Quantum entanglement and beyond:

Entanglement, discord and harnessing noise

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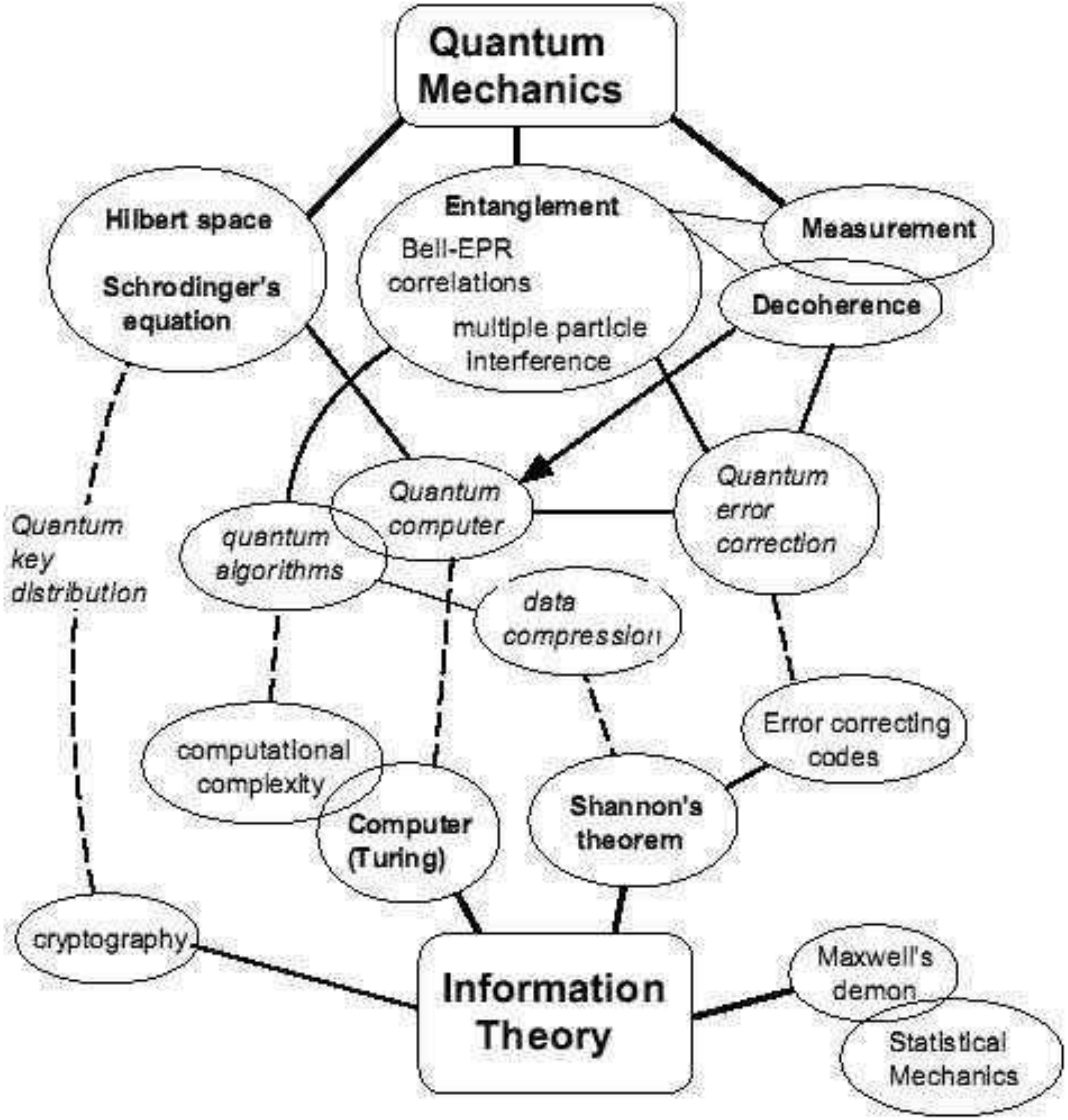
Quantum Optics &
Quantum Information

SUPA



QUANTUM

INFORMATION

Quantum entanglement: definition and some textbook examples

Going macroscopic: quantum information in infinite-dimensional Hilbert space

Quantum correlations beyond entanglement: quantum mixed states, quantum discord

Role of dissipation: harnessing noise

Quantum entanglement: definition and some textbook examples

Entanglement – nonlocal bi-partite superposition state

$$\hat{\rho} = \sum_i p_i \hat{\rho}_{1i} \otimes \hat{\rho}_{2i}$$ **- separable**

$$\hat{\rho} \neq \sum_i p_i \hat{\rho}_{1i} \otimes \hat{\rho}_{2i}$$ **- entangled**

$$|\psi_{in}\rangle = a|0\rangle + b|1\rangle$$

- state to teleport

Initial global state

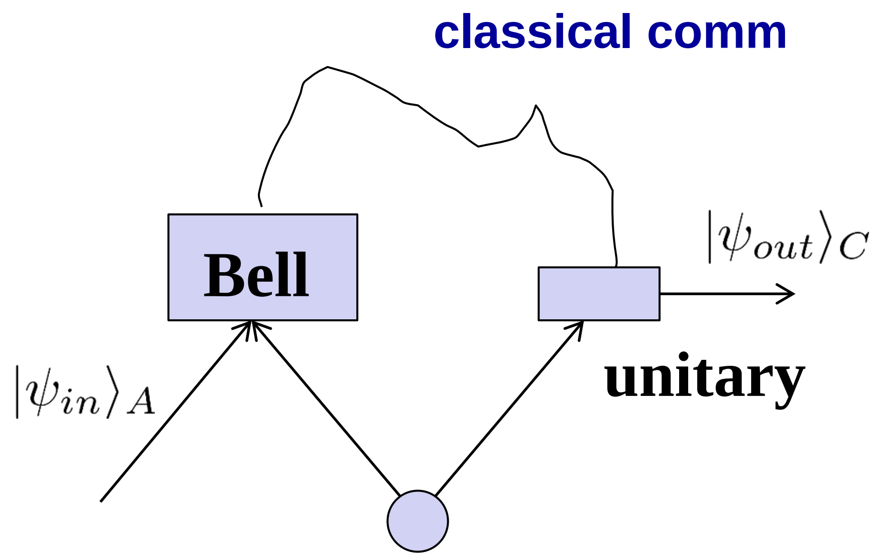
$$|\psi\rangle_{ABC} = |\psi_{in}\rangle_A \otimes \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)_{BC}$$

Bell basis

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$$

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)$$

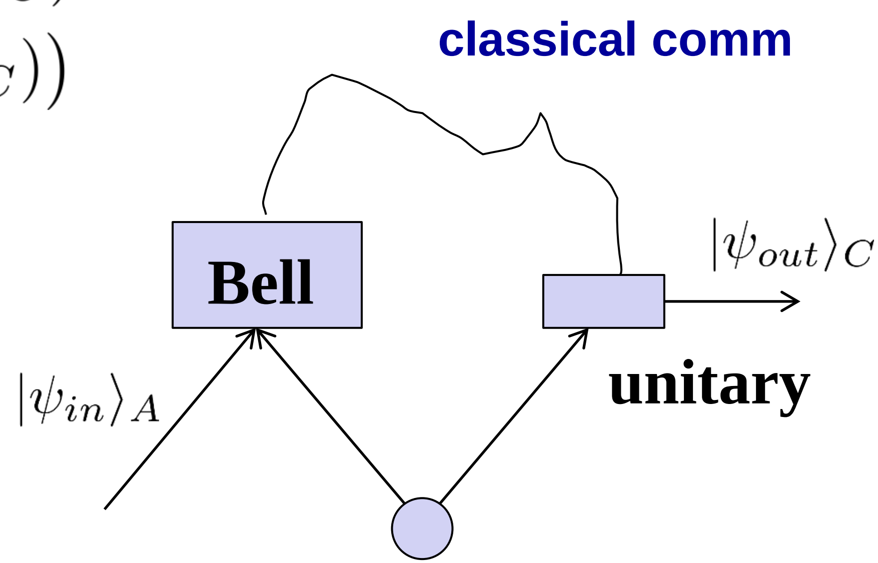
Quantum teleportation in few lines



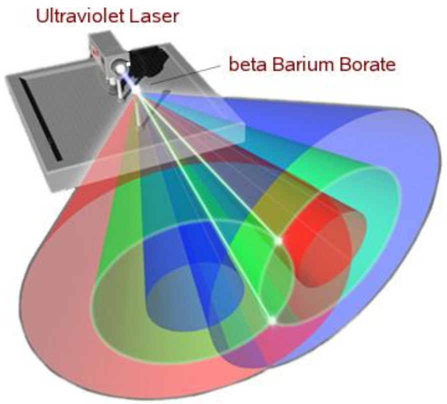
$$|\Phi^{+}\rangle_{BC} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{BC}$$

$$|\psi\rangle_{ABC} = |\psi_{in}\rangle_A \otimes \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)_{BC}$$

$$\begin{aligned}|\psi\rangle_{ABC} = \frac{1}{2}\Big(& \,|\Phi^+\rangle_{AB}(a|0\rangle_C + b|1\rangle_C) \\ & +|\Phi^-\rangle_{AB}(a|0\rangle_C - b|1\rangle_C) \\ & +|\Psi^+\rangle_{AB}(a|1\rangle_C + b|0\rangle_C) \\ & +\,|\Psi^-\rangle_{AB}(a|1\rangle_C - b|0\rangle_C)\Big)\end{aligned}$$

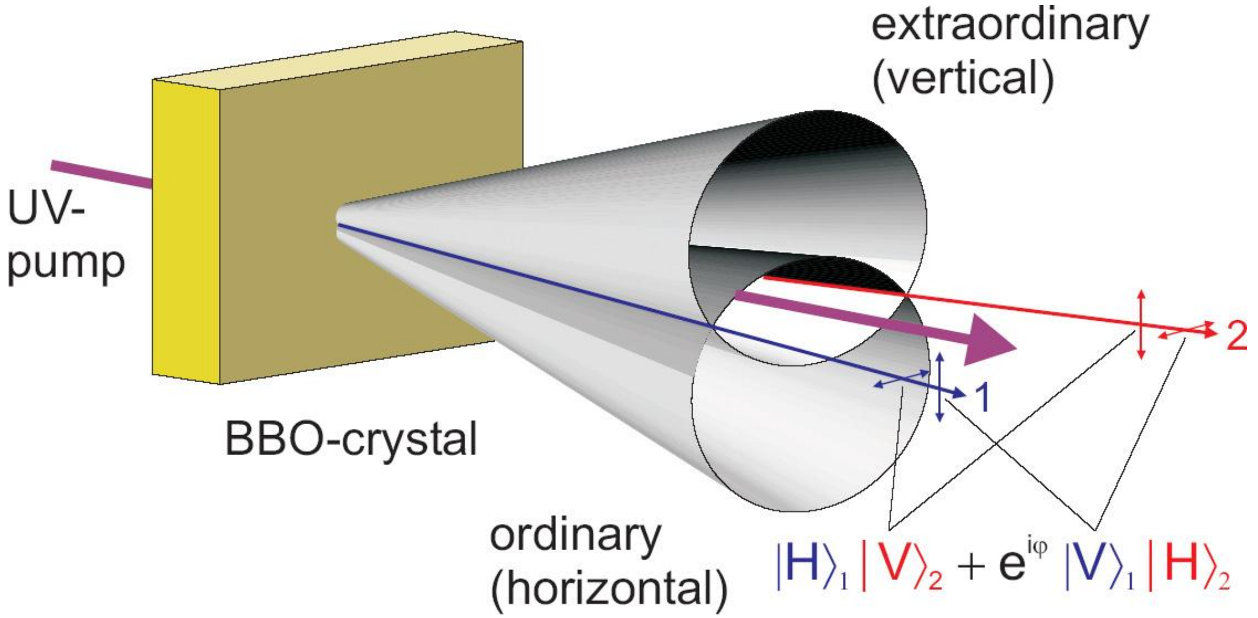


$$|\Phi^+\rangle_{BC} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{BC}$$



Example of photonic entanglement

$$\hat{H} = i\hbar\kappa\{\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\hat{a}\}$$



Going macroscopic:

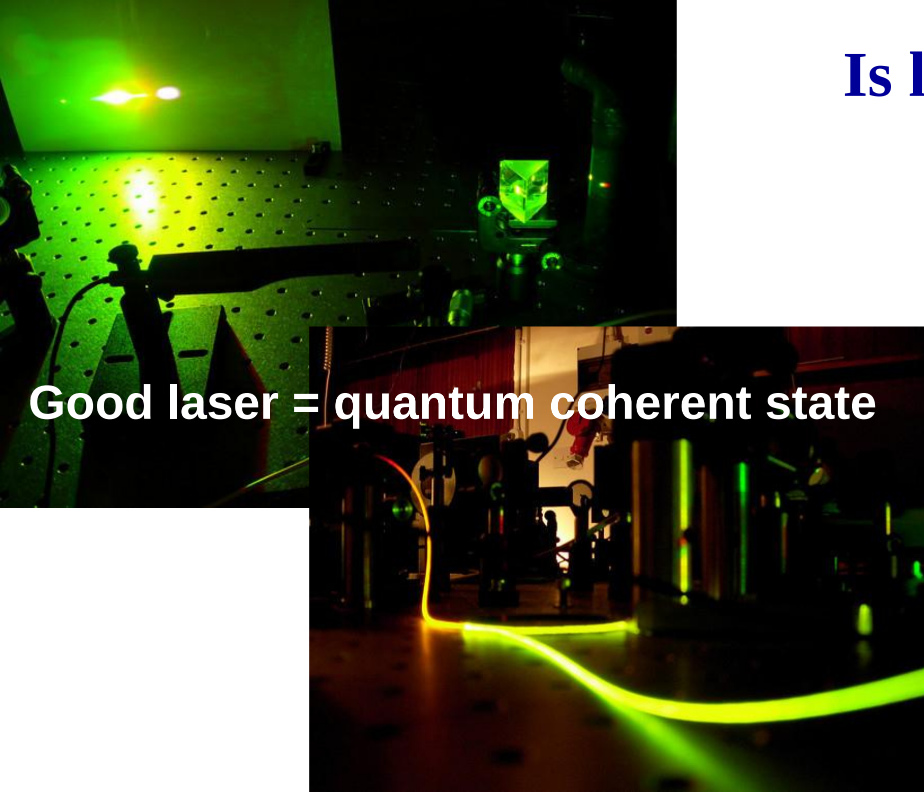
quantum information in infinite-dimensional Hilbert space

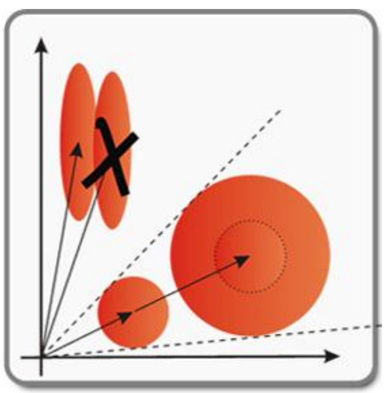
Not only single quanta are quantum:

Quiet light, tangled atoms and beating eavesdropper with merely a laser beam

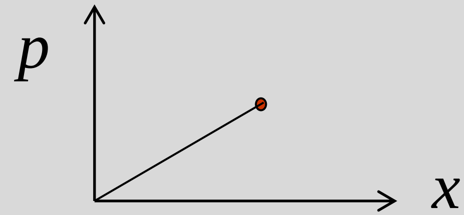
Not only single quanta are quantum

Is laser light quantum or classical?

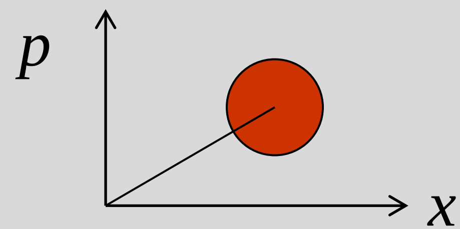




Coherent light cannot be amplified without introducing additional noise – Q info!



