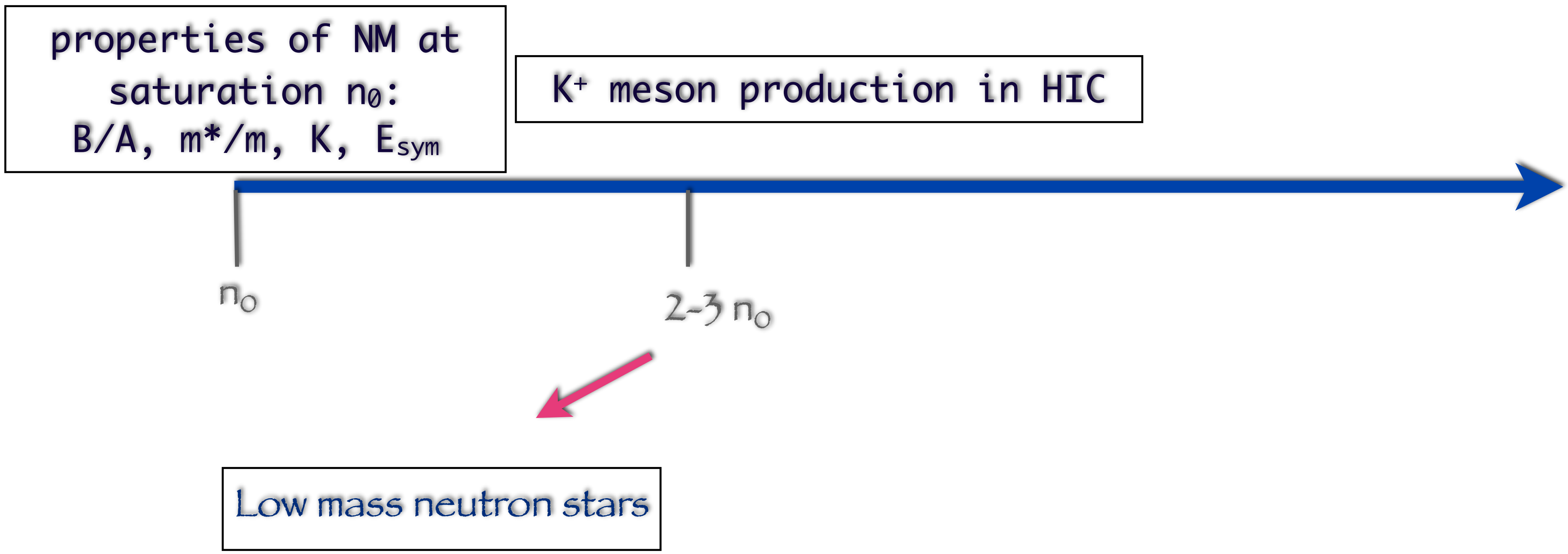

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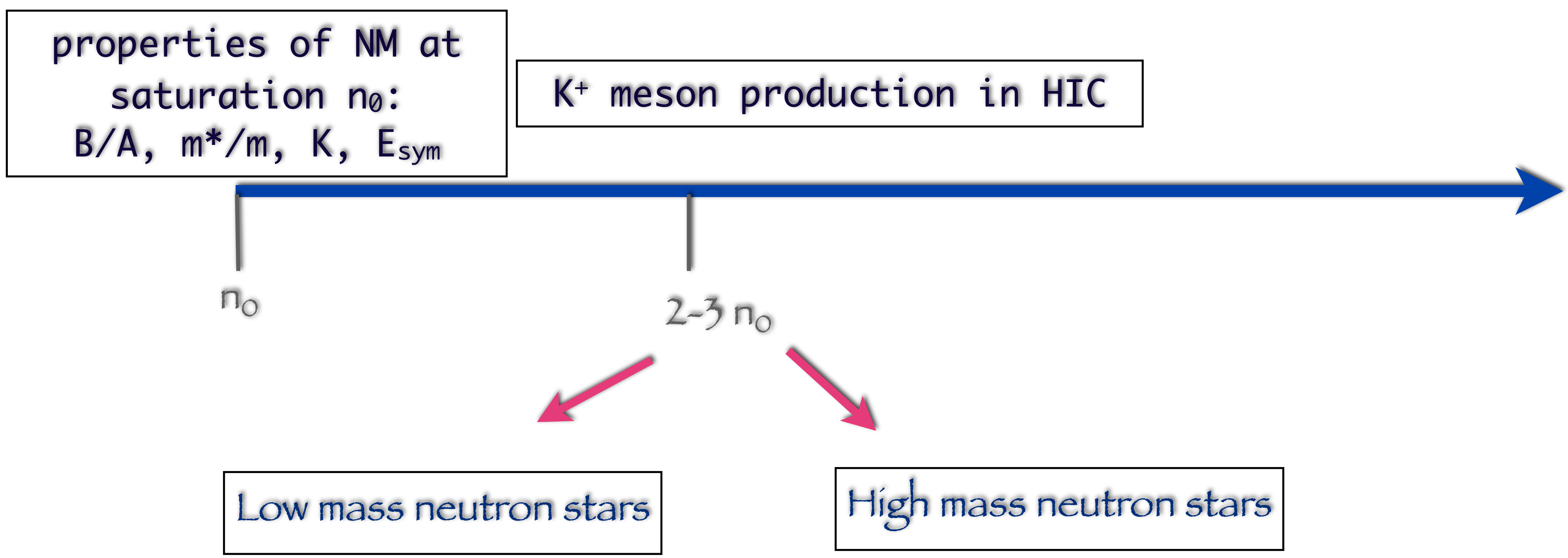

Phenomenological EoS for NS core

$$\frac{E}{A} = m_n(1-Y_p) + m_p Y_p + E_0 u^{\frac{2}{3}} + B\frac{u}{2} + D\frac{u^\sigma}{(\sigma+1)}$$
$$+ \ (1-2Y_p)^2 \left[\left(2^{\frac{2}{3}}-1\right) E_0 \left(u^{\frac{2}{3}} - F(u)\right) + S_0 u^\gamma\right], \quad (1)$$

Skyrme EoS

- *E_0 = binding energy of SNM at n_0*
- *baryon number density $u = n/n_0$*
- *Y_p = proton fraction*
- *density dependence of symmetry energy chosen as a power law with u^γ*
- *parameters σ, B,D (2BF, 3BF)*

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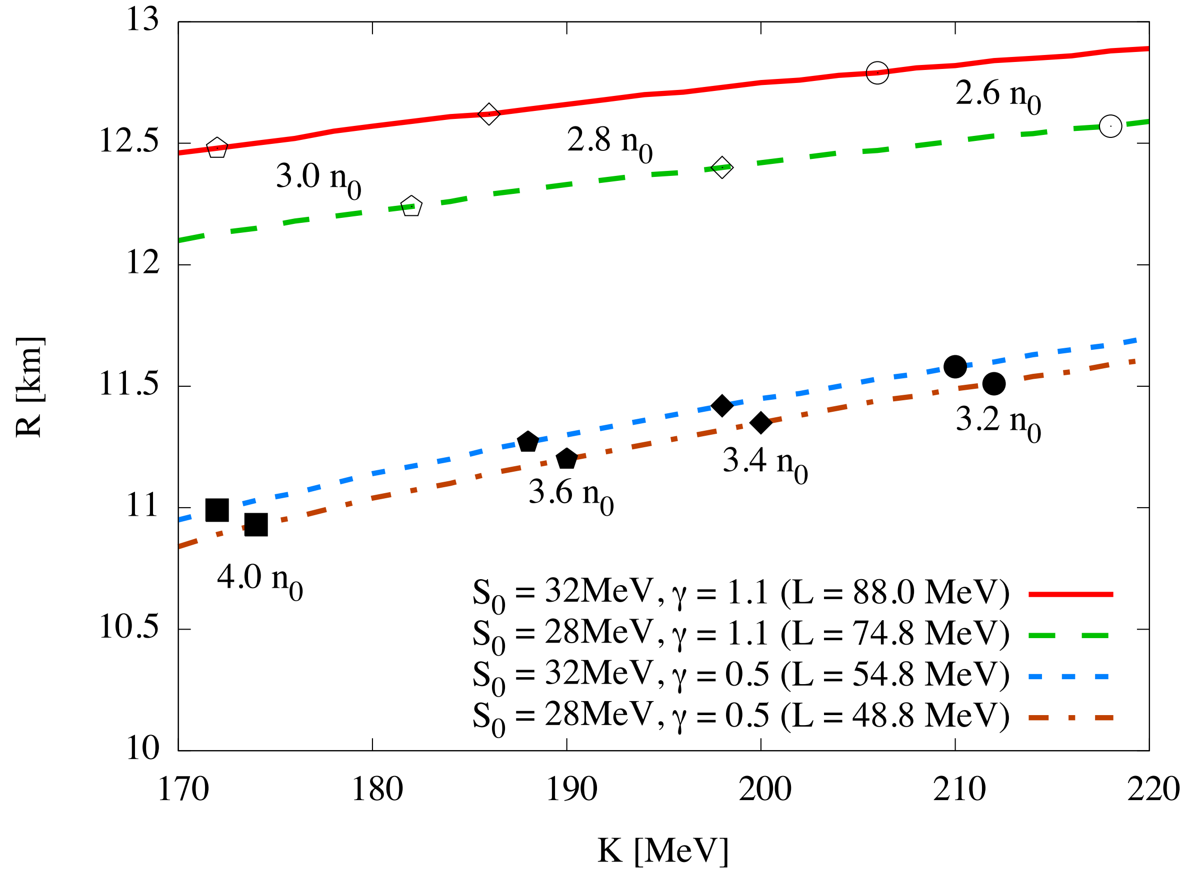
- Parameters fitted to reproduce saturation density, binding energy, stiffness parameter
- Variation of values:
  - K = 170 - 220 MeV
  - S_0 = 28 - 32 MeV
  - γ = 0.5 - 1.1 (motivated by heavy-ion experiments)
- $M = 1.25\ M_{sol}$: lightest pulsar mass deduced from observations

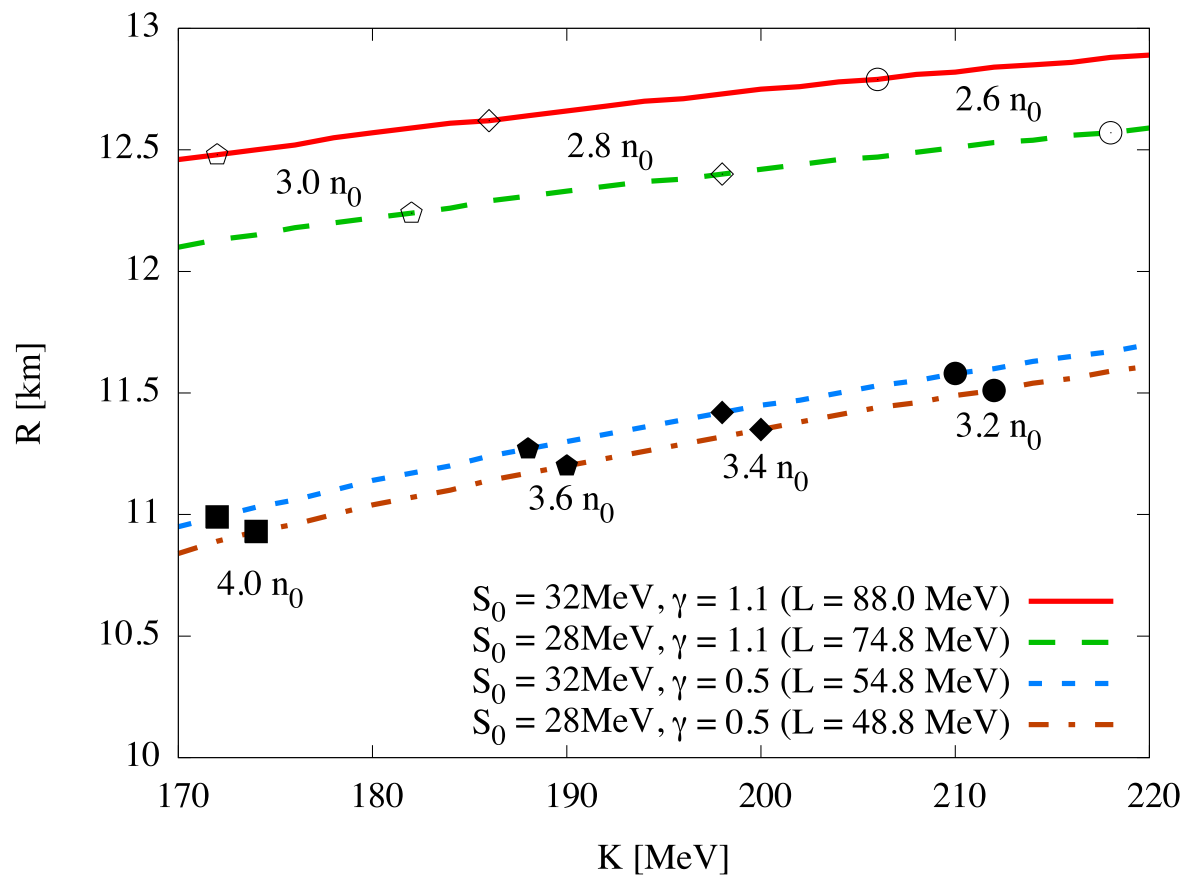
Radius of light neutron stars



Radii and central densities of 1.25 M_{sol} neutron stars with K, for different values of S_0 and γ