Simple Harmonic Oscillator

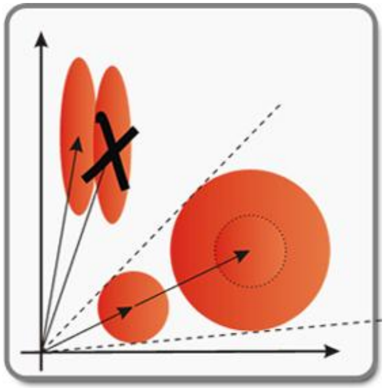


Min uncertainty wave packet

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/2} \mathrm{e}^{-\frac{m\omega x^2}{2\hbar}}$$

$$(\Delta x)^2 = \frac{\hbar}{2m\omega}, (\Delta p)^2 = \frac{\hbar m\omega}{2},$$

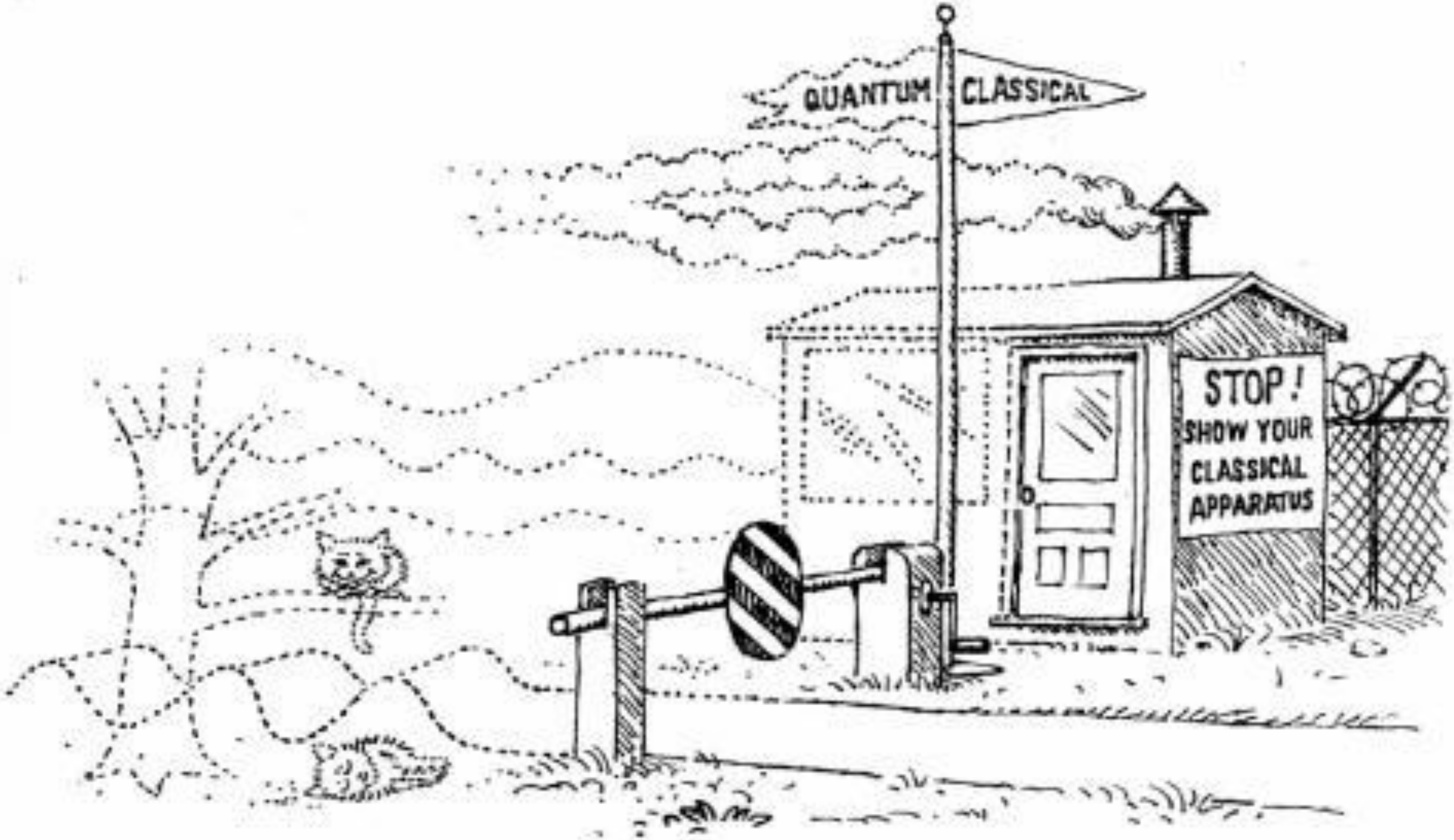
$$\Delta x \Delta p = \hbar/2$$



Quantum information:

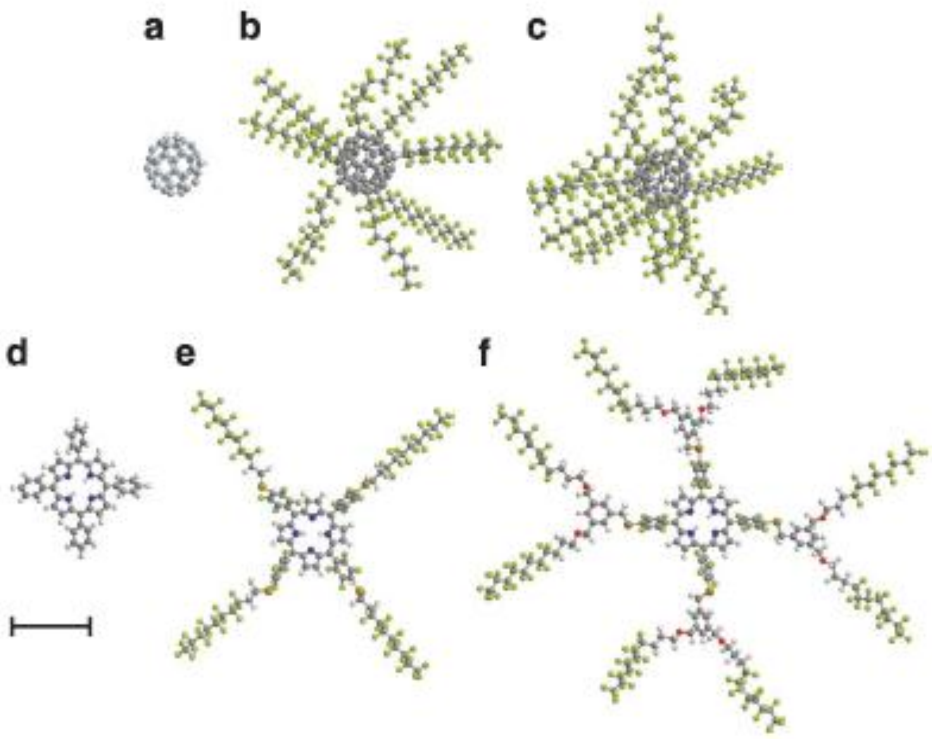
- **cannot be copied (non-cloning theorem)**
- **cannot be read/manipulated without disturbance**

This holds also for such macroscopic states as bright laser beams or atomic ensemble with millions atoms



Delineating the border between the quantum realm ruled by the Schrödinger equation and the classical realm ruled by Newton's laws is one of the unresolved problems of physics. **Figure 1**

Zurek, Physics Today (1991)

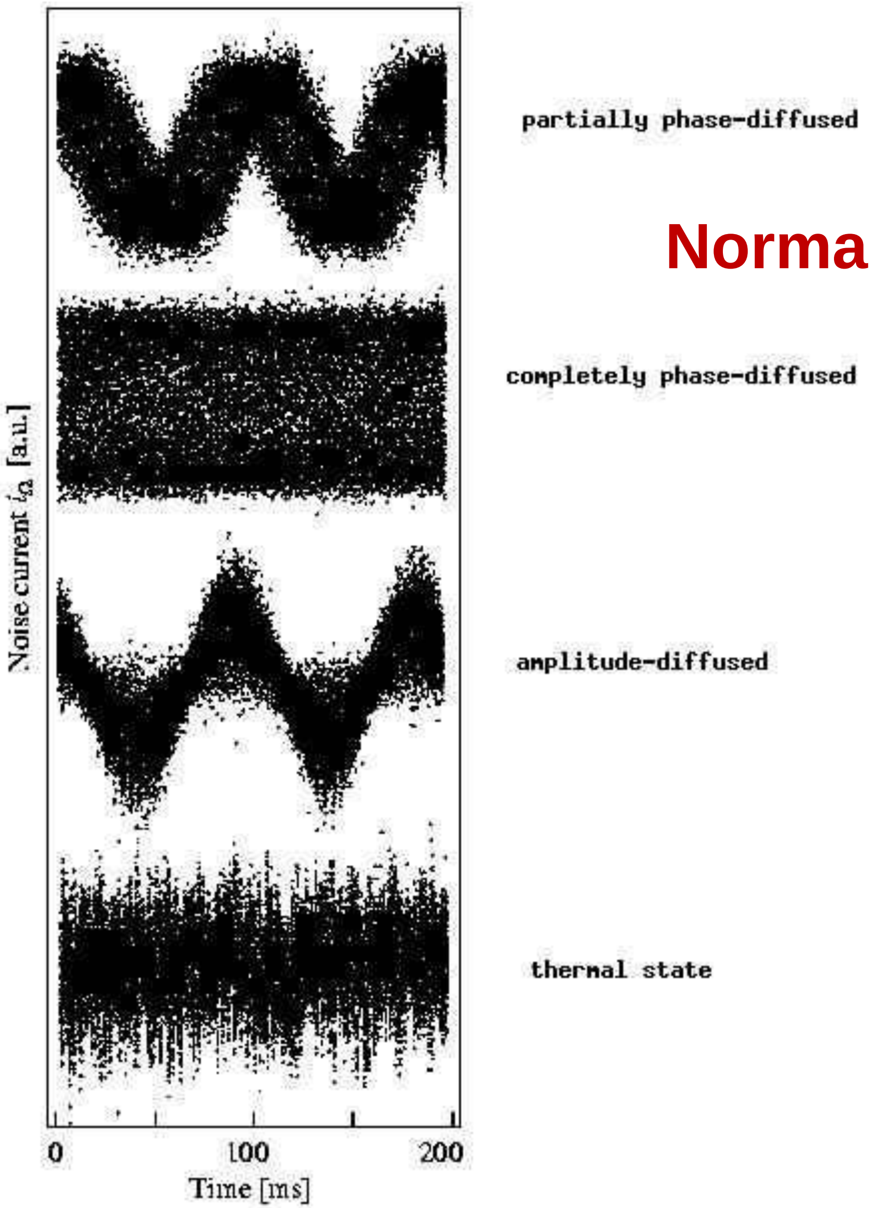


Our experiments prove the quantum wave nature and delocalization of compounds composed of up to 430 atoms, with a maximal size of up to 60 Å, masses up to m = 6,910 AMU We show that even complex systems, with more than 1,000 internal degrees of freedom, can be prepared in
quantum states that are sufficiently well isolated from their environment to avoid decoherence and to show almost perfect coherence.

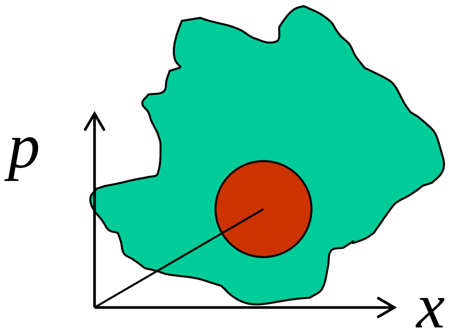
Figure 1 | Gallery of molecules used in our interference study. (**a**) The fullerene C_{60} (m = 720 AMU, 60 atoms) serves as a size reference and for calibration purposes;

Quantum interference of large organic molecules, group of Markus Arndt, Vienna; see also talk on Thu

Quiet and strange light



Normally light is quite loud and noisy

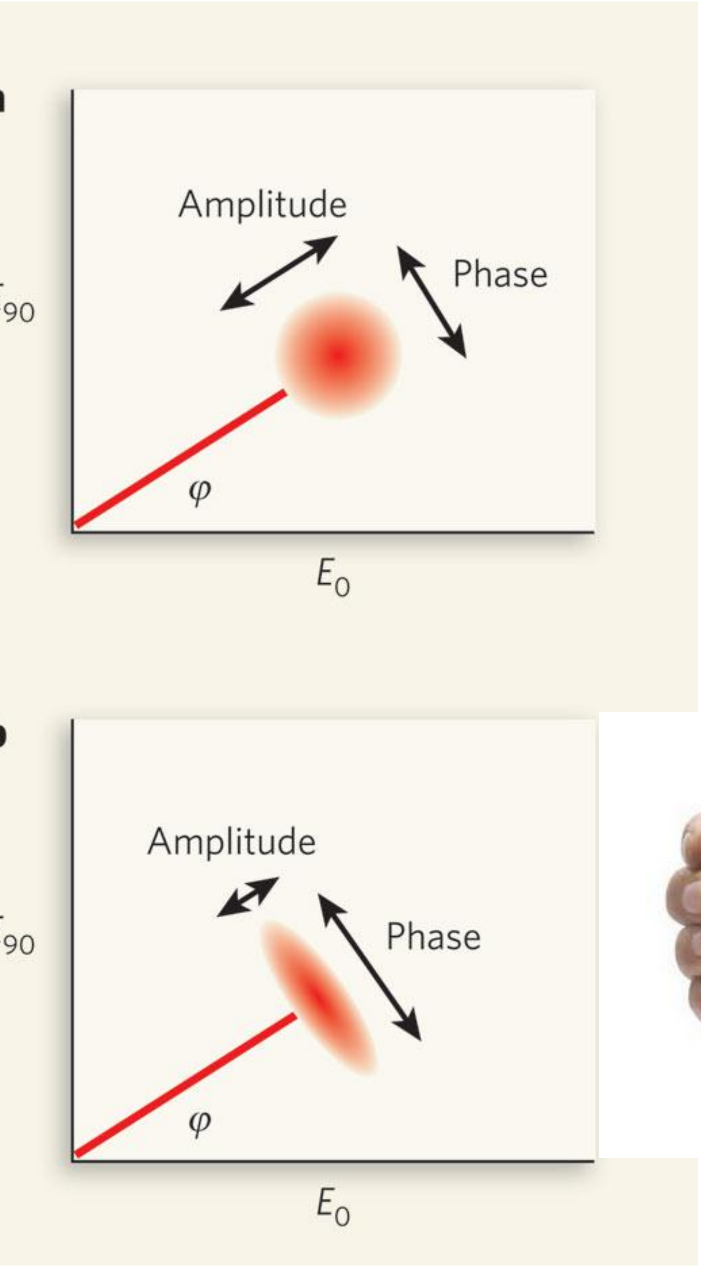


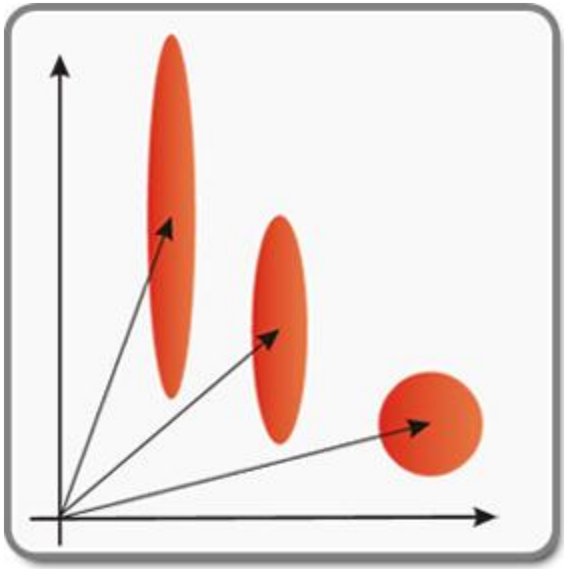
Minimum uncertainty wave packet and thermal light