I. Sagert, L.Tolos, D. C., J. Schaffner-Bielich and C. Sturm,, arXiv :1112.0234

Radius of light neutron stars



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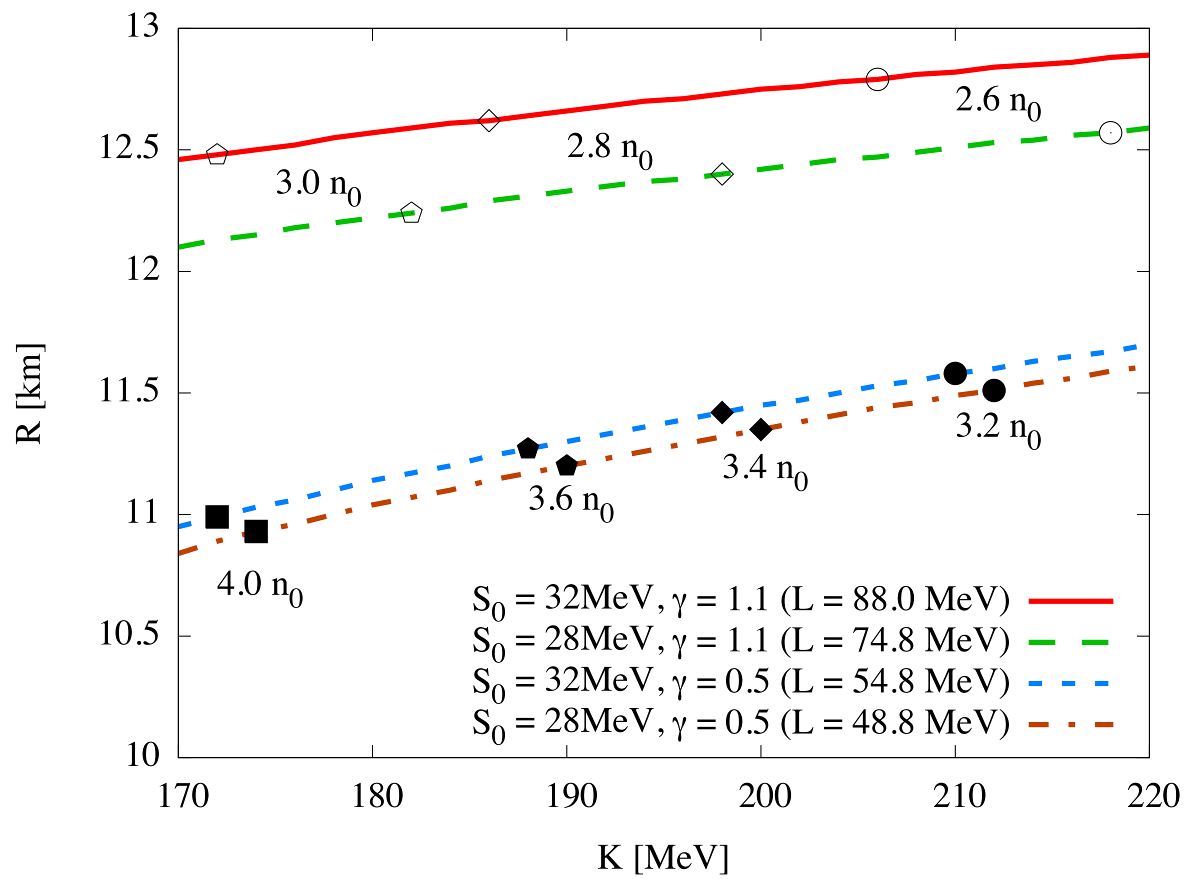

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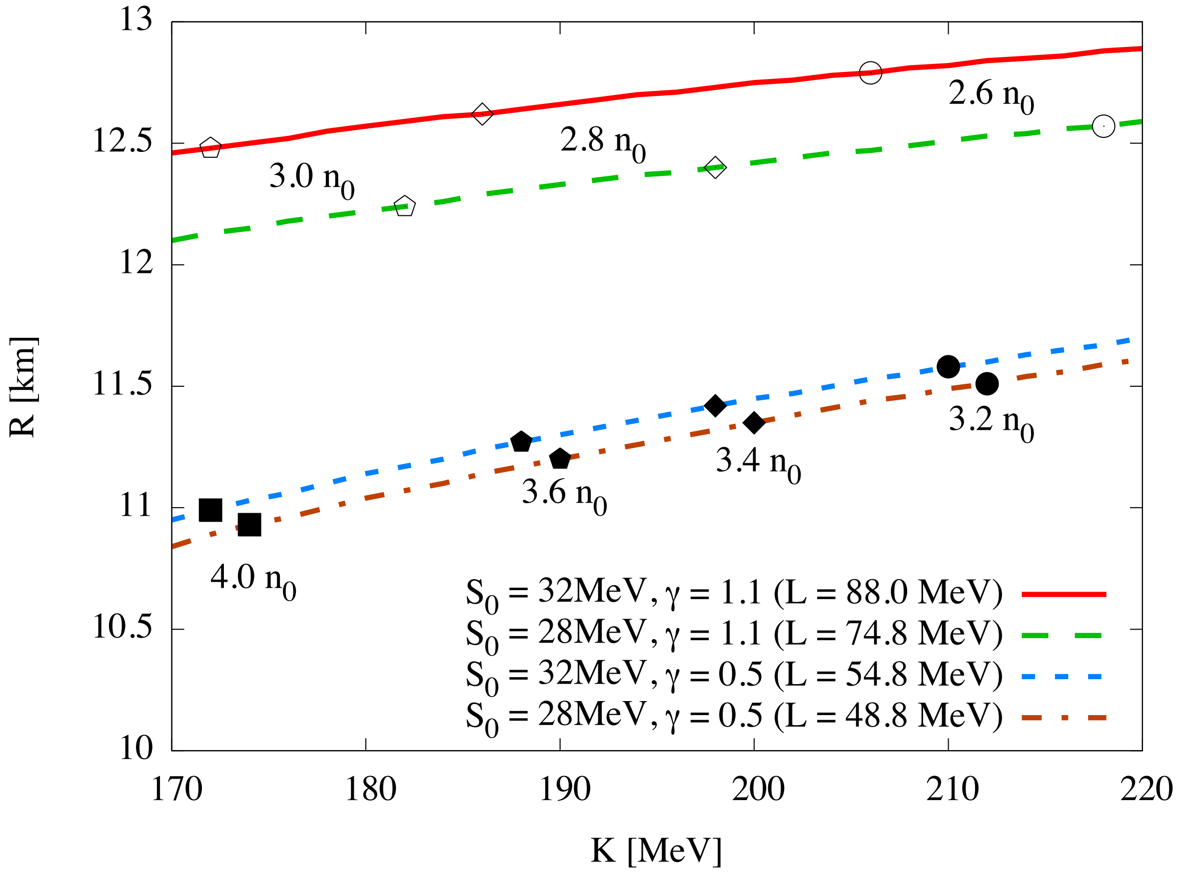

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- *At K ~ 200 MeV, stiff and soft symmetry energy configurations lead to a difference in the neutron star radius of around $\Delta R \sim (1 - 1.5)$ km.*

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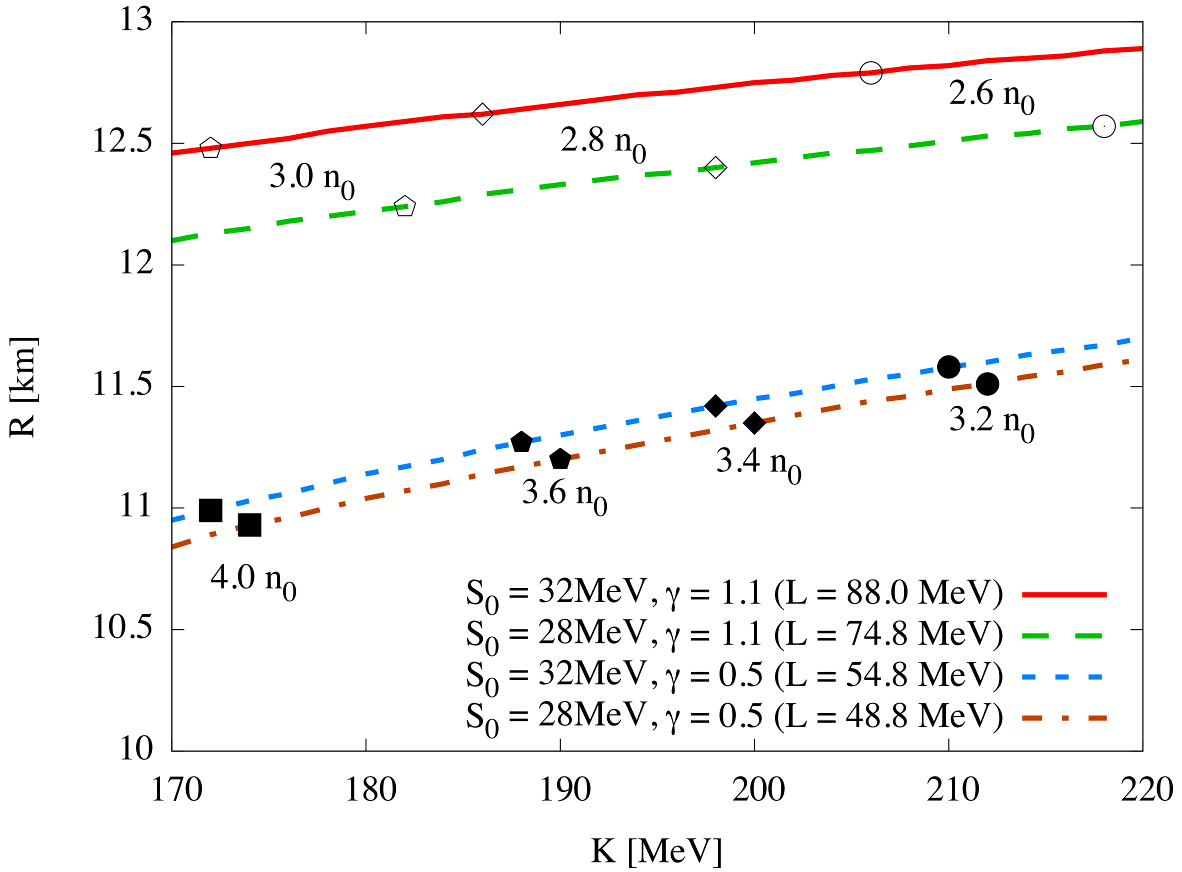