Quantum squeezing: Cheating QM

$$\Delta x \Delta p = \hbar/2, \quad \Delta x = \Delta p$$ **coherent**

$$\Delta x \Delta p = \hbar/2, \quad \Delta x \neq \Delta p$$ **squeezed**



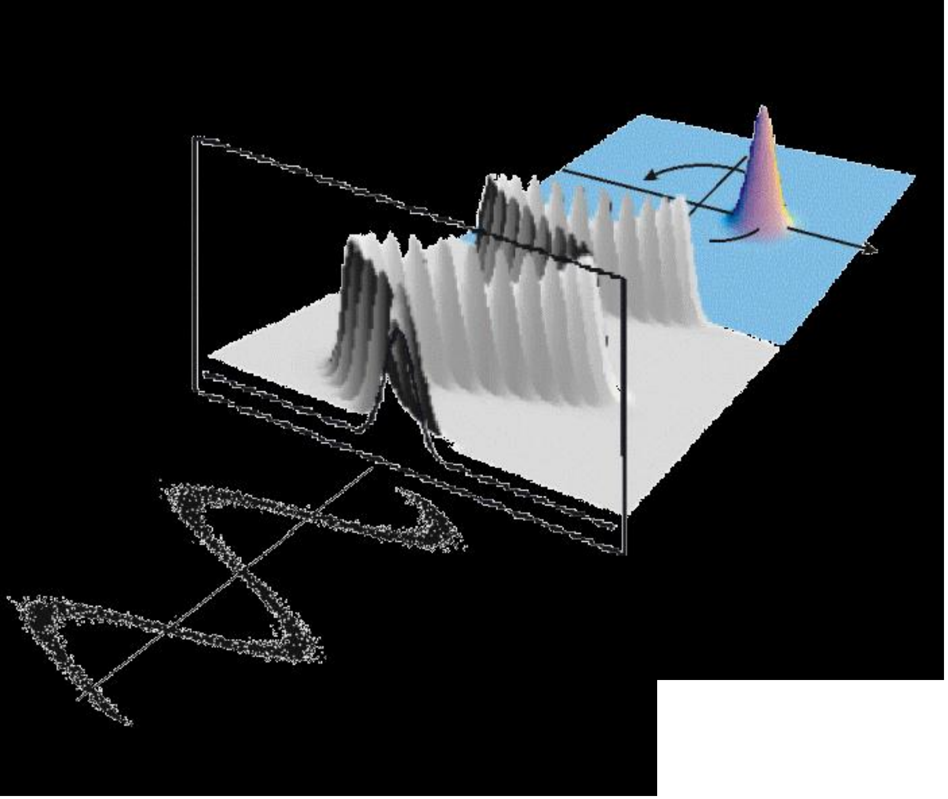


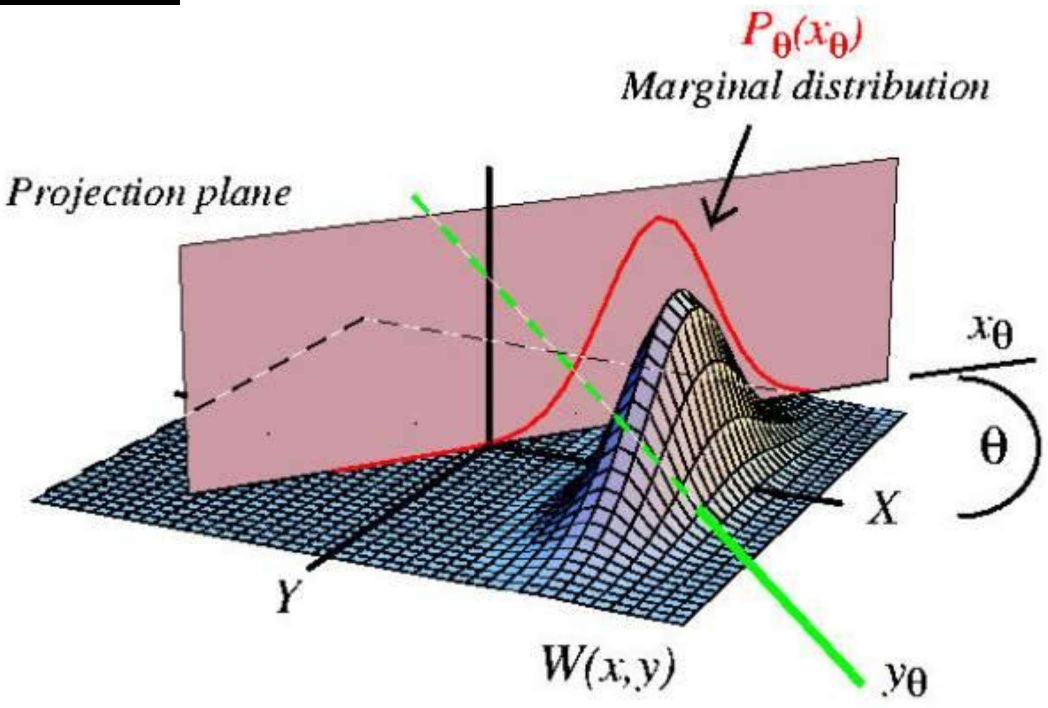
Quiet light

Description:

- Evolution of a wave packet;

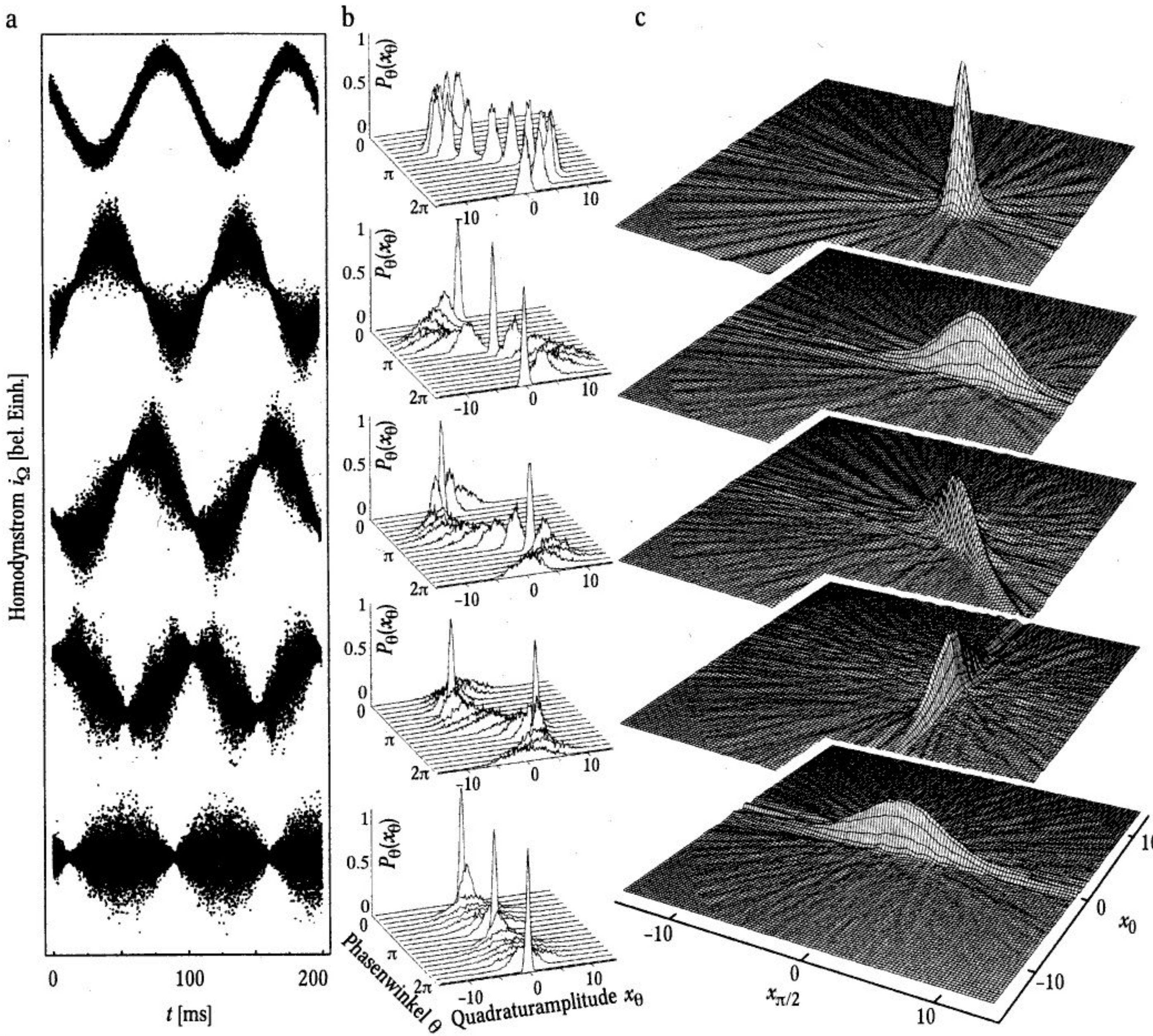
- Phase-space quasi-probability distributions (e.g. Wigner)

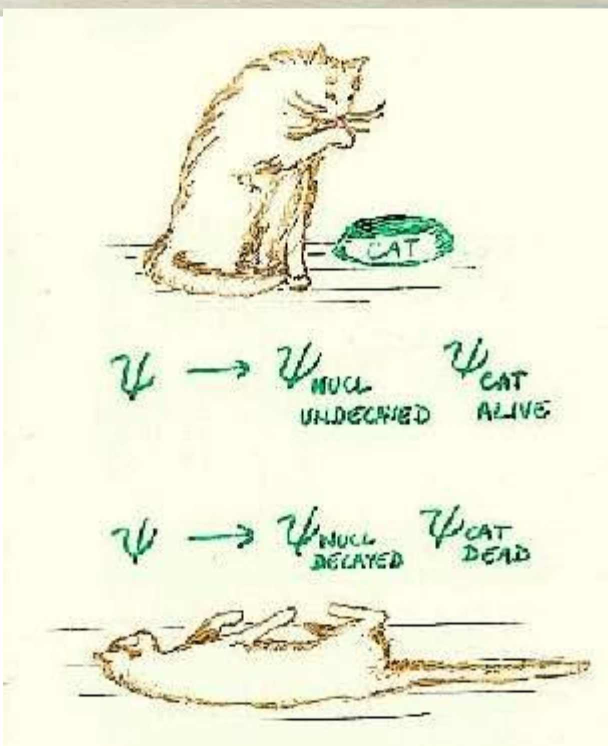


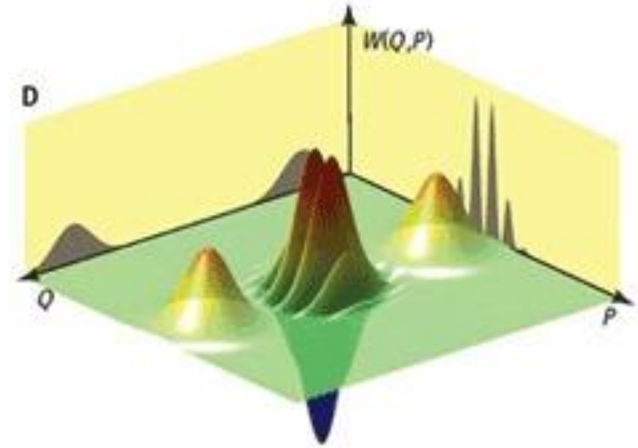


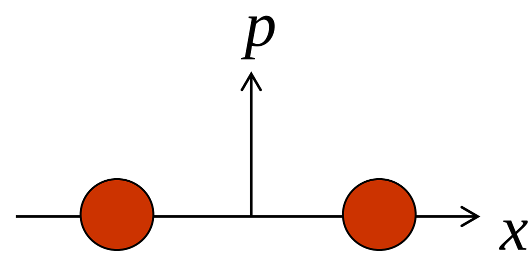
Quiet light

Gallery of quantum states









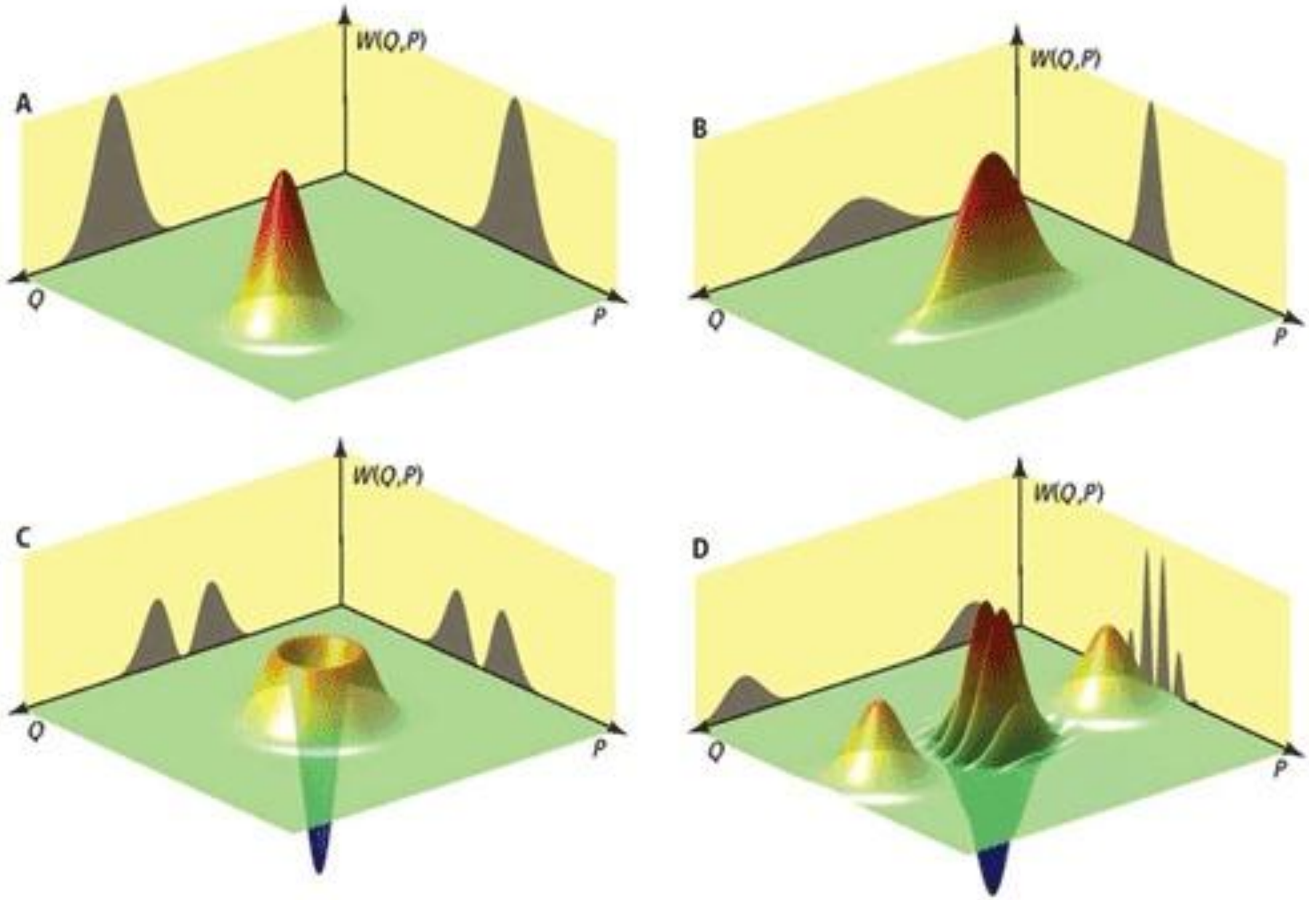
$$|\Psi\rangle = N\{|+\alpha\rangle + |-\alpha\rangle\}$$

Schroedinger cat-like state:

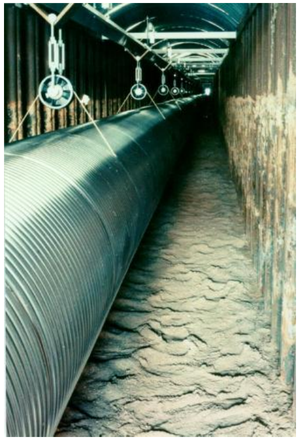
Macroscopically distinguishable superposition

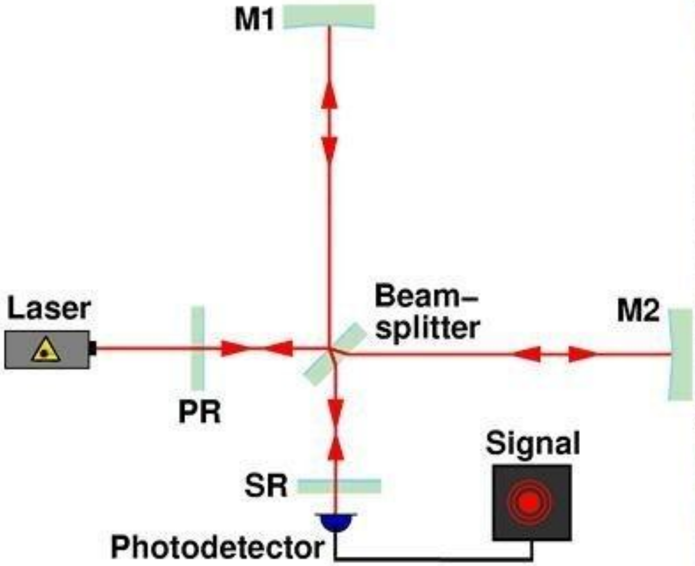
Strange light

P Grangier et al, Science 2011;332:313-314



(A) the coherent state, (B) a squeezed state,
(B) (C) the single-photon state, and (D) a Schrödinger's-cat state.