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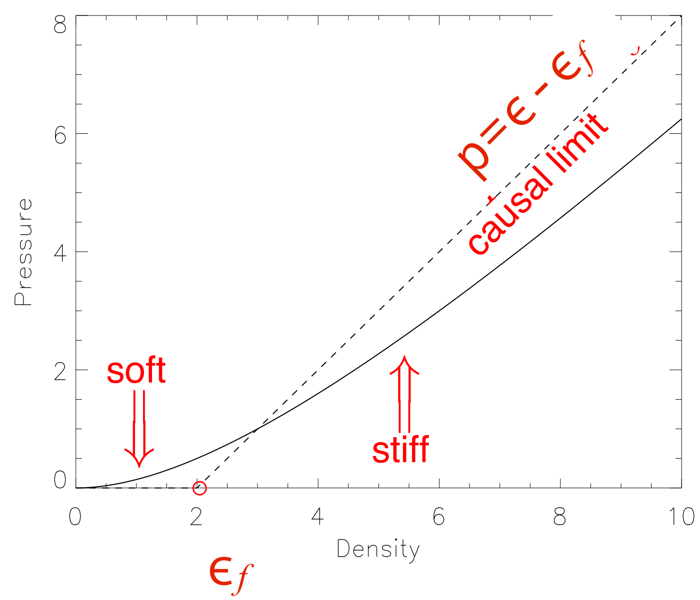
- *The radii largely depend on the density dependence of the symmetry energy γ*
- *At K ~ 200 MeV, stiff and soft symmetry energy configurations lead to a difference in the neutron star radius of around ΔR ~ (1 − 1.5) km.*
- *radii of light neutron stars with M ~ 1.25 M_{sol} are strong candidates for a direct cross check between heavy-ion experiments and astrophysical observations*

I. Sagert, L.Tolos, D. C., J. Schaffner-Bielich and C. Sturm,, arXiv :1112.0234

Massive neutron stars



$$M_{\rm max} = 4.2 M_\odot (\epsilon_0/\epsilon_f)^{1/2}$$

Rhoades & Ruffini (1974)
Hartle (1978)

- *Stiffest causal EoS: $p = \epsilon - \epsilon_f$ above the fiducial density ϵ_f*
- *at high densities, smooth transition to the stiffest EoS*
- *gives the highest possible mass of a compact star*
- *At low densities, EoS should satisfy KaoS constraint*

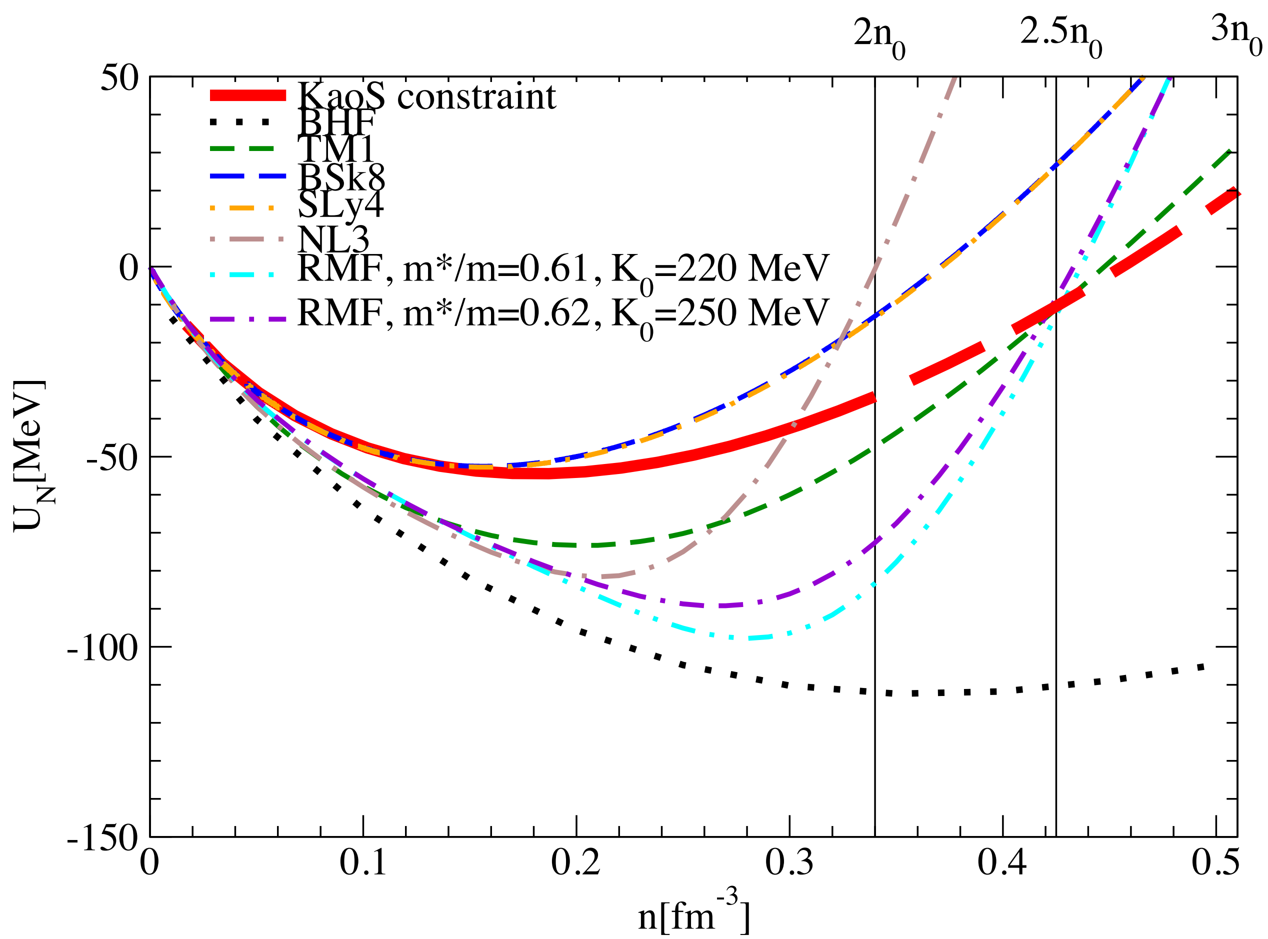
⇒ ***new upper mass limit of 3 M_{sol} from heavy-ion data***

Equations of State

Densities around and above saturation

- *Relativistic Mean Field Models (RMF)*
  - with non-linear interaction of mesons, fitted to bulk nuclear matter (GL, TM1)
  - fitted to properties of nuclei (NL3)
- *Brueckner Hartree Fock models (BHF)*
  - realistic N-N interactions
- *Phenomenological models*
  - Skyrme interactions (Bsk8, SLy4)

the KaoS constraint



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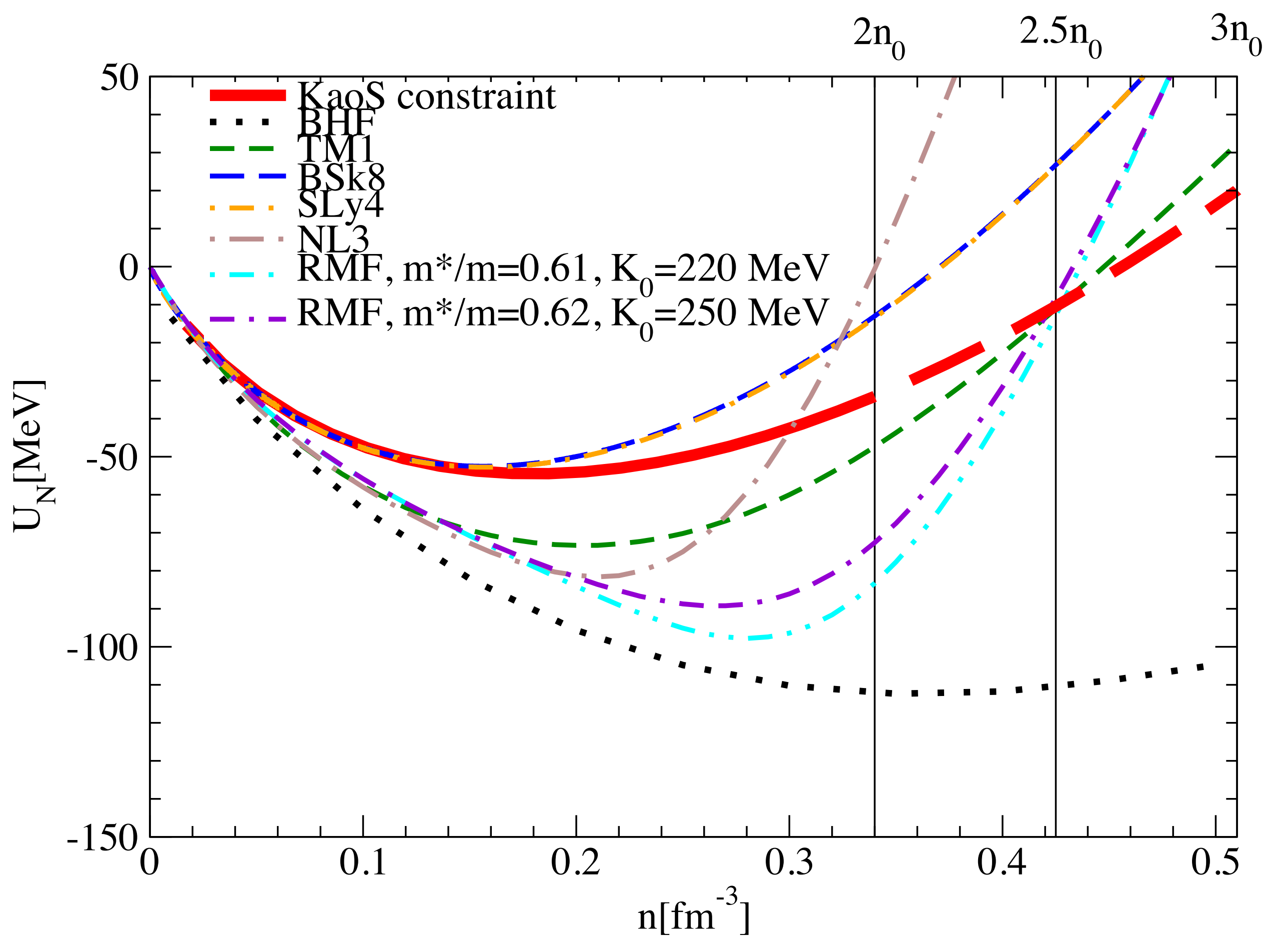

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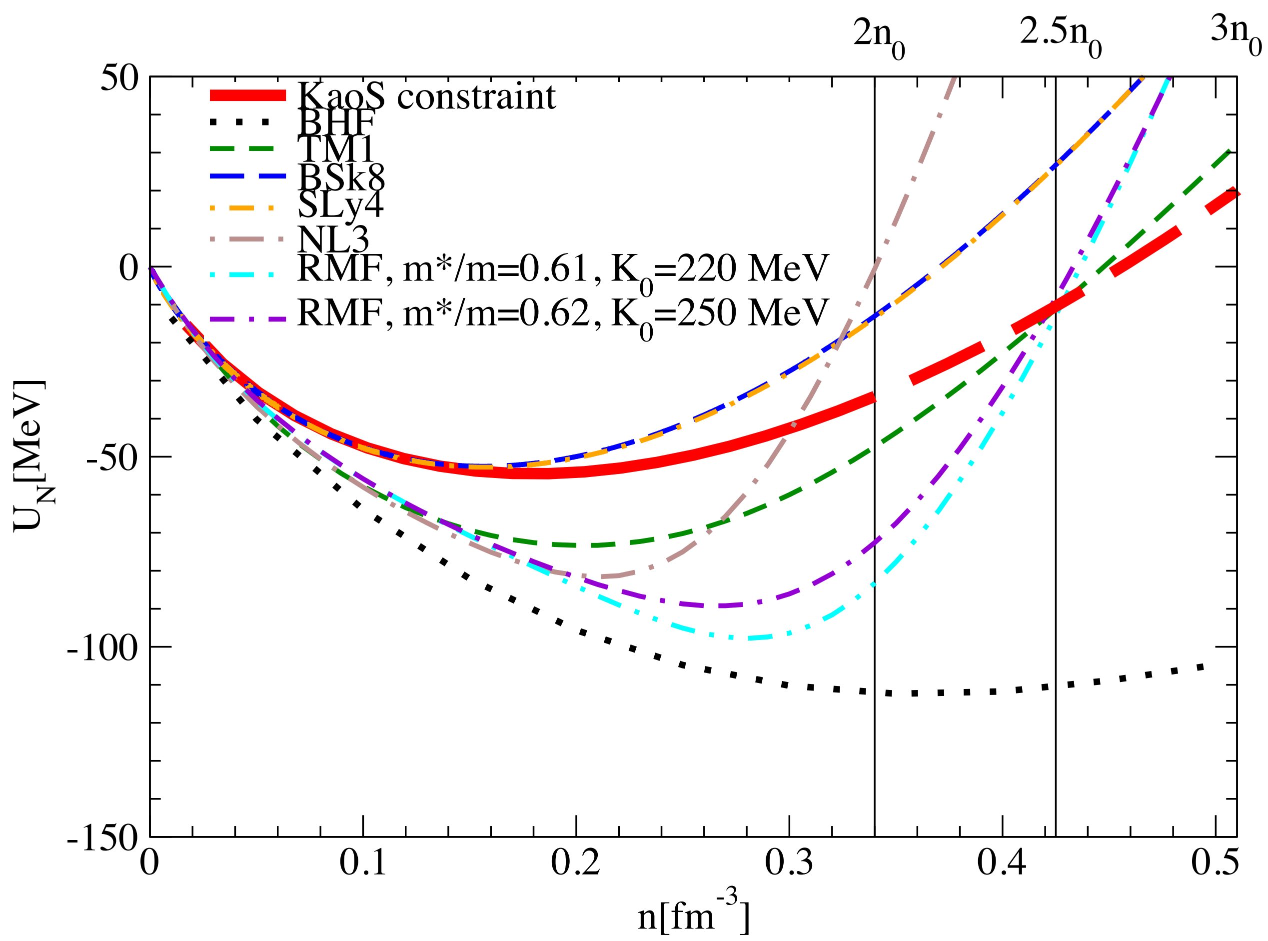
the KaoS constraint