Interaction of Gravitational Wave with M1
or M2 will produce interference pattern

Applications of
quiet and strange light:

Metrology:

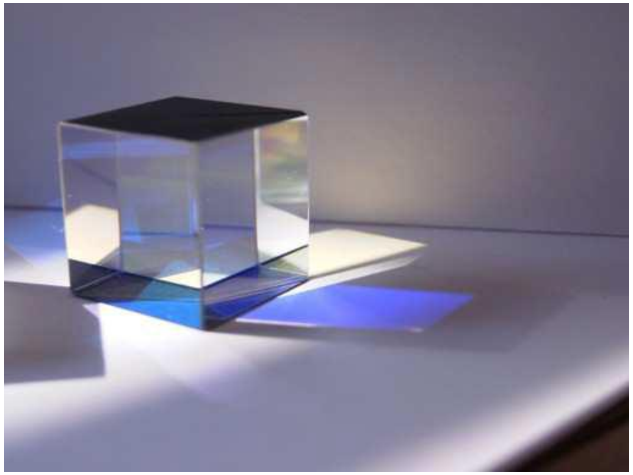
High-precision measurement (interferometry)
including gravitional wave detection (GEO600)

Quantum information:

entanglement generation,
quantum computation (e.g. with cluster states),
quantum (secure) communication
Quantum teleportation, dense coding etc

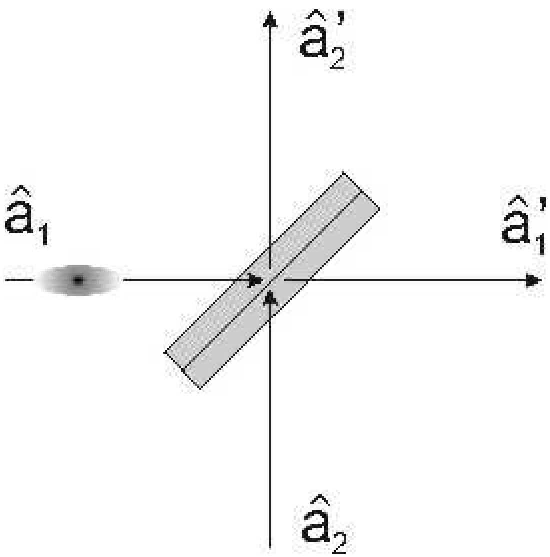
Mesoscopic entanglement

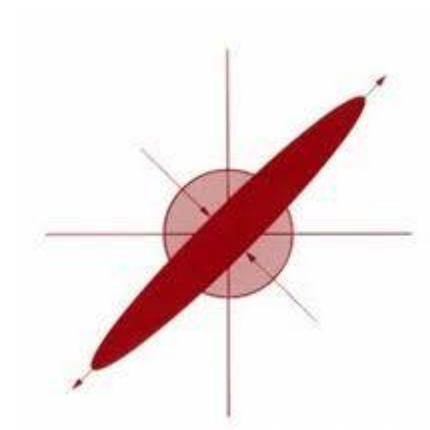
(continuous variable or infinite dimensional entanglement)



Tools:
Squeezers, beamsplitters …

Optical fiber,
Nonlinear cristal etc

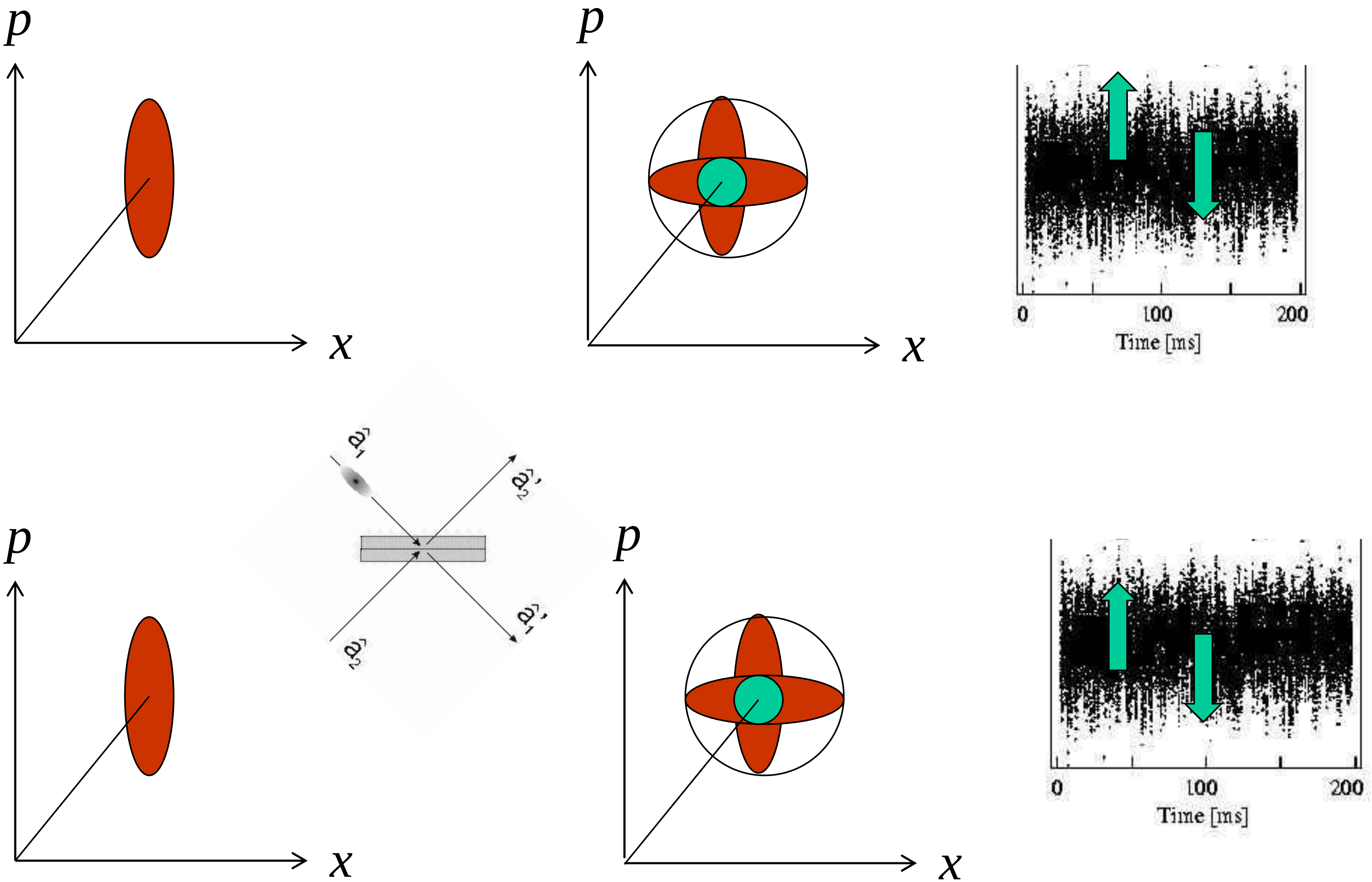




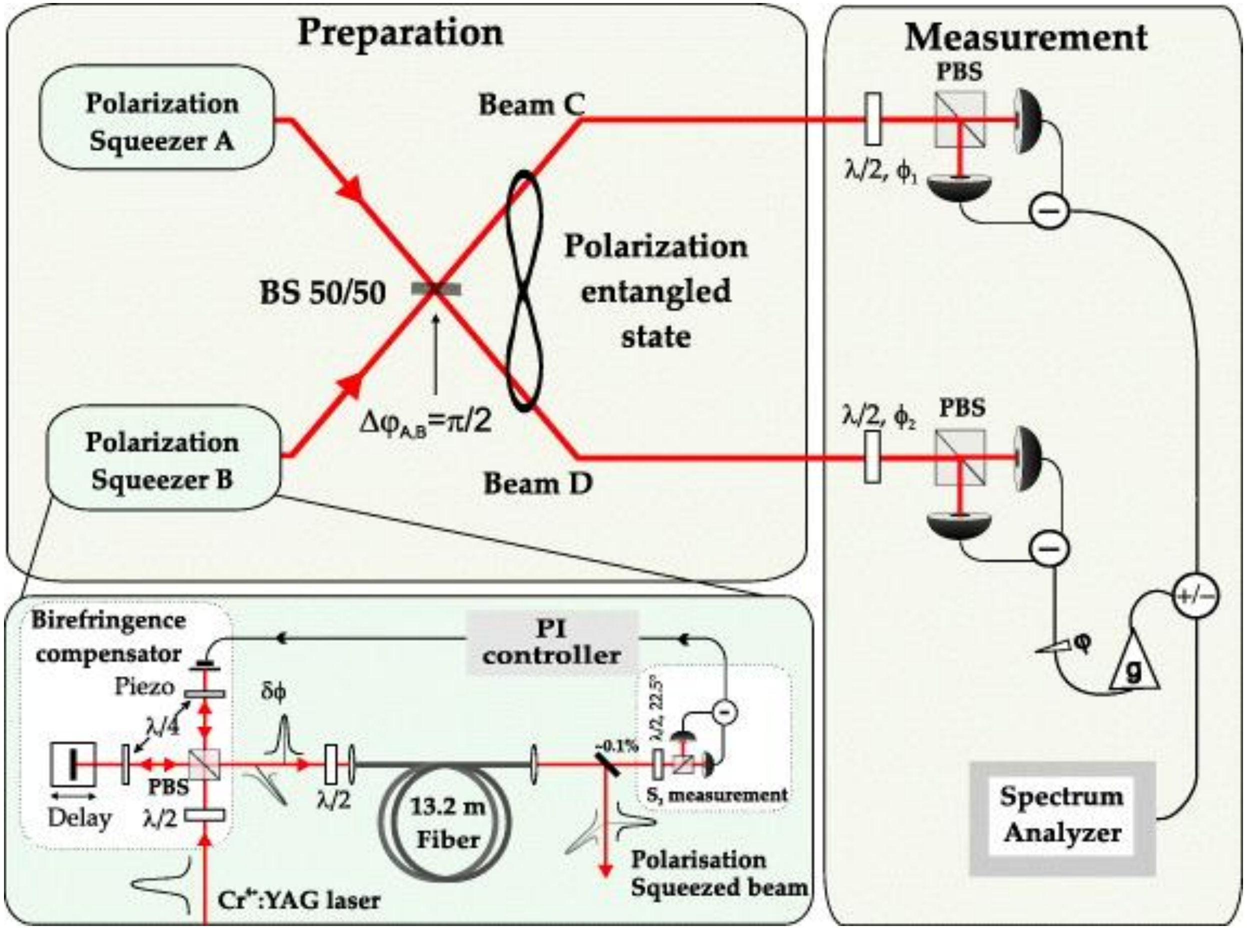
Macroscopic EPR entanglement

Example:

Entangled femtosecond pulses 10^8 photons each (optical soliton pulses in fibers)



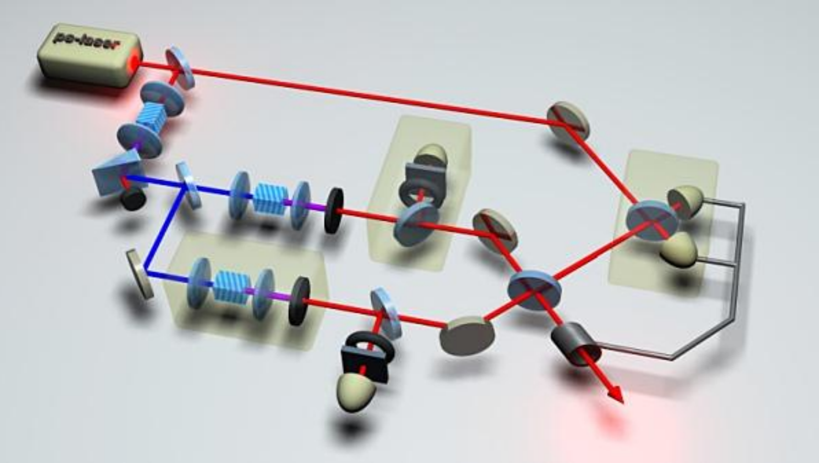
Idea: G. Leuchs, T. Ralph, Ch. Silberhorn, N. Korolkova (1999)
First realization: Ch. Silberhorn, P. K. Lam, O. Weiss, F. Koenig, N. Korolkova, G. Leuchs Phys. Rev. Lett. (2001)



Resord quantum noise supression: factor 10

Realization of a macroscopic triplet-like state

New J. Phys. 11 (2009) 113040
Triplet-like correlation symmetry of continuous variable entangled states
Gerd Leuchs, Ruifang Dong and Denis Sych

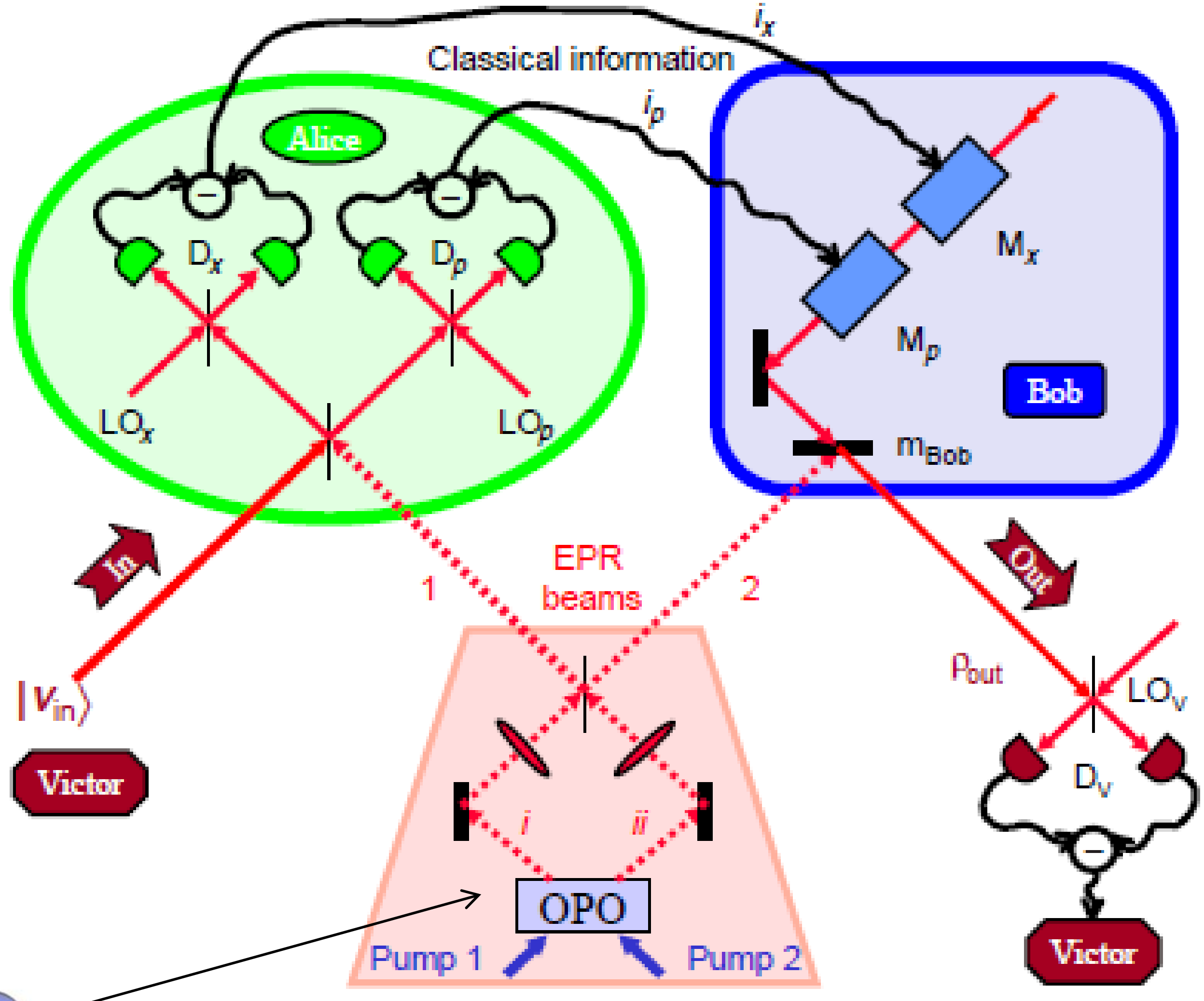


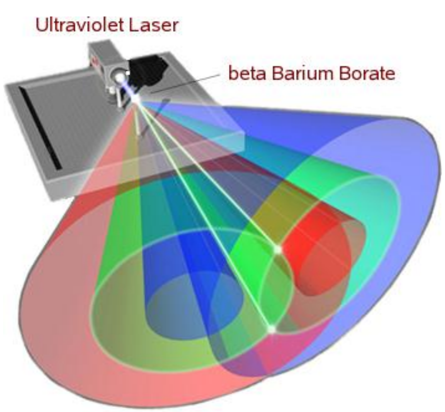
Applications:

Quantum information:

quantum computation
quantum simulations
quantum teleportation
quantum (secure) communication

Fig. 1. Schematic of the experimental apparatus for teleportation of an unknown quantum state $|v_{in}\rangle$ from Alice's sending station to Bob's receiving terminal by way of the classical information (i_x, i_p) sent from Alice to Bob and the shared entanglement of the EPR beams (1, 2).

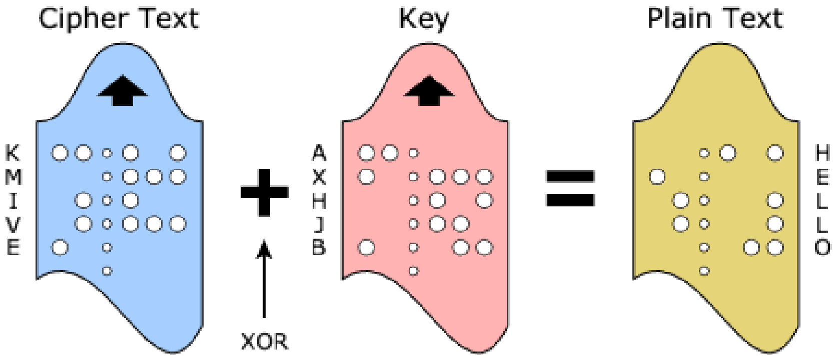




Beating eavesdropper with merely a laser light

Ling-distance free-space optical communication (beacon)

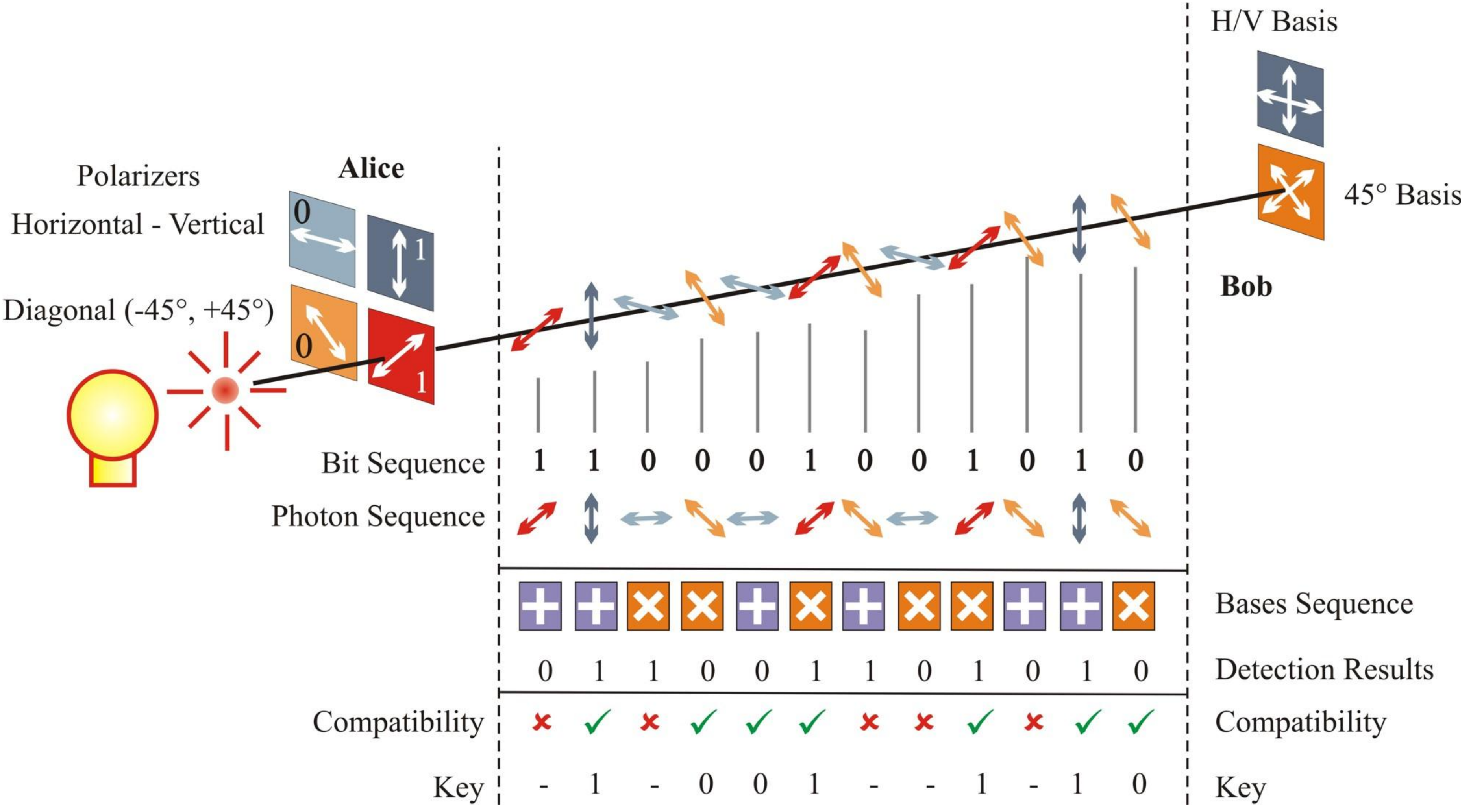
Vernam cipher: one-time pad; Unconditionally secure but needs large amounts of “key”
(same length as message and used only one time)



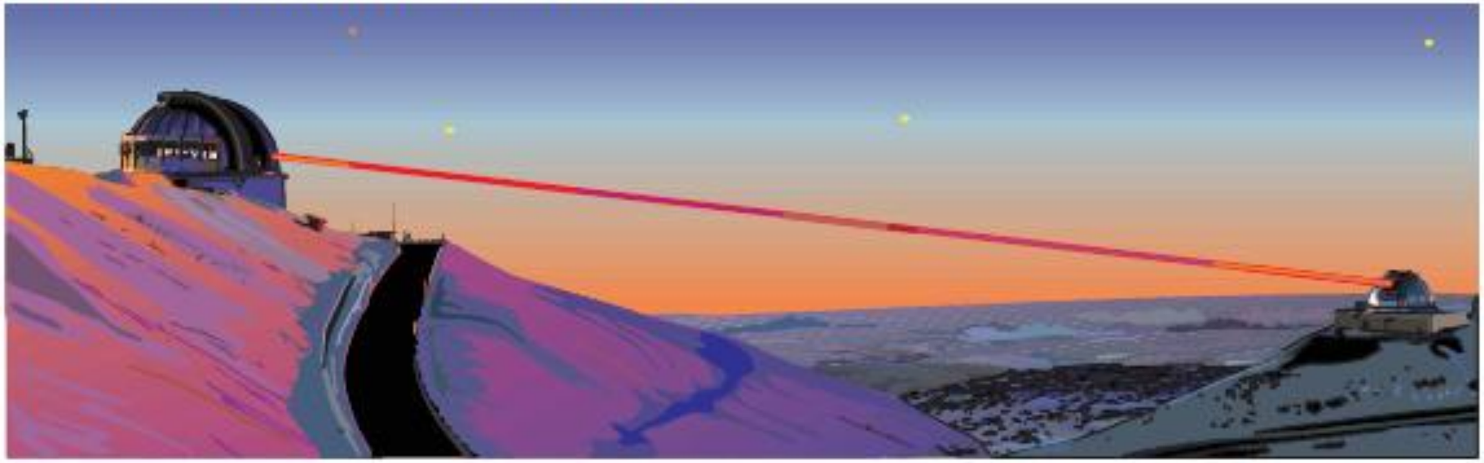
Quantum cryptography is actually quantum key distribution

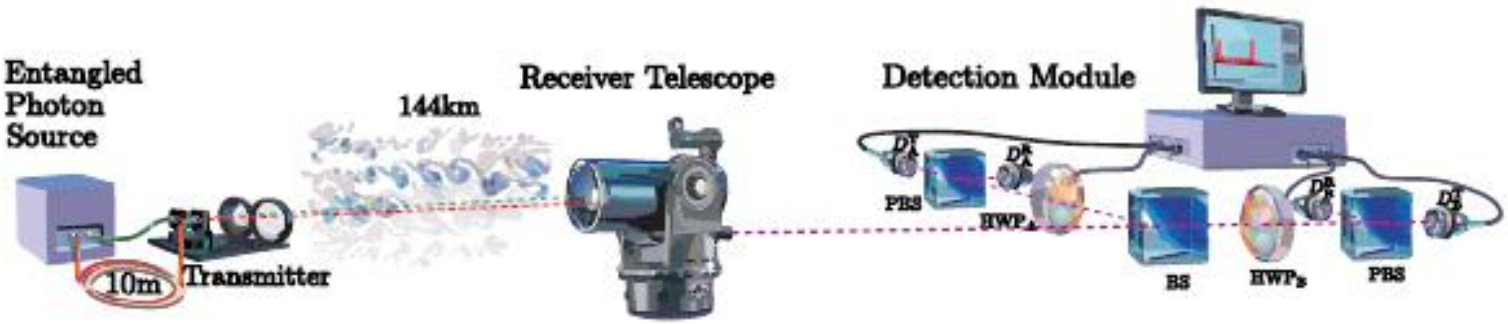
QKD

Quantum key distribution using single (nearly!) photons



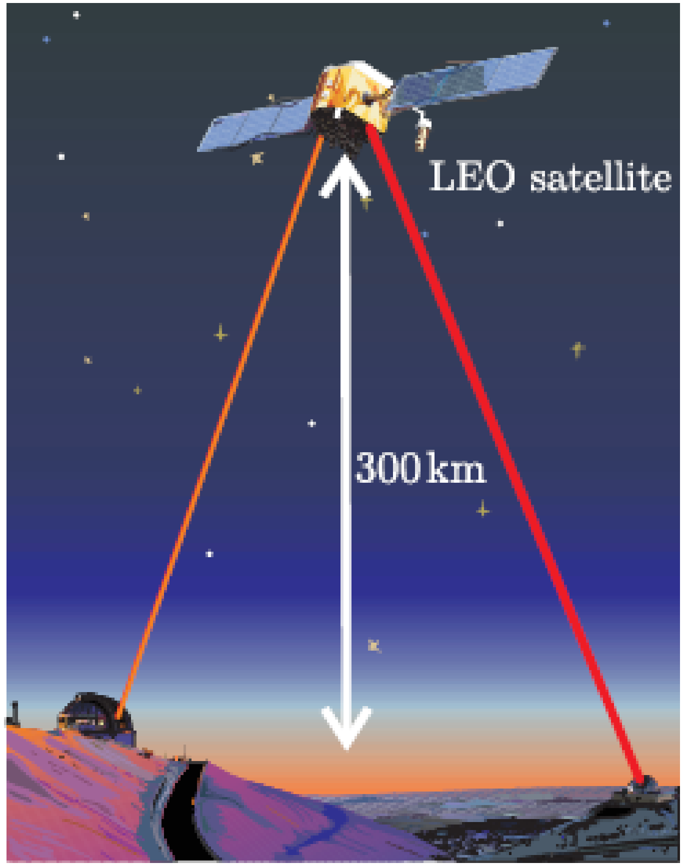
Terrestrial transmission link





A. Fedrizzi et al., Nature Phys. **5**, 389 (2009).

Satellite-Earth transmission



R. Kaltenbaek et al., Proc. of SPIE **5161** 252 (2003).

Space QUEST:

www.quantum.at/quest

esa

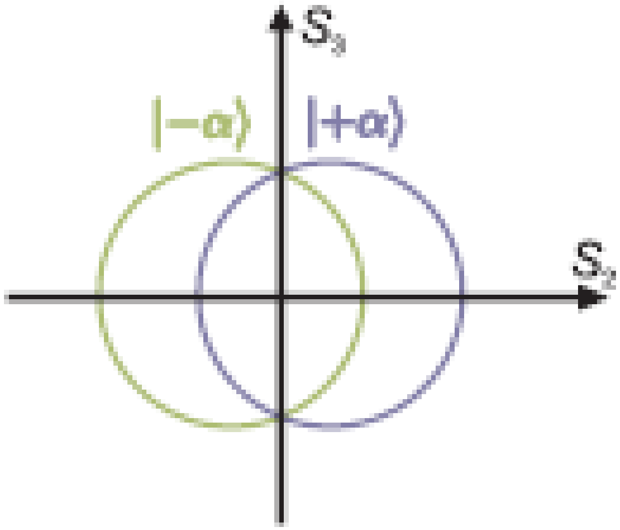
Key ingredient: incompatible measurements

If somebody "reads" quantum info, he leaves a trace

Do we really need troublesome single photons?

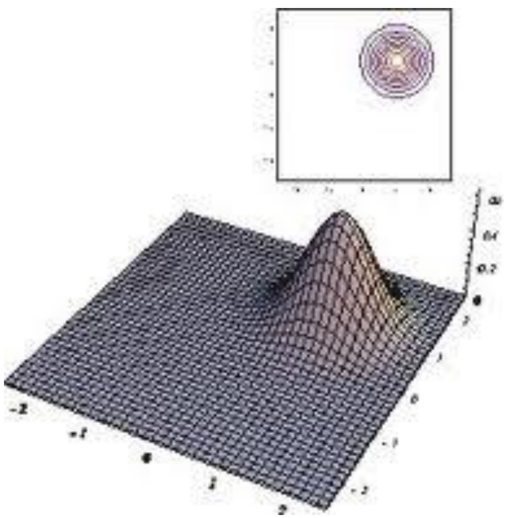
Amplitude and phase of quantum light are also subject to commutation relation;

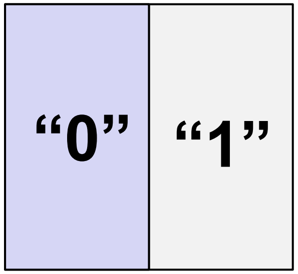
Coherent states overlap – Gaussian bells have infinite "tails"