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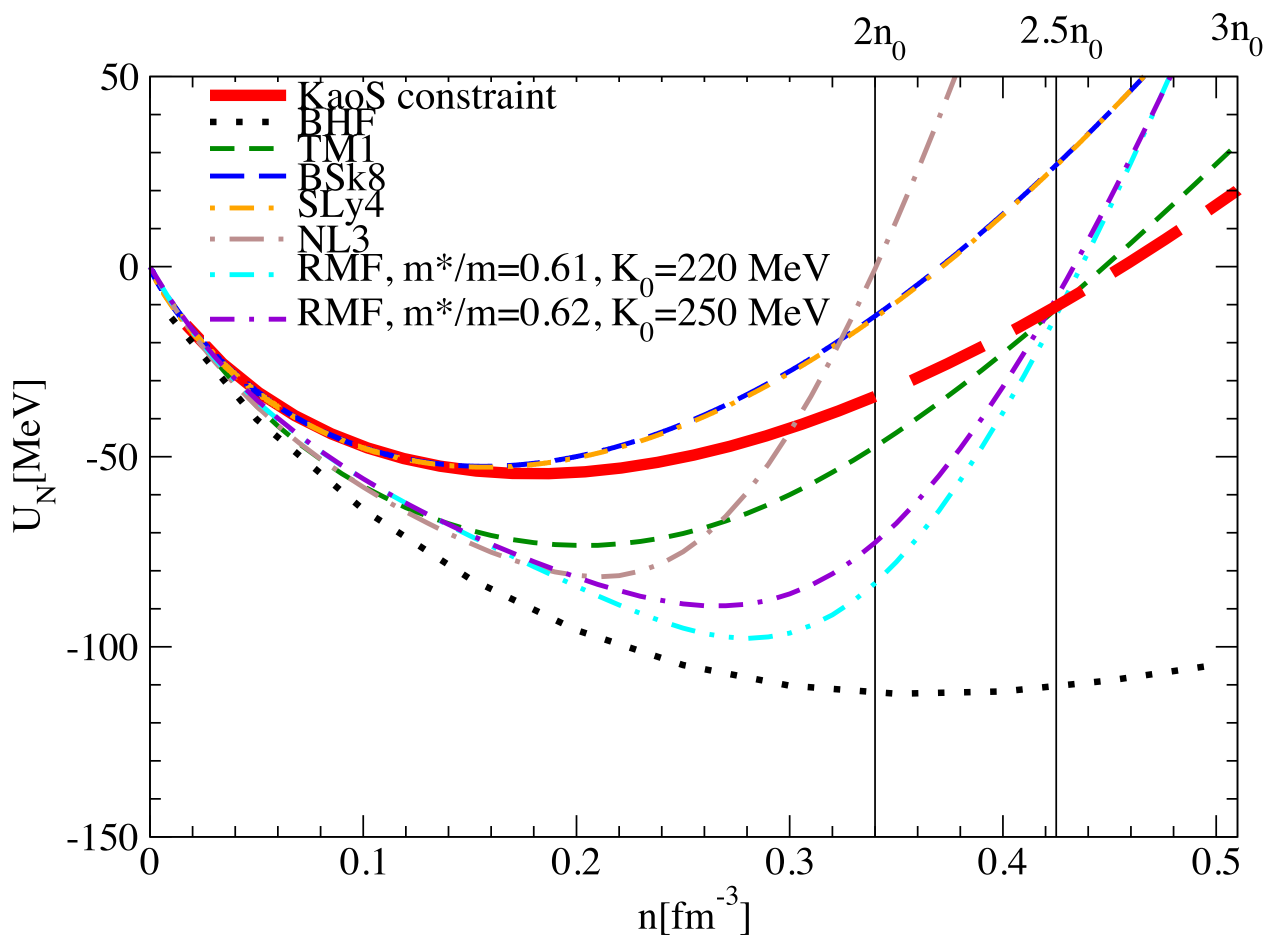
the KaoS constraint



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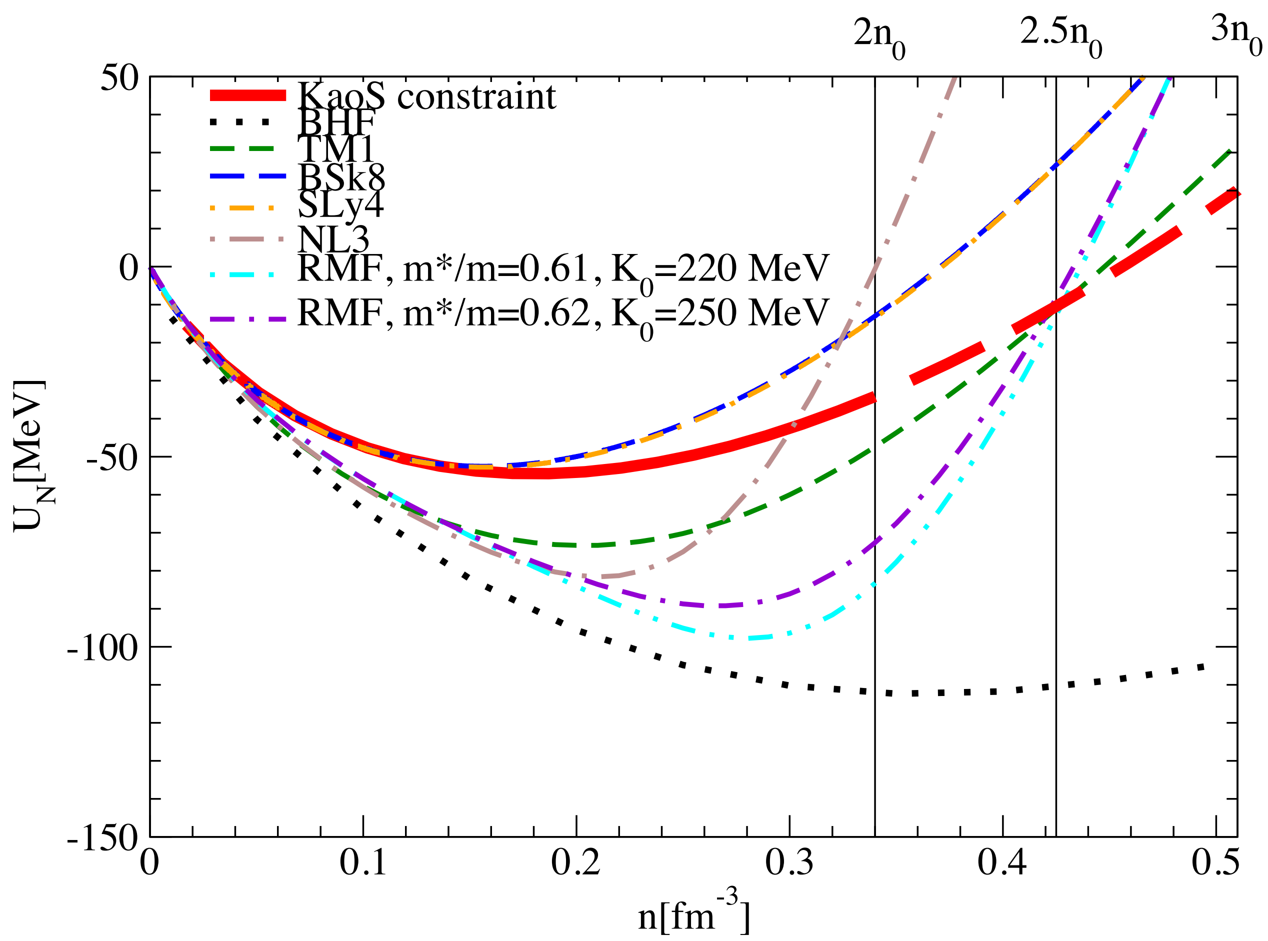

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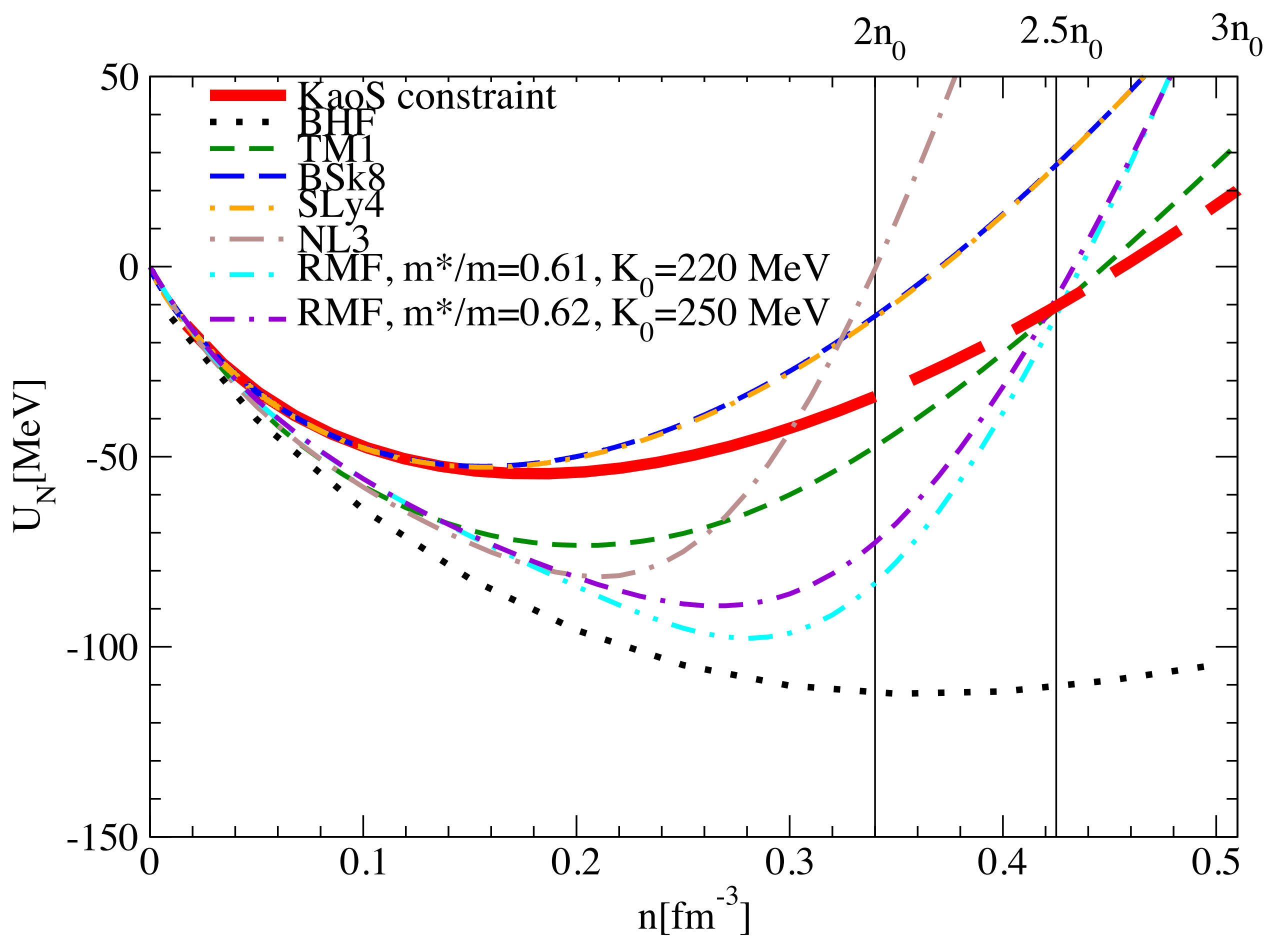