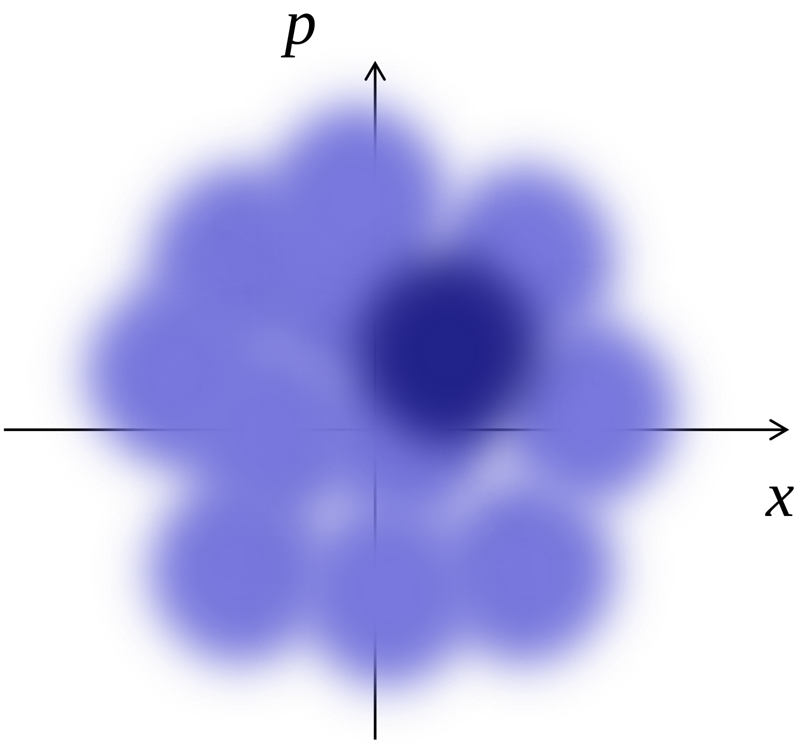


Protocol based on Gaussian distributed coherent states

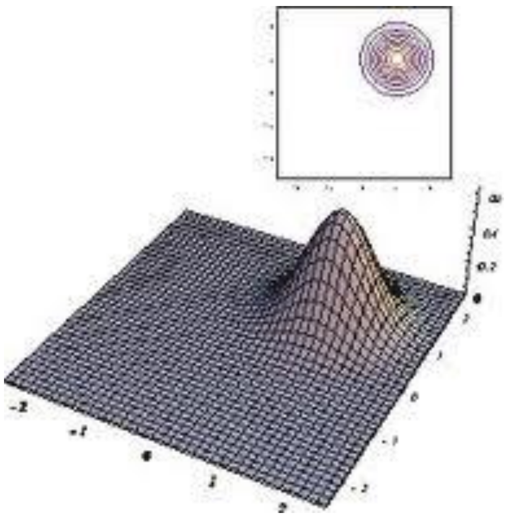
Grangier group

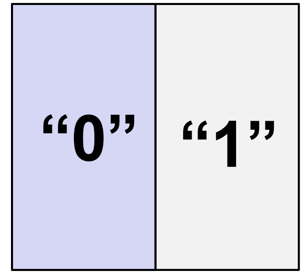


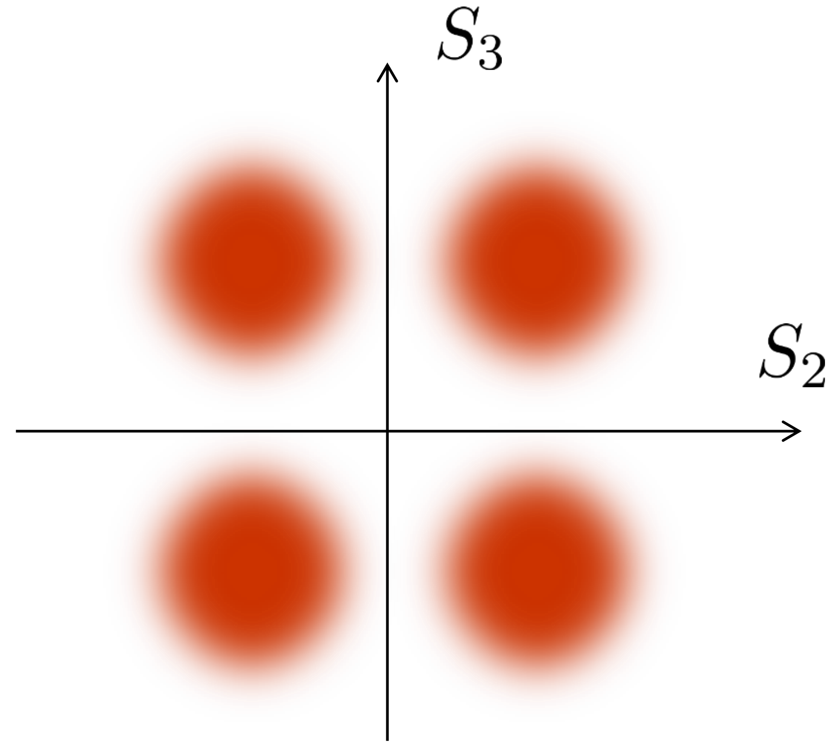


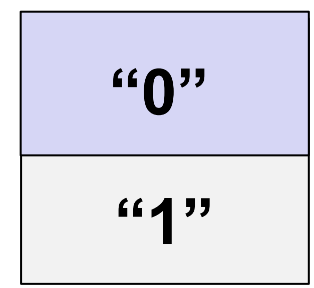


Protocol







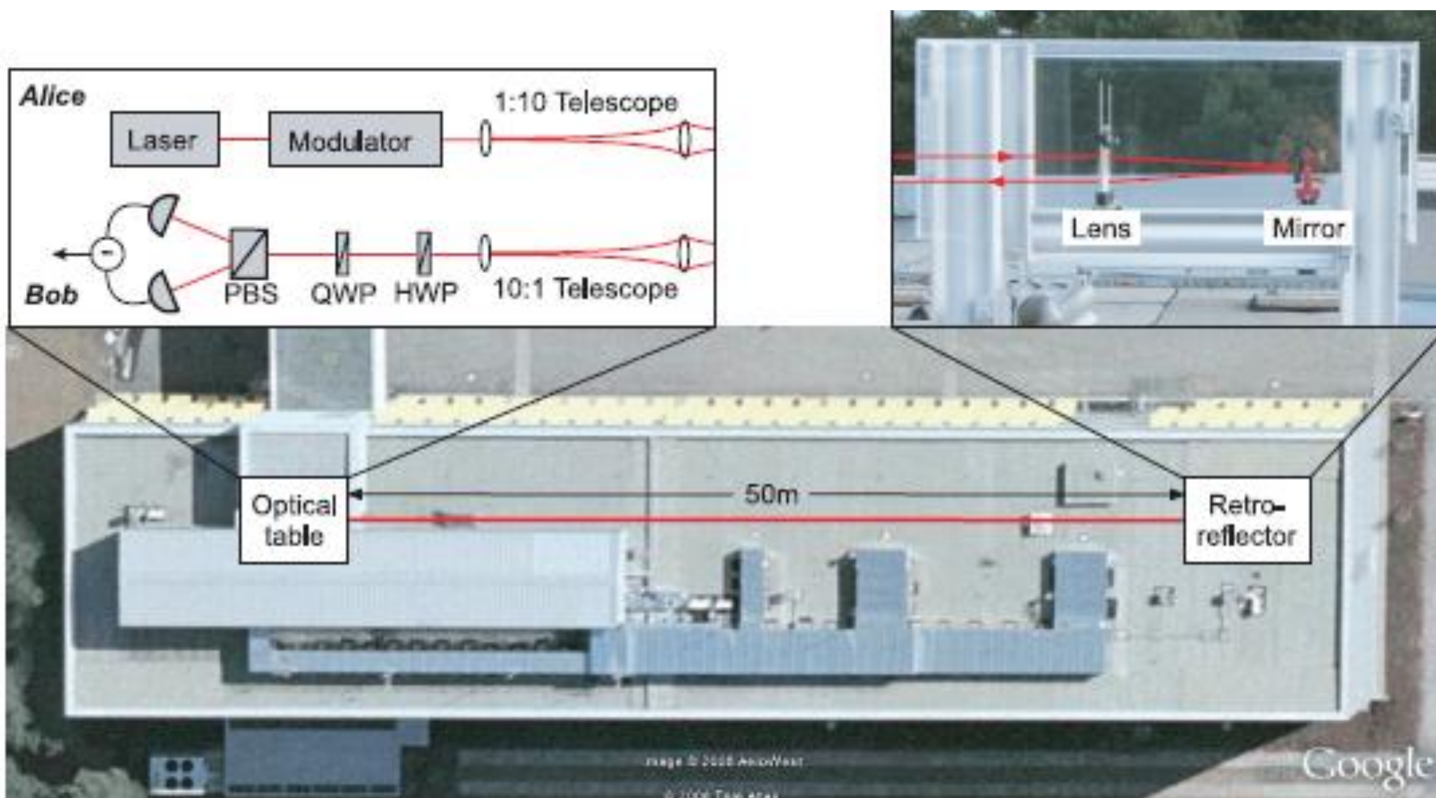


Alice’s bit	0	1	1	0	1	0	0	1
Alice’s basis	+	+	X	+	X	X	X	+
Alice’s polarization	↑	→	↖	↑	↖	↗	↗	→
Bob’s basis	+	X	X	X	+	X	+	+
Bob’s measurement	↑	↗	↖	↗	→	↗	→	→
Public discussion								
Shared Secret key	0		1			0		1

Korolkova, Lorenz, Leuchs,
Appl. Phys. B (2003)

Max Plank Insititute for the Science of Light, Erlangen, Germany

Free-space urban link over 2 km

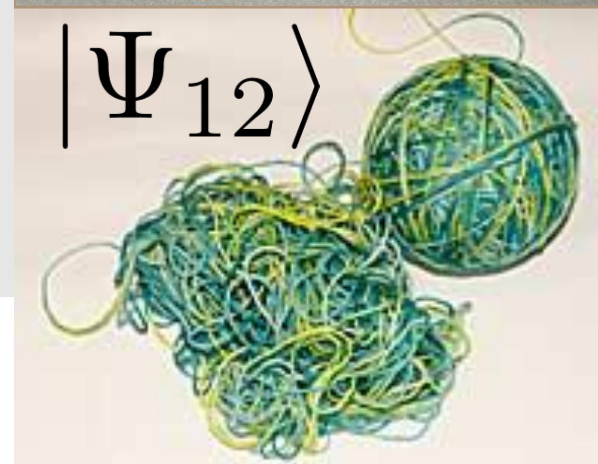


Quantum correlations beyond entanglement:

quantum mixed states, quantum discord

Role of dissipation

$$\hat{\rho}_{12}$$



$$|\Psi_{12}\rangle$$

Pure states: entangled or separable

The world of mixed quantum states is richer and closer to real-world QIP applications

How good do we know quantum resources there?

Mixed states can be arbitrary more non-classical … *(e.g. Piani et al, Phys. Rev. Lett. 106, 220403 (2011))*

Pure states: quantum correlations $=$ entanglement

Mixed states: quantum correlations $\neq$ entanglement

Entanglement $\longleftrightarrow$ Superposition

Quantumness $\longleftrightarrow$ Noncommutativity of observables

measurements !

Classically - equivalent definitions of mutual information:

$$\begin{aligned} I(A:B) &= H(A)+H(B)-H(A,B) = \\ J(A:B) &= H(A)-H(A|B) = \\ J(B:A) &= H(B)-H(B|A) \end{aligned}$$

Shannon entropy: $H(A)$

Conditional: $H(A|B)$

Quantum – they are not equivalent; mutual information:

$$\mathcal{I}_q(\hat\rho_{AB}) = \mathcal{S}(\hat\rho_A)+\mathcal{S}(\hat\rho_B)-\mathcal{S}(\hat\rho_{AB}) \quad \longleftrightarrow \quad I(A:B)$$

von Neumann entropy: $S(\hat\rho_{AB})$

$$\mathcal{D}^{\leftarrow}(\hat\rho_{AB}) = \mathcal{I}_q(\hat\rho_{AB}) - \mathcal{J}^{\leftarrow}(\hat\rho_{AB}) = \mathcal{S}(\hat\rho_B)-\mathcal{S}(\hat\rho_{AB}) + \inf_{\{\hat\Pi_i\}} \mathcal{H}_{\{\hat\Pi_i\}}(A|B)$$

Quantum discord: (quantum mutual information) - (one way classical correlation)

Protocols without entanglement outperforming classical ones???

For mixed states:

- NMR computing;

- DQC1 - deterministic quantum computation with one quantum bit, Knill and Laflamme, Phys. Rev. Lett. **81**, 5672 (1998).

NB: For pure-state:
To achieve exponential speed up over classical computation the unbounded growth of entanglement with the system size is required (Jozsa, Linden 2003)

Zoology of the non-classicality measures

Quantum discord

H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001);
L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001)

G. Adesso and A. Datta, Phys. Rev. Lett. 105, 030501 (2010) - CV
P. Giorda and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010)

Measurement-induced disturbance (MID)

S. Luo, Phys. Rev. A 77, 022301 (2008)

Ameliorated MID (AMID)

CV *-L. Mista, R. Tatham, D. Girolami, N. Korolkova, G. Adesso, quant-ph (2010), accepted by Phys. Rev. A (2011)*

etc ….

Quantifiers of non-classical correlations (quantum discord, etc):

link coherence with disspipation

link the emergence of quantum correlations with changes in entropy in quantum systems coupled to the environment

Koashi-Winter inequality:

$$\mathcal{S}\left(\hat{\rho}_A\right) \geq \mathcal{E}_F\left(\rho_{AB}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AC}\right)$$

Koashi, Winter, PRA 69, 022309 (2004)

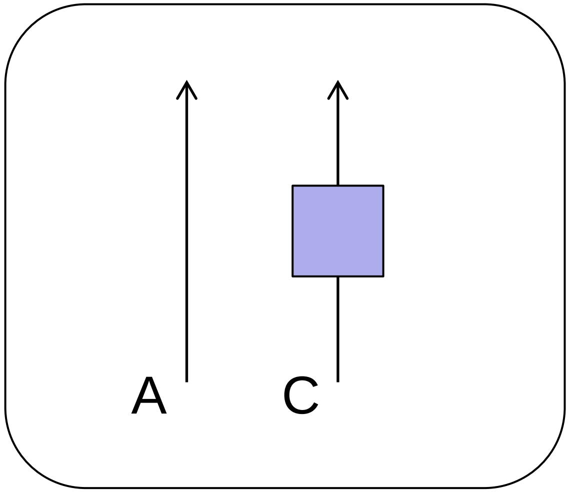
Monogamy of entanglement: a quantum system being entangled with another one limits its possible entanglement with a third system

Interplay between entanglement and classical correlation.

Perfect entanglement and perfect classical correlation are mutually exclusive

A simple but counterintuitive example:

Emergence of quantum correlations under local non-unitary measurement



Local noisy channels, nonunital (!) – e.g. dissipation.

DV: Streltsov, Kampermann, Bruss, PRL 107, 170502 (2011);

CV: Ciccarello, Giovannetti, PRA 85, 010102 (2012)

Streltsov et al paper: "Are there any noisy channels that might even *increase* the amount of quantum correlations? How does dissipation influence quantum correlations, and how are they affected by decoherence?"

Tatham, Quinn, Korolkova, in preparation

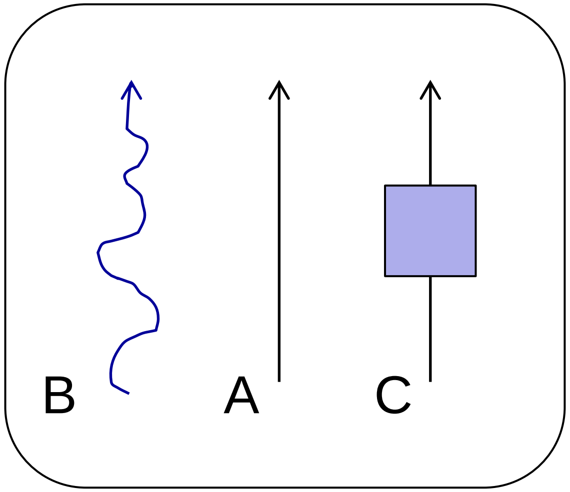
A mixed state is not a fundamental object, but a sign of our ignorance.

Purification:

every mixed state acting on finite dimensional Hilbert spaces can be viewed as the reduced state of some pure state.

Ansatz: mixed quantum state + environment = pure quantum state

Local non-unitary measurement



$$\hat{\rho}_{AC} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$$

after measurement on C:

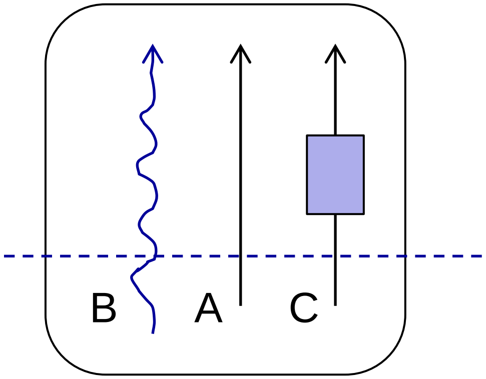
$$\hat{\rho}_{A'C'} = \frac{1}{2}\left(|00\rangle\langle 00| + |1+\rangle\langle +1|\right)$$

with $|+\rangle_C = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Purification: $|\psi\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ - GHZ – state, max entangled

Initially: entanglement across any bipartition (GHZ)

$$\mathcal{E}_F\left(\hat{\rho}_{AB,C}\right) = \mathcal{E}_F\left(\hat{\rho}_{A,BC}\right) = \mathcal{E}_F\left(\hat{\rho}_{B,AC}\right) = 1$$



Now: any subsystem traced out –

no entanglement btw two remaining ones:

$$\mathcal{E}_F\left(\hat{\rho}_{AB}\right) = \mathcal{E}_F\left(\hat{\rho}_{AC}\right) = \mathcal{E}_F\left(\hat{\rho}_{BC}\right) = 0$$

classical correlations btw those are ***maximal:***

$$\mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AC}\right) = \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{AC}\right) = 1$$

$$\mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AB}\right) = \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{AB}\right) = 1$$

$$\mathcal{J}^{\leftarrow}\left(\hat{\rho}_{BC}\right) = \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{BC}\right) = 1$$

$$\mathcal{S}\left(\hat{\rho}_{A}\right) = \mathcal{S}\left(\hat{\rho}_{B}\right) = \mathcal{S}\left(\hat{\rho}_{C}\right) = 1$$

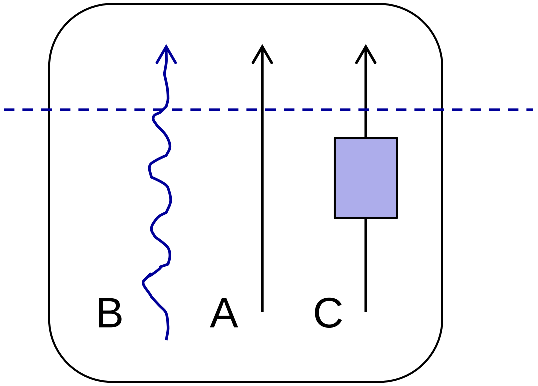
Koashi-Winter:

$$\mathcal{S}\left(\hat{\rho}_{A}\right) = \mathcal{E}_F\left(\rho_{AB}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AC}\right)$$

The entropy of marginal $\hat{\rho}_A$ quantifies capacity of Alice's state to form correlations

$$\mathcal{S}\left(\hat{\rho}_A\right) = \mathcal{E}_F\left(\rho_{AB}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AC}\right)$$

Local non-unitary measurement on C



$$\mathcal{S}\left(\hat{\rho}_{A'}\right) = \mathcal{S}\left(\hat{\rho}_A\right) = 1$$

$$\mathcal{S}\left(\hat{\rho}_{B'}\right) = \mathcal{S}\left(\hat{\rho}_B\right) = 1$$

Alice's and Bob's capacities to create correlations remain unchanged

Charlie's capacity to create correlations decrease upon the measurement on C:

$$\mathcal{S}\left(\hat{\rho}_{C'}\right) = \frac{\ln[8] - \sqrt{2}\coth^{-1}\left[\sqrt{2}\right]}{\ln[4]} = S_0^C$$

$$\mathcal{S}\left(\hat{\rho}_{C'}\right) = \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{B'C'}\right) = \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{A'C'}\right) = S_0^C < \mathcal{S}\left(\hat{\rho}_C\right)$$

(using Koashi-Winter)

After non-unitary measurement on *C:*

$$\mathcal{S}\left(\hat{\rho}_{A'}\right) = \mathcal{E}_F\left(\hat{\rho}_{A'B'}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{A'C'}\right) = 1$$

classical correlations decrease

$$\mathcal{J}^{\leftarrow}\left(\hat{\rho}_{A'C'}\right) = 1 - S_0^C$$

capacity for Alice's correlations must be filled up $\mathcal{E}_F\left(\hat{\rho}_{A'B'}\right) = S_0^C$

Alice and Bob become entangled

The discord between A and C arises as a side effect of this entanglement formation between the subsystem unaffected by the measurement and environment

Tatham, Quinn, Korolkova, in preparation

Koashi-Winter inequality:

$$\mathcal{S}\left(\hat{\rho}_A\right) = \mathcal{E}_F\left(\rho_{AB}\right) + \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{AC}\right)$$

- for pure tripartite system

$$\mathcal{E}_F\left(\rho_{AB}\right) = \lim_{\{p_i,|\psi_i\rangle\}} \sum_i p_i S(\mathrm{Tr}_B[|\psi_i\rangle\langle\psi_i|])$$

$$\{p_i, |\psi_i\rangle\} : \quad \sum_i p_i|\psi_i\rangle\langle\psi_i|] = \rho_{AB}$$

- min taken over all ensembles satisfying this

Quantum Discord:

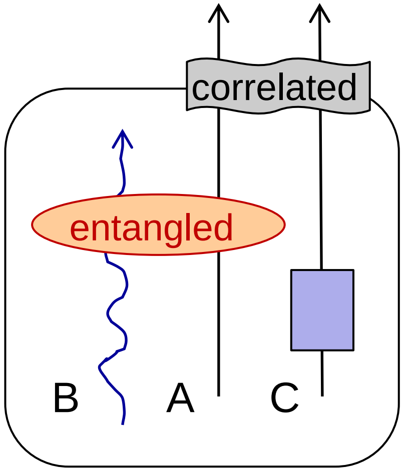
$$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AC}) = \mathcal{I}_q(\hat{\rho}_{AC}) - \mathcal{J}^{\leftarrow}(\hat{\rho}_{AC})$$

$$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AC}) = \mathcal{S}(\hat{\rho}_C) - \mathcal{S}(\hat{\rho}_{AC}) + \inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|C)$$

The discord between A and C arises as a side effect of the entanglement formation between the subsystem unaffected by the measurement and environment

$$\mathcal{D}^{\leftarrow}\left(\hat{\rho}_{A'C'}\right) = \mathcal{I}\left(A':C'\right) - \mathcal{J}^{\leftarrow}\left(\hat{\rho}_{A'C'}\right) = 2S_0^C - 1$$

$$\mathcal{D}^{\rightarrow}\left(\hat{\rho}_{A'C'}\right) = \mathcal{I}\left(A':C'\right) - \mathcal{J}^{\rightarrow}\left(\hat{\rho}_{A'C'}\right) = 0$$



Quantum discord is not really a fundamental phenomenon but a side effect of all the changes in local entropy in a quantum system coupled to the environment

Dissipation-induced coherence

non-classical correlations, such as quantified by quantum discord, unite coherence with changes in entropy in quantum systems coupled to the environment

link coherent and dissipative dynamics together

Quantum optics: correlations, coherence, correlation functions, ability to interfere ….

Quantum information: quantum entropy, entropy of entanglement ….

Quantum discord:
correlations via changes in local entropy across dissipative system

linking coherence and correlations to the entropy flow in a global system

Quantum resource:

correlations induced by controlled dissipation

Tailored dissipation as a tool:

(quantum) environment becoming your friend

Emerging new field:

Entanglement via dissipation

Correlations via dissipation

Correlated noise to counteract decoherence

Dissipatevely-driven quantum computation

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Quantum Optics
Quantum Information
in St. Andrews

Theoretical Quantum Optics group
(U. Leonhardt, S. Horsley, T. Philbin, S. Sahebdivani, W. Simpson)

Theoretical Quantum Information group
(N. Korolkova, S. Ivanov, D. Vasylyev, L. Mista*, D. Milne, R. Tatham, N. Quinn)

Experimental Quantum Optics group
(F. Koenig, S. Kehr, J. McLenaghan, M. Rybak)

*- *regular visitor from Olomouc*

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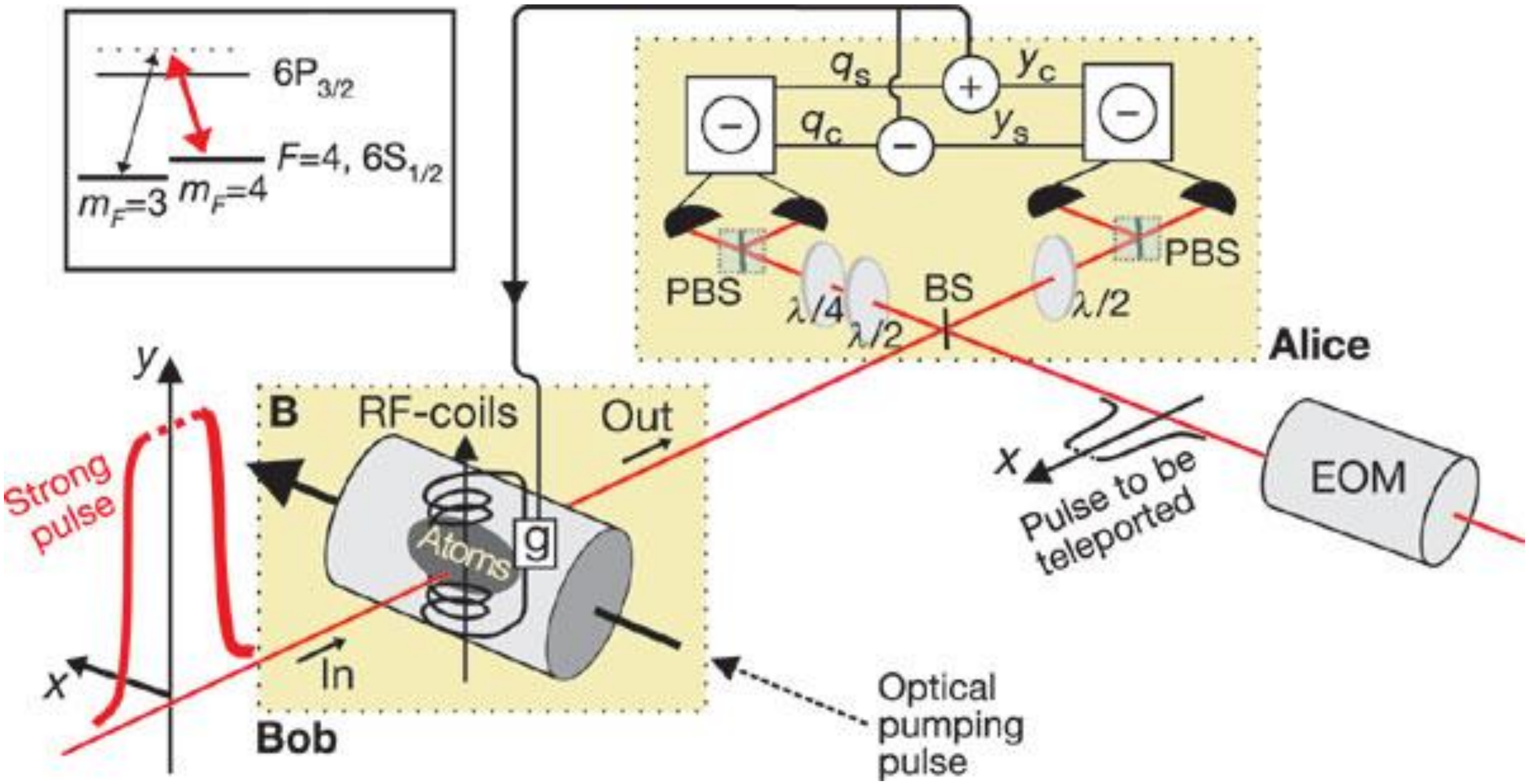


Experimental set-up for teleportation of light onto an atomic ensemble (Polzik group, NBI, Copenhagen)

$$\mathcal{D}^{\leftarrow} = f\left[\sqrt{\beta}\right] - f[V_+] - f[V_-] + f\left[\sqrt{\det \epsilon_{\text{inf}}^{\leftarrow}}\right]$$

$$\mathcal{D}^{\rightarrow} = f\left[\sqrt{\alpha}\right] - f[V_-] - f[V_+] + f\left[\sqrt{\det \epsilon_{\text{inf}}^{\rightarrow}}\right]$$

$$f(x) = \left(\frac{x+1}{2}\right)\ln\left(\frac{x+1}{2}\right) - \left(\frac{x-1}{2}\right)\ln\left(\frac{x-1}{2}\right)$$

$$\det \epsilon_{\text{inf}}^{\leftarrow} = \begin{cases} \frac{2\gamma^2+(\beta-1)(\delta-\alpha)+2|\gamma|\sqrt{\gamma^2+(\beta-1)(\delta-\alpha)}}{(\beta-1)^2} \\ \text{if} \quad (\delta-\alpha\beta)^2 \leq (1+\beta)\,\gamma^2\,(\alpha+\delta)\,, \\ \frac{\alpha\beta-\gamma^2+\delta-\sqrt{\gamma^4+(\delta-\alpha\beta)^2-2(\gamma^2)(\alpha\beta+\delta)}}{2\beta} \\ \text{Otherwise.} \end{cases}$$

$$\det \epsilon_{\text{inf}}^{\rightarrow} = \begin{cases} \frac{2\gamma^2+(\alpha-1)(\delta-\beta)+2|\gamma|\sqrt{\gamma^2+(\alpha-1)(\delta-\beta)}}{(\alpha-1)^2} \\ \text{if} \quad (\delta-\beta\alpha)^2 \leq (1+\alpha)\,\gamma^2\,(\beta+\delta)\,, \\ \frac{\beta\alpha-\gamma^2+\delta-\sqrt{\gamma^4+(\delta-\beta\alpha)^2-2\gamma^2(\beta\alpha+\delta)}}{2\alpha} \\ \text{Otherwise.} \end{cases}$$

Mutual information = total correlations btw A and B

Classically - equivalent definitions of mutual information:

$$\begin{aligned} I(A:B) &= H(A)+H(B)-H(A,B) = \\ J(A:B) &= H(A)-H(A|B) = \\ J(B:A) &= H(B)-H(B|A) \end{aligned}$$

Shannon entropy: $H(A)$

Conditional: $H(A|B)$

Quantum – they are not equivalent; mutual information:

$$\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB})$$

von Neumann entropy: $S(\hat{\rho}_{AB})$

↔ $I(A:B)$

$J(A:B), J(B:A)$?

measurements!

$$I(A:B) = H(A) + H(B) - H(A,B) =$$
$$J(A:B) = H(A) - H(A|B) =$$

$$\mathcal{J}^{\leftarrow}(\hat{\rho}_{AB}) = \underbrace{\mathcal{S}(\hat{\rho}_A)}_{\text{Total info about A}} - \underbrace{\inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)}_{\text{Quantum correlation}}$$

- one way classical correlation

Total info about A

Quantum correlation:
Info about A inferred via quantum measurement on B

$$\mathcal{H}_{\{\hat{\Pi}_i\}}(A|B) \equiv \sum_i p_i \mathcal{S}(\hat{\rho}^i_{A|B})$$

Quantum conditional entropy related to $\hat{\rho}^i_{A|B}$ upon POVM $\{\hat{\Pi}_i\}$ on B.

$$\hat{\rho}^i_{A|B} = \mathrm{Tr}_B[\hat{\Pi}_i \hat{\rho}_{AB}]/p_i$$

Infimum: optimization to single out the least disturbing measurement on B

Quantum discord:

$$\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB})$$

$-$

$$\mathcal{J}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) - \inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)$$

quantum mutual information | **one way classical correlation**

$$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{I}_q(\hat{\rho}_{AB}) - \mathcal{J}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB}) + \inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)$$

$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) = 0$ - classical

$0 < \mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) < 1$ - quantum, separable

$\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}) > 1$ - entangled

$$\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB})$$ **—** $$\mathcal{J}^{\leftarrow}(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) - \inf_{\{\hat{\Pi}_i\}} \mathcal{H}_{\{\hat{\Pi}_i\}}(A|B)$$

quantum mutual information **one way classical correlation**

QD = quantum mutual info - the classical mutual info of outcomes;

QD = total correlations - classical correlations; ← **not unique**

\+ operational; conceptually easy; optimized over POVMs

\- hard to compute; asymmetric

DV: H. Ollivier and W. H. Zurek, Phys. Rev. Lett. 88, 017901 (2001);
L. Henderson and V. Vedral, J. Phys. A 34, 6899 (2001)

CV: G. Adesso and A. Datta, Phys. Rev. Lett. 105, 030501 (2010)
P. Giorda and M. G. A. Paris, Phys. Rev. Lett. 105, 020503 (2010)
L. Mista, R. Tatham, D. Girolami, N. Korolkova, G. Adesso, Phys. Rev. A. (2011)

$$\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB})$$ **−** $$\mathcal{I}(A:B)$$

quantum mutual information — **non-Gaussian classical correlation**

MID = quantum mutual info - the classical mutual info of outcomes of local Fock-state detectors;

+ symmetric

- no optimizations over local measurements

$\Longrightarrow$ often overestimates quantum correlations

Measurement-induced disturbance (MID)

S. Luo, Phys. Rev. A 77, 022301 (2008)

$$\mathcal{I}_q(\hat{\rho}_{AB}) = \mathcal{S}(\hat{\rho}_A) + \mathcal{S}(\hat{\rho}_B) - \mathcal{S}(\hat{\rho}_{AB})$$

quantum mutual information

–

$$\mathcal{I}_c(\hat{\rho}_{AB}) = \sup_{\hat{\Pi}_A \otimes \hat{\Pi}_B} \mathcal{I}(A:B)$$

maximal classical correlation extractable by local (Gaussian) processing

AMID (Gaussian) = quantum mutual info - the *maximal* classical mutual info obtainable by (Gaussian) local measurements

AMID – optimized as discord and symmetric as MID

L. Mista, R. Tatham, D. Girolami, N. Korolkova, G. Adesso, Phys. Rev. A (2011)

Protocols without entanglement outperforming classical ones???