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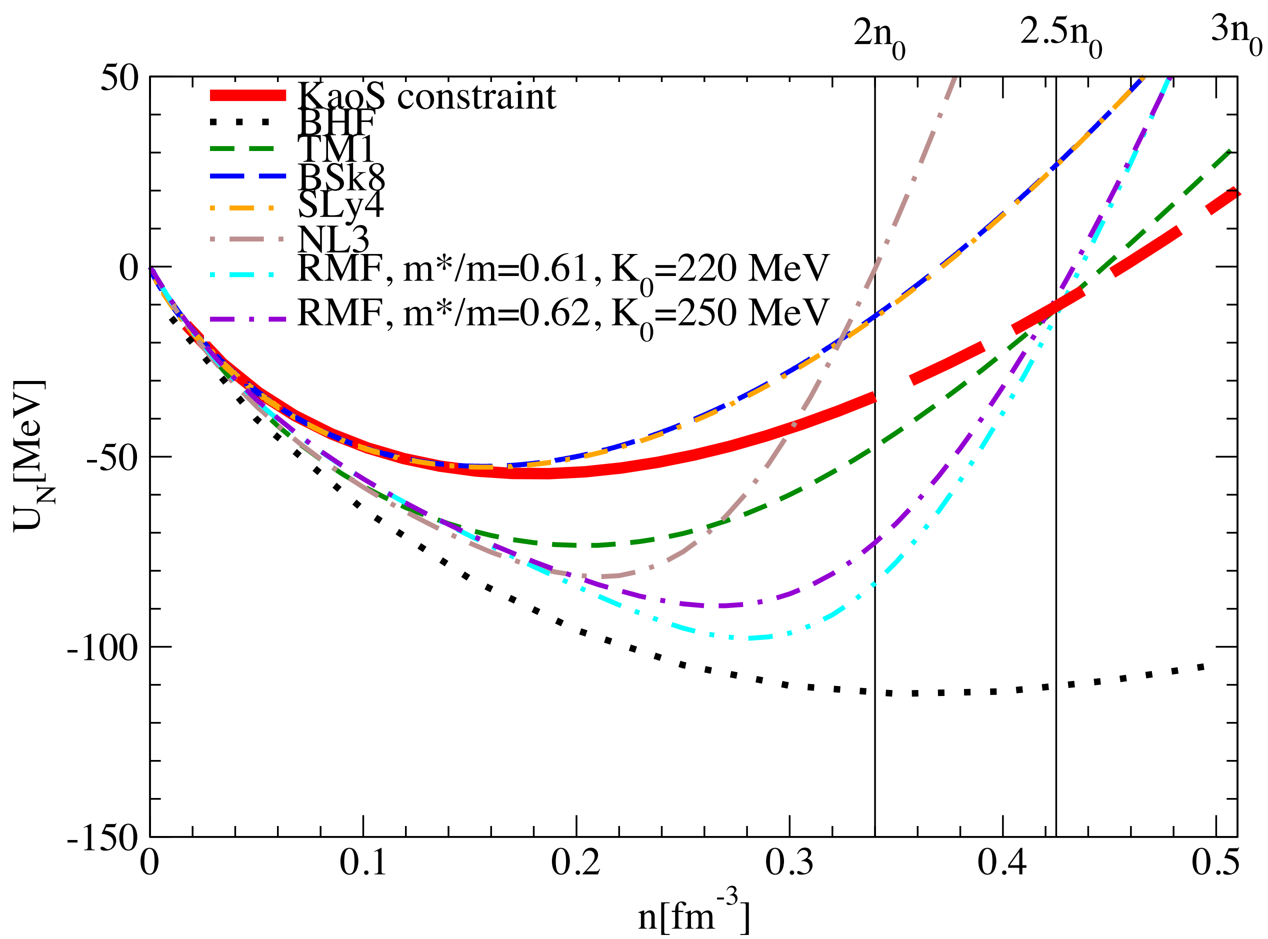

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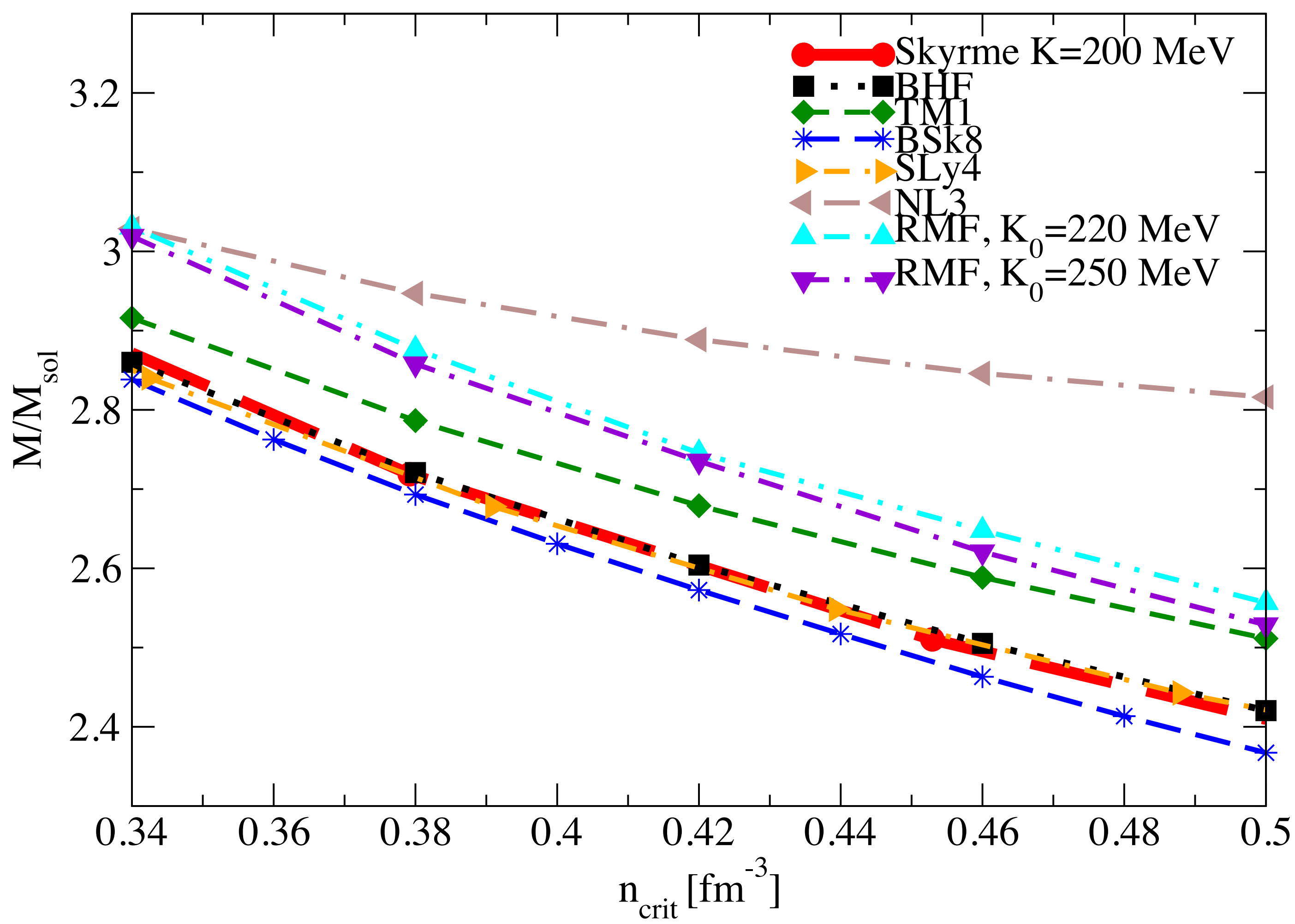
Maximally allowed gravitational mass



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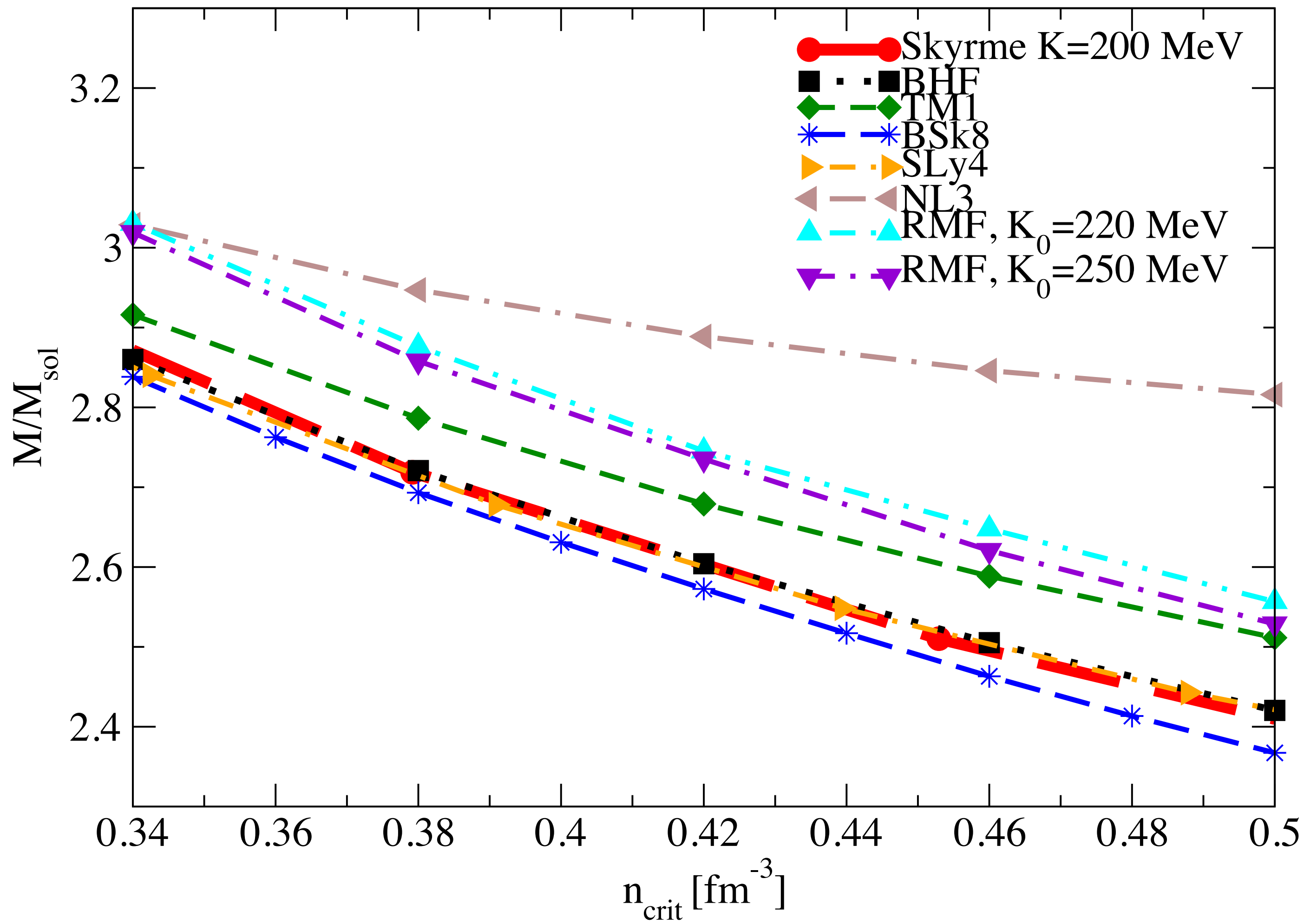

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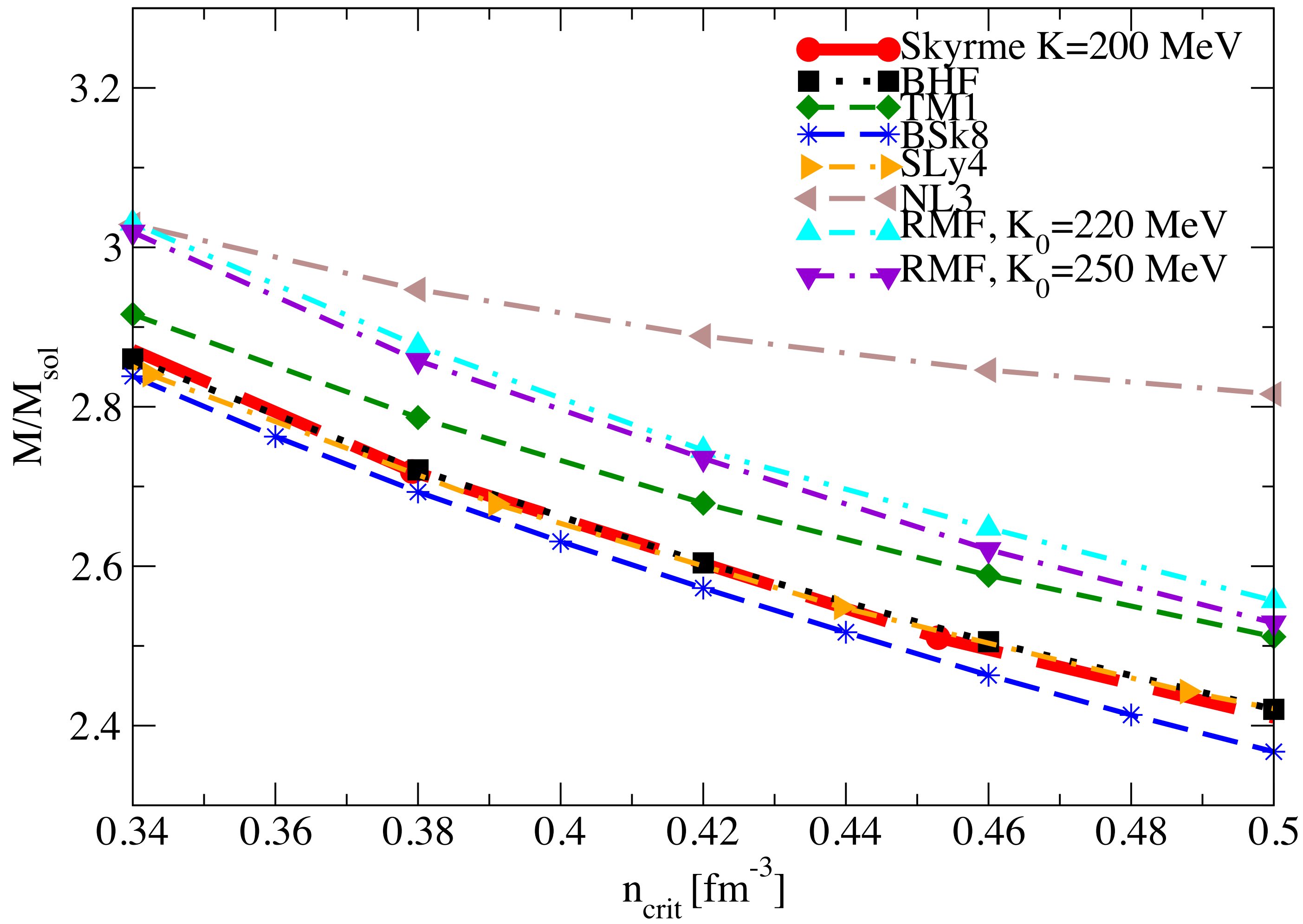

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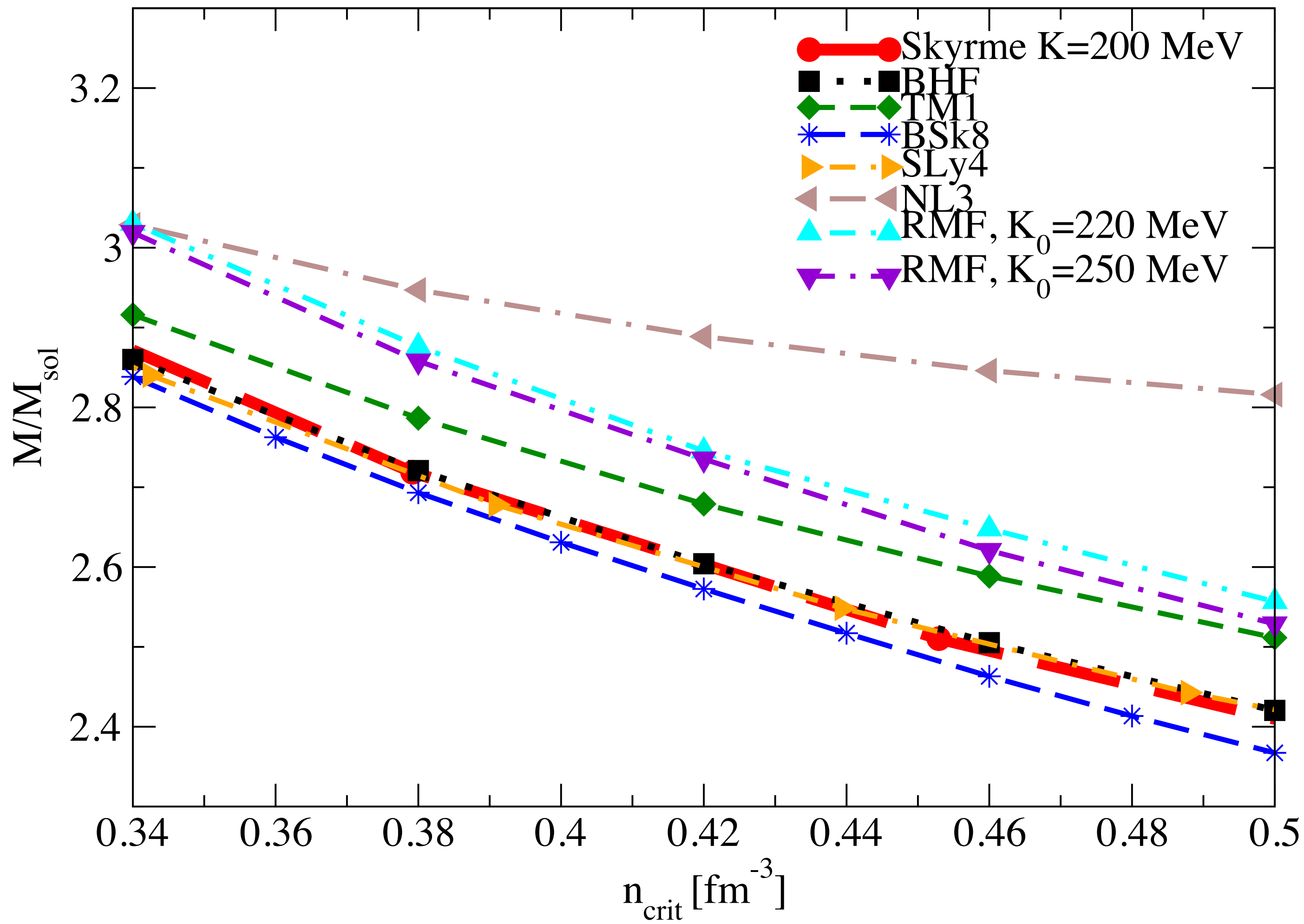

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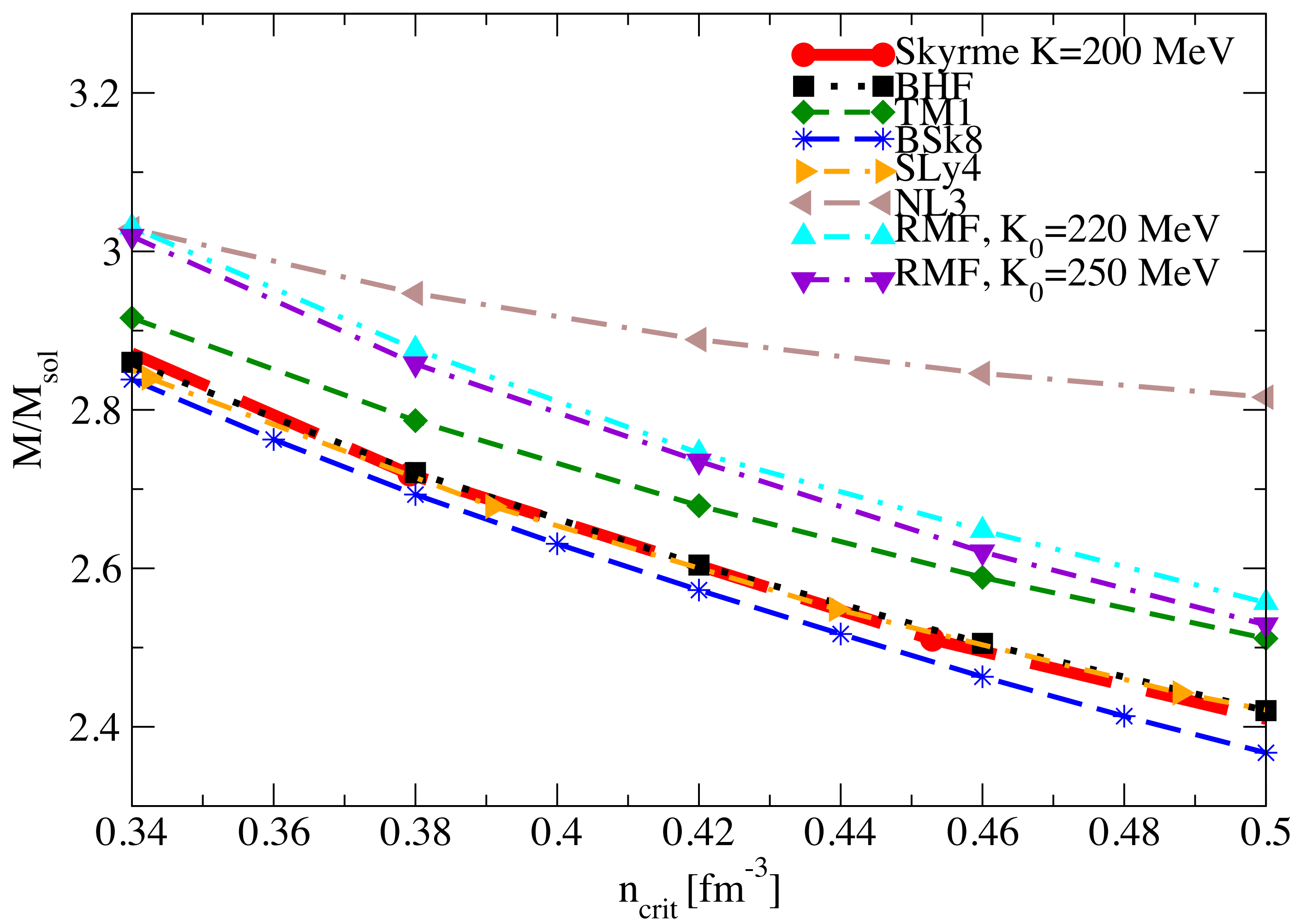