For mixed states:

- NMR computing;

- DQC1 - deterministic quantum computation with one quantum bit, Knill and Laflamme, Phys. Rev. Lett. **81**, 5672 (1998).

NB: For pure-state:
To achieve exponential speed up over classical computation
the unbounded growth of entanglement with the system size is required
(Jozsa, Linden 2003)

Symmetrical nonclassicality indicators: comparison

$$\mathcal{D}^{\leftrightarrow}(\hat{\rho}_{AB}) = \max\{\mathcal{D}^{\leftarrow}(\hat{\rho}_{AB}), \mathcal{D}^{\rightarrow}(\hat{\rho}_{AB})\}$$ **- "two-way discord"**

$$\mathcal{M}(\hat{\rho}_{AB}) = \mathcal{I}_q(\hat{\rho}_{AB}) - \mathcal{I}(A:B)$$ **- measurement-induced disturbance**

amelorated MID:

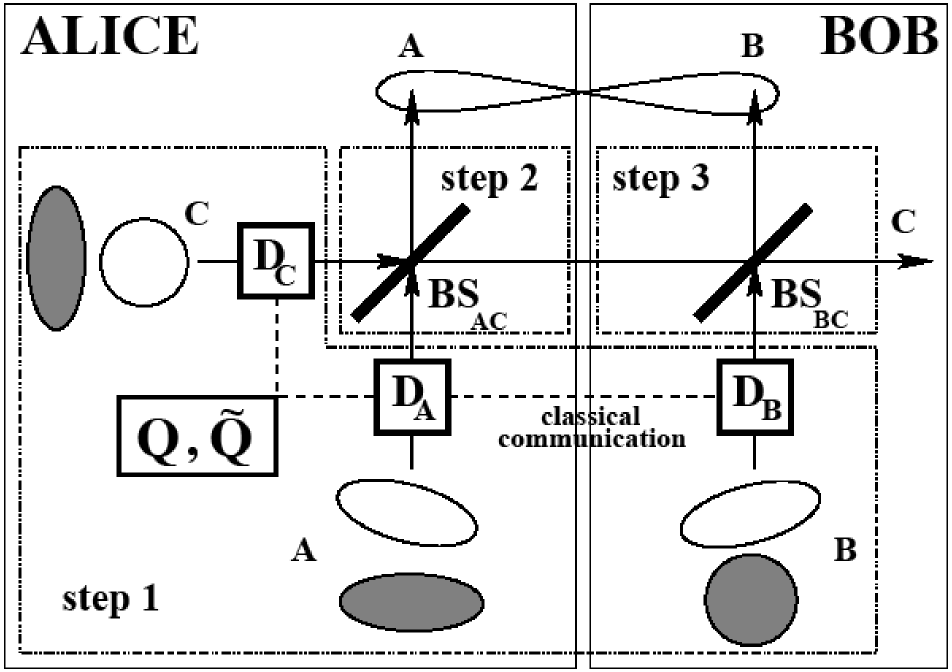
$$\mathcal{A}(\hat{\rho}_{AB}) = \mathcal{I}_q(\hat{\rho}_{AB}) - \mathcal{I}_c(\hat{\rho}_{AB}), \qquad \mathcal{I}_c(\hat{\rho}_{AB}) = \sup_{\hat{\Pi}_A \otimes \hat{\Pi}_B} \mathcal{I}(A:B)$$

$$\mathcal{A}^G(\hat{\rho}_{AB}) = \mathcal{I}_q(\hat{\rho}_{AB}) - \mathcal{I}_c^G(\hat{\rho}_{AB}), \qquad \mathcal{I}_c^G(\hat{\rho}_{AB}) = \sup_{\hat{\Pi}_A^G \otimes \hat{\Pi}_B^G} \mathcal{I}(A:B)$$

- Gaussian amelorated MID

L. Mista, R. Tatham, D. Girolami, N. Korolkova, G. Adesso, Phys. Rev. A (2011)

Entanglement distribution using separable ancilla



$$\gamma_A \oplus \gamma_B \oplus \gamma_C \Rightarrow$$
$$\gamma_1(x) = \gamma_{AB} \oplus \mathrm{I}_C$$

$$+x(q_1 q_1^T + q_2 q_2^T)$$
$$q_{1,2} = q_{1,2}(squeezing)$$

$$\mathcal{A}^G(\hat{\rho}_{AB}) = \mathcal{I}_q(\hat{\rho}_{AB}) - \mathcal{I}_c^G(\hat{\rho}_{AB})$$

L. Mista and N. Korolkova Phys. Rev. A 80, 032310 (2009).

Entanglement ⟷ Secrecy

quantum entanglement	secret correlations
quantum communication	private communication
classical communication	public communication
local operations	local actions

J. Bae, T. S. Cubitt, A. Acin, Phys. Rev. A 79, 032304 (2009);
P. W. Shor, J. Preskill, Phys. Rev. Lett. 85, 441 (2000); etc

Quantum correlations beyond entanglement can be used for distribution of secrecy

Non-secret correlations can be used to distribute secrecy

$P(A, B, C)$

Probability distribution contains secret correlations iff it cannot be distributed using only local operations and public communication.

All entangled states can be mapped by single copy measurements onto *P(A,B,C)* with secret correlations

A. Acin and N. Gisin, Phys. Rev. Lett. 94, 020501 (2005)

Consider Eve;

The distribution is secret if:

$$P(A = B = 0) = P(A = B = 1) = \frac{1}{2},$$

$$P(A, B, E) = P(A, B)P(E)$$

Gaussian multipartite bound information

Analog of bound entanglement; cannot be distilled but can be activated:

A Gaussian distribution $P(A, B, C)$ can be distilled to a secret key using reversed reconciliation protocol (*Van Assche, Grosshans, Grangier, Cerf*) if

$$\max\left(\Delta I_{DR}, \Delta I_{RR}\right) > 0$$

$$\Delta I_{DR} = I_{AB} - I_{AE} \qquad \Delta I_{RR} = I_{AB} - I_{BE}$$

L. Mista, N. Korolkova, Gaussian multipartite bound information, submitted (2010)

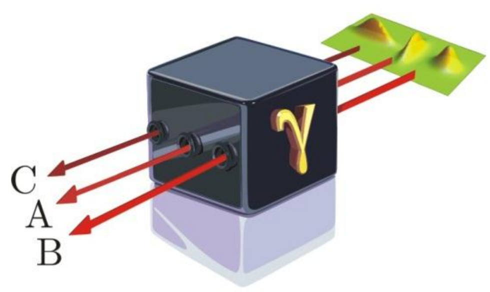
Distribution of secrecy by non-secret correlations

Step 1: construct purification:

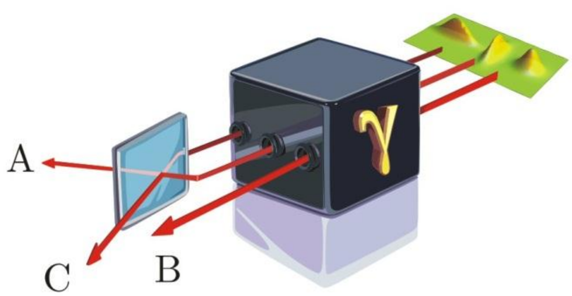
$$|\psi\rangle = \int \sqrt{\mathcal{P}(u,v)} \left|-i\frac{u}{2};+r\right\rangle_A \left|\frac{v+iu}{\sqrt{2}};0\right\rangle_B \left|\frac{v}{2};-r\right\rangle_C |v\rangle_{E_1}^{(x)} |u\rangle_{E_2}^{(p)} \, du dv$$

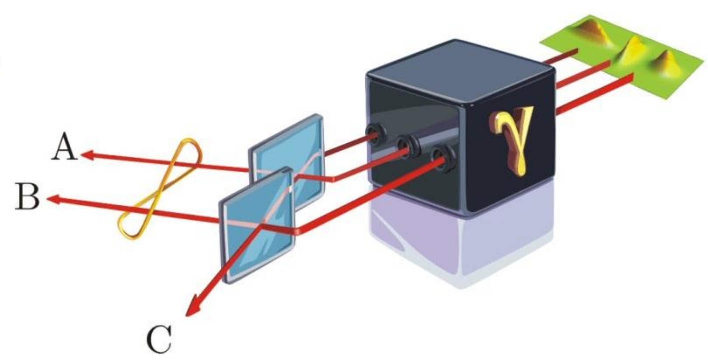
A, *B*, *C* modes as above + two Eve modes

map it onto Gaussian probability distribution with no secret correlations across any bipartition of honest parties by measuring *x*-quadrature on all 5 modes



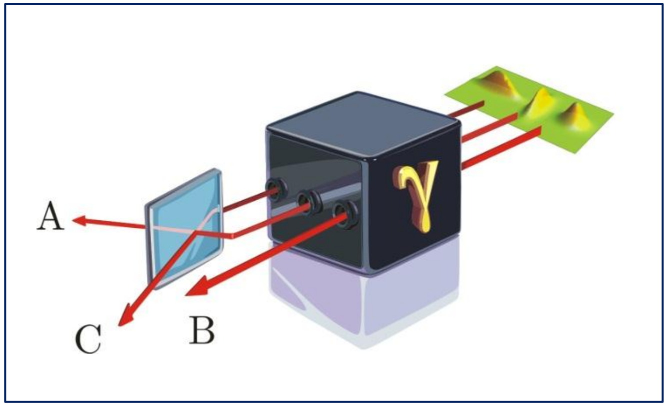
+ 2 Eve’s modes





Step 2: Alice transforms her random variables (BS)

$$x_{A,C} \to (x_A \pm x_C)/\sqrt{2}$$



The new distribution Π_2 contains Gaussian bound information!

no secret correlations across *B-(AC)* and *C-(AB)*.

any two honest parties cannot to establish a secret key
(even when Alice is allowed to collaborate either with Bob or Clare)

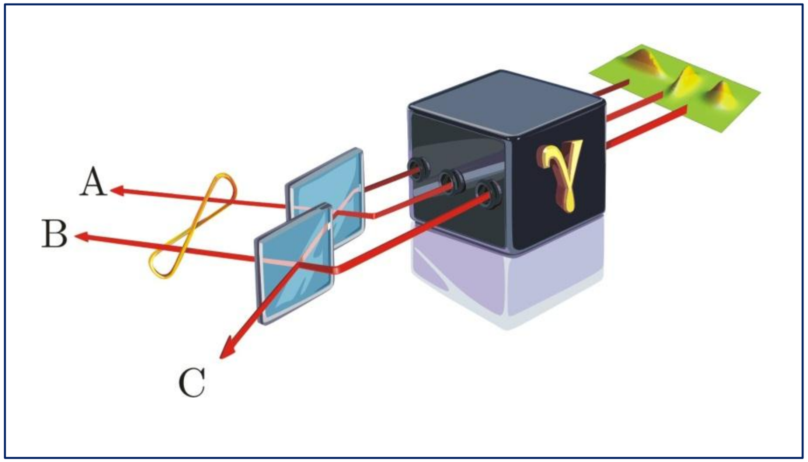
But the distribution Π_2 **cannot be created by LOPC** (secret correlations *A-(BC)*).

The correlations are not detected by the criterion $\max\left(\Delta I_{DR}, \Delta I_{RR}\right) > 0$

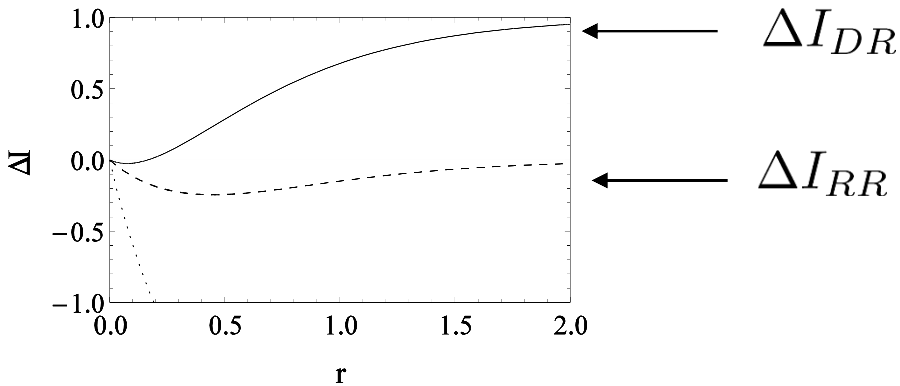
Bound information can be activated

Step 3: Alice transforms her random variables (2nd BS)

$$x_{B,C} \rightarrow \left(x_B \pm x_C\right)/\sqrt{2}$$



Can distil a secret key using the reverse reconciliation: $\max\left(\Delta I_{DR}, \Delta I_{RR}\right) > 0$



Gaussian bound information can be activated and used for secret communication!

$$\mathcal{D}^{\leftrightarrow}, \mathcal{A}, \mathcal{M}, \mathcal{A}^{G}(\hat{\rho}_{AB})$$

Q correlations beyond entanglement in step 2 are responsible for creation of BI and can be used to distribute secrecy

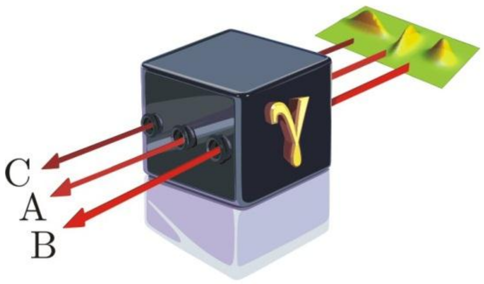
L. Mista, N. Korolkova, Gaussian multipartite bound information, submitted (2010)

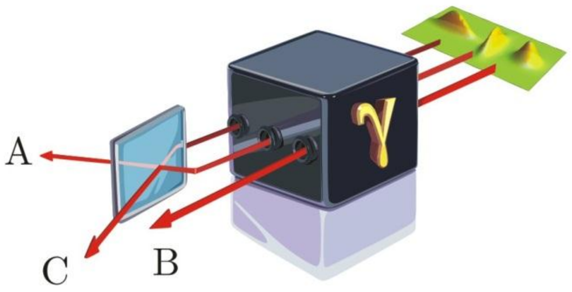
See also:
M. Piani et al, All non-classical correlations can be activated into distillable entanglement, Phys. Rev. Lett. 106, 220403 (2011); A. Streltsov, H. Kampermann, D. Bruss, Linking Quantum Discord to Entanglement in a Measurement, Phys. Rev. Lett. 106, 160401 (2011)

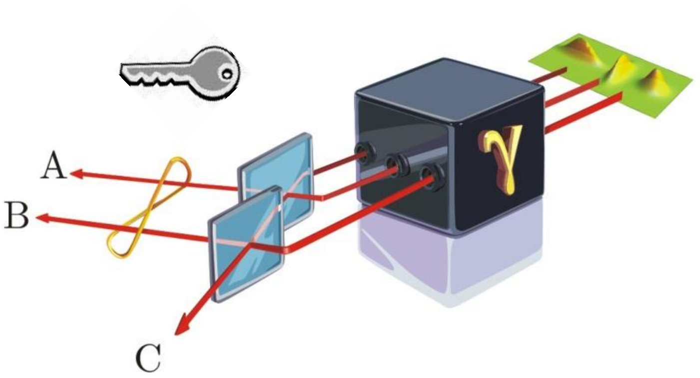
(1) 3-partite mixed fully separable state

(2) Separable state with non-zero quantum correlations

(3) Separable ancilla *C* renders entanglement between *A* and *B*







(2) Bound information is created

(3) Bound information is activated into secret key

Gaussian ameliorated measurement-induced disturbance

accurate measure of quantum correlations for mixed and strongly correlated states

Entanglement distribution by separable ancilla

C never entangled but non-zero quantum correlations

Distribution of secrecy using non-secret correlations

uses quantum correlations beyond entanglement

Gaussian multipartite bound information

can be activated into secret correlations/entanglement

Quantum correlations beyond entanglement for Gaussian states

Thank you!

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(N. Korolkova, L. Mista*, D. Milne, R. Tatham, N. Quinn, NN – postdoc position open)

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(F. Koenig, S. Kehr, J. McLenaghan, S. Rohr)

*- *regular visitor from Olomouc*

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