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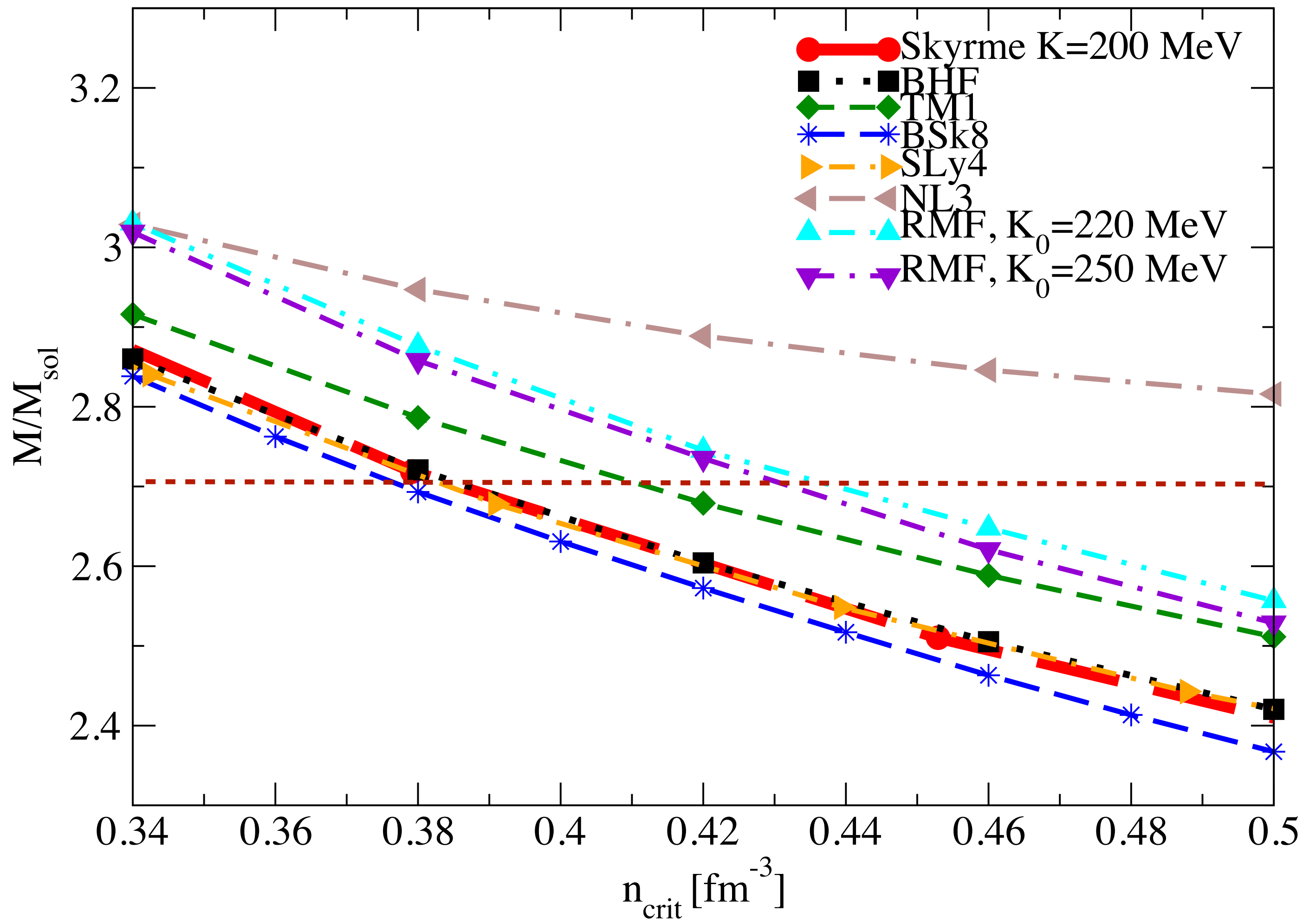

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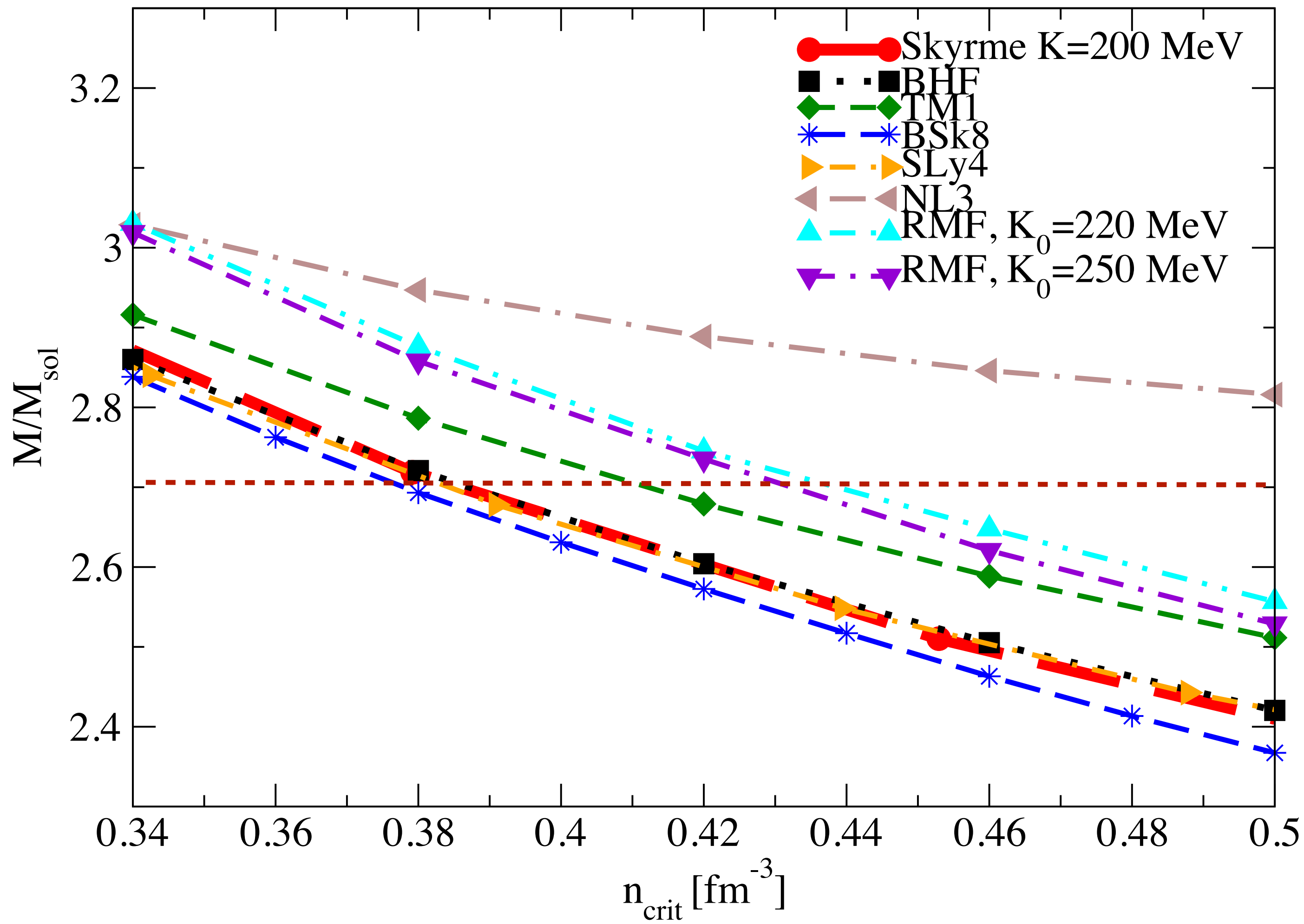

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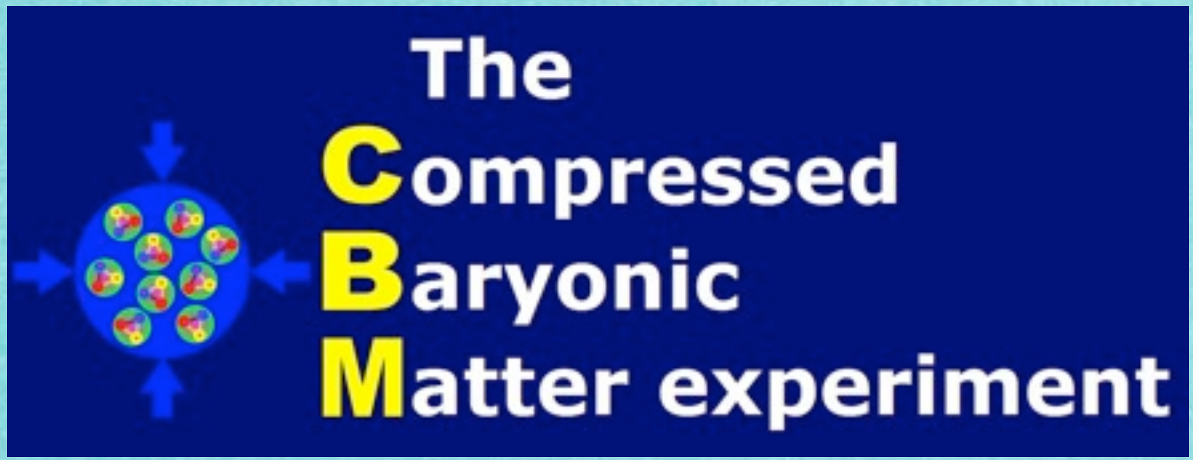
- *Since the maximum mass configuration is dominated by the causal high density EoS, the symmetry energy has very little influence on M_{max} : $\Delta M \approx 0.02M_{sol}$*

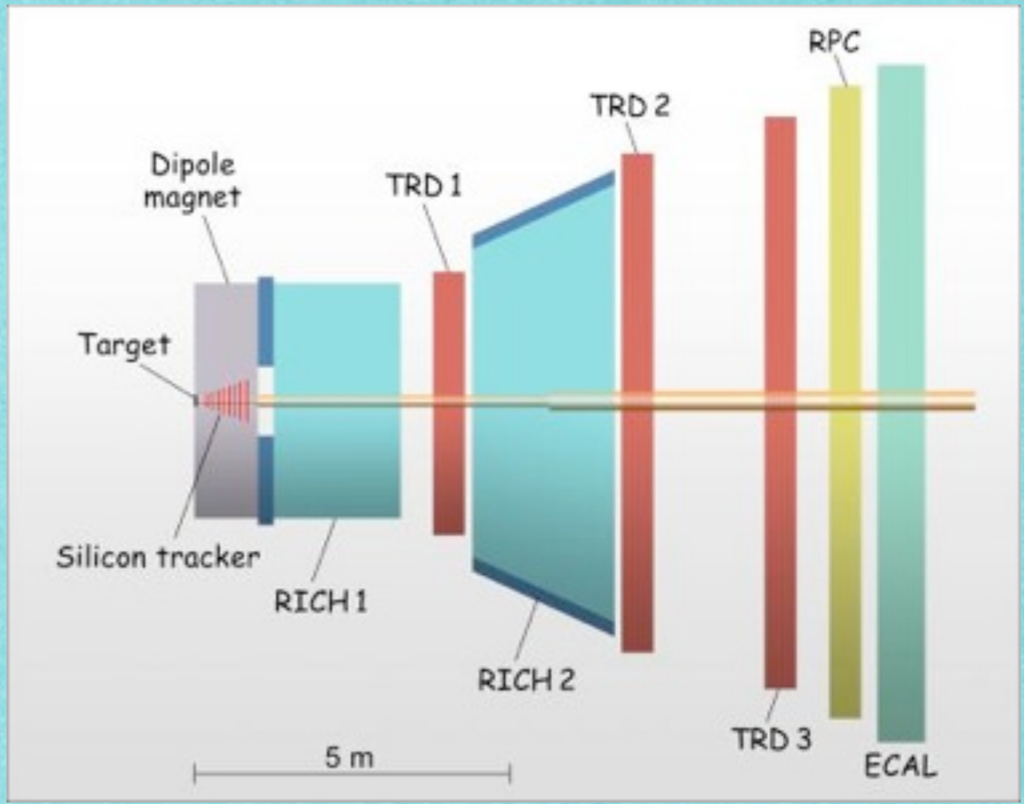
Summary

- K^+ multiplicities from heavy-ion collisions indicate a soft nuclear EoS for densities of 2-3 n_0
- We test the implications of results on neutron stars
- Light neutron stars with $M \sim 1.25\ M_{sol}$ have central densities ~ 2-3 n_0
- Measurement of radii of low mass neutron stars can test KaoS results
- To test if soft nuclear EoS is compatible with massive neutron stars, we apply KaoS results at densities up to 2-3 n_0, and then introduce the stiffest possible causal EoS to calculate the highest allowed maximum neutron star mass
- KaoS results indicate highest possible neutron star mass of 3 M_{sol}
- The massive pulsar of 2.7 M_{sol} requires an onset of the stiffest possible EoS at a fiducial density of ~2.2 - 2.5 n_0.

"Soft equation of state from heavy-ion data and implications for compact stars"
I. Sagert, L. Tolos, D.C., J. Schaffner-Bielich and C, Sturm, arXiv :1112.0234

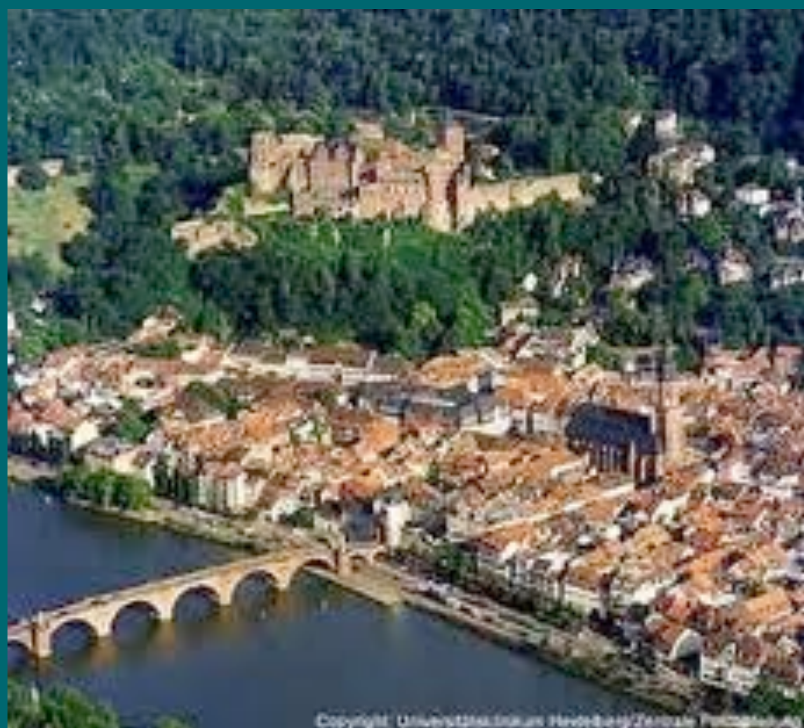
Outlook





The CBM (Condensed Baryonic Matter) experiment at FAIR will probe densities beyond 3 n_0, using rare probes such as D-meson and provide better constraints on the maximum neutron star mass.

THANK YOU FOR YOUR ATTENTION!



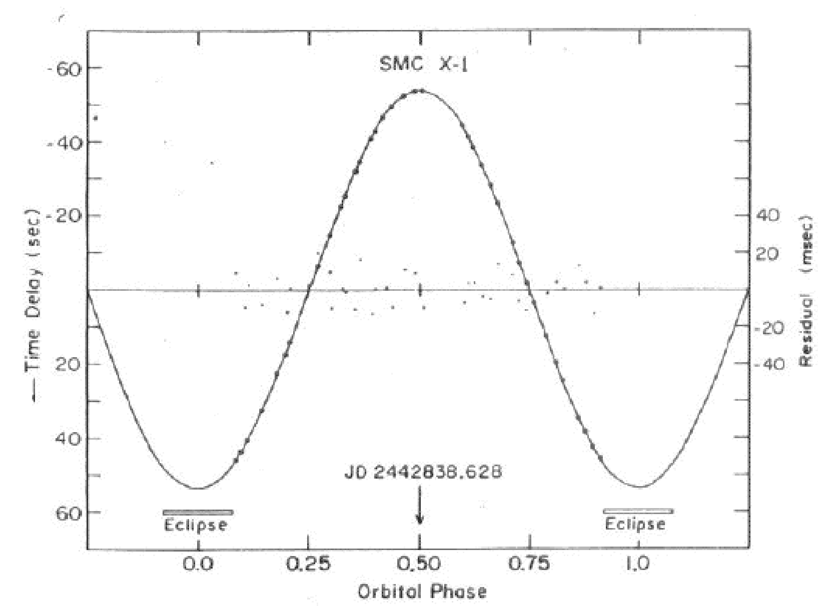
Mass measurements: *in eclipsing X-ray Binaries*

Mass function:

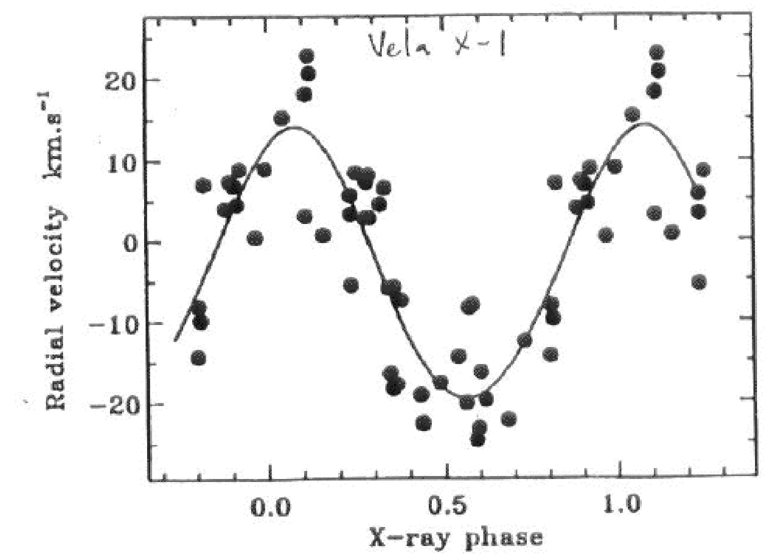
$$f(M_D) = \frac{P(V_x \sin i)^3}{2\pi G} = \frac{(M_D \sin i)^3}{(M_x + M_D)^2}$$

- *Doppler shifts of X-ray pulse period* $\Rightarrow$ $V_x \sin i$
- *Doppler shifts of Companion's spectral features* $\Rightarrow$ $V_D \sin i$

  $\Rightarrow$ $f(M_X)/f(M_D)$
- *eclipse* $\Rightarrow$ $\sin i \sim 1$

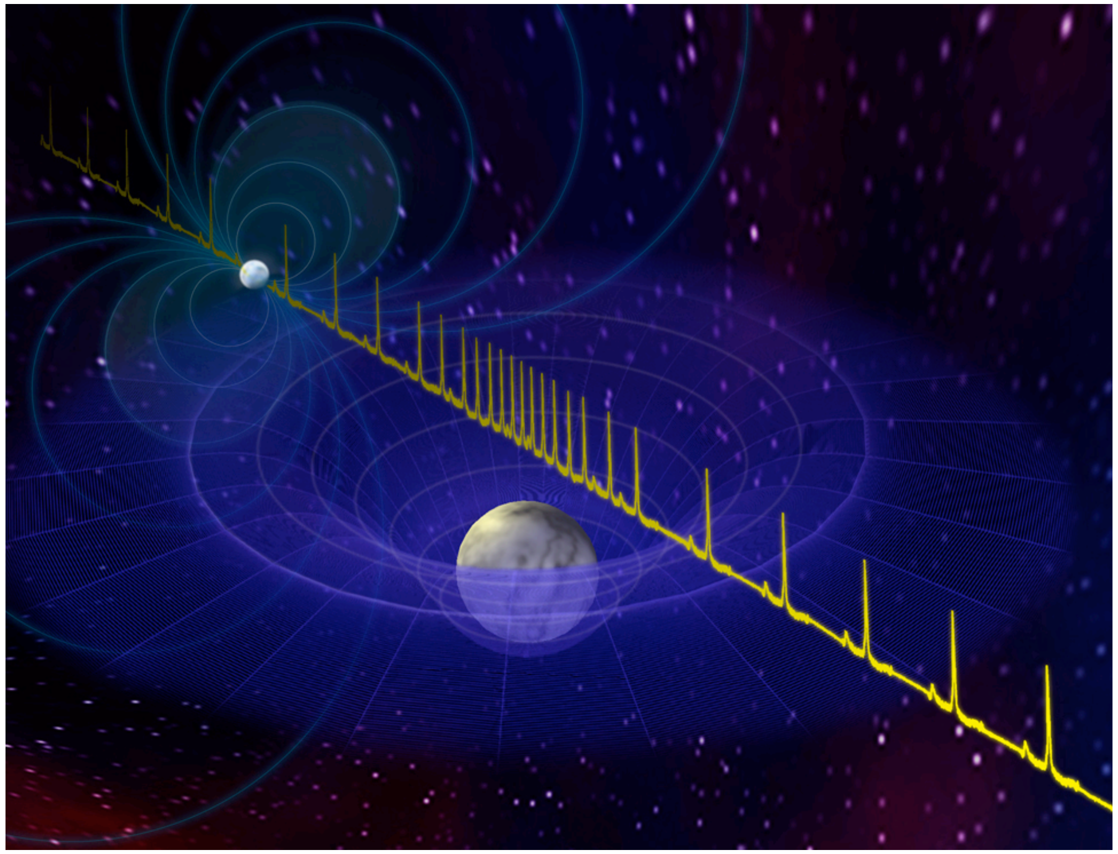


Radial velocity of X-ray pulsar



Radial velocity of optical companion

Shapiro Delay



Credit: Bill Saxton/NRAO

- If there is just one pulsar in the system, it is only possible to measure the mass ratio of a pulsar to its companion M_p/M_c
- If the system is nearly edge on, as the pulse train passes close to the companion, it experiences *Shapiro* delay in the pulses.
- The magnitude and duration of the delay episode is related to the inclination of the binary orbit to the line of sight, and the mass of the companion.
- This completely determines the mass of the pulsar.

Relativistic Mean Field Model

$$
\begin{aligned}
\mathcal{L} = & \sum_B \bar{\psi}_B(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu \\
& -\frac{1}{2}g_{\rho B}\gamma_\mu\vec{\tau}_B\cdot\vec{\rho^\mu})\psi_B \\
& +\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma{}^2\sigma^2\right) - U(\sigma) \\
& -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega{}^2\omega_\mu\omega^\mu \\
& -\frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}m_\rho{}^2\rho_\mu\cdot\rho^\mu + \mathcal{L}_{YY} \\
& + \sum_{e^-,\mu^-}\bar{\psi}_\lambda\left(i\gamma_\mu\partial^\mu - m\right)\psi_\lambda.
\end{aligned}
$$

where, $U(\sigma) = \frac{1}{3}bm_N\,(g_{\sigma N}\sigma)^3 + \frac{1}{4}c\,(g_{\sigma N}\sigma)^4$.

Properties of asymmetric nuclear matter at saturation

- *saturation density* $n_0 = 0.17\ fm^{-3}$
- *binding energy per nucleon* $B/A = -16.3$ *MeV*
- *incompressibility* $K_o = 200\text{-}250$ *MeV*
- *symmetry energy* $E_{sym} = 32.5$ *MeV*
- *effective nucleon mass* $m^*/m = 0.55\text{-}0.8$

TM1 Model

$$\mathcal{L} = \sum_B \bar{\Psi}_B \left(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - g_{\rho B}\gamma_\mu \mathrm{t}_B \cdot \boldsymbol{\rho}^\mu\right)\Psi_B$$
$$+\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - U(\sigma) + U(\omega)$$
$$-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu.$$

$$U(\sigma) = \frac{1}{3}b\sigma^3 + \frac{1}{4}c\sigma^4,$$

$$U(\omega) = \frac{1}{4}d(\omega_\mu\omega^\mu)^2,$$

Nuclear equations of state

Phenomenological:

$$U(n_b) = \frac{A}{2}\left(\frac{n_b}{n_0}\right) + \frac{B}{\sigma+1}\left(\frac{n_b}{n_0}\right)^\sigma$$
$$E_{sym}(n_b) \sim S_0\left(\frac{n_b}{n_0}\right)^\alpha$$

- BE=-16 MeV, $n_0 \sim 0.16$ fm^{-3}
- $K_0 = (160 - 240)$ MeV
- $S_0 = (28 - 32)$ MeV, $\alpha = 0.7 - 1.1$

Skyrme and rel. mean field:

	S_0 [MeV]	K_0 [MeV]
Bsk8 (NPA 750 (2005))	28.0	230.2
Sly4 (NPA 635 (1998))	32.0	229.9
TM1 (NPA 579 (1994))	36.9	281